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Final Report

## A STUDY OF UNMANNED <br> MISSION OPPORTUNITIES TO COMETS AND ASTEROIDS

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## SUMMARY

Several unmanned multiple-target mission opportunities to comets and asteroids have been studied. The targets investigated include Grigg-Skjellerup, Giacobini-Zinner, Tuttle-Giacobini-Kresak, Borrelly, Halley, Schaumasse, Geographos, Eros, Icarus, and Toro, and the trajectories consist of purely ballistic flight, except that powered swingbys and deep space $b \cdot r n s$ are employed when necessary.

Optimum solar electric rendezvous trajectories to the comets GiacobiniZinner/85, Borrelly/87, and Tempel(2)/83 and /88 employing the 8.67 kw Sert III spacecraft modified for interplanetary flight have also been investigated.

The problem of optimizing electric propulsion hellocentric trajectories, including the effects of geocentric launch asymptote declination on launch vehicle performance capability, has been formulated and a solution has been developed using variational calculus techniques. Comparison cases have been run on the computer and the results have been presented as a paper at an AIAA conference.

Major improvements have been made to the HILTOP trajectory optimization computer program, and a report detailing the changes is being published concurrently.

An error analysis of high-thrust maneuvers involving spin-stabilized spacecraft has been developed and applied to the Synchronous Meteorological Satellite mission.

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## I. INTRODUCTION

This study of unmanned multiple-target mission opportunities to comets and asteroids begins with a presentation of performance requirements for singletarget comet rendezvous missions via solar electric propulsion.

Optimum solar electric rendezvous trajectories to the comets GiacobiniZinner/85, Bnrrelly/87, and Tempel(2)/83 and /88 employing the 8.67 kw Sert III spacecraft $r$ !ified for interplanetary flight have been investigated and the results are presenter in Section II and in Appendix A. Launched by either a Titan III E/ Centaur or a Shuttle/Transtage combination, the Sert III spacecraft performs rendezvous missions to one of the three comets at various points in the comet's orbit, after which the spacecraft remains with the comet indefinitely, possibly performing scientific measurements throughout the comet's period of revolution. Performance requirements and thruster throttling considerations are discussed, and an overview of the missions is presented.

Section III consists of a presentation of several multiple-target mission opportunities to comets and asteroids. Each mission investigated involves two targets and consists of an all-ballistic trajectory which returns to the vicinity of the Earth after having intercepted the initial target. At the first passage of the spacecraft by the Earth, a swingby maneuver perturbs the heliocentric trajectory either to send the spacecraft directly to the second target or to re-target the spacecraft back to the vicinity of the Earth for one or more additional swingby maneuvers. The key parameters defining the existence of these double-target-via-Earth-swingby solutions are presented in tabular form.

Section IV consists of a solution to the problem of optimizing electric propulsion heliocentric trajectories having high geocentric launch asymptote declinations. Comparison cases have been run on the computer and the results have been published [9].

Section V describes the major improvements which nave been $r$ ade to the HILTOP trajectory optimization compuier program. A report detailing the changes is being concurrently published [10]. The program changes include the incorporation of power degradation, housekeeping power, and declination optimization into the model. The ballistic swingby-continuation simulation capability has been significantly expanded.

Section VI summarizes the transfer of the QUICKTOP III and CHEBYTOP III trajectory optimization computer programs from the NASA Ames Research Center to the Goddard Space Flight Center, and their subsequent usage.

Section VII presents the results of an error analysis of high-thrust maneuvers involving spin-stabilized spacecraft, with application to the Synchronous Meteorological Satellite mission. In addition to the simulation of the burn, an algorithm predicts the expected post-burn errors. The procedure incorporates navigational uncertainties in the pre-burn state, and attitude and magnitude errors during the burn to estimate errors in the resulting orbital elements.

## II. SERT III COMET RENDEZVOUS MISSIONS

Performance requirements for rendezvous missions to three highly interesting comets which have perihelia near one astronomical unit from the sun have been investigated. Launched by either a Titan III E/Centaur or a Shuttle/ Transtage combination, an 8.671 kilowatt Sert III spacecraft, modified for interplanetary flight, performs rendezvous missions to one of the three comets at various points in the comet's orbit, after which the spacecraft remains with the comet indefinitely, possibly performing scientific measurements throughout the comet's period of revolution. The comets are Giacobini-Zinner, having a perihelion passage on September 5, 1985; Borrelly, having a perihelion passage on December 18, 1987; and Tempel II, having perihelion passages on June 1, 1983 and September 16, 1988. The missions of interest are referenced to these perihelia passages.

The proposed interplanetary version of the Sert III spacecraft was simulated using the spacecraft model employed by the HILTOP trajectory optimization program ${ }^{[1]}$, using parameter values suggested by the Lewis Research Center, Cleveland, Ohio ${ }^{[2]}$. In terms of the engineering parameters used by the HILTOP program, the total thrust subsystem, including power conditioners, of the Sert III spacecraft has an efficiency $\eta=.63376$, which includes the $12^{\circ}$ thrust canting angle of the three operating thrusters; this thrust subsystem accepts a (reference) power of 8.671 kilowatts, input to the power conditioners, when all three thrusters are operating at full power. The trajectory simulations of interest here do not require more than three* full-throttle thrusters operating at one time; any spare thrusters and power conditioners are therefore included in the net spacecraft mass. The Sert III spacecraft is assumed to have a constant specific impulse of 2900 seconds; in this approximation, both the efficiency and specific impulse are maintained constant with throttling ratio, whereas in reality they vary slightly with throttling ratio. The propellant tankage weight is assumed to be ten percent of the propellant weight, and these two weights vary with each

[^0]trajectory solution; reserve propellant and tankage must be accounted for in the net spacecraft mass.

The following mass components are assumed to be components of the Sert III spacecraft. Three thrusters at 16 pounds per thruster; 48 pounds: three power conditioners at 45 pounds per power conditioner; 135 pounds: solar array, 340 pounds: solar array structure and gimbals, 20 pounds: solar array orientation mechanism, 14 pounds: sun sensor on solar array, 5 pounds: computer for thruster operations, 10 pounds and cabling, 7 pounds. These mass components add up to 579 pounds, or 262.6 kilograms, which is the "propulsion system mass" assumed by the HILTOP trajectory optimization computer program. All other mass is either propellant, tankage, or net spacecraft mass. A switching matrix between power conditioners and thrusters is omitted. Variation of the above mass component assumptions does not alter the basic results of this study; inert mass component values may be juggled between the net spacecraft mass and propulsion system mass, keeping the sum constant.

The thrust vector is assumed to be optimally oriented (along the primer vector) at each point of a given trajectory, and thrust angle limits are given in Appendix A. The optimization criterion for this study is maximum net spacecraft mass. Coast phases are optimized. Launch date, departure excess speed, and departure asymptote declination are also optimized. The effect of degradation of the solar array is ignored. Launches are from the Eastern Test Range, with an assumed latitude of $28^{\circ} .5$; the Titan III E/Centaur is assumed to have a range safety constraint corresponding to a parking orbit inclination of $32^{\circ} .5$, and the Shuttle/Transtage is assumed to have one corresponding to an inclination of $57^{\circ}$.

The six standard orbital elements defining the two-body motion of each comet in the ecliptlc system, in astronomical units and degrees, are given by the following table:

|  | Giacobinl-Zinner |  | Borrelly | Tempel II |  |
| ---: | ---: | ---: | ---: | ---: | :---: |
|  |  |  | 1983 | 1988 |  |
| a | 3.516014 | 3.610604 | 3.035350 | 3.036684 |  |
| e | .7075535 | .6242364 | .544894 | .544428 |  |
| l | 31.87374 | 30.32472 | 12.4374 | 12.4317 |  |
| $\Omega$ | 195.20738 | 75.26815 | 119.1582 | 119.1183 |  |
| $\omega$ | 172.48223 | 353.33262 | 190.9242 | 191.0410 |  |
| $t_{p}$ | $9 / 5.6958 / 85$ | $12 / 18.2683 / 87$ | $6 / 1.5342 / 83$ | $9 / 16.7175 / 88$ |  |

Representative trajectory proflles, projected onto the ecliptic, are displayed in Figs. 1 to 3, for missions rendezvousing at perihelion. The optimum launch dates for these trajectories are shown. Missions tend to fall into multipleyear classes, since locally optimum launch dates usually occur each year, when the Earth is in its most favorable position in its orbit, which may be approximately determined intuitively by considering the geometrical configuration of the transfer trajectory. All trajectories considered in this study are single-revolution trajectories, such as those depicted in Figs. 1 to 3, since such trajectories, to the comets considered, have minimum solar distances essentially at one astronomical unit, which considerably eases the spacecraft thermal design problem. All trajectory profles shown are four year missions, except for one three year proflle, the innermost one of Fig. 3. For trajectorles which rendezvous at the same time but which depart in different years, the longer-flight-time trajectorles will obviously move farther away from the sun than the shorter ones.

It can be seen that all three comets pass relatively close to the Earth's orbit when the comets are at perihelion; Giacobini-Z inner and Borrelly both recede to about six astronomical units from the sun, whereas Tempel II recedes to about 4.7 AU. In Figs. 1 and 2, the spacecraft trajectories to GiacobiniZinner and Borrelly have maximum solar distances of about 4 AU , and in Fig. 3 the maximum solar distances are approximately in the 3 to $3 \frac{1}{2} \mathrm{AU}$
Figure 1
COMET GIACOBINI-ZINNER RENDEZVOUS MISSION


Figure 2

## COMET BORRELLY RENDEZVOUS MISSION USING SERT III TECIINOLOGY

Ecliptic Projections


Figure 3

## COMET TEMPEL II RENDEZVOUS MISSION USING SERT III TECHNOLOGY


range. Detailed values of the maximum and minimum spacecraft solar distances are given In Appendix A.

Figs. 4 through 6 display the power curves, normalized to the reference power, corresponding to the trajectory proflles of Figs. 1 through 3. Gaps in the curves represent optimum coast phases. These power profiles represent the normalized solar power available to the spacecraft for propulsion along the given trajectory, since this is in accordance with the method by which the HILTOP computer program simulates trajectories. Housekeeping power is not represented, and therefore housekeeping power considerations must be included in the net spacecraft mass. The HILTOP program trajectory simulation does not employ explicit engine throttling, shutdown, and start-up along the powered portion of a trajectory, since these operations may be done implicitly, so long as the input power to the power conditioners equals the product of the reference power ( 8.671 kw ) and the power ratio. Engine throttling considerations are therefore accomplished by utilizing normalized power curves such as those shown, and, in general, there may be more than one throttling solution available along a given trajectory. As a crude example of this, for a spacecraft having three thrusters, a power ratlo of one-third at a given point along a trajectory may be satisfied, In the approximation considered here, by either one thruster at full throttle or two thrusters at one-half throttle. The particular solution having the least number of thrusters operating at any given instant is usually considered to be the best, since this solution implies a hlgher ihrottling ratio, which in turn implies slightly improved performance in the real-world model.

If the engines on board the Sert III spacecraft have a minimum throttling ratio of one-half, this corresponds to a one-half times one-third equals onesixth, or about . 167, normalized power cutoff value, below which the spacecraft must coast since there is not enough power to operate even one thruster. Considering the power proflles corresponding to the four-year missions in Figs. 4 through 9. It can be seen that this power cutoff value zesults in imposed

Figure 4
COMET GIACOBLNI-ZIN`ER RENDEZVOUS MISSION USING SERT III TECHNGLOGY

Arrival at Perihelion on 5 SEP 1985

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Figure 5
COMET BORRELLY RENDEZVOUS MISGION USING SERT III TECHNOLOGY

Figure 6
COMET TEMPEL II RENDEZVOUS MISSION USING SERT III TECHNOLOGY
Arrival at Perihelion on 16 SEP 1988

coast phases lasting for 800 to 850 days during the portion of flight which is farthest from the sun, and results in an imposed coast phase of about 300 days for the three-year mission of Fig. 6. Due to the HILTOP computer program simulation, however, the spacecraft was assumed to thrust during these times, the thrust being obviously quite weak; it is felt that this approximation does not greatly affect the results of this performance analysis study, in which the spacecraft masses, primarily the net mass, are of greatest significance. The spacecraft masses corresponding to real-world trajectories, having imposed coast phases near their aphelia, are therefore assumed to be in good approximation to those given in this report, although other mission parameters such as launch date may have to be adjusted. Also, some trajectories in this study have very brief periods after the launch time, such as in Fig. 5, dur ing which the power ratio exceeds unity, a condition which cannot be physically satisfied by the assumed 8.671 kw Sert III spacecraft unless a fourth thruster is briefly turned on. Such optimum spacecraft trajectories could easily be adjusted, without degrading the performance significantly, so that a maximum of only three thrusters would be required.

The total propulsion system on-time for each trajectory generated is tabulated in Appendix A; however, these tabulated values merely correspond to the duration for which the spacecraft is considered to be thrusting by the HILTOP computer program, and therefore are not a realistic assessment of the actual thruster on-time, which must be determined by again considering the representative power-ratio curves as given by Figs. 4 through 6 together with the throttling lower limit of a single thruster. If this lower-limit is assumed to be one-half, then, for example, the Borrelly rendezvous power curve of Fig. 5 may be analyzed approximately as follows:

Case (1): With Imposed Coast Phase
In this case, there is an imposed coast phase from about -1140 to about -320 days from comet rendezvous, due to the lack of power available
to operate even one thruster. Since there are three thrusters, the pertinent values of power ratio are two-thirds $\cong .667$ and one-third $\cong .333$. For approximately the first 100 days, 3 thrusters must operate (after which a maximum of only two are required for the remainder of the mission), for approximately the next 100 days, 2 thrusters are on, and then one thruster operates for about 160 days up to the start of the imposed coast phase. Following the imposed coast phase, one thruster operates for about 150 days and then 2 thrusters are on for the final 80 days. This amounts to $(3 \times 100)+(2 \times 100)+$ $(160) \div(150)+(2 \times 80)=970$ single-thruster-days, which, if siared equally by the three thrusters, implies that each thruster operates for only 323 days.

## Case (2): Without Imposed Coast Phase

This case is identical to the above except for an additional 820 single-thruster-days, which increases the total single-thruster-days to 1790 , which, when divided by three (thrusters), yields 597 operating days per thruster.

The Case (1) value is considered to be more representative, but may have to be adjusted upward slightly when the imposed coast phase is actually introduced into the simulation. The Borrelly-rendezvous power-curve discussed as an example is representative of most power profiles for four-year missions involved in this comet rendezvous study, and, since thruster on-time values are generally less for three year and two year missions, the general conclusion may be drawn that the maximum thruster on-time for each thruster of the Sert III spacecraft will be "about one year", a situation which will of course improve if a fourth "spare" thruster is available to equally share the load.

Were a fourth thruster to have been assumed in the HILTOP trajectory simulation, then the reference power assumed by HILTOP would have been 4/3 as great as that actually assumed. Consequently, the optimization algorithm, sensing a more powerful electric propulsion spacecraft, would have directed the launch vehicle to inject more initial mass (and, for low power levels, more net mass) into heliocentric space at less departure excess speed, requiring the
electric propulsion spacecraft to bear more of the burden in effecting the rendezvous, and involving some adjustment to the trajectory profile. This process of adding more thrusters, or discrete steps of reference power, increases the net mass up to a point, which would be as close as the discrete referencepower variation could get to the true optimum value of reference power, which varies with launch vehicle.

