FORMULAS FOR PRECESSION

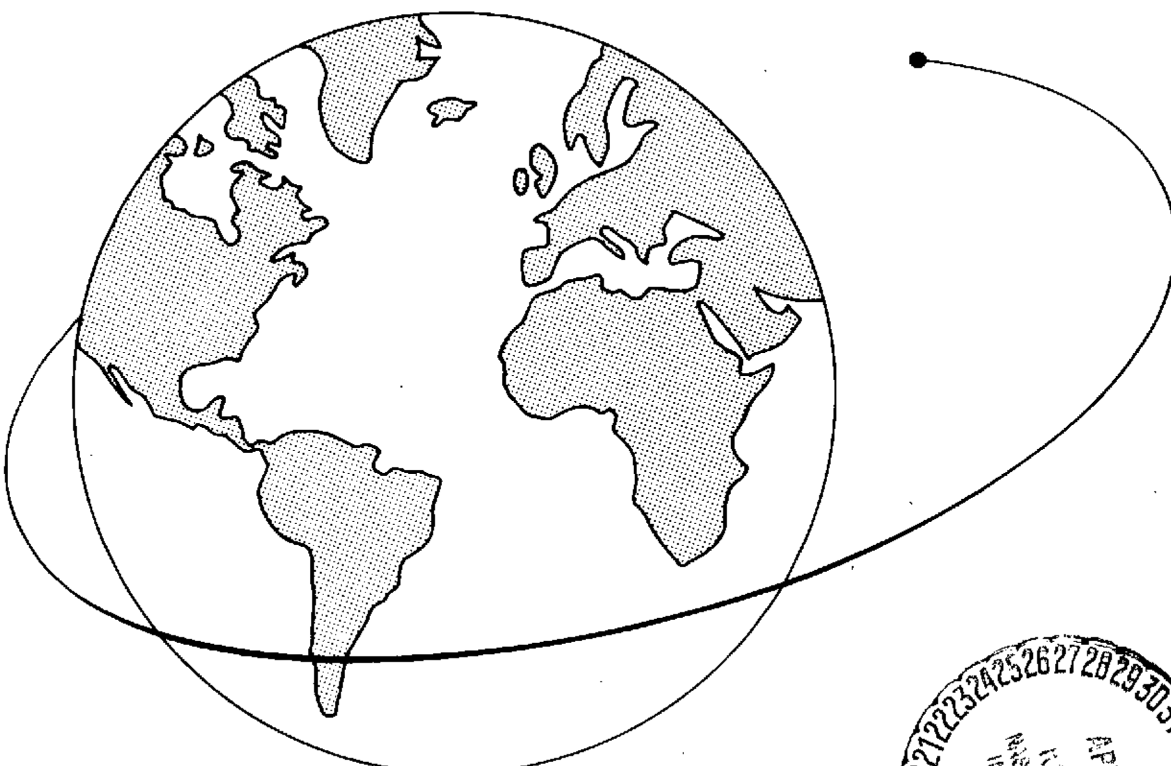
H. KINOSHITA

(NASA-CR-142301) FORMULAS FOR PRECESSION (Smithsonian Astrophysical Observatory) 31 p HC $3.75 CSCL 03B

N75-19098

Unclas

G3/90 12476



Smithsonian Astrophysical Observatory

SPECIAL REPORT 364

Research in Space Science
SAO Special Report No. 364

FORMULAS FOR PRECESSION

Hiroshi Kinoshita

February 28, 1975

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

411-091

TABLE OF CONTENTS

ILLUSTRATIONS

TABLES

ABSTRACT

Literal expressions for the precessional motion of the mean equator referred to an arbitrary epoch are constructed. Their numerical representations, based on numerical values recommended at the working meeting of the International Astronomical Union Commission held in Washington in September 1974, are obtained. In constructing the equations of motion, the second-order secular perturbation and the secular perturbation due to the long-periodic terms in the motions of the moon and the sun are taken into account. These perturbations contribute more to the motion of the mean equator than does the term due to the secular perturbation of the orbital eccentricity of the sun. In this paper, we use the correction $\Delta f = 1\overset{''}{.}10$ to the speed of Newcomb's lunisolar precession, the correction $\Delta g = -0\overset{''}{.}03$ to the speed of Newcomb's planetary precession, and the geodesic precession $f_g = 1\overset{''}{.}915$.

RESUME

On construit des expressions littérales du mouvement précessionnel de l'équateur moyen en prenant comme référence une époque arbitraire. Leurs représentations numériques sont obtenues en se basant sur les valeurs numériques recommandées aux séances de travail de la Commission de l'Union Astronomique Internationale tenues à Washington en septembre 1974. On a tenu compte, en construisant les équations du mouvement, de la perturbation séculaire du second ordre et de la perturbation séculaire dûe aux termes à longue période dans les mouvements du soleil et de la lune. Ces perturbations contribuent plus au mouvements de l'équateur moyen que ne le fait le terme dû à la perturbation séculaire de l'excentricité de l'orbite du soleil. Nous employons, dans cet exposé, la correction Δf = 1,10" pour la vitesse de précession lunisolaire de Newcomb, la correction Δg = -0,03" pour la vitesse de la précession planétaire de Newcomb et la précession géodésique f_g = 1,915"

КОНСПЕКТ

Составлены буквенные выражения прецессионного движения среднего экватора относительно произвольно заданной эпохи. Получены их численные выражения, основанные на численных значениях, которые были рекомендованы на рабочем совещании комиссии международного астрономического союза, состоявшемся в сентябре 1974 г. в Вашингтоне. При составлении уравнений движения были учтены вековое возмущение второго порядка и вековое возмущение, вызванное долгопериодическими членами в уравнениях движения луны и солнца. Эти возмущения больше влияют на уравнение движения среднего экватора, чем член, учитывающий вековое возмущение орбитального эксцентриситета солнца. В настоящей работе мы пользуемся поправкой на скорость лунно-солнечной прецессии Ньюкомба $\Delta f = 1{,}10''$, поправкой на скорость планетарной прецессии Ньюкомба $\Delta g = -0{,}03''$ и геодезической прецессией $f_g = 1{,}915''$.

FORMULAS FOR PRECESSION

Hiroshi Kinoshita

1. HAMILTONIAN FOR PRECESSION

In this paper, we basically adopt the conventions and notations used by Andoyer (1911). The Hamiltonian K for the precessional motion of the mean equator is as follows (Kinoshita, 1975):

$$K = F_s + E,$$

$$\begin{aligned} F_s = {} & \frac{\kappa^2 M_☾}{a_☾^3} \frac{2C - A - B}{2} \left(1 - \frac{3}{2} \sin^2 J\right) \left[-\frac{1}{2} M_0 (3 \cos^2 I - 1) - \frac{3}{2} M_1 \sin 2I - \frac{3}{4} M_2 \sin^2 I \right. \\ & \left. + 3M_3 \frac{M_☾}{M_☾ + M_\oplus} \frac{n_☾^2}{\omega n_\Omega} \frac{2C - A - B}{2C} \cos 2I \cos I \right] \qquad (1) \\ & + \frac{\kappa^2 M_\odot}{a_\odot^3} \frac{2C - A - B}{2} \left(1 - \frac{3}{2} \sin^2 J\right) \left[-\frac{1}{2} (S_0 + t\,\Delta S_0)(3 \cos^2 I - 1) - \frac{3}{4} S_2 \sin^2 I \right] , \end{aligned}$$

$$E = G \sin I \left[-\sin (h - \phi) \frac{dk}{dt} + \sin k \cos (h - \phi) \frac{d\phi}{dt} \right] + H(1 - \cos k) \frac{d\phi}{dt} ,$$

where $H = G \cos I$ and $G = C\omega$. The angle J between the axes of the figure and the angular-momentum axis of the earth is of the order of 10^{-6}. Therefore, we can neglect the term $\frac{3}{2} \sin^2 J$ in this paper. Here, the ecliptic at initial epoch t = 1850 is adopted as the reference plane of the inertial system, and the mean equinox at 1850 is the origin for longitude. The longitude h of the mean equinox of date is measured from the fixed mean equinox of 1850 along the fixed ecliptic of 1850 and then along the ecliptic

This work was supported in part by Grant NGR 09-015-002 from the National Aeronautics and Space Administration.

