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February 1975

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ABSTRACT

The motion of charged particles is examined in the case of a homogeneous magnetic field $\underline{B}$ together with an orthogonal electric field $\underline{E}$, which has a gradient $\underline{\nabla}E$ parallel to $\underline{E}$. If

$$\frac{B^2 q^2}{m^2} - \frac{q\nabla E}{m} > 0$$

the particles drift at right angles to $\underline{E}$ and $\underline{B}$ with a modified gyrofrequency and produce a current in that direction. If

$$\frac{B^2 q^2}{m^2} - \frac{q\nabla E}{m} < 0$$

the particles not only drift in the direction of $\underline{E} \times \underline{B}$ but are also accelerated in the direction of $\underline{E}$, in which direction they also produce a current.

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$\underline{\nabla E} \times \underline{B}$ EFFECTS ON CHARGED PARTICLE MOTION

INTRODUCTION

The motion of a charged particle in orthogonal electric ($\underline{E}$) and magnetic ($\underline{B}$) fields is considered in the case where $\underline{\nabla}E$ is non-zero and parallel to $\underline{E}$. Two regimes of interest emerge. In one of these particles rate of gyration is changed from the conventional gyrofrequency. In the other, acceleration of the particle takes place. Applied to a plasma the theory predicts new electric currents orthogonal to magnetic fields. The theory has wide application and pertains to the outer magnetosphere of the earth, the solar atmosphere and to many astrophysical situations of interest.

A MODEL

For a model calculation let the magnetic field be homogeneous and parallel to the Ox axis of a Cartesian frame of reference (x, y, z). Let $\underline{E}$ and $\underline{\nabla}E$ be parallel to the Oz axis, and suppose

$$\underline{E} = \underline{E}_o + (\underline{\nabla}E)z \tag{1}$$

where $\underline{E}_o$ = constant and $\underline{\nabla}E$ is assumed to be constant over the region of interest. Consider the motion of a charged particle of mass m and charge q in this field configuration. The equation of motion is (in emu)

$$m\frac{d\underline{v}}{dt} = q[\underline{E}_o + (\underline{\nabla}E)z] + q\,\underline{v} \times \underline{B} \tag{2}$$

In a frame of coordinates given by $X = x$, $Y = y - \underline{E}_o \times \underline{B}t/B^2$, $Z = z$ the equation of motion yields

$$m\frac{d^2Y}{dt^2} = q\frac{dZ}{dt}B \tag{3}$$

and

$$m\frac{d^2Z}{dt^2} = q(\nabla E)Z - q\frac{dY}{dt}B \tag{4}$$

It follows that

$$\frac{d^2Z}{dt^2} + \left(\omega^2 - \frac{q\nabla E}{m}\right)Z = -\omega C \tag{5}$$

where

$$C = \left(\frac{dY}{dt}\right)_{Y=0}$$

and

$$\omega = \frac{Bq}{m}$$

<u>Case I</u>

$$\omega^2 - \frac{q\nabla E}{m} > 0 \tag{6}$$

Let

$$\Omega^2 = \omega^2 - \frac{q\nabla E}{m} \tag{7}$$

The solution of Equation 5 in the original (x, y, z) system becomes

$$u = u^1 \tag{8a}$$

$$v = \frac{E_o}{B} + \left(v^1 - \frac{E_o}{B}\right)\left[1 - \frac{\omega^2}{\Omega^2} + \frac{\omega^2}{\Omega^2}\cos \Omega t\right] + \frac{\omega}{\Omega} w^1 \sin \Omega t \tag{8b}$$

$$w = -\frac{\omega}{\Omega}\left(v^1 - \frac{E_o}{B}\right)\sin \Omega t + w^1 \cos \Omega t \tag{8c}$$

together with

$$x = u^1 t \tag{8d}$$

$$y = \left[v^1 - \frac{\omega^2}{\Omega^2}\left(v^1 - \frac{E_o}{B}\right)\right] t + \left(v^1 - \frac{E_o}{B}\right)\frac{\omega^2}{\Omega^3}\sin \Omega t + \frac{\omega}{\Omega^2} w^1 (1 - \cos \Omega t) \tag{8e}$$

$$z = \frac{w^1}{\Omega}\sin \Omega t - \left(v^1 - \frac{E_o}{B}\right)\frac{\omega}{\Omega^2}(1 - \cos \Omega t) \tag{8f}$$

where $\underline{v} = (u, v, w)$ and (u^1, v^1, w^1) are the components of velocity at time $t = 0$ at the point (0, 0, 0). Equations 8 reduce to the well-known results when $\nabla E = 0$ i.e., $\Omega = \omega$.