Some of the comet rendezvous mission simulations employed a new theory for the optimization of the launch asymptote declination, which is discussed in Section IV. Most of the Borrelly missions, some of the Giacobini-Zinner missions, but none of the Tempel II misisions made use of the new launch optimization model. The results of the launch phase optimization are summarized by the tabulations of the launch asymptote declination $\delta$ and the parking orbit inclination $i$ in Appendix A.

A performance overview for the Sert III comet rendezvous missions is displayed in Fig. 7, in which the top of each bar represents the net spacecraft mass deliverable to perihelion rendezvous, each notch below the top corresponds ', the net mass rendezvous capability at $50,100,150, \ldots$ etc. days before perihelion, and each bar above the top shows the net mass rendezvous capability at $50,100,150, \ldots$ etc. days after perihelion.

The most striking feature of this bar graph is the relatively large net masses which the little Sert III spacecraft is capable of rendezvousing with Temuel II during the 1988 opportunity. As solar system targets go, these th. , ee comets are relatively easy to get to, since they pass somewhat near to the Earth's orbit. However, as may be seen by examining the comet's orbital elements listed earlier in this section, Tempel II is inclined only about $12^{\circ}$ to the ecliptic, whereas Giacobini-Zinner and Borrelly are in the $30^{\circ}$ range. Moreover, the value of Tempel II's semi-major axis, which is proportional to its ciergy, is closer to the Earth's value than those of the other two comets. These facts are considered to be the major contributing factors in the explanation of the greater Tempel II/1988 net mass capability.

Considering the TEM/88 TITAN bars of Fig. 7, there is only a small penalty in net mass between the 4-year mission and the 3-year mission, rendezvousing at perihelion; however, when considering rendezvousing fifty, or one hundred, days before perihelion, the penalty is seen to be severe for the 3 -year mission, but small for the 4 -year mission, and this is basically because the 3 -year mission is more difficult for the little Sert III spacecraft to accomplish. Clearly there is a trade-off between mission duration and payload dellvered, to produce a minimum-cost mission for accomplishing given scientific objectives.

Another obvious fact derivable from Fig. 7 is the lesser interplanetary capability of the Shuttle/Transtage relative to the Titan III E/Centaur; what is not obvious is the relative launch costs, which hopefully would be less for the Shuttle.

Only the 2-year Tempel II/1983 mission was investigated, due to the unlikelyhood of a comet rendezvous electric propulsion mission being launched before 1981; hence the emphasis on post-perihelion rendezvous, which not only yield greater net masses delivered but also are associated with later launch dates.

Detailed tabular data pertaining to these comet rendezvous missions may be found in Appendix A.


## III. BALLISTIC MULTIPLE-TARGET FLYBY MISSIONS*

The work described in this section consisted of confirming the existence of ballistic multiple-target mission opportunities to comets and asteroids, in particular those which pass relatively near to the Earth's orbit, such that one or more Earth-swingby maneuvers can be employed to perturb the heliocentric trajectory. Details of the Earth swingby technique and complete descriptions of the multiple-target mission profiles are discussed by Farquhar, et. al. $[11,12]$.

All missions involve two targets and consist of trajectories which return to the vicinity of the Earth after having intercepted the first triget. At the first passage of the Earth, a swingby maneuver perturbs the heliocentric trajectory either to send the spacecraft directly to the second target or to re-target the spacecraft back to the vicinity of the Earth for one or more additional swingbys, in which the final swingby sends the spacecraft to the second target. Quite often there is more than one way to swing past the Earth at a given Earth-passage, in order to produce the desired re-targeting of the spacecraft.

Thls work was accompllshed using the HILTOP trajectory optimization computer program $[1,10]$. This program generates swingby maneuvers under the assumption of the patched-conic approximation, such that the swingby planet's sphere-of-influence is assumed to have zero radius as seen from interplanetary space and infinite radius as seen from the planetary vantage point. The passage time in the swingby planet's sphere-of-influence is neglected, i.e., taken to be zero in the heliocentric reference frame.

When, for a particular multiple-target mission, no trajectory solution could be found having entirely unpowered swingby maneuvers, powered swingby maneuvers were employed, and the minimum $\Delta V$ solution was determined in each case by optimizing the post-swingby-leg flight time. Figure 8 depicts the relation of unpowered swingby solutions to the wider class of powered swingby

[^1]
solutions. The upper curve represents the case in which no unpowered swingbys exist, and the lower curve shows how unpowered swingbys occur in pairs, even though one or both of the corresponding trajectories may hit the swingby planet.

Each powered swingby maneuver is constrained to occur at the mutual perlfoci of the approach and departure hyperbolic trajectory segments within the swingby planet's sphere of influence; the burn is assumed to be impulsive with the thrust collinear to the velocity at closest approach.

The first-target (of a double-target sequence) investigated in this study was either the 1977 apparition of the comet Grigg-Skjellerup (GS/77), the 1982 apparition of the same (GS/82), or the 1985 apparition of the comet GiacobiniZinner ( $\mathrm{GZ} / 85$ ). The spacecraft thus launches from the Earth, flies past one of these comets at considerable relative speed, and returns to the vicinity of the Earth to perform a swingby maneuver onto a subsequent target. For convenience, the following abbreviations are used for three of the comets:

```
GS Grigg-Skjellerup
GZ Giacobini-Zinner
TGK Tuttle-Giacobini-Kresak
```

An overview of the double-target missions investlgated is given in Table III-1, and the six standard orbital elements defining the two-body motion of the targets involved, in the ecliptic system, expressed in astronomical units and degrees, are given in Table III-2.

Table III-3 contains the parameters which characterize the initial heliocentric trajectory segments including launch from Earth, flyby of the initial target, and arrival back at Earth, for the three cases investigated (GS/77, GS/82, GZ/85). In each case, the initial target is approximately crossing the ecliptic at the time of encounter, and the trajectory segments are essentially $360^{\circ}$ in-ecliptic transfers. The trajectory segments defined by Table III-3 apply to all of the associated missions to second-targets, summarized by Table III-1 (except for the unpowered swingby to Halley, in which the spacecraft returns to swingby the Earth after three

TABLE III - 1
OVERVIEW OF MISSIONS INVESTIGATED

| First <br> Target | Second Target | No. of Earth Swingbys | Comments |
| :---: | :---: | :---: | :---: |
| GS/77 | GZ/79 ICARUS/78 | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | Min $\Delta \mathrm{V}$ powered swingby. Two unpowered swingby possibilities. |
| GS/82 | TGK/84 | 1 | Two unpowered swingby possibilities. |
|  | GEOGRA PHOS/83 | 1 | Three unpowered swingby possibilities, one having relatively low flyby speed at Geographos ( $9.3 \mathrm{~km} / \mathrm{sec}$ ). |
|  | TORO/83 | 1 | Possibility of unpowered or min $\Delta \mathrm{V}$ powered swingby. |
|  | ICARUS/87 | 1 | Min $\triangle \mathrm{V}$ powered swingby. |
|  | SCHAUMASSE/84 | 1 | Min $\triangle V$ Earth powered swingby followed by Min $\Delta V$ Venus powered swingby. |
|  | HALLEY | 1 | Min $\triangle \mathrm{V}$ powered swingby (or lone unpowered swingby using a three year transfer to Earth). |
| GZ/85 | BORRELLY/87 | 2 | First swingby is standoff encounter; second Earth encounter has two unpowered swingby possibilities. |
|  | TORO/87 | 1 | Powered swingby having Earth passage distance 1.2 radii. |
|  | EROS/87 | 1 | Family of powered swingbys having low flyby speeds at Eros ( 4 to $6 \mathrm{~km} / \mathrm{sec}$ ). |
|  | ICARUS/87 | 1 | Lone unpowered swingby. |
|  | GEOGRAPHOS/87 | 1 | Lone unpowered swingby. |

TABLE III - 2
TARGET ORBITAL ELEMENTS

| Target | a | e | i | $\Omega$ | $\omega$ | $\mathbf{t}_{\mathbf{p}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BORRELLY/87 | 3.61060400 | . 6242364 | 30.32472 | 75.26815 | 353.33262 | 12/18.26830/87 |
| EROS/87 | i. 45820548 | . 2227330 | 10.82860 | 304.30050 | 178.55830 | 5/23.75410/87 |
| GEOGRAPHOS/83 | 1.24472390 | . 3420300 | 13.33410 | 337.23520 | 276.54530 | 5/18.77180/83 |
| GEOGRAPHOS/87 | 1.24495016 | .3420090 | 13.32880 | 337.26210 | 276.54740 | 7/18.76370/87 |
| GS/77 | 2.96273200 | . 6647010 | 21.10145 | 213.03235 | 359.31610 | 4/11.00088/77 |
| GS /82 | 2.95889800 | . 6656651 | 21.13392 | 213.09221 | 359.32122 | 5/14.99629/82 |
| GZ/79 | 3.49071500 | . 7146548 | 31.69660 | 195.47830 | 171.96939 | 2,12.91951/79 |
| GZ/85 | 3.51601400 | .7075535 | 31.87374 | 195.20738 | 172.48223 | 9/5.69583/85 |
| HALLEY | 17.94348000 | . 9672770 | 162.24046 | 58.67151 | 111.87092 | 2/ 9.39474/86 |
| ICARUS/78 | 1.07785100 | . 8267308 | 22.92507 | 87.95981 | 31.10711 | 5/28.72346/78 |
| ICARUS/87 | 1.07795840 | . 8266640 | 22.88890 | 88.00800 | 31.18870 | 5/11.87700/87 |
| SCHAUMASSE/84 | 4.08725300 | . 7032880 | 11.84050 | 80.42600 | 57.38400 | 12/7.36721/84 |
| TGK/84 | 3.14693400 | . 6430245 | 9.93305 | 153.73014 | 49.52093 | 7/28.41270/84 |
| TORO/83 | 1.36737860 | . 4511330 | 9.37360 | 274.27700 | 126.81560 | 12/16.39710/83 |
| TORO/87 | 1.36736120 | . 4511070 | 9.37290 | 274.30820 | 126.80780 | 2/26.47011/87 |

Orbital elements obtained from Dr. D. K. Yeomans of Computer Sciences Corporation.

TABLE III - 3
PARAMETERS CHARACTERIZING INITLAL TRAJECTORY SEGMENTS

| EARTH LAUNCH | INITIAL TARGET FLYBY | EARTH ARRIVAL |
| :---: | :---: | :---: |
| $\begin{aligned} \text { Date } & =10 / 30 / 76 \\ C_{3} & =2.3 \mathrm{kmn}^{2} / \mathrm{sec}^{2} \\ \delta & =-18^{\circ} .5 \end{aligned}$ | GS/77 <br> Date $=4 / 11 / 77$ <br> Flyby at $15.2 \mathrm{~km} / \mathrm{sec}$ $\begin{aligned} & \mathrm{CD}=.20 \mathrm{AU} \\ & \mathrm{CA}=82^{\circ} \end{aligned}$ | $\begin{aligned} \text { Date } & =10 / 30 / 77 \\ \mathrm{~V}_{\infty} & =1.53 \mathrm{~km} / \mathrm{sec} \end{aligned}$ |
| $\begin{aligned} \text { Date } & =10 / 23 / 81 \\ \mathrm{C}_{3} & =8.0 \mathrm{~km}^{2} / \mathrm{sec}^{2} \\ \delta & =9^{\circ} .9 \end{aligned}$ | $\begin{aligned} & \quad \underline{\mathrm{GS}} / 82 \\ & \text { Date }=5 / 15 / 82 \\ & \text { Flyby at } 15.3 \mathrm{~km} / \mathrm{sec} \\ & \mathrm{CD}=.37 \mathrm{AU} \\ & \mathrm{CA}=76^{\circ} \end{aligned}$ | $\begin{aligned} \text { Date } & =10 / 24 / 82 \\ \mathrm{~V}_{\infty} & =2.83 \mathrm{~km} / \mathrm{sec} \end{aligned}$ |
| $\begin{aligned} \text { Date } & =3 / 10 / 85 \\ \mathrm{C}_{3} & =12.3 \mathrm{~km}^{2} / \mathrm{sec}^{2} \\ \delta & =-4^{0} .1 \end{aligned}$ | $\mathrm{GZ} / 85$ <br> Date $=9 / 11 / 85$ <br> Flyby at $20.6 \mathrm{~km} / \mathrm{sec}$ $\begin{aligned} & \mathrm{CD}=.46 \mathrm{AU} \\ & \mathrm{CA}=80^{\circ} \end{aligned}$ | $\begin{aligned} \text { Date } & =3 / 10 / 86 \\ V_{\infty} & =3.51 \mathrm{~km} / \mathrm{sec} \end{aligned}$ |
| NOMENCLATURE |  |  |
| 6 -Departure asymptote declination <br> CD-Communication distance <br>  |  |  |

years instead of one, with the trajectory to GS being the same). Therefore, it would be theoretically possible to send several spacecraft, perhaps using a single launch, to, say, GS/82 to obtain several different vlewpoit.. of the comet, and each spacecraft could subsequently swing past the Earth differently and continue on to a different target.

Parameters defining the trajectory segments which continue from Earth swingby to the final targets are given in Tables III-4, III-5, and III-6. These tables, together with Table III-3, completely define the double-target missions which have bion identified. It is helpful to refer back to the comments in Table III-1 in order to better understand the latter tables. Multiple swingby f , ssibilities (at the same time) are specified by multiple-values, for certain parameters, which are separated by slashes.

The GZ/85: Eros/87 double-target mission consists of a family of pussibllities having relatively slow flybys of Eros. Thls famiiy of missions, in which the post-swingby-leg transfer tıme is varied, is summarized by Figure 9.