411-091

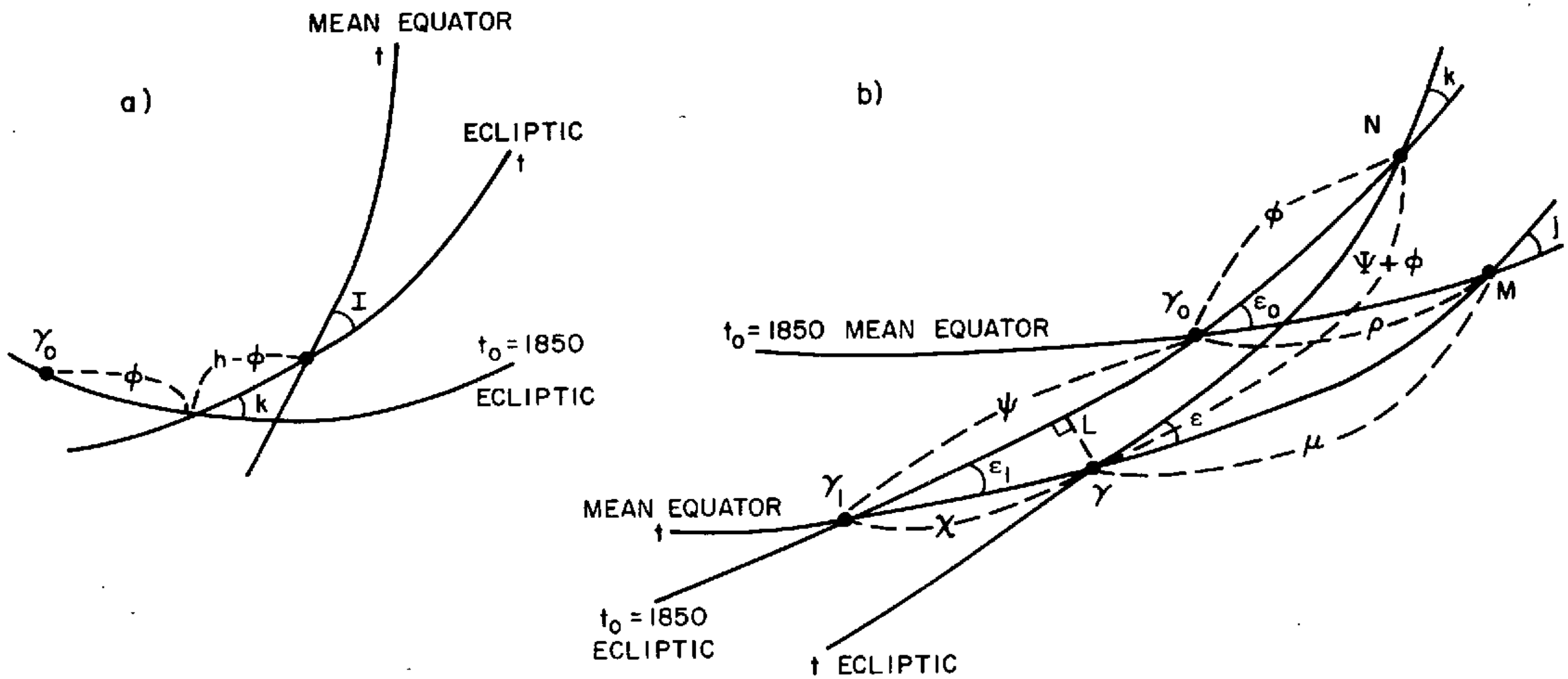


Figure 1. a) Definitions of h and I. b) Precessional motions referred to fundamental $t_0 = 1850$.

of date, and the inclination I of the mean equator is referred to the ecliptic of date (see Figure 1a); h and I, respectively, correspond to the general precession in longitude and the obliquity, but their signs are opposite from conventional usage, since we are using a right-hand system here. The principal moments of inertia of the earth are denoted by A, B, and C, and the angular velocity of the rotation of the earth is ω. The masses of the sun, the earth, and the moon are denoted by $M_{\odot}$, $M_{\oplus}$, and $M_{☾}$; the sidereal mean motions of the sun and the moon by $n_{\odot}$ and $n_{☾}$; and the mean motion of the node of the moon by n_{Ω}. The first term F_s in the Hamiltonian K is the secular term due to the direct lunisolar perturbation. The factors M_0, M_1, and M_2 in the equation for F_s are the secular terms of $\frac{1}{2}(a/r)^3(1 - 3\sin^2\beta)$, $(a/r)^3 \sin\beta\cos\beta\sin\lambda$, and $(a/r)^3\cos^2\beta\cos 2\lambda$, respectively, where λ and β are the longitude and latitude of the moon referred to the ecliptic of date and the mean equinox of date. The factors S_0 and S_2 are the same quantities as for the sun, and ΔS_0 is due to the secular perturbation of the orbital eccentricity of the sun. The term with M_3 comes from the second-order secular perturbation. The quantities M_0, M_1, M_2, M_3, S_0, S_2, and ΔS_0 depend only on the orbital elements of the moon and the sun, and their numerical values (Kinoshita, 1975) are obtained from Brown's theory of the moon as improved by Eckert, Walker, and Eckert (1966) and Newcomb's theory of the sun. The terms having M_1, M_2, and M_3 as factors are not included in Newcomb's precessional theory; M_1, M_2, and S_2 come from the long-periodic terms in the motions of the moon and the sun. The second term, E, in the Hamiltonian K represents the effect due to the motion of the ecliptic, which is caused by planetary perturbations. The form of F_s is independent of whether or not the reference system is inertial. Newcomb (1906) gave the motion of the ecliptic of date in the following form:

$$
\begin{aligned}
\sin k \sin\phi &= pt + p't^2 + p''t^3 = 5''.341t + 0''.1935t - 0''.00019t \quad , \\
\sin k \cos\phi &= qt + q't^2 + q''t^3 = -46''.838t + 0''.563t + 0''.00035 \quad ,
\end{aligned}
\tag{2}
$$

in which t = tropical centuries from 1850, k is the inclination of the ecliptic of date on the ecliptic of 1850, and ϕ is the geocentric longitude of the heliocentric ascending node of the ecliptic of date of 1850. If the ecliptic does not move, the additive term E disappears, and both the obliquity and the rate of the general precession in longitude are constant because the canonical variable h conjugate to H is cyclic.

