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DYNAMIC RESPONSE AND STABILITY OF A GAS-LUBRICATED RAYLEIGH-STEP PAD

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16. Abstract
The quasi-static, pressure characteristics of a gas-lubricated thrust bearing with shrouded, Rayleigh-step pads are determined for a time-varying film thickness. The axial response of the thrust bearing to an axial forcing function or an axial rotor disturbance is investigated by treating the gas film as a spring having nonlinear restoring and damping forces. These forces are related to the film thickness by a power relation. The nonlinear equation of motion in the axial mode is solved by the Ritz-Galerkin method as well as the direct, numerical integration. Results of the nonlinear response by both methods are compared with the response based on the linearized equation. Further, the gas-film instability of an infinitely wide Rayleigh step thrust pad is determined by solving the transient Reynolds equation coupled with the equation of the motion of the pad. Results show that the Rayleigh-step geometry is very stable for bearing number Λ up to 50. The stability threshold is shown to exist only for ultrahigh values of $\Lambda \geq 100$, where the stability can be achieved by making the mass heavier than the critical mass.

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NOMENCLATURE

A	array of coefficients for pressure Eqs. (4.16) and (4.23).
A_o, A	amplitudes of zero and first order Fourier series representation of response.
$B = \frac{C_1}{m\delta(\omega^*)^2}$	nondimensional gas film force constant of stiffness.
B	array of constants for pressure Eqs. (4.16).
B	width of Rayleigh step pad.
B_1, B_2	width of Rayleigh step pad with gas film.
$C = \frac{C_2\bar{\omega}}{m\,\omega^*}$	nondimensional constant of gas film damping coefficient.
C	array of coefficients for pressure Eq. (4.16).
D	array of constants for pressure Eq. (4.23).
D_1, D_2	conventional constants in relations (3.30) and (3.40).
$E_n = \varepsilon n/\delta$	nondimensional amplitude of Fourier series expansion of disturbance.
F, G	algebraic equations of A and A_o.
$H = h/\delta$	nondimensional gas film thickness.
$\dot{H} = \frac{dH}{dT}$	nondimensional velocity of gas film thickness.
$H_{s,i} = (H_i + H_{i-1})^3$	array of coefficients in (4.10).
I_1, I_4, I_5	nonlinear integrals in (3.26)(3.27) and (3.28) respectively.
$I = \int_0^1 P_R dX$	in phase gas film reaction of complex pressure profile.
N	number of grids in Chapter II.

$P = P/P_a$	nondimensional pressure of gas film.
P_o, P_c	zero and first order of P in (4.12).
P_I, P_R	imaginary and real part of P_c.
$Q = PH$	pressure field parameter in (2.27).
$Q = q/m\delta\omega^{*2}$	nondimensional force excitation.
R	universal gas constant.
$R = r/r_o$	nondimensional radius.
R	residue in (3.13).
$R = \int_o^1 P_I dX$	out of phase gas film reaction of complex pressure profile.
S_1, S_2	parameters in (3.29).
$T, \tau = \omega t$	nondimensional time.
T	torque in Section 2.3.
$U = \omega r$	velocity of the driving surface.
$W_o = \frac{C_1}{H_o^{2.5}}$	load acted on the ring to get equilibrium at $H = H_o$.
$X = H_o - H$	coordinate and response of gas film in (3.2).
$X = x/B$	coordinate for infinite-width pad.
$X = \left[P_R, P_I\right]$	vector of unknown $P_{R,i}$ and $P_{I,i}$ in (4.23).
$a_o, b_o, c_o, d_o, e_o, f_o$	arrays of coefficients in pressure Eqs. (4.17).
$a_s, b_s, c_s, a_n, b_n, c_n$, d,e,f	arrays of coefficients in pressure Eqs. (4.15).
c_1, c_2	constants in gas film force approximation (2.16).
$\epsilon = H - H_o$	disturbance of gas film thickness in (4.11).
h	gas film thickness.
j	imaginary number in (4.21).

$k = \frac{5}{2} \frac{c_1}{H_o^{3.5}}$		stiffness of gas film force.
m		mass of the ring in (3.1); mass of the pad per unit length in (4.26).
m_c		critical mass for stability in (4.29).
$n = 2\pi\omega$		number of revolution of the motor per second.
n_1, n_2		powers of H for gas film force in (2.16).
p		pressure of gas film.
p_a		ambient pressure of gas film
q_i		mass flow rate.
q		magnitude of excitational force.
r		radius
t		time
x		coordinate in the length of infinitely wide step pad.
	Dimensions	
ρ	FT^2L^{-4}	density of gas film
μ	FTL^{-2}	coefficient of viscosity
δ	L	step deepth.
$\Lambda = \frac{6\mu L/B}{P_a \delta^2}$	0	bearing number
$\Delta = H_{i,j}^{+} - H_{i,j}^{-}$	0	stepped discontinuity at (i,j) grid point.
θ	θ	angular position
θ_G	θ	angle of ground area of step pad.
θ_L	θ	angle of landing area of step pad.

ω	θT^{-1}	rotational velocity.
ω_f	θT^{-1}	frequency of excitational force.
$\omega^* = \sqrt{\frac{k}{m}}$	θT^{-1}	natural frequency based on the linearization of Eq. (3.1).
$\bar{\omega} = \frac{\omega_f}{\omega^*}$	0	Normalized facing frequency.
ϵ_n	L	magnitude n-th order excitation.
α		phase difference between the response and the excitation.
Δ	0	determinent defined in (3.35).
ν	θT^{-1}	threshold frequency.
$\sigma = \frac{12\mu\nu B^2}{\delta^2 P_a}$	0	squeeze number in (4.9).

I. INTRODUCTION

1.1 Introduction

Among many geometrical profiles which generate hydrodynamic pressure in fluid-film bearings, the step geometry is one of the most proficient methods to achieve this purpose. This discovery was first made by Lord Rayleigh in 1918 (Ref. 1), and subsequently there have been many contributions (Refs. 2 to 8) on the prediction of the performance of the Rayleigh-step thrust as well as journal bearings.

Recently, there has been another significant application of the Rayleigh-step bearing in the field of dynamic sealing in advanced air-breathing propulsion systems (Ref. 9). In this application, a series of Rayleigh-step pads is employed on the high-pressure side of a face seal in order to maintain a small steady gap (in the order of 0.001") between a one-tooth laybrinth and the high-speed rotor surface (Fig. 1). The flat step is shrouded in order to minimize the side leakage. These pads function strictly as hydrodynamic thrust bearings operate in the high-ambient pressure, and provide the necessary stiffness to maintain a steady gap. The satisfactory operation of this type of seals depends critically on the dynamic performance of these thrust pads in the presence of an oscillatory force or a disturbance due to rotor run-out or rotor unevenness.

So far, the investigations on shrouded, Rayleigh-step bearings have been restricted to the prediction of the static performance (Ref. 10) only. The dynamic characteristics of this type of thrust-pads have not been given much attention. It is the main objective of this report to study the influence of the nonlinear, gas-film, restoring and damping force upon the response of the pad to a given forcing function or disturbance.

Specifically, the work reported herein fulfills the following objectives:

1. To develop a gas-film analysis of purely hydrodynamic, Rayleigh-step pad to calculate the quasistatic stiffness and damping, which depend not only on the operating conditions but also on the vibration of the system.
2. To extend the analysis to the nonlinear axial response of the stationary ring due to any external force excitation or any disturbances induced by the rotor misalignment or surface waviness.
3. To develop a stability analysis of the infinitely wide, single-step pad to explore whether there exists any thresholds of stability for the system.

1.2 Historical Survey

In 1918, after Reynolds' theory of thick film lubrication became generally accepted, Lord Rayleigh (Ref. 1) first applied the theory and discovered that an optimum profile for the load capacity of a slider bearing is a flat step. The method of the calculus of variations was used to optimize the shape of slider bearing to an infinitely long, incompressible film. From then on, any hydrodynamic bearings composing of two sections of parallel surface film are called Rayleigh-step bearings.

The optimized geometry of a Rayleigh-step slider and its corresponding optimized, non-dimensional, load capacity for an incompressible film can be found in most textbooks on lubrication (Ref. 11). For compressible gas film, the optimized, one-dimensional, Rayleigh-step bearing was analyzed by Wylie and Maday in 1970 (Ref. 2). The load capacity of an optimized step bearing was found to be slightly lower at low bearing number but much higher at higher bearing number than the load capacity of an incompressible, optimized, slider bearing.

The problem of a two-dimensional, Rayleigh-step, thrust bearing has not received much attention until 1954, C. F. Kettleborough (Ref. 3) solved the pressure profile and calculated the load capacity for the step thrust bearing by Relaxation methods. In 1955 (Ref.17)he applied the analogy method contributed by A. Kingsbury (Ref. 12) to investigate the pressure profile for an oil-lubricated step bearing by an electrolytic tank.

In 1959, using air as the lubricant, K. C. Kochi (Ref. 6) showed the characteristics of an infinitely-wide, Rayleigh-step, thrust pad by the use of semi-graphical method. He demonstrated that an analytical solution to express the pressure profile explicitly is extremely difficult, because the Reynolds equation for a compressible film is nonlinear.

In 1961, J. S. Ausmann (Ref. 7) made certain approximations to linearize the Reynolds equation for a compressible film, and obtained a series solution of the pressure and load for a self-acting, stepped, sector, thrust bearing by the aid of Neumann polynomial. He obtained the numerical solutions for the optimized number of sectors and step depth for the maximum load carrying capacity.

Recently, Cheng, Chow and Wilcock (Ref. 8) presented some results for shrouded, Rayleigh-step pad used as a flexible face seal. The pressure generation and static stability of this type of surface profiles using as a flexible seal ring was discussed, and the effectiveness of hydrodynamic action was confined to the static stiffness characteristics of gas film. The influence of the nonlinear gas-film forces and the question of gas-film stability of the hydrodynamic, Rayleigh-step pad as not been investigated. Therefore, to ensure a safe operation of ayleigh-step, thrust pads, it is necessary to conduct a full-scale nonlinear tudy, which is the major objective of this work.

II. GAS FILM FORCES

2.1 Statement of the Problem

The major concern in this section is the determination of the time-dependent, pressure distribution within the gas film between two annular surfaces containing a series of Rayleigh-step pockets as shown in Fig. 1. This problem is formulated within the framework of the conventional theory of lubrication for a compressible lubricant. The major assumptions commonly used for gas-lubrication theories are:

1. The pressure across the film is constant
2. The flow is laminar
3. Inertia forces are neglected
4. The film is isothermal
5. The flow is Newtonian.

Under these assumptions, the governing equation for a transient, continuous, pressure field becomes the well-known, transient, Reynolds equation for a compressible fluid. Various analytical and numerical methods for the solution to this equation have been outlined by Castelli and Peviric (Ref. 13). For the present problem, the abrupt geometry introduced by the Rayleigh pocket makes it rather difficult to solve the pressure equation by any analytical methods. For this reason, a numerical method is employed in solving the discretized pressure field based on the solution of a system, non-linear algebraic equations by the Newton-Raphson procedure. The numerical integration of the discretized pressure field gives rise to the time-dependent gas-film forces which are required for investigating the nonlinear response of this type gas-film to a prescribed forcing function or disturbance function.

2.2 Pressure Equation

The discretized pressure equation is derived by considering a flow balance within an element of the gas film as shown in Fig. 2. The mass flow rate into the left boundary, AD, and the bottom boundary, AB, are designated as q_1 and q_2. The mass flow rates out of the boundaries, DC and BC are, likewise, designated as q_3 and q_4. In addition, there are mass stored in the volume which is designated as q_5.

Based on the assumption in 2.1, the velocity is parabolic across the film. Integrating the velocity across the film, one may express the mass flow rate in terms of the boundary velocities and the pressure gradients. These can be written as

$$q_1 = \left[\rho \frac{\omega r h}{2} - \frac{\rho h^3}{12\mu r} \frac{\partial p}{\partial \theta}\right]_{j-\frac{1}{2},i} \left(\frac{r_{i+1} - r_{i-1}}{2}\right) \tag{2.1}$$

$$q_2 = \left[\rho \frac{U_r h}{2} - \frac{\rho h^3}{12\mu} \frac{\partial p}{\partial r}\right]_{i-\frac{1}{2},j} \left(\frac{\theta_{j+1} - \theta_{j-1}}{2}\right) r_{i-\frac{1}{2}} \tag{2.2}$$

$$q_3 = \left[\rho \frac{\omega r h}{2} - \frac{\rho h^3}{12\mu r} \frac{\partial p}{\partial \theta}\right]_{j+\frac{1}{2},i} \left(\frac{r_{i+1} - r_{i-1}}{2}\right) \tag{2.3}$$

$$q_4 = \left[\rho \frac{U_r h}{2} - \frac{\rho h^3}{12\mu} \frac{\partial p}{\partial r}\right]_{i+\frac{1}{2},j} \left(\frac{\theta_{j+1} - \theta_{j-1}}{2}\right) r_{i+\frac{1}{2}} \tag{2.4}$$

$$q_5 = \frac{\partial(\rho h)}{\partial t} \left(\frac{r_{i+1} - r_{i-1}}{2}\right) r_i \left(\frac{\theta_{i+1} - \theta_{i-1}}{2}\right) \tag{2.5}$$

The flow balance requires

$$q_1 + q_2 = q_3 + q_4 + q_5 \tag{2.6}$$

Introducing the following nondimensional variables:

$$P = \frac{p}{p_a} \ , \quad H = \frac{h}{\delta} \ , \quad R = \frac{r}{r_o} \ , \quad \tau = \omega t \ , \ Q = PH \ , \text{ and } \Lambda = \frac{6\mu\omega r_o^2}{p_a \delta^2}$$

and using the equation of state $\rho = \frac{p}{RT}$, Equation (2.6) becomes

$$\left[\left(\Lambda RH \frac{Q}{\sqrt{Q}} - \frac{H^3}{2R} \frac{\partial Q}{\partial \theta}\right)_{i,j-\frac{1}{2}} - \left(\Lambda RH \frac{Q}{\sqrt{Q}} - \frac{H^3}{2R} \cdot \frac{\partial Q}{\partial \theta}\right)_{i,j+\frac{1}{2}} \right] \left(\frac{R_{i+1} - R_{i-1}}{2}\right)$$

$$+ \left[\left(- \frac{H^3}{2} \frac{\partial Q}{\partial R}\right)_{i-\frac{1}{2},j} R_{i-\frac{1}{2}} + \left(\frac{H^3}{2} \frac{\partial Q}{\partial R}\right)_{i+\frac{1}{2},j} R_{i+\frac{1}{2}} \right] \left(\frac{\theta_{j+1} - \theta_{j-1}}{2}\right)$$

$$= \left[2\Lambda \left(\frac{\partial PH}{\partial \tau}\right)_{i,j} \left(\frac{R_{i+1} - R_{i-1}}{2}\right) R_i \left(\frac{\theta_{j+1} - \theta_{j-1}}{2}\right) \right]$$

Further simplification leads to,

$$\frac{1}{(\theta_{j+1} - \theta_{j-1})} \left[\left(\Lambda RH \frac{Q}{\sqrt{Q}} - \frac{H^3}{2R} \frac{\partial Q}{\partial \theta}\right)_{i,j-\frac{1}{2}} - \left(\Lambda RH \frac{Q}{\sqrt{Q}} - \frac{H^3}{2R} \frac{\partial Q}{\partial \theta}\right)_{i,j+\frac{1}{2}} \right]$$

$$+ \frac{1}{(R_{i+1} - R_{i-1})} \left[-\left(\frac{H^3}{2} \frac{\partial Q}{\partial R}\right)_{i-\frac{1}{2},j} R_{i-\frac{1}{2}} + \left(\frac{H^3}{2} \frac{\partial Q}{\partial R}\right)_{i+\frac{1}{2},j} R_{i+\frac{1}{2}} \right]$$

$$= 2\Lambda \left(\frac{\partial PH}{\partial \tau}\right)_{i,j} R_i \tag{2.7}$$

Introducing the following finite difference approximations:

$$\left(\frac{\partial Q}{\partial \theta}\right)_{i,j-\frac{1}{2}} = (Q_j - Q_{j-1})_i / (\theta_j - \theta_{j-1})_i$$

$$\left(\frac{\partial Q}{\partial R}\right)_{i-\frac{1}{2},j} = (Q_i - Q_{i-1})_j / (R_i - R_{i-1})_j \tag{2.8}$$

$$Q_{i,j-\frac{1}{2}} = \frac{Q_{i,j} + Q_{i,j-1}}{2}$$

one obtains a system of first order differential equations in time for the discretized pressures.

To obtain the exact, time-dependent, gas-film forces, it is necessary to solve this equation simultaneously with the equations of motion of the supported mass by an explicit or implicit, numerical procedure for the initial-valued problems. This procedure necessitates the calculation of the pressure field at each time interval during the transient, and it is extremely uneconomical and cumbersome.

In the case of a high, ambient pressure, the effect produced by the term containing $\frac{\partial P}{\partial \tau}$ becomes insignificant comparing to that by $\frac{\partial H}{\partial \tau}$, one may neglect the $\frac{\partial P}{\partial \tau}$ effect, and thus decoupled the gas-film force calculation from the dynamics of the supporting mass. Ignoring the contribution by $\frac{\partial P}{\partial \tau}$, the pressure distribution can be solved independently as a function of H and $\frac{\partial H}{\partial \tau}$ or $\dot{H}$. The assumption of a negligible $\frac{\partial P}{\partial \tau}$ effect is made in the subsequent analysis. Expanding the terms in Equation (2.7) by using relations (2.8) one obtains

$$a_3Q_{i,j-1} + a_1\sqrt{Q_{i,j-1}} + a_5Q_{i-1,j} - (a_3 + a_4 + a_5 + a_6)Q_{i,j} + a_6Q_{i+1,j}$$

$$+ \left[\frac{a_1}{H_{i,j-1}}\Delta - a_6\right]\sqrt{Q_{i,j}} + a_4Q_{i,j+1} - a_2\sqrt{Q_{i,j+1}} = 0 \tag{2.9}$$

where $\Delta = H_{i,j}^{+} - H_{i,j}^{-} = \begin{cases} 1 & \text{at the end of the pocket} \\ 0 & \text{otherwise} \end{cases}$

and

$$
\begin{aligned}
(a_1)_{i,j} &= \frac{\Lambda R_i}{2(\theta_{j+1} - \theta_{j-1})} (H_{i,j-1}) \\
(a_2)_{i,j} &= \frac{\Lambda R_i}{2(\theta_{j+1} - \theta_{j-1})} (H_{i,j+1}) \\
(a_3)_{i,j} &= \frac{1}{2R_i(\theta_{j+1} - \theta_{j-1})(\theta_j - \theta_{j-1})} (H^3)_{i,j-\frac{1}{2}} \\
(a_4)_{i,j} &= \frac{1}{2R_i(\theta_{j+1} - \theta_{j-1})(\theta_{j+1} - \theta_j)} (H^3)_{i,j+\frac{1}{2}} \\
(a_5)_{i,j} &= \frac{(R_i + R_{i-1})}{4(R_{i+1} - R_{i-1})(R_i - R_{i-1})} (H^3)_{i-\frac{1}{2},j} \\
(a_6)_{i,j} &= \frac{(R_{i+1} + R_i)}{4(R_{i+1} - R_{i-1})(R_{i+1} - R_i)} (H^3)_{i+\frac{1}{2},j} \\
(a_8)_{i,j} &= \Lambda R_i \dot{H}_i = \Lambda R_i \frac{\partial H}{\partial \tau}
\end{aligned}
\tag{2.10}
$$

2.3 Method of Solution

The set of nonlinear, algebraic equations to be solved for the pressure field, $Q_{i,j}$, is

$$
\begin{aligned}
\Phi_{i,j} = {} & a_3 Q_{i,j-1} + a_1 \sqrt{Q_{i,j-1}} + \\
& + a_5 Q_{i-1,j} - (a_3 + a_4 + a_5 + a_6) Q_{i,j} + \left[\frac{a_1}{H_{i,j-1}} \Delta - a_8\right] \sqrt{Q_{i,j}} + \\
& + a_6 Q_{i+1,j} + a_4 Q_{i,j+1} - a_2 \sqrt{Q_{i,j+1}} = 0
\end{aligned}
\tag{2.9}
$$

where $\Phi_{i,j}$ is the implicit function for the node point (i,j). Newton-Raphson method has been employed here to solve these equations.

Based on this method, $Q^n_{i,j}$ can be improved by calculating the difference $\Delta Q^n_{i,j}$ which is found from first order approximation of Taylor's expansion for $\Phi_{i,j}$. Using Taylor's expansion, eq. (2.9) becomes

$$\Phi^{n+1}_{i,j} = \Phi^n_{i,j} + \left(\frac{\partial\Phi_{i,j}}{\partial Q_{i,j-1}}\right) \Delta Q^n_{i,j-1} + \frac{\partial\Phi_{i,j}}{\partial Q_{i,j}} \Delta Q^n_{i,j} + \frac{\partial\Phi_{i,j}}{\partial Q_{i,j+1}} \Delta Q^n_{i,j+1}$$

$$+ \left(\frac{\partial\Phi_{i,j}}{\partial Q_{i-1,j}}\right)\Delta Q^n_{i-1,j} + \left(\frac{\partial\Phi_{i,j}}{\partial Q_{i+1,j}}\right)\Delta Q^n_{i+1,j} + O(\epsilon^2) = 0 \quad \cdots \quad (2.11)$$

From Equations (2. 9) and (2.11), one obtains

$$- \Phi^n_{i,j} = \left(a_3 + \frac{a_1}{\sqrt{Q^n_{i,j-1}}}\right)\Delta Q^n_{i,j-1} + a_5 \Delta Q^n_{i-1,j}$$

$$+ \left\{ -(a_3 + a_4 + a_5 + a_6) + \left[\frac{a_1}{H_{i,j-1}}\Delta - a_8 \right]\frac{1}{\sqrt{Q^n_{i,j}}} \right\} \Delta Q^n_{i,j}$$

$$+ a_6 \Delta Q^n_{i+1,j} + \left(a_4 - \frac{a_2}{2\sqrt{Q^n_{i,j+1}}}\right) \Delta Q^n_{i,j+1} \quad \cdots \quad (2.12)$$

Since $Q^n_{i,j}$ are known from the previous iteration, the set of linear simultaneous equations (2,12) are solved for the difference $\Delta Q_{i,j}$, by Gaussian Elimination Method.

The new iterated Q becomes,

$$Q^{n+1}_{i,j} = Q^n_{i,j} + \Delta Q_{i,j}$$

and the procedure is repeated until all $\Delta Q_{i,j}$ becomes less than a prescribed convergence error.

A computer-program RSS-FILM has been written to calculate $Q_{i,j}$, for a given nondimensionaless, gas-film thickness H and a nondimensionaless, velocity $\dot{H}$. The nondimensional, pressure distribution can be obtained by dividing Q by the nondimensional gas film thickness H. The horsepower required to overcome the friction resistance is calculates as

$$T = \int r\ \tau_s dA = \frac{\mu\omega r_o^3}{h} \int_{R_1}^{R_2} R^3 dR \int_o^{\theta_G+\theta_L} \frac{1}{H}\, d\theta \tag{2.13}$$

and

$$HP = \frac{n \times T \times N \times 60}{63000} \tag{2.14}$$

Also, the load per pad is obtained by the following integration

$$W = p_a \int_A (P - 1) r\ d\theta dr$$

$$= p_a r_o^2 \int_{R_i}^{R_o} RdR \int_o^{\theta_G+\theta_L} (P - 1) d\theta \tag{2.15}$$

Figures 3, to 7 show the effects of change in H and $\dot{H}$ on the pressure distributions in a shrouded, Rayleigh-step gas pad.

2.4 Approximation for Gas-Film Forces

The gas-film forces are shown to be dependent upon H, $\frac{\partial H}{\partial \tau}$, and the following operating and geometrical parameters

$$\Lambda = \frac{6\mu\omega r_o^2}{p_a \delta^2} \quad = \text{Bearing Parameter}$$

$$\frac{\theta_L}{\theta_G + \theta_L} \quad = \text{Step Length Parameter}$$

$$\frac{2B}{R_o - R_i} \quad = \text{Shroud Width Parameter}$$

$$\frac{2\pi R_o}{N(R_o - R_i)} \quad = \text{Length to Width Ratio}$$

Table 1 lists the gas-film forces for an annular bearing surface containing 20 Shrouded-Rayleigh-Step pads, with geometrical dimensions shown in Fig. 1 . By plotting data on a log-log scale in Fig.8 & Fig.9 it is found that they can be fitted with the following function for a wide range of dimensionless thickness, H.

$$F = \frac{C_1}{H^{n_1}} - \frac{C_2}{H^{n_2}} \delta\omega \frac{dH}{dT} \tag{2.16}$$

Tables 2 list the approximate,gas-film force based on Eq. (2.16) and the actual values interpolated from Table 1. The errors introduced by using the fitted Equation (2.16) are also listed in these tables.

The values for C_1, C_2, n_1, and n_2 for the gas-film forces listed in

Table 1 are found to be

$C_1 = 2.32 \quad lb$

$C_2 = 0.76 \quad ^{lb}/(in/sec)$

$n_1 = 2.5$

$n_2 = 2.5$

The above constants, of course, only apply to the operating and geometrical parameters shown in Fig.1 . For other parameters, a different set of C_1, C_2, n_1 and n_2 will be required to approximate the gas-film forces.

Letting H_o be the equilibrium nondimensional gas film thickness, and defining x as the nondimensionalized displacement from H_o, positive for a decreasing of H, the gas film force can be alternatively expressed as,

$$F = \frac{C_1}{(H_o - X)^{n_1}} + \frac{C_2}{(H_o - X)^{n_2}} \omega\delta \frac{dX}{dT} \tag{2.17}$$

where $H = H_o - X$

and $\frac{dH}{dT} = - \frac{dX}{dT}$

III. NONLINEAR AXIAL RESPONSE

3.1 Mathematical Modeling

The main problem in this chapter is to determine the dynamic response of the stationary ring to any external force excitation or to any disturbances produced by the rotor misalignment and by the rotor surface waviness. Knowing the detailed motion of the stationary ring with respect to the rotor motion, one may calculate time-dependent gap distribution between the surfaces. In general, the dynamic response of the stationary ring is measured by the axial translation and by the rotations about two mutually perpendicular diameters of the ring. However, for a stationary ring with a narrow width and a large diameter, the response in the axial mode is very weakly coupled with the oscillation in the angular mode. Furthermore, the equation governing the angular oscillation is very nearly the same as that governing the axial motion. Thus, it is only necessary to concentrate the analysis on the non-linear characteristics of the motion in the axial mode. This reduces the problem from a complicated, three-degrees-of-freedom dynamical system to a single-degree-of-freedom problem for which a more thorough analysis can be afforded. The weak coupling between the axial and angular oscillation is demonstrated analytically in Reference 14.

Figure 10 shows the mathematical modelling of the non-linear vibration of the stationary ring in the axial direction. It consists of a stationary ring of mass m subjected to a steady load W_O. The back face of the ring is flexibly mounted to the frame through a soft spring of stiffness K_s and the front face is supported on a very stiff, non-linear gas-film whose restoring force is represented by the power relations formulated in Chapter 2, Eq. (2.17). The major problem here is to investigate the motion of the stationary ring for one of the two following conditions:

a, a force excitation, $q \cos \omega_f t$, acting on the stationary ring.

b, a prescribed rotor disturbance characterized by a Fourier series $\sum_{n=1}^{N} \epsilon_n \cos n\omega_f t$.

3.2 Equation of Motion

3.2.1 Force-Excited Motion

The equation of motion due to a force excitation $q \cos \omega t$ is considered first. Recalling from Eq. (2.16), the force balance on the stationary ring gives the following equation,

$$m \frac{d^2x}{dt^2} + \frac{C_1}{H^{2.5}} - \frac{C_2}{H^{2.5}} \frac{dh}{dt} - W_o + k_s x = q \cos \omega_f t \tag{3.1}$$

where $k_s x$ is small comparing with W_o and other terms.

Introducing the following nondimensional variables

$$X = {}^{x}/\delta$$

$$T = \omega_f t$$

$$\bar{\omega} = \frac{\omega_f}{\omega^*}$$

$$B = \frac{C_1}{m\delta(\omega^*)^2}$$

$$C = \frac{C_2 \bar{\omega}}{m\omega^*}$$

$$Q = \frac{q}{m\delta(\omega^*)^2}$$

where ω^* is the natural frequency of the system based on the linearized equation of Eq. (3.1). Equation (3.1) becomes

$$\bar{\omega}^2 \ddot{X} + C\dot{X}\frac{1}{(H_o - X)^{2.5}} + B\left[\frac{1}{(H_o - X)^{2.5}} - \frac{1}{H_o^{2.5}}\right]$$

$$= Q \cos T \tag{3.2}$$

where

$$H = H_o - X$$

$$W_o = \frac{C_1}{H_o^{2.5}}$$

$$\cdot = \frac{d}{dT}$$

3.2.2 Displacement-Excited Motion

Consider now the disturbance function $\sum_{n=1}^{N} \epsilon_n \cos n\omega_f t$ resulted from the rotor misalignment, commonly known as the run out, and the non-flatness of the rotor-surface. The film thickness H will be perturbed by this disturbance, and becomes

$$H = H_o - X + \sum_{n=1}^{N} E_n \cos n\,T \tag{3.3}$$

where $E_n = \epsilon_n/\delta$.

The equation of motion in the absence of the excitation $q \cos \omega_f t$ becomes

$$m \frac{dx^2}{dt^2} + \frac{C_1}{H^{2.5}} - \frac{C_1}{H_o^{2.5}} - \frac{C_2}{H^{2.5}} \frac{dh}{dt} = 0 \tag{3.4}$$

Substituting Eq. (3.3) into (3.4) and letting $X' = X - \sum_{n=1}^{N} E_n \cos n T$, one obtains

$$\bar{\omega}^2 \ddot{X}' + C\dot{X}' \frac{1}{(H_o - X')^{2.5}} + B \left[\frac{1}{(H_o - X')^{2.5}} - \frac{1}{H_o^{2.5}}\right]$$

$$= \sum_{n=1}^{N} Q'_n \cos n T \tag{3.5}$$

where

$$Q'_n = n^2 \bar{\omega}^{2} E_n \tag{3.6}$$

Equation (3.5) is identical to (3.2) with the exception that X, and $Q \cos \omega t$ are replaced by X' and $\sum_{n=1}^{N} Q'_n \cos nT$ respectively. Thus, the solution for the force-excited oscillations is also applicable to the displacement-excited motion provided the proper substitutions are made for X' and Q'_n.

3.3 Linearized Solution

If the motion of the stationary ring is such that the resulting gap variation, $H - H_o$, is only a small fraction of the equilibrium film thickness H_o, the response can be estimated from the solution of the linearized equation about the equilibrium film thickness H_o. Linearizing Eq. (3.2) for the force-excited motion, one obtains

$$\bar{\omega}^2 \ddot{X} + \frac{C}{H_o^{2.5}} \dot{X} + X = Q \cos T \qquad (3.7)$$

Similarily, linearization of Eq. (3.4) for the displacement-induced motion leads to

$$\bar{\omega}^2 \ddot{X} + \frac{C}{H_o^{2.5}} \dot{X} + X = \varepsilon \cos T \qquad (3.8)$$

Eqs. (3.7) and (3.8) are clearly the standard, damped vibration equation for a single mass, and its solution can be readily written as

$$A = \frac{Q}{\left[(1 - \bar{\omega}^2)^2 + \frac{C^2}{H_o^5}\right]^{1/2}} \qquad (3.9)$$

$$\alpha = \tan^{-1} \frac{(1 - \bar{\omega}^2) H_o^{2.5}}{C} \qquad (3.10)$$

where $X = A \cos (T - \alpha)$

3.4 Non-Linear Solution

Two methods have been employed to obtain the non-linear response characterized by the solution to Eqs. (3.2). The first is the method of Galerkin (Ref.18) which gives an approximate solution to the nonlinear equation. The degree of approximation is governed by the number of terms considered in the assumed function in the Galerkin procedure. The second method is the direct, step by step, numerical integration using a Runge-Kutta procedure. Details of these two methods are given next.

3.4.1 Method of Galerkin

The non-linear equation in question, Eq. (3.2), can be represented, implicitly as

$$f(\ddot{X}, \dot{X}, X, T) = 0$$

where

$$f = \bar{\omega}^2 \ddot{X} + \frac{C\dot{X} + B}{(H_o - X)^{2.5}} - \frac{B}{H_o^{2.5}} - Q \cos T = 0 \tag{3.11}$$

According to the method of Galerkin, one may assume that the unknown response X(T) can be represented approximately by a truncated Fourier series,

$$X = \sum_{n=0}^{N} a_n \cos nT + b_n \sin nT \tag{3.12}$$

The substitution of (3.12) into the differential equation gives arise to the residue function,

$$R(T) = f(\ddot{X}, \dot{X}, X, T) \tag{3.13}$$

The R(T) will not vanish unless X(T) exactly satisfy the differential equation. The Galerkin method provides a set of equations by which one can solve for the constants a_n and b_n for which the residue function will be made extremely small. These conditions are obtained by requiring that the integration of the residue function as weighted by each individual Fourier component (cos nT or sin nT) be made equal to zero. Stating mathematically, one obtains,

$$\left.\begin{aligned} &\int_o^{2\pi} R(T) \cos nT \, dT = 0 \\ &\int_o^{2\pi} R(T) \sin nT \, dT = 0 \end{aligned}\right. \tag{3.14}$$

for n = 0, 1,2....N

where

$$R(T) = \sum_{n=0}^{N} - \bar{\omega}^{-2}\, n^2 (a_n \cos nT + b_n \sin nT)$$

$$+ \frac{B + C \sum_{n=0}^{N} n(-a_n \sin nT + b_n \cos nT)}{\left[H_o - \sum_{n=0}^{N} (a_n \cos nT + b_n \sin nT)\right]^{5/2}} - \frac{B}{H_o^{5/2}} \tag{3.15}$$

$$- Q \cos T$$

For $N > 1$, integration of Equation (3.14) involves definite integrals of $-\frac{5}{2}$ th power of the Fourier series. A gallant attempt was made in reducing these integrals in some manageable form, but was unsuccesful. Thus, the inclusion of any terms beyond $N = 1$ was not made.

