# NASA CONTRACTOR REPORT 

## NASA CR-120700

## A MONTE CARLO INVESTIGATION OF THRUST IN: BALANCE OF SOLID ROCKET MOTOR PAIRS

By Richard H. Sforzini, Winfred A. Foster, Jr., and Joseph S. Johnson Auburn University<br>Engineering Experiment Station<br>Auburn, AL 36830

November 1974

Prepared for

# NASA-GEORGE C. MARSHAL.I SPACE FLIGHT CENTER Marshall Space Flight Center, AL 35812 


17. KEY WORDS

Thrust imbalance
Grain design
Internal ballistics
Monte Carlo technique
Space Shuttle
Thrust reproducibility
Impulse reproducibility
19. SECURITY CLASSIF. (of this report)

UNCLASSIFIED

| 20. SECURITY CLASSIF. (of this page) UNCLASSIFIED | 21. NO. OF PAGES $149$ | 22. PRICE NTIS |
| :---: | :---: | :---: |

FINAL REPORT

A MONTE CARLO INVESTIGATION OF THRUST IMBALANCE OF SOLID ROCKET MOTOR PAIRS
by
Richard H. Sforzini Professor

Winfred A. Foster, Jr. Assistant Professor and

Joseph S. Johnson, Jr. Application EDP Programmer

Aerospace Engineering Department

Prepared under
Modification No. 12 to the Cooperative Agreement Dated October 6, 1969, between

George C. Marshall Space Flight Center National aeronautics and space administration


AUBURN UNIVERSITY

Engineering Experiment Station
Auburn University
Auburn, Alabama 36830
November 1974

## ACKNOWLEDGEMENTS

The authors express appreciation to personnel at the George C. Marshall Space Flight Center for their many useful suggestions which materially aided this investigation and in particular to B. W. Shackelford, Jr. (NASA Project Coordinator) who additionally provided data for analysis and helpful encouragement to this effort.

The participation of Edwin C. Word, Jr., Graduate Research Assistant in Aerospace Engineering, Auburn University, whose efforts in the early phases of the investigation provided some valuable insights for the ovality analysis, is also acknowledged. Finally, the authors express grateful appreciation to Mrs. Marjorie N. McGee who typed the manuscript.

## table of contents

Acknowledgements ..... 11
List of Figures ..... iv
List of Tables ..... viii
Nomenclature ..... ix
I. Introduction and Summary ..... 1
II. Analysis ..... 4
Internal Ballistics (Main Program). ..... 4
Internal Ballistics (Area Subroutine) ..... 7
Ovality and Malalignment (Oval Subroutine) ..... 7
Statistical Analysis ..... 10
III. Discussion of Input and Output ..... 16
IV. The Computer Program ..... 36
Input Data ..... 36
Program Listing ..... 44
V. Sample Study ..... 102
VI. Concluding Remarks ..... 118
References ..... 119
Appendix ..... 120

## LIST OF FIGURES

Figure II-1. Geometry for analysis of ovality and malalignment. ..... 8
Figure II-2. General flowchart of setup and input subroutines ..... 12
Figure II-3. Example of the two general types of CDFs ..... 13
Figure III-1. Basic motor dimensions ..... 21
Figure III-2. Standard star grain cross-section ..... 25
Figure III-3 Truncated (slotted tube) star grain cross-section ..... 26
Figure III-4 Wagon-wheel grain cross-section. Calculated variables are circled ..... 27
Figure IV-1. Schematic of data deck ..... 45
Figure IV-2. Block diagram of computer program ..... 50
Figure V-1. Thrust versus time for one pair of SRMs of sample case (CalComp plot reduced size) ..... 110
Figure V-2. Thrust during tailoff versus time for one pair of SRMs of sample case (CalComp plot-reduced size). . ..... 111
Figure V-3. Thrust inbalance versus time for one pair of SRMs of sample case (CalComp plot) ..... 112
Figure V-4. Impulse imbalance versus time for one pair of SRMs of sample case (CalComp plot) ..... 113
Figure V-5. Absolute impulse imbalance versus time for one pair of SRMs of sample case (CalComp plot). ..... 114
Figure V-6. Histogram of absolute thrust imbalance values for fifty motor pairs from the sample study ..... 117
Figure A-1. Thrust versus time for two SRMs with propellant density $\rho$ difference of $3 \%$ ..... 121
Figure A-2. Thrust versus time for two SRMs with burning rate coefficient, $a_{1}$ and $a_{2}$, difference of $3 \%$ ..... 122