2. EQUATIONS OF MOTION AND THEIR SOLUTIONS

Equations of motion are given from the Hamiltonian (1) as

$$\frac{dh}{dt} = \frac{\partial K}{\partial H} \ , \quad \frac{dH}{dt} = -\frac{\partial K}{\partial h} \ ,$$

or (3)

$$\frac{dh}{dt} = -\frac{1}{C\omega \sin I}\frac{\partial K}{\partial I} \ , \quad \frac{dI}{dt} = \frac{1}{C\omega \sin I}\frac{\partial K}{\partial h} \ .$$

Expressing the mean obliquity as ε ($\varepsilon = -I$) and the general precession in longitude as Ψ ($\Psi = -h$), which is reckoned westward, equations (3) become

$$\frac{d\Psi}{dt} = \frac{R(\varepsilon)}{\sin \varepsilon} - \cot \varepsilon \left[\sin (\Psi + \phi)\frac{dk}{dt} + \sin k \cos (\Psi + \phi)\frac{d\phi}{dt}\right] - (1 - \cos k)\frac{d\phi}{dt} \ , \tag{4}$$

$$\frac{d\varepsilon}{dt} = \cos (\Psi + \phi)\frac{dk}{dt} - \sin k \sin (\Psi + \phi)\frac{d\phi}{dt}$$

$$= \cos \Psi \frac{d}{dt}(\sin k \cos \phi) - \sin \Psi \frac{d}{dt}(\sin k \sin \phi) + 2 \sin^2 \frac{k}{2} \cos (\Psi + \phi)\frac{dk}{dt} \ , \tag{5}$$

where

$$R(\varepsilon) = (P_0 + P_1 t) \sin \varepsilon \cos \varepsilon + Q_0 \cos 2\varepsilon + V_0 (6 \cos^2 \varepsilon - 1) \sin \varepsilon \ ,$$

with

$$P_0 = k_{☾}\left(M_0 - \frac{1}{2}M_2\right) + k_{\odot}\left(S_0 - \frac{1}{2}S_2\right) \ ,$$

$$Q_0 = k_{☾} M_1 \ ,$$

PRECEDING PAGE BLANK NOT FILMED

$$V_0 = -k_{☾} M_3 \frac{M_{☾}}{M_{☾} + M_{\oplus}} \frac{n_{☾}^2}{\omega n_{\Omega}} \frac{2C - A - B}{2C} ,$$

$$P_1 = k_{\odot} \Delta S_0 ,$$

$$k_{☾} = 3 \frac{\kappa^2 M_{☾}}{a_{☾}^3 \omega} \frac{2C - A - B}{2C} = 3 \frac{M_{☾}}{M_{☾} + M_{\oplus}} \frac{1}{F_2^3} \frac{n_{☾}^2}{\omega} \frac{2C - A - B}{2C} ,$$

$$k_{\odot} = 3 \frac{\kappa^2 M_{\odot}}{a_{\odot}^3 \omega} \frac{2C - A - B}{2C} = 3 \frac{M_{\odot}}{M_{\odot} + M_{\oplus} + M_{☾}} \frac{n_{\odot}^2}{\omega} \frac{2C - A - B}{2C} .$$

Although equations (4) and (5) are nonlinear, their solutions can be obtained by a successive approximation, assuming that Ψ and ε can be expanded into power series of time with the initial conditions $\Psi = 0$ and $\varepsilon = \varepsilon_0$ at t = 1850. After we solve for Ψ and ε, the other quantities – the obliquity referred to the fixed ecliptic ε_1, the lunisolar precession in longitude ψ, and the planetary precession χ – are derived with the aid of the trigonometric relation in the triangle $\gamma_1 N \gamma$ (see Figure 1b). To simplify the expressions and for convenience in systematic numerical calculations, we solve the following equations together with equation (5):

$$\frac{d\varepsilon_1}{dt} = R(\varepsilon) \sin \chi , \tag{6}$$

$$\sin \varepsilon_1 \frac{d\psi}{dt} = R(\varepsilon) \cos \chi , \tag{7}$$

$$\sin \varepsilon_1 \sin \chi = \sin k \sin (\Psi - \phi) , \tag{8}$$

$$\tan \frac{1}{2} (\Psi - \psi) = \frac{- \tan \chi \cos (\varepsilon + \varepsilon_1)/2}{\cos (\varepsilon - \varepsilon_1)/2} , \tag{9}$$

where equations (6) and (7) are easily derived from equations (4) and (5) with the aid of the trigonometric relation in the triangle $\gamma_1 N \gamma$. Equations (6) and (7) represent the motion of the pole of the mean equator in the ecliptic system and are equivalent to those obtained by Andoyer (1911) if $R(\varepsilon)$ is replaced by $P \cos \varepsilon$, or, in another words, if $Q_0 = V_0 = 0$.

We can express ε, ε_1, ψ, χ, Ψ, and $R(\varepsilon)$, in the form

$$
\begin{aligned}
\varepsilon &= \varepsilon_0 + at + a't^2 + a''t^3 \quad , \\
\varepsilon_1 &= \varepsilon_0 + bt + b't^2 + b''t^3 \quad , \\
\psi &= ft + f't^2 + f''t^3 \quad , \\
\chi &= gt + g't^2 + g''t^3 \quad , \\
\Psi &= ht + h't^2 + h''t^3 \quad , \\
R(\varepsilon) &= R_0(\varepsilon_0) + R_1(\varepsilon_0)t + R_2(\varepsilon_0)t^2 \quad .
\end{aligned}
\tag{10}
$$

Substituting these into equations (5) through (9), and equating the coefficients of like powers of t, we get, to the third power of t,

$$
\begin{aligned}
a &= q \quad , \\
a' &= q' - \frac{1}{2} ph \quad , \\
a'' &= q'' - \frac{1}{6} q(h^2 - p^2 - q^2) - \frac{1}{3} (2p'h + ph') \quad , \\
b &= 0 \quad , \\
b' &= \frac{1}{2} R_0 g = \frac{1}{2} pf \quad , \\
b'' &= \frac{1}{3} (R_1 g + R_0 g') = \frac{1}{3} (g'f + 2gf') \sin \varepsilon_0 \quad , \\
g &= p \operatorname{cosec} \varepsilon_0 \quad , \\
g' &= (p' + qh) \operatorname{cosec} \varepsilon_0 \quad , \\
g'' &= (p'' + qh' + q'h - \frac{1}{2} ph^2) \operatorname{cosec} \varepsilon_0 + \frac{1}{6} g^3 - gb' \cot \varepsilon_0 \quad , \\
h &= f - g \cos \varepsilon_0 \quad ,
\end{aligned}
\tag{11}
$$

[eq. (11) cont. on next page]

$$h' = f' - g' \cos \varepsilon_0 + \frac{1}{2} pq \quad ,$$

$$h'' = f'' - g'' \cos \varepsilon_0 + \frac{1}{2} g(a' + b') \sin \varepsilon_0 + \frac{1}{2} ag' \sin \varepsilon_0 - \frac{1}{12} p^3 \cot \varepsilon_0 \quad ,$$

$$f = R_0 \operatorname{cosec} \varepsilon_0$$

$$= P_0 \cos \varepsilon_0 + Q_0 \cos 2\varepsilon_0 \operatorname{cosec} \varepsilon_0 + V_0 (6 \cos^2 \varepsilon_0 - 1) \quad ,$$

$$f' = \frac{1}{2} R_1 \operatorname{cosec} \varepsilon_0$$

$$= \frac{1}{2} P_0 a \cos 2\varepsilon_0 \operatorname{cosec} \varepsilon_0 + \frac{1}{2} P_1 \cos \varepsilon_0 - 2Q_0 a \cos \varepsilon_0 \tag{11}$$

$$+ \frac{1}{2} V_0 a (9 \cos 2\varepsilon_0 - 4) \cot \varepsilon_0 \quad ,$$

$$f'' = \frac{1}{3} (R_2 \operatorname{cosec} \varepsilon_0 - \frac{1}{2} fg^2 - fb' \cot \varepsilon_0)$$

$$= \frac{1}{3} \Big\{ P_0 (a' \cos 2\varepsilon_0 - a^2 \sin 2\varepsilon_0) + P_1 a \cos 2\varepsilon_0 - 2Q_0 (a' \sin 2\varepsilon_0 + a^2 \cos 2\varepsilon_0)$$

$$+ V_0 \Big[a' (9 \cos 2\varepsilon_0 - 4) \cos \varepsilon_0 - \frac{1}{2} a^2 \sin \varepsilon_0 (14 + 27 \cos 2\varepsilon_0) \Big] \Big\} \operatorname{cosec} \varepsilon_0$$

$$- \frac{1}{6} fg^2 - \frac{1}{3} fb' \cot \varepsilon_0 \quad ,$$

which can be written in many different ways. We have chosen these in such a way that the quantities P_0, P_1, Q_0, and V_0 appear only in the lunisolar precession in longitude. As a result, except for the lunisolar precession, they coincide with Andoyer's (1911).