For $N = 1$,

$$X = a_o + a_1 \cos T + b_1 \sin T \tag{3.16}$$

Further simplification is made by representing X with

$$X = A \cos (T - \alpha) - A_o \tag{3.17}$$

where

$$A_o = - a_o$$

$$A = (a_1{}^2 + b_1{}^2)^{1/2}$$

$$\alpha = \tan^{-1} \frac{b_1}{a_1}$$

Substituting (3.17) into (3.14), one obtains

$$\int_0^{2\pi}\left[-\bar{\omega}^2 A\cos T+\frac{B-CA\sin T}{\left[H_o+A_o\right]^{2.5}\left[1-\frac{A}{H_o+A_o}\cos T\right]^{2.5}}\right.$$

$$\left.-\,Q(\cos\alpha\cos T-\sin\alpha\sin T)\right]\begin{Bmatrix}1\\ \sin T\\ \cos T\end{Bmatrix}dT=0 \tag{3.18}$$

Integrating Eqs. (3.18) and noting the following relations:

$$\int_0^{2\pi}\begin{Bmatrix}\cos T\\ \sin T\\ \sin T\cos T\end{Bmatrix}dT=0 \tag{3.19}$$

$$\int_0^{2\pi}\begin{Bmatrix}\cos^2 T\\ \sin^2 T\end{Bmatrix}dT=\pi \tag{3.20}$$

$$\int_0^{2\pi}\frac{\sin T\cos T}{\left(1-\frac{A}{H_o+A_o}\cos T\right)^{5/2}}dT=0 \tag{3.21}$$

$$\int_0^{2\pi}\frac{\sin T}{\left(1-\frac{A}{H_o+A_o}\cos T\right)^{5/2}}dT=0 \tag{3.22}$$

Equation (3.18) reduces to the following algebraic equations

$$-A\,\bar{\omega}^2+\frac{B}{\pi}I_1-Q\cos\alpha=0 \tag{3.23}$$

$$-\frac{CA}{\pi}I_4+Q\sin\alpha=0 \tag{3.24}$$

$$I_5-2\pi=0 \tag{3.25}$$

where

$$I_1 = \frac{2}{D_1} \int_0^{\frac{\pi}{2}} S_1 \cos T \, dT \tag{3.26}$$

$$I_4 = \frac{2}{D_1} \int_0^{\frac{\pi}{2}} S_2 \sin^2 T \, dT \tag{3.27}$$

$$I_5 = \frac{2H_o^{\ 2.5}}{D_1} \int_0^{\frac{\pi}{2}} S_2 dT \tag{3.28}$$

$$\begin{Bmatrix} S_1 \\ S_2 \end{Bmatrix} = \frac{1}{\left(1 - \frac{A}{H_o + A_o} \cos T\right)^{2.5}} \begin{Bmatrix} - \\ + \end{Bmatrix} \frac{1}{\left(1 + \frac{A}{H_o + A_o} \cos T\right)^{2.5}} \tag{3.29}$$

$$D_1 = (H_o + A_o)^{5/2} \tag{3.30}$$

Eliminating α from (3.23) and (3.24), one obtains a system of two non-linear equations to be solved for A and A_o. These two equations are

$$F(A,A_o) = \left(\frac{BI_1}{\pi} - A \bar{\omega}^2\right)^2 + \left(\frac{CAI_4}{\pi}\right)^2 - Q^2 = 0 \tag{3.31}$$

$$G(A,A_o) = I_5 - 2\pi = 0 \tag{3.32}$$

Using Newton-Raphson procedure, the successive corrections ΔA, ΔA_o can be expressed in terms of F, G, $\frac{\partial F}{\partial A}$, $\frac{\partial F}{\partial A_o}$, $\frac{\partial G}{\partial A}$, and $\frac{\partial G}{\partial A_o}$, evaluated at the last interates of A and A_o.

$$\Delta A = \frac{1}{\Delta} \begin{vmatrix} -F & \frac{\partial F}{\partial A_o} \\ -G & \frac{\partial G}{\partial A_o} \end{vmatrix} \tag{3.33}$$

$$\Delta A_o = \frac{1}{\Delta} \begin{vmatrix} \frac{\partial F}{\partial A} & -F \\ \frac{\partial G}{\partial A} & -G \end{vmatrix} \tag{3.34}$$

$$\text{where } \Delta = \begin{vmatrix} \frac{\partial F}{\partial A} & \frac{\partial F}{\partial A_o} \\ \frac{\partial G}{\partial A} & \frac{\partial G}{\partial A_o} \end{vmatrix} \tag{3.35}$$

$$\frac{\partial F}{\partial A} = 2\left(\frac{BI_1}{\pi} - A\,\bar{\omega}^2\right)\left(\frac{B}{\pi}\frac{\partial I_1}{\partial A} - \bar{\omega}^2\right) + 2\,\frac{C^2 A I_4}{\pi^2}\left(I_4 + A\frac{\partial I_4}{\partial A}\right) \tag{3.36}$$

$$\frac{\partial F}{\partial A_o} = 2\left(\frac{BI_1}{\pi} - A\,\bar{\omega}^2\right)\frac{B}{\pi}\frac{\partial I_1}{\partial A_o} + 2\,\frac{C^2 A^2}{\pi^2}\,I_4\,\frac{\partial I_4}{\partial A_o} \tag{3.37}$$

$$\frac{\partial G}{\partial A} = \frac{\partial I_5}{\partial A} \tag{3.38}$$

$$\frac{\partial G}{\partial A_o} = \frac{\partial I_5}{\partial A_o} \tag{3.39}$$

The differentials of nonlinear integrals can be derived from equations (3.36) - (3.28). They are

$$\frac{\partial I_1}{\partial A} = \frac{1}{D_2}\left\{5\int_0^{\frac{\pi}{2}} \cos^2 T\; S_3\, dT\right\}$$

$$\frac{\partial I_1}{\partial A_o} = \frac{-1}{D_2}\left\{5\int_0^{\frac{\pi}{2}} \cos T\; S_4\, dT\right\} \tag{3.40}$$

Equation (3.40) cont'd.

$$\frac{\partial I_4}{\partial A} = \frac{1}{D_2}\left\{5\int_o^{\frac{\pi}{2}} \sin^2 T \cos T\; S_4 dT\right\}$$

$$\frac{\partial I_4}{\partial A_o} = \frac{-1}{D_2}\left\{5\int_o^{\frac{\pi}{2}} \sin^2 T\; S_3 dT\right\}$$

$$\frac{\partial I_5}{\partial A} = \frac{H_o^{\;2.5}}{D_2}\left\{5\int_o^{\frac{\pi}{2}} \cos T\; S_3 dT\right\}$$

$$\frac{\partial I_5}{\partial A_o} = -\frac{H_o^{\;2.5}}{D_2}\left\{5\int_o^{\frac{\pi}{2}} S_4 dT\right\}$$

where $D_2 = (H_o + A_o)^{3.5}$

and

$$\begin{Bmatrix} S_3 \\ S_4 \end{Bmatrix} = \frac{1}{\left(1 - \frac{A}{H_o + A_o}\cos T\right)^{3.5}} \begin{Bmatrix} - \\ + \end{Bmatrix} \frac{1}{\left(1 + \frac{A}{H_o + A_o}\cos T\right)^{3.5}}$$

Being provided with values of A, and A_o, α can be solved from Equations (3.23) and (3.24) as

$$\alpha = \tan^{-1}\frac{CAI_4}{BI_1 - \pi A\,\bar{\omega}^2} \tag{3.41}$$

A computer program has been written to solve for A, A_o and α, from which, X_{max} and X_{min} can be determined by

$$\begin{aligned} X_{max} &= A - A_o \\ X_{min} &= -A - A_o \end{aligned} \tag{3.42}$$

The integrals I_1, I_4, I_5 and their derivatives with respect to A and A_o are evaluated numerically. Tables have been prepared for various values of $A/(H_o + A_o)$ varying in the range, [0,1] at intervals of 0.01.

3.4.2 Direct Integration

The step-by-step numerical integration of Eq. (3.2) is achieved by splitting it into two first order equations:

$$\dot{X} = Y$$

$$\dot{Y} = \frac{1}{\bar{\omega}^2}\left[\frac{B}{H_o^{2.5}} - \frac{B + CY}{(H_o - X)^{2.5}} + Q \cos T\right]$$

The popular Runge-Kutta method (Ref. 16) has been employed in obtaining the solution for X and Y with a given set of initial conditions X_o, Y_o. The response is represented by the trajectories in the phase space plot (X,Y). The response is considered to have reached a steady state if the trajectory approaches a limit cycle, which could be a single or multiple-looped cycle. A library subroutine at the Vogelback computer center has been used for this numerical integration. Fortran listings for the Computer program, RSGALN, which calculates A, A_o, and by the Galerkin method, and the computer program, RSRKIT, which calculates the trajectories in the phase space, are included in the Appendices and .

3.5 Results of Non-Linear Response

The results of the nonlinear response are presented in two parts. The first part is obtained from the method of Galerkin one-term approximation, and the second part from the Runge-Kutta direct integration. Since the gas-film is an unsymmetrical spring, i.e., the relation between the displacement and restoring force is not symmetric with respect to the equilibrium position, the amplitude of response during an upstroke is different from that during a downstroke. During an upstroke, the gas film stiffness is softer, and the amplitude is greater than that during a downstroke when the gas-film is considerably stiffer. For this reason, the response in the upstroke and downstroke are plotted separately against the non-dimensional excitation frequency in Figs. 11 to 19 .

3.5.1 Results by Method of Galerkin

Referring to the equation of motion, Eq.(3.2`,it is seen that the parameters affecting the dynamic response are:

B = stiffness parameter $= \dfrac{C_1}{m\delta(\omega^*)^2}$

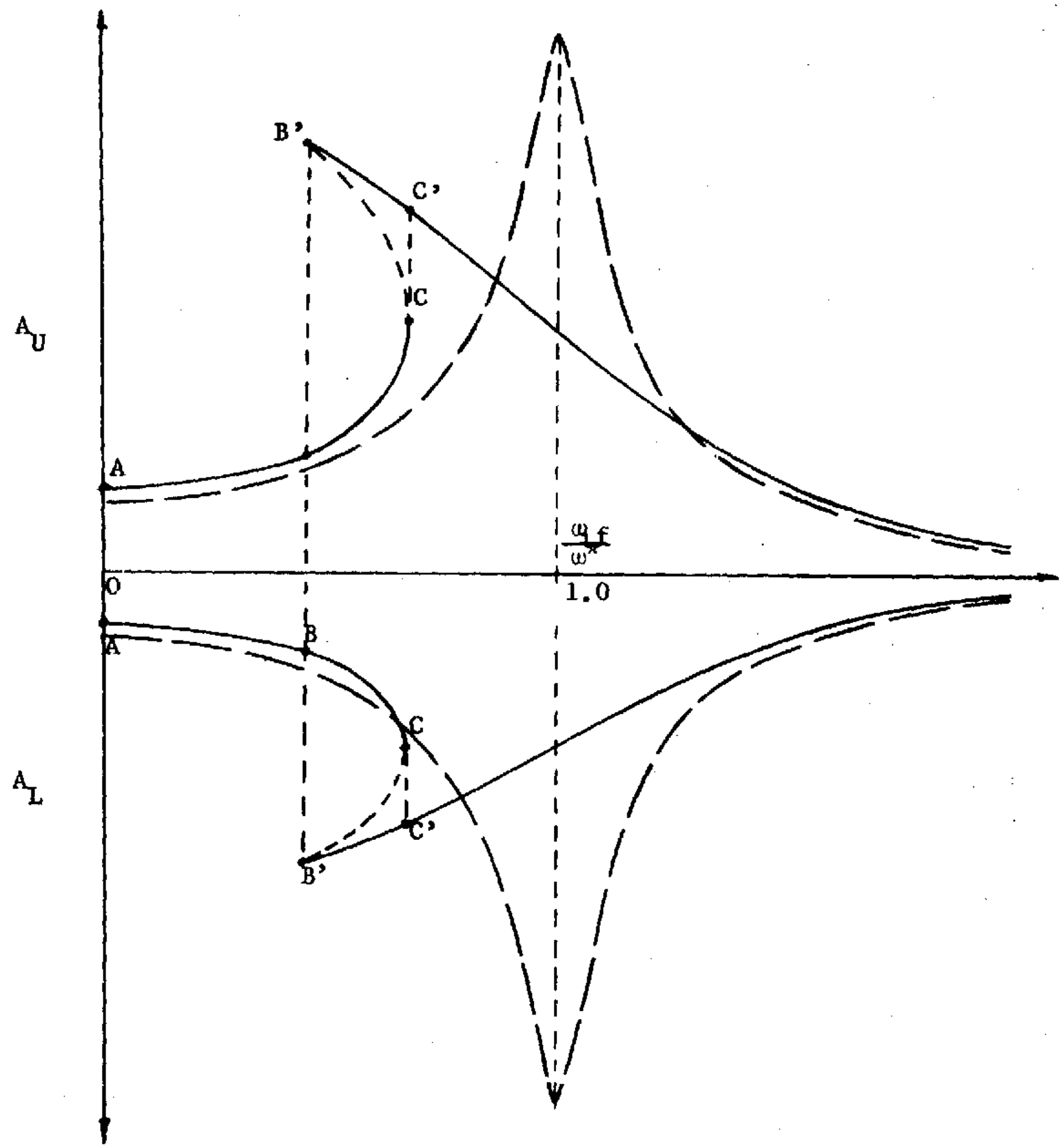
C = damping parameter $= \dfrac{C_2\omega_f}{m\omega^*}$

H_o = static-film parameters $= h_o/\delta$

Q = forcing intensity parameter $= \dfrac{q}{m\delta(\omega^*)^2}$

$\bar{\omega}$ = frequency parameter $= \omega_f/\omega^*$

The general characteristics of the upper and lower amplitudes as a function of excitation frequency are illustrated in the sketch below.



The nonlinear response based on one-term Galerkin method is shown as solid lines except with a small portion of the unstable oscillations, which are shown as dotted lines. The dashed lines show the response predicted from the linearized solution. In the region AB, the excitation freque...

is smaller than the natural frequency, ω^*, based on the linear theory, and the non-linear solution predicts a smaller lower amplitude but a greater upper amplitude comparing to the linear response. As ω_f increases to the region BC, the non-linear theory yields three possible solutions, one along the path BC, one along the path B'C, and another one along the path B'C'. The solution along BC is in-phase with the forcing function and is the most stable mode of response; the solution along B'C is unstable and only exists mathematically; and the solution along B'C' is out-of-phase with the excitation and is less stable than the solution along BC. For excitation frequency beyond the region BC, the characteristics of the non-linear response are similar to that of the linear response in the region where $\omega_f > \omega^*$. In this region, the pad is insensitive to the excitation and would not track any disturbance introduced by rotor runout or waviness.

The non-symmetrical nonlinear gas-film produces a response characteristics which resemble more to the response due to a symmetrical, soft, nonlinear spring, for which the resonance occurs at a frequency considerably lower than the natural frequency based on the linear theory. This correlation is really not surprising, since the mean position of the oscillation shifts to the region of softer stiffness, and the nonlinear oscillations are dominated by the softer part of the gas-film stiffness.

Fig. 11 shows typical response curves for the following dimensional parameters:

$h_o = 0.0005$ inches

$\delta = 0.001$ inches

$m = 0.2$ slug or 6.44 lbm

$C_1 = 2.32$ lb

$C_2 = 0.76$ $^{lb}/(^{in}/_{sec})$

$q = 1, 2,$ and 5 lb

The corresponding, non-dimensional parameters are listed in Fig. 11 . The inward bending of the resonant peak in the region $\bar{\omega} < 1$ is clearly visible in all three cases. Fig. 12 shows the effect of increasing the mass of the pad from 0.2 slug to 1.0 slug. The increase in mass does not alter the parameters, B, H_o, and Q, since $(\omega^*)^2$ is inversely proportional to m. The only parameter affected by changing of m is the non-dimensional damping parameter. A five fold increase in mass is equivalent to a $\sqrt{5}$ times reduction in the effective damping factor C_2. The more peaky response near the resonance is clearly visible in Fig. 12 when the mass is increased by five fold.

Figs. 13 and 14 shows the effect of increasing the equilibrium film thickness from 0.5 to 0.75. The natural frequency is reduced sharply by the increase in the film thickness, and the level of response is also much greater with a thick film than with a thin film for the same forcing function.

To investigate the effect of damping the value of C_2 has been doubled and halved from the case shown in Fig. 11 . The curves in Fig. 15 show that when the damping is doubled, the response near the resonance is considerably suppressed. The opposite effect is introduced if the damping is halved as shown in Fig. 16 .

3.5.2 Results by Direct Integration

Both the upper and lower amplitudes obtained by using the step-by-step, Runge-Kutta, direct integration are plotted against the excitation frequencies in Figs. 17 and 18 . A case of heavy mass, small equilibrium film thickness, and large excitation force has been selected to illustrate the nonlinear effects. The linear response curve and the approximate nonlinear response by Galerkin method are also plotted as dashed and dotted lines for comparison. It is seen that the agreement between the Runge-Kutta results and the Galerkin results is good near $\bar{\omega} = 1$. This clearly shows that even with one term approximation the Galerkin method yields a reasonably accurate prediction for the synchronous response. For $\bar{\omega} < 1$, the Runge-Kutta results show a series of superharmonic resonances at $\bar{\omega}$ approximately equal to 1/2, 1/3, 1/4, etc. The magnitude of these superharmonic amplitudes is, of course, governed by the damping factor.

Fig. 19 shows the trajectory in the phase-space plot for condition near the second superharmonic resonance. The final limit cycle forms a two-loop orbit showing typical characteristics of a superharmonic response. Other trajectories at the third and fourth superharmonic resonances are also shown in Figs. 20 to 22 . A subharmonic resonance is also found for $\bar{\omega} \sim 2.0$, but the amplitude is small and harmless. The Characteristics of the phase space trajectories near $\bar{\omega} = 1$ are plotted in Figs. 23 to 25.

IV. STABILITY OF AN INFINITELY-WIDE RAYLEIGH-STEP PAD

4.1 Statement of the Problem

It is well known in hydrodynamic lubrication that a dynamic system involving any fluid-film supports may, under certain conditions, encounter detrimental self-excited oscillations commonly known as dynamical instability of fluid-film bearings. The gas-film bearings are particularly susceptible to this type of instability. The fractional frequency **whirl** of a shaft supported on gas-bearings and the pneumatic hammer in externally pressurized gas-bearings are two of the prominent examples of the fluid-film instability. For journal bearings, the gas-film instability usually occurs if either the rotating frequency or the supported mass becomes large. There have been considerable data available to predict the threshold speed or critical mass of the journal bearing. However, for gas-lubricated thrust pads, the problem of instability is relatively unexplored. Since present trends in gas-bearing are always toward higher and higher speeds, it is important to determine whether there exists any stability threshold associated with a gas-lubricated thrust pad.

This chapter is devoted to the stability analysis of a thrust pad with a Rayleigh-step. The geometry of such a thrust pad is shown in Fig.26 . In order not to impose excessive burdens on the analysis, the assumption of an infinitely wide pad has been made. Moreover, the motion of the pad is assumed to be restricted in the transverse direction only. With these two assumptions, the problem is reduced to the stability of a single-degree-of-freedom dynamical system with restoring pressures governed by a partial differential equation in space and time.

4.2 Governing Equations

The one-dimensional, time-dependent pressure distribution is governed by the Reynolds equation,

$$\frac{\partial}{\partial x}\left(ph^3 \frac{\partial p}{\partial x}\right) = 6\mu u \frac{\partial(ph)}{\partial x} + 12\mu \frac{\partial(ph)}{\partial t} \tag{4.1}$$

where $h = \delta + h_o + e(t)$ for $0 < x < B_1$

$h = h_o + e(t)$ for $B_1 < x < B$

$e(t)$ is the upward motion of the pad.

The boundary conditions for Eq. (4.1) are

$$p = p_a \text{ at } x = 0 \text{ and } x = B \tag{4.2}$$

The equation governing the pad motion is,

$$m \frac{d^2e}{dt^2} = \int_o^B (p - p_a)dx - W_o \tag{4.3}$$

where m is the mass per unit length of the pad, B the length of the pad, and W_o the static load imposed on the pad. The dynamic system represented by the coupled equations, (4.1) and (4.3) are to be investigated for the stability for an equilibrium position.

4.3 Method of Solution

The time-dependent, nonlinear, pressure equation, Eq. (4.1), is difficult to solve analytically. A numerical approach has been used here in solving a set of discretized, time-dependent, pressures along the coordinate X.

The discretization of pressure is achieved by considering the flow balance in an elemental volume within the gas film as shown in Fig. 26 for the i-th element. The flow rates in and out the element are respectively q_1 and q_2, and

$$q_1 = \left(-\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x} + \frac{u_1 + u_2}{2}\ \rho h\right)_{i-\frac{1}{2}}$$
$$q_2 = \left(-\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x} + \frac{u_1 + u_2}{2}\ \rho h\right)_{i+\frac{1}{2}} \tag{4.4}$$

where $u_1 = u$ and $u_2 = 0$

Considering the flow balance,

$q_1 - q_2$ = rate of mass stored within the elemental volume

It follows,

$$-\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right)_{i-\frac{1}{2}} + \left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right)_{i+\frac{1}{2}}$$
$$+ \frac{u}{2}\left[(\rho h)_{i-\frac{1}{2}} - (\rho h)_{i+\frac{1}{2}}\right]$$
$$= \frac{\partial}{\partial t}\left[\rho_i\left(h_{i-\frac{1}{2}}\frac{\Delta x_{i-1}}{2} + h_{i+\frac{1}{2}}\frac{\Delta x_i}{2}\right)\right] \tag{4.5}$$

Using the isothermal relation,

$$\frac{p}{\rho} = R\ T_e = \text{constant} \tag{4.6}$$

Introducing the non-dimensional parameters,

$$P = {}^{p}/{p_a}$$
$$X = {}^{x}/{B} \tag{4.7}$$

Equation (4.7) cont'd

$$H = \frac{h}{\delta} = \frac{h}{h_2 - h_1}$$

$$\Lambda = \frac{6\mu BU}{p_a \delta^2}$$

$$\sigma = \frac{12\mu\nu B^2}{p_a \delta^2}$$

$$T = \nu t$$

ν = excitation frequency

and using the following finite approximations for $\left(\frac{\partial P}{\partial X}\right)_{i-\frac{1}{2}}$, $P_{i-\frac{1}{2}}$, $H_{i-\frac{1}{2}}$

$$\left(\frac{\partial P}{\partial X}\right)_{i-\frac{1}{2}} = \frac{P_i - P_{i-1}}{X_i - X_{i-1}} = \frac{P_i - P_{i-1}}{\Delta X_{i-1}}$$

$$P_{i-\frac{1}{2}} = \frac{1}{2}(P_i + P_{i-1}) \qquad (4.8)$$

$$H_{i-\frac{1}{2}} = \frac{1}{2}(H_i + H_{i-1})$$

the discretized pressure equation becomes

$$\frac{1}{4\Delta X_i}\left(P_{i+1}^2 - P_i^2\right) H_{s,i} - \frac{1}{4\Delta X_{i-1}}\left(P_i^2 - P_{i-1}^2\right) H_{s,i-1}$$

$$- \Lambda (P_{i+1} + P_i)(H_{i+1} + H_i) + \Lambda (P_i + P_{i-1})(H_i + H_{i-1})$$

$$= \sigma \left\{\frac{\partial}{\partial T}\left[P_i(H_i + H_{i-1})\right] \Delta X_{i-1} + \frac{\partial}{\partial T}\left[P_i(H_{i+1} + H_i)\right] \Delta X_i\right\} \qquad (4.9)$$

where

$$H_{s,i} = \left(H_{i+1} + H_i\right)^3 \qquad (4.10)$$

where $A_i = a_{o,i}P_{i+1,o} + d_{o,i}$

$$B_i = b_{o,i}P_{i,o} + e_{o,i}$$

$$C_i = c_{o,i}P_{i-1,o} + f_{o,i}$$

$$a_{o,i} = \frac{H_{s,i}}{4\Delta X_i}, \qquad d_{o,i} = -\Lambda(H_{i+1,o} + H_{i,o})$$

$$c_{o,i} = \frac{H_{s,i-1}}{4\Delta X_{i-1}}, \qquad f_{o,i} = \Lambda(H_{i,o} + H_{i-1,o})$$

$$b_{o,i} = -a_{o,i} - c_{o,i}, \qquad e_{o,i} = d_{o,i} + f_{o,i} \tag{4.17}$$

and

$$P_{1,o} = P_{n,o} = 1 \tag{4.18}$$

Since the algebraic equations (4.16) are nonlinear, Taylor's expansion is used to reach the following simultaneous equations for $\Delta P_{i,o}$,

$$\Delta \Phi_i = -\Phi_i(P_{i,o}) \tag{4.19}$$

where
$$\begin{aligned}\Delta \Phi_i = {} & (A_i + a_{o,i}P_{i+1,o})\Delta P_{i+1,o} \\ & + (B_i + b_{o,i}P_{i,o})\,\Delta P_{i,o} \\ & + (C_i + C_{o,i}P_{i,o})\Delta P_{i-1,o}\end{aligned}$$

Equations(4.19) are inverted directly for successive $\Delta P_{i,o}$ until the convergence is reached.

In equation (4.9), both the discretized pressure P_i and H_i are dependent on time. For transient studies, they have to be solved simultaneously with the dynamic equation of motion, Eq. (4.3). However, for small oscillations and stability analysis, the variation of H and P with time can be considered as small perturbations about the equilibrium solution, H_o and P_o. These small perturbed quantities can be expressed as

$$H_i(T,X) = H_{i,o}(X) + \epsilon(T) \tag{4.11}$$

$$P_i(T,X) = P_{i,o}(X) - \epsilon(T)\, P_{i,c}(X) \tag{4.12}$$

where $\epsilon = {}^{e}/\delta$

$$\left|\frac{\epsilon}{H_i}\right| << 1$$

The minus sign in (4.12) is due to the fact that an increase in thickness leads to a decrease of pressure of the gas film. Substituting (4.11) and (4.12) into Equation (4.9), the equilibrium equation and the first order perturbed equations can be obtained. They are

$$\frac{1}{4\Delta X_i}\left(P_{i+1,o}^{\ 2} - P_{i,o}^{\ 2}\right) H_{s,i} - \frac{1}{4\Delta X_{i-1}}\left(P_{i,o}^{\ 2} - P_{i-1,o}^{\ 2}\right) H_{s,i-1}$$

$$- \Lambda\,(P_{i+1,o} + P_{i,o})(H_{i+1,o} + H_{i,o}) + \Lambda\,(P_{i,o} + P_{i-1,o})(H_{i,o} + H_{i-1,o})$$

$$= 0 \tag{4.13}$$

and

$$(P_{i+1,o}a_{s,i} + a_{n,i})P_{i+1,c} + (P_{i,o}b_{s,i} + b_{n,i})P_{i,c}$$

$$+ (P_{i-1,o}C_{s,i} + C_{n,i}) - d_i$$

$$= \frac{1}{\epsilon}\frac{\partial\epsilon}{\partial T}\left[P_{i,o}f_i + e_iP_{i,c}\right] \tag{4.14}$$

where

$$a_{s,i} = \frac{H_{s,i}}{8\Delta X_i} \quad , \qquad a_{n,i} = -\frac{\Lambda}{4}(H_{i+1,o} + H_{i,o})$$

$$c_{s,i} = \frac{H_{s,i-1}}{8\Delta X_{i-1}} \quad , \qquad c_{n,i} = \frac{\Lambda}{4}(H_{i,o} + H_{i-1,o})$$

$$b_{s,i} = -a_{s,i} - c_{s,i} \quad , \qquad b_{n,i} = a_{n,i} + c_{n,i}$$

$$d_i = \frac{1}{4\Delta X_i}\left(P_{i+1,o}^{\ 2} - P_{i,o}^{\ 2}\right)\left(H_{i+1,o}^{\ 2} + H_{i+1,o}H_{i,o} + H_{i,o}^{\ 2}\right)$$

$$- \frac{1}{4\Delta X_{i-1}}\left(P_{i,o}^{\ 2} - P_{i-1,o}^{\ 2}\right)\left(H_{i,o}^{\ 2} + H_{i,o}H_{i-1,o} + H_{i-1,o}^{\ 2}\right)$$

$$- \frac{\Lambda}{2}(P_{i+1,o} - P_{i,o})$$

$$e_i = \frac{\sigma}{4}\left[(H_{i+1,o} + H_{i,o})\Delta X_i + (H_{i,o} + H_{i-1,o})\Delta X_{i-1}\right]$$

$$f_i = -\frac{\sigma}{2}(\Delta X_{i-1} + \Delta X_i) \tag{4.15}$$

Equations (4.13) are nonlinear simultaneous algebraic equations for the pressure distribution $P_{i,o}$. They are solved numerically by Newton-Raphson method. The boundary conditions are $P_o(1,n) = 1$.

Rewrite equation (4.13) as

$$\Phi_i(P_{i,o}) = A_i P_{i+1,o} + B_i P_{i,o} + C_i P_{i-1,o} = 0 \tag{4.16}$$

for $i = 2,\ldots,n-1$.

After the equilibrium pressure distribution is found, Equations (4.14) are solved for the first order pressure distribution, $P_{i,c}$. In Equations (4.14), both $P_{i,c}$ and $\epsilon(T)$ are complex quantities, and they are assumed to be

$$P_{i,c} = P_{i,R} + jP_{i,I} \tag{4.20}$$

and

$$\epsilon = \epsilon_o e^{j\,T} \tag{4.21}$$

The boundary conditions are $P_{i,c} = 0$ at entrance and exit, which gives

$$P_{1,R} = P_{n,R} = P_{1,I} = P_{n,I} = 0 \tag{4.22}$$

Substituting Eqs. (4.20) and (4.21) into Eq. (4.14), one obtains a system of 2n - 4 simultanous, linear equations,

$$A_{i+1,i}X_{i+1} + A_{i,i}X_i + A_{i-1,i}X_{i-1} + A_{i+n-2,i}\,X_{i+n-2} = D_i \qquad \text{for } i= 1,\ldots\ldots\, n-2$$

$$A_{i-n+2,i}\,X_{i-n+2} + A_{i+1,i}\,X_{i+1} + A_{i,i}\,X_i + A_{i-1,i}\,X_{i-1} = D_i \qquad \text{for } i = n-1,\ldots\ldots,2n-4 \tag{4.23}$$

In the above, A and X for $i = 1,\ldots\ldots\ldots, n-2$ are defined by, $j = i+1$

$$\begin{aligned}
A_{i+1,i} &= P_{j+1,o}a_{s,j} + a_{n,j} , && X_{i+1} = P_{i+2,R} \\
A_{i,i} &= P_{j,o}b_{s,j} + b_{n,j} , && X_i = P_{i+1,R} \\
A_{i-1,i} &= P_{j-1,o}C_{s,j} + C_{n,j} , && X_{i-1} = P_{i,R} \\
A_{i+n-2,i} &= e_j , && X_{i+n-2} = P_{i+1,I} \\
D_i &= d_j
\end{aligned} \tag{4.24}$$

and for $i = n - 1,\ldots,2n - 4$, they are defined by,

$$j = i-n+3$$

$$A_{i+1,i} = P_{j+1,o}a_{s,j} + a_{n,j} \quad , \quad X_{i+1} = P_{i-n+4,I}$$

$$A_{i,i} = P_{j,o}b_{s,j} + b_{n,j} \quad , \quad X_{i} = P_{i-n+3,I}$$

$$A_{i-1,i} = P_{j-1,o}C_{s,j} + C_{n,j} \quad , \quad X_{i-1} = P_{i-n+2,I} \tag{4.25}$$

$$A_{i-n+2,i} = - e_j \quad , \quad X_{i-n+2} = P_{i-n+3,R}$$

$$D_i = f_j P_{j,o}$$

The boundary conditions become

$$\left.\begin{array}{l} X_o = P_{1,R} = 0 \\ X_{n-1} = P_{n,R} = 0 \end{array}\right\} \text{ for equations with } i = 1,\ n - 2, \text{ respectively} \tag{4.26}$$

and

$$\left.\begin{array}{l} X_{n-2} = P_{1,I} = 0 \\ X_{2n-3} = P_{n,I} = 0 \end{array}\right\} \text{ for equations with } i = n - 1,\ 2n - 4 \text{ respectively} \tag{4.27}$$

These simultaneous equations (4.23) can now be solved for [X] vector by direct matrix inversion. A computer program has been written to solve for $P_{i,o}$ and $P_{i,c}$ for different real values of ν, and the Fortran listing of this program is included in Appendix D.

Once the real and imaginary part of the gas film pressure are determined, the integration of $P_{i,R}$ and $P_{i,I}$ gives respectively the in-phase and out-of-phase bearing forces. The in-phase force $\int_0^1 P_R \, dX$, can be interpreted as the stiffness of the film, whereas the out-of-phase, $\int_0^1 P_I \, dX$, represents the damping factor of the film. It should be emphasized that both the in-phase and out-of-phase perturbed pressure are dependent upon σ, or the excitation frequency ν of the mass. The frequency-dependent characteristics of the gas-film reactions is a direct consequence of the inclusion of the term $\frac{\partial P}{\partial T}$. The use of these frequency-dependent bearing forces in determining the dynamic stability of the gas-film and pad system is described in the next section.

4.4 Stability Criterion

The stability of the gas-film and pad is governed by the equation of motion, Eq. (4.3), which, in its non-dimensional form, appears as

$$\frac{d^2\varepsilon}{dT^2} = \frac{p_a B}{m\delta\nu^2} \int_0^1 (P - 1)\,dX - \frac{W_o}{m\delta\nu^2} \tag{4.28}$$

Recalling the pressure is the summation of the equilibrium pressure, P_o, and the dynamic pressure, $-\epsilon P_c$, one obtains

$$\frac{p_a B}{m\delta\nu^2} \int_0^1 (P - 1)\,dX - \frac{W_o}{m\delta\nu^2} = -\frac{p_a B}{m\delta\nu^2}\epsilon \int_0^1 P_c \,dX \tag{4.29}$$

It follows that

$$\left(\frac{m\delta\nu^2}{p_a B}\right) \frac{d^2\epsilon}{dT^2} + \epsilon \int_0^1 P_c \,dX = 0 \tag{4.30}$$

Mathematically, Eq. (4.30) represents a free oscillation problem which contains stiffness and damping factors depending upon the frequency of the oscillations. A direct approach in determining of the stability of this single-degree of freedom problem is to look for the eigenvalue of this system. If the real part of the eigenvalue is negative, the system is stable; otherwise it is unstable. If the eigenvalue is purely imaginary, then the system is at its threshold of instability.

Alternatively, one can also determine the stability thresholds by assuming that the eigenvalue is a purely imaginary number and inquire what would be the mass parameter, $\frac{m\delta\nu^2}{p_a B}$, for a purely imaginary eigenvalue.

Let the eigenvalue be represented by $\lambda + j\nu$, and for a pure imaginary eigenvalue, $\lambda = 0$. It follows that

$$\epsilon = \epsilon_o e^{j\nu t} = \epsilon_o e^{jT} \tag{4.31}$$

Substituting Eq. (4.27) into Eq. (4.26), and separating the real from the imaginary part, one obtains

$$\int_o^1 P_I(\nu)\,dX = 0 \tag{4.32}$$

$$\int_o^1 P_R(\nu)\,dX - \frac{m\delta\nu^2}{p_a B} = 0 \tag{4.33}$$

where $P_I(\nu)$ and $P_R(\nu)$ are solutions of Equations (4.23) for a given value of ν. The pure imaginary eigenvalue ν may be determined by evaluating the integral, $\int_o^1 P_I(\nu)\,dX$, for various values of ν until the integral changes its sign. The exact eigen frequency ν may be calculated by a linear interpolation. Once the eigen frequency is found, the critical mass at the threshold of instability can be determined by Eq. (4.28), and

$$m_{cr} = \frac{p_a B}{\delta\nu^2} \int_o^1 P_R(\nu)\,dX \tag{4.34}$$

Equation (4.29) predicts a quantitative value of the critical mass, but does not furnish any information on which side the stable region lies. To determine the region of stability, one may use the criterion developed by Malanowski and Pan (ref. 17). Their criterion can be stated in the following manner.

Stability Criterion - The system consisting of a thrust bearing of mass, m, with Rayleigh Shrouded-Step Seal is in a state of self-sustained oscillation at frequency ν_o when and only when the out-of-phase component of the bearing reaction, $\int_o^1 P_I(\nu)dX$, vanishes, and when the mass, m_o has the critical magnitude to be in resonance with the in-phase bearing reaction, Equation (4.28). The system will become unstable if the mass exceeds the critical value provided the out-of-phase bearing reaction increases with the frequency in the algebraic sense in the neighborhood of the critical frequency ν_o , and conversely.

4.5 Results

The steady state pressure distribution, P_o, has been calculated numerically for the following parameters:

$$^{B}1/_{B} = \quad 0.5, \text{ and } 0.75$$

$$H = \frac{h}{\delta} = 0.5 \text{ and } 0.75$$

$$\Lambda = \frac{6\mu bU}{p_a \delta^2} = 8.4,\ 42,\ \text{and } 100$$

The resulting pressure curves are plotted in Fig. 27 , and they are in excellent agreement with the analytical solution provided by Kochi (Ref. 6) This comparison confirms the accuracy of the present numerical solution of the steady state pressure distribution, P_o.

For each steady-state pressure distribution, the dynamic pressure distributions, P_R and P_I are calculated for a series value of σ. Figs.28 to 35 show a typical series of the dynamic pressure profiles for $B_1/B = 0.5$ and 0.75 , $H = 0.5$, and $\Lambda = 42$. For small excitation frequencies, the real part of the dynamic pressure is dominated by the bearing parameter Λ, and the profile is similar to the static pressure distribution, and is relatively independent of σ. As σ increases, the pressure distribution, P_R, becomes slightly wavy at both edges. The waviness penetrates deeper as σ further increases. As σ approaches infinity, the effect of Λ disappears, and P_R approaches the asymptotic solution,

$$(P_R)_{\sigma \to \infty} = \frac{P_o}{H_o} \tag{4.35}$$

The imaginary part of the dynamic pressure takes a wavy pattern even for small values of σ. For σ approaches infinity, the values for P_I vanishes throughout the entire region.

The in-phase and out-of-phase forces are plotted as a function of σ in Fig. 36 , and also listed in Tables 3 to 5. It is seen that for small or moderate values of Λ, the out-of-phase force never becomes negative. This indicates that for nearly incompressible cases, there exists no stability threshold, and all equilibrium solutions are stable. As Λ becomes extremely large, the out-of-phase force does become negative at a fairly high value of σ. This crossing-over of zero line indicates that for a highly compressible film, there does exist a stability threshold, and the gas film will exhibit a self-excited oscillation at a fairly high frequency. Moreover, the criterion in Reference 16 suggests that the stable region lies in the area where the mass of the pad is greater than the critical mass calculated according to Eq. (4.34).

V. SUMMARY OF RESULTS

1. The gas-film restoring forces in a Rayleigh-Shrouded-Step thrust pad can be determined numerically by solving the discretized time-dependent Reynolds equation with irregular grid-spacings to account for any abrupt changes of pressure at the step and at the exit edge.

2. For conditions of high ambient pressure, for which the term of $h \frac{\partial p}{\partial t}$ can be neglected in comparison with other terms at the right side of the Reynolds equation, the gas-film force is found to be approximately inversely proportional to nth power of the film thickness and directly proportional to the squeeze-film velocity. The exact value of n is a function of the step geometry. In general, n lie between 2 and 3.

3. The axial, non-linear response of the Rayleigh-Shrouded-Step pad to a sinusoidal, axial forcing function or a sinusoidal disturbance due to the rotor misalignment or surface waviness can be determined by one of the following two methods:

   a, By assuming the response to be a truncated Fourier series in multiples of the excitation frequency, the Ritz-Galerkin procedure can be employed to predict the non-linear behavior of the pad motion.

   b, By integrating directly the equation of motion of the thrust pad using a step-by-step, numerical routine, the Runge-Kutta procedure.

4. Results obtained by using the Ritz-Galerkin method with the first harmonic terms show considerably departures from the linear response curve as the frequency approaches the resonance based on the linear theory. The asymmetric spring characteristics of the gas film result into a non-linear response similar to that caused by a symmetric, soft, non-linear

spring. The resonating peak bends inward and occurs at a frequency less than the resonating frequency based on the linear theory. The peak can be suppressed by decreasing the mass, increasing the damping, and increase the stiffness.

5. Results obtained by the step-by-step direct integration confirms the approximate solution in the vicinity of the resonance. The direct integration also predicts a number of additional peaks at frequencies less than the resonating frequency known as the superharmonic resonance.

6. The gas-film instability of an infinitely-wide Rayleigh step thrust pad canbe determined by solving the complete, time dependent, Reynolds equation coupled with the equation of the motion of the pad. Results show that for bearing numbers, Λ, up to 50, the Rayleigh step geometry is very stable, and no stability threshold has been discovered. For ultra high values of $\Lambda \geq 100$, a stability threshold is shown to exist, and the stability can be achieved by making the mass heavier than the critical mass.

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TABLE 1

GAS FILM FORCES (lb_f)

$d_o = 6.46$ ins $d_i = 5.96$ ins

$\delta = 0.001$ in (step)

$\omega = 277$ rev./sec.

