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MULTIPLE RENDEZVOUS MANEUVERS AT SYNCHRONOUS ALTITUDE

BY

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NOMENCLATURE

a	Semi-major axis
CEP	Cotangential elliptic phasing transfer
DVT	Scalar sum of velocity increments
E_T	Transfer orbital energy per unit mass
G	Universal gravitational constant
Δi	Inclination difference between initial tug orbit and target orbit
Δi_1	Inclination change from first velocity increment
Δi_2	Inclination change from second velocity increment
$\vec{i}, \vec{j}, \vec{k}$	Unit vectors in local coordinate system
M	Mass of earth
n	Number of revolutions or mean motion of target orbit
r_a	Radius at apogee
r_p	Radius at perigee
r_s	Radius of synchronous orbit
t	Time for maneuver between velocity increments
ΔV_1	First velocity increment
ΔV_2	Second velocity increment
$\Delta\vec{V}_s$	Total velocity increment vector
V_s	Synchronous velocity
$\vec{V}_{s(F)}$	Velocity vector of target orbit
$\vec{V}_{s(I)}$	Velocity vector of initial tug orbit
$\vec{V}_T$	Velocity vector of transfer orbit at cotangent point
β_1	Angle that $\Delta\vec{V}_1$ is out of the tug's initial orbital plane
β_2	Angle that $\Delta\vec{V}_2$ is out of the transfer orbital plane

NOMENCLATURE (continued)

γ	$180^{\circ} - \beta_1 - \Delta i_1$
ϕ	Phase angle
ψ	Longitudinal separation angle
τ_s	Period of synchronous orbit
τ_T	Period of transfer orbit
μ	GM

ABSTRACT

Rendezvous techniques between synchronous satellites are found using a cotangential elliptic phasing (CEP) transfer. This transfer is a closed elliptic orbit which is cotangent to the synchronous orbits of the chase vehicle and target satellite. The cotangent point lies on the line of apsides of the transfer orbit which coincides with the line of intersection of the synchronous orbits. The rendezvous maneuvers accomplish plane changes and phase shifts simultaneously. These maneuvers are found to be optimal when the plane change is split equally between the velocity increments. The CEP transfer can be extended to several revolutions for a further reduction of fuel usage. A set of synchronous satellites is used in a simulated mission. An optimal sequence for rendezvous among these satellites is established such that the total sum of velocity increments is within the realm of estimated 1980 technology. The techniques developed here fill the void in current rendezvous techniques between two circular orbits of equal radii.

I. INTRODUCTION

1.1 General Introduction

The emergence of the Space Transportation System (STS), the Space Shuttle and the orbit-to-orbit tug, will add a new dimension to the space program. Two recent reports have indicated how the STS will have an important influence on future satellite programs. At the 1974 Electronics and Aerospace Systems Convention (EASCON) a report[1] on the future synchronous satellite programs cited that the STS will allow: the use of larger and more powerful satellites; an increase in the number of simpler satellites, particularly the R&D type; and the use of on-orbit servicing. This report predicted that initially on-orbit servicing would be used only for the correction of unanticipated design and construction defects or the replacement of completed R&D experiments. A more optimistic view of on-orbit servicing is found by another author,[2] where cost effectiveness studies indicate that of four different mission approaches considered, on-orbit servicing costs the least. Ground servicing of retrieved satellites was found to be the next best approach.

Early studies[3] were conducted on the retrieval of satellites from orbit. Techniques developed in these studies are easily extended to synchronous satellites once the tug is positioned close to its target satellite.

1.2 Objective

The increased use of synchronous satellites and the on-orbit servicing and retrieval of them will require rendezvous maneuvers at that altitude. Although the rendezvous maneuver has played a major role throughout the space program, most rendezvous maneuvers to date have taken place in low altitude orbits. The orbits involved in these low altitude rendezvous maneuvers have been of various forms and sizes, but each orbit in a maneuver was easily distinguishable from the others. When considering high altitude orbits, attention is primarily upon a unique form and size of orbit, the synchronous orbit. Although there are numerous synchronous satellites, they are all in very closely related orbits. Rendezvous techniques used to date are not suitable for missions operating at synchronous altitude.

It is a major task, both financially and operationally, to get to a synchronous orbit, therefore, an obvious cost reduction is realized if more than one satellite is serviced or retrieved on each mission. This type mission will entail rendezvous with several synchronous satellites in some sequential order. The objective of this thesis is to develop rendezvous techniques which will minimuze the fuel usage specifically in synchronous orbits. Procedures can then be developed to properly sequence the rendezvous maneuvers between several synchronous satellites to optimize the use of the space tug's fuel capacity. Although the approach of this effort is focused on synchronous orbits, the results should apply to any set of circular orbits whose radii are equal.

II. PREVIOUS INVESTIGATIONS

2.1 Rendezvous

Rendezvous maneuvers generally involve two phases, one which takes the transfer vehicle to the close proximity of the target and the second, normally called terminal rendezvous, which ends at a predocking configuration. The earliest studies of rendezvous were concerned with the terminal rendezvous phase,[4,5] with the initial rendezvous included in the launch to orbit maneuver.

As the need arose for rendezvous between two vehicles in orbit, minimum fuel trajectories were found to be Hohmann transfers. These involved two impulse maneuvers with a 180 deg. coasting arc between the perigee and the apogee of the transfer ellipse. When low thrust cases were considered, the optimal transfer between coplanar circular orbits was found as a power series of the small parameter ε, which is the ratio of the initial and the final circular radii. The linearized form of the actual system was represented by the first order terms of ε. The Pontryagin Maximum Principle was used to indicate that the minimum fuel trajectory consists of a two burn maneuver similar to the Hohmann transfer provided the thrust level is greater than a certain minimum value. Below this value more burn maneuvers are needed as the thrust level approaches zero, resulting in spiralling paths of several revolutions.[6]

The actual Gemini and Apollo missions used a common rendezvous procedure based on a technique known as "coelliptic sequencing" or "constant differential height." In this technique, the active vehicle maneuvers into an orbit coelliptic with that of the passive, or target, vehicle. Coelliptic orbits by definition are concentric and have a nearly constant altitude difference. After the orbits were made coelliptic, the terminal rendezvous phase was initiated during which the active vehicle leaves its coelliptic orbit on a path that will let it intercept the passive vehicle. A major advantage of this procedure is the large margin of safety provided by a low closing rate and the numerous opportunities for initiation of the terminal phase. But the price paid for this large safety margin is an increased fuel expenditure.[7] Edelbaum[8] developed a linearized theory for minimum fuel guidance in the neighborhood of a minimum fuel space trajectory. The fuel is minimized by determining the trajectory which requires the minimum total velocity change when summed over all impulses. Although all of these studies were applied to rendezvous between circular orbits or in the vicinity of near circular orbits, the case of both the initial and final orbits having equal altitudes was not considered.

2.2 Perturbations

A discussion of the perturbations on synchronous satellites is relevant to this study in two respects. The initial configuration of the target satellites will be determined by the long term perturbations, since most target selections will be in an inactive state. Secondly, an estimate of the short term perturbation effects should be made before an accurate rendezvous maneuver is planned. The perturbations of synchronous satellites is a well documented area. This thesis is not intended to add to this area, but only summarize the major effects.

The primary perturbations on the synchronous satellite are caused by the lunar and solar gravitational attractions, the solar radiation pressure, and the tangential component of the earth's gravitational geopotential. The combined effects of these perturbing forces cause a passive satellite to deviate from the ideal conic orbital motion. Because the nature of a rendezvous involves a specific time and position relationship, the deviations encountered may be of major importance.

The first deviation to be discussed involves the time factor, which would appear in the period of the orbit. The deviation of the orbital period from true synchronous is directly related to the deviation of the semi-major axis. Zee[9] gives an expression for the oscillating semi-major axis of a synchronous satellite perturbed by an oblate earth, the sun, the moon, and solar radiation. The oscillations are of a yearly cycle. Substitution of appropriate values into this expression and calculation of the maximum deviation in the orbital period of a synchronous satellite yields a deviation of approximately 18 seconds. This would be the maximum deviation to occur in the yearly cycle. The average deviation which might be encountered over several weeks would be at least two or three orders of magnitude smaller than the rendezvous times. Therefore, even the most detailed studies of rendezvous can justify neglecting these deviations.

The deviations with respect to the position of the synchronous satellite will be considered by their polar components. The radial deviations are closely related to the semi-major axis and the period mentioned above, and therefore, will be bypassed. The two other components will be distinguished as longitudinal and inclinational deviations.

The longitudinal deviations of a passive synchronous satellite have been expressed by Cassara[10] as a combination of diurnal oscillations induced by the lunisolar potential and the rotation of the earth, monthly oscillations due to the lunar potential, and a long-period libration about the closest "stable point" due to the tesseral and sectorial harmonics in the earth potential. Cassara also concluded that the long period libration or the diurnal lunisolar effects are of major importance in the long or short term investigations, respectively. The lunisolar gravitational effects, which amount to less than one percent of the total longitudinal drift, result in 0.003 deg. diurnal oscillations. Figure 1 shows the longitudinal position change of a synchronous satellite initially over the equator at 90 deg W longitude. This longitude illustrates the effects of libration with or without lunisolar influence. The small rate of change of the long-period libration and the small magnitude of the diurnal oscillations will not seriously affect the accuracy of the rendezvous maneuver. However, the initial configuration must take into account the long-period libration.

The effects of perturbation forces on the inclination of synchronous satellites have also been studied by Zee.[11] He found that the inclination reaches its maximum value of 14 deg. 43 min. after 26.84 years and the initial rate of change approximately 0.90 deg. per year. If a representative rate of change of 0.0025 deg. per day is assumed, the effects will not significantly influence the rendezvous results.