It is worth noting that the obliquity referred to the fixed ecliptic does not have a secular term in the first power of time, although it does have the terms t^2, t^3, $\cdots$. However, Woolard (1953) derived a secular term $-0\overset{''}{.}0256t$, which has an effect amounting to 10% of the generally accepted value of Ort's constant B. This discrepancy occurs because Woolard neglected some terms in constructing the equations of motion (Kinoshita, 1975).

According to Newcomb's measure (1906), the general precession in longitude is the longitude of the mean equinox of date, referred to the fixed mean equinox and ecliptic of epoch: $\Psi_N = \psi - \widehat{\gamma_1 L}$, where L is the orthogonal projection of the mean equinox of date onto the fixed ecliptic (Figure 1b). This measure is favored by Plummer (1916) and Woolard and Clemence (1966). From the triangle $\gamma_1 L\gamma$, we have

$$\tan \widehat{\gamma_1 H} = \tan \chi \cos \varepsilon_1 \tag{12}$$

and then, to the third order of t,

$$\Psi_N = h_N t + h'_N t^2 + h''_N t^3 \quad , \tag{13}$$

where

$$\begin{aligned} h_N &= f - g \cos \varepsilon_0 \quad , \\ h'_N &= f' - g' \cos \varepsilon_0 \quad , \\ h''_N &= f'' - g'' \cos \varepsilon_0 + gb' \sin \varepsilon_0 - \frac{1}{3} p^3 \cot \varepsilon_0 \quad . \end{aligned} \tag{14}$$

The expressions for the general precession in longitude according to Newcomb's measure differ in the second and third orders of t from those [see eqs. (11)] according to Andoyer's definition, and the numerical differences between them are only of the order of $0\rlap{.}''0005t^2$ and $0\rlap{.}''00001t^3$.

Formulas for ϕ and k can be obtained from equations (2):

$$\begin{aligned} \phi &= \phi_0 + vt + v't^2 \quad , \\ k &= wt + w't^2 + w''t^3 \quad , \end{aligned} \tag{15}$$

where

$$
\begin{aligned}
\phi_0 &= \tan \frac{p}{q} \quad , \\
v &= \frac{p' \cos \phi_0 - q' \sin \phi_0}{w} \quad , \\
v' &= \frac{p'' \cos \phi_0 - q'' \sin \phi_0 - w'v}{w} \quad , \\
w &= \sqrt{p^2 + q^2} \quad , \\
w' &= p' \sin \phi_0 + q' \cos \phi_0 \quad , \\
w'' &= p'' \sin \phi_0 + q'' \cos \phi_0 + \frac{1}{2} wv^2 + \frac{1}{6} w^3 \quad .
\end{aligned}
\tag{16}
$$

Next we calculate the precessional quantities in the equatorial system. From equations (4) and (5) and with the aid of the trigonometric relation in the triangle $\gamma_0 M \gamma_1$ (Figure 1b), the motion of the pole of the mean equator in this system is expressed as

$$
\sin j \frac{d\rho}{dt} = R(\mathcal{E}) \cos \mu \quad , \tag{17}
$$

$$
\frac{dj}{dt} = R(\mathcal{E}) \sin \mu \quad . \tag{18}
$$

The spherical triangle $\gamma_0 M \gamma_1$ (Figure 1b) gives the following relations:

$$
\tan \frac{\mu + \rho + \chi}{2} = \frac{- \tan (\psi/2) \sin [(\mathcal{E}_1 + \mathcal{E}_0)/2]}{\sin [(\mathcal{E}_1 - \mathcal{E}_0)/2]} \quad , \tag{19}
$$

$$
\tan \frac{\mu - \rho + \chi}{2} = \frac{\tan (\psi/2) \cos [(\mathcal{E}_1 + \mathcal{E}_0)/2]}{\cos [(\mathcal{E}_1 - \mathcal{E}_0)/2]} \quad . \tag{20}
$$

As t approaches the epoch in equations (19) and (20), we have $\tan (\mu + \rho)/2 \rightarrow \infty$ and $\tan (\mu - \rho)/2 \rightarrow 0$, because $\chi \rightarrow gt$, $\psi \rightarrow ft$, $(\mathcal{E}_1 + \mathcal{E}_0)/2 \rightarrow \mathcal{E}_0$, and $(\mathcal{E}_1 - \mathcal{E}_0)/2 \rightarrow b't^2/2$. Therefore, we can express $\mu + \rho$, $\mu - \rho$, and j as follows:

$$\mu + \rho = 180 + ut + u't^2 + u''t^3 \quad ,$$

$$\mu - \rho = rt + r't^2 + r''t^2 \quad , \tag{21}$$

$$j = st + s't^2 + s''t^3 \quad .$$

Substituting these into equations (17), (18), and (20), and equating the coefficients of like powers of t, we get, to the third power,

$$s = R_0 = f \sin \varepsilon_0 \quad ,$$

$$s' = \frac{1}{2} R_1 = f' \sin \varepsilon_0 \quad ,$$

$$s'' = \frac{1}{3} R_2 - \frac{1}{24} R_0 \, r^2 = f'' \sin \varepsilon_0 + \frac{1}{24} f \sin \varepsilon_0 \, (3g^2 + 6fg \cos \varepsilon_0 - f^2 \cos^2 \varepsilon_0) \quad ,$$

$$u = 0 \quad ,$$

$$u' = \frac{sr' - rs'}{3s} \quad , \tag{22}$$

$$u'' = \frac{sr'' - 2u's' - rs''}{2s} - \frac{rs^2}{24} - \frac{r^3}{48} \quad ,$$

$$r = f \cos \varepsilon_0 - g \quad ,$$

$$r' = f' \cos \varepsilon_0 - g' \quad ,$$

$$r'' = f'' \cos \varepsilon_0 - g'' + \frac{1}{12} f^2 \sin^2 \varepsilon_0 \, (f \cos \varepsilon_0 - 3g) \quad .$$

The quantities ζ, ζ_0, and z introduced by Newcomb (1906) are related to ρ, μ, and χ by $\zeta = \mu + \chi - 90°$, $\zeta_0 = 90° - \rho$, and $z = \mu - 90°$.

In order to obtain formulas for epoch t′ referred to an arbitrary epoch t, we must first derive the position of the ecliptic of epoch t′ referred to the ecliptic and the mean equinox of t in terms of the coefficients for the fundamental epoch t = 1850. Denoting the quantities for epoch t′ referred to an arbitrary epoch t by (k), (ϕ), (ε), ⋯ and their coefficients by (p), (p′), (p″), (q), (q′), (q″), (a), (a′), (a″), ⋯, we have

$$\sin(k)\sin(\phi) = (p)\,\theta + (p')\,\theta'^2 + (p'')\,\theta'^3 \quad ,$$

$$\sin(k)\cos(\phi) = (q)\,\theta + (q')\,\theta^2 + (q'')\,\theta^3 \quad ,$$

$$(\mathcal{E}) = (\mathcal{E}_0) + (a)\,\theta + (a')\,\theta^2 + (a'')\,\theta^3 \quad , \tag{23}$$

$$\cdot$$
$$\cdot$$
$$\cdot \quad ,$$

where $\theta = t' - t$. It is clear that $(P_0) = P_0 + P_1 t$ and $(\mathcal{E}_0) = \mathcal{E}$. From the triangle N(N)N′ (Figure 2), we get

$$\begin{aligned} (p) &= p + p_1 t + p_2 t^2 \quad , & (q) &= q + q_1 t + q_2 t^2 \quad , \\ (p') &= p' + p_1' t \quad , & (q') &= q' + q_1' t \quad , \\ (p'') &= p'' \quad , & (q'') &= q'' \quad , \end{aligned} \tag{24}$$

in which

$$\begin{aligned} p_1 &= 2p' + qh \quad , & q_1 &= 2q' - ph \quad , \\ p_2 &= 3p'' + 2q'h + qh' - \frac{1}{2}p(h^2 - p^2 - q^2) \quad , & q_2 &= 3q'' - 2p'h - ph' - \frac{1}{2}q(h^2 - p^2 - q^2) \quad , \\ p_1' &= 3p'' + q'h + \frac{1}{2}p(p^2 + q^2) \quad , & q_1' &= 3q'' - p'h + \frac{1}{2}q(p^2 + q^2) \quad , \end{aligned} \tag{25}$$

which were derived by Andoyer (1911).