$P_a = P_o = 315$ psia.: Ambient Pressure

H_{min} \ dh/dt	2.0	1.5	1.0	0.75	0.50	0.30	0.20
-1 in/sec.	0.50635	1.10134	3.2606	6.9965	19.949	65.138	139.729
-0.5	0.45104	0.97913	2.8878	6.1765	17.499	56.041	115.531
-0.25	0.42338	0.91801	2.7015	5.7667	16.275	51.529	103.734
+0.25	0.36806	0.79580	2.3290	4.9477	13.833	42.576	80.750
+0.5	0.34040	0.73470	2.1428	4.5384	12.615	38.135	69.559
+1	0.28509	0.61250	1.7704	3.7204	10.182	29.325	47.778
0	0.3957	0.8569	2.515	5.357	15.06	47.124	92.8
$\frac{F_{g1} - F_{g2}}{\dot{h}_2 - \dot{h}_1} =$	0.1106	0.2444	0.7450	1.6380	4.8836	17.907	45.975

$$\mu = 6 \times 10^{-9} \frac{lb_f - sec}{in^2}$$

$\Lambda = 8.30076$

TABLE 2

LOAD (lb_f) CALCULATED AND ERROR OCCURRED (%)

$\dot{h}$ in/sec. \ H_{min}	2.0	1.5	1.0	0.75	.50	.30	0.20
-1	0.544	1.116	3.08	6.33	17.403	62.7	171.2
	+ 1.5%	+ 1.45%	- 5.53%	- 9.5%	- 12.5%	- 3.7%	+ 22.5%
-0.5	0.477	0.978	2.70	5.55	15.28	54.95	150.1
	+ 5.78%	- 0.1%	- 6.5%	- 10.1%	- 6.29%	+ 6.65%	+ 30%
-0.25	0.4435	0.909	2.51	5.16	14.205	51.075	139.05
	+ 4.78%	- 0.98%	- 7.07%	- 10.5%	- 12.3%	- 0.89%	+ 33%
+0.25	0.3765	0.771	2.13	4.38	12.055	43.325	118.95
	+ 2.34%	- 3.01%	- 8.2%	- 11.5%	- 12.9%	+ 1.76%	47.4%
0.5	0.344	0.703	1.94	3.99	10.98	39.45	107.9
	+ 1.18%	- 4.22%	- 9.35%	- 11.8%	- 12.9%	+ 3.46%	55.2%
1	0.276	0.564	1.56	3.21	8.83	31.7	86.8
	- 3.15%	- 7.85%	- 11.85%	- 13.7%	- 15%	+ 8.2%	+ 82%

TABLE 3

DYNAMIC BEARING REACTION FOR

$B_1/B = .75 \quad H_2 = 0.5$

$\Lambda = 8.4$

σ	$\int_0^1 P_I dX$	$\int_0^1 P_R dX$
20	.13373	1.1836
50	.21003	1.2694
100	.27600	1.4097
200	.24248	1.5930
300	.18121	1.6590
450	.12761	1.6887
	$\Lambda = 42$	
1	.005975	1.5018
10	.057536	1.5144
50	.11984	1.6844
100	.020667	1.7192
200	.031105	1.5013
300	.26865	1.5269
500	.37916	1.8392
2×10^3	.079279	2.0095
10^4	.045201	2.0296
	$\Lambda = 100$	
10	.05897	1.0295
50	.28043	1.1019
100	.47831	1.3014
200	.47977	1.7884
300	.13904	1.9829
500	-.12737	1.5789

TABLE 4

DYNAMIC BEARING REACTION FOR

$B_1/B = .75 \quad H_2 = 0.75$

$\Lambda = 8.4$

σ	$\int_0^1 P_I dX$	$\int_0^1 P_R dX$
10	.15200	.55179
50	.16382	.78737
100	.15032	.84853
200	.14273	.92439
300	.12283	.96835
500	.08910	1.0045

$\Lambda = 42$

1	.009264	.61777
10	.090506	.63251
50	.27479	.86727
100	.17722	1.0606
200	.052296	1.0238
300	.10195	.99666
500	.15700	1.0797

$\Lambda = 100$

1	.004522	.46999
10	.045139	.47216
50	.21611	.52320
100	.37708	.66548
200	.42895	1.0325
300	.22267	1.2315
500	-.036699	1.0725

TABLE 5

DYNAMIC BEARING REACTION FOR

$B_1/B = 0.5 \quad H_2 = 0.5$

$\Lambda = 8.4$

σ	$\int_0^1 P_I dX$	$\int_0^1 P_R dX$
1	.04398	.9242
10	.40067	1.0473
50	.65404	1.9523
100	.34743	2.2234
200	.17872	2.2381
300	.14173	2.2543
500	.10469	2.2785
800	.07845	2.2960

$\Lambda = 42$

1	.00519	2.3313
10	.05146	2.3349
50	.22100	2.3947
100	.38003	2.4791
200	.66419	2.7731
300	.65252	3.1907
500	.22427	3.3674

$\Lambda = 100$

1	.00727	1.9138
10	.07261	1.9170
50	.34848	1.9943
100	.61342	2.2089
200	.73899	2.7584
300	.49508	3.0561
500	.29231	2.8927

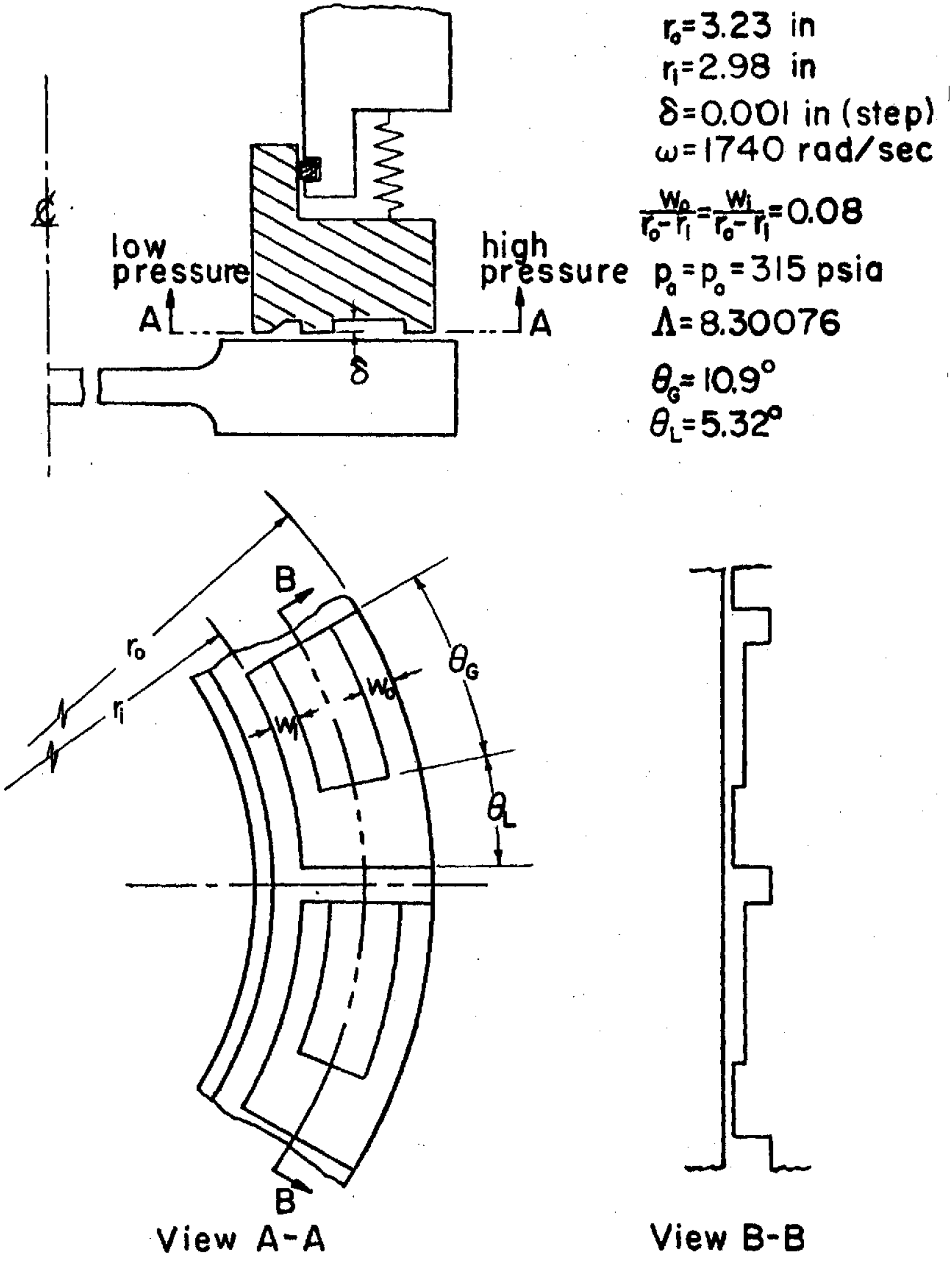


Fig.1 The geometry of a shrouded, Rayleigh-step thrust bearing

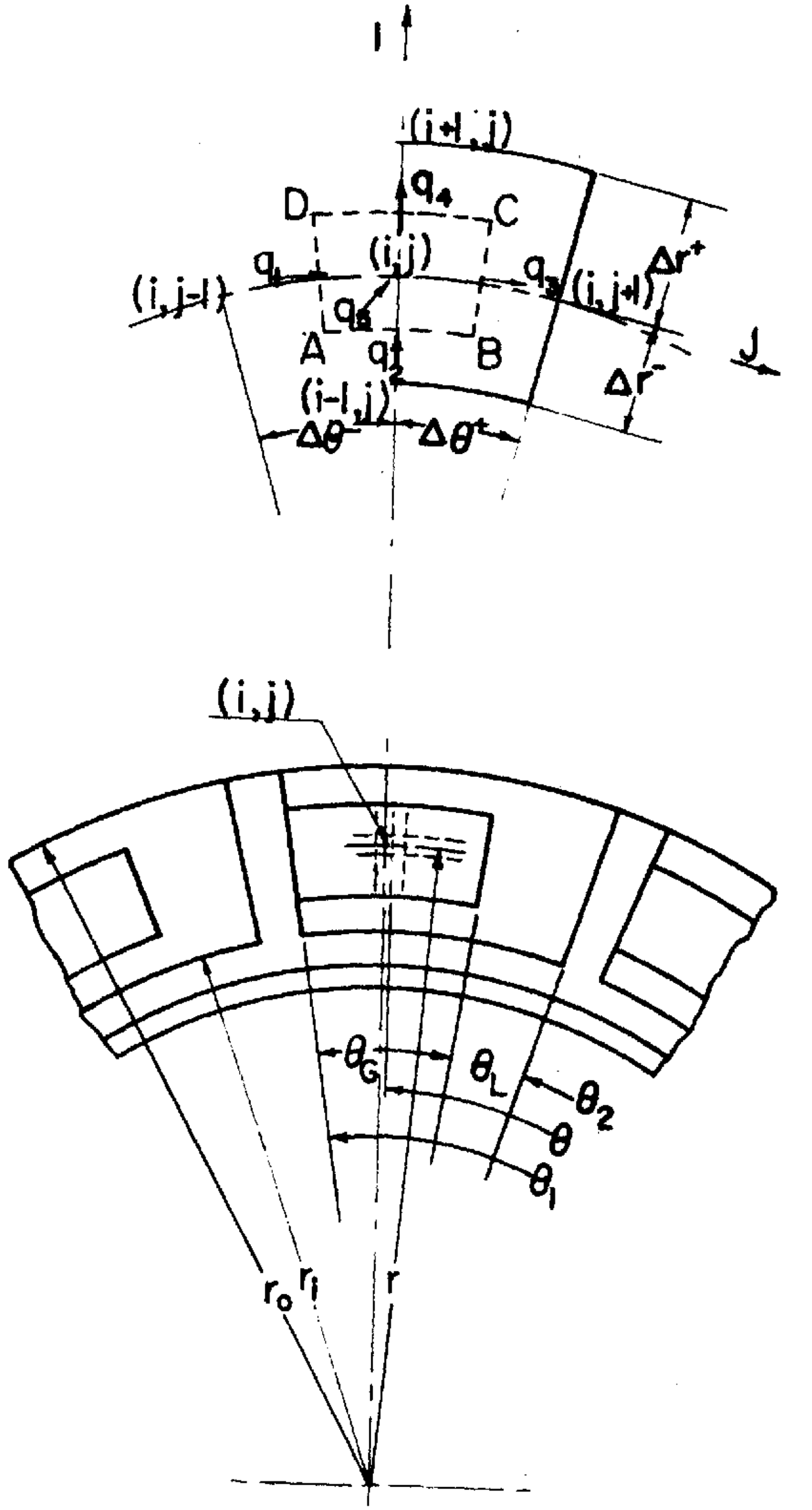


Fig.2 Flow balance around a typical grid point

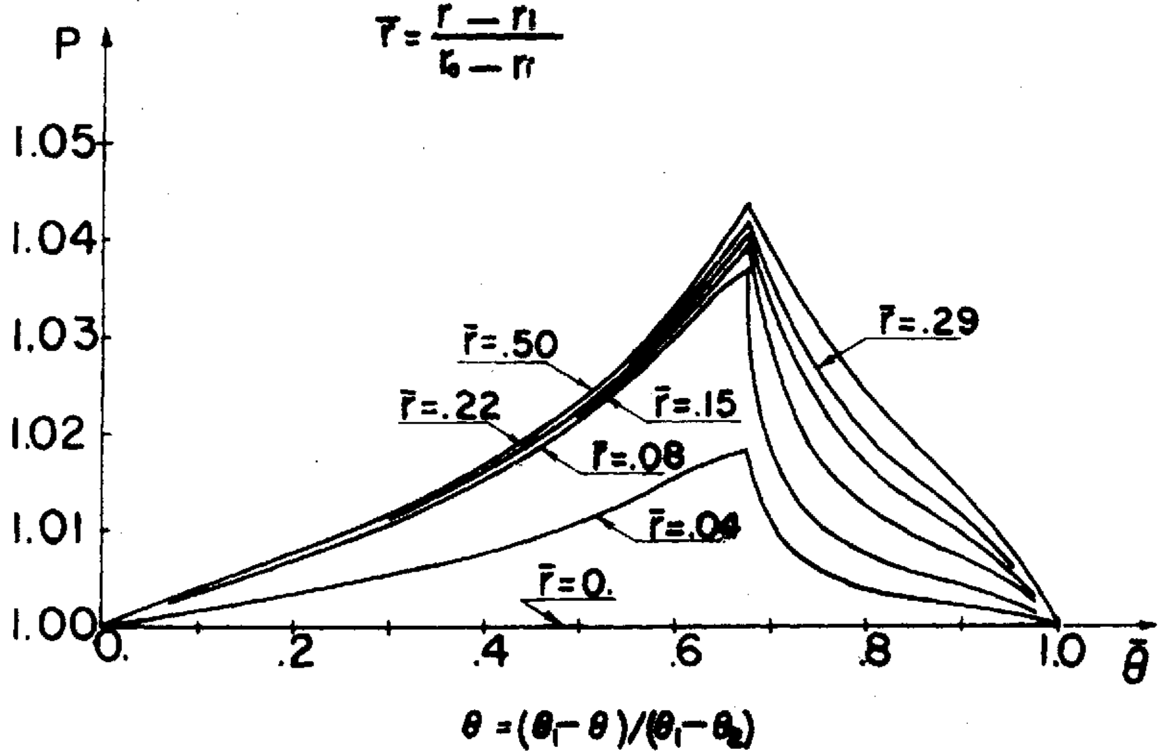


Fig. 3 Pressure distribution for h=-Lips., H_{min}= 0.5

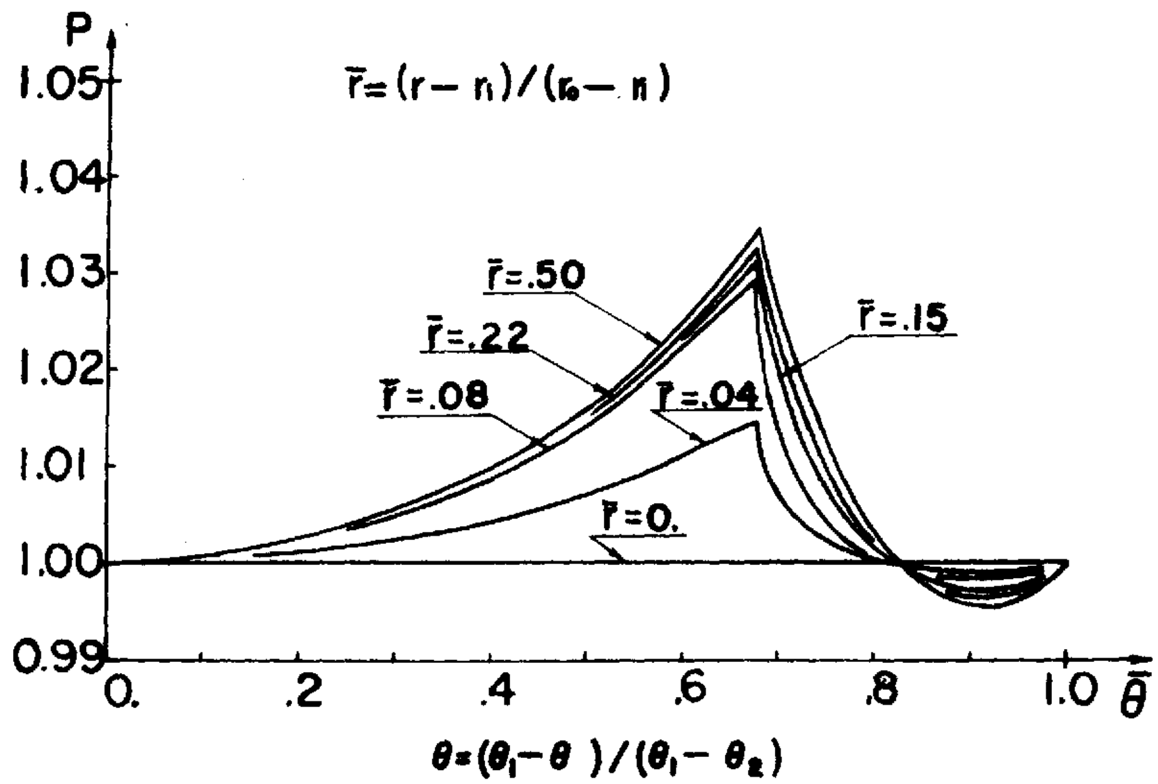


Fig. 4 Pressure distribution for h = 1.ips. H_{min} = 0.5

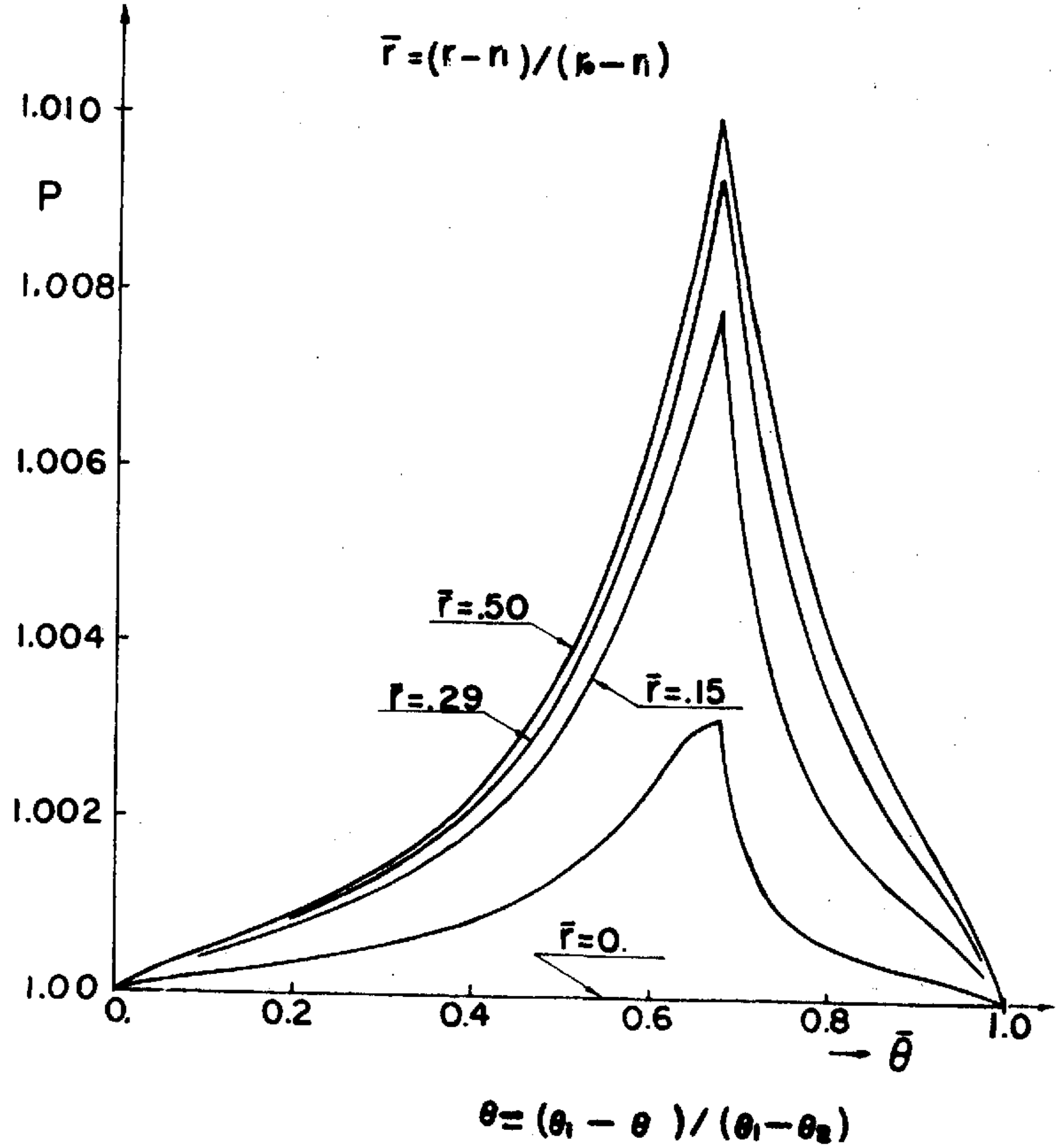


Fig. 5 Pressure distribution for h= -1. ips. H_{min}= 1.

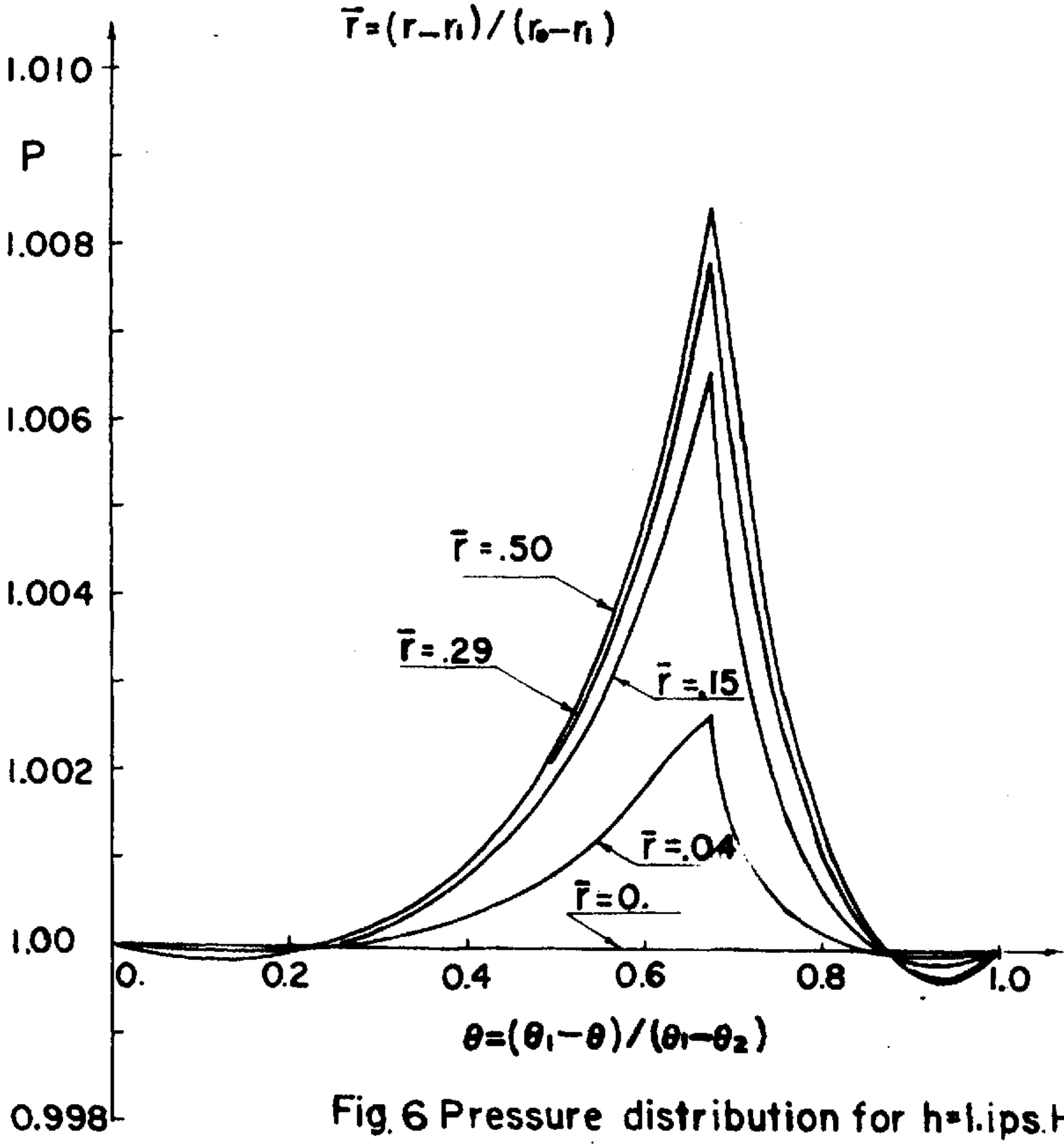


Fig. 6 Pressure distribution for h=1.ips. H=1

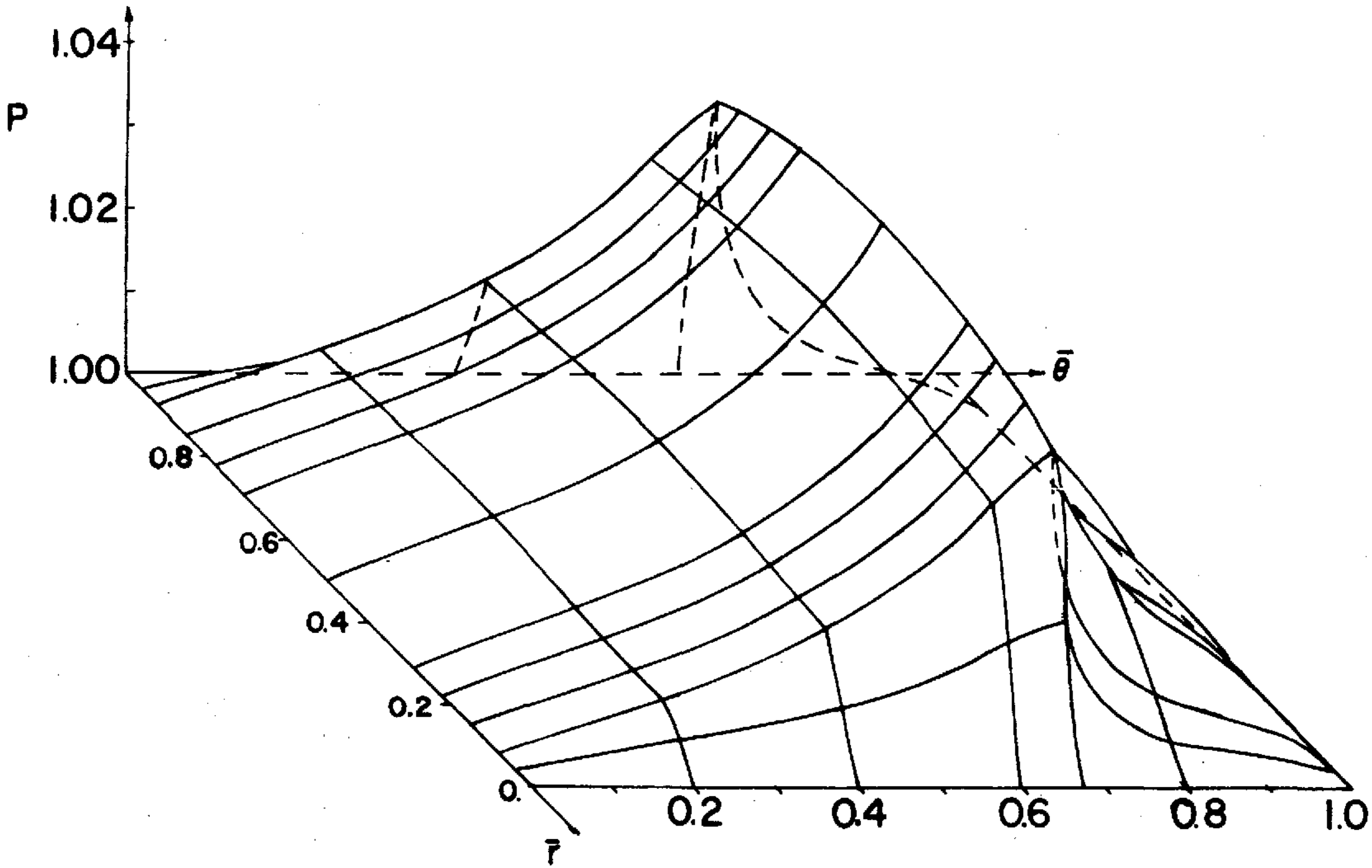


Fig. 7 CONTOUR MAP FOR PRESSURE DISTRIBUTIONS

for $\dot{h}$ = -1 in/sec, H_{min}=0.5

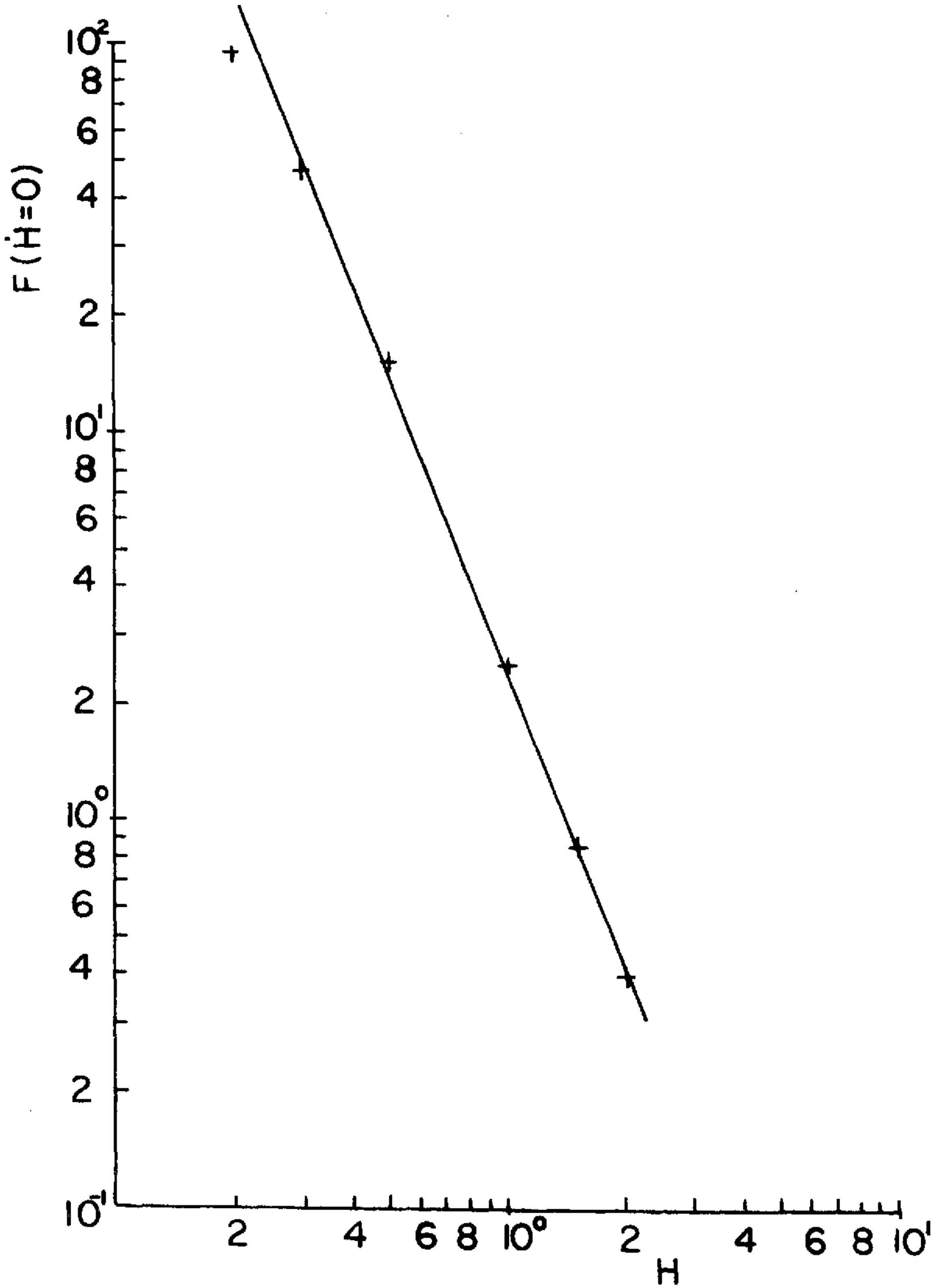


Fig.8 Variation of static, gas film force ($\dot{H}=0$) with the normalized film thickness

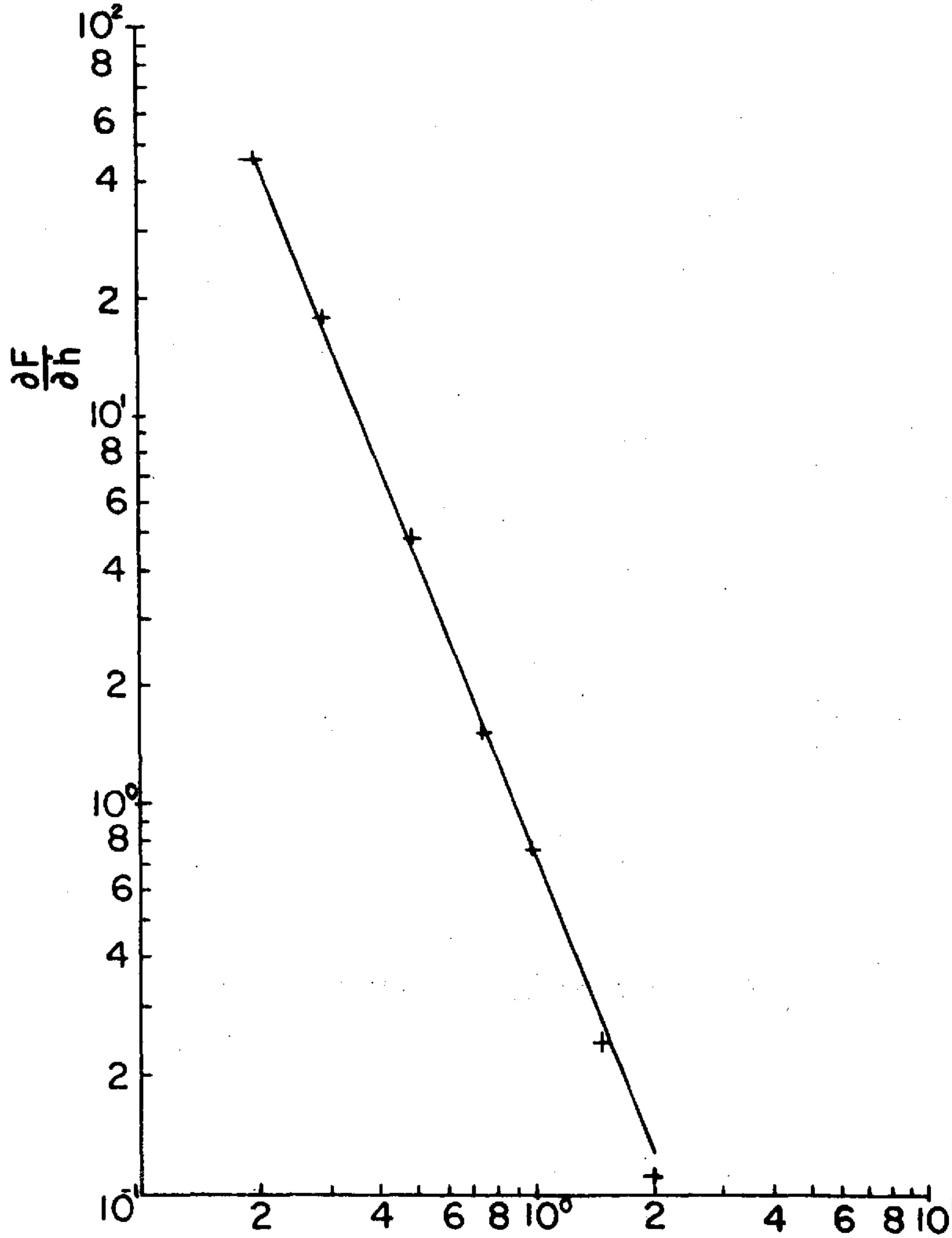


Fig.9 Variation $\frac{\partial F}{\partial h}$ with the normalized film thicknes