## LIST OF FIGURES (CONT'D)

Figure A-3. Thrust versus time for two SRMs with burning rate exponent, $n_{1}$ and $n_{2}$, difference of $3 \%$. ..... 123
Figure A-4. Thrust versus time for two SRMs with characteristic velocity $C^{*}$ difference of $3 \%$ ..... 124
Figure A-5. Thrust versus time for two SRMs with ratio of specific heats $\gamma$ difference of $3 \%$ ..... 125
Figure A-6. Thrust versus time for two SRMs with nozzle exit diameter $D_{e}$ difference of $3 \%$ ..... 126
Figure A-7. . Thrust versus time for two SRNs with nozzle initialthroat diameter $D^{*}$ difference of $3 \%$. . . . . . . . 127
Figure A-8. Thrust versus time for two SRMs with nozzle exitdivergence angle $\alpha_{n}$ difference of $0.3^{\circ}(2.7 \%)$. . . 128
Figure A-9. Thrust versus time for two SRMs with aft end graintapered length $\mathrm{L}_{\mathrm{T}_{\mathrm{a}}}$ difference of $3 \%$. . . . . . 129
Figure A-10. Thrust versus time for two SRMs with aft end radial displacement of tapered grain $\mathrm{x}_{\mathrm{Ta}}$ difference of 0.3 inches (9.9\%) . . . . . . . . . . . . . . 130
Figure A-11. Thrust versus time for two SRMs with radial displacement of the main portion of the grain due to bore taper $z_{0}$ difference of 0.3 inches (12.4\%) :. . 131
Figure A-12. Thrust versus time for two SRMs with radial displacement of the main portion of the grain due to exterior taper difference $z_{c}$ of 0.3 inches (0 base value)132
Figure A-13. Thrust versus time for two SRMs with out-ofround conditions of the grain exterior at the aft end $\Delta R_{c n}$ of 0 and 0.5 inches ( $\alpha_{a n}=0$ ) . . . . 133
Figure A-14. Thrust versus time for two SRMs with out-of-round conditions of the grain exterior at the aft end $\Delta \mathrm{R}_{\mathrm{cn}}$ of 0 and 0.5 inches $\left(\alpha_{\mathrm{an}}=90^{\circ}\right)$. . . . . . 134
Figure A-15. Thrust versus time for two SRMs with out-of-round conditions of the grain bore at the aft end $\Delta \mathrm{R}_{\mathrm{gn}}$ of 0 and 0.5 inches . . . . . . . . . . . . . 135

## LIST OF FIGURES (CONT'D)

Figure A-16. Thrust versus time for two SRMs with out-of-round conditions of the grain bore at the head end $\Delta R_{g h}$ of 0 and 0.5 inches ..... 136
Figure A-17. Thrust versus time for two SRMs with eccentricities of grain exterior and interior at the aft end $e_{x n}$ of 0 and 0.5 inches ..... 137
Figure A-18. Thrust versus time for two SRMs with eccentricities of centers of grain exterior and interior at the head end $e_{x h}$ of 0 and -0.5 inches ..... 138
Figure A-19. Thrust versus time for two SRMs with eccentricities of centers of grain exterior and interior at the head end $e_{x h}$ of 0 and -0.5 inches ( $e_{x n}$ for Figs. A-18 and A-19 is 0 and +0.5 , respectively) ..... 139
Figure A-20. Thrust versus time for two SRMs with radial nozzle throat erosion rate $E_{r e f}$ difference of $20 \%$. . 140
Figure A-21. Thrust versus time for two SRMs with bulk propellant temperature $\mathrm{T}_{\mathrm{gr}}$ difference of $3^{\circ} \mathrm{F}$. ..... 141
Figure A-22. Thrust versus time for two SRMs with ignition delay $T_{i g r}$ difference of 0.4 sec ..... 142
Figure A-23. Thrust versus time for two SRMs with average grain outside diameter $D_{0}$ difference of 0.3 inches ( $0.2 \%$ ) ..... 143
Figure A-24. Thrust versus time for two SRMs with average grain inside diameter $D_{i}$ difference of 0.3 inches ( $0.5 \%$ ) . 144Figure A-25. Thrust versus time for two SRMs with angle ofaft burning surface $\theta_{G}$ difference of $3^{\circ}$(28.8\%)145
Tigure A-26. Thrust versus time for two SRMs with length of circular perforated grain $L_{G c i}$ difference of 3.0 inches ( $0.3 \%$ ) ..... 146
Figure A-27. Thrust versus time for two SRMs with length ofgrain $L_{G n i}$ associated with $\theta_{G}$ difference of $3 \%$147

## LIST OF FIGURES (CONT'D)

Figure A-28. Thrust versus time for two SRMs with length of star grain $\mathrm{L}_{\mathrm{Gsi}}$ difference of 3.0 inches ( $1.6 \%$ ) . . . 148

Figure A-29. Thrust versus time for two SRMs with radius of star grain fillet difference $f$ of 0.3 inches (2.4\%) . . . . . . . . . . . . . . . . . . . . 149

Figure A-30. Thrust versus time for two SRMs with average initial radius of star grain truncation $\mathrm{R}_{\mathrm{p}}$ difference of 0.3 inches ( $2.4 \%$ ) . . . . . . . . . 150

Figure A-31. Thrust versus time for two SRMs with average web thickness of truncated star grain $\tau_{s}$ of 0.3 inches (3.7\%) . . . . . . . . . . . . . . . . 151