The relationships among the coefficients for epoch t′ referred to an epoch t are the same as those for t referred to the fundamental epoch t = 1850:

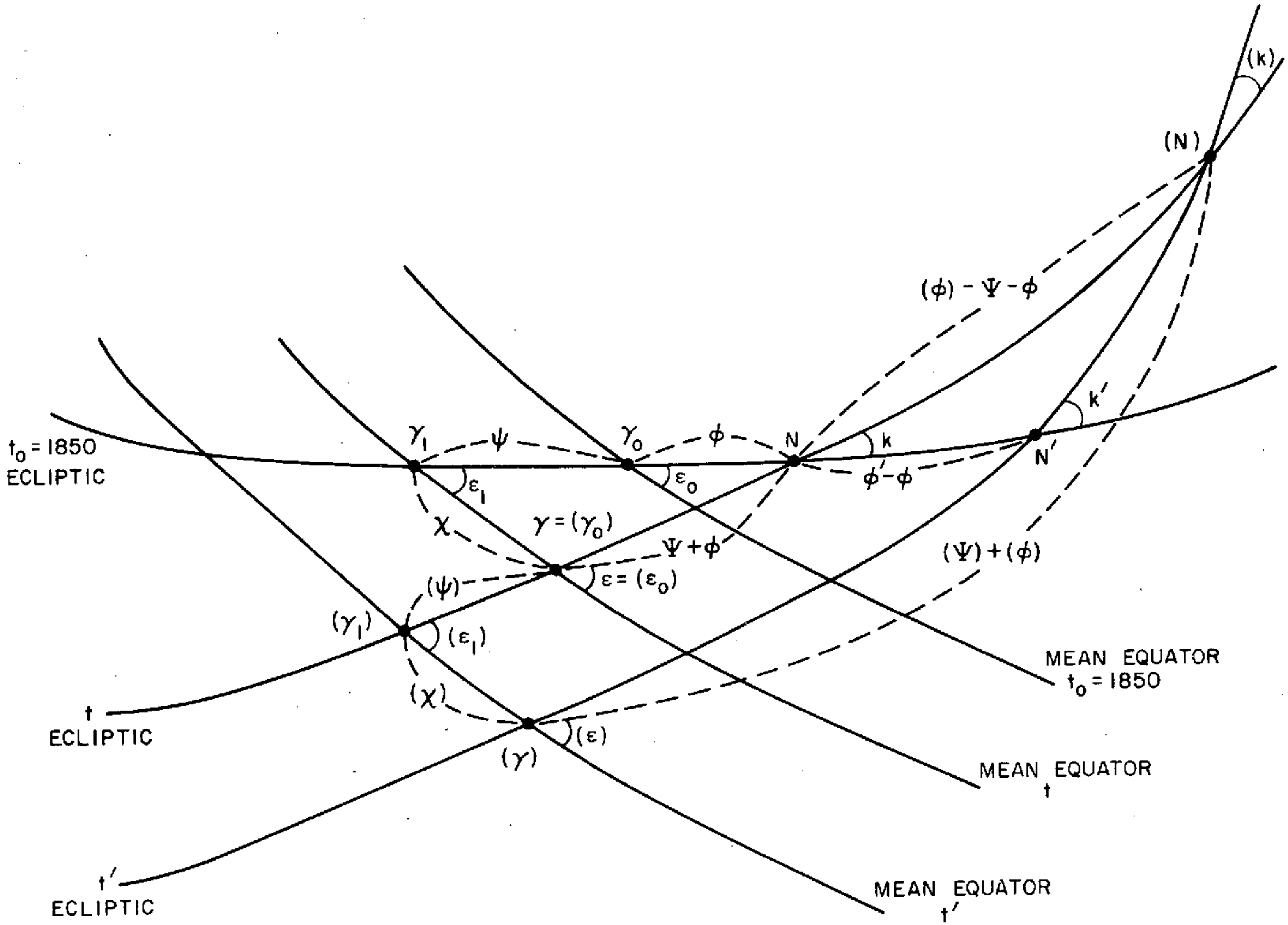


Figure 2. Precessional motions referred to an arbitrary epoch t.

$$(a) = a + a_1 t + a_2 t^2 \quad , \quad (a') = a' + a_1' t \quad , \quad (a'') = a'' \quad ,$$

$$(b) = 0 \quad , \quad (b') = b' + b_1' t \quad , \quad (b'') = b'' \quad , \tag{26}$$

$$(g) = g + g_1 t + g_2 t^2 \quad , \quad (g') = g' + g_1' t \quad , \quad (g'') = g'' \quad ,$$

$$\begin{array}{c} \cdot \\ \cdot \\ \cdot \;, \end{array}$$

where

$$a_1 = q_1 \quad ,$$

$$a_2 = q_2 \quad ,$$

$$a_1' = q_1' - \frac{1}{2}(ph_1 + p_1 h) \quad ,$$

$$b_1' = \frac{1}{2}(p_1 f + pf_1) \quad ,$$

$$g_1 = -\,ap \cos \varepsilon_0 \operatorname{cosec}^2 \varepsilon_0 + p_1 \operatorname{cosec} \varepsilon_0 \quad , \tag{27}$$

$$g_2 = \frac{1}{2} p \left[(2 - \sin^2 \varepsilon_0)\, a^2 - a' \sin^2 \varepsilon_0 \right] \operatorname{cosec}^3 \varepsilon_0 - p_1 \cos \varepsilon_0 \operatorname{cosec}^2 \varepsilon_0 + p_2 \operatorname{cosec} \varepsilon_0 \quad ,$$

$$g_1' = -a(p' + qh) \cos \varepsilon_0 \operatorname{cosec}^2 \varepsilon_0 + (p_1' + qh_1 + q_1 h) \operatorname{cosec} \varepsilon_0 \quad ,$$

$$h_1 = f_1 - g_1 \cos \varepsilon_0 + ag \sin \varepsilon_0 \quad ,$$

$$h_2 = f_2 - g_2 \cos \varepsilon_0 + g(a' \sin \varepsilon_0 + \frac{1}{2} a^2 \cos \varepsilon_0) + g_1 a \sin \varepsilon_0 \quad ,$$

$$h_1' = f_1' - g_1' \cos \varepsilon_0 + g'a \sin \varepsilon_0 + \frac{1}{2}(pq_1 + p_1 q) \quad ,$$