## TABLE III - 4

PARAMETERS CHARACTERIZING FINAL TRAJECTORY SEGMENTS
FOR GS/77 MISSIONS

| EARTH SWINGBY | SECOND-TARGET FLYBY |
| :---: | :---: |
|  | GZ/79 |
| $\begin{aligned} & \mathrm{R}=3.21 \\ & \Delta \mathrm{~V}=392 \mathrm{~meters} / \mathrm{sec} \end{aligned}$ | $\begin{aligned} & \text { Date }=2 / 19 / 79 \\ & \text { Flyby at } 20.9 \mathrm{~km} / \mathrm{sec} \\ & \mathrm{CD}=1.83 \mathrm{AU} \\ & \mathrm{CA}=23^{\circ} \end{aligned}$ |
|  | ICARUS/78 |
| $\begin{aligned} & R=\ldots 96 / 31.27 \\ & \Delta V=\text { zero } / \text { zero meters } / \mathrm{sec} \end{aligned}$ | $\begin{aligned} & \text { Dates }=(7 / 15 / 78) /(7 / 24 / 78) \\ & \text { Flyby at } 29.5 / 26.3 \mathrm{~km} / \mathrm{sec} \\ & \mathrm{CD}=.51 / .65 \mathrm{AU} \\ & \mathrm{CA}=82^{\circ} / 87^{\circ} \end{aligned}$ |

## NOMENCLATURE

$\mathrm{R} \quad$ Passage distance (Earth radii)
$\Delta V \quad$ Powered swingby incremental velocity
CD Communication distance
$\mathrm{CA} \quad$ Communication angle (Sun $=0^{\circ}$ )
(Also applies to Tables III - 5 and III - 6)


TABLE III-5

## PARAMETERS CHARACTERIZING FINAL TRAJECTORY SEGMENTS

FOR GS/82 MISSIONS

| EARTH SWINGBY | SECOND-TARGET FLYBY |
| :---: | :---: |

## TGK/84

Dates $=(6 / 12 / 84) /(6 / 21 / 84)$
$\mathrm{R}=14.97 / 1.46$
$\Delta V=$ zero/zero meters/sec
Flyby at $14.5 / 15.6 \mathrm{~km} / \mathrm{sec}$
$\mathrm{CD}=1.80 / 1.77 \mathrm{AU}$
$\mathrm{CA}=43^{\circ} / 42^{\circ}$

|  | GEOGRAPHOS $/ 83$ |
| :--- | :--- |
|  | Dates $=(3 / 16 / 83) /(7 / 2 / 83) /(11 / 29 / 84)$ |
| $\mathrm{R}=3.07 / 3.76 / 12.34$ | Flyby at $13.9 / 9.3 / 12.5 \mathrm{~km} / \mathrm{sec}$ |
| $\Delta \mathrm{V}=$ zero $/$ zero $/$ zero meters $/ \mathrm{sec}$ | $\mathrm{CD}=.10 / .62 / 1.53 \mathrm{AU}$ |
|  | $\mathrm{CA}=120^{\circ} / 66^{\circ} / 39^{\circ}$ |


|  | TORO $/ 83$ <br>  <br> $\mathrm{R}=1.38 / 6.47$ <br> $\Delta \mathrm{~V}=63 /$ zero meters $/ \mathrm{sec}$ |
| :--- | :--- |
|  | Dates $=(9 / 29 / 83) /(2 / 13 / 84)$ |
|  | Flyby at $10.1 / 12.8 \mathrm{~km} / \mathrm{sec}$ |
|  | CD $=1.19 / .35 \mathrm{AU}$ |
| CA $=65^{\circ} / 90^{\circ}$ |  |


|  | ICARUS $/ 87$ |
| :--- | :--- |
|  | Date $=4 / 1,84$ |
| $R=1,81$ | Flyby at $25.0 \mathrm{~km} / \mathrm{sec}$ |
| $\Delta \mathrm{V}=1014$ meters $/ \mathrm{sec}$ | $\mathrm{CD}=1.79 \mathrm{AU}$ |
|  | $\mathrm{CA}=60^{\circ}$ |

TABLE III - 5 (continued)

| FARTH SWINGBY | SECOND-TARGET FLYBY |
| :---: | :---: |


| Earth Swingby to Venus |  |
| :--- | :--- |
| $R=8.86$ |  |
| $\Delta V=404$ meters $/ \mathrm{sec}$ | SCLIAUMASSE $/ 84$ |
|  | Date $=10 / 17 / 84$ |
| Venus Swingby | Flyby at $12.2 \mathrm{~km} / \mathrm{sec}$ |
| Date $=2 / 10 / 84$ | CD $=1.37 \mathrm{AU}$ |
| $\mathrm{R}=1.42$ Venus radii | CA $=69^{\circ}$ |
| $\Delta \mathrm{V}=950$ meters $/ \mathrm{sec}$ |  |


|  |  |
| :--- | :--- |
| $R=17.27 / 7.10$ | HALLEY |
| $\Delta V=120 /$ zero meters $/ \mathrm{sec}$ | Dates $=(3 / 15 / 85) /(3 / 8 / 85)$ |
| Flyby at $71.8 / 78.2 \mathrm{~km} / \mathrm{sec}$ |  |

TABLE III - 6
PARAMETERS CHARACTERIZING FINAL TRAJECTORY SEGMENTS FOR GZ/8.5 MISSIONS

| EARTH SWINGBY | SECOND-TARGET FLYBY |
| :---: | :---: |
| First Fassage |  |
| $\mathbf{R}=64.05$ | BORRELLY $/ 87$ |
| $\Delta \mathrm{~V}=$ zero meters $/ \mathrm{sec}$ | Dates $=(12 / 25 / 87) /(12 / 30 / 87)$ |
| Second Passage | Flyby at $17.3 / 17.3 \mathrm{~km} / \mathrm{sec}$ |
| Date $=8 / 20 / 87$ | CD $=.53 / .56 \mathrm{AU}$ |
| $\mathrm{R}=3.54 / 3.45$ | CA $=125^{\circ} / 122^{\circ}$ |
| $\Delta \mathrm{V}=$ zero $/$ zero meters $/ \mathrm{sec}$ |  |


|  | TORO/87 |
| :--- | :--- |
|  | Date $=4 / 20 / 87$ |
| $\mathrm{R}=1.20$ | Flyby at $15.7 \mathrm{~km} / \mathrm{sec}$ |
| $\Delta \mathrm{V}=17$ meters $/ \mathrm{sec}$ | $\mathrm{CD}=1.42 \mathrm{AU}$ |
|  | $\mathrm{CA}=44^{\circ}$ |


|  | EROS/87 |
| :--- | :--- |
|  | Date $=5 / 6 / 87$ |
| $R=5.39$ | Flyby at $\epsilon .3 \mathrm{~km} / \mathrm{sec}$ |
| $\Delta \mathrm{V}=220$ meters $/ \mathrm{sec}$ (Min) | CD $=1.84 \mathrm{AU}$ |
| (see Figure 9) | CA $=33^{\circ}$ |


|  | ICARUS $/ 87$ |
| :--- | :--- |
|  | Date $=6 / 19 / 87$ |
| $R=13.78$ | Flyby at $30.7 \mathrm{~km} / \mathrm{sec}$ |
| $\Delta V=$ zero meters $/ \mathrm{sec}$ | CD $=.16 \mathrm{AU}$ |
|  | CA $=59^{\circ}$ |


|  | GEOGRAPHOS $/ 87$ |
| :--- | :--- |
|  | Date $=9 / 26 / 87$ |
| $R=3.79$ | Flyby at $12.9 \mathrm{~km} / \mathrm{sec}$ |
| $\Delta V=$ zero meters $/ \mathrm{sec}$ | CD $=.40 \mathrm{AU}$ |
|  | CA $=90^{\circ}$ |

SWINGBY PASSAGE DISTANCE (ER)



## IV. LAUNCH ASYMPTOTE DECLINATION OPTIMIZATION

Introduction. Preliminary performance studies of heliocentric electric propulsion missions require some means of correlating initial spacecraft mass $m_{0}$ and launch hyperbolic excess velocity $V_{\infty}$. In most studies to date, this has been accomplished by equating $m_{o}$ to the launch vehicle (LV) payload $m_{l}$, which is represented as a non-linear function of the scalar quantity $v_{\infty}$, the hyperbolic excess speed. With few exceptions, this LV payload capability assumed has been that corresponding to a due-East launch fronı the ETR. The direction of the launch hyperbolic excess velocity is usually left unspecified and is determined as part of the solution to the optimization problem. Using the indirect optimization technique, the solution dictates that $V_{\infty}$ be directed along the initial primer vector, a requirement that may be in conflict with the assumed LV payload capability.

By properly choosing the point of the (coplanar) iniection from a circular parking orbit, the geocentric declination $\delta$ of the hyperbolic excess velocity may lie within the range

$$
\begin{equation*}
-i \leq \delta \leq i \tag{1}
\end{equation*}
$$

where $i$ is the equatorial inclination of the parking orbit established by the launch vehicle. If the launch excess velocity asymptote declination, as determined by the solution to the optimization problem, falls within this range and if the LV payload capability is compatible with the orbit inclination $i$, then the solution is consistent within the assumptions made and the results are valid. However, if $|\delta|>i$, then the basic assumptions regarding LV capability are in conflict, and it is necessary to formulate the optimization problem to account for the dependence of LV payload on direction of the launch excess velocity asymptote.

Although the questionable validity of published high asymptote declination solutions has been recognized for some time, no formal treatment of the problem

[^2]has been noted in the literature. The authoris had previously developed ${ }^{[1]}$ a technique for adjusting the LV payload to account for the non-coplanar injection maneuver required to achieve the geocentric declination of the primer vector, which was colinear with $\mathrm{V}_{\infty}$, but this a posteriori correction approach has proven unacceptable because the original transversality conditions (TC's) were no longer valid. Typically, these TC's resulted in "solutions" which were not stationary points. This condition arose because the alignment of $\mathrm{V}_{\infty}$ with the initial primer was no longer a necessary condition of optimality, but rather an imposed constraint which was in violation of the assumptions used in originally formulating the solution.

In this discussion, a unified treatment of the high asymptote declination problem is presented. The LV payload capability is modeled as a function not only of the magnitude of $\mathrm{V}_{\infty}$ but also of the inclination of the circular parking orbit and of the declination of the launch asymptote. The formulation permits the optimization of both the parking orbit inclination and asymptote declination or of the asymptote declination subject to a limitation on parking orbit inclination to satisfy range safety constraints. The necessary conditions of optimality are derived for a typical comet or asteroid rendezvous problem.

Problem Formulation. High asymptote declinations frequently arise in missions to targets that have orbits highly inclined to the ecliptic, such as those to certain comets and asteroids. Therefore, we select, for illustrative purposes, an optimal rendezvous mission to a single, massless target whose path is defined by a specified ephemeris. The extension of the results derived here to other missions of interest, such as flybys, orbiters, and multiple-target missions, is straightforward. We shall also assume a propulsion system of fixed size i. terms of mass and reference power Also, overall propulsion system efficien:y and the specific impulse of the thruster subsystem are assumed given and are held constant throughout the mission.

The assumed spacecratt and propulsion system models are as described in [1] and will not be repeated herc. The launch vehicle payload capability is assumed to follow the simple exponential law

$$
\begin{equation*}
m_{l}=b_{1} e^{-\left(v_{c} / b_{2}\right)}-b_{3} \tag{2}
\end{equation*}
$$

where $b_{1}, b_{2}$, and $b_{3}$ are pre-determined constants for each launch vehicle and $v_{c}$ is a characteristic speed representative of the energy required to achieve a specific escape trajectory. For example, for a due-East launch from ETR and a coplanar injection maneuver, $v_{c}$ is defined to be the speed required at departure from a low altitude circular reference orbit to achieve a specified hyperbolic excess speed $v_{\infty}$, i.e.,

$$
\begin{equation*}
v_{c}=\sqrt{v_{\infty}^{2}+2 v_{o}^{2}}, \tag{3}
\end{equation*}
$$

where $v_{0}$ is the circular satellite speed in the reference orbit. Thus, for due-East launches and coplanar injection maneuvers, $m_{\ell}$ is a function only of $\mathbf{v}_{\infty}$ for a given launch vehicle. Performance data for a large selection of existing and potential launch vehicles are presented graphically in Reference [3] as a function of $v_{c}$, as defined above, with the reference orbit altitude being 185 kilometers. The authors have found a least-squares curve fit to the exponential law above using, say, 7-10 data points from a given payload curve to be a quite adequate and accurate representation of a launch vehicle's performance capability.

To accomodate large launch as: nptote declinations, the same exponential law for launch vehicle payload may be used, but the definition of characteristic speed must be expanded to reflect the additional energy required to rotate the asymptote. This new definition of $v_{c}$ is taken to be that given above plus the velocity penalties associated with the asymptote rotation. The rotation is assumed to be accomplished by first choosing a launch azimuth which establishes a given reference orbit inclination $i$ followed by a non-coplanar injection
maneuver from that circular reference orbil to the desired asymptote, as Illustrated in the figure on the following page. The velocity penalty incurred with non-due-East launches from the ETR is shown graphically in Reference [3] as a function of the orbit inclination. This velocity penalty, which we will denote $\Delta v_{i}$ is adequately approximated with a quadratic curve fit of the form

$$
\begin{equation*}
\Delta v_{i}=c_{1} i^{2}+c_{2} i+c_{3} . \tag{4}
\end{equation*}
$$

Normal range safety limitations restrict the range of inclinations achievable through varying the launch azimuth alone. The referenced graph indicates that the maximum allowable northerly azimuth will yield an orbital inclination of about 48.5 degrees while the maximum allowable southerly azimuth will yield an inclination of about 32 degrees. Now, given a reference orbit inclination $i$, it remains to define the velocity penalty $\Delta v_{g}$ associated with a non-coplanar departure from this circular orbit to the desired hyperbolic excess velocity at a declination $\delta$. Assuming the line of nodes of this reference orbit is an open variable, one may choose this variable to minimize the angle between the excess velocity and the orbital plane. This minimum angle is $\delta-i$. Gunther ${ }^{[4]}$ has shown that the minimum incremental velocity required to achieve a given $\mathbf{v}_{\infty}$ along an asymptote not lying in the orbital plane from a specified circular orbit is obtained from the solution to a quartic equation in the sine of the out-ofplane angle. Defining

$$
\begin{gather*}
s=\sin (\delta-i) ; \rho=v_{\infty} / v_{o} ; \\
p=s^{2}\left(\rho^{2}+4\right) ; \\
q=s^{2}\left(1-s^{2}\right) \rho^{2} ;  \tag{5}\\
x=\left[\sqrt{(q / 2)^{2}+(p / 3)^{3}}+q / 2\right]^{\frac{1}{3}}-\left[\sqrt{(q / 2)^{2}+(p / 3)^{3}}-q / 2\right]^{\frac{1}{3}} ;
\end{gather*}
$$

## LAUNCH GEOMETRY


(A) NON DUE EAST LAUNCH INTO PARKING ORBIT
(B) OPTIMUM NON COPLANAR BURN OUT OF PARKING ORBIT

$$
\begin{gather*}
y=\sqrt{\rho^{2} / 4-x} ; \\
w=\frac{1}{2}\left[\rho / 2+y+\sqrt{(\rho / 2+y)^{2}+4\left(x / 2+\sqrt{x^{2} / 4+s^{2}}\right)}\right] \tag{5}
\end{gather*}
$$ cont.

then Gunther's solution for the magnitude of the minimum velocity impulse required to accomplish the maneuver is

$$
\begin{equation*}
v_{g}=v_{0} \sqrt{\rho^{2}+3-2 \sqrt{\left(a+\rho w-w^{2}\right)(2+\rho w)}} \tag{6}
\end{equation*}
$$

and the penalty $\Delta v_{g}$ is the difference between $v_{g}$ and the velocity increment required if the out-of-plane angle were zero, i.e.,

$$
\Delta v_{g}=v_{g}-\left(\sqrt{v_{\infty}^{2}+2 v_{0}^{2}}-v_{o}\right)
$$

Thus, the definition of the characteristic speed for those cases in which the asymptote declination lies outside the interval (1) is

$$
\begin{gather*}
v_{c}=\sqrt{v_{\infty}^{2}+2 v_{o}^{2}}+\Delta v_{i}+\Delta v_{g} \\
=v_{0}+v_{g}+\Delta v_{i} \tag{7}
\end{gather*}
$$

Optimality Conditions. The state and adjoint equations for the problem under consideration are precisely as formulated in Ref. [1]. The only difference in the optimality conditions is the format and content of certain of the transversality conditions. Specifically, these differences are due solely to the new definition of $v_{c}$ which is now a function of the direction of $V_{\infty}$ as well as its magnitude. Whereas before the differential of $v_{c}$ was, simply

$$
d v_{c}=\left(v_{\infty} / v_{c}\right) d v_{\infty}
$$

we see from (7) that the equivalent formula now is

$$
\begin{aligned}
d v_{c} & =\left(\partial v_{g} / \partial v_{\infty}\right) d v_{\infty}+\left(\partial v_{g} / \partial \delta\right) d \delta \\
& +\left(\partial v_{g} / \partial i+\partial \Delta v_{i} / \partial i\right) d i
\end{aligned}
$$

where, from (4)

$$
\partial \Delta v_{i} / \partial i=2 c_{1} i+c_{2},
$$

and, from (5) and (6)

$$
\partial \mathbf{v}_{\mathbf{g}} / \partial i=-\partial \mathbf{v}_{\mathbf{g}} / \partial \delta .
$$

The derivation of the partial derivatives $\partial v_{g} / \partial v_{\infty}$ and $\partial v_{g} / \partial \delta$ is straightforward although somewhat cumbersome. The equations are presented in Appendix B.