It is seen that the rate of gyration of the particle is changed by the amount $q\nabla E\ m^{-1}$, being decreased from ω to Ω when $q\nabla E$ is positive, and increased from ω to Ω when $q\nabla E$ is negative. This effect should change the characteristic frequencies involved in propagation emission and absorption of waves in a plasma in the magnetic field particularly at low frequencies. This effect will not be developed further here.

It is noted that Equations 8 imply perturbations of the ion-velocity distribution. The original ion velocity distribution function $f^1(u^1, v^1, w^1, 0)$ is modified to become

$$f(u, v, w, t) = f^1(u^1, v^1, w^1, 0)\ \frac{\partial(u^1, v^1, w^1)}{\partial(u, v, w)}$$

In Case I,

$$\frac{\partial(u^1, v^1, w^1)}{\partial(u, v, w)} = \left(1 - \frac{\omega^2}{\Omega^2}\right)\cos \Omega t + \frac{\omega^2}{\Omega^2}$$

Oscillations of the ion-veolcity distribution at the new frequency Ω will take place. Also the usual $E_o \times B/B^{-2}$ drift is modified (see Eq. 8b) to yield the mean drift of a particle along the Oy axis

$$v_D = \frac{\omega^2}{\Omega^2}\ \frac{E_o}{B} + v^1\left(1 - \frac{\omega^2}{\Omega^2}\right) \tag{9}$$

In the frame of reference moving with the drift velocity v_D along the Oy axis, the particle performs elliptical gyrations described by

$$Y^* = \left(v^1 - \frac{E_o}{B}\right)\frac{\omega^2}{\Omega^3}\ \sin \Omega t - \frac{\omega w^1}{\Omega^2}\cos \Omega t$$

$$Z^* = \frac{\omega}{\Omega^2}\left(v^1 - \frac{E_o}{B}\right)\cos \Omega t + w^1\ \sin \Omega t$$

The sizes of the two axes of the ellipse are

$$\left(v^1 - \frac{E_o}{B}\right)^2\ \frac{\omega^4}{\Omega^6} + \frac{\omega^2 w^{1\,2}}{\Omega^4}$$

and Ω^2/ω^2 times this value. The ellipse is rotated about the Ox axis by an amount

$$\tan^{-1}\left[-\ \frac{w^1}{\left(v^1 - \frac{E_o}{B}\right)}\cdot\frac{\Omega}{\omega}\right]$$

It follows from Equation 9 that for an assemblage of particles in a plasma, differential drift occurs for particles of different ω and different v^1. The effect of v^1 could be expressed as a temperature effect in this case. In a uniform neutral plasma consisting of one species of singly charged positive ions and electrons this drift will cause an electric current in the y direction given by

$$j_{y|z=0} = ne\left\{\frac{E_0}{B}\left[\frac{\omega_i^2}{\Omega_i^2} - \frac{\omega_e^2}{\Omega_e^2}\right] + \left[\langle v_i^1\rangle\left(1 - \frac{\omega_i^2}{\Omega_i^2}\right) - \langle v_e^1\rangle\left(1 - \frac{\omega_e^2}{\Omega_e^2}\right)\right]\right\} \qquad (10)$$

provided both Ω_i and $\Omega_e > 0$. In some situations, $\langle v_i^1\rangle$ and $\langle v_e^1\rangle$ may be expected to be zero. This would be the case if the velocity distributions were Maxwellian before time $t = 0$ at which time the electric field might be assumed to be "switched on."