[eq. (27) cont. on next page]

$$\phi_1 = \frac{1}{q}\,(p_1 \cos^2 \phi_0 - q_1 \sin \phi_0 \cos \phi_0) \quad ,$$

$$\phi_2 = \frac{1}{q}\,(p_2 \cos^2 \phi_0 - \phi_1^2 q \tan \phi_0 - q_1\phi_1 - q_2 \sin \phi_0 \cos \phi_0) \quad ,$$

$$v_1 = \frac{1}{w}\Big[p_1' \cos \phi_0 - p'\phi_1 \sin \phi_0 - q'\phi_1 \cos \phi_0 - q_1' \sin \phi_0$$

$$- \frac{w_1}{w}\,(p' \cos \phi_0 - q' \sin \phi_0)\Big] \quad ,$$

$$w_1 = \frac{1}{w}\,(pp_1 + qq_1) \quad ,$$

$$w_2 = \frac{1}{w}\left(\frac{p_1^2 + q_1^2}{2} + pp_2 + qq_2\right) - \frac{1}{2w^3}\,(pp_1 + qq_1)^2 \quad ,$$

$$w_1' = p'\phi_1 \cos \phi_0 + p_1' \sin \phi_0 - q'\phi_1 \sin \phi_0 + q_1' \cos \phi_0 \quad ,$$

$$r_1 = f_1 \cos \varepsilon_0 - af \sin \varepsilon_0 - g_1 \quad ,$$

$$r_2 = -f\,(a' \sin \varepsilon_0 + \frac{1}{2} a^2 \cos \varepsilon_0) - af_1 \sin \varepsilon_0 - g_2 + f_2 \cos \varepsilon_0 \quad , \qquad (27)$$

$$r_1' = f_1' \cos \varepsilon_0 - f'a \sin \varepsilon_0 - g_1' \quad ,$$

$$s_1 = f_1 \sin \varepsilon_0 + fa \cos \varepsilon_0 \quad ,$$

$$s_2 = f_2 \sin \varepsilon_0 + f_1 a \cos \varepsilon_0 + f(a' \cos \varepsilon_0 - \frac{1}{2} a^2 \sin \varepsilon_0) \quad ,$$

$$s_1' = f_1' \sin \varepsilon_0 + f'a \cos \varepsilon_0 \quad ,$$

$$u_1' = \frac{1}{3s}\,(sr_1' + s_1 r' - s'r_1 - s_1' r - 3u's_1) \quad ,$$

$$f_1 = P_1 \cos \varepsilon_0 - P_0 a \sin \varepsilon_0 + \frac{1}{2} Q_0 a\,(\cos 3\varepsilon_0 - 3 \cos \varepsilon_0) \operatorname{cosec}^2 \varepsilon_0$$

$$- 6V_0\, a \sin 2\varepsilon_0 \quad ,$$

[eq. (27) cont. on next page]

$$
\begin{aligned}
f_2 = & -P_0(a' \sin \varepsilon_0 + \tfrac{1}{2} a^2 \cos \varepsilon_0) - P_1 a \sin \varepsilon_0 - 6V_0 (a' \sin 2\varepsilon_0 + a^2 \cos 2\varepsilon_0) \\
& + Q_0 \left[\frac{a^2}{8} (9 - 2 \cos 2\varepsilon_0 + \cos 4\varepsilon_0) \operatorname{cosec}^3 \varepsilon_0 + \frac{a'}{2} (\cos 3\varepsilon_0 - 3 \cos \varepsilon_0) \right. \\
& \left. \operatorname{cosec}^2 \varepsilon_0 \right] \quad , \\
f'_1 = & -\tfrac{1}{2} P_1 a \sin \varepsilon_0 - \tfrac{1}{2} P_0 a q (4 \cos \varepsilon_0 + \cos 2\varepsilon_0 \cos \varepsilon_0 \operatorname{cosec}^2 \varepsilon_0) \\
& + \tfrac{1}{2} (P_1 q + P_0 q_1) \cos 2\varepsilon_0 \operatorname{cosec} \varepsilon_0 - 2Q_0 (q_1 \cos \varepsilon_0 - q^2 \sin \varepsilon_0) \\
& + \tfrac{1}{2} V_0 a (9 \cos 4\varepsilon_0 - 18 \cos 2\varepsilon_0 - 1) \operatorname{cosec}^2 \varepsilon_0 \quad .
\end{aligned}
\tag{27}
$$

3. NUMERICAL REPRESENTATION OF PRECESSIONAL MOTION

In order to obtain the numerical representation of precessional motion, it is necessary to determine values of P_0, P_1, Q_0, and V_0, which depend on the value of the speed of the lunisolar precession. We adopt the following:

$$
\begin{aligned}
&f_{1900} = 5037''.08 + 1''.10 + 1''.915 = 5040''.095/\text{tropical century}, \\
&M_0 - \tfrac{1}{2} M_2 = 496303.4 \times 10^{-6} \quad , \\
&M_1 = -20.7 \times 10^{-6} \quad , \\
&M_3 = 2992 \times 10^{-6} \quad , \\
&(S_0 - \tfrac{1}{2} S_2)_{1900} = 500210.0 \times 10^{-6} \quad , \\
&\Delta S_0 = -1.056 \times 10^{-6} \quad , \\
&M_{☾}/M_{\oplus} = 0.02130 \quad , \\
&M_{\odot}/(M_{\oplus} + M_{☾}) = 328900.2 \quad , \\
&n_{☾}^2/\omega = 63238908''.5/\text{tropical century}, \\
&n_{\odot}^2/\omega = 35386''.791/\text{tropical century}, \\
&n_{☾}^2/\omega n_{\Omega} = -9.082445 \quad , \\
&F_2 = 0.9990931420 \quad , \\
&\varepsilon_{1900} = 23°27'8''.28 \quad , \\
&\Delta p = -0''.012 \quad , \qquad \Delta q = 0''.037 \quad .
\end{aligned}
\tag{28}
$$

The rate of lunisolar precession at 1900, f_{1900}, is obtained by applying the proper-motion correction $\Delta f = 1''10$ (Fricke, 1971) and the geodesic precession $f_g = 1''915$ (de Sitter, 1938) to Newcomb's value 5037''08. Values of M_0, M_1, M_2, M_3, S_0, S_2, and ΔS_0 were calculated by Kinoshita (1975) by using Brown's theory of the moon as improved by Eckert et al. (1966) and Newcomb's theory of the sun. Values for $M_☾/M_⊕$ and $M_⊙/(M_☾ + M_⊕)$ were recommended at the working meeting of the International Astronomical Union held in Washington in September 1974. The meeting also recommended a new system of planetary masses. Since the motion of the ecliptic depends on the planetary masses, a change in mass causes a change in p, q, p′, q′, p″, and q″ in equations (2). We adopt here $\Delta p = -0''012$ and $\Delta q = 0''037$ as corrections to Newcomb's $p = 5''341$ and $q = -46''838$, which were calculated by Laubscher (1972) from the planetary masses proposed by Klepczynski, Seidelman, and Duncombe (1971). Klepczynski et al.'s values are slightly different from those recommended, but the changes are too small to affect Δp and Δq. The correction to planetary precession due to Δp is $-0''03$ per tropical century. The value of the obliquity ε_{1900} includes the effect of Δq. Using values from equations (28) and solving the following equation [see eqs. (11)] with respect to the dynamical ellipticity of the earth [2(-A - B)/2C],

$$
\begin{aligned}
f_{1900} &= 5040.095 \\
&= 3\times \frac{2C - A - B}{2C} \left\{ \frac{M_☾}{M_☾ + M_⊙} \frac{1}{F_2^3} \frac{n_☾^2}{\omega} \left[\left(M_0 - \frac{1}{2} M^2 \right) \cos \varepsilon \right.\right. \\
&\quad \left. + M_1 \frac{\cos 2\varepsilon}{\sin \varepsilon} + M_4 \frac{M_☾}{M_☾ + M_⊕} \frac{n_☾^2}{\omega n_\Omega} \frac{2C - A - B}{2C} (6 \cos^2 \varepsilon - 1) \right] \\
&\quad \left. + \frac{M_⊙}{M_⊙ + M_⊕ + M_☾} \frac{n_⊙^2}{\omega} \left(S_0 - \frac{1}{2} S_2 \right) \cos \varepsilon \right\}_{1900} ,
\end{aligned}
$$

we get

$$\frac{2C - A - B}{2C} = 0.003273952 \quad ,$$

and

$$
\begin{aligned}
k_{☾} &= 7567''.55998/\text{tropical century}, \\
k_{\odot} &= 3475''.32347/\text{tropical century}, \\
P_0 &= 5494''.1988/\text{tropical century}, \\
P_1 &= -0''.00367/\text{tropical century}^2, \\
Q_0 &= -0''.1566/\text{tropical century}, \\
V_0 &= 0''.00818/\text{tropical century}.
\end{aligned}
\tag{29}
$$