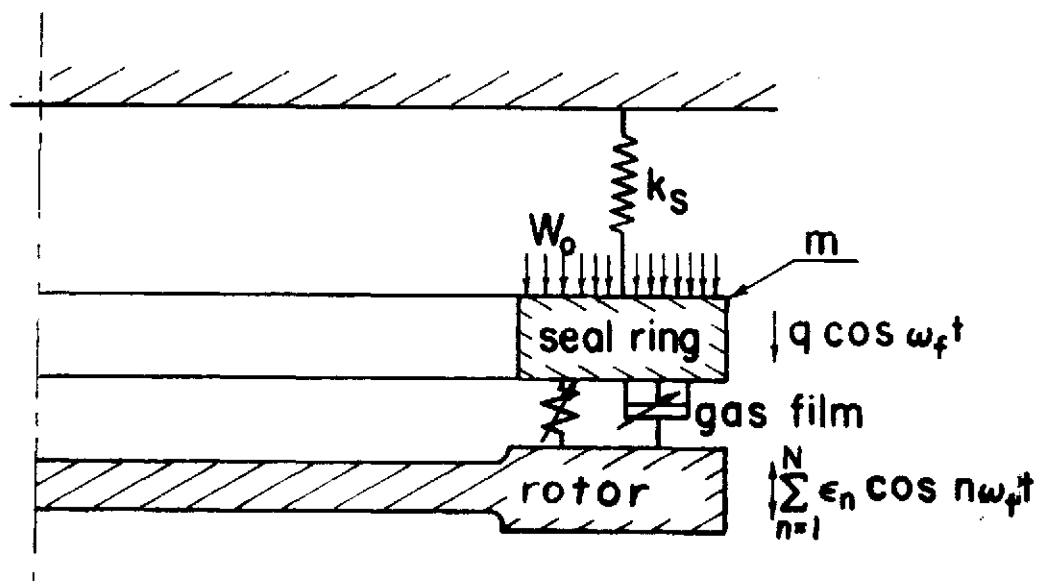


Fig.10 Simplified seal ring-rotor system

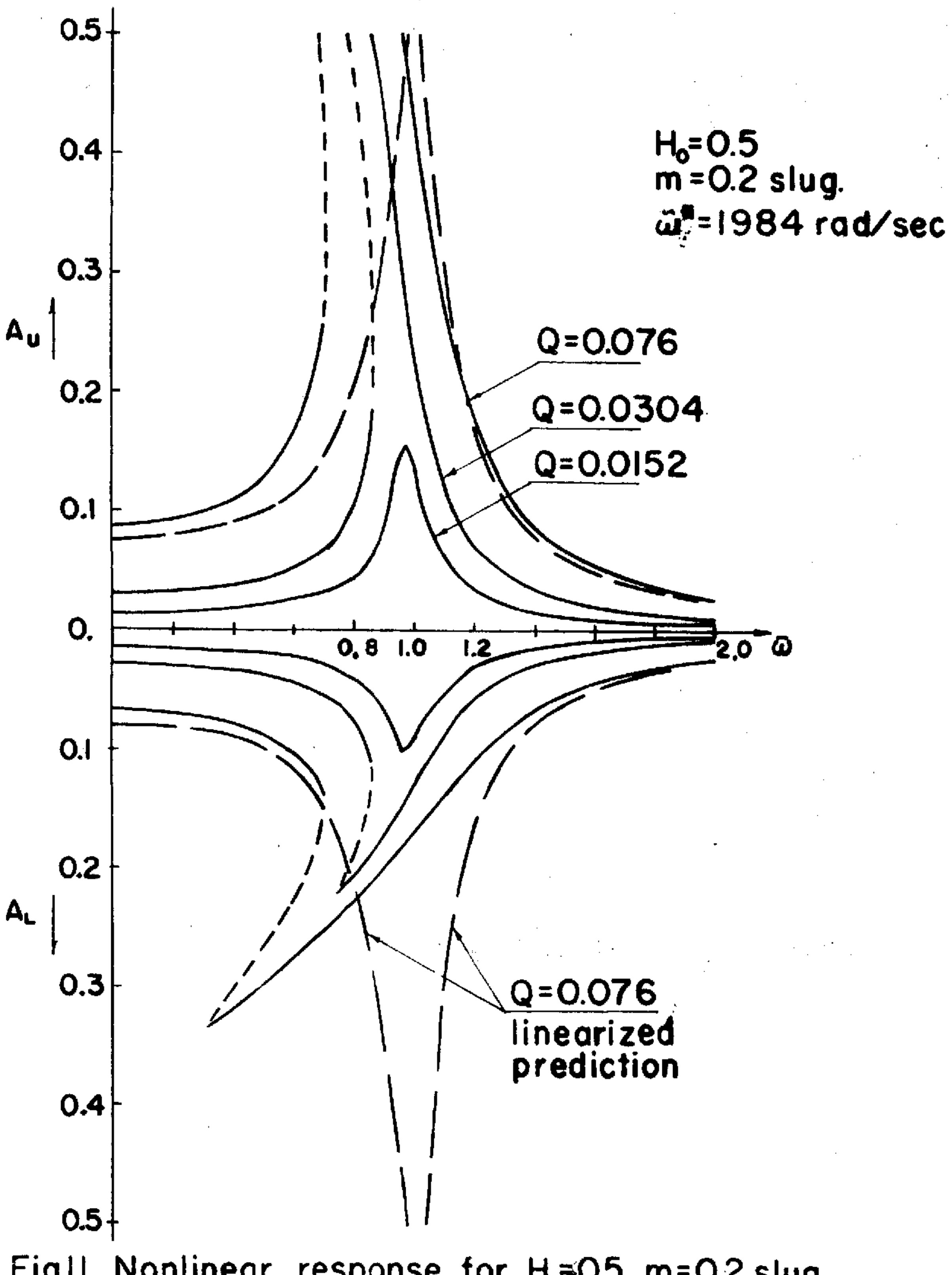


Fig.11 Nonlinear response for H_0=0.5, m=0.2 slug

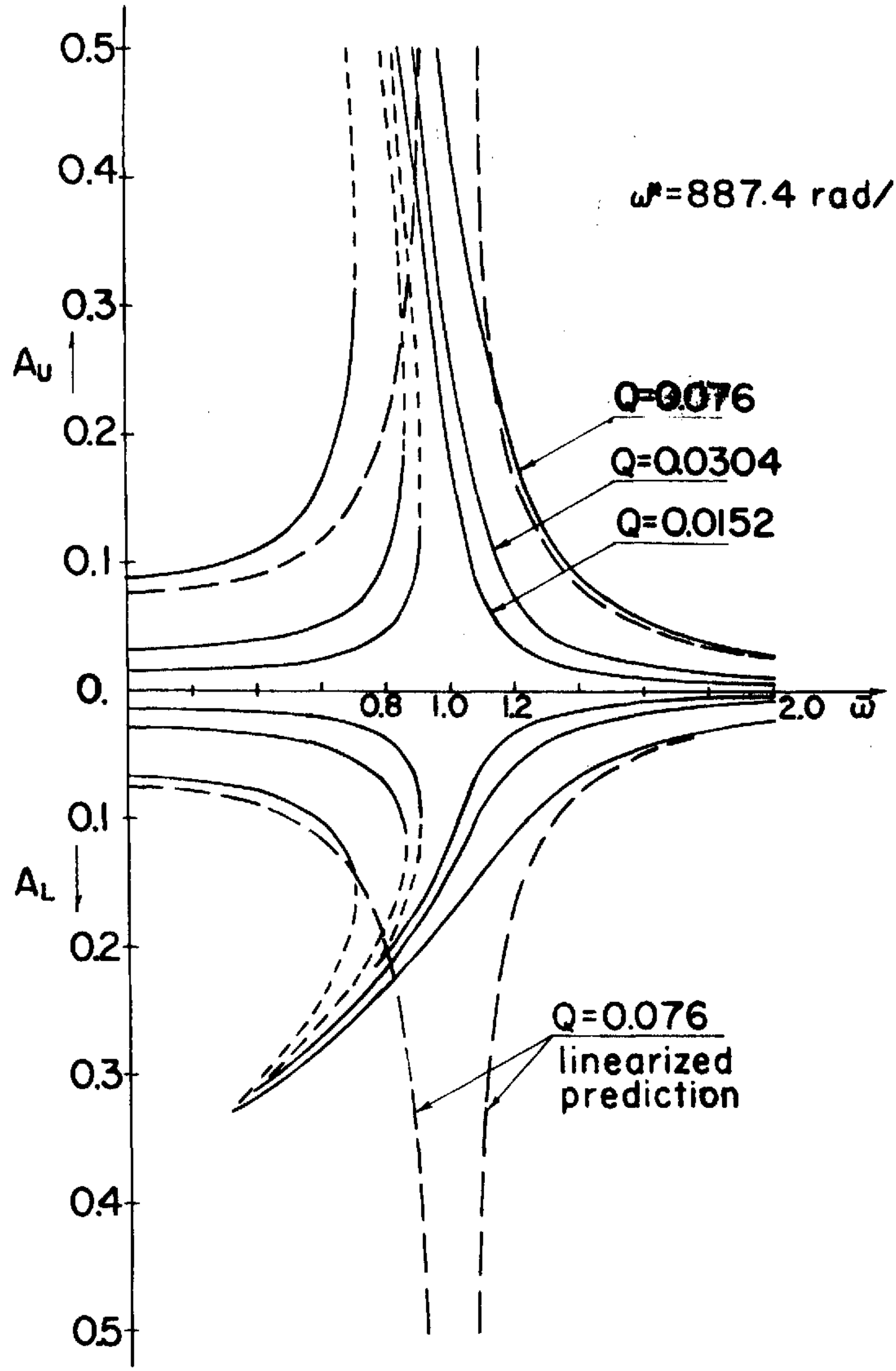


Fig.12 Nonlinear response for H_σ=0.5, m=1 slug

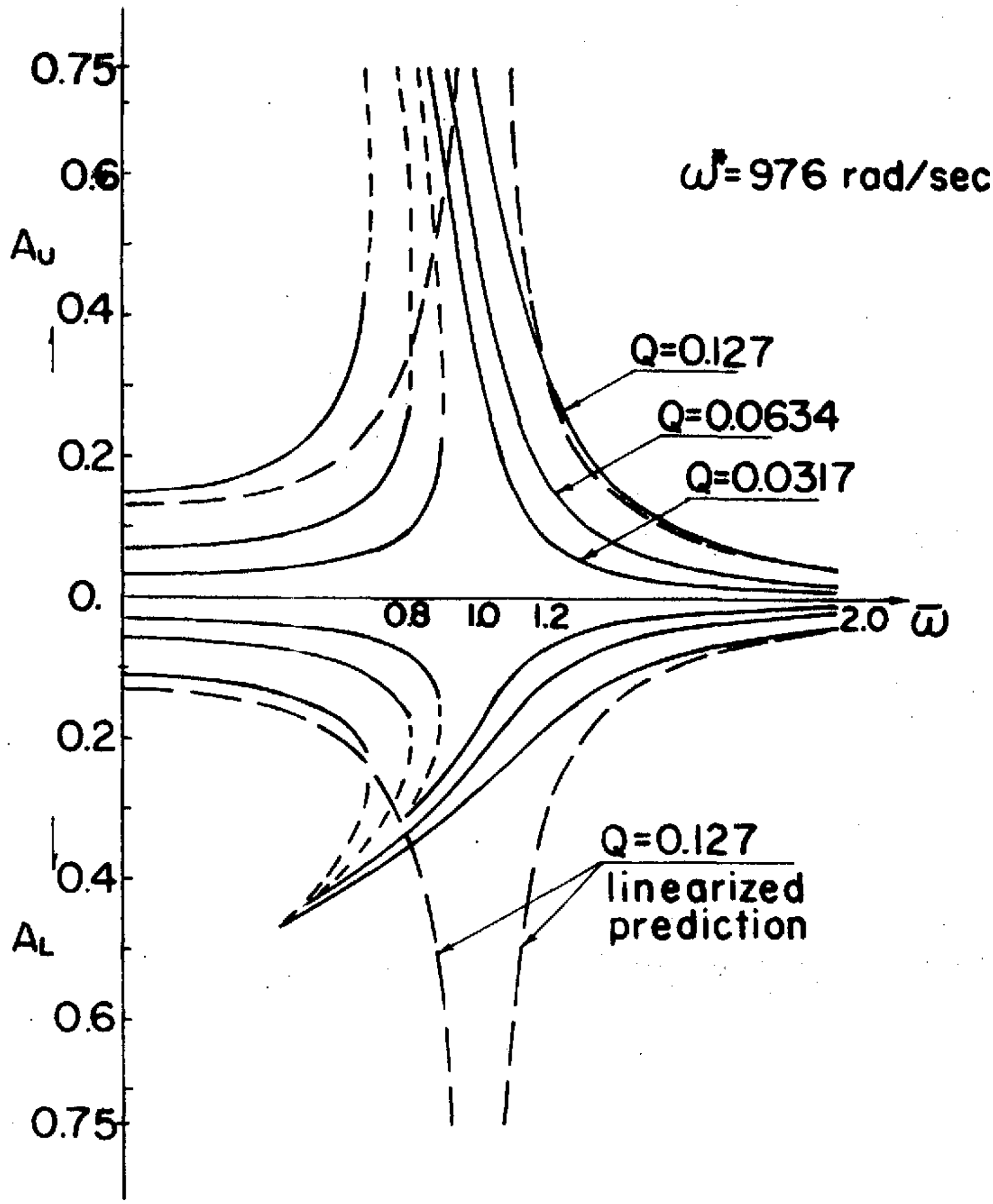


Fig.13 Nonlinear response for H_0=0.75, m=0.2 slug

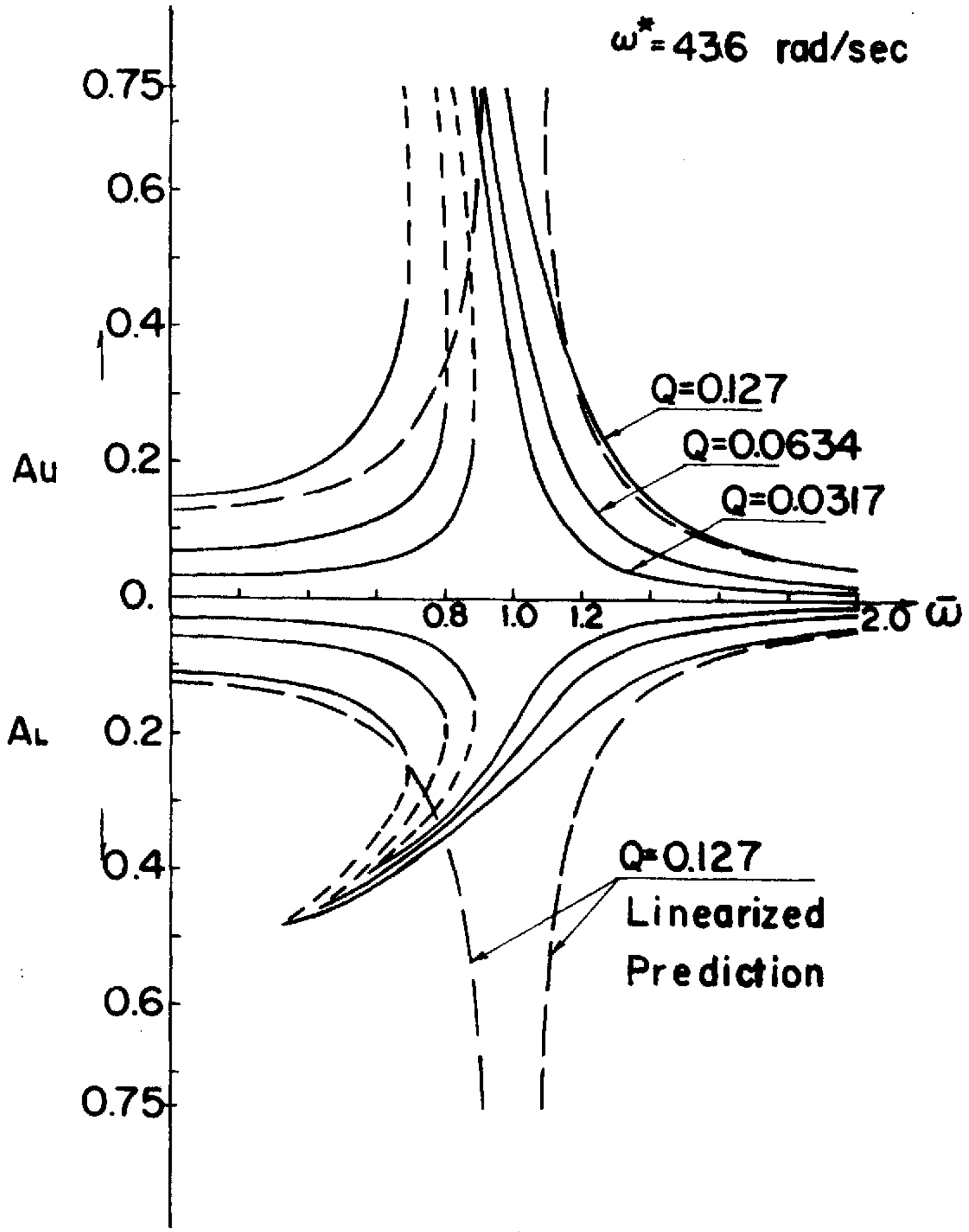


Fig.14 Nonlinear response for H_o=0.75, m=1 slug

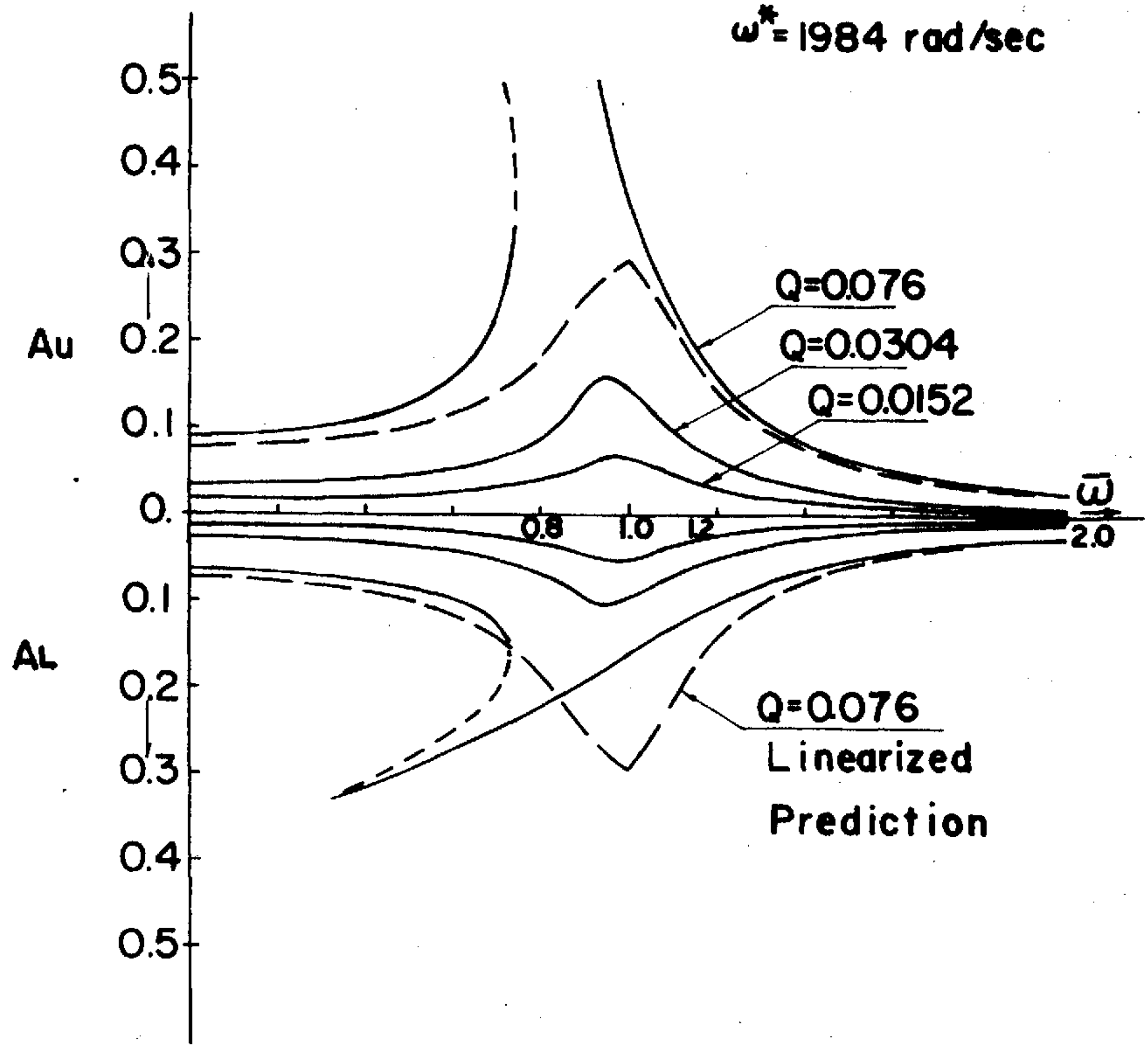


Fig.15 Nonlinear response for

$H_o = 0.5$, $m = 0.2$ slug, $C = 1.52$ lb/in/sec)