Therefore, the rendezvous velocity increments found when neglecting all perturbations should be accurate first order approximations of the

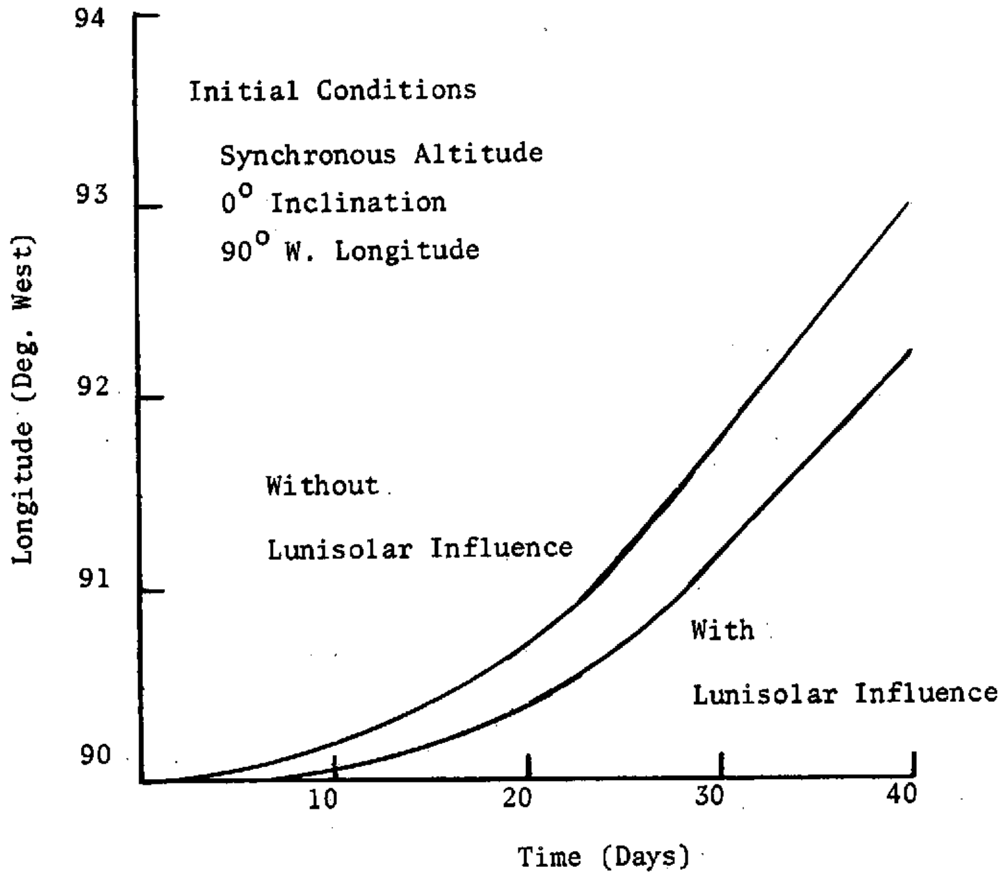

Figure 1. Longitudinal Drift Over Short Period.

(From Reference 10)

actual velocity increments needed. However, the perturbation effects will be a significant factor in determining the initial configuration of the target satellites.

III. DESCRIPTION OF APPROACH

3.1 Initial Configuration

The problem being studied is not limited to one target, but must consider multiple target satellites. The tug must rendezvous with each of these targets in some optimum sequence. The set of satellites from which sample targets will be chosen is given in Table 1. This list includes names, longitudinal positions relative to the earth's surface, inclinations, eccentricities, and semi-major axes of the unclassified synchronous satellites listed in the Satellite Situation Report[12] and through correspondence with Goddard Space Flight Center.[13] Information on the orientation of the lines of apsides was not available to be included in this report. Some of the satellites listed are still in an active state but by the time an operational system of this type is available, all of these satellites will be candidates for retrieval or repair.

The orientation of these satellites with respect to each other is shown in Figure 2, a two dimensional view as seen from the North Pole. The scenario would begin with the tug in the position of the first target. Selection of this first target will influence the remaining rendezvous maneuvers and will also be a function of launch and sub-orbit criteria. For the purposes of this report the selection of the first target will be based only on its effect on the remaining maneuvers. The criterion for sequencing the remaining targets is the fuel minimum required for rendezvous between the tug and each of the remaining targets. This fuel minimum can be expressed in terms of the relative positions of the tug and the target and type of transfer orbit employed.

TABLE 1

Synchronous Satellite Sample Data

Satellite Name	Longitude (Deg)	Inclination (Deg)	Semi-Major Axis (KM)	Eccentricity
Syncom 3	5 East	8.2	42193	.00021
Early Bird	152 West	8.4	42167	.00038
ATS 1	149 West	6.1	42165	.00120
Intelsat 2F2	159 East	4.5	42166	.00088
Intelsat 2F3	52 West	5.3	42177	.00166
Intelsat 2F4	161 West	5.3	42164	.00026
ATS 3	69 West	4.4	42165	.00306
Intelsat 3F2	76 West	4.3	42166	.00093
Intelsat 3F3	59 East	2.5	42164	.00013
Intelsat 3F4	170 West	4.0	42172	.00021
ATS 5	105 West	1.8	42165	.00158
Intelsat 3F6	178 East	2.9	42163	.00042
Intelsat 4F2	20 West	0.1	42165	.00017
Intelsat 4F3	24 West	0.3	42166	.00012
Intelsat 4F4	173 East	0.4	42167	.00014
Intelsat 4F5	61 East	0.5	42164	.00015
Intelsat 4F7	31 West	0.5	42161	.00012
Westar-A	99 West	0.0	42166	.00008
SMS1	53 West	2.0	42162	.00132
ATS 6	93 West	1.7	42160	.00007

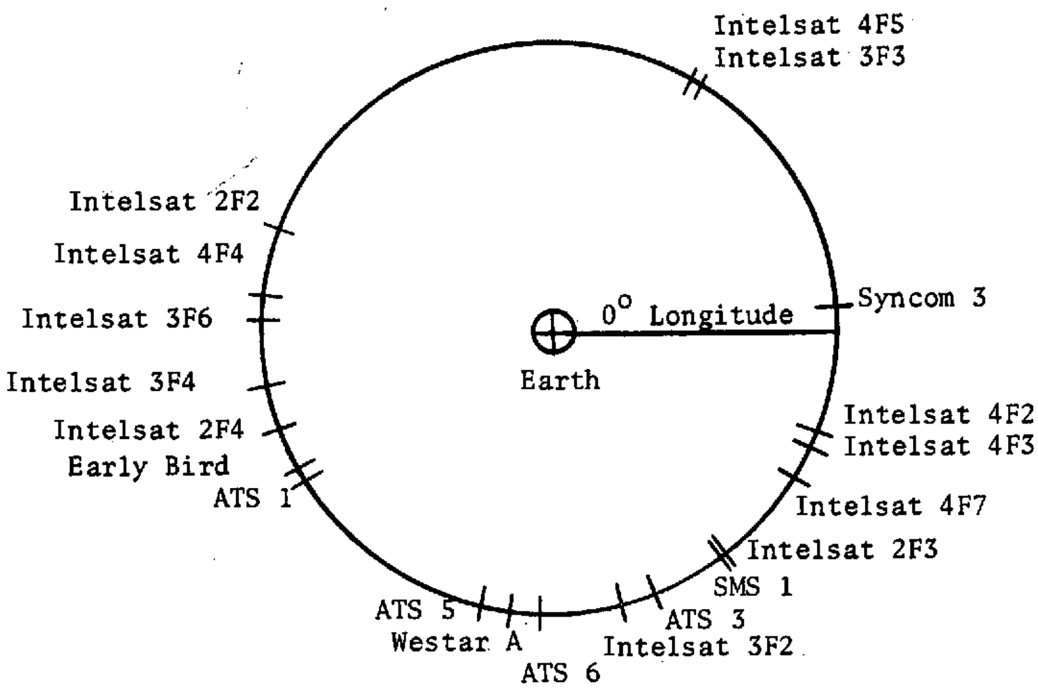

Figure 2. Initial Configuration.

3.2 Transfer Orbits

This thesis proposes to accomplish the rendezvous using a transfer orbit which can be described as a modified Hohmann type transfer. Whereas the Hohmann transfer (Figure 3) consists of a half orbit trajectory between two different radii, the transfer proposed (Figure 4) consists of a complete, possibly multiple revolution orbit returning to the same radius. The reason for classifying the proposed transfer with the Hohmann transfer is the use of tangential thrusts.

For lack of a better name the proposed transfer will be called a cotangential elliptic phasing (CEP) transfer. The transfer is effected by an elliptic orbit cotangent to the synchronous orbit at either its apogee or its perigee. An additional advantage of the CEP transfer is its fuel versus time tradeoff capability by use of multiple revolutions on the same elliptic orbit.

The use of tangential velocity increments for the Hohmann transfer has been proven to be an optimum condition. It can also be proven that tangential impulses are optimum when the initial and final radii are equal.[14] Therefore, the extension to the CEP transfer should also prove to be an optimum.

The CEP transfer can also accomplish plane changes without any additional thrust sequences. The common point of application for each velocity increment simplifies the orientation of the transfer plane. The three planes involved, the initial orbit plane, the transfer plane, and the final orbit plane, all intersect at a common line through this point of application.