We see that Q_0 and V_0 are larger than $P_1 t$ within the period in which Newcomb's theory for the ecliptic is valid. Strictly speaking, as seen from equations (10) and (11), the obliquity at 1900 depends somewhat on P_0, P_1, Q_0, and V_0 through the second and third orders of t, but the effect on ε_{1900} due to them amounts to only 0''.002 and, therefore, does not have a numerical effect on the determination of the dynamical ellipticity and of P_0, P_1, Q_0, and V_0.

Applying equations (29) to equations (11), (16), (22), (25), and (27), we derive numerical representations for precessional motion; these are given in Table 1. Table 2 shows the precessional formulas, in which t′ and t are reckoned from 2000 January $1^d 12^h$ ET in units of Julian centuries. The geodesic precession f_g, which is a direct motion of the zero point of longitude along the ecliptic of date, is subtracted from f, h, r, and s in equations (11) and (22) to obtain f_0, h_0, r_0, and s_0 in Tables 1 and 2, as follows: $f_0 = f - f_g$, $h_0 = h - f_g$, $r_0 = r - f_g \cos \varepsilon_0$, and $s_0 = s - f_g \sin \varepsilon_0$. For comparison, Tables 3 and 4 show precessional formulas based on Newcomb's values of $P_0 = 5490''.66$, $P_1 = -0''.00364$, and $Q_0 = V_0 = 0$.

Table 1. Precessional formulas: $\theta = t' - t$, where t' and t are reckoned from 1850 in units of tropical centuries.

$$\sin(k)\sin(\phi) = (5''.3290 - 0''.75374t + 0''.000335t^2)\,\theta + (0''.19350 + 0''.000802t)\,\theta^2 - 0''.000190\,\theta^3$$

$$\sin(k)\cos(\phi) = (-46''.8010 - 0''.01729t + 0''.005489t^2)\,\theta + (0''.05630 - 0''.003668t)\,\theta^2 + 0''.000350\,\theta^3$$

$$(\phi) = 173°30'14''.36 + 3288''.00125t + 0''.576239t^2 + (-869''.78374 - 0''.554964t)\,\theta + 0''.024576\,\theta^2$$

$$(k) = (47''.1034 - 0''.06809t + 0''.000569t^2)\,\theta + (-0''.03405 + 0''.000569t)\,\theta^2 + 0''.000050\,\theta^3$$

$$(\varepsilon) = 23°27'31''.68 - 46''.80100t - 0''.008645t^2 + 0''.00183t^3 + (-46''.8010 - 0''.01729t + 0''.005489t^2)\,\theta + (-0''.00865 + 0''.005490t)\,\theta^2 + 0''.001830\,\theta^3$$

$$(\varepsilon_1) = 23°27'31''.68 - 46''.80100t - 0''.008645t^2 + 0''.00183t^3 + (0''.06510 - 0''.009202t)\,\theta^2 - 0''.007733\,\theta^3$$

$$(\psi) = (5037''.9337 + 0''.49264t - 0''.000039t^2)\,\theta + (-1''.05099 - 0''.001513t)\,\theta^2 - 0''.001530\,\theta^3$$

$$(\chi) = (13''.3864 - 1''.88640t - 0''.000144t^2)\,\theta + (-2''.37947 - 0''.001554t)\,\theta^2 - 0''.001661\,\theta^3$$

$$(\Psi) = (5025''.6537 + 2''.22191t + 0''.000264t^2)\,\theta + (1''.13120 + 0''.000212t)\,\theta^2 + 0''.000102\,\theta^3$$

$$(\mu) + (\rho) = 180° + (0''.79223 + 0''.000648t)\,\theta^2 + 0''.000328\,\theta^3$$

$$(\mu) - (\rho) = (4608''.1448 + 2''.79355t + 0''.000118t^2)\,\theta + (1''.41535 + 0''.000071t)\,\theta^2 + 0''.036393\,\theta^3$$

$$(j) = (2005''.5487 - 0''.85290t - 0''.000363t^2)\,\theta + (-0''.41839 - 0''.000384t)\,\theta^2 - 0''.041877\,\theta^3$$

Table 2. Precessional formulas: $\theta = t' - t$, where t' and t are reckoned from 2000 in units of Julian centuries.

$$\sin(k)\sin(\phi) = (4''.1992 - 0''.75277t + 0''.000335t^2)\,\theta + (0''.19471 + 0''.000802t)\,\theta^2 - 0''.000190\,\theta^3$$

$$\sin(k)\cos(\phi) = (-46''.8156 - 0''.00082t + 0''.005489t^2)\,\theta + (0''.05080 - 0''.003668t)\,\theta^2 + 0''.000350\,\theta^3$$

$$(\phi) = 174^\circ 52' 27''.66 + 3289''.80023t + 0''.576264t^2 + (-870''.63478 - 0''.554988t)\,\theta + 0''.024578\,\theta^2$$

$$(k) = (47''.0036 - 0''.06639t + 0''.000569t^2)\,\theta + (-0''.03320 + 0''.000569t)\,\theta^2 + 0''.000050\,\theta^3$$

$$(\varepsilon) = 23^\circ 26' 21''.47 - 46''.81559t - 0''.000412t^2 + 0''.00183t^3 + (-46''.8156 - 0''.00082t + 0''.005489t^2)\,\theta + (-0''.00041 + 0''.005490t)\,\theta^2 + 0''.001830\,\theta^3$$

$$(\varepsilon_1) = 23^\circ 26' 21''.47 - 46''.81559t - 0''.000412t^2 + 0''.00183t^3 + (0''.05130 - 0''.009203t)\,\theta^2 - 0''.007734\,\theta^3$$

$$(\psi) = (5038''.7802 + 0''.49254t - 0''.000039t^2)\,\theta + (-1''.05331 - 0''.001513t)\,\theta^2 - 0''.001530\,\theta^3$$

$$(\chi) = (10''.5567 - 1''.88692t - 0''.000144t^2)\,\theta + (-2''.38191 - 0''.001554t)\,\theta^2 - 0''.001661\,\theta^3$$

$$(\Psi) = (5029''.0946 + 2''.22280t + 0''.000264t^2)\,\theta + (1''.13157 + 0''.000212t)\,\theta^2 + 0''.000102\,\theta^3$$

$$(\mu) + (\rho) = 180^\circ + (0''.79323 + 0''.000648t)\,\theta^2 + 0''.000328\,\theta^3$$

$$(\mu) - (\rho) = (4612''.4339 + 2''.79402t + 0''.000118t^2)\,\theta + (1''.41551 + 0''.000071t)\,\theta^2 + 0''.036395\,\theta^3$$

$$(j) = (2004''.3113 - 0''.85403t - 0''.000363t^2)\,\theta + (-0''.41898 - 0''.000384t)\,\theta^2 - 0''.041880\,\theta^3$$

Table 3. Precessional formulas based on Newcomb's values: $\theta = t' - t$, where t' and t are reckoned from 1850 in units of tropical centuries.