$\omega^* = 1984$ rad/sec

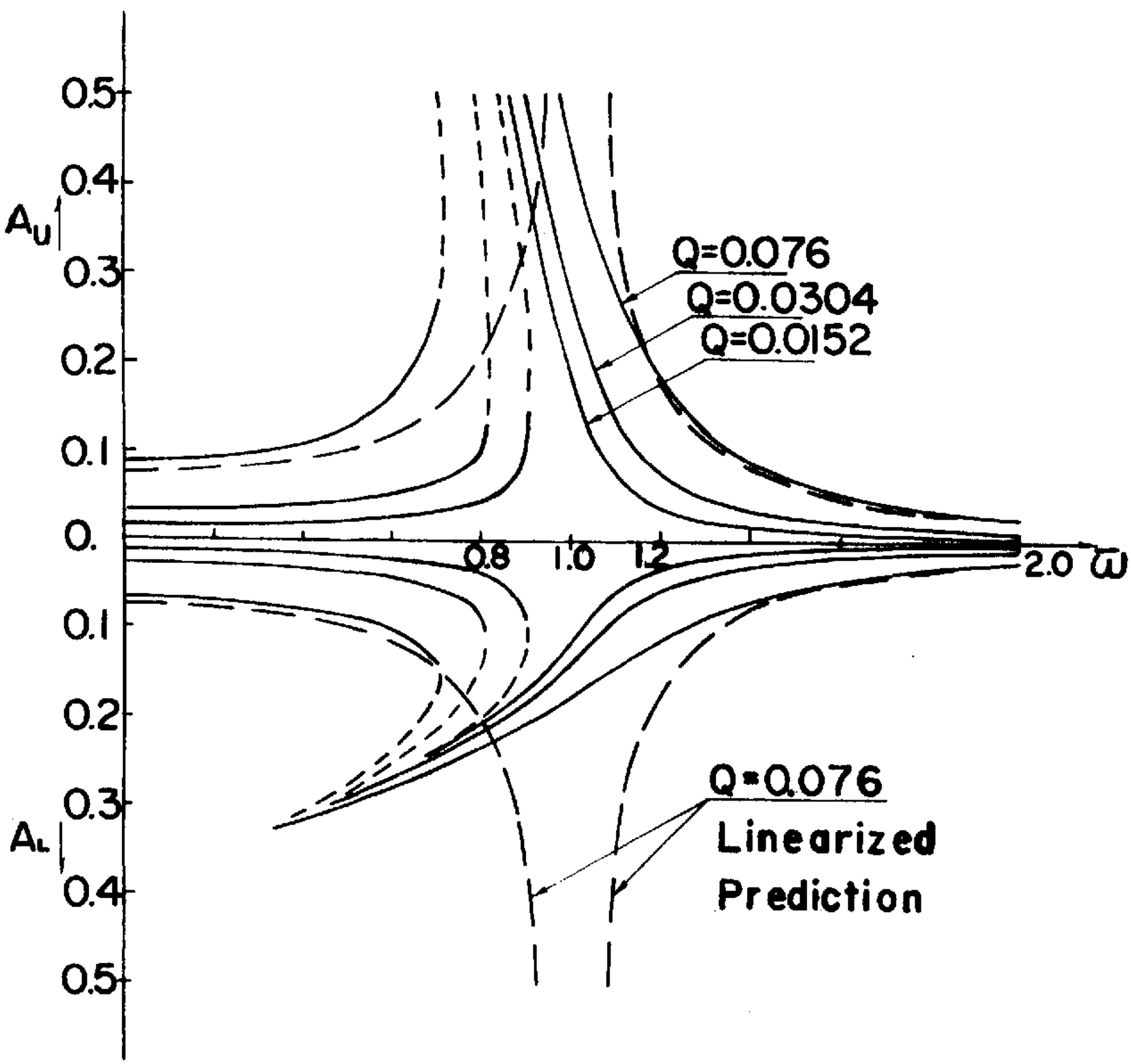


Fig. 16 Nonlinear response for $H_0 = 0.5$

$m = 0.25$ slug $C_2 = 0.38$ lbf/ips

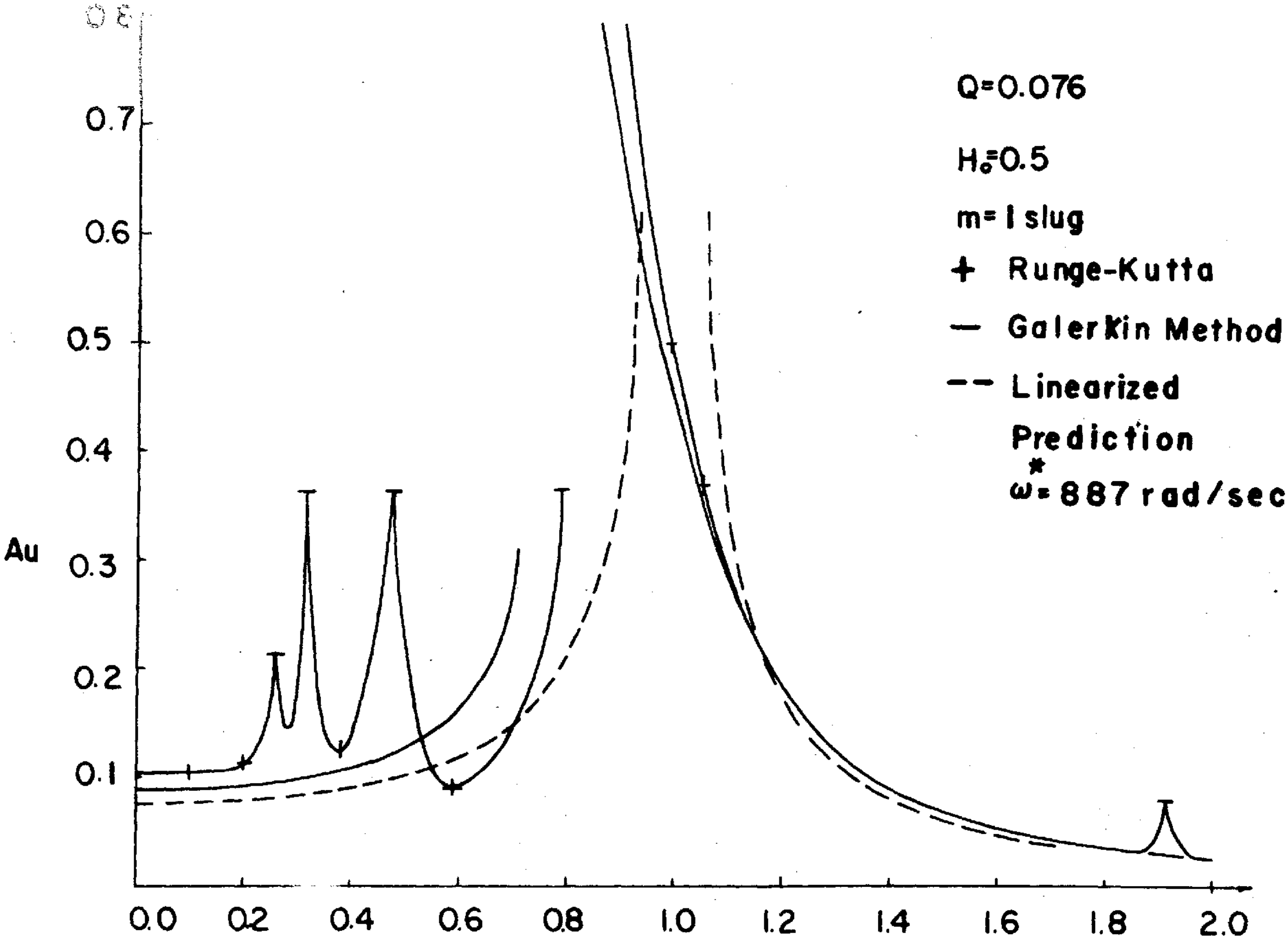


Fig. 17 Comparison of the approximate and exact upward amplitudes of response

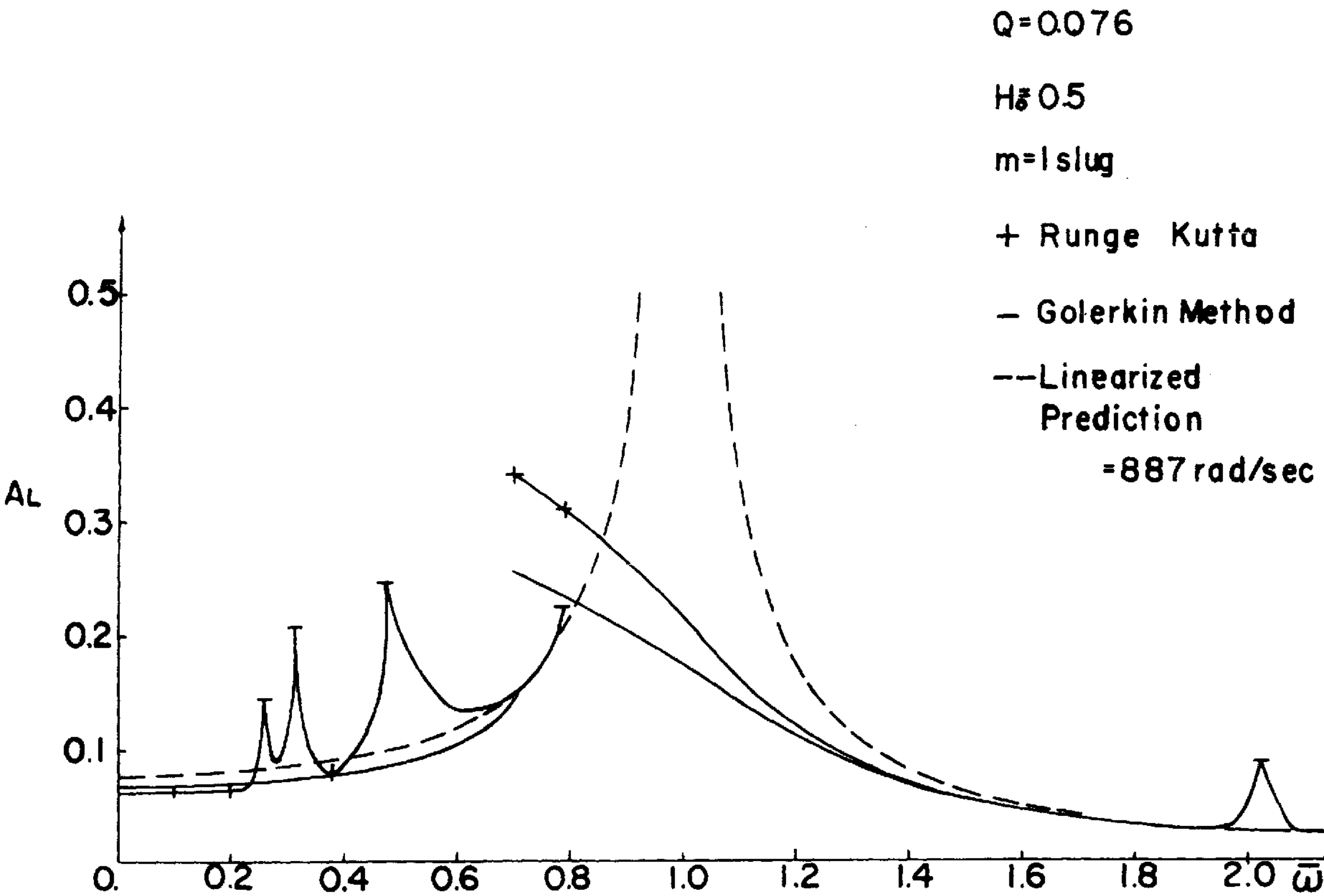


Fig. 18 Comparison of the approximate and exact downward amplitude of response

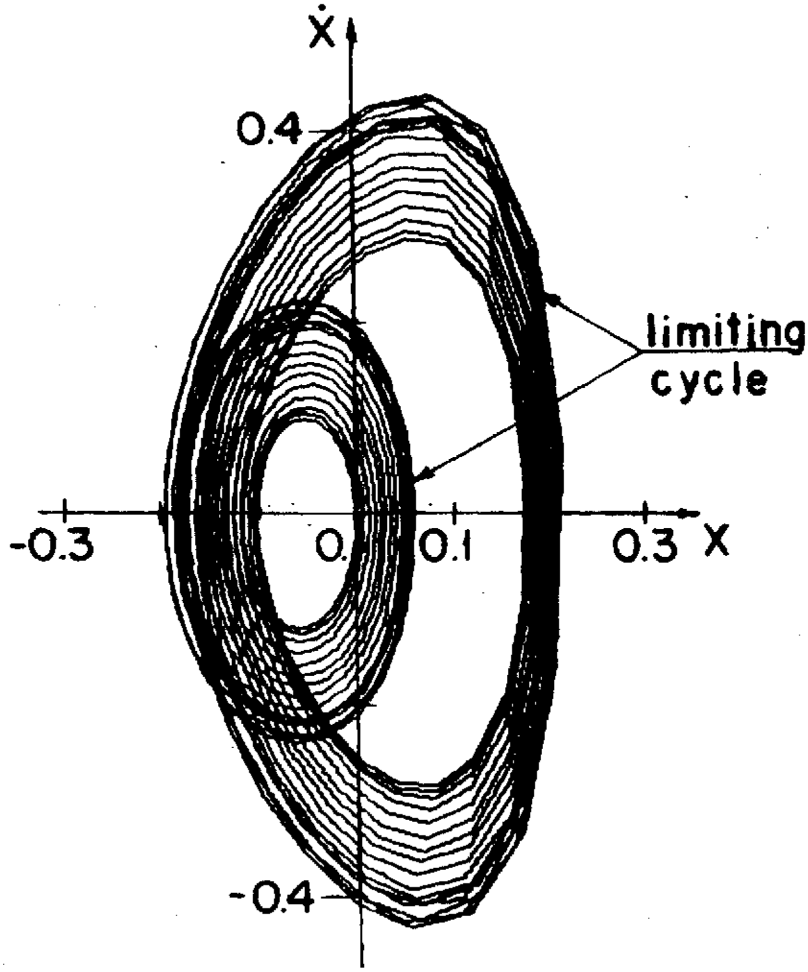


Fig.19 Phase plot with second harmonics
$\tilde{\omega}$=0.507

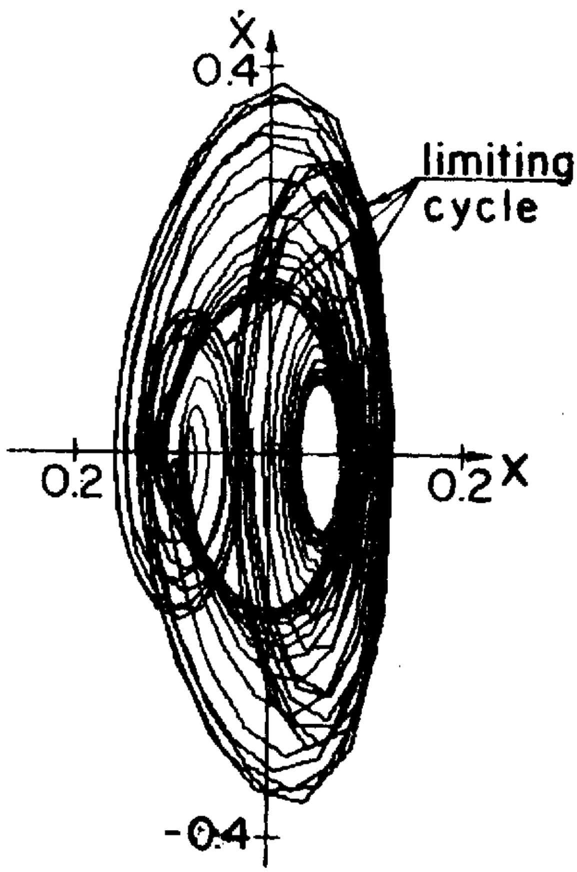


Fig.20 Phase plot with third harmonics
$\bar{\omega}$=0.338

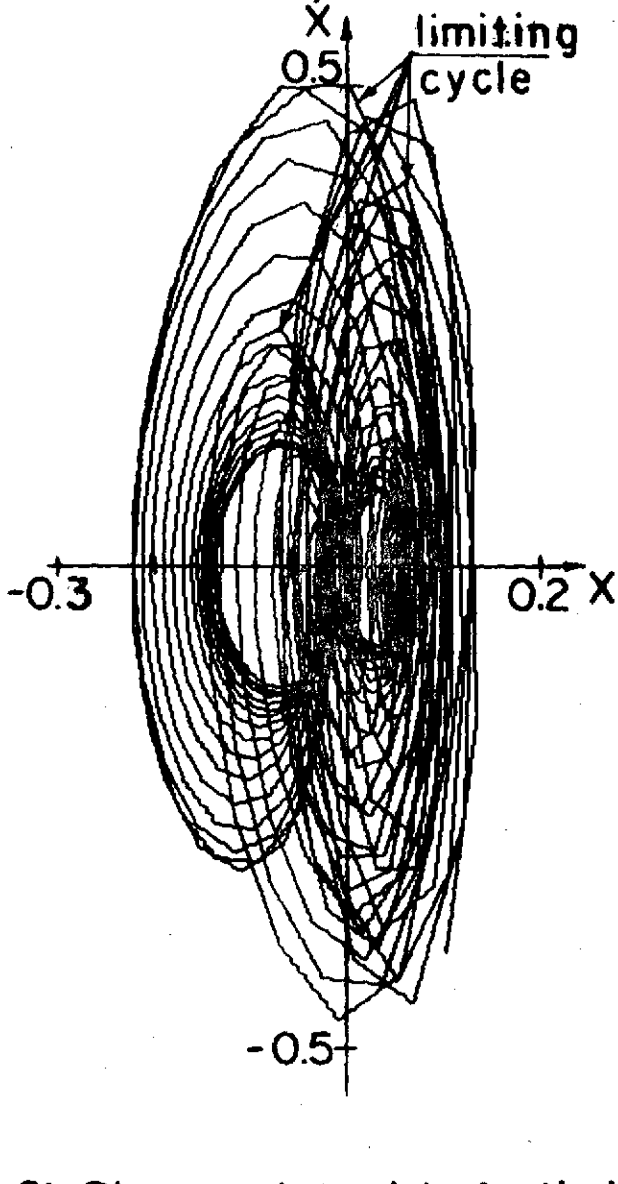


Fig.21 Phase plot with forth harmonics
$\bar{\omega}=0.248$

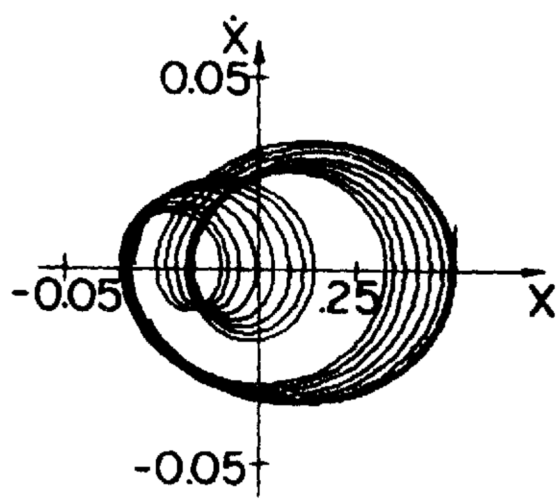


Fig.22 Phase plot with 2nd order subharmonics
$\bar{\omega}=2.096$

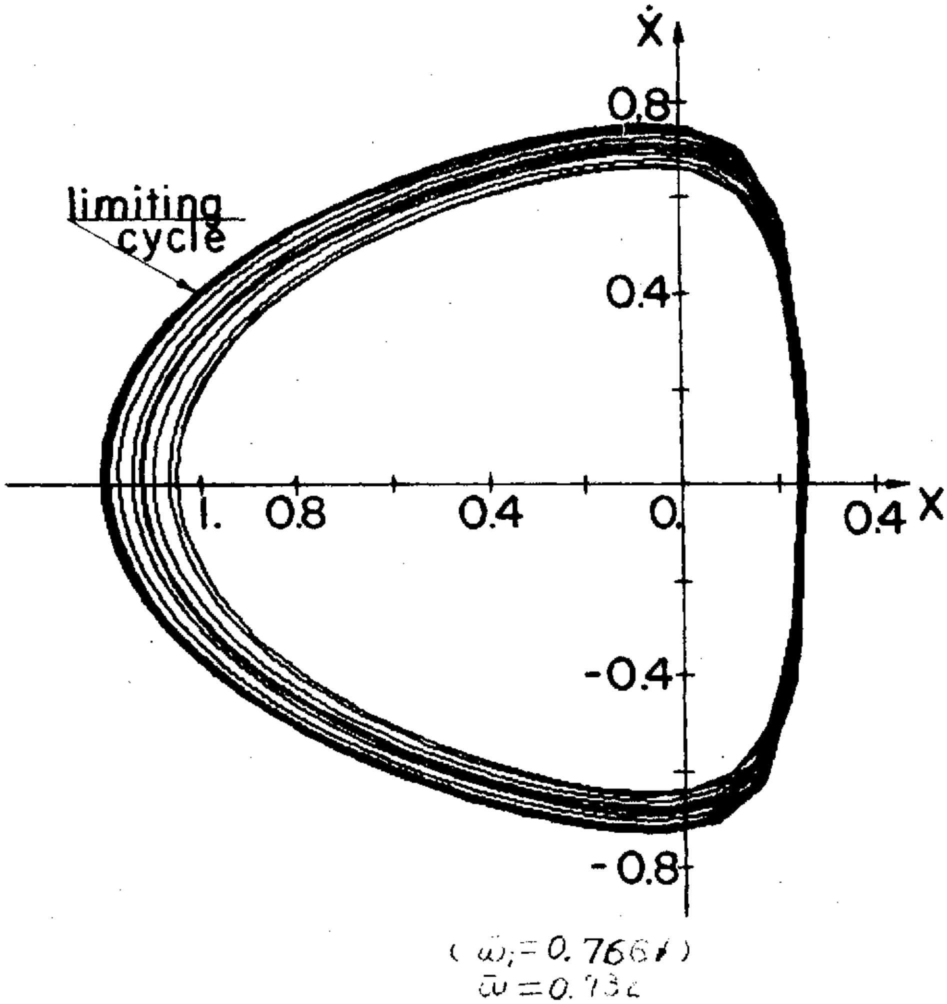


Fig.2.3 Phase plot with limiting cycle of large amplitude
$\bar{\omega}=0.732$

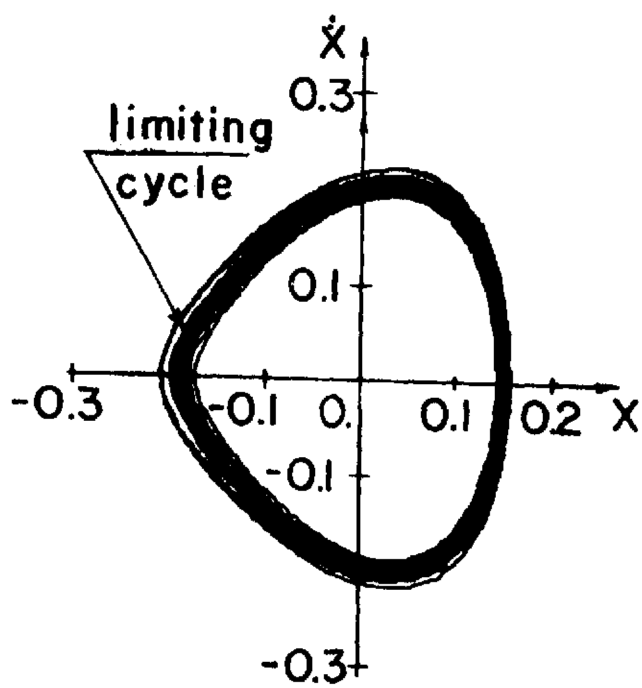


Fig.24 Phase plot with limiting cycle of small amplitude $\bar{\omega}=0.745$

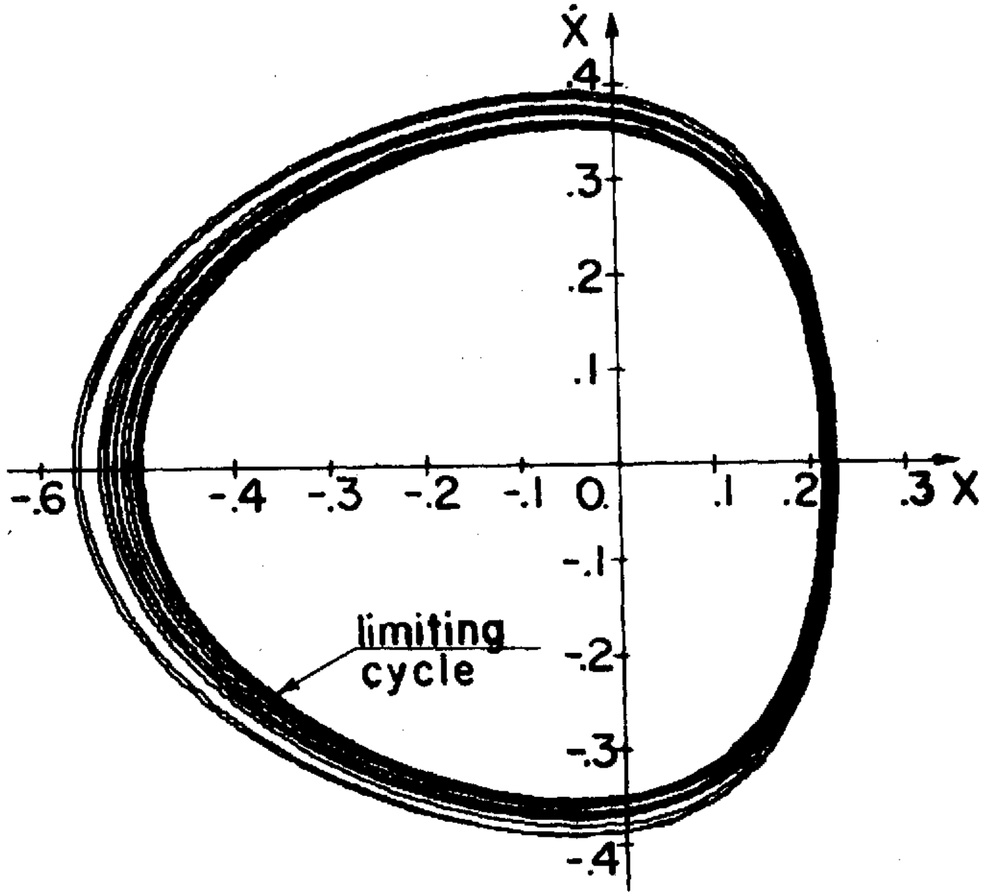


Fig.25 Phase plot at natural frequency $\bar{\omega}=1.0$

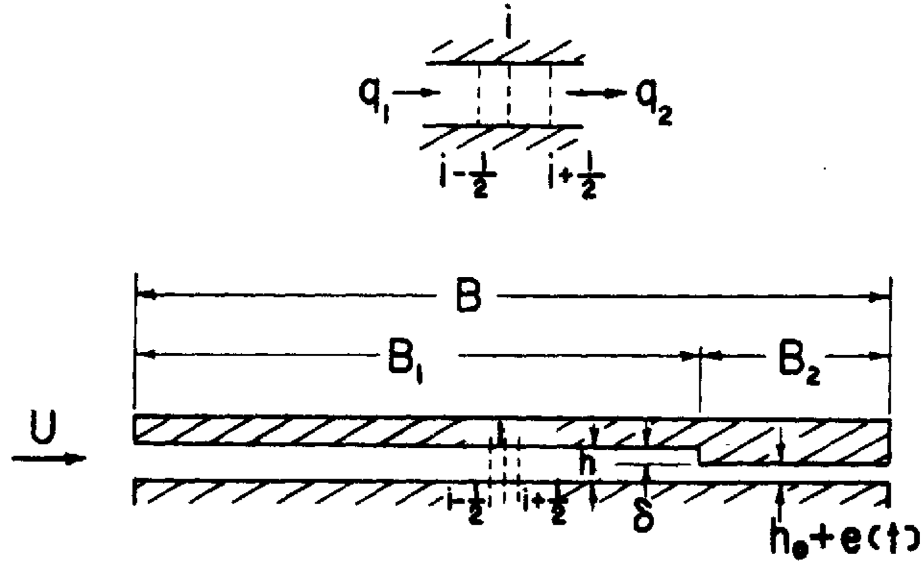


Fig.26 Flow balance and geometry of a infinitely wide Rayleigh step pad

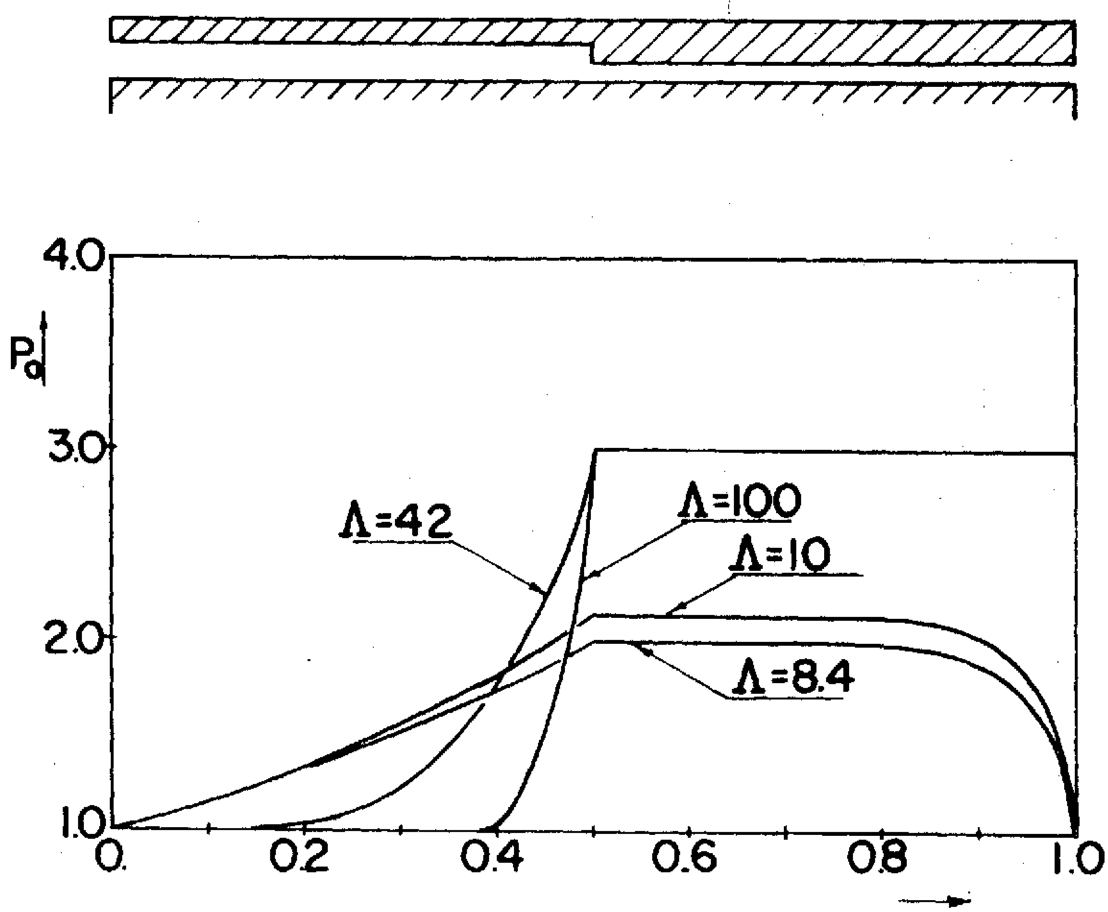


Fig.27 Pressure profile for $B_1/B=0.5$, $H_2=0.5$

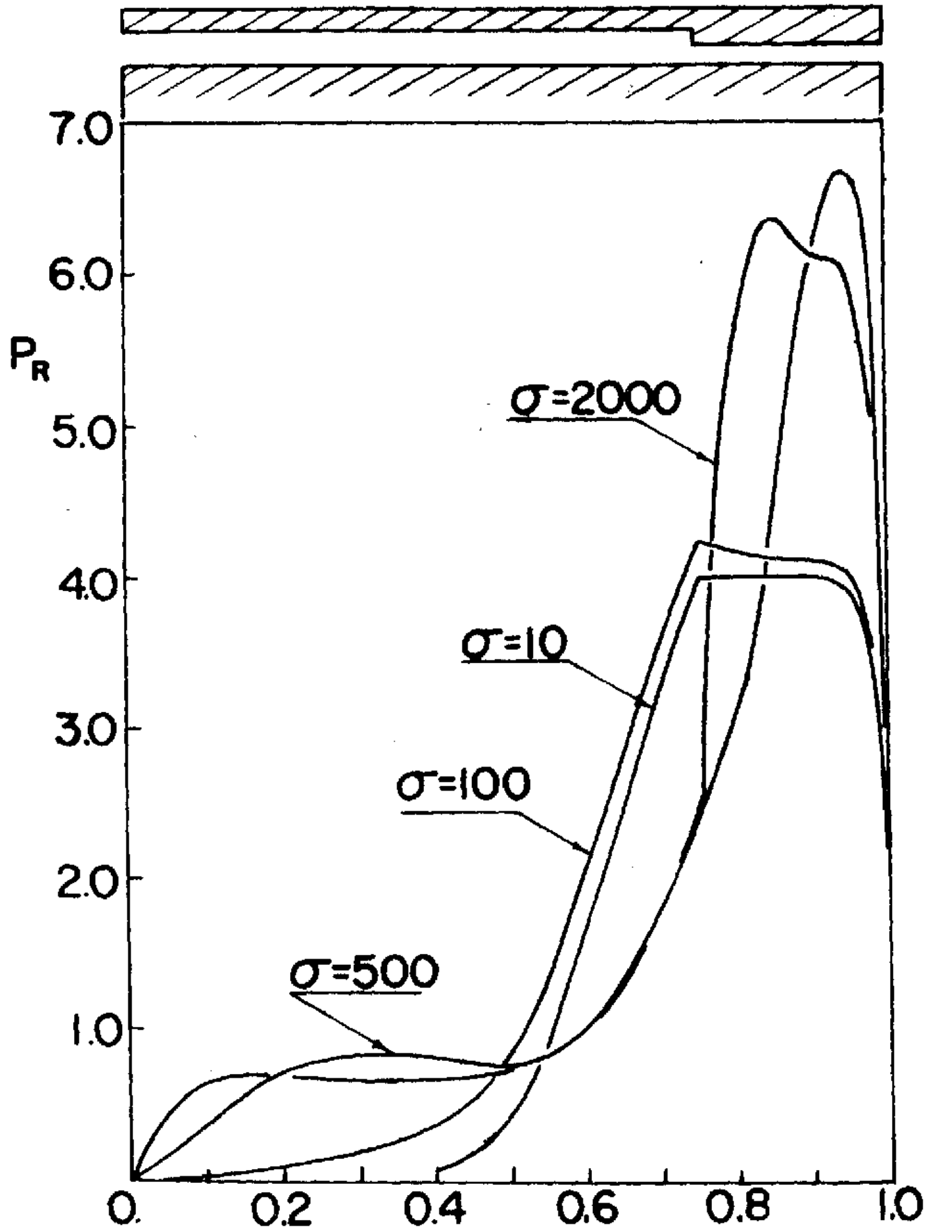


Fig.28 Real pressure profile for Λ=42, H_2=0.5, B_1/B=0.75

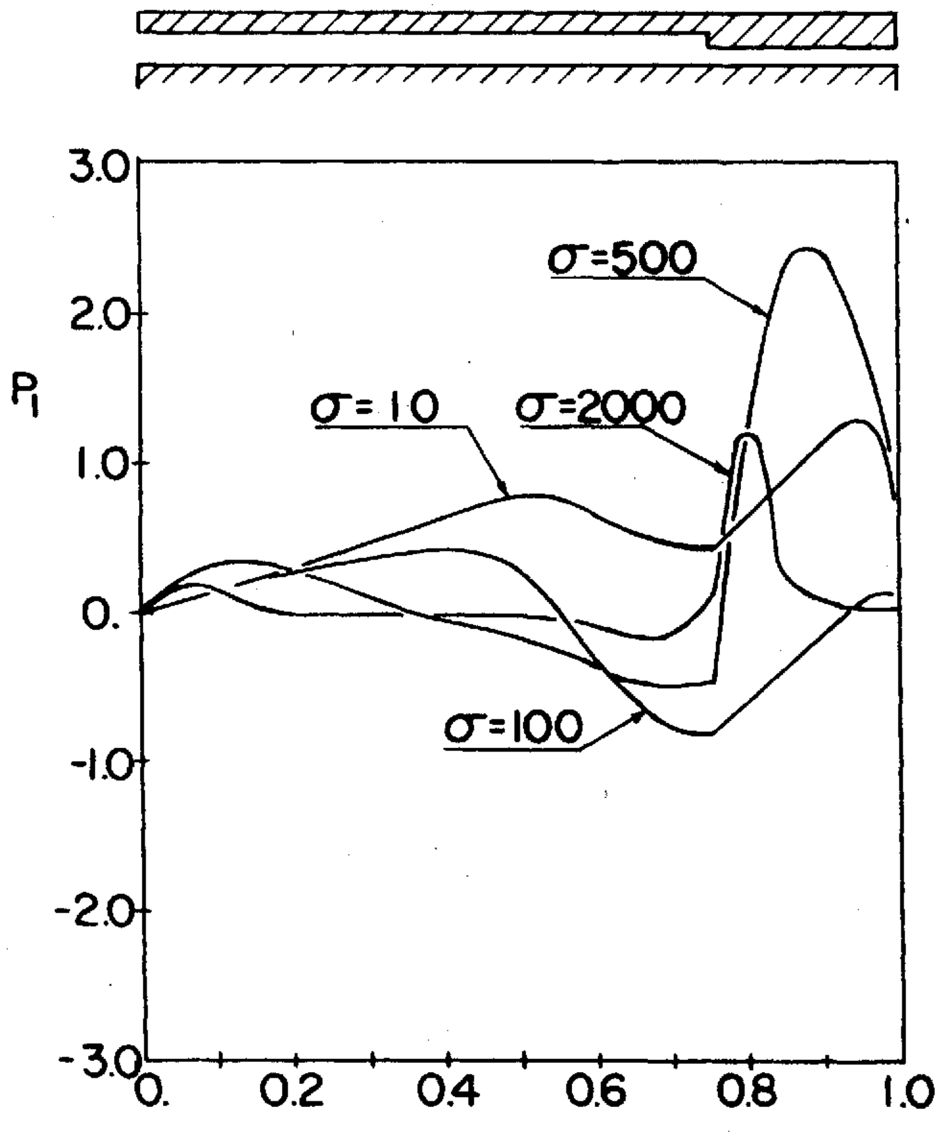


Fig.29 Imaginary pressure profile for Λ=42, H=0.5, B_1/B=0.75

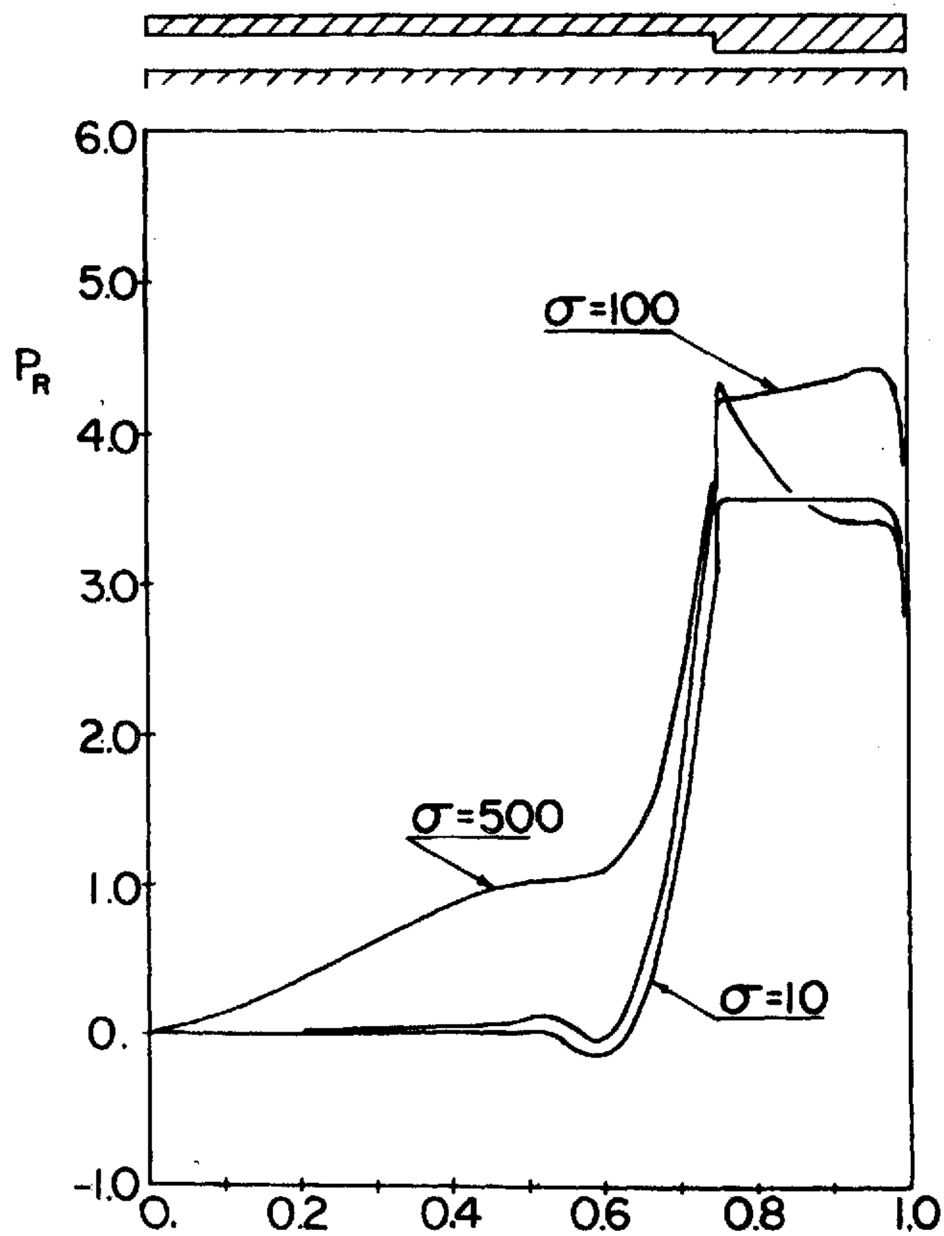


Fig.30 Real pressure profile for Λ=100, H_2=0.5, B_1/B=0.75

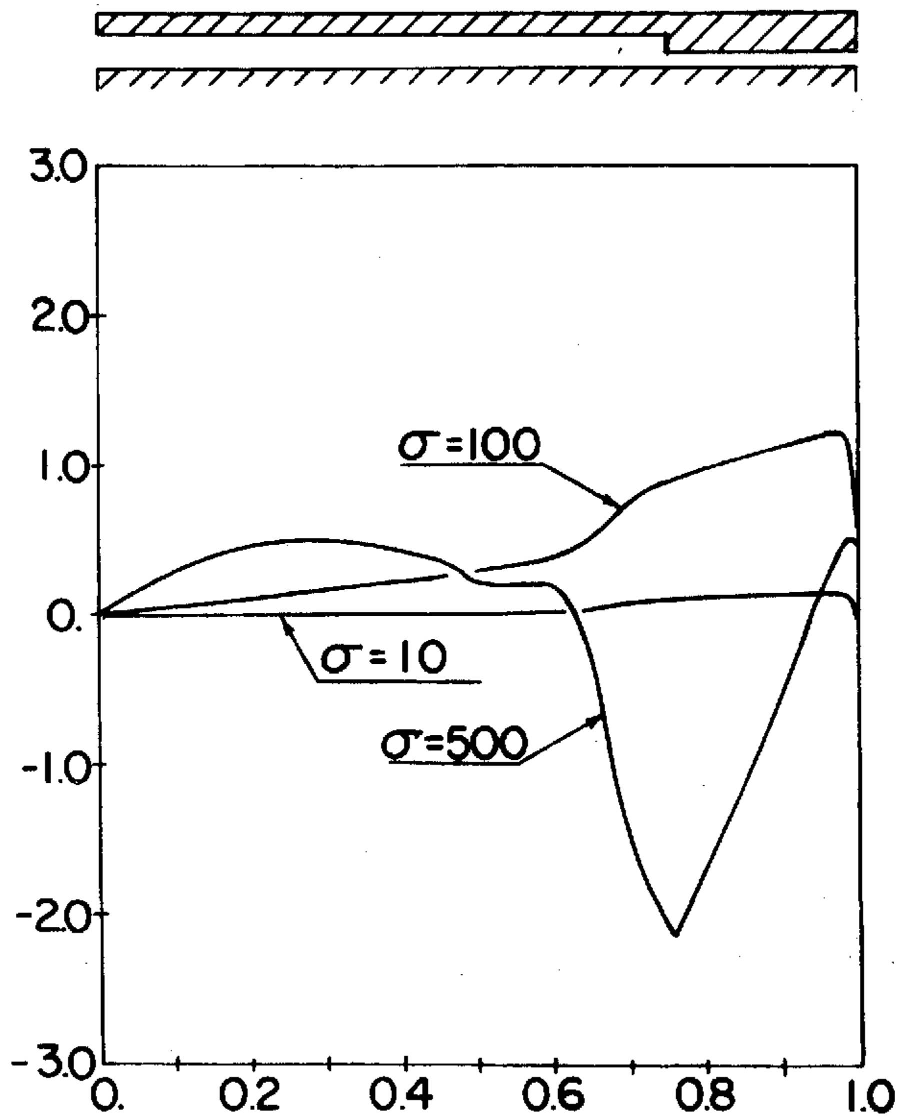
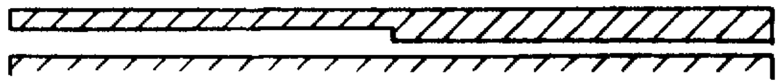