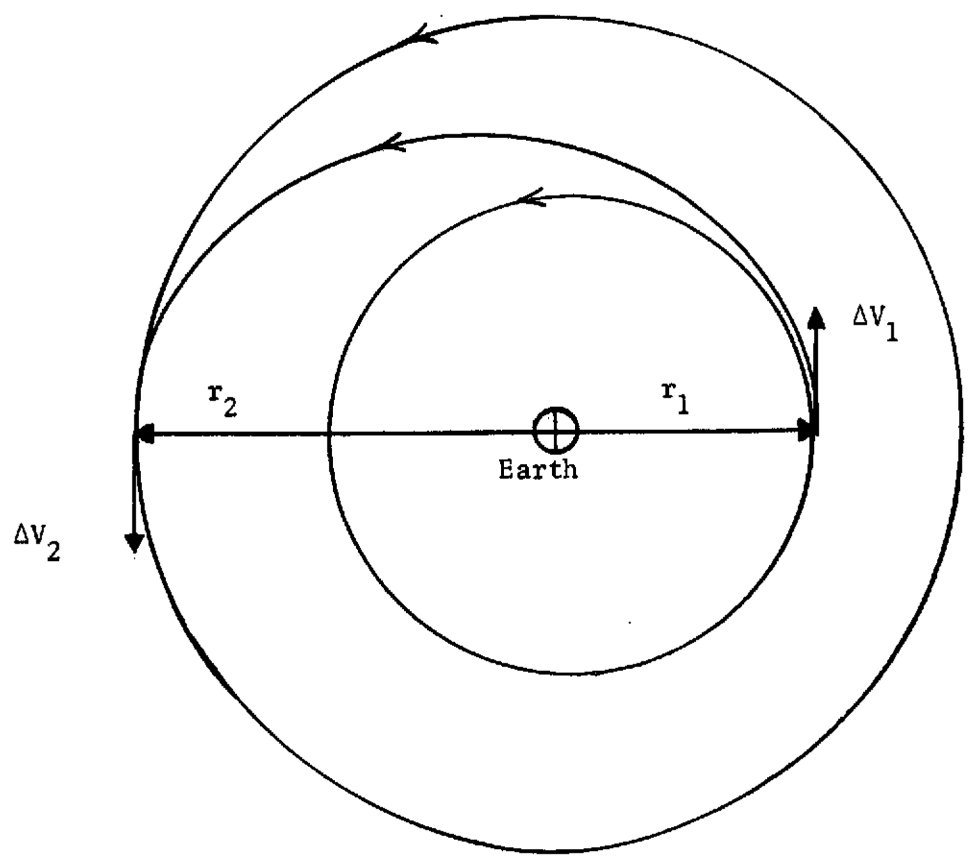

Figure 3. Hohmann Transfer.

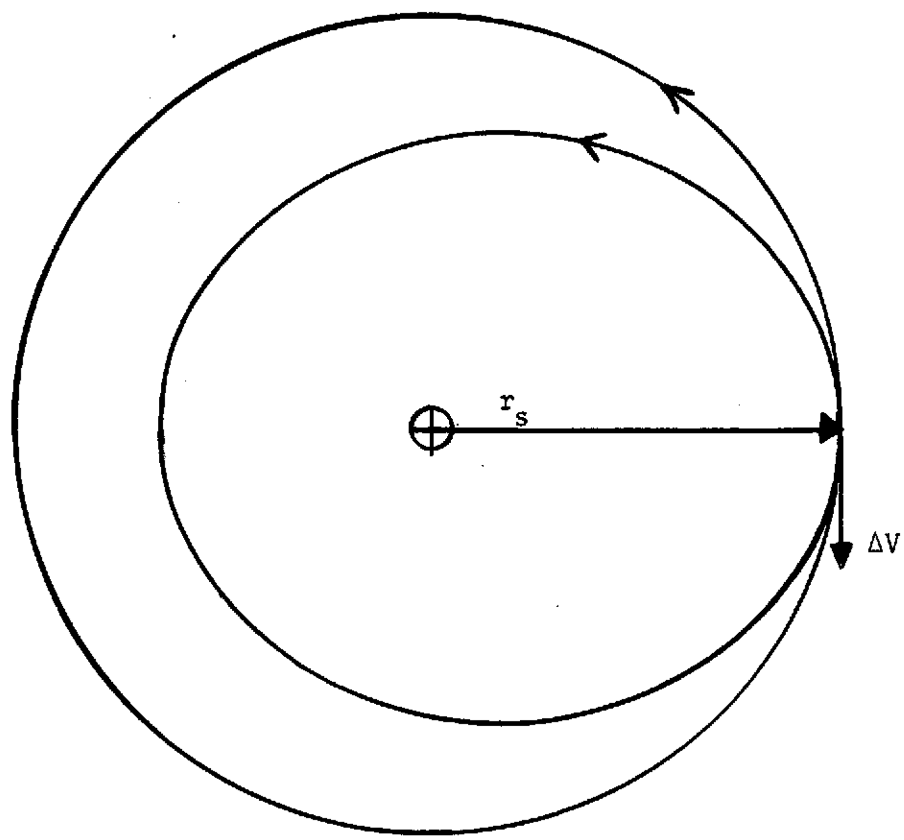

Figure 4a. Inner CEP Transfer Approaches Target from Behind.

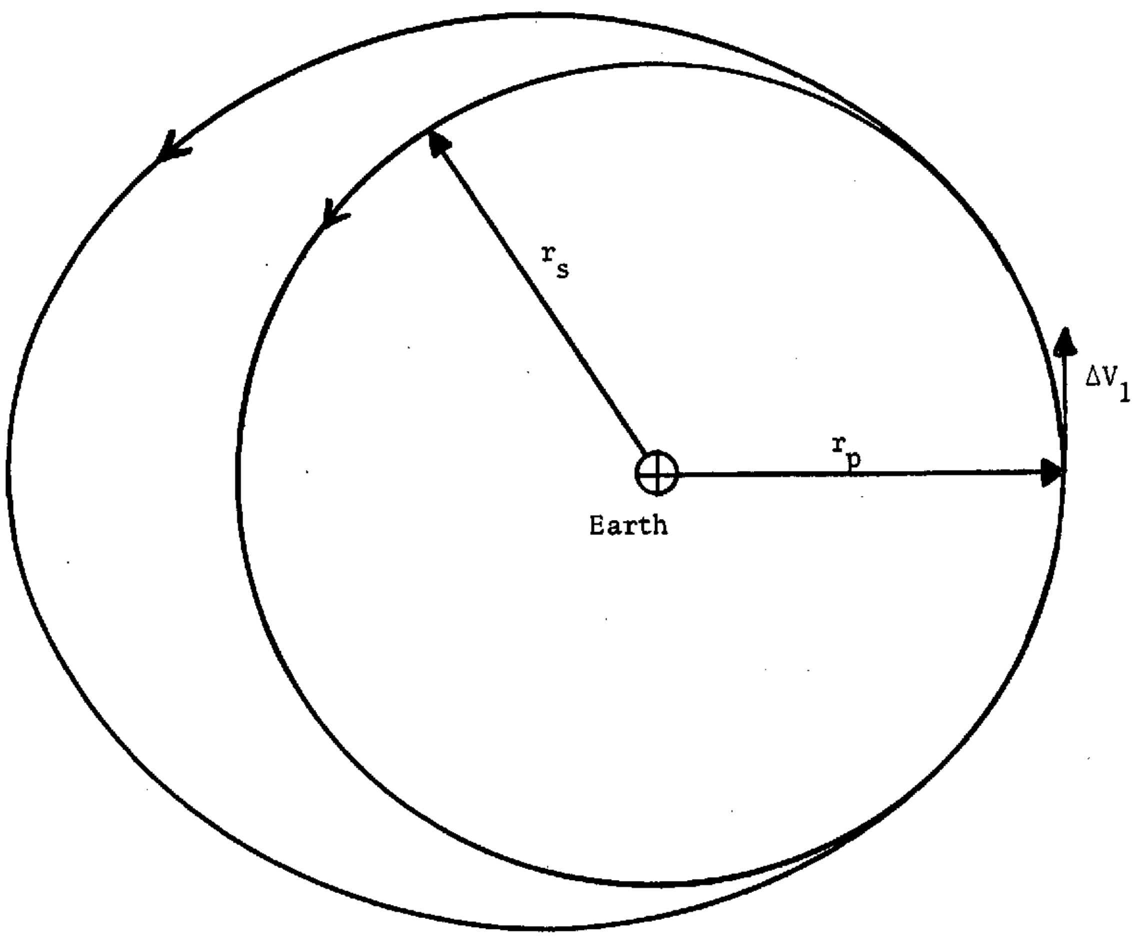

Figure 4b. Outer CEP Transfer Approaches Target from in Front.

3.3 Assumptions and Definitions

Before an analytical development is attempted several assumptions must be made and some terminology defined. The first assumption is that impulsive thrust will be used throughout. This will allow the addition of velocity increments with a change in the position vector. The second assumption, as discussed previously, is that the perturbation effects on the transfer orbit will be neglected. The third assumption is that all the synchronous satellites are in circular orbits with equal periods. This assumption will greatly simplify the orbital equations without sacrificing a great deal to reality, as can be seen from Table 1. This list of the unclassified synchronous satellite orbital data shows that the eccentricities are all very small and their semi-major axes are all within a few kilometers of true synchronous. Any inaccuracies resulting from these assumptions can be corrected during the terminal phase of rendezvous. A brief example of this will be shown later.

Since the transfer considered is between circular orbits of equal radii, the distinguishing factors in the orbits are a longitudinal angle and an inclination angle (see Figure 5a, 5b). The longitudinal angle (ψ) will be defined as the angle separating the tug and the target prior to initiation or after termination of the maneuver. The inclination angle (Δi) is that separating the orbit normals of the initial orbits of the tug and the target satellite. To effect an optimum inclination change the maneuver must be initiated in the orbital plane of the target. This restricts the point of initiation to the two points on the line of intersection of the tug and the target orbits. This may mean a waiting time of up to twelve hours.