After noting that the differential of $g$, the reference thrust acceleration, is

$$
\mathrm{dg}=-\left(\mathrm{g} / \mathrm{m}_{\ell}\right) \mathrm{d} \mathrm{~m}_{\ell}
$$

where

$$
d m_{l}=-\left(b_{1} / b_{2}\right) e^{-\left(v_{c} / b_{2}\right)} d v_{c},
$$

and that the differential of $V_{\infty}$ may be written

$$
d V_{\infty}=\left(V_{\infty} / v_{\infty}\right) d v_{\infty}+\left(\bar{n}_{p} \times V_{\infty}\right) d \alpha+\left[\left(V_{\infty} \times \bar{n}_{p}\right) \times V_{\infty} /\left|V_{\infty} \times \bar{n}_{p}\right|\right] d \delta .
$$

where $\bar{n}_{p}$ is a unit vector along the North Pole $: 1 \alpha$ is the geocentric right ascension of $V_{\infty}$, one may write the transversality conditions associated with $\mathrm{V}_{\infty}$ by inspection.

For optimum launch parking orbit inclination:

$$
\begin{equation*}
f\left(\partial \Delta v_{i} / \partial i-\partial v_{\mathbf{g}} / \partial \delta\right)=0 \tag{8}
\end{equation*}
$$

For optimum launch excess speed:

$$
\begin{equation*}
f\left(\partial \mathbf{v}_{\mathbf{g}} / \partial \mathbf{v}_{\infty}\right)-\left(\Lambda_{0} \cdot{v_{\infty}}\right) / \mathbf{v}_{\infty}=0 \tag{9}
\end{equation*}
$$

For optimum launch asymptote declination:

$$
\begin{equation*}
f\left(\partial v_{\mathbf{g}} / \partial \delta\right)-\Lambda_{0} \cdot\left[\left(V_{\infty} \times \bar{n}_{p}\right) \times v_{\infty} /\left|v_{\infty} \times \bar{n}_{p}\right|\right]=0 \tag{10}
\end{equation*}
$$

For optımum launch asymptote right ascension:

$$
\begin{equation*}
-\Lambda_{0} \cdot\left(\bar{n}_{p} \times V_{\infty}\right)=0 \tag{11}
\end{equation*}
$$

In the above equations $\Lambda_{0}$ is the initial primer vector and $f$ is

$$
f=\left[k_{s}+k_{t}-\left(1+k_{t}\right) \nu_{f}-g \lambda_{g} / m_{l}\right] d m_{l} / d v_{c},
$$

where $k_{s}$ and $k_{t}$ are the structural and tan'age factors, respectively, $\nu_{f}$ is the final mass ratio, and $\lambda_{g}$ is the final value of the Lagrange multiplier assoclated with reference thrust acceleration.

Equation (11) implies that the right ascension of $V_{\infty}$ must be equal to, or 180 degrees from, that of $\Lambda_{o}$; i.e., $V_{\infty}$ must lie in the plane of $\Lambda_{o}$ and $\bar{n}_{p}$. If the first term in (10) were zero, which is the result obtained when the effects of declination are ignored in the formulation, then ore obtains from (10) and (11) the familiar result that $V_{\infty}$ must be colinear with $\Lambda_{0}$. Usually it is assumed that $V_{\infty}$ is aligned with $\Lambda_{0}$, however, cases have been found ${ }^{[5]}$ for which the optimum solution resulted in $V_{\infty}$ being diametrically opposed to $\Lambda_{0}$. The fact that the first term in (10) is non-zero means that $V_{\infty}$ will be offset from $\Lambda_{0}$ by a finite angle. This offset, as noted above, must be in the plane of $\Lambda_{0}$ and $\bar{n}_{p}$ and, intuitively, we know it must be in the direction of the equator so as to reduce $v_{c}$. The amount of the offset of $v_{\infty}$ from $\Lambda_{0}$ may not be determined from (11) as an initial value problem since the variable $f$ is a function of variables ( $\nu$ and $\lambda_{g}$ ) evaluated at the final time. Thus, $\delta$ must be treated as an additional independent parameter, and (10) becomes another condition to be satisfied in the boundary value problem.

The satisfaction of (8) requires that the term within parentheses vanishes since $f$ will normally be a non-zero quantity. Therefore, since the two partial derivatives are functions only of initial conditions, one may solve for the $i$ that causes the parenthesized term to vanish and thereby eliminate the condition (8)
from the boundary valus problem. Due to the complexity of the equations defining $\partial v_{g} / \partial \delta$, this solution for i must be obtained using an iterative techn'pue.

The approach to the solution of the problem as formulated above differs In three basic respects from that of the problem whe re asymptote declination is ignored: (1) the condition (8) must be solved for the optim'm parking erbit inclination, given values of $\mathrm{v}_{\infty}$ and $\delta$; (2) the asymptote declination $\delta$ must be Introduced as an independent parameter and (10) added as an end condition of the problem; and (3) the cevaluation of $V_{\infty}$ becomes somewhat more involved. The computetion of $\mathrm{V}_{\infty}$ given $\mathrm{v}_{\infty}, \Lambda_{0}$ and $\delta$, procceds as follows. Denote as $\epsilon$ the obliquity of the ecliptic such that the matrix

$$
\Phi=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \epsilon & -\sin \epsilon \\
0 & \sin \epsilon & \cos \epsilon
\end{array}\right]
$$

operating on a vector expressed in ecliptic Cartesian coordinates yields the same vector in Earth equatorial coordinates. Then the right ascension $\alpha_{\lambda}$ of the initial primer $\Lambda_{0}$ may be written

$$
\alpha_{\lambda}=\tan ^{-1}\left[\left(\lambda_{y 0} \cos \epsilon-\lambda_{z o} \sin \epsilon\right) / \lambda_{y 0}\right]
$$

where $\lambda_{\text {xo }}, \lambda_{y o}, \lambda_{z o}$ are the given ecliptic coordinates of $\Lambda_{0^{\circ}}$. Then, the right ascension of the asymptote is set

$$
\alpha=\alpha_{\lambda} \text { or } \alpha=\alpha_{\lambda}+\pi
$$

and $V_{\infty}$ is evaluated

$$
V_{\infty}=V_{\infty} \Phi^{T}\left[\begin{array}{c}
\cos \alpha \cos \delta \\
\sin \alpha \cos \delta \\
\sin \delta
\end{array}\right]
$$

This may be contrasted with the usual definition of $\mathrm{V}_{\infty}$;

$$
v_{\infty}=v_{\infty} \Lambda_{0} / \mid \Lambda_{0}!.
$$

## V. HILTOP COMPUTER PROGRAM IMPROVEMENTS

This section describes the modifications and improvements made to the HILTOP electric propulsion trajectory optimization computer program [1]. New program features include the simulation of power degra dation, housekeeping power, launch asymptote declination optimization, and powered and unpowered ballistic multiple swingby missiuns with an optional deep space burn.

Power Degradation. The power degradation model has been hypothesized by the authors in earlier publications [5, 8]. The model allows a single parameter (denoted "characteristic degradation time") to describe the power degradation behavior of an electric propulsion spacecraft to a degree which fundamentally affects the solution to the trajectory optimization problem. In short, the power generated is degraded by a multiplicative damage factor $q$ of the form

$$
q=e^{-s / \tau} d
$$

where $\tau_{d}$ is the charactoristic degradation time and $s$ is the degradation time, which is computed in the HILTOP program as the time integral of the density of damaying particles impinging on the solar arrays (for solar electric propulsion). The density of damaging particles is assumed to be a function of solar distance and array orientation.

The characteristic degradation time $\tau_{d}$ is an engineering parameter that is determined experimentally. For example, by exposing a solar cell to the particle emission of a solar simulator and measuring the performance of the cell over a period of time, a reasonable value of $\tau_{d}$ can be estimated.

The assumed exponential form of the degradation factor, although intended for use with SEP systems, is applicable for NEP systems as well. The principal difference is in the definition of $\dot{s}$. The exponential form permits the evaluation of radio-isotope systems by defining $\dot{s}=1$ and letting

## PRECEDING PAgE BLANR NOT FTMMFD

$\tau_{\mathrm{d}}$ represent the time for the radioactivity to dissipate to $1 / \mathrm{e}$ of its initial level. A more complete exposition of this subject is given in $[8]$.

Housekeeping Power. An option of simulating spacecraft housekeeping power has been added to the program. This option applies to solar electric propulsion with specified reference power. The housekeeping power is a specifled constant power generated by the solar arrays and shunted away from the thruster power-conditioners and directly to the spacecraft payload for "housekeeping" purposes. The spacecraft model has been expanded by deleting the old, total propulsion system specific mass and replacing it with the specific mass of the solar arrays and the specific mass of the thruster subsystem.

Declination Optimization. The program has been expanded to include the optimization of launch asymptote declination. The launch asymptote declination optimization model was first hypothesized by the authors in the appendix of [1], and later a more thorough treatment of the subject was put forth in [9]. A solution to the problem of optimizing electric propulsion heliocentric trajectories, including the effects of geocentric launch asymptote declination on launch vehicle performance capability, he = been developed using variational calculus techniques. The model of the launch vehicle performance includes a penalty associated with a non-easterly launch plus another penalty arising from a non-coplanar launch from the parking orbit. Provisions for range safety constraints are included. Optimal trajectories will generally have the launch excess velocity offset from the initial primer vector. The analysis describing the launch asymptote declination optimization model is found in Section IV of this document.

Swingby Continuation. Additional optional computations have been provided in which ballistic swingbys past the primary target may be simulated.

In one mode of program operation, single swingbys past the primary target may be simulated to up to ten post-swingby targets per case. This mode of operation was already in existence.

In another mode of program creration, multiple swingbys along a single trajectory may be simulated, first swinging past the primary target and then subsequently swinging past more targets downstream along the trajectory. One multiple swingby trajectory may be simulated per case. This mode of operation was recently added to the program.

In either mode of operation, the following basic assumptions are made. The swingby continuation computations are independent of the trajectory leg leading up to the swingby target, which may consist of an optimized electric propulsion trajectory segment (if the swingby target is the primary target), except that the arrival $\bigvee_{\infty}$ and arrival time at the swingby target are used in the determination of the swingby passage conditions. Each swingby mancuver is calculated יnder the assumption of the patched-conic approximation, and the swingby planet's sphere-of-influence is assumed to have zero radius as seen from interplanetary space and infinite radius as seen from the planetary vantage point. The passage time in the swingby planet's sphere-of-influence is neglected (taken to be zero in the heliocentric frame).

Each swingby maneuver may be either unpowered or powered. The unpowered swingby solutions are embedded in the wider class of powered-swingby solutions, tending to appear in pairs which are separated by a region of braking powered swingbys.

The powered swingby maneuver is restricted to onexr at the mutual perifoci of the approach and departure hyperbolic arcs; the powered phase is impulsive and the thrust is collinear (pro or con) to the velocity at closest approach. Whether the swingby is powered or unpowered, the trajectory segment leading up to the swingby planet has been pre-determined, this being the method by which the program has been designed to obtain swingby solutions. Therefore the swingby time and the arrival hyperbolic excess velocity are linown.

A basic assumption of the powered swingby algorithm used by the program is that the flight time from the swingby planet to the next target is
specified. This being so, the program is able to converge, by iteration, on some ballistic trajectory from the swingby planet to the next target having the specified transfer time, implying that the departure hyperbolic excess velocity at the swingly planet is thereby determined. Therefore, the heliocentric trajectory before and after the swingby planet is determined, and it then remains to perform the required computations pertaining to the hyperbolic arcs within the swingby planet's sphere of influence. The computatinns are outlined in a companion document [10] which is being published concurrently with this report.

The unpowered swingby maneuver is considered to be a powered swingby having $\Delta V=0$. The program adjusts the post-swingby heliocentric trajectory segment, by iteration, until the swingby departure $\mathrm{V}_{\infty}$ magnitude equals the given arrival $\mathrm{V}_{\infty}$ magnitude. The primary independent variable in this iteration is the post-swingby transfer time to the specified target, which was held constant in the powered swingby case.

The program can generate multiple-revolution ballistic arcs, and a particular solution obtained by the program may not be unique, even for the same transfer time. All solutions are reachable, however, by means of inputting an appropriate initial velocity guess for the trajectory segment in question.

Deep Space Burn. In simulations of trajectories which are all-ballistic, the program is now capable of simulating a single deep space burn, or impulsive velocity-change, at any point prior to arrival at the primary target. The three components of the incremental velocity $\Delta \mathrm{V}$ are independent variables of the boundary value problem, such that, at a specified time, the spacecraft velocity is incremented:

$$
\dot{R}^{+}=\dot{R}^{-}+\Delta V
$$

The optional existence of a deep space burn provides greater targeting flexibility in simulations of multiple target missions,
${ }_{-\infty}$ Optimization in LVI Mode. The optimization of the launch excess velocity $\mathrm{V}_{\infty}$ when using the Launch Vehicle Independent (LVI) mode of simulation is accomplished when the initial primer vector is forced to vanish:

$$
\Lambda_{0}=0
$$

This is accomplished by setting the values of these three independent variables of the boundary value problem to zero and turning their triggers off; the three components of the departure heliocentric velocity become independent variables instead. The program has been augmented with special logic to circumvent the numerical singularity associated with the null primer vector.

Print Expansion. The standard trajectory block print has been expanded to include target-relative position and velocity coordinates of the spacecraft, and comet nuclear and total magnitudes as seen by both the spacecraft and Earth. A more detailed description is given in [10].

Extra-Ecliptic Missions. Extra-ecliptic mission simulations now involve launches from the Earth in which the Earth's ephemeris is generated by the program's analytic ephemeris capability; previously, extra-ecliptic missions were generated simply by starting the trajectory on the x -axis at one astronomical unit from the sun. This improved extra-ecliptic capability allows the launch date to be optimized together with the launch asymptote declination. Also, an additional set of boundary conditions has been added to the program for simulating extra-ecliptic missions, and these are given in [10?.

Miscellaneous Improvements. Ephemerides for several additional comets and asteroids have been included in the program's analytic ephemeris capability, increasing the total number of possible targets to fifty-one. The ephemeris arrays have been expanded to allow up to seventy targets.

The program option of generating a ballistic trajectory as an initial guess for an electric propulsion mission has been expanded to allow multiplerevolution trajectories.

The capability has been added of monitoring the proximity of any given spacecraft trajectory to any object in the solar system.

Another capability has been added of specifying enforced coast phases for selected intervals throughout a given mission.

Finally, a major size-reduction of the HILTOP program was completed, from 436 K to 326 K hexadecimal locations, a reduction of $25 \%$, allowing faster turn-around time on the GSFC IBM 360/91.

## VI. QUICKTOP III/CHEBYTOP III COMPUTER PROGRAM USAGE

The QUICKTOP III ${ }^{[6]}$ and CHEBYTOP III ${ }^{[7]}$ computer programs have been obtained from the NASA Ames Research Center, Moffett Field, California, and converted to run on the IBM 360/91 computer at the Goddard Space Flight Center. These trajectory optimization computer programs were used initially in the Sert III comet rendezvous missien study discussed in Section II, and aided in establishing the feasibility of using the Sert III spacecraft for these comet rendezvous missions. However, it was determined early in the study that the QUICKTOP/CHEBYTOP prograins were consuming as much or more machine time as the HILTOP ${ }^{[1]}$ program. For this reason, a decision was made to solely use the HILTOP program for the comet rendezvous study, since the authors are more familiar with this computer program. The QUICKTOP/ CHEBYTOP computer programs remain ready for use at the Goddard Space Flight Center by any interested persons.