## LIST OF TABLES

TABLE IV-1. EXAMPLE DATA SHEETS ..... 46
TABLE IV-2. MONTE CARLO PERFORMANCE ANALYSIS OF SRM PAIRS. . 51
TABLE V-1. FIXED INPUT VALUES OF SAMPLE CASE ..... 103
TABLE V-2. INPUT DATA FOR STATISTICAL VARIABLES OF SAMPLE CASE ..... 104
TABLE V-3. PRINTOUT OF STATISTICAL INPUT DATA FROM SAMPLE CASE ..... 107
TABLE V-4. PRINTOUT OF INPUT VARIABLES FOR ONE SRMFROM SAMPLE CASE108
TABLE V-5. PORTION OF PRINTOUT OF TRANSIENT VALUES FORONE SRM FROM SAMPLE CASE . . . . . . . . . . . . 109
TABLE V-6. OUTPUT VALUES FOR FINAL MOTOR PAIR AND STATISTICAL CHARACTERISTICS OF TWENTY-FIVE MOTOR PAIRS FROM SAMPLE STUDY . . . . . . . . . . . . . . . . . . 115
$\begin{array}{ll}\text { TABLE V-7. } & \text { SELECTED STATISTICAL CHARACTERISTICS OF FIFTY } \\ & \text { MOTOR PAIRS FROM THE SAMPLE STUDY • . . . . } 116\end{array}$

An asterisk before the symbols indicates an input variable. All input subscripted and non-subscripted variables are listed separately.

| Eng1ish <br> Symbol | Definition U | Units Used |
| :---: | :---: | :---: |
| ${ }^{\text {c }}$, ${ }^{\text {c }}$ c | Major and minor semiaxis, respectively, of grain exterior used in the ovality analysis. | in. |
| $*_{1}{ }_{1}{ }^{*}{ }_{2}$ | Propellant burning rate coefficient above and below the transition pressure, respectively. | in/sec-psi ${ }^{\text {n }}$ |
|  | Pressure sensitivity of $C *$ and $\gamma$, respectively. | - |
| ${ }^{*}{ }_{c *}{ }^{\text {r }}$ | Temperature sensitivity of C* | $1{ }^{\circ} \mathrm{F}$ |
| $\mathrm{a}_{\mathrm{g}}, \mathrm{b}_{\mathrm{g}}$ | Major and minor semiaxis of grain interior, respectively, used in ovality analysis. | in. |
| $A_{b n}, A_{b p}, A_{b s}$ | Total buming surface associated with nozzle end, port and slot surfaces, respectively. | in ${ }^{2}$ |
| $*_{\text {A }}^{\text {bnT }}$ | Burning surfaces (as a function of $y$ ) at nozzle end for tabular input. | in ${ }^{2}$ |
| ${ }^{*} \mathrm{AbpT}$ | Burning surfaces (as a function of $y$ ) of port sides for tabular input. | $\mathrm{in}^{2}$ |
| ${ }^{*} \mathrm{AbsT}^{\text {b }}$ | Burning surfaces (as a function of $y$ ) of slots for tabular input. | s in ${ }^{2}$ |
| $*^{\text {ph }}$, ${ }^{*} \mathrm{~A}_{\mathrm{pnT}}$ | Controlling port areas at head and aft ends, respectively (as a function of $y$ ) for purely tabular inputs. | $1 \mathrm{n}^{2}$ |
| $c_{v}$ | Coefficient of variation; i.e., the ratio of the standard deviation to the mean. | - |



## NOMENCLATURE (Continued)

| English <br> Symbol | Definition Un | Units Used |
| :---: | :---: | :---: |
| *Ieo | Integer designating option of ovality and malalignment calculations. | - |
| ${ }^{\text {I }}$ I | Number of steps in the integrations of perimeters in ovality analysis subroutine. | - |
| $*_{\mathrm{op}}$ | Integer designating type of program input. | - |
| ${ }^{* I_{\mathrm{po}}}$ | Integer designating options of plotting results and obtaining special outputs. | - |
| $\mathrm{I}_{\text {Sp }}, \mathrm{I}_{\mathrm{T}}$ | Specific and total impulse, respectively. lbf | f-sec/lbm, |
| $*_{\text {xi }}, I_{x f}$ | The initial and final seed numbers, respectively, for the random number generator. | - |
| K | Criterion value in Pearson's system for determining frequency curves. | - |
| $*_{1}, * \ell_{2}$ | The length of the two parallel sides of the first and second set of points, respectively, of a wagon-wheel grain. |  |
| $\ell_{f}$ | Distance between center line of motor and fillet center of standard star grain. | in. |
| $\ell_{s}$ | Length of sides of truncated star point excluding fillets. | in. |
| ${ }^{*}{ }_{\text {TP }}$ | Initial length of termination passages between centers of gravity of perimeters of bases. | n in. |
| *L | Total initial perforated grain length including gaps at slots. | ng in. |
| $\mathrm{L}_{\text {Gc }}$ * ${ }^{\text {L }}$ Gci | Instantaneous and initial total axial lengths, respectively, of circular perforated grain (not including gaps). | , in. |
| ${ }^{\text {L }} \mathrm{Gni}^{\text {a }}$ | Initial slant length of grain surface at the nozzle end. | in. |
| ${ }^{\text {L }} \mathrm{GSI}$ | Initial total length of star-shaped perforated grain. | d in. |