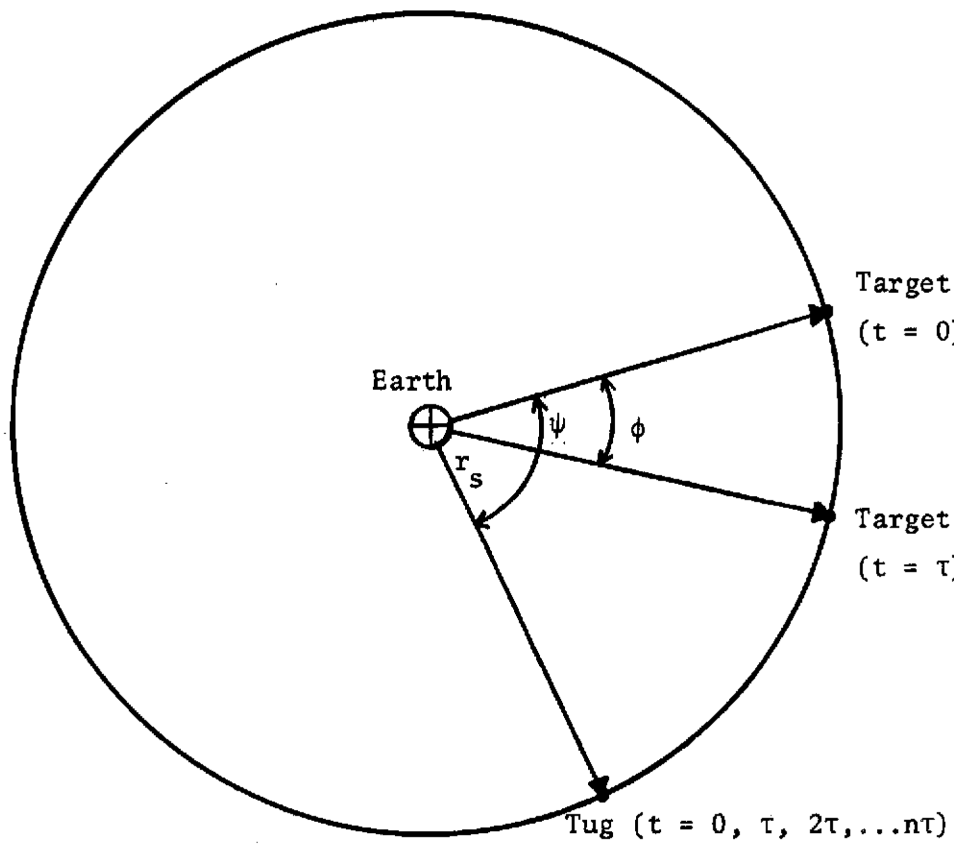

Figure 5a. Angles ψ and ϕ.

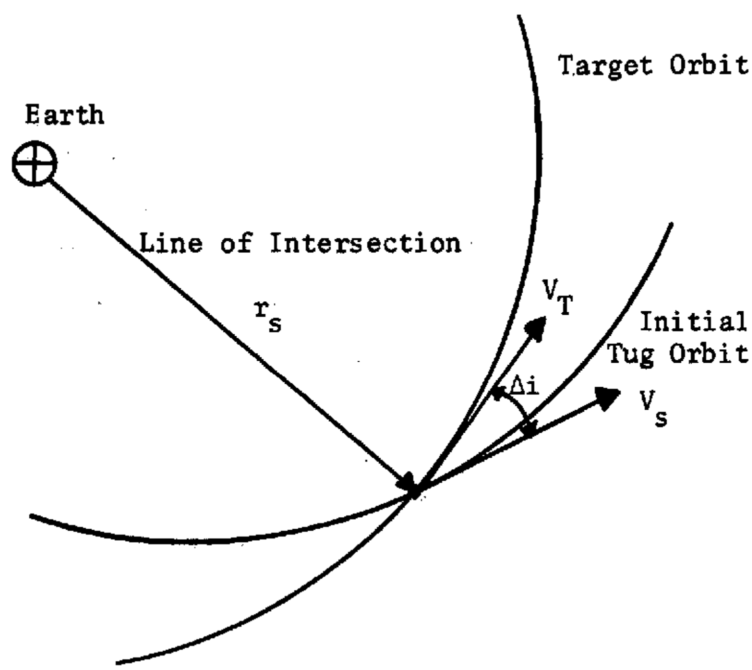

Figure 5b. Inclination Change Δi.

The rendezvous is accomplished by phasing the tug so that the longitudinal angle becomes zero. A useful angle in this analysis is one which will be called the phase angle (ϕ). The phase angle is defined as the angle that a synchronous satellite will subtend in the time difference between the transfer orbital period and the synchronous orbital period. This is shown in Figure 5a. A simplier explanation might be that the target will travel through that angle $360^{o} - \phi$ in the time (τ) equal to the period of the transfer orbit.

IV. ANALYTICAL DEVELOPMENT

4.1 Coordinates and Constraints

In general, the orbits and transfer maneuvers will be expressed in geocentric polar coordinates. The vector analysis of the thrust applications, which involves velocity increments, will utilize a local coordinate system as illustrated in Figure 6. This local system is fixed to the tug and is defined by the unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$. The unit vectors form radial, transverse, and orbit normal coordinates, respectively. The transverse axis is positive in the direction of motion.

There are several constraints basic to the type of transfer being considered that can be enumerated before the formal analysis begins. They are:

1) Velocity increments will be in the $\vec{j}$, $\vec{k}$ plane, thus satisfying the co-tangential requirement of this type transfer.
2) Velocity increments will only take place at one point on the transfer orbit, that being the point where $r = r_s$ (radius of the synchronous orbit).
3) The transfer orbit will take one of two forms, either inside or outside of the synchronous circle.

The first two constraints come from the definition of the CEP transfer, while the third follows from the other two since the transfer orbit and the synchronous orbit cannot touch at more than one point.

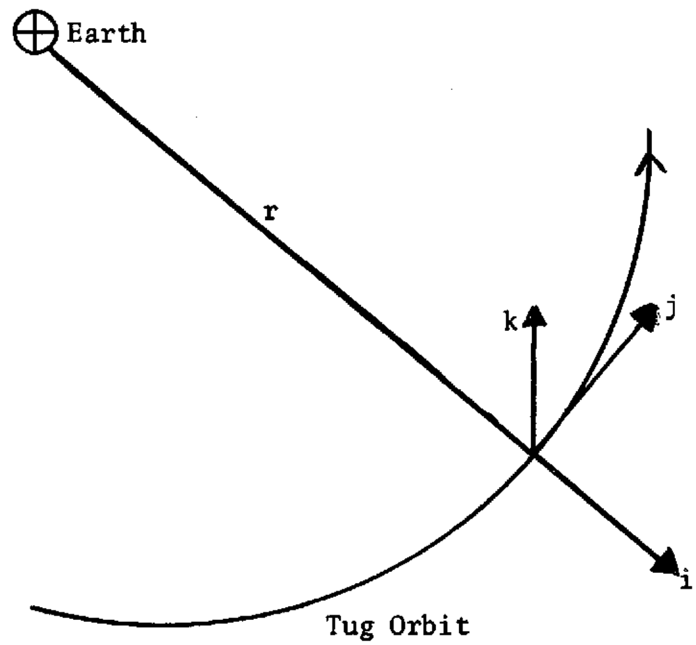

Figure 6. Local Coordinates.

4.2 Transfer Equations

Using the constraints just stated, the properties of the transfer orbit can be derived at the point of cotangency. The radial distance at this point for the inner and outer CEP orbits, respectively, is

$$r_a = r_s \text{ (Inner CEP)}, \qquad 4.1$$

$$r_p = r_s \text{ (Outer CEP)}. \qquad 4.2$$

The subscripts refer to the apogee and the perigee of the transfer orbit. Basic orbital mechanics states that the velocity at apogee is less than the circular velocity at that point and similarly the velocity at perigee is greater than circular velocity. Therefore, if the CEP orbit velocity at the cotangent point is the vector ($\vec{V}_T$), then its magnitude compared to the magnitude of the circular velocity vector at this point (V_s) will determine the type of CEP transfer. This will allow the analysis of both types simultaneously.

The use of impulsive thrust permits an analysis of these velocities from the first velocity increment vector ΔV_1. This vector is chosen such that a desired $\vec{V}_T$ will result. The magnitude of the resultant $\vec{V}_T$ will determine the phase shift properties of the CEP transfer. And the direction of $\vec{V}_T$ will determine the inclination change of the transfer. The vector diagram illustrated in Figure 7 is used to compute $\vec{V}_T$ from the first velocity increment (ΔV_1). The vector equation is

$$\vec{V}_T = \vec{V}_{s(I)} + \Delta\vec{V}_1, \qquad 4.3$$

where $\vec{V}_{s(I)}$ is the synchronous velocity vector of the initial orbit of the tug. Using the unit vectors this becomes

$$\vec{V}_T = (V_s + \Delta V_1 \cos \beta_1)\, \vec{j} + \Delta V_1 \sin \beta_1 \vec{k}, \qquad 4.4$$

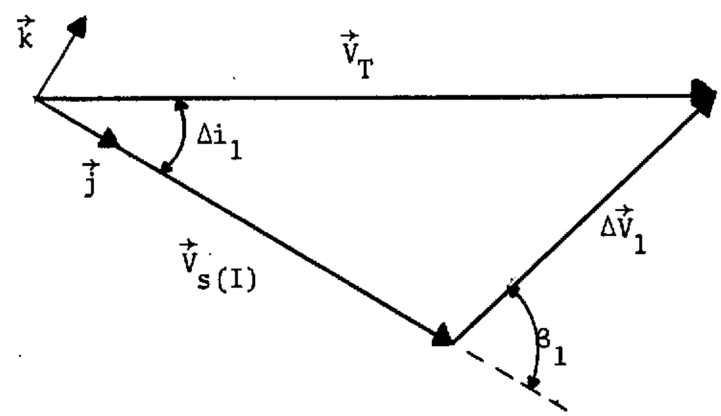

Figure 7. Vector Diagram $\vec{V}_T = \vec{V}_s + \Delta\vec{V}_1$.

where β_1 is the angle out of the initial orbit plane at which ΔV_1 is directed. This equation is consistent with the constraints listed above since there is no radial component of velocity.