## VII . ERROR ANALYSIS OF HIGH-THRLST MANEUVFLS

High thrust burns can result in large postburn errors when applied to spin-stabilized spacecraft. In addition to the simulation of the burn, an algorithm predicts the expected errors after high thrust mancuvers. The procedure incorporates navigational uncertainties in the pre-burn state, and attitude and magnitude errors during the burn to estimate errors in the resulting orbital elements. The work was performed by W. Bjorkman.

To add to the concreteness of the analytical development, the Synchronous Meteorological Satellite (SMS) mission is discussed as a sample case. The apogee burn of the SMS mission is performed by firing the solid rocket of a spin-stabilized spacecraft near apogee of the transfer orbit. The rocket burns for about 23 seconds, imparting a delta-velocity of about $1723 \mathrm{~m} / \mathrm{sec}$ along the direction of the spin axis. The time of firing the rocket and the attitude of the spin axis are controllable. Appropriate values for these control parameters (firing time and attitude angles) can be determined from a scanning procedure. Any procedure used to establish control parameters will use 'best estimates" of the transfer orbit state and the expected delta-velocity from the rocket. The estimated transfer orbit state will always differ from the true orbit state because of navigation errors (i.e., errors caused by neglected error sources, incomplete modeling or measurement errors in the orbit determination process). The solid rocket will not deliver exactly the delta-velocity stated by the manufacturer. In addition to these errors, the knowledge of spin-axis attitude will be in error by uncertainties in the attitude determination process. An algorithm was developed for assessing the effects of navigation errors and burn errors (i.e., delta-velocity and attitude determination errors) on the achievenient of mission objectives. Attitude control errors are not considered, nor are errors in firing time. The algorithm makes use of an impulsive burn model and linear propagation of errors through the burn. A particular set of mission objectlves is assumed without assuming a linear relationship between mission constraint parameter variations and navigation/burn errors. The validity of the linear error propagation assumption is
demonstrated with a numerical example.
Navigation errors are specified by means of a $6 x 6$ covariance matrix, $P_{o}$, referred to th anchor epoch, ${ }^{\dagger}{ }_{0}$, and a suitable coordinate frame. $P_{o}$ should include uncertainties due to all orbit determination errors, and not simply those uncertainties caused by measurement noise. The Cartesian state error, $\tilde{\mathbf{x}}_{\mathrm{o}}$, is the difference between the true state at epoch and the "anchor vector". We assume

$$
\begin{equation*}
\varepsilon\left(\tilde{x}_{0}\right)=0 \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
\varepsilon\left(\tilde{x}_{0} \tilde{x}_{0}^{T}\right)=P_{0} \tag{2}
\end{equation*}
$$

where $\varepsilon$ is the expectation operator. References to equations in this section of the report apply to equations in this section only. Small deviations in the state propagate linearly along the transfer orbit between $t_{0}$ and the firing time, $\mathrm{t}_{\mathrm{f}}$, by means of the "mean conic" state transition matrix, $\varphi\left(\mathrm{t}_{\mathrm{f}} ; \mathrm{t}_{\mathrm{o}}\right)$.

$$
\begin{equation*}
\tilde{x}_{f}=\varphi\left(t_{f} ; t_{o}\right) \tilde{x}_{o} \tag{3}
\end{equation*}
$$

This transition matrix is evaluated from the anchor vector and the osculating state (integrated from the anchor vector) at $t_{f}$, using the average reciprocal seml-major axis at the two terminals in calculation of the incremental regularizing anomaly. Table VII -1 illustrates numerically the adequacy of equation (3) for propagation of errors along the SMS-A transfer orbit. Linearity errors may be smaller than those shown in the table, because a coordinate system inconsistency was detected after the tabulated data were generated. The transition matrix is deterministic and therefore commutes with the expectation operator.

$$
\begin{align*}
& \varepsilon\left(\varphi \tilde{x}_{0}\right)=\varphi \mathcal{E}\left(\tilde{x}_{0}\right)=0  \tag{4}\\
& P_{f}=\mathcal{E}\left(\tilde{x}_{f} \tilde{x}_{f}^{T}\right)=\varphi \mathcal{E}\left(\tilde{x}_{0} \tilde{x}_{0}^{T}\right) \varphi^{T}=\varphi \mathrm{P}_{0} \varphi^{T} \tag{5}
\end{align*}
$$

The effect of navigation errors on the state at firing time is thus given by $P_{f}$ in equation (5).






Apogee burn errors are specified as "proportional" and "pointing" errors. The proportional error is a delta-velocity error which is proportional to the expected totai delta-velocity, $\overline{\Delta \mathrm{V}}$. Random behavior is assigned to the proportional error.

$$
\begin{align*}
& \varepsilon\left(k_{v}\right)=0  \tag{6}\\
& \varepsilon\left(k_{v}^{2}\right)=\sigma_{v}^{2}|\overline{\Delta V}|^{2} \tag{7}
\end{align*}
$$

Attitude errors contribute a delta-velocity crror approximately in the plane normal to $\overline{\Delta \mathrm{V}}$. Unit basis vectors in that plane are defined by

$$
\hat{E}=\frac{1}{\sqrt{{\overline{\delta v_{1}}}_{1}^{2}+\overline{\delta v}_{2}^{2}}}\left(\begin{array}{c}
\bar{\delta}_{\mathrm{v}}  \tag{8}\\
\overline{\delta v}_{1} \\
0
\end{array}\right)
$$

and

$$
\begin{equation*}
\hat{\mathrm{N}}=\frac{\overline{\Delta \mathrm{V}}}{|\overline{\Delta \mathrm{~V}}|} \times \hat{\mathrm{E}} \tag{9}
\end{equation*}
$$

Pointing error is a circular error probability number assigned to attitude determination accuracy. It is treated as having a normal distribution about zero mean in each of two orthogonal directions.

$$
\begin{align*}
& \varepsilon\left(k_{e}\right)=\varepsilon\left(k_{n}\right)=0  \tag{10}\\
& \varepsilon\left(k_{e}^{2}\right)=\varepsilon\left(k_{n}^{2}\right)=\sigma_{a}^{2}|\overline{\Delta v}|^{2} \tag{11}
\end{align*}
$$

The delta-velocity vector (with errors included) may be written

$$
\begin{equation*}
\Delta v=\left(|\overline{\Delta V}|+k_{v}\right)\left\lceil\frac{\overline{\Delta V}+k_{e} \hat{E}+k_{n} \hat{N}}{-\overline{\Delta V}+k_{e} \hat{E}+k_{n} \hat{N} \mid}\right] \tag{12}
\end{equation*}
$$

To linearize the burn we write the delta-velocity error in the form

$$
\tilde{\Delta} v=M\left(\begin{array}{l}
r_{v}  \tag{13}\\
r_{e} \\
r_{n}
\end{array}\right)=M \tilde{r}
$$

where $r_{v}, r_{e}$ and $r_{n}$ are uncorrelated random numbers of zero mean and unit varlance, defining

$$
\left(\begin{array}{l}
k_{v}  \tag{14}\\
k_{e} \\
k_{n}
\end{array}\right)=|\bar{\Delta} \bar{v}|\left(\begin{array}{cc}
\sigma_{v} r & v \\
\sigma_{a} & r \\
\sigma_{a} & r_{n}
\end{array}\right)
$$

The covariance matrix of delta-velocity errors is

$$
\begin{equation*}
\mathcal{E}\left(\tilde{\Delta V} \tilde{\Delta V}^{T}\right)=M \mathcal{E}\left(\tilde{\mathrm{r}}^{\mathrm{r}}\right) \mathrm{M}^{\mathrm{T}}=\mathrm{M} M^{\mathrm{T}} \tag{15}
\end{equation*}
$$

where M is defined by the following equation

$$
\begin{align*}
M & =\left.\frac{\partial \Delta V}{\partial\left(r_{v}, r^{\prime} r_{n}\right)}\right|_{r_{v}=e_{e}=r_{n}=0} \\
& =|\overline{\Delta V}|\left(\left.\sigma_{v} \frac{\overline{\Delta V}}{\mid \bar{\Delta} \bar{v}} \right\rvert\, \quad \sigma_{a} \hat{E} \quad \sigma_{a} \hat{N}\right) \tag{16}
\end{align*}
$$

The adequacy of the linear model for burn errors is illustrated numertcally in Table VII -2. The comparisons were obtained by multiplying random samples of $\tilde{\mathfrak{r}}$ by $M$ of equation (16) and differencing these with ( $\Delta V$ of equation (12) $-\overline{\Delta V})$. The input standard deviations were: $\sigma_{v}=.0025$ and $c_{a}=(.43 / 57.3)^{\circ}$.

Table VII -2
Linear Propagation Verification: Burn Errors

$$
\begin{aligned}
& X A=M \tilde{r} \\
& X E=X A-(\Delta V-\overline{\Delta V})
\end{aligned}
$$

(In meters/second)

| Sample |  | $\varepsilon_{x}$ | $\varepsilon_{y}$ | $\varepsilon_{\mathrm{z}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | XA | . 3336 | -. 0109 | -. 3015 |
|  | XE | -. 0000 | -. 0001 | . 0000 |
| 2 | XA | -. 8110 | 1.5135 | . 7079 |
|  | XE | -. 0001 | -. 0001 | -. 0000 |
| 3 | XA | 2.6601 | -7.2964 | -3.6459 |
|  | XE | . 0022 | . 0004 | . 0010 |

The post-burn state is computed impulsively by adding $\Delta V$ to the preburn velocity.

$$
\begin{equation*}
x^{+}=x_{f}+\binom{0}{\Delta v} \tag{17}
\end{equation*}
$$

Errors are added the same way,

$$
\begin{equation*}
\tilde{x}^{+}=\tilde{x}_{f}+\binom{\tilde{0}}{\Delta v} \tag{18}
\end{equation*}
$$

The covariance matrix of post-burn errors, $\mathrm{p}^{+}$, is then computed adding $\varepsilon\left(\tilde{\Delta} V \tilde{\Delta V}^{T}\right.$ ) of equation (15) to $P_{f}$ of equation (5). Navigation and burn errors are assumed uncorrelated.

$$
\mathrm{P}^{+}=\mathrm{P}_{\mathrm{f}}+\left[\begin{array}{cc}
0 & 0  \tag{19}\\
0 & \mathrm{MM}^{\mathrm{T}}
\end{array}\right]
$$

A vector, $\psi$, of mission constraint parameters was defined for the SMS mission. Each elemert of $\psi$ can be computed unambiguously from $\mathrm{X}^{+}$. The parameters are:
$\psi_{1} \quad$ required three-impulse trim velocity required to circularize at synchronous radius with a specified inclination and norde
$\psi_{2} \quad$ longltudinal drift rate
$\psi_{3} \quad$ eccentricity of the pre-trim orbit
$\psi_{4} \quad$ inclination of the pre-trim orbit
$\psi_{5} \quad$ ascending node of the pre-trim orbit
Table VII -3 compares constraint vector errors computed from lincarlypropagated errors with errors propagated by "non-linear" means.

Table VII -3

Linear Propagation Verification: Constraint Parameters

$$
\begin{aligned}
& \mathrm{PSI}=\psi\left(\overline{\mathrm{X}}^{+}+\varphi \ddot{\mathrm{X}}_{\mathrm{o}}+\binom{0}{\mathrm{M} \tilde{\mathrm{r}}}, \text { linear } \psi\right. \\
& \mathrm{ERR}=\mathrm{XA}-\psi\left(\mathrm{X}^{+}\right) \quad \text { linear } \psi \text { error }
\end{aligned}
$$

| Sample |  | TRIMV | DRIFT | e | 1 | $\Omega$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | PSI | 253.59 | 64.97 | . 057456 | . 659 | 151.932 |
|  | ERR | . 06 | . 02 | . 000013 | . 000 | - . 008 |
| 2 | PSI | 284.83 | 66.78 | . 045527 | 1.117 | 136.212 |
|  | ERR | . 06 | . 02 | . 000020 | . 000 | - . 003 |
| 3 | PSI | 317.76 | 68.99 | . 041745 | 1.616 | 13C. 577 |
|  | ERR | . 07 | . 02 | . 000026 | . 000 | - . 001 |
| 4 | PSI | 252.05 | 54.81 | . 058739 | . 645 | 153.628 |
|  | ERR | . 13 | . 04 | . 000027 | . 000 | - . 020 |
| 5 | PSI | 283.13 | 66.59 | . 046765 | 1.097 | 136.928 |
|  | ERR | . 13 | . 04 | . 000041 | . 000 | - . 006 |
| 6 | PSI | 315.89 | 68.77 | . 042661 | 1.592 | 130.961 |
|  | ERR | . 14 | . 04 | . 000052 | . 000 | - . 002 |

Variation of sume of these parameters with $\tilde{\mathbf{x}}^{+}$is very non-linear, so a Monte Carlo procedure was implemented as part of the error analysis algorithm. The valid assumption of linearity of error propagation from anchor epoch through the burn (equations (3), (13), and (18)) makes is reasonable to sample post-burn errors directly from $\mathrm{P}^{+}$of equation (19).

Statistical characteristics of the constraint parameter errors are computed from the Monte Carlo samples. These characteristics (as implemented to test the algorithm) a1』:

1. minimum value
2. maximum value
3. mean
4. standard deviztion
5. probability that $|\tilde{\psi}| \leqslant \varepsilon$ ( $\varepsilon$ input range)
6. probability that $|\tilde{\psi}| \leq 2 \varepsilon$
7. probability that $|\tilde{\psi}| \leq 3 \varepsilon$

A frequency histogram of 20 equal algorithm intervals about zero was also implemented for the test. All tests indicate that the algorithm as described is a valid one for error analysis of high thrust maneuvers.

The algorithm for error analysis of high-thrust maneuvers may thus be stated:

1. Propagate the navigation error covariance matrix to firing time (equation (5)).
2. Add the velocity-impulse error covariance matrix, $M$, to the navigation error covariance matrix at firing time (equation (19)).
3. Sample post-burn state errors from the resulting state error covariance matrix using a random number generator.
4. Add the sampled state errors to the errorless post-burn state.
5. Compute the mission constraint parameter error by differencing the value computed with the erroneous state al.d that computed with the errorless state.
6. Accumulate statistics of constraint parameter errors.
7. Return tc step 3 a specified number of times.
8. Display statistics of constraint parameter errors.

## APPENDIX A

This appendix consists of detailed tabular data pertaining to Section II on comet rendezvous missions. A glossary of the tabulated parameters precedes the tables, which have headings sufficient to define their contents.