| English Symbol | Definition Un | Units Used |
| :---: | :---: | :---: |
| ${ }^{*} \mathrm{~L}_{\mathrm{Ta}}$ | Estimated length of grain at the aft end at start of first tailoff having an additional taper not represented by $z_{0}$ or $\theta_{G}$. | in. |
| n | Number of observations of a statistically distributed variable or burning rate exponent. |  |
| $*_{1}{ }_{1} *_{n}$ | Burning rate exponents above and below the transition pressure, respectively. |  |
| $*_{n}$ | Number of burning flat end surfaces of a star grain located at the extreme nozzle end of the chamber. | - |
| $*^{\prime} \mathrm{n}_{\mathrm{p}}$ | Number of star points. |  |
| ${ }^{\prime} n_{s}$ | Number of burning flat end surfaces of a star grain not located at the nozzle end of the chamber. | - |
| *N(j) | Integer designating whether or not a specific output plot is desired. | c |
| P | Pressure. | $1 \mathrm{bf} / \mathrm{in}^{2}$ |
| $P_{h}$ max | Maximum head end chamber pressure calculated by the program. | psia |
| ${ }^{*} \mathrm{P}_{\text {ref }}$ | Reference average nozzle stagnation pressure used in the nozzle throat erosion equation. | psia |
| $*_{\text {tran }}$ | Transition pressure at which the burning rate coefficient and exponent change. | e psia |
| ${ }^{*} \mathrm{op}$ | Integer designating grain arrangement. | - |
| $\mathbf{r}$ | Burning rate. | in/sec |
| $*_{\text {is }}$, *R ${ }_{\text {iws }}$, *R ${ }_{\text {iww }}$ | Initial inside radius of the propellant web of a standard star, truncated star, and wagon-wheel grain, respectively. | in. |
| $\mathrm{R}_{7}$ | Distance from center of curvature of a spherical end of circular perforated grain to the burning surface associated with $\theta_{G}$. | in. |
| $*_{\text {R }}$ | Outside radius of a star grain. | in. |
| $*^{\text {R }}$ OAl | The propellant oxidizer to aluminum weight ratio. |  |

## NOMENCLATURE (Continued)

| English <br> Symbol | Definition U | Units Used |
| :---: | :---: | :---: |
| ${ }_{*} \mathrm{R}_{\mathrm{P}}$ | Initial radius of truncated star points. | in. |
| *S | Number of burning flat slot sides of a circular perforated grain not including the nozzle end. | - |
| ${ }^{*} \mathrm{~S}_{\mathrm{op}}$ | Integer designating type of star grain. | - |
| t | Time | sec. |
| $t_{b}$ | Calculated total operating time of motor. | sec. |
| ${ }^{*} t_{b 1}$ | Estimated time at burnout. | sec. |
| ${ }^{*} \mathrm{t}_{\mathbf{i g r}}$ | Ignition delay at $60^{\circ} \mathrm{F}$ grain temperature. | sec. |
| ${ }^{*} t_{\max } q$ | Estimated time at which maximum dynamic pressure on the vehicle occurs. | sec. |
| $t_{t i}$ | Earlier time at which tailoff begins in a motor pair. | sec. |
| ${ }^{*} \mathrm{~T}_{\mathrm{gr}}$ | Bulk temperature of the propellant grain. | ${ }^{\circ} \mathrm{F}$ |
| ${ }^{*} V_{c i T}$ | Initial volume of chamber gases associated with tabular input. | in ${ }^{3}$ |
| $\mathrm{W}_{\mathrm{pl}}, \mathrm{W}_{\mathrm{p} 2}, \mathrm{~W}_{\mathrm{p}}$ | Total masses of propellant burned based on mass discharge rate, volume calculations and the arithmetic average of $W_{p 1}$ and $W_{p 2}$. | 1bm. |
| x | Value of general statistical variable. | Units vary |
| $x_{c}, y_{c}$ | Coordinates of the grain exterior used in the ovality analysis. | in. |
| $\mathrm{x}_{\mathrm{g}},{ }^{\prime}{ }_{\mathrm{g}}$ | Coordinates of the grain interior used in the ovality analysis. | in. |
| $*_{x_{\text {out }}}$ | Distance burned at which propellant breaks up. | . in. |
| ${ }^{*}{ }_{\text {T }}$ | Difference in web thicknesses at ends of $\mathrm{L}_{\mathrm{Ta}}$ 。 | in. |
| ${ }^{*}{ }_{\text {Tz }}$ | Difference between the initial circular perforated grain diameter and the nozzle end of $\mathrm{L}_{\mathrm{Gci}}$ and the nominal value of $\mathrm{D}_{\mathrm{i}}$ less $\mathrm{z}_{\mathrm{o}}$ and less twice $\mathrm{x}_{\mathrm{Ta}}$. | in. |
| y | Distance propellant has burned from initial surface. | in. |