The magnitude of $\vec{V_T}$ can be found by taking the dot product of Equation 4.4 with itself,

$$V_T^2 = \vec{V_T} \cdot \vec{V_T}$$

$$= V_s^2 - 2\Delta V_1 V_s \cos\beta_1 + \Delta V_1^2 \cos^2\beta_1$$

$$+ \Delta V_1^2 \sin^2 \beta_1 \quad . \qquad 4.5$$

Using the trigonometric identity

$$\cos^2\beta + \sin^2\beta = 1,$$

and rearranging yields

$$\Delta V_1^2 + 2\Delta V_1 V_s \cos\beta_1 + V_s^2 - V_T^2 = 0 \ . \qquad 4.6$$

Since non-coplanar cases are being considered, the inclination of the transfer orbit must be determined. The simplest and least costly plane change maneuver is done at the intersection of the two planes. Therefore the CEP transfer orbit should intersect both the initial tug orbit and the target orbit along the same line, which is called the line of intersection. The actual inclination of the transfer orbit with the equatorial plane will depend on the location of this line. The primary interest here is the change in inclination (Δi) caused by the first velocity increment as shown in Figure 7.

From geometry, the normal components of the vectors $\vec{V_T}$ and $\Delta\vec{V_1}$ must be equal. In equation form this can be expressed as

$$\Delta V_1 \sin\beta_1 - V_T \sin \Delta i_1 = 0. \qquad 4.7$$

At this point the CEP transfer has been initiated by the first velocity increment (ΔV_1). If more than two velocity increments are used, the intermediate velocity increments must take place at the cotangent point along the line of intersection by definition of the CEP transfer orbit. This means that the multiple-impulse case can be reduced to the two-impulse case where $\Delta\vec{V}_1$ becomes the vector sum of all velocity increments except the last one.

The second (or last) velocity increment ($\Delta\vec{V}_2$) must return the tug to a synchronous orbit matching that of the target. This involves a reversal of the transverse component of the first velocity increment(s) and the plane change needed between the transfer orbit and the target orbit. The vector equation, as illustrated in Figure 8, is written

$$\vec{V}_{s(F)} = \vec{V}_T + \Delta\vec{V}_2, \qquad 4.8$$

where $\vec{V}_{s(F)}$ is the synchronous velocity vector of the target orbit. Using the unit vectors this becomes

$$\vec{V}_{s(F)} = (V_T + \Delta V_2 \cos\beta_2)\vec{j}$$

$$+ \Delta V_2 \sin\beta_2 \vec{k}. \qquad 4.9$$

The conservation of energy principle insures that $\vec{V}_T$ remains the same each time the tug passes the cotangent point. Thus, it is the same vector defined in Equation 4.4, but the local coordinates have rotated with the tug by the angle Δi_1.

By taking the dot product of Equation 4.9 and simplifying as before, the following equation can be written:

$$\Delta V_2^{\,2} + 2\Delta V_2 V_T \cos\beta_2 + V_T^{\,2} - V_s^{\,2} = 0. \qquad 4.10$$

Also as before, the normal components of $\vec{V}_{s(F)}$ and $\Delta\vec{V}_2$ are equal by the geometry of Figure 8. Therefore, another equation can be written

$$\Delta V_2 \sin\beta_2 - V_s \sin\Delta i_2 = 0. \qquad 4.11$$

Thus far only the velocity vector $\vec{V}_T$ is known for the transfer orbit. In order to calculate the phase shift properties of the transfer, the period of the orbit must be known. The energy equation for the transfer orbit must be used to derive the equation for the transfer orbital period from the given conditions. Expressed in terms of the unit mass of the tug this becomes

$$E_T = \frac{V_T^{\,2}}{2} - \frac{\mu}{r_s}, \qquad 4.12$$

where E_T is the orbital energy of the transfer orbit and μ equals GM, the universal gravitational constant times the mass of the earth. This energy is constant for each position on the orbit. By realizing that

$$\frac{\mu}{r_s} = V_s^{\,2}, \qquad 4.13$$

Equation 4.12 can be written as

$$E_T = \frac{1}{2}\left(V_T^{\,2} - 2V_s^{\,2}\right) \quad . \qquad 4.14$$

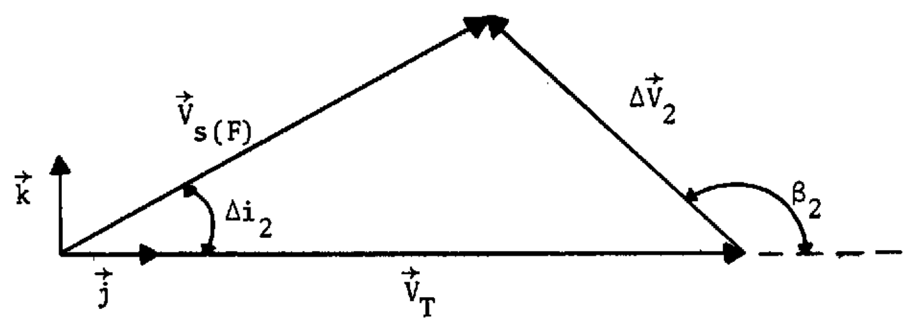

Figure 8. Vector Diagram $\vec{V}_s = \vec{V}_T - \Delta\vec{V}_2$.

Note: Unit vectors have rotated with the tug from Figure 7 by the angle Δi_1.

The orbital energy can also be expressed in terms of the semi-major axis (a) of the orbit

$$E_T = \frac{\mu}{2a} \qquad 4.15$$

Equating Equations 4.14 and 4.15 and solving for a yields

$$a = \frac{\mu}{(2V_s^2 - V_T^2)} . \qquad 4.16$$

The period of a closed orbit is found from

$$\tau = 2\pi \sqrt{a^3/\mu} \quad . \qquad 4.17$$

Substitution of Equation 4.16 into this expression will give the orbital period (τ_T) of the CEP transfer,

$$\tau_T = 2\pi\mu \ (2V_s^2 - V_T^2)^{-\frac{3}{2}}. \qquad 4.18$$

This orbital period will determine the amount of phase shift which can be accomplished on each revolution of the transfer orbit.

4.3 Rendezvous Equations

Rendezvous of the tug with a target involves two steps: the longitudinal phasing through the angle ψ and the inclination change Δi. By using the previously defined phase angle (ϕ), the first step can be expressed as

$$\psi = n\phi \qquad 4.19$$

where n is a positive integer representing the number of revolutions the tug will remain in the transfer orbit. The second step is the total plane change from both velocity increments. Using subscripts to distinguish the two increments, this second step is

$$\Delta i = \Delta i_1 + \Delta i_2 \ . \qquad 4.20$$

No specification has been made as yet for the fraction of the inclination change from each increment. To make optimum use of the velocity increments the two steps are accomplished simultaneously.

The phase angle was defined as that angle which a synchronous satellite will subtend in the time difference between the periods of the CEP transfer orbit and the synchronous orbit. This can be written in equation form as

$$\phi = 2\pi \frac{(\tau_T - \tau_s)}{\tau_s} = 2\pi \left(\frac{\tau_T}{\tau_s} - 1\right) \qquad 4.21$$

where ϕ is in radians and τ_s is the synchronous orbital period. Since the semi-major axis of the synchronous orbit is the radius r_s, Equations 4.13 and 4.17 can be used to express τ_s as

$$\tau_s = \frac{2\pi r_s}{V_s} \qquad 4.22$$

Now by using τ_T from Equation 4.18 the phase angle can be expressed as a function of the transfer velocity at the cotangent V_T,

$$\phi = 2\pi \left[\frac{2\pi\mu(2V_s^2 - V_T^2)^{-\frac{3}{2}}}{2\pi \ r_s/V_s} - 1\right] \ . \qquad 4.23$$

This can be simplified by using Equation 4.13 and substituting this into Equation 4.19 to yield

$$\psi = 2n\pi \left| \frac{V_s^{\ 3}}{(2V_s^{\ 2} - V_T^{\ 2})^{\frac{3}{2}}} - 1 \right| . \qquad 4.24$$

The time necessary to accomplish the transfer portion of the rendezvous, i.e., the time between the velocity increments, is found from n the number of revolutions and τ_T the orbital period of the transfer from Equation 4.18

$$t = 2n\pi\mu \ (2V_s^{\ 2} - V_T^{\ 2})^{\frac{3}{2}}. \qquad 4.25$$

There may be a waiting period in the initial tug orbit before the transfer is initiated so that the tug will be at the line of intersection.