The explanations and comments below may be helpful in clarifying the definitions and usefulness of some of the tabulated quantities. Refer also to the glossary mentioned in the preceding paragraph. These comments are in no particular order.
(1) The quantities $r_{\text {com }}$ and $\theta_{\text {com }}$ may be used to determine (on all but the first table "Launch Date Variation") the rendezvous communication distance and angle along the orbit of the comet, i.e., after the rendezvous is accomplished and while the spacecraft is traveling along with tive comet. Also, the $p_{t}$ tabulation effectively displays the power available to the
 spacecraft as it moves along with the comet.
(2) The first table, labelled "Launch Date Variation", may be used to obtain a rough idea of the mission window width for the Giacobini-Zinner/1985 rendezvous mission using a Titan III E/Centaur launch vehicle; the results of a mission window analysis would depend on the analyst's criteria.
(3) In the burn time, $t_{b}$, tabulation, "cont." means "continuous" which stands for continuous thrusting throughout the pre-rendezvous portion of the mission: $t_{b}=t_{f}$. See the discussion in Section II of the main report concerned with single-engine throttling for the proper method of computing individual engine on-time requirements.
(4) Pertaining to $\phi_{\text {min }}$ and $\phi_{\text {max }}$, the thrust cone angle $\varnothing$ is depicted by the following sketch: THRUST

(5) The initial spacecraft mass is given by

$$
m_{o}=m_{p s}+m_{p}+m_{t}+m_{n e t}
$$

where $m_{p s}$ is the propulsion system mass (see main text, Section II, for discussion), equal to 262.6 kilograms, $m_{p}$ and $m_{n e t}$ are tabulated, and $m_{t}=0.1 \times m_{p} \cdot m_{o}$ was computed by the HILTOP program as a function of the launch characteristic speed $v_{c}$ according to the expression:

$$
m_{o}=b_{1} e^{\left[-v_{c} / b_{2}\right]}-b_{3}
$$

For $m_{o}$ in kilograms and $v_{c}$ in meters per second, the coefficients are:


| Titan III E/Centaur | 167238.95 | 3480.2038 | 1753.6965 |
| :--- | ---: | ---: | ---: |
| Shuttle/Transtage | 2859382.94 | 1715.7632 | 1199.9231 |

(6) The importance of the launch asymptote declination $\delta$ and parking orbit inclination $i$ are discussed in Section IV of the main text. When $|\delta| \leq 28^{\circ} .5$, the launch is due-east, and, for possible comparison with other studies, the tabulated $m_{0}$ and $v_{\infty}$ columns give the dependence of launch vehicle injected mass $m_{0}$ on departure excess speed $v_{\infty}$. This is not true when $|\delta|>28^{\circ} .5$ because then other factors contribute to the launch phase characteristic speed.

## GLOSSARY OF TABULATED PARAMETERS

AD - Arrival date, days from perihelion passage.
$r_{\text {min }}$ - Minimum solar distance encountered along trajectory, AU.
$r_{\text {max }}$ - Maximum solar distance encountered along trajectory, AU.
$r_{\text {com }}$ - Earth-spacecraft distance at arrival, AU.
$\theta_{\text {com }}$ - Sun-Earth-spacecraft configuration angle at arrival, degrees.
$\phi_{\text {min }}$ - Minimum thrust cone angle required, degrees.
$\Phi_{1 . d x}$ - Maximum thrust cone angle required, degrees.
$\Delta \lambda \quad$ - Change in longitude during the mission, degrees.
$\delta$ - Launch asymptote declination, degrees.
i - Inclination of launch parking orbit, degrees.
LD - Launch date, calendar date.
$\mathrm{t}_{\mathrm{f}}$ - Mission duration, days.
$t_{b} \quad-\quad$ Total propulsion system on time, days.
$m_{0} \quad$ - Initial spacecraft mass, kilograms.
$m_{p} \quad$ - Low thrust propellant mass, kilograms.
$m_{\text {net }}$ - Net spacecraft mass, kilograms.
$p_{t} \quad-\quad$ Power developed by arrays at target arrival, kilowatts.
$\mathbf{v}_{\infty} \quad$ - Launch hyperbolic excess speed, meters per second.

AD

## Glacobini-Zinner(85) Rendezvous

| $\mathbf{r}_{\mathrm{mln}}$ | $r_{\text {max }}$ | $\mathrm{r}_{\mathrm{com}}$ | $\theta_{\text {com }}$ | $\phi_{\mathrm{mln}}$ | $\Phi_{\max }$ | $\Delta \lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . 844 | 3.992 | . 45 | 79.6 | 50.9 | 159.3 | 470.7 |
| . 834 | 4.045 | . 45 | 79.6 | 49.5 | 163.7 | 461.2 |
| .8.40 | 4.073 | . 45 | 79.6 | 46.1 | 166.7 | 451.6 |
| . 855 | 4.095 | . 45 | 79.6 | 41.1 | 166.1 | 442.1 |
| . 878 | 4.082 | . 45 | 79.6 | 35.2 | 160.2 | 432.5 |
| . 905 | 4.067 | . 45 | 79.6 | 29.2 | 151.0 | 423.0 |
| . 935 | 4.041 | . 45 | 79.6 | 24.0 | 139.6 | 413.4 |
| . 964 | 4.011 | . 45 | 79.6 | 20.9 | 127.4 | 403.8 |
| . 985 | 3.990 | . 45 | 79.6 | 20.3 | 116.0 | 394.2 |
| . 996 | 3.98 .4 | . 45 | 79.6 | 21.0 | 106.7 | 334.5 |
| 1.001 | 3.902 | . 45 | 79.6 | 21.7 | 101.9 | 374.8 |
| 1.001 | 4.003 | . 45 | 70.6 | 21.8 | 103.5 | 365.0 |
| 1.000 | 4.026 | . 45 | 79.6 | 21.4 | 101.4 | 355.2 |
| . 997 | 4.045 | .45 | 79.6 | 20.4 | 99.3 | 345.3 |
| . 99.4 | 4.065 | . 45 | 79.6 | 19.1 | 97.5 | 335.3 |
| . 901 | 4.088 | . 45 | 79.6 | 17.7 | 96.1 | 325.3 |
| . 999 | 4.114 | . 45 | 79.6 | 16.8 | 95.4 | 315.3 |
| 987 | 4.149 | 45 | 79.6 | 16.8 | 96.0 | 305. 2 |

Launch Date Variation

TItan III E/Centaur/Sert III

| 8 | 1 | LD | $t_{f}$ | $t b$ |
| :---: | :---: | :---: | :---: | :---: |
| 44.27 | 32.50 | 6/8.5/81 | 1550 | 1394.5 |
| 38.44 | 32.50 | 6/18.5/81 | 1540 | 1406.9 |
| 31.67 | 30.98 | 6/28.5/81 | 1530 | 1417.7 |
| 23.93 | 28.50 | 7/8.5/8i | 1520 | 1424.1 |
| 15.10 | 23.50 | 7/18.5/81 | 1510 | 1430.2 |
| 5.24 | 28.59 | 7/28.5/81 | 1500 | 1433.7 |
| -5.32 | 23.50 | 8/7.5/81 | 1490 | 1434.5 |
| -15.26 | 29.50 | 8/17.5/81 | 1480 | 1432.6 |
| -22.48 | 28.50 | 8/27.5/81 | 1470 | 1427.5 |
| -26.17 | 28.50 | 9/6.5/81 | 1460 | 1418.9 |
| -27.16 | 28.50 | 9/16.5/81 | 1450 | 1407.3 |
| -26.42 | 28.50 | 3/26.5/81 | 1.140 | 1393.6 |
| -24.50 | 28.50 | 10/6.5/81 | 1430 | 1379.6 |
| -21.52 | 28.50 | 10/16.5/81 | 1.120 | 1367.6 |
| -17.32 | 28.50 | 10/26.5/81 | 1410 | 1558.1 |
| -11.46 | 24.50 | 11/5.5/81 | 1400 | 1351.2 |
| -3.75 | 28.50 | 11/15.5/81 | 1390 | 1318.4 |
| 5.04 | 28.50 | 11/25.5/81 | 1380 |  |

$28.50 \quad 11 / 25.5 / 81 \quad 1380 \quad 1353.9$
$m_{0}$


1
1127.7
1134.0
1135.5
1137.5

| $m_{p}$ | $m_{n e t}$ | $p_{t}$ | $\nabla_{0}$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| 469.5 | 348.6 | 8.37 | 8053.7 |
| 464.9 | 360.0 | 8.37 | 8619.6 |
| 459.9 | 367.0 | 8.37 | 8813.5 |
| 453.5 | 375.9 | 8.37 | 8829.7 |
| 448.7 | 391.6 | 8.37 | 8809.8 |
| 444.1 | 416.4 | 8.37 | 8772.0 |
| 439.7 | 452.6 | 8.37 | 8712.4 |
| 435.1 | 501.4 | 8.37 | 81929.6 |
| 429.5 | 560.0 | 8.37 | 8531.0 |
| 422.5 | 619.6 | 8.37 | 8.33 .7 |
| 414.2 | 667.6 | 8.37 | 8363.3 |
| 405.5 | 691.8 | 8.37 | 8336.5 |
| 397.8 | 654.6 | 8.37 | 8365.3 |
| 391.7 | 645.6 | 8.37 | 8149.4 |
| 336.4 | 581.6 | 8.37 | 8579.3 |
| 381.1 | 503.7 | 8.37 | 8737.7 |
| 376.3 | 422.2 | 8.37 | 390.5 |
| 373.9 | 343.2 | 8.37 | 9064.5 |



3-Year Mlesions

| 0 | . 999 | 3.135 | . 46 | 79.3 | 30.7 | 123.4 | 360.9 | -34.88 | 32.50 | 10/1.4/82 | 1070.1 | cont. | 1336.7 | 434.4 | 596.3 | 8.37 | 8392.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -10 | . 998 | 3.143 | . 48 | 79.8 | 32.6 | 127.0 | 350.8 | -35.29 | 32. 50 | 9/30.9/82 | 1060.6 | cont. | 1306.9 | 423.0 | 579.0 | 8.26 | 8436.9 |
| -20 | . 996 | 3.159 | . 52 | 81.5 | 34.2 | 130.4 | 340.6 | -35.65 | 32.50 | 9/30.5/82 | 1051.0 | cont. | 1272.4 | 411.7 | 557.0 | 7.97 | 8489.9 |
| -30 | . 992 | 3.190 | . 57 | 84.0 | 34.0 | 133.6 | 329.9 | -35.88 | 3:., 50 | 9/30.8/82 | 1040.7 | cont. | 1227.5 | 399.9 | 525.0 | 7.52 | 8567.2 |
| -40 | . 985 | 3.235 | . 64 | 86.9 | 31.1 | 136.8 | 318.7 | -35.87 | 3:50 | 10/1.9/82 | 1029.6 | cont. | 1167.9 | 387.9 | 478.6 | 6.99 | 8682.4 |
| -50 | . 977 | 3.285 | . 72 | 89.8 | 26.7 | 139.7 | 308.2 | -35.61 | 32.50 | 10/2.9/82 | 1018.6 | cont. | 1098.8 | 376.6 | 422.0 | 6.42 | 8826.1 |
| -60 | . 967 | 3.333 | . 80 | 92.5 | 22.2 | 142.2 | 298.9 | -35.11 | 32. 50 | 10/3.4/82 | 1008.1 | cont. | 1028.0 | 566.4 | 362.3 | 5.86 | 8980.7 |
| -70 | . 955 | 3.378 | . 89 | 94.8 | 18.1 | 144.4 | 291.1 | -34.41 | 32.50 | 10/3.2/82 | 998.3 | cont. | 900.6 | 357.5 | 304.8 | 5.33 | 9131.5 |
| -80 | . 943 | 3.420 | . 98 | 96.6 | 14.3 | 146.2 | 284.5 | -33.55 | 32.50 | 10/2.4/82 | 989.0 | cont. | 893.6 | 349.7 | 251.4 | 4.85 | 9260.6 |



Borrelly(87) Rendezvous

Titan III E/Centaur/Sort III
4-Year Miscions

|  | AD | $\tau_{\mathrm{mln}}$ | $r_{\max }$ | $\mathrm{r}_{\mathrm{com}}$ | $\theta$ com | $\Phi_{\mathrm{mln}}$ | $\oplus_{\max }$ | $\Delta \boldsymbol{\lambda}$ | 8 | I | LD | $t_{1}$ | $t$ | $\mathrm{m}_{0}$ | ${ }^{\text {m }}$ | $\mathrm{m}_{\text {net }}$ | $\mathbf{p}_{t}$ | * |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | . 978 | 4.026 | . 51 | 127.5 | 36.7 | 90.0 | 360.0 | 37.83 | 32.50 | 12/1.8/83 | 1477.7 | cont. | 1386.7 | 423.7 | 658.1 | 5. 62 | 8167.1 |
|  | -10 | . 978 | 4.029 | . 49 | 131.8 | 36.6 | 90.1 | 353.3 | 37.87 | 32.50 | 12/1.6/83 | 1467.9 | cont. | 1371.6 | 416.2 | 651.2 | 5.59 | 8192.9 |
|  | -20 | . 978 | 4.031 | . 50 | 133.2 | 36.7 | 90.1 | 346.6 | 37.92 | 32.50 | 12/1.3/83 | 1458.2 | cont. | 1356.9 | 408.8 | 644.6 | 5.50 | 8218.0 |
|  | -30 | . 977 | 4.034 | . 53 | 132.1 | 36.9 | 90.3 | 340.1 | 37.96 | 32.50 | 11/30.9/83 | 1448.6 | cont. | 1342.9 | 401.7 | 638.4 | 5.35 | 8242.0 |
|  | -40 | . 977 | 4.036 | . 58 | 129.5 | 37.2 | 90.9 | 333.8 | 38.01 | 32.50 | 11/30.4/83 | 1439.1 | cont. | 1329.6 | 394.9 | 632.6 | 5.16 | 8264.9 |
|  | -50 | . 976 | 4.039 | . 65 | 126.6 | 37.5 | 93.7 | 327.6 | 38.05 | 32.50 | 11/29.9/83 | 1429.6 | cont. | 1317.0 | 388.3 | 627.2 | 4.94 | 8286.6 |
|  | -60 | . 976 | 4.042 | . 72 | 124.0 | 37.9 | 97.7 | 321.6 | 38.08 | 32.50 | 11/29.3/83 | 1420.2 | cont. | 1304.8 | 382.1 | 621.9 | 4.69 | 8307.9 |
|  | -70 | . 975 | 4.046 | . 80 | 121.6 | 38.0 | 102.6 | 315.8 | 38.11 | 32.50 | 11/28.9/83 | 1410.6 | cont. | 1292.4 | 376.0 | 616.2 | 4.44 | 8329.9 |
|  | -80 | . 975 | 4.052 | . 89 | 119.4 | 37.6 | 108.3 | 309.9 | 38.12 | 32.50 | 11/29.7/83 | 1400.8 | cont. | 1278.8 | 369.9 | 609.3 | 4.19 | 8355.6 |
|  | -90 | . 974 | 4.062 | . 98 | 117.3 | 36.0 | 114.4 | 303.7 | 38.08 | 32.50 | 11/29.1/83 | 1390.4 | cont. | 1261.6 | 363.4 | 599.3 | 3.9: | 8391.5 |
|  | -100 | . 974 | 4.079 | 1.08 | 115.1 | 32.8 | 119.8 | 296.8 | 37.95 | 32.50 | 11/30.5/83 | 1379.0 | cont. | 1238.1 | 356.4 | 583.5 | 3.70 | 8445.7 |
|  | -110 | . 374 | 4.101 | 1.18 | 112.7 | 28.5 | 124.3 | 289.8 | 37.71 | 32.50 | 12/2.3/83 | 1367.2 | cont. | 1207.2 | 349.1 | 560.6 | 3.47 | 8520.1 |
| $\underset{\sim}{8}$ | -120 | . 974 | 4.126 | 1.29 | 110.1 | 24.0 | 128.0 | 282.9 | 37.39 | 32.50 | 12/4.3/83 | 1355.2 | cont. | 1170.2 | 342.1 | 531.3 | 3.26 | 8610.3 |
|  | -130 | . 973 | 4.152 | 1.41 | 107.2 | 19.9 | 131.1 | 276.6 | 37.00 | 32.50 | 12/6.1/83 | 1343.4 | cont. | 1129.5 | 335.6 | 497.7 | 3.06 | 8710.3 |
|  | -140 | . 972 | 4.178 | 1.54 | 104.0 | 16.4 | 133.6 | 270.8 | 36.55 | 32.50 | 12/7.6/83 | 1331.9 | cont. | 1087.0 | 329.6 | 461.8 | 2.88 | 8814.6 |
|  | -150 | . 971 | 4.204 | 1.67 | 100.4 | 13.5 | 135.9 | 265.6 | 36.05 | 32.50 | 12/8.8/83 | 1320.7 | cont. | 1044.5 | 324.1 | 425.3 | 2.71 | 8919.7 |