Some interesting consequences of Equation 10 follow by considering the direction of $\underline{\nabla}E$ with respect to that of $\underline{E}$. If ∇E is positive i.e., in the direction of Oz, then

$$\Omega_i < \omega_i \quad \text{and} \quad \Omega_e > \omega_e$$

Consequently the current given by Equation 10 flows in the direction of Oy in regions of positive E and the opposite direction in regions of negative E. On the other hand if ∇E is in the opposite direction to Oz the current flows in the negative y direction in regions of positive E_0 and the opposite direction in regions of negative E_0. These consequences can be stated as follows: that the current flows in the direction $\pm\underline{\nabla}E \times \underline{B}$ according as $\underline{\nabla}E$ is in the same or opposite direction to $\underline{E}$. The current may appropriately be referred to as the $\underline{\nabla}E \times \underline{B}$ current.

If boundaries impede the flow of this current a secondary electrostatic field would be set up which would cause a component of plasma drift in the Oz for similarly directed E_0 and ∇E and in the negative z direction for oppositely directed E and ∇E.

Case II

In the case of extremely large values of ∇E we may have

$$\omega^2 - \frac{q\nabla E}{m} < 0 \qquad (11)$$

Let

$$\phi^2 = \frac{q\nabla E}{m} - \omega^2$$

The solution of Equation 5 in the original system (x, y, z) is

$$u = u^1 \tag{12a}$$

$$v = \frac{E_o}{B} + \left(v^1 - \frac{E_o}{B}\right)\left[1 + \frac{\omega^2}{\phi^2} - \frac{\omega^2}{\phi^2}\cosh\phi t\right] + w^1 \frac{\omega}{\phi}\sinh\phi t \tag{12b}$$

$$w = -\frac{\omega}{\phi}\left(v^1 - \frac{E_o}{B}\right)\sinh\phi t + w^1 \cosh\phi t \tag{12c}$$

together with

$$x = u^1 t \tag{12d}$$

$$y = \left[v^1 + \frac{\omega^2}{\phi^3}\left(v^1 - \frac{E_o}{B}\right)\right] t - \left(v^1 - \frac{E_o}{B}\right)\frac{\omega^2}{\phi^3}\sinh\phi t + \frac{\omega}{\phi^2} w^1(\cosh\phi t - 1) \tag{12e}$$

$$z = \frac{w^1}{\phi}\sinh\phi t - \left(v^1 - \frac{E_o}{B}\right)\frac{\omega}{\phi^2}(\cosh\phi t - 1) \tag{12f}$$

where again (u^1, v^1, w^1) are the components of velocity at time $t = 0$ at the point (0, 0, 0). Again Equations 12 reduce to the well known results when $\nabla E = 0$ i.e., $\phi = \iota\omega$.

It is observed that the particle does not gyrate but is continually accelerated in the electric field. The original velocity distribution $f(u^1, v^1, w^1, 0)$ of a group of similar particles would be modified to become

$$f(u, v, w, t) = f(u^1, v^1, w^1, 0)\left\{1 + \frac{\omega^2}{\phi^2}\cosh\phi t - \frac{\omega^2}{\phi^2}\right\}$$

A group of similar particles will contribute the following amounts to electric current in the Oy and Oz directions respectively.

$$\begin{aligned} j_{iy}(\bar{x}, \bar{y}, \bar{z}, t) = {} & \frac{n_i e E_o}{B}\frac{\omega_i^2}{\phi_i^2}(\cosh\phi_i t - 1) + \langle v_i^1\rangle\left[1 + \frac{\omega_i^2}{\phi_i^2} - \frac{\omega_i^2}{\phi_i^2}\cosh\phi_i t\right] \\ & + \langle w_i^1\rangle \frac{\omega_i}{\phi_i}\sinh\phi_i t \end{aligned} \tag{13a}$$

$$j_{iz}(\bar{x}, \bar{y}, \bar{z}, t) = \frac{n_i e E_o}{B}\frac{\omega_i}{\phi_i}\sinh\phi_i t + \langle v_i^1\rangle \frac{\omega_i}{\phi_i}\sinh\phi_i t + \langle w_i^1\rangle\cosh\phi_i t \tag{13b}$$

where E_o v^1 and w^1 are defined at time $t = 0$ at the point (0, 0, 0) and $\bar{x}$ $\bar{y}$ $\bar{z}$ are the mean coordinates