## NOMENCLATURE (Continued)

| English <br> Symbol | Definition | Units Used |
| :---: | :---: | :---: |
| ${ }_{*} z_{0}, z$ | Initial and instantaneous differences, respectively, between web thicknesses due to grain bore taper at head and nozzle ends of controlling grain length, excluding any initial length associated with $\mathrm{L}_{\mathrm{Ta}}$ or $\theta_{\mathrm{G}}$. | in. |
| ${ }^{*}{ }_{c}$ | Initial difference between web thicknesses due to grain exterior taper at the head and aft ends of the controlling grain length. | in. |
| Greek <br> Symbol |  |  |
| $*_{\text {ah }}$ * $\alpha_{\text {an }}$ | The angular orientation of the ovality of the grain interior with respect to the grain exterior at the head and nozzle end of the grain, respectively. | degrees |
| $*_{1}, * \alpha_{2}$ | The angle between the slant sides of a wagonwheel grain point and the center line of the point for the first and second set of points, respectively. | degrees |
| ${ }^{*}{ }_{\text {eb }}$ | Erosive burning coefficient in the $\mathrm{in}^{2} \cdot 8_{-f t}$ Robillard-Lenoir rule. | $t^{0.8} / \mathrm{sec}^{1.8} 1 \mathrm{bf} f^{0.8}$ |
| ${ }^{*}{ }_{n}$ | Nozzle exit half angle. | degrees |
| * $\beta$ | Erosive burning pressure coefficient in the Robillard-Lenoir rule. | - |
| $\beta_{1},{ }^{\beta} 2$ | Ratio of the square of the third moment about the mean to the cube of the second moment and the ratio of the fourth moment to the square of the second moment, respectively, for a statistically distributed variable. | $\square$ |
| $\Delta P_{c} / \Delta y$ | Rate of change of chamber pressure with respect to distance burned. | $1 \mathrm{bf} / \mathrm{in}^{3}$ |
| * ( $\Delta \mathrm{P} / \Delta \mathrm{y})$ | Depressurization rate at which propellant is extinguished for computation control purposes | . $1 \mathrm{bf} / \mathrm{in}^{3}$ |
| $* \Delta \mathrm{R}_{\mathrm{ch}}, * \Delta \mathrm{R}_{\mathrm{cn}}$ | One half the difference between the maximum and minimum diameter of the grain exterior at the head and nozzle end reference planes, respectively. | in |

Greek

| Symbol | Definition | Units Used |
| :---: | :---: | :---: |
| $* \Delta R_{g h}, * \Delta R_{g n}$ | One half the difference between the maximum and minimum diameter of the grain bore at the head and nozzle end reference planes, respectively. | in. |
| $\Delta y, * \Delta y_{1}$ | Incremental distance burned and initial value of same, respectively. | in. |
| ${ }^{*} \zeta_{\mathrm{F}}$ | Thrust loss coefficient. |  |
| ${ }^{*} \theta_{\mathrm{cn}}{ }^{* *} \theta_{\mathrm{ch}}$ | Approximate acute angle bonded circular perforated grain makes with motor center line at the head and nozzle closures, respectively. Also referred to as the grain contraction angles. | degrees |
| ${ }^{*}{ }_{\mathbf{f}}$ | Angular location of fillet center of standard star from line of symmetry. | degrees |
| $\theta_{\text {fw }}$ | Angular location of fillet centers with respect to radial center line of wagonwheel grain points. | radians |
| ${ }^{*}{ }_{\mathbf{G}}$ | Angle burning surface element of circular grain makes with longitudinal axis of motor at the nozzle end of the chamber. | degrees |
| $*_{n}$ | Nozzle cant angle. | degrees |
| ${ }^{*} \theta_{p}$ | Angle of standard star point. | degrees |
| $\theta_{s}$ | Half angle of star grain sector. | radians |
| ${ }^{\text {a }}$ S 1 | Angular location of slot side of truncated star grain. | radians |
| ${ }^{*} \theta_{\text {TP }}$ | Acute angle between axis of thrust termination passage and motor axis. | degrees |
| $\lambda$ | Volumetric loading density; i.e., initial volume occupied by propellant divided by empty case volume. | - |
| $\mu_{2},{ }_{3},{ }^{\mu} 4$ | Second, third and fourth moments, respectively, of a statistically distributed variable about its mean. | Units vary |

## NOMENCLATURE (Continued)

| Greek Symbol | Definition | Units Used |
| :---: | :---: | :---: |
| * ( $\left.\Pi_{\mathrm{P}}\right)_{\mathrm{K}}$ | Temperature sensitivity coefficient of pressure at constant $K=A_{3} / A^{*}$. | $1{ }^{\circ} \mathrm{F}$ |
| ${ }^{*}{ }_{p}$ | Solid propellant density. | slugs/in ${ }^{3}$ |
| $\sigma$ | The standard deviation of a statistically distributed variable; i.e., the square root of the second moment about its mean value. | Units vary |
| $\sigma_{1}, \sigma_{2}$ | The square root of the second moment of a statistically distributed variable about zero $\left(\sigma_{1}\right)$ and $\sigma_{1} / \sqrt{2}$, respectively. | Units vary |
| ${ }^{\tau}$ | Thickness of propellant web at slot bottom of truncated star grain. | in. |
| ${ }^{*} \boldsymbol{T}_{\text {Teff }}$ | Estimated "effective" web thickness of termination port. | in. |
| ${ }^{*}{ }_{w}$ | Web thickness of main propellant grain. | in. |
| ${ }^{\boldsymbol{\tau}}$ ws | Web thickness of standard star (same as $\tau_{w}$ except for some combination grains). | in. |
| ${ }^{\tau}$ ww | Web thickness of wagon-wheel grain. | in. |