4.4 Terminal Rendezvous

The analysis so far has been concerned with rendezvous maneuvers which cancel the phase angle and inclination angle differences between the two vehicles, the tug and the target. The assumption that the initial and final synchronous orbits are circular was used in this analysis. If a maneuver as discussed in successfully completed, the tug will be positioned on the same radial line with the target. However, the radial distance of the tug will be the same as it was in its initial orbit at that phase position. This means that any radial differences between the initial and final orbits, either from the eccentricities or the semi-major axes, will result in a radial distance between the tug and the target after the maneuver.

The terminal rendezvous phase must eliminate these radial discrepancies. A convenient analysis of the terminal rendezvous can be done using Hill's equations of relative motion. Hill's equations are written in a local coordinate system fixed to the target. The x direction is measured radially, the y direction is measured tangentially in the orbit plane, and the z direction is normal to the orbit plane. These equations for a force-free case are

$$\ddot{x} - 2n\dot{y} - 3n^2x = 0$$
$$\ddot{y} + 2n\dot{x} = 0 \qquad 4.26$$
$$\ddot{z} + n^2z = 0$$

where n is the mean motion of the target in its orbit,

$$n = \sqrt{\mu/r_s^{\,3}} \qquad 4.27$$

A closed form solution can be obtained for these equations in terms of the initial conditions x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, and $\dot{z}_o$:

$$x(t) = \frac{\dot{x}_o}{n}\sin(nt) - (\frac{2\dot{y}_o}{n} + 3x_o)\cos(nt) + (\frac{2\dot{y}_o}{n} - 4x_o)$$

$$y(t) = \frac{2\dot{x}_o}{n}\cos(nt) + (\frac{4\dot{y}_o}{n} - 6x_o)\sin(nt) + (y_o - \frac{2\dot{x}_o}{n})$$
$$- (3\dot{y}_o + 6nx_o)t$$

$$z(t) = z_o\cos(nt) - \frac{\dot{z}_o}{n}\sin(nt). \qquad 4.28$$

The initial position components that result from the previous rendezvous maneuver are

$$x_o = \Delta r$$
$$y_o = 0$$
$$z_o = 0\ . \qquad 4.29$$

Since the out-of-plane z-component is uncoupled and the initial position is in the orbit plane, the velocity $\dot{z}_o$ must be zero. The x- and y-components are coupled for the in-plane transfer motion. The values of $\dot{x}_o$ and $\dot{y}_o$ must be such that x and y simultaneously approach zero at some reasonable time. These values can be easily found from Equations 4.28 and 4.29

$$\dot{x}_o = \frac{-nx_o}{\sin nt}\left[\frac{4(1 - \cos nt)\ (15 - 13 \cos nt) - 3nt \sin nt\ (8 - 7 \cos nt)}{8(1 - \cos nt) - 3nt \sin nt}\right]$$

$$\dot{y}_o = 2nx_o\left[\frac{7(1 - \cos nt) - 3nt \sin nt}{8(1 - \cos nt) - 3nt \sin nt}\right] . \qquad 4.30$$

These equations will give the initial velocity components necessary to transfer the tug through the radial distance x_o in the time t. For example, the initial velocity components must be

$$\dot{x}_o = - 4.425 \text{ m/sec.}$$
$$\dot{y}_o = 3.707 \text{ m/sec.}$$

if the radial separation between the tug and the target is 50 km. and the terminal rendezvous is to take one hour.

V. DISCUSSION OF RESULTS

5.1 Numerical Results

The rendezvous maneuver using a CEP transfer has been found in the form of a set of non-linear equations. This set of equations and their variables are summarized in Table 2. The two variables ψ and Δi are known from the initial conditions of the tug and the specific target involved. The variables n and t are limited by the maximum time available for the maneuver; n is also limited to positive integers. The other variables listed must be resolved from the given set of equations.

The variables n and t are determined primarily from other mission constraints. They must be included in this study to keep the results within a realistic context. The minimization of fuel will normally mean using the maximum time available; but where the fuel minimum can be achieved with different transfer times, the shortest of these times is desired. This thesis uses seven days as a maximum time for each maneuver.

The set of non-linear equations can now be solved by use of a digital computer. The IBM 370/Model 168 system was used in this case. The first attempt at a solution employed an iteration of the variable Δi. For each iteration Δi_1 assumed a different fractional value of the total Δi. The results of this program, which are summarized in Table 3, proved that a minimum total velocity increment is achieved when $\Delta i_1 = \Delta i_2 = \frac{1}{2} \Delta i$ and $\Delta V_1 = \Delta V_2$. This proved to be a general case for all initial conditions.

TABLE 2

Non-linear Equations and Variables

	Equations
4.6	$\Delta V_1^{\,2} + 2\Delta V_1 V_s \cos\beta_1 + V_s^{\,2} - V_T^{\,2} = 0$
4.7	$\Delta V_1 \sin\beta_1 - V_T \sin\Delta i_1 = 0$
4.10	$\Delta V_2^{\,2} + 2\Delta V_2 V_T \cos\beta_2 + V_T^{\,2} - V_s^{\,2} = 0$
4.11	$\Delta V_2 \sin\beta_2 - V_s \sin\Delta i_2 = 0$
4.20	$\Delta i = \Delta i_1 + \Delta i_2$
4.24	$\psi = 2n\pi \left[\dfrac{V_s^{\,3}}{(2V_s^{\,2} - V_T^{\,2})^{\frac{3}{2}}} - 1 \right]$
4.25	$t = 2n\pi\mu\,(2V_s^{\,2} - V_T^{\,2})^{\frac{3}{2}}$

Variables	
ΔV_1 - unknown	Δi_2 - unknown
ΔV_2 - unknown	Δi - known
β_1 - unknown	ψ - known
β_2 - unknown	n - constraint
V_T - unknown	t - constraint
Δi_1 - unknown	

TABLE 3

Results from Iteration of $\Delta i_1/\Delta i$

Δi_1 DEG	Δi_2 DEG	β_1 DEG	β_2 DEG	ΔV_1 M/SEC	ΔV_2 M/SEC	DVT M/SEC
0.00	5.00	0.000	-79.781	36.607	272.297	308.904
0.25	4.75	20.362	-79.505	39.016	258.942	297.958
0.50	4.50	36.653	-79.185	45.482	245.601	291.082
0.75	4.25	48.257	-78.813	54.582	232.274	286.856
1.00	4.00	56.359	-78.382	65.223	218.966	284.189
1.25	3.75	62.146	-77.878	76.767	205.681	282.448
1.50	3.50	66.424	-77.288	88.861	192.422	281.284
1.75	3.25	69.695	-76.592	101.310	179.197	280.507
2.00	3.00	72.272	-75.766	113.996	166.014	280.010
2.25	2.75	74.355	-74.775	126.849	152.884	279.733
2.50	2.50	76.076	-73.576	139.882	139.822	279.644
2.75	2.25	77.525	-72.105	152.884	126.849	279.733
3.00	2.00	78.766	-70.272	166.014	113.997	280.011
3.25	1.75	79.842	-67.945	179.197	101.310	280.508
3.50	1.50	80.788	-64.924	192.422	88.862	281.284
3.75	1.25	81.628	-60.896	205.681	76.767	282.448
4.00	1.00	82.382	-55.358	218.966	65.224	284.190
4.25	0.75	83.063	-47.506	232.274	54.583	186.857
4.50	0.50	83.685	-36.152	245.601	45.482	291.083
4.75	0.25	84.255	-20.112	258.942	39.016	297.958

Initial Conditions

Δi = 5.0 DEG n = 3

ψ = 40.0 DEG V_T = 3111.3 M/SEC

Figure 9 illustrates the vectors for the minimum total velocity increment condition. It is important to note the distinction between the total velocity increment and the total velocity increment vector ($\Delta\vec{V}_s$). The total velocity increment, denoted by DVT, is the scalar addition of two scalars

$$DVT = \Delta V_1 + \Delta V_2, \qquad 5.1$$

while the vector ($\Delta\vec{V}_s$) involves the vector addition

$$\Delta\vec{V}_s = \Delta\vec{V}_1 + \Delta\vec{V}_2. \qquad 5.2$$

The optimization implicated in this thesis is the minimization of DVT. The magnitude and direction of $\Delta\vec{V}_s$ are functions of only the inclination change.

The vectors $\Delta\vec{V}_s$ and $\vec{V}_T$ can be shown to be perpendicular in Figure 9 from simple geometry. Also by comparing similar triangles the angle γ is found

$$\gamma = 180^o - \beta_1 - \Delta i_1. \qquad 5.3$$

The perpendicular components of the velocity increments can now be shown to be functions of phase and inclination independently,

$$\Delta V_1 \cos\gamma = \Delta V_2 \cos\beta_2 = f(\phi) \qquad 5.4$$

$$\Delta V_1 \sin\gamma = \Delta V_2 \sin\beta_2 = f(\Delta i). \qquad 5.5$$

The general results of DVT for various values of the initial conditions are given in Figures 10 and 11. Figure 10 shows the variations of the DVT versus phase angle curves for one degree changes in Δi. The curves are shown to flatten as Δi is increased. Figure 11 shows the variations of the DVT versus inclination curves for ten degree changes in phase angle. These curves also tend to flatten as ϕ is increased. This flattening is a result of accomplishing both steps

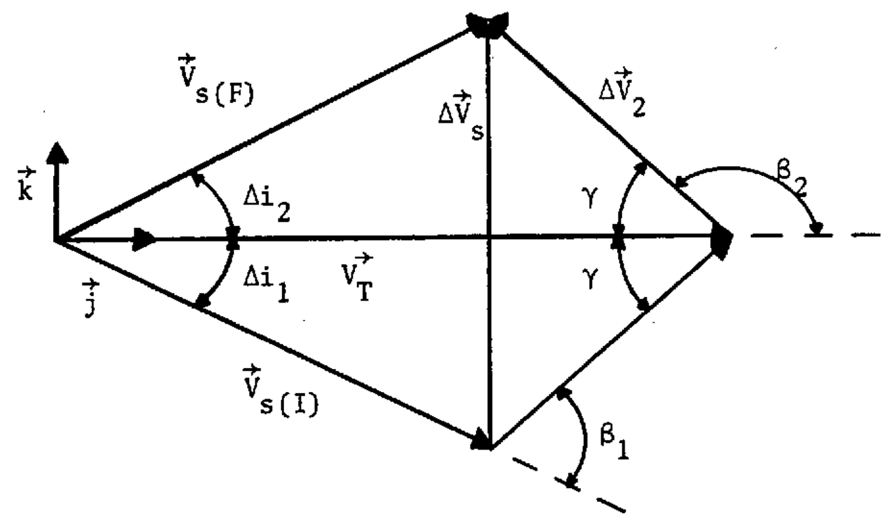

Figure 9. Vector Diagrams with $\Delta i_1 = \Delta i_2$.