Fig. 31 Imaginary pressure profile for Λ=100, H_2=0.5, B_1/B=0.75



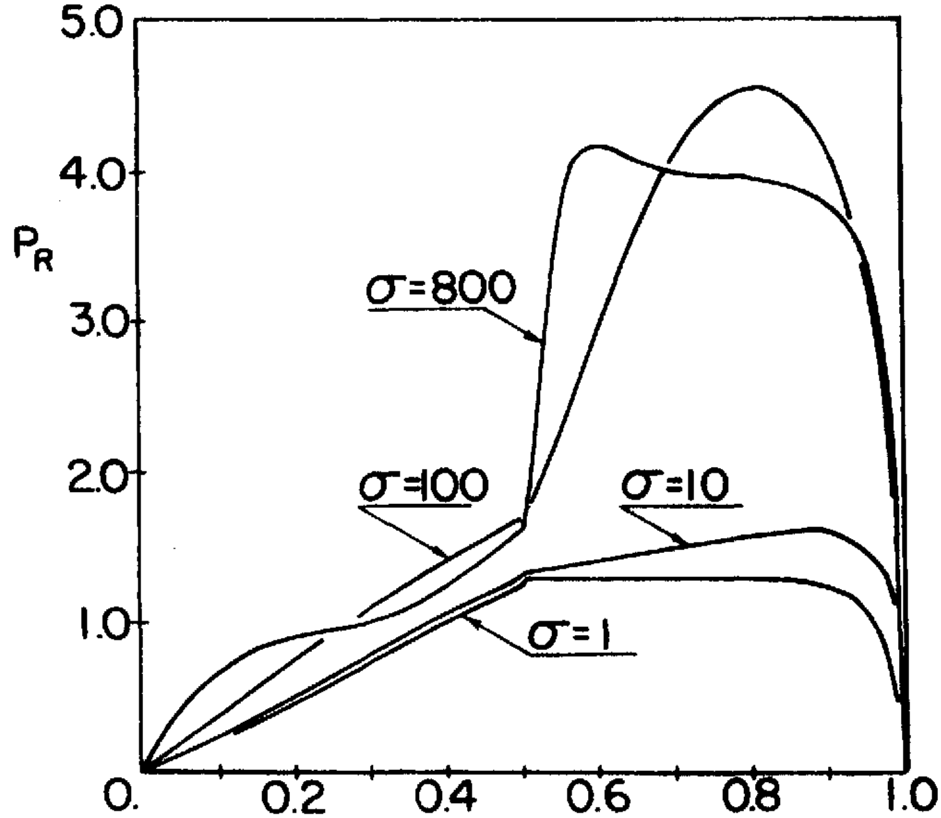


Fig.32 Real pressure profile for $\Lambda=8.4, H_2=0.5, B_1/B=0.5$

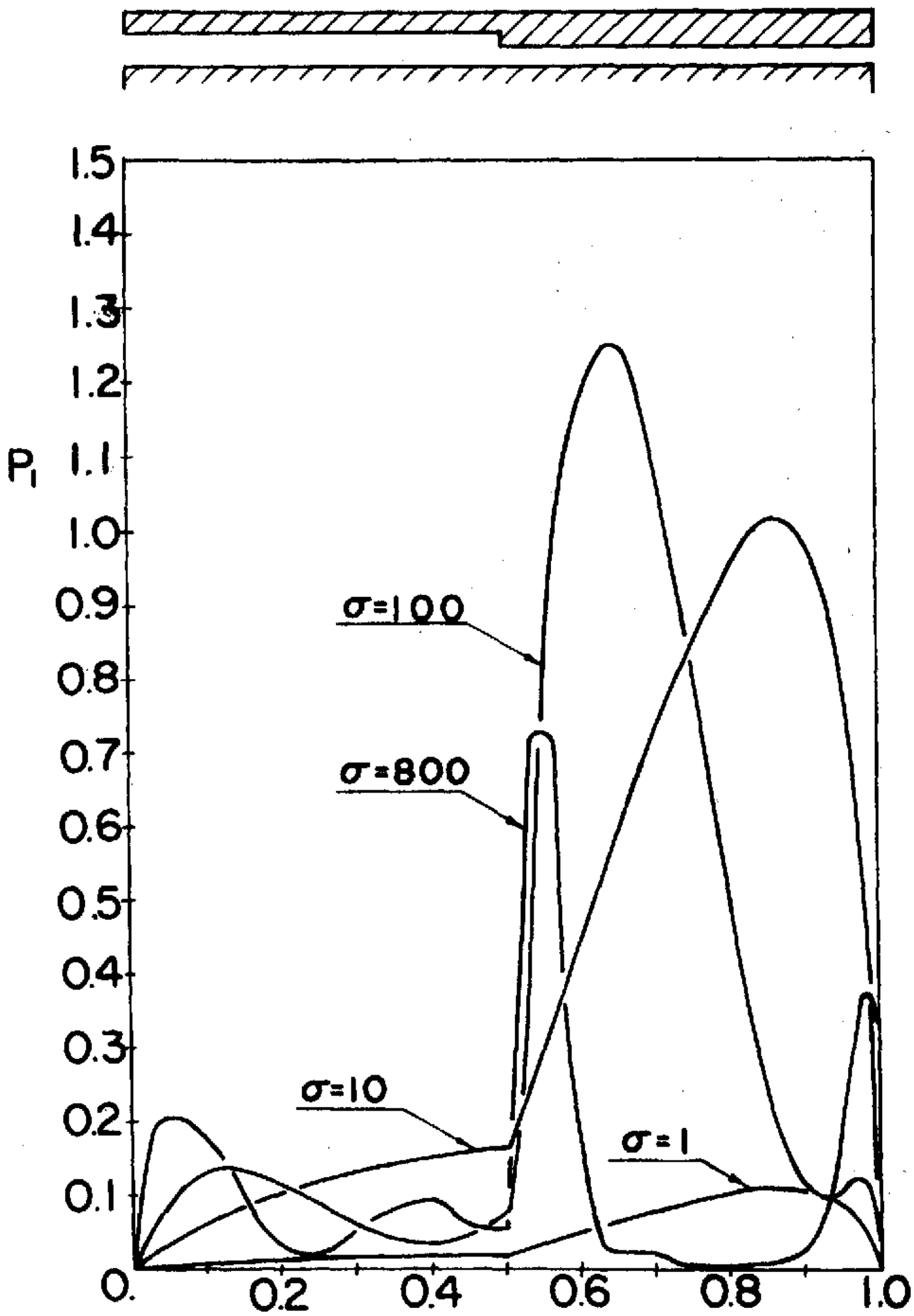


Fig.33 Imaginary pressure profile for Λ=8.4, H_2=0.5, B_1/B=0.5

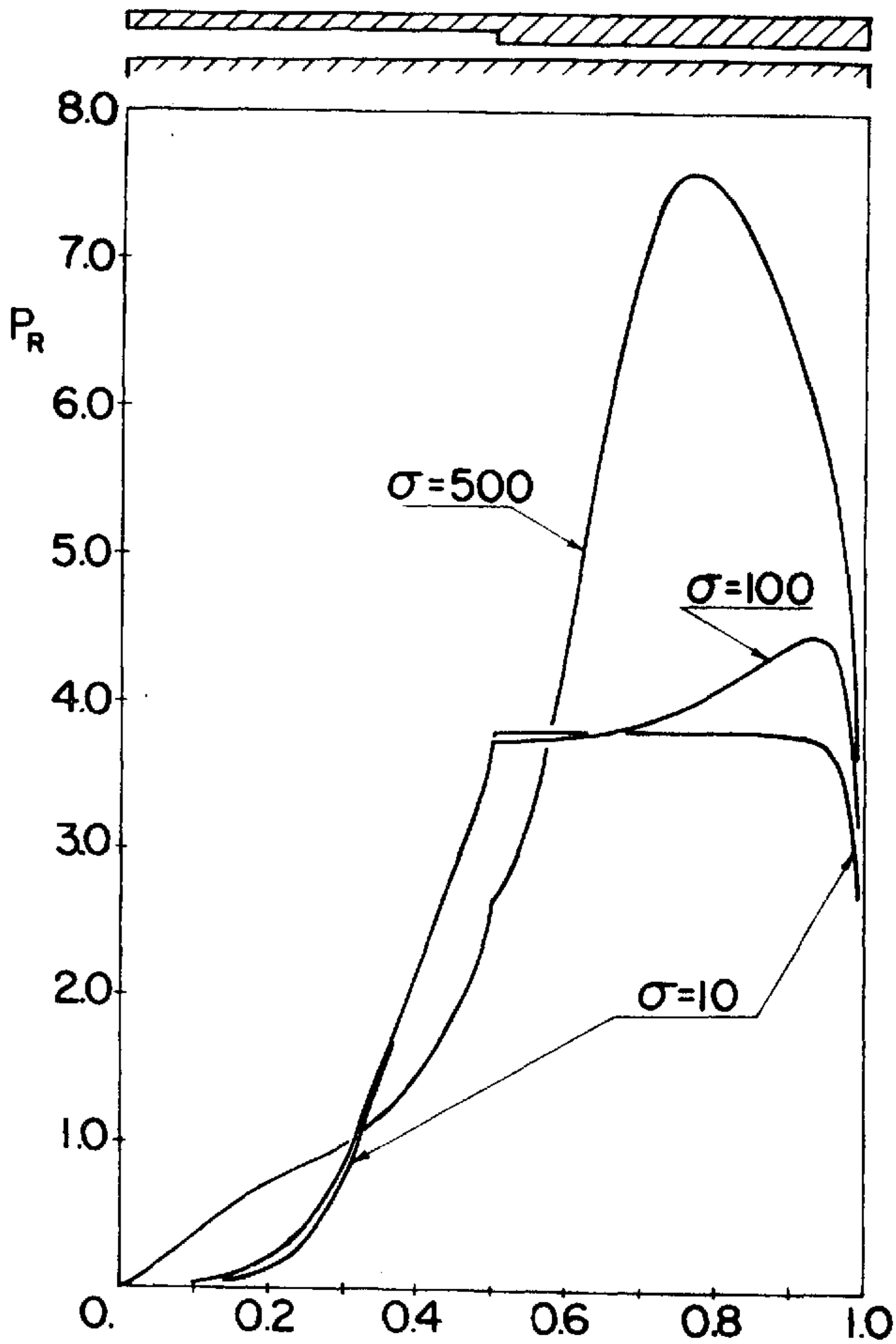


Fig.34 Real pressure profile for $\Lambda=42$, $H_2=0.5$, $B_1/B=0.5$

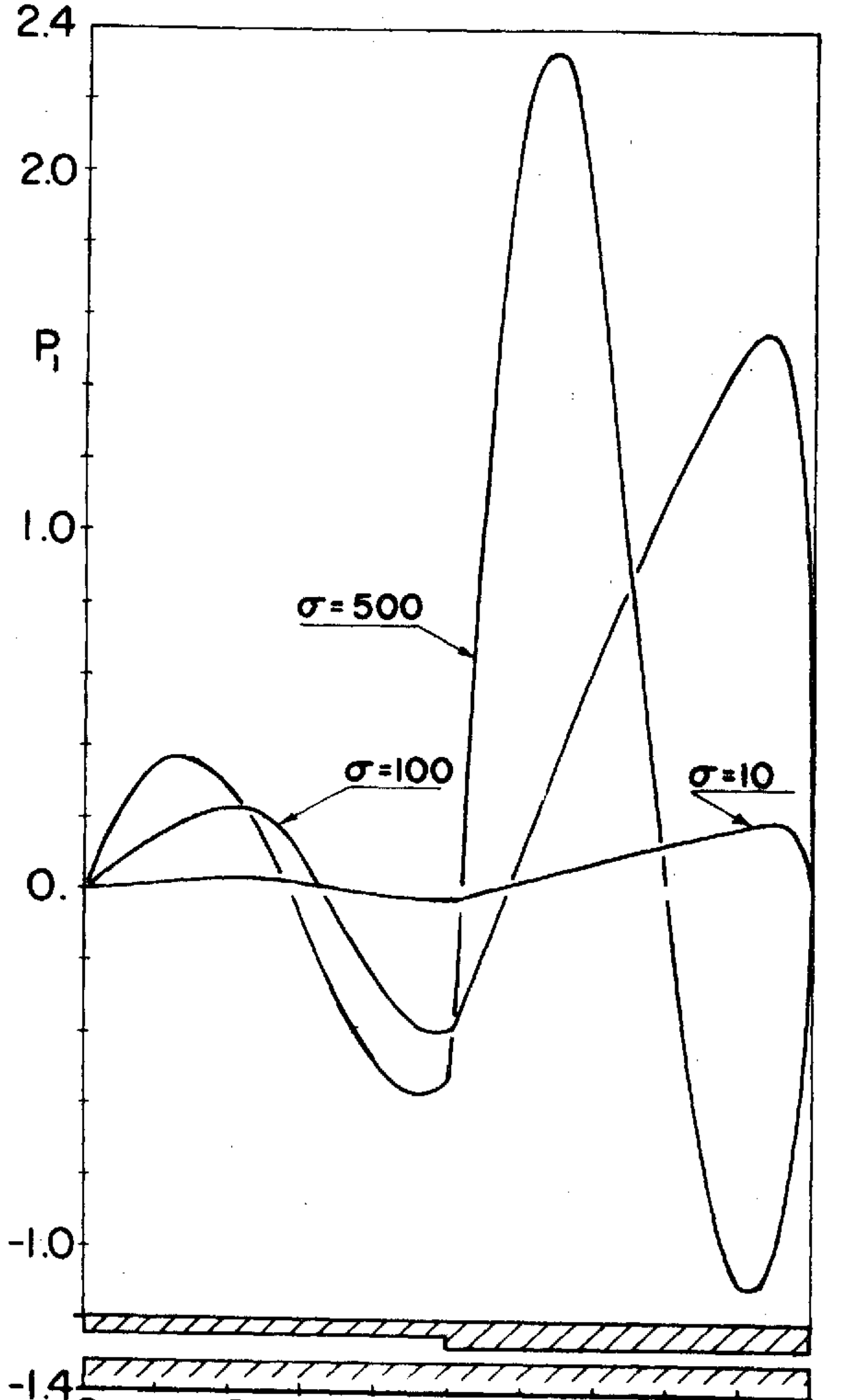


Fig.35 Imaginary pressure profile for Λ=42, H_2=0.5, B_1/B=0.5

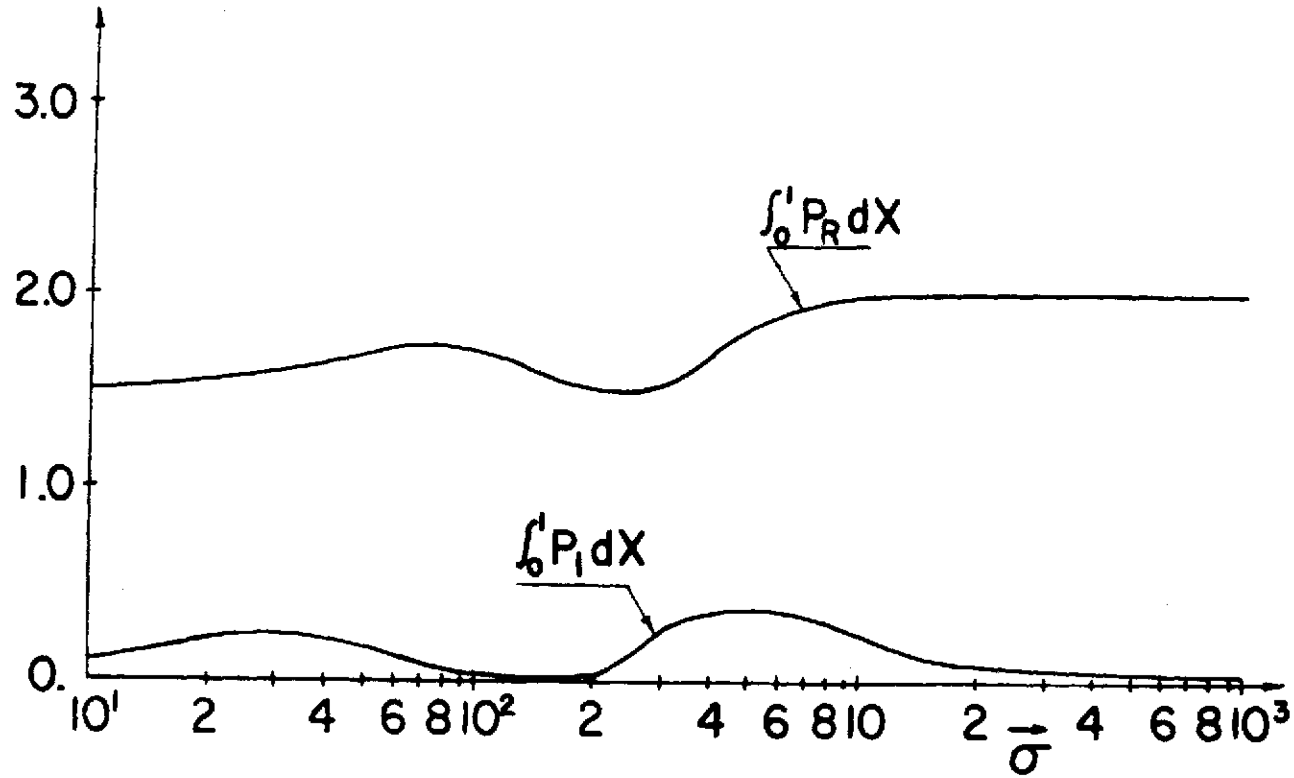


Fig. 36 Variation of dynamic bearing forces with the excitation frequency, σ for $B_1/B=0.75$, $\Lambda=42$, $H_2=0.5$

```
      PROGRAM RSEAL(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     RAYTH
      DIMENSION DR(17),DTH(33),HMINA(10),RPSA(10),PAA(10),VISA(10),TH(
     1),R(17),FRC(17),H(17,33),A1(17,33),A2(17,33),A3(17,33),A4(17,33)
     2 A5(17,33),A6(17,33),A8(17,33),B(17),C(17),A(17,33),F(17),QSMA(1
     317),E(17,17,33),G(17,33),DQ(17,33),P(17,33),Q(17,33),QQQ(33),
     4PP(17),HOUT(10),HDOT(10)
    1 FORMAT(72H
     1                         )
    2 FORMAT(16I5)
    3 FORMAT(8F10.7)
      +O-MAT (+5,5X,7E10.4/(8E10C4DD
    5 FORMAT(6X,4HLAST,6X,4HNPAD,9X, 1HM, 9X, 1HN,8X, 2HIH,8X, 2HJH,7X
     1 3HIHH,7X, 3HJHH, /)
    6 FORMAT(8I10,/)
    7 FORMAT(4X, 6HLKOUNT,5X 5HNDIAG,6X 4HIRRG, 6X,4HNPRE,/)
    8 FORMAT(/55H OUTSIDE DIAMETER(INCHES)
     1,E12.5/)
    9 FORMAT(55H INSIDE DIAMETER(INCHES)
     1 E12.5/)
   10 FORMAT(55H THE ANGLE EXTENDING THE POCKET REGION(DEG.)
     1 E12.5/)
   11 FORMAT(55H THE ANGLE EXTENDING THE LAND REGION(DEG.)
     1 E12.5/)
   12 FORMAT(55H STEP DEPTH(INCHES)
     1 E12.5/)
   13 FO-MAT(55+ OUTE- WIDTH OF THE SH-OUD(INCHES)
      +12+5/D
   14 FORMAT(55H INNER WIDTH OF THE SH-OUD(INCHES)
     1 E12.5/)
   15 FORMAT(55H CONVERGENCE ERROR
     1 E12.5/)
   16 FORMAT(54H GRID SPACINGS IN THE RADIAL DIRECTION(INCHES)
   17 FORMAT(54H GRID SPACINGS IN THE CIRCUMFERENTIAL DIRECTION(DEG)
   18 FORMAT(55H MINIMUM FILM THICKNESS (INCHES)
     1 E12.5/)
   19 FORMAT(55H REVOLUTIONS PER SECOND
     1 E12.5/)
   20  O--1T(1H , E12.5)
   2                              -+2S -EO-S++
     1 E12.5/)
   22 FORMAT(55H VISCOSITY(LB-SEC/+N$$2)
     1 E12.5/)
   23 FORMAT(55H LAMDA(6.*VIS$6.28$RPS$(RO/HMIN)**2/PA)
     1 E12C5/D
   24 FORMAT(12H RB(I),TH(J)      )
   25 FO-MAT(25H HL,H-,HLH,HRH,HTH,HBH           )
   2    -- 1TU15+ A1U+TJDTETC          D
   27 FO-MAT(45+ F(-),BU+D,C(+DTA(+TI-1),A(I,1),A(+,++1)        ,10X,2H
     1 ,I5,10X,2HJ=,I5)
   28 FORMAT( 25H MATRIX IS SINGULAR AT J= I3,16H,CASE ABANDONED./1
   29 FORMAT(29H0FINAL PRESSURE DISTRIBUTION. //)
   30 FORMAT( //18H CASE CONVERGES TO F9.6,
     1                                   14H          AFTER I3,11H ITER
```

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OF POOR QUALITY

```
  1ONS)
31 FORMAT( /                    10(1X,F11.7))
33 FORMAT(55H FILM THICKNESS AT O.D (INCHES)
  1 E12.5/)
34 FORMAT(/55H TOTAL LOAD(LBS)
  1,E12.5/)
35 FORMAT(55H DIMENSIONLESS LOAD=LOAD/(AREA*PA)
  1 E12.5/)
36 FORMAT(55H HORSEPOWE- LOSS
  1 E12.5/,1H1)
37 FORMAT(55H  STATIC SQUEEZE FILM VELO (IN/SEC)
  1 E12.5/)
38 FORMAT (I5,5X,(8E10.4))
3   +O-MATU55+ D+MENS+-NLESS STAT+C F+LM VELOC+TY W+TH HO=STEP DEPTH
  1 E12.5/)
    NR=5
    NW=6
    READ(NR,1)
90  READ(NR,2)LAST,NPAD,M,N,IH,JH,IHH,JHH,LKOUNT,NDIAG,IRRG,NPRE ,NSYM
   1,ND1
    READ(NR,3)DO,DI,THG,THL,STEPD,WO,WI,ERROR
    NN=N-1
    MM=M-1
    WRITE (NW,1)
    IF(IRRG.NE.1)GO TO 91
    READ(NR,3)(DR(I),I=1,MM)
    READ(NR,3)(DTH(J),J=1,NN)
    WRITE(NW,16)
    WRITE(NW,20)(DR(I),I=1,MM)
    WRITE(NW,17)
    WRITE(NW,20)(DTH(J),J=1,NN)
    TWD=(DO-DI)*0.5
    IF(NSYM.EQ.1) TWD=TWD*0.5
    TLG=THG+THL
    DO 700 I=1,MM
700 DR(I)=DR(I)*TWD
    DO 701 J=1,NN
701 DTH(J)=DTH(J)*TLG
    GO TO 95
 91 FMM=M-1
    FNN=N-1
      IHH1=IHH-1
     AIHH1=IHH-IH
    BIH1=M-IHH
    IH1=IH-1
    AIH1=IH1
    IF(NSYM.EQ.1) GO TO 710
    IF(IH .EQ. 1) GO TO 709
    DO 92  I=1,IH1
 92 DR(I)=WI/AIH1
709 DO 97 I=IH,IHH1
 97 DR(I)=((DO-DI)*.5-WI-WO)/AIHH1
    GO TO 720
710 FIHH1=IHH1
```

```
      TEMP=((DO-DI)*0.25-WO)/FIHH1
      DO 711  I=1,IHH1
  711 DR(I)=TEMP
  720 DO 98 I=IHH,MM
   98 DR(I)=WO/BIH1
        JHH1=JHH-1
      AJHH1=JHH1
      BJH=N-JHH
       DO 93 J=1,JHH1
   93 DTH(J)=THG/AJHH1
      DO 99 J=JHH,NN
   99 DTH(J)=THL/BJH
   95 CONTINUE
      READ (NR,4) NVISM,(VISA(I),I=1,NVISM)
      READ(NR,4)NRPSM,(RPSA(I),I=1,NRPSM)
      READ(NR,4)NPAM,(PAA(I),I=1,NPAM)
      READ(NR,4)NHMM,(HMINA(I),I=1,NHMM)
C  WRITE INPUT DATA
      READ (NR,3)        (HOUT(I),I=1,NHMM)
      READ (NR,4)   NHDTM,(HDOT(I),I=1,NHDTM)
      WRITE(NW,5)
      WRITE(NW,6)LAST,NPAD,M,N,IH,JH,IHH,JHH
      WRITE(NW,7)
      WRITE(NW,6)LKOUNT,NDIAG,IRRG,NPRE
      WRITE(NW,8)DO
      WRITE(NW,9)DI
      WRITE(NW,10)THG
      WRITE(NW,11)THL
      WRITE(NW,12)STEPD
      WRITE(NW,13)WO
      WRITE(NW,14)WI
      WRITE(NW,15)ERROR
      IF(NDIAG)731,731,730
  730 WRITE(NW,16)
      WRITE(NW,20)(DR(I),I=1,MM)
      WRITE(NW,17)
      WRITE(NW,20)(DTH(J),J=1,NN)
  731 CONTINUE
      DO 1000 NVIS=1,NVISM
      VIS=VISA(NVIS)
      DO 1000 NRPS=1,NRPSM
      RPS=RPSA(NRPS)
      DO 1000 NPA=1,NPAM
      PA=PAA(NPA)
      DO 96 I=1,M
      DO 96 J=1,N
   96 Q(I,J)=1.
      DO 1000 N+DT31TN+DTM
      DO 1000 NHM=1,NHMM
      HMIN=HMINA(NHM)
        CONE=HOUT(NHM)/HMIN
      CONE1=CONE-1.
      WRITE (NW,33)  HOUT(NHM)
      WRITE(NW,18)HMIN
```

```
      WRITE(NW,19)RPS
      WRITE(NW,21)PA
      WRITE(NW,22)VIS
      O-+TE UNOT3 D +DOTUN+DTD
      PI=3.1416
      RO=DO/2.0
      RI=DI/2.
      PLAM=6.*VIS*RPS*2.*PI*((RO/HMIN)**2.)/PA
      CHDOT=HDOT(NHDT)/(2.*PI*RPS*HMIN)
      CHDT=HDOT(NHDT)/(2.*PI*RPS*STEPD)
      WRITE(NW,39) CHDT
      STEP=STEPD/HMIN
      WRITE(NW,23)PLAM
      STEPH=STEP*0.5
      CC=0.01745329
C   GENERATE COORDINATES
      TH(1)=0.
      NN=N-1
      DO 100 J=1,NN
  100 TH(J+1)=TH(J)+DTH(J)*CC
      R(1)=DI/DO
      MM=M-1
      DO 105 I=1,MM
  105 R(I+1)=R(I)+DR(I)/RO
      DIDO=DI/DO
      ODIDO=1.-DIDO
      L6=1
      K8=1
      ADMY=1.
  109  DO 107 J=1,N
  107 H(L6,J)=ADMY
       IF (K8.EQ.2)  GO TO 108
      L6=M
      K8=2
       ADMY=CONE
         GO TO 109
  108 DO 106 I=2,MM
      RS9=R(I)
      DO 106 J=1,N
  106 H(I,J)=1.+CONE1*((RS9-DIDO)/ODIDO)**2
      IF(NDIAG) 114,114,110
  110 WRITE(NW,24)
      WRITE(NW,20)(R(I),I=1,M)
      WRITE(NW,20)(TH(J),J=1,M)
C   GENERATE A1(I,J) TO A6(I,J)
      DO 112       I=1,M
  112 WRITE (NW,20)  (H(I,J),J=1,N)
  114 DO 140 J=2,NN
      DZ1=TH(J+1)-TH(J-1)
      DZ2=2.*DZ1
      DZ3=DZ2*(TH(J)-TH(J-1))
      DZ3=1./DZ3
      DZ4=1./(DZ2*(TH(J+1)-TH(J)))
      DO 140 I=1,MM
```

```
      HL=H(I,J-1)
      HR=H(I,J+1)
      HLH=(H(I,J-1)+H(I,J))*0.5
      HRH=(H(I,J+1)+H(I,J))*0.5
      HTH=(H(I+1,J)+H(I,J))*0.5
      HTEMP=H(I-1,J)
      IF(I.EQ.1)HTEMP=H(I,J)
      HBH=(HTEMP    +H(I,J))*0.5
      +F(J-JHH) 111,120,130
   POINTS AT THE LEFT SIDE OF THE STEP
  111 IF(I-IH) 130,115,116
C  POINTS ON THE BOTTOM EDGE
  115 IF(NSYM .EQ. 1) GO TO 116
  735 HL=HL+STEPH
      HR=HR+STEPH
      HLH=HLH+STEPH
      HRH=HRH+STEPH
      IF(I.EQ.IH)HTH=HTH+STEP
      IF(I.EQ.IHH)HBH=HBH+STEP
      GO TO 130
  116 IF(I-IHH) 117,735,130
C  POINTS IN THE POCKET
  117 HL=HL+STEP
      HR=HR+STEP
      HLH=HLH+STEP
      HRH=HRH+STEP
      HTH=HTH+STEP
      HBH=HBH+STEP
      GO TO 130
  120 IF(+-IH) 130,125,126
C  BOTTOM OR TOP CORNER
  125 HL=HL+STEPH
      HLH=HLH+STEPH
      IF(I.EQ.IH) HTH=HTH+STEPH
      IF(I.EQ.IHH) HBH=HBH+STEPH
      GO TO 130
  126 IF(I-IHH) 127,125,130
C  POINTS ALONG THE VERTICAL EDGE
  127 HL=HL+STEP
      HLH=HLH+STEP
      HTH=HTH+STEPH
      HBH=HBH+STEPH
      GO TO 130
  130 IF(NDIAG) 132,132,131
  131 WRITE(NW,25)
      WRITE(NW,20)HL,HR,HLH,HRH,HTH,HBH
C  GENERATE A1(I,J),ETC
  132 RTEM=R(I-1)
      DRT=DR(I-1)
      IF(I.EQ.1)DRT=DR(I)
      IF(I.EQ.1)RTEM=R(I)
      TEMP1=PLAM*R(I)/DZ2
      TEMP2=    1.0        /(4.0*(DR(I)+DRT      )/RO)
      A1(I,J)=TEMP1*HL
```

```
      A2(I,J)=TEMP1*HR
      A3(I,J)=DZ3*HLH**3/R(I)
      A4(I,J)=DZ4*HRH**3/R(I)
      A5(I,J)=TEMP2*HBH**3/(DRT     /RO) *(R(I)+RTEM  )
      A6(I,J)=TEMP2*HTH**3/(DR(I)/RO)    *(R(I)+R(I+1))
      A8(I,J)=PLAM*R(I)*CHDOT
      IF(NDIAG) 140,140,133
  133 WRITE(NW,26)
      WRITE(NW,20)A1(I,J),A2(I,J),A3(I,J),A4(I,J),A5(I,J),A6(I,J) ,
     1 A8(I,J)
  140 CONTINUE
      KOUNT=1
  141 DO 143 I=1,M
      G(I,1)=0.
      DO 142 K=1,M
  142 E(I,K,1)=0.
  143 CONTINUE
      DO 370 J=1,N
      DO 310 I=1,M
      IF(J.EQ.1.OR. J.EQ.N) GO TO 210
      IF( I .EQ. 1) GO TO 725
      IF( I .EQ. M) GO TO 210
      GO TO 726
  725 IF(NSYM .NE.1) GO TO 210
  726 SQP=SQRT(ABS(Q(I,J+1)))
      SQM=SQRT(ABS(Q(I,J-1)))
      SQQ=SQRT(ABS(Q(I,J)))
      B(I)=A3(I,J)+A1(I,J)/(2.0*SQM)
      C(I)=A4(I,J)-A2(I,J)/(2.0*SQP)
      ASUM=A3(I,J)+A4(I,J)+A5(I,J)+A6(I,J)
      F(I)=A3(I,J)*Q(I,J-1)+A1(I,J)*SQM
      QTT=Q(I-1,J)
      IF(I .EQ. 1 .AND. NSYM .EQ. 1) QTT=Q(I+1,J)
      F(I)=F(I)+A5(I,J)*QTT       -ASUM*Q(I,J)+A6(I,J)*Q(I+1,J)+A4(I,J)*Q
     1(I,J+1)-A2(I,J)*SQP-A8(I,J)*SQQ
      F(I)=-F(I)
      DO 150 K=1,M
      A(I,K)=0.0
      IF(K.EQ.I) A(I,K)=-ASUM-A8*0.5/SQQ
      A6TT=A6(I,J)
      +F(+ .EQ. 1 .AND. NSYM .EQ. 1) A6TT=A6(I,J)+A5(I,J)
      +F(K.EQ.++1)A(I,K)=A6TT
      IF(K.EQ.I-1)A(I,K)=A5(I,J)
  150 CONTINUE
      IF(J.NE.JHH) GO TO 305
      IF(I.EQ.IH.OR.I.EQ.IHH) GO TO 151
      IF(I.GT.IH.AND.I.LT.IHH) GO TO 152
      GO TO 305
  151 DMSP=STEP*0.5
      GO TO 155
  152 DMSP=STEP
  155 SQQ=SQRT(ABS(Q(I,J)))
      TEMP=A1(I,J)*DMSP/(H(I,J-1)+DMSP)
      F(I)=  F(I)-TEMP*SQQ
```

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```
      A(I,I)=A(I,I)+TEMP*0.5/SQQ
  305 IF(NDIAG) 310,310,306
  306 WRITE(NW,27)I,J
      WRITE(NW,20)F(I),B(I),C(I),A(I,I-1),A(I,I),A(I,I+1)
      GO TO 310
  210 B(I)=0.
      C(I)=0.
      F(I)=0.
      DO 211 K=1,M
      A(I,K)=0.
      IF(I.EQ.K)A(I,K)=1.
  211 CONTINUE
      GO TO 305
  310 CONTINUE
      DO 320 I=1,M
      DO 320 K=1,M
  320 QSMA(I,K)=A(I,K)+B(I)*E(I,K,J)
      CALL MATINV(QSMA,M,BB,0,DET,ID)
      GO TO (340,330),ID
  340 DO 360 I=1,M
      G(I,J+1)=0.
      DO 360 K=1,M
      G(I,J+1)=G(I,J+1)+QSMA(I,K)*(F(K)-B(K)*G(K,J))
      E(I,K,J+1)=-QSMA(I,K)*C(K)
  360 CONTINUE
  370 CONTINUE
      DMA=0.
      DO 380 I=1,M
      DMA=AMAX1(DMA,ABS(G(I,N+1)))
  380 DQ(I,N)=G(I,N+1)
      DO 400 JJ=2,N
      J=N+2-JJ
      DO 400 I=1,M
      DUM=0.
      DO 3 0 K=1,M
  390 DUM=DUM+E(I,K,J)*DQ(K,J)
      DUM=DUM+G(I,J)
      DMA=AMAX1(DMA,ABS(DUM))
  400 DQ(I,J-1)=DUM
      DO 401 I=1,M
      DO 401 J=1,N
  401 Q(I,J)=Q(I,J)+DQ(I,J)
      +F(ND+AG)405,405,402
  402 DO 403 J=1,N
  403 WRITE(NW,20)(DQ(I,J),I=1,M)
      DO 404 J=1,N
  404 WRITE(NW,20)(Q(I,J),I=1,M)
  405 CONTINUE
       GO TO 560
  330 WRITE(NW,28)J
      GO TO 1000
  560 KOUNT=KOUNT+1
      IF(KOUNT.GE.LKOUNT) GO TO 561
      IF(DMA.GT.ERROR) GO TO 141
```

```
561 WRITE(NW,30)ERROR,KOUNT
    IF(NPRE.EQ.1) WRITE(NW,29)
    DO 576 J=1,N
    DO 576 I=1,M
    P(I,J)=SQRT(ABS(Q(I,J)))
576 CONTINUE
    DO 585 I=1,M
    IF(NPRE.EQ.1)WRITE(NW,31)(P(I,J),J=1,N)
    DO 580 J=1,N
580 QQQ(J)=P(I,J)-1.
    NN=N-1
    PP(I)=0.
    DO 581 J=1,NN
581 PP(I)=PP(I)+DTH(J)*(QQQ(J)+QQQ(J+1))*0.5
    PP(I)=R(I)*CC*PP(I)
    IF(ND1)585,585,800
800 WRITE(NW,31)PP(I)
585 CONTINUE
    FPAD=NPAD
    MM=M-1
    WLOAD=0.
    DO 589 I=1,MM
589 WLOAD=WLOAD+DR(I)*(PP(I)+PP(I+1))*0.5
    WLOAD=WLOAD*RO*PA*FPAD
    IF(NSYM.EQ.1)WLOAD=2.0*WLOAD
    WBAR=WLOAD/(3.1416*(RO**2-RI**2)*PA)
    IF(NSYM.EQ.1)WBAR=2.*WBAR
    DO 600 I=1,M
    IF(I.EQ.IH.OR.I.GT.  IH) GO TO 593
591 FRC(I)=0.
    NN=N-1
    DO 592 J=1,NN
592 FRC(I)=FRC(I)+DTH(J)/H(I,J)
    FRC(I)=FRC(I)*CC*R(I)**3
    GO TO 600
593 IF (I.GT.IHH)  GO TO 591
    FRC(I)=0.
    NN=N-1
    DO 594 J=1,NN
    IF(J.EQ.JHH.OR.J.GT.JHH) TEMP=1.0/H(I,J)
    IF(J.LE.JHH)TEMP=1.0/(H(I,J)+STEP)
594 FRC(I)=FRC(I)+DTH(J)*TEMP
    FRC(I)=FRC(I)*CC*R(I)**3
600 CONTINUE
    MM=M-1
    HPOW=0.
    DO 605 I=1,MM
605 HPOW=HPOW+FRC(I)*DR(I)
    HPOW=HPOW*VIS*6.2832*RPS*-O**3
    HPOW=HPOW*FPAD*RPS*60./63000.0
    HPOW=HPOW/HMIN
    IF(NSYM.EQ.1) HPOW=2.0*HPOW
    WRITE(NW,34)WLOAD
    WRITE(NW,35)WBAR
```

```
      WRITE(NW,36)HPOW
 1000 CONTINUE
      IF(LAST)90,90,1001
 1001 STOP
      END


      SUB-OUT+NE MATINVUATN1,BTM1,DETE- ,ID)
      MATRIX INVERSION WITH ACCOMPANYING SOLUTION OF LINEAR EQUATIONS
      NOVEMBER 1962    S GOOD   DAVID TAYLOR MODEL BASIN    AM MAT1

      DIMENSION A(17,17),B(17,1),INDEX(17,3)

      GENERAL FORM OF DIMENSION STATEMENT

      EQUIVALENCE (IROW,JROW), (ICOLU ,JCOLU ), (AMAX, T, SWAP)

      INITIALIZATION

      M=M1
      N=N1
   10 DETER =1.0
   15 DO 20 J=1,N
   20 INDEX(J,3) = 0
   30 DO 550 I=1,N

      SEARCH FOR PIVOT ELEMENT

   40 AMAX=0.0
   45 DO 105 J=1,N
      IF(INDEX(J,3)-1) 60, 105, 60
   60 DO 100 K=1,N
      +F(INDEX(K,3)-1) 80, 100, 715
   80 IF (      AMAX -ABS (A(J,K))) 85, 100, 100
    5 +-OW3J
    0 +COLU =K
      AMAX=ABS UA(J,K))
  100 CONTINUE
  105 CONTINUE
      INDEX(+COLU ,3) = INDEX(+COLU ,3) +1
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLU

      INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL

  130 IF (IROW-ICOLU ) 140, 310, 140
  140 DETER =-DETER
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLU ,L)
  200 A(ICOLU ,L)=SWAP
      +F(M) 310, 310, 210
```

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```
210 DO 250 L=1, M                                                MAT10048
220 SWAP=B(+-OW,L)                                               MAT10049
230 B(IROW,L)=B(ICOLU ,L)
250 B(ICOLU ,L)=SWAP
                                                                 MAT10052
    DIV+DE PIVOT ROW BY PIVOT ELEMENT                            MAT10000
                                                                 MAT10053
310 PIVOT   =A(ICOLU ,ICOLU )
    DETER=DETER*PIVOT
330 A(+COLU ,+COLU )=1.0
340 DO 350 L=1,N                                                 MAT10057
350 A(ICOLU ,L)=A(ICOLU ,L)/PIVOT
355 +F(M) 380, 380, 360                                          MAT10059
360 DO 370 L=1,M                                                 MAT10060
370 B(ICOLU ,L)=B(ICOLU ,L)/PIVOT
                                                                 MAT10062
    -EDUCE NON-PIVOT -OWS                                        MAT10063
                                                                 MAT10064
380 DO 550 L1=1,N                                                MAT10065
390 IF(L1-ICOLU ) 400, 550, 400
400 T=A(L1,ICOLU )
420 A(L1,ICOLU )=0.0
    IF(T)430,550,430
430 DO 450 L=1,N                                                 MAT10069
450 A(L1,L)=A(L1,L)-A(ICOLU ,L)*T
455 IF(M) 550, 550, 460                                          MAT10071
460 DO 500 L=1,M                                                 MAT10072
500 B(L1,L)=B(L1,L)-B(ICOLU ,L)*T
550 CONTINUE                                                     MAT10074
                                                                 MAT10075
    INTERCHANGE COLUMNS                                          MAT10076
                                                                 MAT10077
600 DO 710 I=1,N                                                 MAT10078
610 L=N+1-I                                                      MAT10079
620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630                     MAT10080
630 JROW=INDEX(L,1)                                              MAT10081
640 JCOLU =INDEX(L,2)
650 DO 705 K=1,N                                                 MAT10083
660 SWAP=A(K,JROW)                                               MAT10084
670 A(K,JROW)=A(K,JCOLU )
700 A(K,JCOLU )=SWAP
705 CONTINUE                                                     MAT10087
710 CONTINUE                                                     MAT10088
    DO 730 K = 1,N                                               MAT10089
    IF(INDEX(K,3) -1) 715,720,715                                MAT10090
720 CONTINUE                                                     MAT10093
730 CONTINUE                                                     MAT10094
    ID=1
740 -ETU-N                                                       MAT10096
715 ID =2                                                        MAT10091
    GO TO 740                                                    MAT10092
    END
```

Card 1   Format (72 H)

Identification Card

Card 2   Format (16I5)

Last	-	Integer to determine whether additional input data are to be read
		Last = 1 , no more input data
		Last = 0 , more input data to be read from statement 90.
NPAD	-	Number of pads (see Fig.A2)
M	-	Number of grids in the radial direction
N	-	Number of grids in the circumferential direction
IH	-	Grid number for the bottom edge of the step (see Fig. A2) For NSYM=1, set IH-1
IHH	-	Grid number for the top edge of the step (see Fig. A2)
JH	-	Set JH=1
JHH	-	Grid number for the left edge of the step
LKOUNT	-	Maximum number of iterations allowed for the pressure calculation (recommended value: 10-20)
NDIAG	-	Control for diagnostics
		NDIAG = 1 , diagnostics print out
		NDIAG = 0 , no diagnostics
IRRG	-	Control for irregular grids
		IRRG = 1 , read in irregular grid spacings
		IRRG = 0 , uniform grid spacing
NPRE	-	Control for printing out pressure profile
		NPRE = 1 , print out pressure
		NPRE = 0 , no pressure print out
NYSM	-	This integer is used to control whether pressure calculation is made for a full pad or half a pad as in the case where the pressure is symetrical about

the center line (see Fig.A2).

NYSM = 0 , For calculation covering full pad

NYSM = 1 , For symmetrical pressure profiles where where calculation is only made for half a pad.

ND1 - Set ND1 = 0 , for normal runs.

<u>Card 3</u> Format (8F10.7)

DO - Outside diameter of thrust brg., d_o in Fig.A1 (in.)

DI - d_i in Fig.A1 (in.)

THG - Angle extending the pocket region, θ_g in Fig.A2, (degrees)

THL - Angle extending the land region, θ_L in Fig.A2, (degrees)

STEPD - Depth of the step (in.)

WO - Outerwidth of the shreud, W_o in Fig.A2, (in.)

WI - Innerwidth of the shroud, W_i in Fig.A2 (in.)

ERROR - Convergence factor for pressure iteration. (recommended value: .001 - .0002)

<u>Card 4</u> Format (8F 10.7)

<u>This card is required only when IRRG = 1</u>

DR(I), I = 1, MM - Dimensionless irregular gird spacings in the radial direction. (MM - M-1)

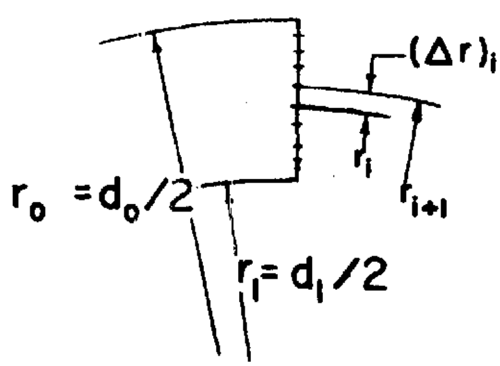

Fig.A1

$$DR(I) = \frac{(\Delta r)_i}{(r_o - r_i)} = \frac{r_{i+1} - r_i}{r_o - r_i}$$

Card 5 Format (8F 10.7)

This card is required only when IRRG = 1

DTH(J), J = 1, NN - Dimensionless grid spacings in the circumferential direction. (NN = N-1)

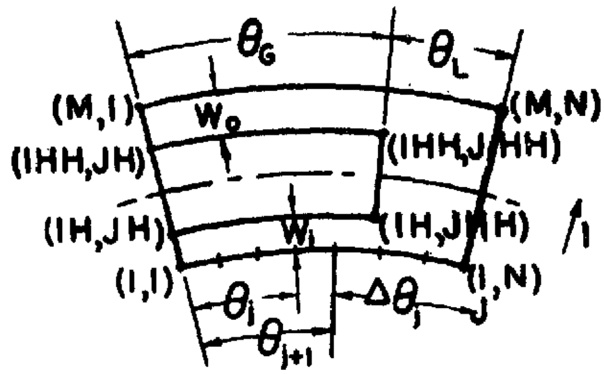

Fig. A-2

$$DTH(J) = \frac{(\Delta\theta)_j}{\theta_L + \theta_G} = \frac{\theta_{j+1} - \theta_j}{\theta_L + \theta_G}$$

Card 6 Format (I5, 5X, 7E 10.3)

NVISM - Total number of viscosities to be investigated in the production run.