$$\bar{x} = \langle x \rangle = \langle u^1 \rangle t \tag{13c}$$

$$\bar{y} = \langle y \rangle = \langle v^1 \rangle \left(1 + \frac{\omega^2}{\phi^2} - \frac{\omega^2}{\phi^3} \sinh \phi t\right) t - \frac{\omega^2}{\phi^2} \frac{E_o}{B} t - \frac{\omega}{\phi^2} \langle w^1 \rangle (\cosh \phi t - 1) \tag{13d}$$

$$\bar{z} = \langle z \rangle = \frac{\langle w^1 \rangle}{\phi} \sinh \phi t - \langle v^1 \rangle \frac{\omega}{\phi^2} (\cosh \phi t - 1) + \frac{E_o}{B} \frac{\omega}{\phi^2} (\cosh \phi t - 1) \tag{13e}$$

Suppose that the plasma is uniform, singly ionized, collisionless and neutral and that both ϕ_i and Ω_e are real. This will be shown in a later section to be a useful special case. Further suppose that $\langle v^1 \rangle$ and $\langle w^1 \rangle$ are zero for both ions and electrons up till time $t = 0$ at which time the electric field is "switched on." It follows from 13 and 9 that the total current after time $t = 0$ at the point ($\bar{x}$, $\bar{y}$, $\bar{z}$, t) is given by

$$j_y(\bar{x}, \bar{y}, \bar{z}, t) = \frac{n_e e E_o}{B} \left[\frac{\omega_i^2}{\phi_i^2} (\cosh \phi_i t - 1) - \frac{\omega_e^2}{\Omega_e^2}\right] \tag{14a}$$

$$j_z(\bar{x}, \bar{y}, \bar{z}, t) = \frac{n_e e E_o}{B} \frac{\omega_i}{\phi_i} \sinh \phi_i t \tag{14b}$$

In the rare event that both ϕ_i and ϕ_e are real the current is given by

$$j_y(\bar{x}, \bar{y}, \bar{z}, t) = \frac{n_e e E_o}{B} \left[\frac{\omega_i^2}{\phi_i^2} (\cosh \phi_i t - 1) - \frac{\omega_e^2}{\phi_e^2} (\cosh \phi_e t - 1)\right] \tag{15a}$$

$$j_z(\bar{x}, \bar{y}, \bar{z}, t) = \frac{n_e e E_o}{B} \left[\frac{\omega_i}{\phi_i} \sinh \phi_i t - \frac{\omega_e}{\phi_e} \sinh \phi_e t\right] \tag{15b}$$

In order then to calculate the distribution of current at some time t the position of the origin of the particles of the current can be found indirectly from Equations 13d and 13e. This gives the value of E_o which must be used to calculate j_y and j_z in Equations 14a and 14b. It is assumed in this calculation that the original electric field configuration is maintained. In practice this may be caused by the flow of charged particles along the magnetic field to and from some reservoir of charge outside the region of interest. The currents j_y and j_z may be appropriately referred to as the $\underline{\nabla} E \times \underline{B}$ and $\underline{\nabla}_\perp E$ currents respectively.

Suppose that, to a sufficient approximation, the magnetic field lines are equipotentials of the electric field ($\underline{E}$) and suppose a constant gradient of $\underline{E}$ exists over a dimension L. The total potential change is then

$$\Delta\Phi = \frac{\nabla E L^2}{2} \tag{16}$$

The values of L which satisfy the condition for acceleration to take place are such that

$$L < B^{-1} \, |2m \, \Delta\Phi/q|^{1/2} \tag{17}$$

An alternative way of expressing the condition is given by

$$L < \left(\frac{E}{B}\right) B^{-1} \, \frac{m}{q} \tag{18}$$

where E is a characteristic value of the field within the system. In words, the inequality 18 states that acceleration becomes possible when the scale of the system becomes less than the gyroradius that one would associate with a particle of speed equal to the well known drift speed (E/B). A general condition for the operation of the acceleration mechanism is

$$q\nabla E > \frac{q^2 B^2}{m} \tag{19}$$

The applications of this theory to structures in the magnetosphere and plasmas in other planetary and astrophysical settings are being investigated by the author.