| 0 | . 989 | 3.180 | . 51 | 128.0 | 40.5 | 102.3 | 360.8 | 38.87 | 32.50 | 11/30.2/84 | 1113.1 | cont. | 1286.0 | 425.6 | 555.3 | 5.62 | 8290.3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -10 | . 968 | 3.186 | .49 | 132.3 | 41.5 | 105.8 | 354.4 | 38.84 | 32.50 | 11/29.7/84 | 1103.6 | cont. | 1265.0 | 418.0 | 542.6 | 5.59 | 8333.4 |
| -20 | . 966 | 3.193 | . 50 | 133.6 | 42.5 | 110.0 | 348.0 | 38.79 | 32.50 | 11/29.2/84 | 1094.1 | cont. | 1242.5 | 410.5 | 528.4 | 5.50 | 8381.4 |
| -30 | . 965 | 3.203 | . 53 | 132.3 | 43.3 | 114.9 | 341.6 | 38.68 | 32.50 | 11/28.7/84 | 1084.6 | cont. | 1217.3 | 402.9 | 511.5 | 5.35 | 8438.7 |
| -40 | . 963 | 3.221 | . 58 | 129.6 | 43.6 | 120.4 | 334.8 | 38.43 | 32.50 | 11/29.6/84 | 1074.6 | cont. | 1186.6 | 394.9 | 489.6 | 5.16 | 8516.7 |
| -50 | . 960 | 3.249 | . 65 | 126.6 | 42.3 | 125.9 | 327.1 | 37.87 | 32.50 | 11/29.6/84 | 1063.6 | cont. | 1146.5 | 385.9 | 459.5 | 4.94 | 8630.1 |
| -60 | . 957 | 3.285 | . 72 | 123.9 | 39.9 | 130.5 | 318.9 | 37.00 | 32.50 | 12/1.2/84 | 1052.0 | cont. | 1097.3 | 376.5 | 420.6 | 4.69 | 8773.7 |
| -70 | . 953 | 3.321 | . 80 | 121.5 | 37.0 | 134.3 | 311.2 | 35.90 | 32.50 | 12/2.6/54 | 10.10 .6 | cont. | 104.3 .7 | 367.7 | 376.6 | 4.44 | 8925.3 |
| -80 | . 948 | 3.356 | . 89 | 119.3 | 34.2 | 137.4 | 304.3 | 34.68 | 32.50 | 12/3.4/84 | 1029.8 | cont. | 989.7 | 359.8 | 331.3 | 4.19 | 9067.8 |



|  | Tempel IIP88) Rendezvous |  |  |  |  |  |  | TItan III E/Centaur/Sert III |  |  |  |  | 4-Year Miselons |  |  |  |  | * |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | AD | $r_{\mathrm{min}}$ | $r_{\text {max }}$ | $r_{\text {com }}$ | ${ }^{6} \mathrm{com}$ | $\mathrm{mln}$ | $\emptyset_{\max }$ | $\Delta \lambda$ | 6 | 1 | LD | $t_{1}$ | $t$ | $m_{0}$ | $\mathbf{m}_{\mathbf{p}}$ | $\mathrm{m}_{\text {net }}$ | $p_{t}$ |  |
|  | 0 | 1.018 | 3.919 | . 96 | 89.4 | 64.1 | 98.0 | 371.0 | -5.39 | 28.50 | 7/21.5/84 | 1518.0 | 1387.1 | 2361.7 | 395.9 | 1663.6 | 5.45 | 6690.9 |
|  | -10 | 1.015 | 3.923 | . 92 | 91.9 | 64.1 | 99.2 | 363.1 | -5.53 | 28.50 | T/22.1/84 | 1507.4 | 1387.3 | 2353.1 | 391.9 | 1659.5 | 5.42 | 6704.9 |
|  | -20 | 1.015 | 3.927 | . 89 | 94.9 | 64.1 | 100.7 | 355.3 | -5.68 | 28.50 | 7/22.7/84 | 1496. 8 | 1388.2 | 2344.1 | 388.1 | 1654.6 | 5.35 | 6719.6 |
|  | -30 | 1.015 | 3.931 | . 86 | 98.6 | 64.1 | 102.6 | 347.6 | -5.82 | 28.50 | 7/23.3/84 | 1486.2 | 1390.1 | 2334.8 | 384.7 | 1649.1 | 5.23 | 6734.8 |
|  | -40 | 1.015 | 3.936 | . 83 | 103.0 | 64.1 | 104.8 | 340.2 | -5.94 | 28.50 | 7/23.8/84 | 1475.7 | 1393.2 | 2325.3 | 381.6 | 1642.9 | 5.08 | 6750.3 |
| $\underset{\sim}{8}$ | -50 | 2.015 | 3.941 | . 81 | 108.2 | 64.1 | 107.3 | 333.1 | -6.04 | 28.50 | 7/24.3/84 | 1465.2 | 1398.1 | 2315.8 | 379.0 | 1636.3 | 4.99 | 6765.9 |
|  | -60 | 1.015 | 3.946 | . 79 | 114.3 | 64.1 | 110.0 | 326.4 | -6.10 | 28.50 | 7/24.9/84 | 1454.6 | 1405.2 | 2306.3 | 376.8 | 1629.2 | 4.69 | 6781.3 |
|  | -70 | 1.015 | 3.952 | .78 | 121.2 | 64.2 | 112.8 | 319.9 | -6.11 | 28.30 | 7/25.4/84 | 1444.1 | 1415.2 | 2297.1 | 375.3 | 1621.7 | 4.47 | 6796.4 |
|  | $-80$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | -90 | 1.015 | 3. 963 | . 78 | 137.3 | 64.5 | 119.5 | 308.3 | -6.37 | 28.50 | 7/26.0/84 | 1423.5 | cont. | 2274.2 | 369.4 | 1605.3 | 4.03 | 6834.0 |
|  | -100 | 1.015 | 3.968 | . 79 | 145.5 | 64.8 | 123.1 | 303.2 | -6.73 | 28.50 | 7/26.1/81 | 1413.4 | cont. | 2257.5 | 363.7 | 1594.8 | 3.81 | 6861.4 |
|  | -110 | 1.014 | 3.976 | . 83 | 152.4 | 65.1 | 126.4 | 299.3 | -7.07 | 28.50 | 7/26.3/84 | 1403.2 | cont. | 2238.1 | 358.3 | 1581.4 | 3.60 | 6893.2 |
|  | -120 | 1.014 | 3.987 | . 98 | 156.1 | 65.0 | 129.4 | 293.6 | -7.43 | 28.50 | 7/25.6/84 | 1392.9 | cont. | 2213.7 | 352.9 | 1562.9 | 3.40 | 6933.4 |
|  | -130 | 1.014 | 4.003 | . 95 | 155.0 | 64.2 | 131.6 | 288.8 | -7.87 | 28.50 | 7/27.3/84 | 1382.2 | cont. | 2180.0 | 347.4 | 1535.3 | 3.22 | 6985.9 |
|  | -140 | 1.014 | 4.025 | 1.04 | 149.8 | 61.7 | 133.3 | 283.9 | -8.51 | 28.50 | 7/28.4/84 | 1371.1 | cont. | 2131.3 | 341.6 | 1492.9 | 3.04 | 7069.5 |
|  | -150 | 1.013 | 4.045 | 1.14 | 142.7 | 57.7 | 135.0 | 279.0 | -9.26 | 28.50 | 7/23.6/84 | 1359.9 | cont. | 20es. 7 | 336.0 | 1436.6 | 2.88 | 7173.7 |
|  | -160 | 1.013 | 4.063 | 1.27 | 134.9 | 52.8 | 136.7 | 274.5 | -9.91 | 28.50 | 7/30.8/84 | 1348.7 | cont. | 1993.2 | 330.6 | 1366.3 | 2.75 | 7300.2 |
|  | -170 | 1.012 | 4.079 | 1.41 | 127.1 | 48.1 | 138.4 | 270.5 | -10.28 | 28.50 | 7/31.5/94 | 1338.0 | cont. | 1916.3 | 326.0 | 1295.2 | 2.59 | 7430.1 |
|  | -180 | 1.010 | 4.093 | 1.56 | 119.4 | 43.6 | 140.0 | 267.2 | -10.38 | 28.50 | 7/31.8/84 | 1327.7 | cont. | 1810.0 | 321.8 | 1223.4 | 2.46 | 7560.2 |
|  | -100 | 1.009 | 4.106 | 1.73 | 111.9 | 39.2 | 141.7 | 264.3 | -10.24 | 28.50 | 7/31.9/84 | 1317.6 | cont. | 1766.0 | 318.2 | 1153.5 | 2.34 | 7687.3 |
|  | -200 | 1.007 | 4.118 | 1.90 | 104.7 | 35.0 | 143.4 | 291.8 | -9.91 | 28.50 | 7/31.6/84 | 1307.9 | cont. | 1695.5 | 314.9 | 1086.5 | 2.23 | 7809.9 |

Tempel $\Pi$ (88) Rendezvous


| Tempel II(88) Rendervous |  |  |  |  |  |  | TItan III E/Centaur/Sert III |  |  |  |  | 3-Year Mlssions |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AD | $r_{m l n}$ | $r_{\max }$ | $\mathbf{r}_{\mathrm{com}}$ | $\theta_{\mathrm{com}}$ | $\omega_{\mathrm{min}}$ | $\max ^{2}$ | $\Delta \lambda$ | 8 | 1 | LD | $t_{1}$ | $t$ | $m_{0}$ | $\mathbf{m}_{\mathbf{p}}$ | $m_{\text {net }}$ | $\mathbf{P}_{\mathbf{t}}$ | $\nabla$ |
| 0 | 1.020 | 3.168 | . 96 | 89.4 | 70.1 | 131.3 | 363.4 | -15.53 | 28.50 | 7/29.7/85 | 1144.8 | cont. | 2272.7 | 428.3 | 1539.0 | 5.45 | 6836.4 |
| -10 | 1. 007 | 3.191 | . 92 | 91.9 | 67.1 | 134.0 | 355.1 | -15.07 | 28.50 | 7/30.7/85 | 1133.8 | cont. | 2172.3 | 419.2 | 1448.6 | 5.43 | 7001.6 |
| -20 | 1.004 | 3.212 | . 89 | 94.9 | 63.1 | 136.7 | 347.4 | -14.19 | 23.50 | 7/31.2/85 | 1123.3 | cont. | 2063.1 | 410.6 | 1348.8 | 5.35 | 7183.1 |
| -30 | 1.000 | 3.233 | . 86 | 98.6 | 58.4 | 139.3 | 340.3 | -12.92 | 28.50 | 7/31.2/85 | 1113.3 | cont. | 1952.0 | 402.8 | 1246.3 | 5.23 | 7369.7 |
| -40 | . 994 | 3.253 | . 83 | 103.0 | 53.4 | 141.8 | $333 .{ }^{\text {n }}$ | -11.36 | 28.50 | 7/30.7/85 | 1103.8 | cont. | 1843.0 | 395.8 | 1145.1 | 5.08 | 7554.9 |
| -60 | . 980 | 3.292 | . 79 | 114.3 | 48.3 | 145.7 | 324.2 | -7.68 | 28.50 | 7/27.4/85 | 1087.1 | 1009.1 | 1626.9 | 372.7 | 954.3 | 4.69 | 7930.2 |
| -70 | . 971 | 3.311 | . 78 | 121.2 | 46.1 | 147.2 | 320.6 | -5.89 | 28.50 | 7/24.9/85 | 1079.6 | 1051.2 | 1527.9 | 362.0 | 867.1 | 4.47 | 8105.9 |
| -80 | . 961 | 3.331 | . 77 | 129.0 | 43.3 | 148.3 | 317.8 | -4.13 | 28.50 | 7/21.8/85 | 1072.7 | 1035.2 | 1437.3 | 353.6 | 745.7 | 4.25 | 8269.4 |
| -9n | . 949 | 3.352 | . 78 | 137.3 | 40.2 | 148.9 | 316.2 | -2.33 | 24.50 | 7/17.0/85 | 1066.5 | 1021.0 | 135.3 .3 | 347.5 | 710.5 | 4.02 | 8419.5 |
| $-100$ | . 934 | 3.373 | . 79 | 145.5 | 37.0 | 148.8 | 316.0 | -0.30 | 24.50 | 7/12.9/85 | 1061.6 | 1009.4 | 1282.5 | 344.2 | 641.2 | 3.81 | 8534.6 |
| -110 | . 015 | 3.395 | . 83 | 152.4 | 34.1 | 147.5 | 318.4 | 2.38 | 24. 50 | 7/5.5/85 | 1059. C | 1003.1 | 1221.4 | 346.2 | 578.0 | 3.60 | 8669.6 |

## 2-Year Missions

| 0 | . 825 | 2. 935 | . 5 S | 89.4 | 54.0 | 128.8 | 414.2 | 7.14 | 28.50 | 6/6.7/86 | 832.8 | cont. | 1346.8 | 462.8 | 575.2 | .. 45 | 8435.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -10 | . 792 | 2.465 | . 92 | 91.9 | 57.5 | 128.7 | 416.6 | 9.21 | 28.50 | 5/27.5/86 | 832.9 | cont. | 1287.9 | 459.7 | 519.6 | 5.43 | 8544.6 |
| -20 | . 761 | 2.497 | . 89 | 94.9 | 58.9 | 129.9 | 417.5 | 10.27 | 28.50 | 5/19.1/86 | 831.4 | cont. | 1289.9 | 454.7 | 466.2 | 5.35 | 8653.3 |
| -30 | .7.1 | 2.531 | . 86 | 98.6 | 59.0 | 169.0 | 417.6 | 10.75 | 28.50 | 5/11.6/86 | 829.9 | cont. | 1171.0 | 448.5 | 415.1 | 5.23 | 8:65.4 |
| -40 | . 701 | 2.596 | . 83 | 103.0 | 58.3 | 168.3 | 417.3 | 10.89 | 29.50 | 5/4.8/86 | 825.7 | cont. | 1114.7 | 4.11 .5 | 366.5 | 5.08 | 8573.6 |
| -50 | . 671 | 2.602 | . 81 | 109.2 | 57.3 | 167.7 | 410.8 | 10.85 | 29.50 | 4/28.5/86 | 822.0 | cont. | 1000.6 | 434.0 | 320.5 | 4.89 | 8979.0 |
| -60 | . 642 | 2.639 | . 79 | 114.3 | 56.0 | 167.3 | 416.2 | 10.73 | 2*. 50 | 4/22.6/86 | 817.9 | cont. | 100*. 9 | 426.2 | 277.5 | 4.69 | 9040.7 |
| -70 | . 612 | 2.676 | . 78 | 121.2 | 5.4 .7 | 166.9 | 415.6 | 10.58 | 28.50 | 4/17.2/86 | 813.3 | cont. | 959.9 | 418.7 | 237.3 | 4.47 | 9179.4 |
| -90 | . 543 | 2.714 | . 77 | 129.0 | 53.4 | 166.6 | 415.0 | 10.43 | 29.50) | 4/12.1/8rs | 808.4 | cont. | 913.6 | 410.0 | 200.0 | 4.25 | 92:1.6 |
| -30 | . 555 | 2751 | . 79 | 137.3 | 52.1 | 166.3 | 414.3 | 10.32 | 2N.51) | 4/7.3/46 | 803.2 | cont. | Nin. 1 | 401.8 | 165.5 | 4.02 | 9360.3 |
| -100 | . 527 | 2.788 | . 79 | 14.5 .5 | 50.9 | 166.1 | 413.7 | 10.26 | 28.50) | 4/2.9/36 | 797.6 | cont. | 829.4 | 393.7 | 133.6 | 3.41 | 944.2 |