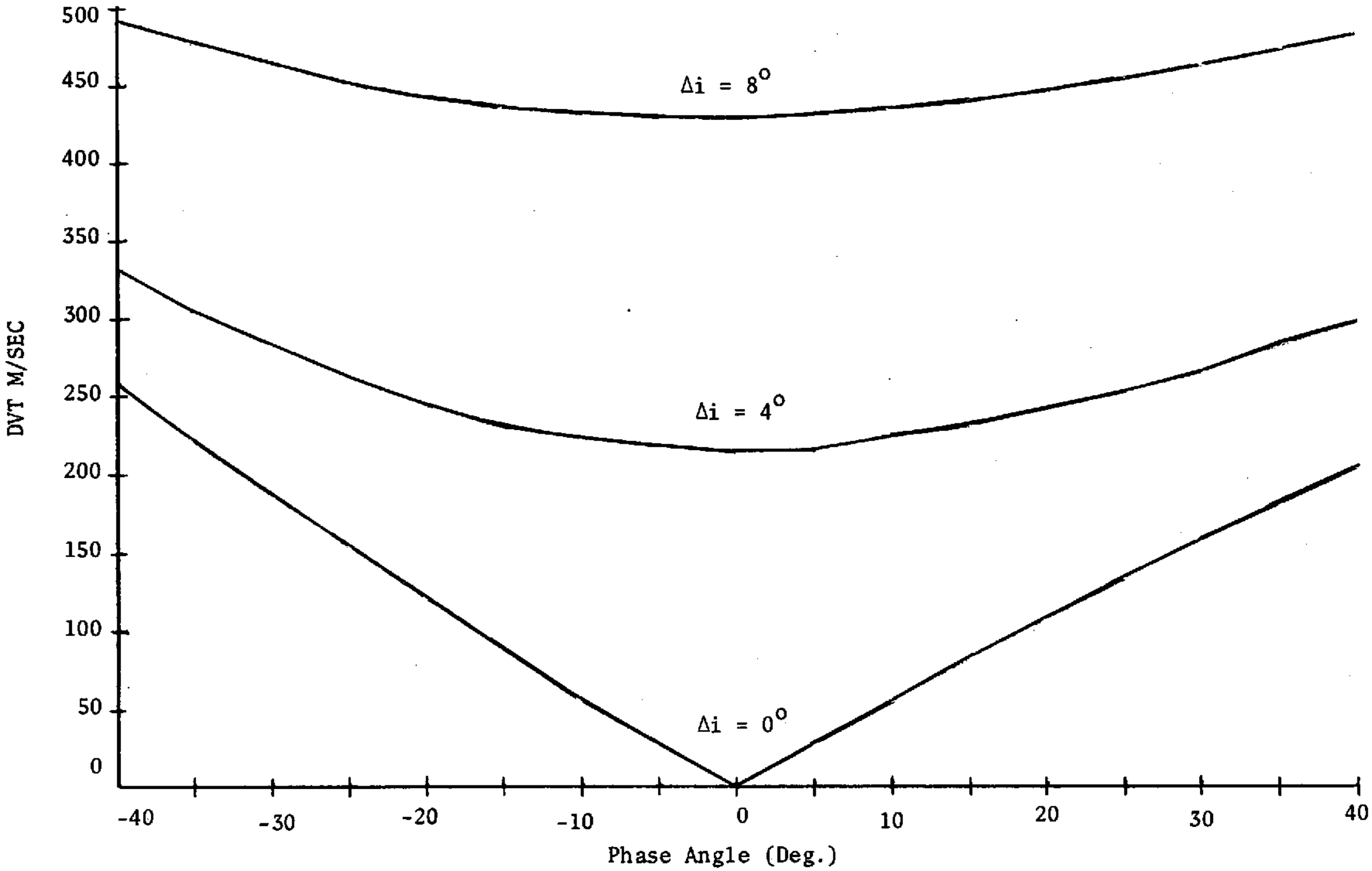

Figure 10. DVT vs. Phase Angle.

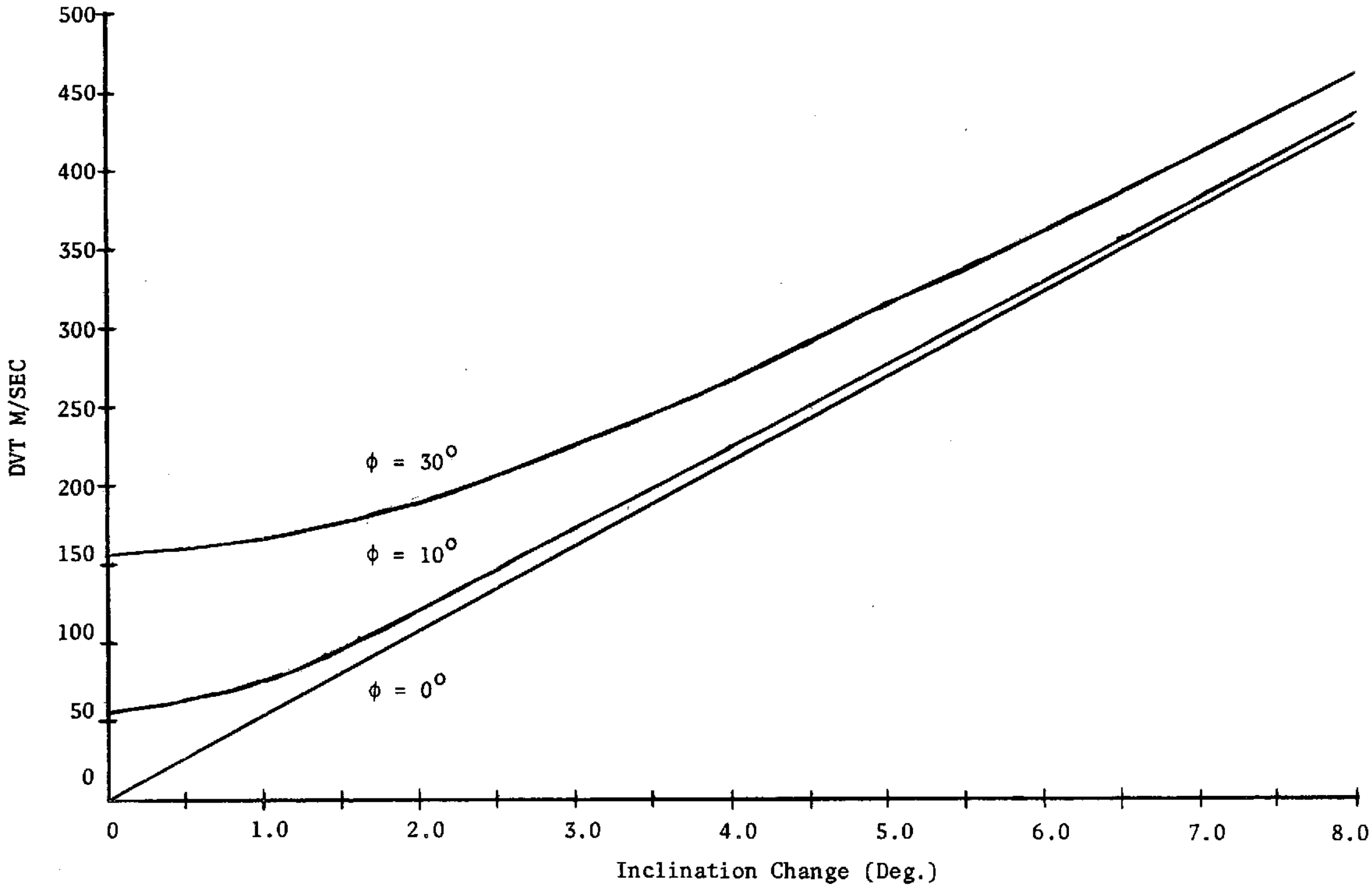

Figure 11. DVT vs. Inclination Change.

simultaneously. A composite graph of the results of Figures 10 and 11 is given in Figure 12, which has inclination changes versus phase angles for constant values of DVT. This graph can be used to select the value of DVT necessary for a rendezvous maneuver at specific values of the initial conditions. A singularity region exists where the phase angle is very small and the inclination change is large. This region results because the best inclination change condition yields a phase shift, i.e., a velocity increment perpendicular to the orbit plane will yield a resultant $\vec{V}_T$ greater than initial velocity. This means a zero phase shift requires a velocity increment directed at more than 90 degrees from the orbit plane in the direction of motion, the result is a less effective plane change.