$$\sin(k)\sin(\phi) = (5''.3410 - 0''.75393t + 0''.000336t^2)\,\theta + (0''.19350 + 0''.000802t)\,\theta^2 - 0''.000190\,\theta^3$$

$$\sin(k)\cos(\phi) = (-46''.8370 - 0''.01750t + 0''.005489t^2)\,\theta + (0''.05630 - 0''.003665t)\,\theta^2 + 0''.000350\,\theta^3$$

$$(\phi) = 173°29'40''.01 + 3286''.28345t + 0''.560350t^2 + (-869''.12350 - 0''.550988t)\,\theta + 0''.025375\,\theta^2$$

$$(k) = (47''.1405 - 0''.06803t + 0''.000567t^2)\,\theta + (-0''.03401 + 0''.000567t)\,\theta^2 + 0''.000050\,\theta^3$$

$$(\varepsilon) = 23°27'31''.68 - 46''.83700t - 0''.008752t^2 + 0''.00183t^3 + (-46''.8370 - 0''.01750t + 0''.005489t^2)\,\theta + (-0''.00875 + 0''.005489t)\,\theta^2 + 0''.001830\,\theta^3$$

$$(\varepsilon_1) = 23°27'31''.68 - 46''.83700t - 0''.008752t^2 + 0''.00183t^3 + (0''.06521 - 0''.009199t)\,\theta^2 - 0''.007730\,\theta^3$$

$$(\psi) = (5036''.8381 + 0''.49299t - 0''.000037t^2)\,\theta + (-1''.07129 - 0''.001478t)\,\theta^2 - 0''.001533\,\theta^3$$

$$(\chi) = (13''.4166 - 1''.88685t - 0''.000142t^2)\,\theta + (-2''.37995 - 0''.001571t)\,\theta^2 - 0''.001657\,\theta^3$$

$$(\Psi) = (5024''.5305 + 2''.22267t + 0''.000264t^2)\,\theta + (1''.11134 + 0''.000263t)\,\theta^2 + 0''.000095\,\theta^3$$

$$(\mu) + (\rho) = 180° + (0''.79236 + 0''.000656t)\,\theta^2 + 0''.000328\,\theta^3$$

$$(\mu) - (\rho) = (4607''.1096 + 2''.79440t + 0''.000118t^2)\,\theta + (1''.39720 + 0''.000118t)\,\theta^2 + 0''.036320\,\theta^3$$

$$(j) = (2005''.1125 - 0''.85294t - 0''.000365t^2)\,\theta + (-0''.42647 - 0''.000365t)\,\theta^2 - 0''.041802\,\theta^3$$

Table 4. Precessional formulas based on Newcomb's values: $\theta = t' - t$, where t' and t are reckoned from 2000 in units of Julian centuries.

$$\sin(k)\sin(\phi) = (4''.2109 - 0''.75296t + 0''.000336t^2)\,\theta + (0''.19471 + 0''.000802t)\,\theta^2 - 0''.000190\,\theta^3$$

$$\sin(k)\cos(\phi) = (-46''.8519 - 0''.00104t + 0''.005490t^2)\,\theta + (0''.05080 - 0''.003665t)\,\theta^2 + 0''.000350\,\theta^3$$

$$(\phi) = 174^\circ 51' 50''.70 + 3288''.03472t + 0''.560374t^2 + (-869''.96857 - 0''.551011t)\,\theta + 0''.025376\,\theta^2$$

$$(k) = (47''.0408 - 0''.06633t + 0''.000567t^2)\,\theta + (-0''.03316 + 0''.000567t)\,\theta^2 + 0''.000050\,\theta^3$$

$$(\varepsilon) = 23^\circ 26' 21''.41 - 46''.85191t - 0''.000519t^2 + 0''.00183t^3 + (-46''.8519 - 0''.00104t + 0''.005490t^2)\,\theta + (-0''.00052 + 0''.005490t)\,\theta^2 + 0''.001830\,\theta^3$$

$$(\varepsilon_1) = 23^\circ 26' 21''.41 - 46''.85191t - 0''.000519t^2 + 0''.00183t^3 + (0''.05142 - 0''.009199t)\,\theta^2 - 0''.007731\,\theta^3$$

$$(\psi) = (5037''.6851 + 0''.49290t - 0''.000037t^2)\,\theta + (-1''.07355 - 0''.001478t)\,\theta^2 - 0''.001533\,\theta^3$$

$$(\chi) = (10''.5862 - 1''.88736t - 0''.000142t^2)\,\theta + (-2''.38240 - 0''.001571t)\,\theta^2 - 0''.001657\,\theta^3$$

$$(\Psi) = (5027''.9724 + 2''.22356t + 0''.000264t^2)\,\theta + (1''.11178 + 0''.000263t)\,\theta^2 + 0''.000095\,\theta^3$$

$$(\mu) + (\rho) = 180^\circ + (0''.79338 + 0''.000656t)\,\theta^2 + 0''.000328\,\theta^3$$

$$(\mu) - (\rho) = (4611''.3999 + 2''.79488t + 0''.000118t^2)\,\theta + (1''.39744 + 0''.000118t)\,\theta^2 + 0''.036322\,\theta^3$$

$$(j) = (2003''.8751 - 0''.85407t - 0''.000365t^2)\,\theta + (-0''.42704 - 0''.000365t)\,\theta^2 - 0''.041805\,\theta^3$$

4. REFERENCES

ANDOYER, H.

1911. Les formules de la précession d'après S. Newcomb. Bull. Astron. Paris, vol. 28, pp. 67-76.

DE SITTER, W.

1938. On the system of astronomical constants. Bull. Astron. Netherlands, vol. 8, pp. 213-216.

ECKERT, W. J., WALKER, M. J., and ECKERT, D.

1966. Transformations of the lunar coordinates and orbital parameters. Astron. Journ., vol. 71, pp. 314-332.

FRICKE, W.

1971. Determinations of precession. Celest. Mech., vol. 4, pp. 150-162.

KINOSHITA, H.

1975. Theory of the rotation of the rigid earth. Smithsonian Astrophys. Obs. Spec. Rep., in preparation.

KLEPCZYNSKI, W. J., SEIDELMAN, P. K., and DUNCOMBE, R. L.

1971. The masses of the principal planets. Celest. Mech., vol. 4, pp. 253-272.

LAUBSCHER, R. E.

1972. A determination of the motion of the ecliptic. Astron. Astrophys., vol. 20, pp. 407-414.

NEWCOMB, S.

1906. A Compendium of Spherical Astronomy. Macmillan Publ. Co., New York, 234 pp.

PLUMMER, H. C.

1916. Note on general precession. Mon. Not. Roy. Astron. Soc., vol. 76, pp. 627-630.

WOOLARD, E. W.

1953. Theory of the rotation of the earth around its center of mass. Astron. Papers, vol. 15, part 1, 128 pp.

WOOLARD, E. W., and CLEMENCE, G. M.

1966. Spherical Astronomy. Academic Press, New York, 237 pp.

BIOGRAPHICAL NOTE

HIROSHI KINOSHITA received his M.S. and D.S. degress in astronomy from Tokyo University in 1967 and 1972, respectively. He joined the staff of Smithsonian Astrophysical Observatory in 1974.

His general field of investigation is celestial mechanics. He is interested in the establishment of the reference system and the rotational motion of the earth.

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

First issued to ensure the immediate dissemination of data for satellite tracking, the reports have continued to provide a rapid distribution of catalogs of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals. The Reports are also used extensively for the rapid publication of preliminary or special results in other fields of astrophysics.

The Reports are regularly distributed to all institutions participating in the U. S. space research program and to individual scientists who request them from the Publications Division, Distribution Section, Smithsonian Astrophysical Observatory, Cambridge, Massachusetts 02138.