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 응운우웅우욱

## Tempel 11 (83) Rendezvous

TItan III E/Centaur/Sert III
E-Year Misstons

| AD | $\mathbf{r}_{\mathrm{m} \\| \mathrm{n}}$ | ${ }^{r_{\text {max }}}$ | $\mathbf{r}_{\mathrm{com}}$ | $\theta_{\text {com }}$ | $\Phi_{\min }$ | $*_{\text {max }}$ | $\Delta \lambda$ | 6 | 1 | LD | $t_{f}$ | $t$ | $\mathrm{m}_{0}$ | ${ }^{m} p$ | $m_{n e t}$ | $p_{t}$ | $\nabla^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 250 | . 982 | 2.001 | 2.59 | 83.4 | 68.9 | 146.7 | 487.2 | -21.97 | 28.50 | 7/17.6/81 | 933.9 | 842.4 | 1918.5 | 488.5 | 1118.6 | 1.79 | 7426.3 |
| 240 | . 981 | 1.999 | 2.40 | 90.6. | 69.5 | 146.7 | 485.3 | -21.61 | 28.50 | 7/17.3/81 | 924.2 | 832.0 | 1902.1 | 485.1 | 1105.9 | 1.86 | 7454.2 |
| 230 | . 979 | 1.997 | 2.21 | 98.1 | 70.1 | 146.5 | 483.4 | -21.22 | 26.50 | 7/17.1/81 | 914.4 | 821.7 | 1884.9 | 481.7 | 1092.5 | 1.94 | 7483.4 |
| 220 | . 977 | 1.995 | 2.03 | 105.8 | 70.6 | 146.4 | 481.3 | -20.79 | 28.50 | 7/16.8/81 | 904.7 | 811.5 | 1867.0 | 478.3 | 1078.3 | 2.03 | 7513.9 |
| 210 | . 974 | 1.994 | 1.85 | 113.8 | 71.2 | 146.1 | 479.2 | -20.32 | 28.50 | 7/16.5/81 | 895.0 | 801.5 | 1848.3 | 474.8 | 1063.4 | 2.13 | 7545.9 |
| 200 | . 972 | 1.993 | 1.68 | 122.2 | 71.6 | 145.8 | 476.9 | -19.81 | 28.50 | 7/16.2/81 | 885.3 | 791.7 | 1828.7 | 471.4 | 1047.6 | 2.23 | 7579.4 |
| 100 | . 969 | 1.992 | 1.52 | 130.7 | 72.1 | 145.4 | 474.6 | -19.25 | 28.50 | 7/15.8/81 | 875.7 | 782.1 | 1808.2 | 468.0 | 1030.8 | 2.34 | 7614.7 |
| 180 | . 996 | 1.992 | 1.39 | 139.4 | 72.5 | 144.9 | 472.1 | -18.65 | 28.50 | 7/15.6/81 | 866.1 | 772.8 | 1786.6 | 464.6 | 1012.9 | 2.46 | 7651.8 |
| 170 | . 963 | 1.993 | 1.26 | 147.6 | 75.3 | 144.3 | 469.5 | -17.98 | 28.50 | 7/14.9/81 | 856.6 | 763.8 | 1763.9 | 461.3 | 993.9 | 2.59 | 7691.0 |
| 160 | . 959 | 1.994 | 1.16 | 154.0 | 73.0 | 143.6 | 466.7 | -17.26 | 28.50 | 7/14.4/81 | 847.1 | 755.0 | 1739.9 | 458.0 | 973.6 | 2.73 | 7732.5 |
| 150 | . 955 | 1.996 | 1.09 | 156.3 | 73.2 | 142.7 | 463.9 | -16.48 | 28.50 | 7/13.8/81 | 837.7 | 746.6 | 1714.6 | 45.4 | 951.8 | 2.88 | 7776.5 |
| 140 | . 950 | 1.998 | 1.03 | 153.0 | 73.2 | 141.6 | 460.9 | -15.62 | 28.50 | 7/13.1/81 | 828.4 | 738.7 | 1687.9 | 451.6 | 928.5 | 3.05 | 7823.1 |
| 130 | . 945 | 2.001 | 1.00 | 145.7 | 7:3 | 140.4 | 457.7 | -14.65 | 28.50 | 7/12.3/81 | 819.2 | 731.1 | 1659.5 | 448.5 | 903.5 | 3.22 | 7872.8 |
| 120 | . 939 | 2.006 | 0.99 | 136.8 | 74.9 | 139. C | 454.5 | -13.68 | 28.50 | 7/11.3/81 | 810.2 | 7:4.0 | 1629.4 | 445.6 | 876.6 | 3.41 | 7925.7 |
| 110 | . 932 | 2.012 | 0.99 | 127.8 | 72.5 | 137.6 | 451.1 | -12.57 | 28.50 | 7/10.1/81 | 801.4 | 717.6 | 1597.5 | 442.9 | 847.7 | 3.60 | 7982.1 |
| 100 | . 924 | 2.019 | 1.00 | 119.2 | 72. | 136.0 | 447.7 | -11.35 | 28.50 | 7/8.7/81 | 792.8 | 711.7 | 1563.7 | 440.3 | 816.7 | 3.81 | 8042.1 |
| 90 | . 915 | 2.027 | 1.01 | 111.4 | 71.2 | 134.5 | 444.2 | -10.00 | 28.50 | 7/7.0/81 | 784.4 | 706.7 | 1527.8 | 438.1 | 783.3 | 4.03 | 8106.1 |
| 80 | . 905 | 2.038 | 1.03 | 104.5 | 70.2 | 133.0 | 440.8 | -8.49 | 28.50 | 7/4.9/81 | 776.5 | 702.8 | 1490.1 | 436.3 | 747.6 | 4.25 | 8173.8 |
| 70 | . 893 | 2.050 | 1.05 | 98.5 | 69.0 | 131.5 | 437.6 | -6.76 | 28.50 | 7/2.3/81 | 769.2 | 700.3 | 1450.8 | 435.2 | 709.4 | 4.47 | 8244.9 |
| 60 | . 878 | 2.065 | 1.08 | 93.3 | 67.5 | 130.0 | 434.8 | -4.75 | 28.50 | $6 / 28.8 / 81$ | 762.6 | 699.9 | 1410.4 | 435.3 | 668.9 | 4.69 | 8318.4 |
| 50 | . 860 | 2.083 | 1.10 | 88.9 | 65.8 | 128.3 | 432.9 | -2.28 | 28.50 | 6/24.1/81 | 757.4 | 703.2 | 1370.4 | 437.7 | 626.3 | 4.90 | 8391.6 |
| 40 | . 836 | 2.106 | 1.12 | 85.1 | 64.0 | 126.4 | 433.0 | 1.00 | 28.50 | 6/16.9/81 | 754.6 | 714.2 | 1334.8 | 445.7 | 581.9 | 5.09 | 8457.2 |
| 30 | . 790 | 2.140 | 1.14 | 81.9 | 65.1 | 121.9 | 444.6 | 7.76 | 28.50 | 5/28.5/81 | 764.0 | cont. | 1336.0 | 487.6 | 537.0 | 5.24 | 8455.1 |
| 20 | . 761 | 2.170 | 1.17 | 79.3 | 67.4 | 121.8 | 445.1 | 9.57 | 28.50 | 5/20.5/81 | 762.0 | cont. | 1286.0 | 482.2 | 493.0 | 5.36 | 8543.0 |
| 10 | . 731 | 2.201 | 1.20 | 77.1 | 68.2 | 122.6 | 444.4 | 10.69 | 28.50 | 5/13.6/81 | 758.9 | cont. | 1234.1 | 475.2 | 448.8 | 5.44 | 8645.5 |
| 0 | . 701 | 2.235 | 1.23 | 75.3 | 67.9 | 166.8 | 443.0 | 11.30 | 28.50 | 5/7.5/81 | 755.0 | cont. | 1181.0 | 467.1 | 401.6 | 5.47 | 8746.2 |
| -13 | . 672 | 2.270 | 1.27 | 73.8 | 66.9 | $165 . ?$ | 441.0 | 11.58 | 28.50 | 5/1.9/81 | 750.6 | cont. | 1127.4 | 453.1 | 360.8 | 5.44 | 8849.1 |
| -20 | -. 6.12 | 2.306 | 1.32 | 72.5 | 65.4 | 163.9 | 438.9 | 11.66 | 28.50 | 4/26.7/81 | 745.8 | cont. | 1074.0 | 448.8 | 317.8 | 5.36 | 8952.7 |
| -30 | . 612 | 2.343 | 1.38 | 71.2 | 63.7 | 162.8 | 436.6 | 11.63 | 28.50 | 4/21.7/81 | 740.8 | cont. | 1021.4 | 439.0 | 275.8 | 5.24 | 9056.0 |
| -40 | . 583 | 2.381 | 1.44 | 69.8 | 61.9 | 161.9 | 434.4 | 11.56 | 28.50 | 4/16.9/81 | 735.6 | cont. | 970.2 | 429.1 | 235.5 | 5.09 | 9157.8 |
| -50 | . 553 | 2.420 | 1.52 | 68.4 | 60.1 | 161.2 | 432.2 | 11.51 | 28.50 | 4/12.4/81 | 730.1 | cont. | 920.8 | 419.2 | 197.1 | 4.90 | 9257.1 |
| -60 | . 524 | 2.459 | 1.61 | 66.7 | 58.4 | 160.7 | 430.2 | 11.49 | 28.50 | 4/8.1/81 | 724.4 | cont. | 873.7 | 409.4 | 160.8 | 4.69 | 9352.9 |
| -70 | . 496 | 2.498 | 1.71 | 64.8 | 56.7 | 160.3 | 428.2 | 11.54 | 28.50 | 4/3.9/81 | 718.5 | cont. | 829.1 | 399.6 | 126.9 | 4.47 | 9444.9 |

## APPENDIX B

The equations for the partial derivatives $\partial \mathrm{v}_{\mathrm{g}} / \partial \mathrm{v}_{\infty}$ and $\partial \mathrm{v}_{\mathrm{g}} / \partial \delta$ are derived from Equations (5) and (6) of Section IV and are listed below:

$$
\begin{gathered}
\frac{\partial v_{g}}{\partial v_{\infty}}=\frac{v_{o}^{2}}{v_{g}}\left\{\rho \frac{\partial \rho}{\partial v_{\infty}}-\frac{w\left(3+2 \rho w-w^{2}\right)\left(\partial \rho / \partial v_{\infty}\right)+\left(3 \rho+2 \rho^{2} w-3 \rho w^{2}-4 w\right)\left(\partial w / \partial v_{\infty}\right)}{2 \sqrt{\left(1+\rho w-w^{2}\right)(2+\rho w)}}\right\}, \\
\frac{\partial v_{g}}{\partial \delta}=-\frac{v_{o}^{2}\left(3 \rho+2 \rho^{2} w-3 \rho w^{2}-4 w\right)}{2 v_{g} \sqrt{\left(1+\rho w-w^{2}\right)(2+\rho w)}} \frac{\partial w}{\partial \delta},
\end{gathered}
$$

where

$$
\begin{aligned}
& \partial \rho / \partial v_{\infty}=1 / v_{o}, \\
& \frac{\partial w}{\partial v_{\infty}}=\frac{1}{2}\left\{\frac{1}{2} \frac{\partial \rho}{\partial v_{\infty}}+\frac{\partial y}{\partial v_{\infty}}+\left[\left(1+\frac{x}{2 \sqrt{x^{2} / 4+s^{2}}}\right) \frac{\partial x}{\partial v_{\infty}}+\left(\frac{\rho}{2}+y\right)\left(\frac{1}{2} \frac{\partial \rho}{\partial v_{\infty}}\right.\right.\right. \\
& \left.\left.\left.+\frac{\partial y}{\partial v_{\infty}}\right)\right] /(2 w-\rho / 2-y)\right\}, \\
& \frac{\partial w}{\partial \delta}=\frac{1}{2}\left\{\frac{\partial \mathrm{v}}{\partial \delta}+\left[\frac{\partial \mathrm{y}}{\partial \bar{\delta}}+\frac{\mathrm{x}(\partial \mathrm{x} / \partial \delta)+4 \mathrm{~s}(\partial \mathrm{~s} / \partial \delta)}{2 \sqrt{x^{2} / 4+s^{2}}}+\left(\frac{\rho}{2}+\mathrm{y}\right) \frac{\partial \mathrm{y}}{\partial \delta}\right] /(2 w-\rho / 2-y)\right\}, \\
& \partial s / \partial \delta=\cos (\delta-i), \\
& \partial y / \partial v_{\infty}=\left[(\rho / 2) \partial \rho / \partial v_{\infty}-\partial x / \partial v_{\infty}\right] / 2 y, \\
& \partial y / \partial \delta=-(\partial x / \partial \delta) / 2 y, \\
& \frac{\partial x}{\partial u}=\frac{1}{6}\left[\frac{(q / 2)(\partial q / \partial u)+(p / 3)^{2}(\partial p / \partial u)}{\sqrt{(q / 2)^{2}+(p / 3)^{3}}}+\frac{\partial q}{\partial u}\right]\left[\sqrt{(q / 2)^{2}+(p / 3)^{3}}+q / 2\right]^{-2 / 3} \\
& -\frac{1}{6}\left[\frac{(q / 2)(\partial q / \partial u)+(p, 3)^{2}(\partial p / \partial u)}{\sqrt{(q / 2)^{2}+(p / 3)^{3}}}-\frac{\partial q}{\partial u}\right]\left[\sqrt{(q / 2)^{2}+(p / 3)^{3}}-q / 2^{7^{-2 / 3}}\right. \\
& \text { with } u=v_{\infty} \text { or } \delta \text {, }
\end{aligned}
$$

$$
\begin{aligned}
& \partial q / \partial v_{\infty}=2 \rho s^{2}\left(1-s^{2}\right)\left(\partial \rho / \partial v_{\infty}\right), \\
& \partial q / \partial \delta=2 \rho^{2} s\left(1-2 s^{2}\right)(\partial s / \partial \delta), \\
& \partial p / \partial v_{\infty}=2 \rho s^{2}\left(\partial \rho / \partial v_{\infty}\right), \\
& \partial p / \partial \delta=2 s\left(\rho^{2}+4\right)(\partial s / \partial \delta) .
\end{aligned}
$$

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[^0]:    *Three is not necessarily the optimum
    iber.

[^1]:    *The multiple-target missions descrihed in this section were originally identified by R.W. Farquhar of the Goddard Space Flight Center.

[^2]:    *Equation numbers referenced in this section of the report pertain to this section only.