5.2 Sequencing Considerations

To offset the initial launch to synchronous orbit expenses, a mission of this type must service or retrieve several satellites. The total velocity increment for the complete mission will be significantly affected by the sequence of the rendezvous maneuvers. The optimum sequence would be one where the tug would start at a particular target, rendezvous with the next target that requires the least fuel for the maneuver, and so forth without selecting the same target more than once. This task would not be difficult if only one degree of freedom were involved; it would then be analogous to finding the shortest distance between several pairs of points. But the problem being studied has two degrees of freedom, inclination and longitude, and the optimum rendezvous maneuvers involve non-linear relationships between the two.

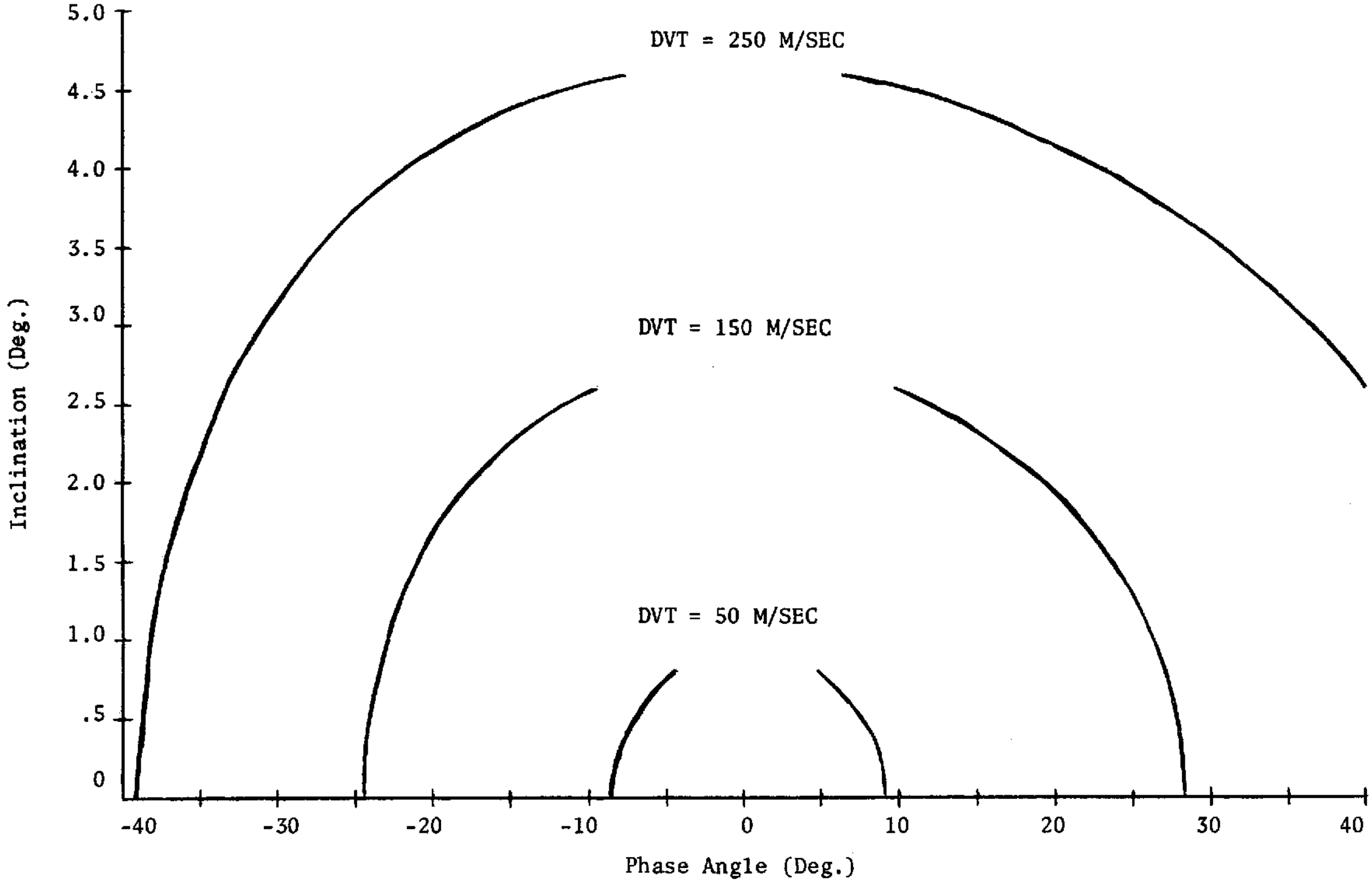

Figure 12. Phase Angle vs. Inclination Change.

The non-linearity of the problem requires that trial and error methods be used to select an optimum sequence. A computer program was written to calculate the optimum rendezvous between each pair of the given targets listed in Table 1. The calculations were performed using the results discussed in the previous section. Each maneuver was then compared to the others and arranged in order of the total velocity increments required. This was done with each target selected as the starting point. Therefore, if the tug was in the position of a given target, the rendezvous maneuvers with each of the other targets were arranged in the sequential order of their total velocity increments. The target with the lowest rank, that requiring the least total velocity increment for rendezvous, was selected next and a new sequencial order was used to continue the process until all targets were selected.

The results of this process are shown in Table 4. The tug started at the position of target #2 for this case, which was found to be the optimum case. By starting at a different target the sum of the velocity increments for the complete mission varied by as much as 400 M/SEC. The case shown in Table 4 required a total velocity increment of 1215 M/SEC and a total time for the rendezvous maneuvers of approximately 124.5 days. Considering the scope of the mission, rendezvous with twenty synchronous satellites, these figures prove the feasibility of synchronous rendezvous.

TABLE 4

Results of Target Sequencing

Tug initially positioned at Target #2 Early Bird

Target	DVT M/SEC	ΔV_1 M/SEC	β_1 DEG	ΔV_2 M/SEC	β_2 DEG	V_T M/SEC	t HOURS
# 1 S-3[a]	32.67	16.33	19.3	16.33	-19.2	3090	145.8
3 ATS-1	116.28	58.14	-74.7	58.14	75.7	3059	165.1
6 I-2F4[b]	44.44	22.22	75.4	22.22	-75.0	3080	144.4
10 I-3F4	70.33	35.16	83.4	35.16	-82.7	3079	144.2
8 I-3F2	81.03	40.51	-11.3	40.51	11.5	3035	161.3
7 ATS-3	7.83	3.92	-43.2	3.92	43.2	3072	167.1
5 I-2F3	50.21	25.11	-73.7	25.11	74.1	3068	166.4
4 I-2F2	74.60	37.30	-34.7	37.30	35.1	3044	162.7
12 I-3F6	87.82	43.91	78.7	43.91	-77.9	3084	144.9
19 SMS-1	70.30	35.15	43.8	35.15	-43.4	3100	147.3
20 ATS-6	40.62	20.31	23.5	20.31	-23.4	3093	146.3
11 ATS-5	12.53	6.27	25.4	6.27	-25.4	3080	144.4
17 I-4F7	93.08	46.54	-47.9	46.54	48.6	3044	162.6
14 I-4F3	12.15	6.08	-61.9	6.08	62.0	3072	167.1
13 I-4F2	11.21	5.61	-73.1	5.61	73.2	3073	167.3
15 I-4F4	27.41	13.70	-35.8	13.70	36.0	3064	165.7
16 I-4F5	95.50	47.75	-3.2	47.75	3.2	3027	160.1
9 I-3F3	107.32	53.66	-88.6	53.66	89.6	3074	167.4
18 W-A[c]	179.42	89.71	-47.1	89.71	48.4	3014	158.2
TOTALS	1214.75						2988.2

[a] Syncom 3 [b] Intelsat 2F4 [c] Westar A

VI. CONCLUSIONS

This thesis endeavored to find techniques for multiple rendezvous in synchronous orbits. An optimization was defined as the minimization of the total velocity increment required by the maneuvers, thus the least propellant usage per mission requirements could be achieved. The rendezvous maneuvers were calculated for a given set of sample synchronous targets and a relative optimum sequence of these maneuvers was found. The type of transfer selected for the rendezvous maneuvers was a cotangential elliptic phasing transfer. This transfer is similar to a Hohmann transfer, but the angle subtended in the CEP transfer is some integer multiple of 360 degrees. The Hohmann transfer could not be used here because the orbits being transversed are of equal radii. The CEP transfer makes optimum use of the velocity increments by the cotangency condition. Although the cotangency condition is not optimum in general, it can be proven to be optimum in special cases of which the equal radii case is one. Another advantage of this transfer is the capability of remaining in the transfer orbit for more than one revolution. This results in a significant savings in velocity increment at the expense of transfer time.

The rendezvous maneuvers were developed for conditions that included plane changes as well as longitudinal phase shifting. The equations derived for these conditions were found to be non-linear if both plane change and phase shift were accomplished simultaneously. These equations were solved by numerical methods. The maneuver which accomplished one half the total plane change with each velocity increment was found to be optimum for all initial conditions.

The optimum rendezvous maneuvers between the sample targets were analyzed to find the best sequence of these maneuvers. Neither a definite pattern nor a general sequence could be found since the rendezvous equations were non-linear. All possible combinations of the samples were computed and the optimum sequence was established by choosing the maneuvers available which require the least total velocity increment. The selection of the starting point in the sequence proved to be an important factor in the analysis.

In summary, this thesis found that the feasibility of rendezvous in synchronous orbit is not beyond the scope of the technology of the next decade. Further study must be made in the areas of perturbation effects and the terminal rendezvous and docking with passive satellites. The possibilities of low-thrust maneuvers at these altitudes is another area requiring further study. It is also recommended that analytical optimization methods be applied to this type of problem in search of the absolute optimum conditions.

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