Subscripts

| $\mathbf{a}$ | Value, during web action time. |
| :--- | :--- |
| $\mathbf{c}$ | Case, grain exterior or chamber value. |
| $\mathbf{f}$ | Final. |
| $\mathbf{g , G}$ | Grain interior. |
| $\mathbf{h}$ | Head end of grain. |
| $\mathbf{i}$ | Initial. |
| $\mathbf{1 g}$ | Ignition. |
| $\mathbf{m a x}$ | Maximum value. |
| $\mathbf{m i n}$ | Minimum value. |
| $\mathbf{n}$ | Nozzle or nozzle end of grain. |
| $\mathbf{0}$ | Stagnation. |
| $\mathbf{q}$ | Dynamic pressure. |
| $\mathbf{t}$ | Value during tailoff. |

## Subscripts (Cont'd)

w

Superscripts
*
-
-

100k Value when thrust has decayed to $100,000 \mathrm{lbs}$.
Value at web time.

Choked throat value.
Arithmetic average value over $\Delta y$ or statistical mean.
Time rate of change.

## I. INTRODUCTION AND SUMMARY

This report presents the results of research performed at Auburn Unviersity during the period May 31 to November 30 , 1974, under modification No. 6 to the Cooperative Agreement, dated October 6, 1969, between NASA Marshall Space Flight Center and Auburn University. The principal objective of the research was to develop a technique for statistically investigating the thrust imbalance of a pair of solid rocket motors (SRMs) firing in parallel.

The study of thrust balance and imbalance is of particular interest with regard to application to the NASA Space Shuttle because two very large solid motors fire in parallel on the Shuttle. Although a similar arrangement was utilized in the Titan program with somewhat smaller motors, because of the differences in the configuration of the Titan and Shuttle, it is more imperative that the thrust on the solid motor booster stage be uniform to assure proper guidance and control for the overall vehicle.

Past analyses of SRM reproducibility have been concerned mainly with characteristics of an entire population of single motors rather than pairs. Usually a non-time varying parameter such as average thrust and total impulse has been of interest, the distributions of variables affecting the parameter are assumed normal in the statistical sense, and crosscorrelation effects are neglected. Reference 1 typifies such an analysis.

For the present investigation the Monte Carlo technique (Reference 2) was selected. Sets of the significant variables are selected on a probability basis using a random sampling technique and the imbalance calculated for a large number of motor pairs using a mathematical model of the internal ballistics. This method is not limited to normal distributions of the input variables. Raw data, histograms, or cumulative distribution functions may be used for any of forty or more controlling variables to specify the statistical input.

Errors arising from neglecting the cross-correlation of variables are minimized in this study by selecting for the most part completely independent variables. However, the analysis may be readily extended to account for the cross-correlations where they are shown to exist and when they are calculable. The imbalance is evaluated in terms of six timevarying parameters as well as eleven single valued ones which themselves are subject to statistical analysis.

Application of the Monte Carlo method, which requires evaluation of a large number of motor pairs, is made practical and economical through the utilization of the simplified computer program for the internal balIistics presented in References 3, 4, and 5. Familiarity of the reader with References 3 and 4 or alternatively with 5 , which is a consolidated
report on 3 and 4, is assumed. Although the simplified program makes use of numerous approximations, the effects of the vast majority of the variables affecting imbalance are represented. Also, the essence of the phenomena controlling the critical tailoff portion of the thrust trace are embodied in the program which has been upgraded during the current effort to refine the analysis and to incorporate additional input variables which may now be statistically distributed. It is fundamental to the approach that, in spite of bias in the performance calculated resulting from the approximations in the ballistic analysis, the bias will reflect equally in each motor of a pair. Thus the difference in performance calculations for a pair of SRMs will be of a higher degree of accuracy than the individual motor calculations assuming that the significant parameters causing the difference are incorporated into the model and their effects precisely determined.

Owing to the complexity of the problem it was not possible to include all variables or give precise representation of the influences of all those that are included. Nevertheless, it is felt that the model developed yields a good first estimate of the difference in performance between a pair of SRMs. Early in the study, the sensitivities of performance to the variation in the various input parameters was evaluated by changing each variable one at a time a small amount between a pair of SRMs and computing the performance of each motor. Partial results are given in the Appendix in the form of pairs of thrust-time traces obtained for each variable. These traces were obtained directly from the computer output using the CalComp plotter. Although only very minor performance differences were obtained in many cases, it was decided that for the Monte Carlo investigation all but a few of the variables would be statistically determined in order to minimize cumulative errors that might result from neglect of a large number of seemingly unimportant variables.