VISA(I), I = 1, NVISM

The arrary of viscosities, (lb-sec/in^2)

Card 7 Format (I5, 5X, 7E 10.3)

NRPSM - Total number of angular speeds to be investigated in the production run.

RPSA(I), I - 1, NRPSM

The array of angular speeds, (Rev. per sec.)

Card 8 Format (I5, 5X, 7E 10.3)

NPAM - Total number of ambient pressure to be investigated in the production run.

PAA(I), I = 1, NPAM

The arrary of ambient pressure, (PSI)

Card 9 Format (I5, 5X, 7E 10.3)

This card reads in the film thickness to be investigated in the production run. In the case of a parallel film, the film thickness in the land region will be read in the array designated as HMINA(I). In the case of non-parallel film with an axisymmetric coning of dishirg, HMINA (I) represents the film thickness at the inside diameter in the land region. The film thickness at the outside diameter is HOUT(I) which read in Card 10.

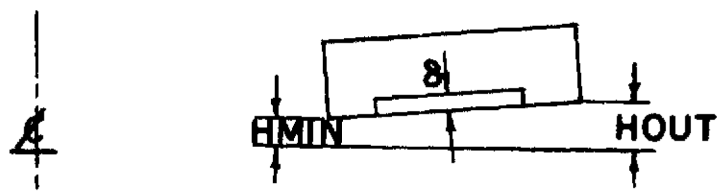

Fig. A3

The variables to be read in this card are:

NHMM - Total number of film thickness to be investigated in the production run.

HMINA(I), I = 1, NAMM

HMINA is the film thickness at the inside diameter in the land region (inch)

Card 10 Format (8F 10.7)

HOUT(I), I = 1, NHMM

- HOUT is the film thickness at the outside diameter in the land region. (inch)

Card 11 Format (I5, 5X, 7E 10.4/(8E10.4)

NHDTM - Total number of velocity or time variation of gas film thickness to be investigated in the production run.

HDOT(I) - Array of velocity or time variation of gas film thickness in in./sec.

```
      PROGRAM RSGALN(INPUT,OUTPUT)
      DIMENSION WN(30), QN(7), AG(9),AOG(9)
      D+MENS+ON AHA(100D, F1A(100), F2A(100), F3A(100), F4A(100), FI(9)
      DIMENSION F5A(100), F6A(100),F7A(100), F8A(100), F9A(100)
      THE TABLES ARE PREPARED IN THE FOLLOWING ORDER THAT (FJA(I),J=1,9)
      = ( D1*I1, D2*I1A, D1*I4, D2*I4A, -D2*I1AO, -D2*I4AO, D2*I5/HO**2.5
      , DI*I5A/HO**2.5, -D2*I5AO/HO**2.5 ).
      DATA F1A(1)/0./,F2A(1)/7.854/,F3A(1)/3.1416/,F4A(1)/0./
       DATA(F1A(I),I=2,60)/ 7.8555281E-02, 1.5720341E-01, 2.3603750E-01,
     2  3.1515123E-01, 3.9463912E-01, 4.7459680E-01, 5.5512133E-01,
     2  6.3631146E-01, 7.1826801E-01, 8.0109411E-01, 8.8489560E-01,
     2  9.6978133E-01, 1.0558636E+00, 1.1432584E+00, 1.2320861E+00,
     2  1.3224715E+00, 1.4145448E+00, 1.5084415E+00, 1.6043034E+00,
     2  1.7022791E+00, 1.8025243E+00, 1.9052031E+00, 2.0104879E+00,
     2  2.1185609E+00, 2.2296146E+00, 2.3438529E+00, 2.4614919E+00,
          2 611E+00T 2C707 04 E+00T 2C83718335E+00, 2. 708745E+00,
     2  3.10 2748E+00, 3.2527020E+00, 3.4014964E+00, 3.5560237E+00,
     2  3.7166766E+00, 3C 8387 1E+00, 4C0580843E+00, 4.23 7876E+00,
     2  4.42 5206E+C0, 4C6278602E+00, 4. 354321E+00, 5.0529163E+00,
     2  5.2810529E+C0, 5.5206483E+00, 5.7725837E+00, 6.0378224E+00,
     2  6.3174205E+C0, 6.6125372E+00, 6.9244476E+00, 7.2545570E+00,
     2  7.6044175E+00, 7.9757465E+00, 8.3704491E+00, 8.7906429E+00,
     2  9.2386874E+00, 9.7172183E+00, 1.0229187E+01, 1.0777908E+01/
       DATAUF1AU+),I=61,100)/
     2  1.1367114E+01, 1.2001018E+01, 1.2684396E+01, 1.3422673E+01,
     2  1.4222038E+01, 1.5089573E+01, 1.6033412E+01, 1.7062939E+01,
     2  1.8189023E+01, 1.942430 E+01, 2.0783580E+01, 2.2284206E+01,
     2  2.3946707E+01, 2.5795465E+01, 2.7859629E+01, 3.0174285E+01,
     2  3.2781965E+01, 3.5734627E+01, 3.9096287E+01, 4.2946530E+01,
     2  4.7385298E+01, 5.2539475E+01, 5.8572095E+01, 6.5695462E+01,
     2  7.4190157E+01, 8.4433212E+01, 9.6940826E+01, 1.1243492E+02,
     2  1.31 5000E+02, 1.57010 5E+02, 1.8994101E+02, 2.3442157E+02,
     2  2.9656997E+02, 3.871653 E+02, 5.2666744E+02, 7.5788976E+02,
     2  1.1832960E+03T 2.1018302E+03, 4.7245976E+03, 1.8878381E+04/
       DATA(F2A(I),I=2,60)/ 7.8586215E+00, 7.8725551E+00, 7.8958239E+00,
     2  7.9284975E+00, 7.9706737E+00, 8.0224795E+00, 8.0840712E+00,
     2  8.1556360E+00, 8.2373922E+00, 8.3295914E+00, 8.4325193E+00,
     2  8.5464976E+00, 8.6718859E+00, 8.8090839E+00, 8.9585338E+00,
     2  9.1207230E+00, 9.2961871E+00, 9.4855135E+00, 9.6893451E+00,
     2  9.9083843E+00, 1.01433 8E+01, 1.0395223E+01, 1.0664772E+01,
     2  1.0953037E+01, 1.1261104E+01, 1.1590152E+01, 1.1941467E+01,
     2  1.2316453E+01, 1.2716640E+01, 1.3143699E+01, 1.3599453E+01,
        1C4085  5E+01, 1C4605207E+01, 1C5159776E+01, 1.575221 E+01,
     2  1.6385407E+01, 1.7062493E+01, 1.7786946E+01, 1.8562586E+01,
     2  1.9393624E+01, 2.0284712E+01, 2.1240995E+01, 2.2268170E+01,
     2  2.3372563E+01, 2.4561200E+01, 2.5841910E+01, 2.7223422E+01,
     2  2.8715497E+01, 3.0329068E+01, 3.2076405E+01, 3.3971315E+01,
     2  3.602 365E+01, 3.8268150E+01, 4.0707613E+01, 4.3370409E+01,
     2  4.6282353E+01, 4.9472941E+01, 5.2975980E+01, 5.6830339E+01/
       DATA(F2A(I),I=61T100)/
     2  6.1080856E+01, 6.5779435E+01, 7.0986379E+01, 7.6772024E+01,
     2  8.3218737E+01, 9.0423387E+01,  .8500418E+01, 1.0758568E+02,
     2  1.1784127E+02, 1.2946160E+02, 1.4268123E+02, 1.5778484E+02,
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2  1.7512021E+02, 1.9511517E+02, 2.1829994E+02, 2.4533690E+02,
2  2.7706071E+02, 3.14532 6E+02, 3.5911751E+02, 4.1258599E+02,
2  4.7726736E+02, 5.5626382E+02, 6.5376773E+02, 7.7553624E+02,
2  9.2961755E+02, 1.12748 5E+03, 1.3858938E+03, 1.7298827E+03,
2  2.1980610E+03, 2.8519737E+03, 3.7937283E+03, 5.2009564E+03,
2  7C400 326E+03, 1C1041001E+04T 1C7522447E+04, 3.0261024E+04,
2  5.9068684E+04, 1.3993135E+05, 4.7198425E+05, 3.7735728E+06/
 ++  UF A0++0+ 2060D/ 3.1419510E+00, 3.1429824E+00, 3.1447027E+00,
2  3.1471142E+00, 3.1502198E+00, 3.1540235E+00, 3.1585298E+00,
2  3.1637446E+00, 3.1696743E+00, 3.1763265E+00, 3.1837096E+00,
2  3.1918331E+00, 3.2007075E+00, 3.2103442E+00, 3.2207558E+00,
2  3.2319562E+00, 3.2439602E+00, 3.2567839E+00, 3.2704448E+00,
2  3.2849616E+00, 3.3003546E+00, 3.3166453E+00, 3.3338569E+00,
2  3.3520144E+00, 3.3711442E+00, 3.3912749E+00, 3.4124369E+00,
2  3+4346625E+00, 3.4579865E+00, 3.4824458E+00, 3.5080801E+00,
2  3.5349315E+00, 3.5630451E+00, 3.5924691E+00, 3.6232549E+00,
2  3.6554576E+00, 3.6891360E+00, 3.7243531E+00, 3.7611763E+00,
2  3.7996778E+00, 3.8399352E+00, 3.8820315E+00, 3.9260561E+00,
2  3.9721049E+00, 4.0202812E+00, 4.0706963E+00, 4.1234702E+00,
2  4.1787323E+00, 4.2366228E+00, 4.2972932E+00, 4.3609078E+00,
2  4.4276449E+00, 4.4976984E+00, 4.5712794E+00, 4.6486182E+00,
2  4.7299663E+00, 4.81559 1E+00, 4.9058188E+00, 5.0009572E+00/
 DATA(F3A(I),I=61,100)/
2  5.1013801E+00, 5.2074911E+00, 5.3197370E+00, 5.4386134E+00,
2  5.5646716E+00, 5.6985265E+00, 5.8408664E+00, 5.9924634E+00,
2  6.1541 75E+00, 6.3270215E+00, 6.5120806E+00, 6.7106352E+00,
2  6.9241382E+00, 7.1542599E+00, 7.4029288E+00, 7.6723848E+00,
2  7.9652429E+00, 8.2845764E+00, 8.6340204E+00, 9.0179069E+00,
2   .44143 0E+00,  .9109212E+00, 1.0434067E+01, 1.1020413E+01,
2  1.1681892E+01, 1.2433635E+01, 1.3295114E+01, 1.4291818E+01,
2  1.5457768E+01, 1.6839404E+01, 1.8501808E+01, 2.0539089E+01,
2  2.3092560E+01, 2.6384528E+01, 3.0785914E+01, 3.6965137E+01,
2  4.6260583E+01, 6.1798892E+01, 9.2971744E+01, 1.8680213E+02/
 +AT1UF4AU+DT+32T60D/ 6. 736515E-02, 1.3755 14E-01, 2.0655024E-01,
2  2.7579871E-01, 3.4539021E-01, 4.1541249E-01, 4.8595458E-01,
2  5.5710716E-01, 6.2896280E-01, 7.0161625E-01, 7.7516481E-01,
2  8.4970863E-01, 9.2535102E-01, 1.0021989E+00, 1.0803631E+00,
2  1.1599589E+00, 1.2411062E+00, 1.3239304E+00, 1.4085626E+00,
2  1.4951401E+00, 1.5838073E+00, 1.6747161E+00, 1.7680265E+00,
2  1.8639077E+00, 1.9625385E+00, 2.0641085E+00, 2.1688189E+00,
2  2.2768835E+00, 2.3885300E+00, 2.5040014E+00, 2.6235570E+00,
2  2.7474742E+00, 2.8760502E+00, 3.0096038E+00, 3.1484776E+00,
2  3.2930400E+00, 3.4436882E+00, 3.6008507E+00, 3.7649907E+00,
2  3.9366095E+00, 4.1162504E+00, 4.3045037E+00, 4.5020110E+00,
2  4.7094711E+00, 4.9276467E+00, 5.1573713E+00, 5.3995573E+00,
2  5.6552054E+00, 5.9254154E+00, 6.2113978E+00, 6.5144879E+00,
2  6.8361620E+00, 7.1780548E+00, 7.5419812E+00, 7.9299605E+00,
2  8.3442447E+00, 8.7873513E+00, 9.2621024E+00, 9.7716694E+00/
 DATA(F4A(I),I=61,100)/
2  1C031 627E+01, 1.0 10015E+01T 1C1547415E+01, 1.2237038E+01,
2  1.2984832E+01, 1.3797611E+01, 1.4683210E+01, 1.5650673E+01,
2  1.6710487E+01, 1.7874863E+01, 1.9158091E+01, 2.0576979E+01,
2  2.2151398E+01, 2.3904986E+01, 2.5866029E+01, 2.8068606E+01,
2  3C0554070E+01, 3C3372  1E+01T 3C6587731E+01, 4.0275 05E+01,
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2   4.4535078E+01, 4.9489238E+01, 5.5297872E+01, 6.2168873E+01,
2   7.0377289E+01, 8.0293107E+01, 9.2423422E+01, 1.0747819E+02,
2   1.2647590E+02, 1.5091947E+02, 1.8310120E+02, 2.2665762E+02,
2   2.8763837E+02, 3.7671568E+02, 5.1416995E+02, 7.4249041E+02,
2   1.1634881E+03, 2.0745753E+03, 4.6822333E+03, 1.8789967E+04/
    DATA(F5A(I),I=1, 59)/ 0.           , 2.7497442E-01,5.5045962E-01,
2   8.2696846E-01, 1.1050180E+00, 1.3851315E+00, 1.6678408E+00,
2   1.9536883E+00, 2.2432295E+00, 2.5370353E+00, 2.8356944E+00,
2   3.13 8161E+00, 3.4500330E+00, 3C7670041E+00, 4.0914178E+00,
2   4.4239953E+00, 4.7654945E+00, 5.1167138E+00, 5.4784962E+00,
2   5.8517341E+00, 6.2373745E+00, 6.6364245E+00, 7.0499568E+00,
2   7.4791171E+00, 7.9251312E+00, 8.3893125E+00, 8.8730716E+00,
2   9.3779257E+00, 9.9055097E+00, 1.0457588E+01, 1.1036068E+01,
2   1C1643017E+01, 1C2280673E+01T 1C2951473E+01T 1.3658065E+01,
2   1.4403336E+01, 1.5190438E+01, 1.6022818E+01, 1.6904250E+01,
2   1C7 38 77E+01, 1C 831251E+01T 1C 886382E+01T 2.1009798E+01,
2   2.2207604E+01, 2.3486560E+01, 2.4854161E+01, 2.6318738E+01,
2   2.7889564E+01, 2.9576990E+01, 3.1392586E+01, 3.3349322E+01,
2   3.5461763E+01, 3.7746313E+01, 4.0221486E+01, 4.2908234E+01,
2   4.5830332E+01, 4.9014836E+01, 5.2492622E+01, 5.6299036E+01/
    DATA(F5A(I),I=60,100)/
2   6.0474671E+01, 6.5066298E+01, 7.0128000E+01, 7.5722544E+01,
2   8.1923058E+01, 8.8815087E+01, 9.6499133E+01, 1.0509380E+02,
2   1.1473976E+02, 1.2560462E+02, 1.3788928E+02, 1.5183581E+02,
2   1.6773775E+02, 1.8595332E+02, 2.0692274E+02, 2.3119103E+02,
2   2.5943838E+02, 2.9252105E+02, 3.3152694E+02, 3.7785238E+02,
2   4.3330926E+02, 5.0027714E+02, 5.8192238E+02, 6.8251978E+02,
2   8.0793374E+02, 9.6635414E+02, 1.1694491E+03, 1.4342207E+03,
2   1.7860852E+03, 2.2641687E+03, 2.9307839E+03, 3.8892080E+03,
2   5.3189242E+03, 7.5502829E+03, 1.1236044E+04, 1.7787769E+04,
2   3.0642697E+04, 5.9664177E+04, 1.4098798E+05, 4.7435606E+05,
2   3.7830331E+06/
    DATA(F6A(I),I=1, 59)/ 7.8540184E+00, 7.8555648E+00,7.8602071E+00,
2   7.8679533E+00, 7.8788175E+00, 7.8928191E+00, 7.9099834E+00,
2   7.9303414E+00, 7.9539300E+00, 7.9807924E+00, 8.0109779E+00,
2   8.0445422E+00, 8C0815478E+00,  .1220643E+00, 8.1661682E+00,
2   8.2139440E+00, 8.265483 E+00, 8.3208885E+00, 8.3802672E+00,
2   8.4437389E+00, 8.5114321E+00, 8.5834860E+00, 8.6600507E+00,
2   8C7412  3E+00,  C 273737E+00T  C 184 52E+00,  .0148555E+00,
2   9.1166732E+00, 9.2241836E+00, 9.3376399E+00, 9.4573150E+00,
2    .583502 E+00,  C7165205E+00,  . 567093E+00, 1.0004438E+01,
2   1.0160104E+01, 1.0324138E+01, 1C0497005E+01, 1.0679206E+01,
2   1.0871287E+01, 1.1073838E+01, 1.1287501E+01, 1.1512970E+01,
2   1.1751005E+01, 1.2002430E+01, 1.2268144E+01, 1.2549132E+01,
2   1.2 46467E+01, 1C316132 E+01, 1C3495011E+01, 1.3848932E+01,
2   1.4224658E+01, 1.4623916E+01, 1.5048615E+01, 1.5500868E+01,
2   1.5983024E+01, 1.64976 3E+01, 1.7047788E+01, 1.7636566E+01/
    DATA(F6A(I),I=60,100)/
2   1.8267678E+01, 1.8945226E+01, 1.9673837E+01, 2.0458740E+01,
    2.1305867E+01, 2.2221971E+01, 2.3214764E+01, 2.4293085E+01,
    2.5467109E+01, 2.6748600E+01, 2.8151209E+01, 2.9690866E+01,
    3.1386243E+01, 3.3259352E+01, 3.5336290E+01, 3.7648184E+01,
2   4C0232417E+01, 4C3134201E+01T 4C640 644E+01T 5.0123481E+01,
2   5.4362733E+01, 5.9231660E+01, 6.4863586E+01, 7.1429421E+01,
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2   7.9151196E+01, 8.8321653E+01, 9.9333228E+01, 1.1272193E+02,
2   1.2 23557E+02, 1C4  4321E+02, 1C7641683E+02, 2.1104560E+02,
2   2.5760616E+02, 3.2235870E+02, 4.1630690E+02, 5.6028454E+02,
2   7.9777874E+02, 1.2326001E+03, 2.1668353E+03, 4.8210180E+03,
2   1. 06 072E+04/
  DATAUF7AU+),+=1, 5 )/ 6C2 32000E+00, 6.2845747E+00,6.2887012E+00,
2   6.2955873E+00, 6.3052457E+00, 6.3176945E+00, 6.3329570E+00,
2   6.3510618E+00, 6.3720431E+00, 6.3959406E+00, 6.4227999E+00,
2   6.4526727E+00, 6.4856166E+00, 6.5216958E+00, 6.5609815E+00,
2   6.6035514E+00, 6C64 4 11E+00, 6.6988937E+00, 6.7518606E+00,
2   6.8085018E+00, 6.8689366E+00, 6.9332940E+00, 7.0017134E+00,
2   7.0743452E+00, 7.1513519E+00, 7.2329084E+00, 7.3192034E+00,
2   7.4104401E+00, 7.5068376E+00, 7.6086319E+00, 7.7160774E+00,
2   7.8294482E+00, 7.9490401E+00, 8.0751721E+00, 8.2081888E+00,
2   8.3484623E+00, 8.4963949E+00, 8.6524219E+00, 8.8170147E+00,
2   8.9906842E+00, 9.1739847E+00, 9.3675184E+00, 9.5719399E+00,
2   9.7879622E+00, 1.0016362E+01, 1.0257989E+01, 1.0513769E+01,
2   1.0784718E+01, 1.1071 4 E+01, 1.1376685E+01, 1.1700271E+01,
2   1.2044190E+01, 1.2410079E+01, 1.2799751E+01, 1.3215214E+01,
2   1.3658700E+01, 1.4132695E+01, 1.4639974E+01, 1.5183642E+01/
 DATA(F7A(I),I=60,100)/
2   1.5767185E+01, 1.6394521E+01, 1.7070076E+01, 1.7798856E+01,
2   1.8586546E+01, 1.9439621E+01, 2.0365486E+01, 2.1372637E+01,
2   2.2470863E+01, 2.36714 1E+01, 2.4987684E+01, 2.6434811E+01,
2   2. 030 11E+01, 2. 7 726 E+01, 3C1759145E+01, 3.3946705E+01,
2   3.6396210E+01, 3.9151565E+01, 4.2266341E+01, 4.5806456E+01,
2   4C  53764E+01T 5C4510 25E+01, 5. 90 111E+01T 6.6212373E+01,
2   7.3640975E+01, 8.2480692E+01, 9.3116396E+01, 1.0607436E+02,
2   1.2208965E+02, 1.4221431E+02, 1.6799712E+02, 2.0179468E+02,
2   2.4733660E+02, 3.1081241E+02, 4.0311463E+02, 5.4489157E+02,
2   7C7 2 615E+02T 1C20 46  E+03, 2C1359358E+03T 4.7745321E+03,
2   1.8975671E+04/
  DATA(F8A(I),I=1, 59)/ 0.            , 2.7497442E-01,5.5045962E-01,
2   8.26 6846E-01, 1.1050180E+00, 1.3851315E+00, 1.6678408E+00,
2   1.9536883E+00, 2.2432295E+00, 2.5370353E+00, 2.8356944E+00,
2   3.13 8161E+00, 3.4500330E+00, 3.7670041E+00, 4.0914178E+00,
2   4.4239953E+00, 4.7654945E+00, 5.1167138E+00, 5.4784962E+00,
2   5.8517341E+00, 6.2373745E+00, 6.6364245E+00, 7.0499568E+00,
2   7.4791171E+00, 7.9251312E+00, 8.3893125E+00, 8.8730716E+00,
2   9.3779257E+00, 9.9055097E+00, 1.0457588E+01, 1.1036068E+01,
2   1.1643017E+01, 1.2280673E+01, 1.2951473E+01, 1.3658065E+01,
2   1.4403336E+01, 1.5190438E+01, 1.6022818E+01, 1.6904250E+01,
2   1.7838877E+01, 1.8831251E+01, 1.9886382E+01, 2.1009798E+01,
2   2.2207604E+01, 2C3486560E+01, 2.4854161E+01, 2.6318738E+01,
2   2.7889564E+01, 2.9576990E+01, 3.1392586E+01, 3.3349322E+01,
2   3.5461763E+01, 3.7746313E+01, 4.0221486E+01, 4.2908234E+01,
2   4.5830332E+01, 4.9014836E+01, 5.2492622E+01, 5.6299036E+01/
 DATA(F8A(+),I=60,100)/
2   6.0474671E+01, 6.5066298E+01, 7.0128000E+01, 7.5722544E+01,
2   8.1923058E+01, 8.8815087E+01, 9.6499133E+01, 1.0509380E+02,
2   1.1473976E+02, 1.2560462E+02, 1.3788928E+02, 1.5183581E+02,
2   1.6773775E+02, 1.8595332E+02, 2.0692274E+02, 2.3119103E+02,
2   2.5943838E+02, 2.9252105E+02, 3.3152694E+02, 3.7785238E+02,
2   4.3330 266E+02, 5.0027714E+02, 5.8192238E+02, 6.8251978E+02,
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  2   8.07 3374E+02, 9.6635414E+02, 1.1694491E+03, 1.4342207E+03,
  2   1.7860852E+03, 2.2641687E+03, 2.9307839E+03, 3.8892080E+03,
  2   5.3189242E+03, 7.5502829E+03, 1.1236044E+04, 1.7787769E+04,
  2   3.0642697E+04, 5.9664177E+04, 1.4098798E+05, 4.7435606E+05,
  2   3.7830331E+06/
     DATAUF AU+DT+31T 5 D/ 1C570 000E+01, 1.5714186E+01,1.5732762E+01,
  2   1.5763777E+01, 1.5807315E+01, 1.5863493E+01, 1.5932463E+01,
  2   1.6014413E+01, 1.6109566E+01, 1.6218185E+01, 1.6340569E+01,
  2   1.6477061E+01, 1.6628045E+01, 1.6793950E+01, 1.6975252E+01,
  2   1.7172478E+01, 1.7386207E+01, 1.7617076E+01, 1.7865781E+01,
  2   1.8133084E+01, 1.8419816E+01, 1.8726884E+01, 1.9055274E+01,
  2   1.9406060E+01, 1.9780411E+01, 2.0179599E+01, 2.0605007E+01,
  2   2.1058140E+01, 2.1540637E+01, 2.2054280E+01, 2.2601014E+01,
  2   2.3182956E+01, 2.3802416E+01, 2.4461916E+01, 2.5164214E+01,
  2   2.5912323E+01, 2.6709545E+01, 2.7559497E+01, 2.8466152E+01,
  2   2.9433873E+01, 3.0467462E+01, 3.1572213E+01, 3.2753965E+01,
  2   3.4019175E+01, 3.5374992E+01, 3.6829344E+01, 3.8391041E+01,
  2   4.0069889E+01, 4.1876827E+01, 4.3824079E+01, 4.5925337E+01,
  2   4.8195974E+01, 5.0653281E+01, 5.3316765E+01, 5.6208481E+01,
  2   5.9353433E+01, 6.2780045E+01, 6.6520729E+01, 7.0612547E+01/
    DATA(F9A(I),I=60,100)/
  2   7.5098017E+01, 8.0026082E+01, 8.5453271E+01, 9.1445119E+01,
  2   9.8077892E+01, 1.0544071E+02, 1.1363815E+02, 1.2279350E+02,
  2   1.3305279E+02, 1.4458987E+02, 1.5761281E+02, 1.7237209E+02,
  2   1C8 17108E+02T 2.0837956E+02, 2.3045146E+02, 2.5594812E+02,
  2   2.8556931E+02, 3.2019491E+02, 3.6094160E+02, 4.0924100E+02,
  2   4.66 4873E+02, 5.3649902E+02, 6.2112740E+02, 7.2519715E+02,
  2   8.5468744E+02, 1.01793 2E+03, 1.2268227E+03, 1.4986157E+03,
  2   1.8591183E+03, 2.3480042E+03, 3.0283905E+03, 4.0047739E+03,
  2   5.4585626E+03, 7.7232913E+03, 1.1457308E+04, 1.8082732E+04,
  2   3.1058803E+04, 6.0301284E+04, 1.4209818E+05, 4.7680527E+05,
  2   3.7926419E+06/
10 FORMAT (9(E14.7),/)
11 FORMAT (4I5)
12 FORMAT (8F10.4)
13 FO-MAT (54H THE AMPL+TUDE OF -ES-ONSE =                               ,
  1 E14.7,/)
14 FORMAT (54H PHASE ANGLE DIFFERENCE (DEGREE) =                         ,
  1 E14.7,///)
15 FORMAT (1H1)
16 FORMAT (54H THE GUESSED AMPLITUDE OF RESPONSE =                       ,
  1 E14.7,/)
17 FORMAT (5(E14.7,6X),/)
18 FORMAT ( 5X6HQ(LB)=,17X3HHO=,12X6HM(SLUG)=,14X6HD(IN)=,8X12HW(RAD/
  1 SEC)=)
19 FORMAT (F10.5,7(E14.7,1X),/)
20 FORMAT (2X3HHA=,5X,6HFI(1)=,9X,6HFI(3)=,9X3HFA=,12X,6HFI(7)=,9X,3H
  1DA=,13X,4HDAO=,11X,2HF=)
21 FORMAT (54H THE CHARACTE-ISTIC  FREQUENCY                             ,
  1 E14.4,/)
22 FORMAT (54H THE NONDIMENSIONALIZED FREQUENCY                          ,
  1 E14C4,/D
23 FO-MAT (+5,5X,(7F10.5))
24 FORMAT (54H THE ZERO-ORDER AMPLITUDE OF RESPONSE                      ,
```

```
  1 E14.7,/)
25 FORMAT (54H THE NOND+MENSIONALIZED UPPER RESPONSE=                    ,
  1 E14.7,/)
26 FORMAT (54H THE NONDIMENSIONALIZED LOWER RESPONSE=                    ,
  1 E14.7,/)
   BKS=C1, BCS=C2, AN1=N1=2.5, AN2=N2=2.5,  TOL IS THE TOLERANCE OF
   ERROR AALLOWED FOR THE SOLUTION, BMASS IS IN SLUG, AND DELTA IS
   STEP IN INCHES.
   AG(I), AND AOG(I) ARE I SERIES OF THE GUESSED VALUES OF AMPLITUDES,
   A AND AO RESPECTIVELY.  QN(I) THE GUESSED VALUES OF THE DYNAMIC
   LOAD IN LBF.  WN(I) ARE SERIES OF THE NONDIMENSIONALIZED FREQUENCY
   TO BE USED IN THE CALCULATION.
   -EAD 12, BKS, BCS, AN1, AN2
   -EAD 12, HO, BMASST DELTAT TOL
   READ 23, IG, (AG(I),I=1,IG)
   -EAD 23, IG, (AOG(I),I=1,+G)
   READ 23, LQ, (QN(I),I=1,LQ)
   IA=0.
   DO 110 IQ=1,LQ
   -EAD 23, LW,(WN(I),I=1,LW)
   PRINT 15
   +AA=0C
   QS=QN(IQ)
   AMASS=BMASS/12.
   AMDEL=DELTA*AMASS
   HON1=HO**AN1
   HON2=HO*MAN2
   HOPN1=HON1*HO
   HOPN2=HON2*HO
   WS2=BKS*AN1/AMDEL/HOPN1
   WS=SQRT(WS2)
   P-+NT 21, WS
   DO 110 +W31,LW
   +A=+Q-0.
   WB=WN(IW)
   W=WB*WS
   PRINT 18
   PRINT 17,QS, HO, BMASS, DELTA, W
   PRINT 22, WB
   WB2=WB*WB
   WO=WS2*AMDEL
   Q=QS/WO
   HOMAX=HO*0.99
   PI=3.1416
   BK=BKS/WO
   BC=BCS*WB/WS/AMASS
   PHO1=PI*HON1
   PHO2=PI*HON2
   B1=BK/PHO1
   B3=BC/PHO2
   THE AMPLITUDE OF A FOR W=0., AO=0. IS ESTIMATED BY LINEAR INTERPOSITION.
   IF (W.GT.0.) GO TO 50
   IAA=1
   AWO=Q*PI*HON1/BK
```

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```
      IF (AWO-F1A(50)) 41, 42, 43
   42 A=0.5*HO
      AO=0.
      GO TO 50
   41 I1=1
      +2=50
      GO TO 44
   43 +1=50
      I2=100
   44 I3=(I1+I2)/2
      ID=I2-I1+0.1
      +F (+D-1D 45, 45, 46
   46 IF (AWO-F1A(I3)) 47, 49, 48
   47 I2=I3
      GO TO 44
   48 I1=I3
      GO TO 44
   45 AI=+1.
      A=(AI+(AWO-F1A(I1))/(F1A(I2)-F1A(+1)))/100.*HO
      AO=0.
      GO TO 50
   4  A=+3/100.MHO
      AO=0.
   50 CONTINUE
      GUESS THE VALUES OF A AND AO.
      IF (IAA-1) 1,6,6
    1 CONTINUE
      +A=+A+1
      IF (IA.GT.IG) GO TO 110
      A=AG(IA)
      AO=AOG(IA)
    6 CONTINUE
      --+NT 16T A
      ITER=0.
  103 CONTINUE
      HAO5=(1.+AO/HO)**2.5
      +AO7=(1.+AO/HO)**3.5*HO
      A+O3A/U+O+AOD
      HA=AHO*100.
      HA=HA+1.
      IHA=HA
              +A+1
              +1 0++AH+O8A-+HA)*(F1A(IHA1)-F1A(IHA))
      F+(2D3F2AU+HA)+(HA-+HA)*UF2AU++A1)-F2A(IHAD)
      FI(3)=F3A(IHA)+(HA-IHA)*(F3A(IHA1)-F3A(IHA))
      FI(4)=F4A(+HA)+U+A-IHA)*(F4A(IHA1)-F4A(IHA))
      F+(5)3F5A(+HA)+(HA-+HA)*UF5A(IHA1D-F5A(IHAU)
      FI(6)=F6A(IHA)+(HA-IHA)*(F6A(IHA1)-F6A(IHA))
      FI(7)=F7A(IHA)+(HA-IHA)*(F7A(IHA1)-F7A(IHA))
      FI(8)=F8A(IHA)+(HA-IHA)*(F8A(IHA1)-F8A(IHA))
      FI(9)=F9A(IHA)+(HA-IHA)*(F9A(IHA1)-F9A(IHA))
      FI(1)= FI(1)/HAO5
      FI(2)= FI(2)/HAO7
      FI(3)= FI(3)/HAO5
```

```
      FI(4)= FI(4)/HAO7
      FI(5)=-FI(5)/HAO7
      FI(6)=-FI(6)/HAO7
      FI(7)= FI(7)/HAO5
      FI(8)= FI(8)/HAO7
      FI(9)=-FI(9)/HAO7
      F=(FI(1)*B1-A*WB2)**2+(F+(3)*B3*A)**2-Q*Q
      FA1=(FI(1)*B1-A*WB2)*(FI(2)*B1-WB2)*2.
      FA2=(FI(3)*B3*A)*(FI(3)*B3+FI(4)*B3*A)*2.
      FA=FA1+FA2
      G=FI(7)-2.*PI
      FAO=2.*(B1*FI(1)-A*WB2)*B1*FI(5)+(B3*A)**2*FI(3)*FI(6)*2.
         DEL=FA*FI(9)-FAO*FI(8)
      DA=(-F*F+( )+G*FAO)/DEL
      DAO=(F*FI(8)-G*FA)/DEL
      IF (ABS(DA).GT.0.5) GO TO 1
      AO=AO+DAO
      A=A+DA
      P-+NT 20
      PRINT 19,HA, FI(1), FI(3), FA, FI(7), DA, DAO, F
      PRINT 10, FAO, DEL, G
      ITER=ITER+1
      IF (A.LE.0.) GO TO 1
      AOB=ABS(AO)
      AAO=ABS(A-AOB)
      IF (AAO.GT.HOMAX) GO TO 1
      IF (ITER.GT.15) GO TO 1
      IF (ABS(DA).GT.TOL) GO TO 103
      IF (ABS(DAO).GT.TOL) GO TO 103
      R=FI(3)*BC*A/PHO2/(FI(1)*BK/PHO1-A*WB2)
      ALPHA=ATAN(R)
      ALPHA=ALPHA*180./3.1416
      PRINT 24, AO
      PRINT 13, A
      AU=A+AO
      ADOWN=A-AO
      PRINT 25, AU
      PRINT 26, ADOWN
      PRINT 14, ALPHA
      +AA=+AA+1
  110 CONTINUE
      END
```

Card 1 Format (8F10.5)

BKS - Value of c_1 for the stiffness of the gas film force in lb_f.

BCS - Value of c_2 for the damping of the gas film force in lb_f/ips.

AN1 - Power n_1 for the stiffness force in terms of gas film thickness.

AN2 - Power n_2 for the damping force in terms of gas film thickness.

Card 2 Format (8F10.5)

HO - Normalized gas film thickness at equilibrium.

BMASS - Mass of the step ring in response in slug.

DELTA - Step depth of the pad in inches.

TOL - Convergence factor for the amplitude iteration.

Card 3 Format (I5, 5X, (7F10.5))

IG - Total number of the guessed amplitude A of the response.

AG(I) - Array of the guessed amplitude A of the response.

Card 4 Format (I5, 5X, (7F10.5))

IG - Total number of the guessed amplitude Ao of the response.

ADG(I) - Array of the guessed amplitudes Ao of the response.

Card 5 Format (I5, 5X, (7F10.5)

LQ - Total number of the force excitation to be investigated in the production run.

QN(I) - Array of amplitude of force excitation in lb_f.

Card 6 Format (I5, 5X, (7F10.5))

LW - Total number of the normalized forcing frequencies to be investigated.

WN(I) - Array of normalized forcing frequencies.

If the sixth input statement is located within the doloop of IQ = 1, LQ, LQ sets of card 6 are required.