Illustrative of the comprehensiveness of the analysis is the capability which has been added to calculate as a computer program option the approximate effects of case and mandrel ovality, eccentricity, and malalignment. On the other hand, the effects of grain temperature gradient (as opposed to bulk temperature) variation between rocket motors are not within the capability of the program to evaluate. However, the ovality analysis would permit simulation of the approximate results of one special situation for temperature gradient from which insight into the effects of this important variable may be gleaned.

The Monte Carlo computer program retains the capability of the programs of References 3, 4, and 5 to treat segmented configurations with both star and circular perforated grains present in various arrangements in the same SRM as well as monolithic grains with either a circular perforation or one of the three most common types of star grains. However, the program is more accurate if most of the grain is circular perforated as in the Space Shuttle. Also, whereas tabular input may still be used to specify portions of the burning surface, the statistical input variables influence only the burning geometry that is computed from the program
equations. Basic assumptions of the program are that the propellant does not break up and is not extinguished except by being completely consumed by burning normal to the propellant surface.

Ignition transients are not calculated in the analysis. However, the variations in the initial equilibrium chamber pressures as calculated by assuming the grains are completely ignited have been incorporated along with statistically determined values of ignition delay. It is believed that this treatment embodies the significant effects of ignition upon the remainder of the trace.

Perhaps the greatest limitation of the analysis is its reliance on the availability of reliable statistical data to specify accurately the distributions of the numerous variables. For example, there simply have not been enough SRMs built in the size class of those in the Space Shuttle stage to provide direct statistical samples. For small motors, where large populations exist from which data may be directly applied or scaled to another SRM, documentation of manufacturing variations is often incomplete or unavailable. In the test cases used in this report, drawing tolerance limits were taken in a number of cases to represent the ranges corresponding to six standard deviations in assumed normally distributed dimensions. Hopefully, this research will provide a stimulus for acquisition of statistical data which will reduce reliance on assumptions of this nature. As suggested earlier and as is demonstrated in the sample case, the computer program is virtually unlimited in its ability to treat the various types of statistical distributions.

The program which is written in FORTRAN IV requires approximately 2,000 computer cards not including the data cards necessary to specify the input variables and their statistical distributions. The compilation time is approximately 1 minute and 10 seconds on the IBM $370 / 155$ computer. Performance computation time for 1 pair of SRMs using recommended increment sizes in the several integration processes involved is approximately 35 seconds using the FORTRAN H compiler.

Section II of this report discusses the changes made to the ballistic performance analysis program and also describes how the statistically distributed variables are treated and sets of random input variables selected. The basic mathematical details of the ovality and malalignment analysis are also given. A discussion of the program input and output is presented in Section III. Although most of the input variables are defined in References 3, 4, and 5, a complete discussion is given here for a ready reference and guide to the user in specification of his problem and the interpretation of the output. Section IV gives the information required to operate the program including data card format and the program listing which contains concise printed definitions of all input variables along with identification of the statistically determined variables to serve as a checklist. In Section V a sample study is presented to demonstrate the setup procedures, format and computational capabilities of the computer program.

## II. ANALYSIS

This section of the report describes the changes and additions that have been made to the ballistic analyses of References 3, 4, and 5, and presents the basic elements of the Monte Carlo analysis. Rationale to the selection of input variables is interspersed throughout the discussion. The section is divided into four parts: the first two parts give the changes to the "main" program and the "area" subroutine, the third part gives the ovality and malalignment analysis and the final part is the statistical presentation. It is assumed the reader is familiar with References 3, 4, and 5, and thus in discussing program reviaions only information sufficient to identify the changes and the bases for them is given. Complete documentation of the program logic and computational changes is given in the program listing in Section IV. Although a number of completely new subroutines have been incorporated into the program, the methods used are straightforward applications of basic statistics and analytical geometry. Therefore, again concise descriptions of the analyses and their capabilities and limitations are given in lieu of detailed mathematical procedures which can be readily identified from the program 1isting.

## Internal Ballistics (Main Program)

Inert parts and ignition. The computation of the weights of inert parts have been removed from the program since the program is now strictly a performance analysis rather than a design analysis. Also, the option of calculating the ignition transient by the methods of References 4 and 5 has been removed because the complications of the ignition transients make a detailed Monte Carlo impractical from a computer requirements standpoint. However, an ignition delay time has been incorporated as a statistical variable subject to both random variation and systematic variation due to the distribution of bulk temperature of the grain. Also, the initial equilibrium pressure is subject to variations arising from a number of sources. It is believed that this treatment of ignition embodies the significant effects of the ignition transient on the remainder of the thrust time trace.