```
      PROGRAM RSRKIT(INPUT,OUTPUT,PUNCH,TAPE99)
      COMMON Y, DY, ATABL, RTABL, IFVD, X, DX,W, BK, BC, Q, AMASS, DELTA
      COMMON HO, TLAST, AY, FAC, WB2, HON1, HON2, HOPN1, HOPN2,AN1,AN2
      COMMON XA, TA, XTA, IA, IA2, DXI, WLB, WB, SLOPE, TLW, WBO, XO
      DIMENSION Y(2), DY(2), ATABL(2), -TABL(2), WO-K(18)
      DIMENSION XA(1000), TA(1000), XTA(1000)
      EXTERNAL DERIV, CNTRL, GRAPH
   11 FORMAT (8F10.5)
   12 FORMAT (54H MASS OF THE SEAL (SLUG)                              ,
     1 E14.3,/)
   13 FORMAT (54H DEEPTH OF STEP DISCONTINUITY (IN)                    ,
     1 E14.3,/)
   14 FORMAT (54H NONDIMENSIONALIZED EQUILIBRIUM POSITION, HO          ,
     1 E14.3,/)
   15 FORMAT (54H FREQUENCY OF PERIODIC FORCE, W (RAD/SEC)             ,
     1 E14.3,/)
   16 FORMAT (54H AMPLITUDE OF THE PERIODIC FORCE APPLIED (LBS)        ,
     1 E14.3,/)
   17 FORMAT (54H NONDIMENSIONALIZED DAMPING COEFFICIENT OF GAS FILM C ,
     1 E14.3,/)
   18 FORMAT (60H NONDIMENSIONALIZED MULTIPLE OF STIFFNESS *  DISPLACEME
     1NT, K,E14.3,/)
   19 FORMAT (54H NONDIMENSIONALIZED DISPLACEMENT                      ,
     1 E14.3,/)
   20 FORMAT (54H THE MULTIPLE OF TIME AND FREQUENCY (RAD)             ,
     1 E14.3,/)
   21 FORMAT (3X7HDEGREE=,4X,11HPHASE(RAD)=,7X13HDISPLACEMENT=,11X9HVELO
     1CITY=,/)
   31 FORMAT (7X,23HDATA (XA(IA),IA=1, 70)/,2(E14.6,1H,))
   32 FORMAT (5X,1H2,1X,4(E14.6,1H,))
   33 FORMAT (7X,22HDATA (TA(IA),IA=1,94)/,4(F9.5,1H,))
   34 FORMAT (5X,1H3,1X,6(F9.5,1H,))
   35 FORMAT (7X26HDATA (XTA(IAP),IAP=1,360)/,2(E14.6,1H,))
   36 FORMAT (5X1H2,1X,4(E14.6,1H,))
   40 FORMAT (52H ERROR RETURN, DX=0                                  ,/)
   41 FORMAT (52H NORMAL RETURN                                       ,/)
   42 FORMAT (52H ERROR RETURN, VARIABLE INTERVAL MODE ONLY           ,/)
      TLAST IS THE FINAL TIME OF THE INTERVAL IN INTEGRATION.
      TLW IS THE FINAL TIME THAT THE SMOOTH VARIATION OF FREQUENCY ENDS.
      WLB IS THE FINAL NONDIMENSIONALIZED FREQUENCY TO BE USED IN THE
      TECHNIC OF SMOOTH VARYING FREQUENCY.
      Y(1), Y(2), X ARE INITIAL CONDITIONS OF X, DX/DT, T RESPECTIVELY.
      BKS=C1.  BCS=C2.  DELTA +S STEP IN INCHES.  QS IS DYNAMIC LOAD IN
      LB.   AM IS MASS OF THE PAD IN SLUG.  WB IS THE NONDIMENSIONALIZED
      FREQUENCY.  AN1=N1=2.5, AN2=N2=2.5 DXI IS THE TIME INCREMENT IN DEGREE.
      READ 11, TLAST, TLW, WLB
      READ 11, Y(1), Y(2), X
      READ 11 , BKS, BCS, QS, AM, DELTA, HO, WB
                              0+
      HON1=HO**AN1
      HON2=HO**AN2
      HOPN1=HON1*HO
      HOPN2=HON2*HO
```

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```
      AMASS=AM/12.
      AMDEL=AMASS*DELTA
      WS2=BKS*AN1/AMDEL/HOPN1
      WS=SQRT(WS2)
      W=WB*WS
      WB2=WB*WB
      WL=WLB*WS
      XO=X
      SLOPE=(WLB-WB)/WB/(TLW-XO)
      WBO3WB
      WO=WS2*AMDEL
      Q=QS/WO
      BK=BKS/WO
      BC=BCS/WS/AMASS
      PRINT 12, AM
      PRINT 13, DELTA
      PRINT 14, HO
      PRINT 15 , W
      PRINT 16, QS
      PRINT 17, BC
      PRINT 18, BK
      DEFINE THE INITIAL VALUE OF Y(I)
      XA(1)=Y(1)
      TA(1)=X
      XTA(1)=Y(2)
      DEFINE X AS Y(1), AND DX/DT AS Y(2)
      THE FOLLOWING IS PREPARED FOR RUNGE-KUTTA NUMERICAL INTEGRATION.
      PI=3.1415926535
      DX=DXI*PI/180.
      FAC=10.**0.2
      NTRY=1
      N=2
      IFVD=1
      IBKP=1
      ATABL(1)=0.001
      ATABL(2)=0.001
      RTABL(1)=0.001
      RTABL(2)=0.001
      IA2=1
      IA=1
      PRINT 21
      CALL RKS3(DERIV,CNTRL,Y,DY,ATABL,RTABL,WORK,X,DX,N,IFVD,IBKP,NTRY,
     1 IERR)
      IF (IERR-0) 4, 5, 6
    4 CONTINUE
      PRINT 40
      GO TO 3
    5 CONTINUE
      PRINT 41
      GO TO 3
    6 CONTINUE
      PRINT 42
    3 CONTINUE
      THE FOLLOWING IS FOR THE PLOTS OF X AND DX/DT V.S. T.
```

```
      CALL NAMPLT
      CALL SCALE (XA,5.0,IA,1)
      CALL SCALE (TA,40.0,IA,1)
      CALL SCALE (XTA,5.0,IA,1)
      CALL AXIS(0.0,0.0,7HT VALUE,7,40.0,0.0,TA(IA+1),TA(IA+2))
      CALL AXIS (0.0,0.0,7HX VALUE,-7,8.,90.0,XA(IA+1),XA(IA+2))
      CALL AXIS (1.0,0.0,8HXT VALUE,-8,8.0,90.0,XTA(IA+1),XTA(IA+2))
      CALL LINE (TA,XA,IA,1,1,0)
      CALL LINE (TA,XTA,IA,1,1,3)
      CALL SYMBOL (3.0,9.0,0.20,16HPLOT OF X V.S. T,0.0,16)
      CALL SYMBOL(4.0,10.0,0.20,20HPLOT OF DX/DT V.S. T,0.0,15)
      CALL ENDPLT
      THE FOLLOWING IS FOR THE PHASE PLOT OF X V.S. DX/DT.
      RIA=IA
      WIDTH=10.
      WSPACE=10.
      CALL SETUPC(0.,0.,0.,1.,-20.,2.,WIDTH,WSPACE)
      CALL PARAM(GRAPH,1.,1.,RIA)
      CALL ENDSURF
      END



       UB-OUTINE DERIV
      DIMENSION Y(2), DY(2), ATABL(2), -TABL(2), WORK(18)
      DIMENSION XA(1000), TA(1000), XTA(1000)
      COMMON Y, DY, ATABL, RTABL, IFVD, X, DX,W, BK, BC, Q, AMASS, DELTA
      COMMON HO, TLAST, AY, FAC, WB2, HON1, HON2, HOPN1, HOPN2,AN1,AN2
      COMMON XA, TA, XTA, IA, IA2, DXI, WLB, WB, SLOPE, TLW, WBO, XO
      DY(1)=DY(1)/DT, DY(2)=DY(2)/DT.
      DY(1)=Y(2)
      DY(2)=(BK/HON1+Q*COS(X)-BK/(HO-Y(1))**AN1-BC*Y(2)*WB/(HO-Y(1))**AN
     12)/WB2
      RETURN
      END



      SUBROUTINE CNTRL(NTRY)
      DIMENSION Y(2), DY(2), ATABL(2), RTABL(2)
      DIMENSION XA(1000), TA(1000), XTA(1000)
      COMMON Y, DY, ATABL, RTABL, IFVD, X, DX,W, BK, BC, Q, AMASS, DELTA
      COMMON HO, TLAST, AY, FAC, WB2, HON1, HON2, HOPN1, HOPN2,AN1,AN2
      COMMON XA, TA, XTA, IA, IA2, DXI, WLB, WB, SLOPE, TLW, WBO, XO
   51 FORMAT (5X,I5,4(E14.6,6X))
   54 FORMAT (54H THE DISPLACEMENT IS OUT OF RANGE                          )
      IF Y(1) IS LESS THAN -4., OR Y(1) IS GREATER THAN HO, TERMINATE
      THE NUMERICAL INTEGRATION.
      AY=ABS(Y(1))
      IF (AY.GT.4.0) GO TO 2
      IF (Y(1)-HO) 1, 2, 2
      CONTINUE
```

```
      PRINT 54
      NTRY=2
    1 CONTINUE
      IF (X-TLAST) 3, 3, 4
    4 NTRY=2
    3 CONTINUE
      IA2=IA2+1
      IADX=IA2*DXI
      IA=IA2/4+0.0001
      XTA(IA)=Y(2)
      XA(IA)=Y(1)
      TA(IA)=X
      PRINT 51 , IADX, X, Y(1), Y(2), WB
      IF (WB-WLB) 11, 12, 12
   11 WB=WBO*(1.+SLOPE*(X-XO))
      GO TO 13
   12 WB=WLB
   13 CONTINUE
      WB2=WB*WB
      RETURN
      END



      SUBROUTINE GRAPH(T,XP,YP,ZP)
      COMMON Y, DY, ATABL, RTABL, IFVD, X, DX,W, BK, BC, Q, AMASS, DELTA
      COMMON HO, TLAST, AY, FAC, WB2, HON1, HON2, HOPN1, HOPN2,AN1,AN2
      COMMON XA, TA, XTA, IA, IA2, DXI, WLB, WB, SLOPE, TLW, WBO, XO
      DIMENSION Y(2), DY(2), ATABL(2), RTABL(2), WORK(18)
      DIMENSION XA(1000), TA(1000), XTA(1000)
      NP=T+.0001
      XP=XA(NP)*5.
      YP=0.0
      ZP=XTA(NP)*5.
      RETURN
      END
```

Card 1 Format (8F10.5)

TLAST - Final normalized time in radian for the integration.

TLW - Final normalized time in radian for the changing of $\bar{W}$ during the integration.

WLB - Final normalized forcing frequency.

Card 2 Format (8F10.5)

Y(1) - The initial value of X.

Y(2) - The initial value of $\dot{X}$, i.e. dX/dT.

X - The initial time, T_i for the integration.

Card 3 Format (8F10.5)

BKS - Value of c_1 in lb_f.

BCS - Value of c_2 in lb_f/ips.

QS - Value of q in lb_f.

AM - Mass of the ring in response in slug.

DELTA - Step depth, δ in inches.

HO - Normalized equilibrium gas film thickness.

WB - The initial normalized excitational frequency.

Card 4 Format (8F10.f)

AN1 - Value of n_1 which is 2.5 in the case being investigated here.

AN2 - Value of n_2 which is 2.5 in the case being investigated here.

DXI - Time increments during the fixed interval of the numerical integration.

```
      PROGRAM RSTAB(INPUT,OUTPUT)
      DIMENSION SA(50), SB(50), SC(50), SD(50), SE(50), SF(50)
      DIMENSION PO(50), PCR(50), PCI(50), DX(50), C(100), A(70,70)
      DIMENSION CI(2,3,50), SUBV(50), VI(20)
      DIMENSION XX(100), B(50,3), BH(50), AH(50), CH(50)
      COMMON B1, ALAM, H1, TOL, NL, N2, B2, NL1, N21, NP, NP1, NL2, NL3
      COMMON H2, H13, H23, HSUM
      COMMON SA, SB, SC, SD, SE, SF
   11 FORMAT (4F10.4,3I5)
   12 FORMAT (8F10.5)
   13 FORMAT (54H THE THRESHOLD FREQUENCY OF THE STEP SEAL                ,
     1 E14.4,/)
   14 FORMAT (54H THE SQUEEZE FILM PARAMETER                              ,
     1 E14.4,/)
   15 FORMAT (54H THE BEARING NUMBER                                      ,
     1 E14.4,/)
   16 FORMAT (54H MASS                                                    ,
     1 E14.4,///)
   17 FORMAT (54H RATIO OF MIN CLEARANCE HEIGHT TO DIFF. IN CLEARANCES ,
     1 E14.4,/)
   18 FORMAT (54H THE 1ST ORDER PRESSURISED LOAD                          ,
     1 E14.4,/)
   19 FORMAT ((8E14.4),/)
   20 FORMAT (54H THE POINTS ARE TOO FEW                                  )
   21 FORMAT (54H A AND/OR B IS OUT OF RANGE OF TABLE                     )
   22 FORMAT (54H THE GUESSED THRESHOLD FREQUENCY OF THE STEP SEAL        ,
     1 E14.4,/)
   23 FORMAT (52H THE GRID DIFFERENCE                                   ,/)
   24 FORMAT (52H THE ZERO ORDER PRESSURE DISTRIBUTION                  ,/)
   25 FORMAT (54H THE VALUE OF SMALLNESS                                  ,
     1 E14.4,/)
   26 FORMAT (54H LOAD DUE TO THE IMAGINARY PART OF COMPLEX PRESSURE    ,
     1 E14.4,/)
   27 FORMAT (5X,I5,(7F10.4))
   29 FORMAT (/,(8E14.4,/))
   30 FORMAT (52H THE IMAGINARY PART OF COMPLEX PRESSURE DISTRIBUTION,/)
   31 FORMAT (52H THE REAL PART OF COMPLEX PRESSURE DISTRIBUTION        ,/)
   32 FORMAT (54H THE RATIO OF WIDTH OF STEP WITH HEIGHT H1               ,
     1 E14.4,/)
      READ 11, B1, ALAM, H1, TOL, NL, N2, NV
      READ 12, (VI(I),I=1,NV)
      B2=1.-B1
      NL1=NL-1
      NL2=NL-2
      NL3=NL-3
      ND=NL2+NL2
      N21=N2-1
      NP=N2+1
      NP1=NP+1
      H12=H1**2
      H13=H1**3
      H2=H1+1.
      H22=H2**2
      H23=H2**3
```

```
      READ 12, (DX(I),I=1,NL1)
      READ 12, (PO(I),I=1,NL)
      HSUM=H13+3.*H1*H2*(H1+H2)+H23
      CALL NEWR(PO,DX)
      CALL INTEG(0.,1.,DX,PO ,NL,WLOAD,CI,IERR)
      PRINT 32, B1
      PRINT 15, ALAM
      PRINT 17, H1
      PRINT 14, SEG
      PRINT 23
      PRINT 19, (DX(I),I=1,NL1)
      PRINT 24
      PRINT 19, (PO(I),I=1,NL)
      PRINT 25, TOL
      A1=-ALAM*H1/2.
      A2=-ALAM*(H1+H2)/4.
      A3=-ALAM*H2/2.
      THE COEFFICIENTS OF SMALL A, B, C, D, E, F AT EACH GRID POINT ARE
      CALCULATED IN THE FOLLOWING AS SA, ... ETC.
      DO 2 I=2,N21
      SA(I)=H13/DX(I)
      SC(I)=H13/DX(I-1)
      SB(I)=-SA(I)-SC(I)
      SD1=(PO(I+1)**2/DX(I)    -PO(I)**2*(1./DX(I)+1./DX(I-1))+PO(I-1)**2
     1 /DX(I-1))*3.*H12/4.
      SD2=ALAM*(PO(I+1)-PO(I-1))/2.
      SD(I)=SD1-SD2
      SE(I)=H1*(DX(I)+DX(I-1))/2.
      SF(I)=(DX(I)+DX(I-1))/2.
      AH(I)=A1
      CH(I)=-A1
      BH(I)=0.
    2 CONTINUE
      SA(N2)=HSUM/8./DX(N2)
      SC(N2)=H13/DX(N21)
      SB(N2)=-SA(N2)-SC(N2)
      SD1=((PO(NP)**2-PO(N2)**2)*(H12+H1*H2+H22)/DX(N2)+(PO(N21)**2-
     1 PO(N2)**2)*3.*H12/DX(N21))/4.
      SD2=ALAM*(PO(NP)-PO(N21))/2.
      SD(N2)=SD1-SD2
      SE(N2)=((H1+H2)*DX(N2)+2.*H1*DX(N21))/4.
      SF(N2)=(DX(N2)+DX(N21))/2.
      AH(N2)=A2
      CH(N2)=-A1
      BH(N2)=AH(N2)+CH(N2)
      DO 3 I=NP1,NL1
      SA(I)=H23/DX(I)
      SC(I)=H23/DX(I-1)
      SB(I)=-SA(I)-SC(I)
      SD1=(PO(I+1)**2/DX(I)-PO(I)**2*(1./DX(I)+1./DX(I-1))+PO(I-1)**2
     1 /DX(I-1))*3.*H22/4.
      SD2=ALAM*(PO(I+1)-PO(I-1))/2.
      SD(I)=SD1-SD2
      SE(I)=H2*(DX(I)+DX(I-1))/2.
```

```
      SF(I)=(DX(I)+DX(I-1))/2.
      AH(I)=A3
      CH(I)=-A3
      BH(I)=0.
    3 CONTINUE
      SA(NP)=H23/DX(NP)
      SC(NP)=HSUM/8./DX(N2)
      SB(NP)=-SA(NP)-SC(NP)
      SD1=((PO(NP1)**2-PO(NP)**2)*3.*H22/DX(NP)+(PO(N2)**2-PO(NP)**2)*
     1 (H12+H1*H2+H22)/DX(N2))/4.
      SD2=ALAM*(PO(NP1)-PO(N2))/2.
      SD(NP)=SD1-SD2
      SE(NP)=((H1+H2)*DX(N2)+2.*H2*DX(NP))/4.
      SF(NP)=(DX(NP)+DX(N2))/2.
      AH(NP)=A3
      CH(NP)=-A2
      BH(NP)=AH(NP)+CH(NP)
       CALCULATE ELEMENTS OF (A) AND (C) IN THE EQUATION AX=C.
      B MATRIX IS USED TO SAVE A(I,J) WHICH ARE INDEPENDENT ON THE
      DO 4 J=1,ND
      DO 4 I=1,ND
      A(I,J)=0.
    4 CONTINUE
      V=VI(1)
      PRINT 22, V
      AMASS=WLOAD/V**2
      PRINT 16, AMASS
      DO 5 J=2,NL3
      I=J+1
      JN=NL2+J
      IN=JN+1
      A(J,I)=SA(I)*PO(I+1)+AH(I)
      A(JN,IN)=A(J,I)
      A(J,J)=SB(I)*PO(I)+BH(I)
      A(JN,JN)=A(J,J)
      A(J,J-1)=SC(I)*PO(I-1)+CH(I)
      A(JN,JN-1)=A(J,J-1)
      A(J,JN)=SE(I)*V
      A(JN,J)=-A(J,JN)
      C(J)=-SD(I)
      C(JN)=SF(I)*PO(I)*V
      B(J,3)=A(J,J+1)
      B(J,2)=A(J,J)
      B(J,1)=A(J,J-1)
    5 CONTINUE
      A(1,1)=SB(2)*PO(2)+BH(2)
      A(NL1,NL1)=A(1,1)
      A(1,2)=SA(2)*PO(3)+AH(2)
      A(NL1,NL)=A(1,2)
      A(1,NL1)=SE(2)*V
      A(NL1,1)=-A(1,NL1)
      C(1)=-SD(2)
      C(NL1)=SF(2)*PO(2)*V
      A(NL2,NL3)=SC(NL1)*PO(NL2)+CH(NL1)
```

```
      A(ND,ND-1)=A(NL2,NL3)
      A(NL2,NL2)=SB(NL1)*PO(NL1)+BH(NL1)
      A(ND,ND)=A(NL2,NL2)
      A(NL2,ND)=SE(NL1)*V
      A(ND,NL2)=-A(NL2,ND)
      C(NL2)=-SD(NL1)
      C(ND)=SF(NL1)*PO(NL1)*V
      B(1,1)=A(1,1)
      B(1,2)=A(1,2)
      B(NL2,1)=A(NL2,NL3)
      B(NL2,2)=A(NL2,NL2)
      DO 61 J=1,ND
      XX(J)=C(J)
   61 CONTINUE
      CALL DETEQ(A,XX,ND,DET)
      IF (DET-0.) 108, 109, 108
  109 STOP
  108 CONTINUE
      PCI(1)=0.
      PCI(NL)=0.
      PCR(1)=0.
      PCR(NL)=0.
      DO 7 J=NL1,ND
      PCI(J-NL3)=XX(J)
    7 CONTINUE
      PRINT 30
      PRINT 29, (PCI(I),I=1,NL)
      CALL INTEG(0.,1.,DX,PCI,NL,PSM1,CI,IERR)
      IF (IERR-1) 113, 114, 115
  114 PRINT 20
      STOP
  115 PRINT 21
      STOP
  113 CONTINUE
      PRINT 26, PSM1
      DO 69 J=1, NL2
   69 PCR(J+1)=XX(J)
      PRINT 29, (PCR(I),I=1,NL)
      PRINT 31
      CALL INTEG(0.,1.,DX,PCR,NL,PRSM,CI,IERR)
      PRINT 18, PRSM
      AMASS=PRSM/V**2
      PRINT 16, AMASS
      DO 121 IV=2,NV
      V=VI(IV)
      PRINT 22, V
      DO 51 J=1,ND
      DO 51 I=1,ND
      A(I,J)=0.
   51 CONTINUE
      DO 52 J=2,NL3
      I=J+1
      JN=NL2+J
      IN=JN+1
```

```
      A(J,J-1)=B(J,1)
      A(J,J)=B(J,2)
      A(J,J+1)=B(J,3)
      A(JN,IN)=A(J,I)
      A(JN,JN)=A(J,J)
      A(JN,JN-1)=A(J,J-1)
   52 CONTINUE
      A(1,1)=B(1,1)
      A(1,2)=B(1,2)
      A(NL2,NL3)=B(NL2,1)
      A(NL2,NL2)=B(NL2,2)
      A(NL1,NL1)=A(1,1)
      A(NL1,NL)=A(1,2)
      A(ND,ND-1)=A(NL2,NL3)
      A(ND,ND)=A(NL2,NL2)
      CALCULATE THE ELEMENTS OF A AND C WHICH VARY WITH RESPECT TO V.
      DO 6 J=1,NL2
      I=J+1
      JN=J+NL2
      A(J,JN)=SE(I)*V
      A(JN,J)=-A(J,JN)
      C(JN)=SF(I)*PO(I)*V
    6 CONTINUE
      DO 62 J=1,ND
      XX(J)=C(J)
   62 CONTINUE
      CALL DETEQ(A,XX,ND,DET)
      IF (DET-0.) 111, 112, 111
  112 STOP
  111 CONTINUE
      DO 8 J=NL1,ND
      PCI(J-NL3)=XX(J)
    8 CONTINUE
      PRINT 30
      PRINT 29, (PCI(I),I=1,NL)
      CALL INTEG(0.,1.,DX,PCI,NL,PSM2,CI,IERR)
      IF (IERR-1)  116,117, 118
  117 PRINT 20
      STOP
  118 PRINT 21
      STOP
  116 CONTINUE
      PRINT 26, PSM2
      DO 9 J=1, NL2
    9 PCR(J+1)=XX(J)
      PRINT 31
      PRINT 29, (PCR(I),I=1,NL)
      CALL INTEG(0.,1.,DX,PCR,NL,PRSM,CI,IERR)
      PRINT 18, PRSM
      AMASS=PRSM/V**2
      PRINT 16, AMASS
  121 CONTINUE
      END
```

```
      SUBROUTINE INTEG(A,B,H,F,NP,VALUE,C,IERR)
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*
      SUBROUTINE INTEG
*     ----+      -----
*
*
      INTEGRATES THE NON EQUIDISTANTLY TABULATED FUNCTION F(X(I))
      BETWEEN THE LIMITS A AND B.
*
      A MODIFIED METHOD OF OVERLAPPING PARABOLAS IS EMPLOYED.
*
      A SECOND ENTRY POINT 'INTEG2' IS PROVIDED FOR MORE THAN ONE
      INTEGRATION ON THE SAME DIVISIONS OF X.  THIS SAVES THE TIME
      OF CALCULATING THE WEIGHTING FUNCTIONS.
*
*     ARGUMENTS -
      A          LOWER LIMIT OF INTEGRATION.
      B          UPPER LIMIT OF INTEGRATION.
*     X          ARRAY OF ARGUMENT VALUES.  MUST BE MONOTONICALLY
                 INCREASING AND MUST BE DIMENSIONED NP.
      F          ARRAY OF FUNCTION VALUES. MUST BE DIMENSIONED NP.
*     NP         NUMBER OF POINTS.  NP MUST BE GREATER THAN 3.
*     VALUE      RESULTANT VALUE OF THE INTEGRATION.
      C          WEIGHTING FUNCTION PASSED TO THE MAIN PROGRAM
                 FOR STORAGE.
*     IERR       RESULTANT ERROR PARAMETER.
*
      REQUIRED SUBPROGRAMS - NONE
*
      COMMON STORAGE -
      THE WEIGHTING FUNCTION C IS STORED IN THE MAIN PROGRAM AND
*     REQUIRES THE FOLLOWING DIMENSION STATEMENT WHERE D.GE.NP.
*                DIMENSION C(2,3,D)
*
      ERROR INDICATIONS -
*     IERR = 0   INDICATES NO ERROR.
      IERR = 1   INDICATES NP IS LESS THAN 4.
      IERR = 2   INDICATES THE LIMITS OF INTEGRATION ARE OUT OF
*                THE RANGE OF THE TABLE.
*
*     EDWARD G. TRACHMAN             8 JULY 1970  M.E. DEPT.  492-5640
*
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION X(50), F(50), C(2,3,50)
      DIMENSION H(50),SUBV(50)
*
*     NP MUST BE GREATER THAN 3
*
      IF (NP.LE.3) GO TO 96
*
*     CALCULATION OF INTERVALS OF X
*
      NH=NP-1
      X(1)=0.
```

```
      DO 10 I=1,NH
   10 X(I+1)=X(I)+H(I)
      DO 20 I=1,NH
      IF (I.EQ.1) GO TO 15

      DEFINE COEFFICIENTS OF FIRST PARABOLA

      C(1,1,I)=-(H(I))**3/(6.*H(I-1)*(H(I-1)+H(I)))
      C(1,2,I)=H(I)*(3.*H(I-1)+H(I))/(6.*H(I-1))
      C(1,3,I)=H(I)*(3.*H(I-1)+2.*H(I))/(6.*(H(I-1)+H(I)))
   15 CONTINUE
      IF (I.EQ.NH) GO TO 20

      DEFINE COEFFICIENTS OF SECOND PARABOLA

      C(2,1,I)=H(I)*(2.*H(I)+3.*H(I+1))/(6.*(H(I)+H(I+1)))
      C(2,2,I)=H(I)*(H(I)+3.*H(I+1))/(6.*H(I+1))
      C(2,3,I)=-(H(I))**3/(6.*H(I+1)*(H(I)+H(I+1)))
   20 CONTINUE

      ENTRY INTEG2


      INITIALIZE SUMMATION VARIABLE

      VALUE=0.0
      IF (B-A) 40,92,30

      B IS GREATER THAN A

   30 ALIM = A
      BLIM = B
      SIGN = 1.0
      GO TO 50

      A IS GREATER THAN B

   40 ALIM = B
      BLIM = A
      SIGN =-1.0
   50 NH=NP-1

      SETTING THE LOWER LIMIT OF INTEGRATION

      DO 63 I=1,NH
      PARTA = 1.0
      IF (ALIM-X(I)) 61,69,63
   61 IF (I.EQ.1) GO TO 97
      ALIM = X(I-1)
      PARTA=(X(I)-ALIM)/(X(I)-X(I-1))
      GO TO 69
   63 CONTINUE
   69 CONTINUE
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
*     SETTING THE UPPER LIMIT OF INTEGRATION                          INTEG108
*                                                                     INTEG109
      DO 73 I=1,NH                                                    INTEG110
      PARTB = 1.0                                                     INTEG111
      IF (BLIM-X(I)) 71,79,73                                         INTEG112
   71 IF (I.EQ.1) GO TO 97                                            INTEG113
      BLIM = X(I)                                                     INTEG114
      PARTB=(BLIM-X(I-1))/(X(I)-X(I-1))                               INTEG115
      GO TO 79                                                        INTEG116
   73 IF (I.EQ.NP) GO TO 97                                           INTEG117
   79 CONTINUE                                                        INTEG118
*                                                                     INTEG119
*     CALCULATION OF INTEGRAL OVER SUBINTERVAL                        INTEG120
*                                                                     INTEG121
      DO 80 I=1,NH                                                    INTEG122
      SUBV(I) = 0.0                                                   INTEG123
      IF (X(I).EQ.ALIM) SUBV(I)=C(2,1,I)*F(I)+C(2,2,I)*F(I+1)+C(2,3,I)* INTEG124
     1F(I+2)                                                          INTEG125
      IF (I-NH) 102, 103, 103
  103 CONTINUE
      ADX=ABS(X(I+1)-BLIM)
      IF (ADX.LT.1.E-5) SUBV(I)=C(1,1,I)*F(I-1)+C(1,2,I)*F(I)+C(1,3,I)* INTEG126
     1 F(I+1)                                                         INTEG127
      GO TO 101
  102 CONTINUE
      IF (X(I).GT.ALIM.AND.X(I+1).LT.BLIM) SUBV(I)=0.5*(C(1,1,I)*F(I-1) INTEG128
     1+(C(1,2,I)+C(2,1,I))*F(I)+(C(1,3,I)+C(2,2,I))*F(I+1)+C(2,3,I)*  INTEG129
     2F(I+2))                                                         INTEG130
  101 CONTINUE
      IF (PARTA.NE.1.OR.PARTB.NE.1) SUBV(I)=PARTA*PARTB*SUBV(I)       INTEG131
*                                                                     INTEG132
*     CALCULATE THE FINAL VALUE OF THE INTEGRAL                       INTEG133
*                                                                     INTEG134
   80 VALUE=VALUE+SUBV(I)                                             INTEG135
      VALUE=SIGN*VALUE                                                INTEG136
*                                                                     INTEG137
*     SET ERROR PARAMETER FOR NORMAL RETURN                           INTEG138
*                                                                     INTEG139
   92 IERR = 0                                                        INTEG140
      RETURN                                                          INTEG141
*                                                                     INTEG142
*     SET ERROR PARAMETER FOR TOO FEW POINTS                          INTEG143
*                                                                     INTEG144
   96 IERR = 1                                                        INTEG145
      RETURN                                                          INTEG146
*                                                                     INTEG147
*     SET ERROR PARAMETER FOR A AND/OR B OUT OF RANGE OF TABLE        INTEG148
*                                                                     INTEG149
   97 IERR = 2                                                        INTEG150
      RETURN                                                          INTEG151
      END                                                             INTEG152
```

```
      SUBROUTINE DETEQ (A,B,N,DET)
      DIMENSION A(70,70),B(70),IPVOT(70)
      DET=1.0
      DO 11 J=1,N
   11 IPVOT(J)=0.0
      DO 121 I=1,N
      T=0.0
      DO 51 J=1,N
      IF (IPVOT(J)-1) 21,51,21
   21 DO 50 K=1,N
      IF (IPVOT(K)-1) 31,50,50
   31 IF (ABS(T)-ABS(A(J,K))) 41,50,50
   41 IROW=J
      ICOL=K
      T=A(J,K)
   50 CONTINUE
   51 CONTINUE
      IF (ABS(T)-1.E-8) 131,131,55
   55 IPVOT(ICOL)=1
      IF (IROW-ICOL) 61,81,61
   61 DET=-DET
      DO 71 L=1,N
      T=A(IROW,L)
      A(IROW,L)=A(ICOL,L)
   71 A(ICOL,L)=T
      T=B(IROW)
      B(IROW)=B(ICOL)
      B(ICOL)=T
   81 TEMP=A(ICOL,ICOL)
      DET=DET*TEMP
      A(ICOL,ICOL)=1.
      DO 91 L=1,N
   91 A(ICOL,L)=A(ICOL,L)/TEMP
      B(ICOL)=B(ICOL)/TEMP
      DO 121 L1=1,N
      IF (L1-ICOL) 101,121,101
  101 T=A(L1,ICOL)
      A(L1,ICOL)=0.
      DO 111 L=1,N
  111 A(L1,L)=A(L1,L)-A(ICOL,L)*T
      B(L1)=B(L1)-B(ICOL)*T
  121 CONTINUE
      RETURN
  131 DET=0.0
      RETURN
      END
```

```
      SUBROUTINE NEWR(PO,DX)
      DIMENSION AA(50), BB(50), CC(50), DD(50), EE(50), FF(50), DX(50)
      DIMENSION A(50), B(50), C(50), PO(50), DP(50), FI(50), FIDP(50,3)
      COMMON B1, ALAM, H1, TOL, NL, N2, B2, NL1, N21, NP, NP1, NL2, NL3
```

```
      COMMON H2, H13, H23, HSUM
      COMMON AA, BB, CC, DD, EE, FF
   11 FORMAT (4F10.4,2I5)
   12 FORMAT (8F10.5)
   13 FORMAT (6XE14.4,3(6X,E14.4),/)
   14 FORMAT (17X3HFI=,3X17HDFI/DP(J),J=1,2,3,/)
   15 FORMAT (3X2HI=,22X3HDX=,17X3HPO=,/)
   16 FORMAT (5XI5,2(6XE14.4),/)
   17 FORMAT (54H PRESSURE IS SMALLER THAN THE AMBIENT PRESSURE
   18 FORMAT (5XI5,3(6XE14.4),/)
   19 FORMAT (1H1)
      DO 2 I=2, N21
      AA(I)=H23/2./DX(I)
      CC(I)=H23/2./DX(I-1)
      BB(I)=-AA(I)-CC(I)
      FF(I)=ALAM*H2/2.
      DD(I)=-FF(I)
      EE(I)=0.
    2 CONTINUE
      AA(N2)=HSUM/DX(N2)/16.
      CC(N2)=H23/2./DX(N21)
      BB(N2)=-AA(N2)-CC(N2)
      FF(N2)=FF(N21)
      DD(N2)=-ALAM*(H2+H1)/4.
      EE(N2)=DD(N2)+FF(N2)
      DO 3 I=NP1, NL1
      AA(I)=H13/2./DX(I)
      CC(I)=H13/2./DX(I-1)
      BB(I)=-AA(I)-CC(I)
      FF(I)=ALAM*H1/2.
      DD(I)=-FF(I)
      EE(I)=0.
    3 CONTINUE
      AA(NP)=H13/2./DX(NP)
      CC(NP)=HSUM/DX(N2)/16.
      BB(NP)=-AA(NP)-CC(NP)
      DD(NP)=DD(NP1)
      FF(NP)=ALAM*(H1+H2)/4.
      EE(NP)=DD(NP)+FF(NP)
  104 CONTINUE
      DO 5 I=2, NL1
      J=I-1
      A(J)=AA(I)*PO(I+1)+DD(I)
      B(J)=BB(I)*PO(I)+EE(I)
      C(J)=CC(I)*PO(I-1)+FF(I)
      FI(J)=A(J)*PO(I+1)+B(J)*PO(I)+C(J)*PO(I-1)
      FI(J)=-FI(J)
    5 CONTINUE
      DO 6 I=3,NL2
      J=I-1
      FIDP(J,1) = CC(I)*2.*PO(I-1)+FF(I)
      FIDP(J,2)=BB(I)*2.*PO(I)+EE(I)
      FIDP(J,3)=AA(I)*2.*PO(I+1)+DD(I)
    6 CONTINUE
```

```
      FIDP(1,2)=BB(2)*2.*PO(2)+EE(2)
      FIDP(1,3)=AA(2)*2.*PO(3)+DD(2)
      FIDP(NL2,1) = CC(NL1)*2.*PO(NL2)+FF(NL1)
      FIDP(NL2,2) = BB(NL1)*2.*PO(NL1)+EE(NL1)
      CALL  TDLEQ (FIDP,FI,DP,NL2)
      DO 7 I=2, NL1
      PO(I)=PO(I)+DP(I-1)
      IF (PO(I)-0.) 101, 101, 7
  101 PRINT 17
      DO 23 IP=1, NL1
      PRINT 18, IP,DX(IP),PO(IP), DP(IP-1)
   23 CONTINUE
      STOP
    7 CONTINUE
      RMAX=TOL/2.
      DO 8 K=1, NL2
      R=DP(K)
      IF (R-RMAX) 8, 8, 102
  102 RMAX=R
    8 CONTINUE
      IF (RMAX-TOL) 103, 103, 104
  103 CONTINUE
      PRINT 15
      DO 9 IP=1,NL1
      PRINT 16, IP, DX(IP), PO(IP)
    9 CONTINUE
      PRINT 19
      RETURN
      END



      SUBROUTINE TDLEQ (A,C,X,N)
      SUBROUTINE SOLVES A TRIDIAGONAL SYSTEM OF LINEAR EQUATIONS
      DIMENSION A(50,3), C(50), X(50), B(50,2), D(50)
      START
      J=N
      B(1,1)=A(1,2)
      B(1,2)=A(1,3)
      D(1)=C(1)
      JJ=J-1
      DO 5 K=2,JJ
      IF (ABS(B(K-1,1))-ABS(B(K-1,2)))3, 4, 4
    3 B(K,1)=A(K,2)*B(K-1,1)/B(K-1,2)-A(K,1)
      B(K,2)=A(K,3)*B(K-1,1)/B(K-1,2)
      D(K)=(C(K)*B(K-1,1)-A(K,1)*D(K-1))/B(K-1,2)
      GO TO 5
    4 B(K,1)=A(K,2)-A(K,1)*B(K-1,2)/B(K-1,1)
      B(K,2)=A(K,3)
      D(K)=C(K)-A(K,1)*D(K-1)/B(K-1,1)
    5 CONTINUE
    7 X(J)=(C(J)*B(J-1,1)-A(J,1)*D(J-1))/(A(J,2)*B(J-1,1)-A(J,1)*B(J-1,2
     1))
```

```
    K=J-1
 15 X(K)=(D(K)-B(K,2)*X(K+1))/B(K,1)
    IF (K-1) 100, 100, 10
 10 K=K-1
    GO TO 15
100 RETURN
    END
```

<u>Card 1</u> Format (5F10.4, 3I5)

B1 - Ratio of B_1/B.

ALAM - Bearing number, Λ.

H1 - Normalized gas film thickness, H_2.

Tol - Smallness for convergence.

NL - Total grid number for the step pad including two end points.

N2 - Total grid number for the left edge of the step pad with $H = H_2$.

<u>Card 2</u> Format (8F10.5)

VI(I) - Array of the squeeze numbers to be solved.

<u>Card 3</u> Format (8F10.5)

DX(I) - Array of increments of X defined as $\Delta X_i = X_{i+1} - X_i$ such that $\sum_{I=1}^{N} DX(I) = 1$.

<u>Card 4</u> Format (8F10.5)

PO(I) - Array of the guessed zero order pressure profile Po, i on the pad with Po, 1 = Po, n = 1.