Throat area. The throat relationship has been modified to account for Reynolds' number and Stanton's number effects of pressure and diameter on heat transfer (Ref. 6) as follows:

$$
\begin{align*}
& E_{n}=E_{n \text { ref }}\left(P_{\text {on }} / P_{\text {ref }}\right)^{0.8}\left(D_{\text {ref }} / D^{*}\right)^{0.2}  \tag{1}\\
& D^{*}=D_{1} *+2.0 \Sigma E_{n} \Delta t  \tag{2}\\
& A^{*}=\pi D^{*} / 4 \tag{3}
\end{align*}
$$

Mach number. To account for throat erosion effects, the Mach number at the nozzle exit from which the thrust coefficient is ultimately derived is now recalculated at each time step rather than just initially.

Characteristic velocity and ratio of specific heats. These variables, $C^{*}$ and $\gamma$, respectively, are calculated based on the oxidizer to aluminum ratio of the propellant $R_{0 A 2}$ which is introduced as a new independent distributed variable. In this way the cross-correlation between $C^{*}$ and $\gamma$ has been incorporated. Data for determining separate relationships for $C^{*}$ and $\gamma$ as a function of $\mathrm{R}_{\mathrm{OA} 2}$ should be obtained from thermochemical analyses. The present program uses typical functional relationships obtained from a linear regression analysis of such data. Similar relationships must be established to fit the propellant system under consideration. More rigorously, additional composition components might be included. However, because of the relatively strong influence of the aluminum on the propellant chemistry the two-component oxidizer to aluminum system should convey the largest effects of composition on $C^{*}$ and $\gamma$.

Systematic variations in C* and $\gamma$ have also been incorporated to give the effects of chamber pressure and propellant bulk temperature on C* and of chamber pressure on $\gamma$. The relationships are as follows:

$$
\begin{align*}
& C^{*}{ }_{r e f}=C^{*}{ }_{n} \exp \left[a_{C^{*} T}\left(T_{g r}-60\right)\right]  \tag{4}\\
& C^{*}=C^{*}{ }_{r e f}\left(P_{o n} / 1000.0\right)^{a} C^{*} p  \tag{5}\\
& \gamma=\gamma_{n}\left(P_{o n} / 1000.0\right)^{a} \gamma_{\gamma p} \tag{6}
\end{align*}
$$

The detailed attention given to these variables was motivated not only by the cross-correlation but also by the high sensitivity of performance to both $C^{*}$ and $\gamma$ as revealed by the sensitivity study discussed in Section III. The effect of $\gamma$ is largely due to its influence on exit Mach number and hence thrust coefficient. In the subsonic flow regime, Mach number is relatively insensitive to variation in $\gamma$ and it is found convenient to neglect the variations in $\gamma$ resulting from pressure variations in calculating the Mach number at the end of the grain.

The introduction of $\mathrm{R}_{\mathrm{OAl}}$ as an independent variable raises the question of possible cross-correlation with other independent propellant variables, especially the burning rate coefficient $a$ and density $\rho$. When such correlations can be identified they may of course be treated in a manner similar to that given for $C^{*}$ and $\gamma$. Although the matter has not been investigated fully as of this writing, it appears that factors such as oxidizer particle size distributions, amount of burning rate catalyst and other composition variables will play a more important role than the oxidizer to
aluminum ratio in fixing the distributions of burning rate coefficient and density; cross-correlation between RoAl and $\rho$ are assumed negligible as have, with somewhat less justification, the correlation between a and $\rho$, themselves. The latter warrants further investigation as it is intuitive$1 y$ clear that at least a weak cross-correlation exists between a and $\rho$ because of the effects of oxidizer particle size distribution on these parameters.

Buming rate coefficient. Bulk temperature has been introduced as an explicit variable subject to statistical variation. It modifies the burning rate coefficient:

$$
\begin{equation*}
a=a_{60^{\circ}} e \times p\left[(1-n)\left(\pi_{p}\right)_{k}\left(T_{g r}-60\right)\right] \tag{7}
\end{equation*}
$$

Also, the program now makes use of two sets of burning rate coefficients and exponents in recognition of the possibility of separate values applying above and below a specified transition pressure. In conjunction with this, the level above which burning of the propellant is permitted was changed from 30 psia to 5 psia as the thrust of some SRMs may still be significant at low pressure. Choked flow is still assumed at the lower pressure levels.

Time increments. Several changes were made in the calculation of time increments for the purpose of obtaining more accurate representation of the thrust-time characteristics of the individual SRM. First, the time increment is now calculated from the burning rate and incremental distance burned using the burning rate that applies during the increment under consideration rather than the previous increment. This is done except for the purpose of computing changes in port areas which must be known before the rate is calculated. The error in the latter case is minor and a time consuming iterative solution is avoided. Also, time values and corresponding output values are now obtained at the precise $y$ positions that the lengths of grain associated with $\mathrm{X}_{\mathrm{Ta}}$ and z begin to burnout. These times are identified in the program printout by "initial tailoff begins" and "final tailoff begins," respectively. It was found desirable in general to use a $y$ increment ( $\Delta y$ ) of approximately 0.001 of the web thickness. This gives good precision in the calculations and also sufficient values to obtain good graphical portrayals. However, the smaller increment size introduces a difficulty in satisfying the mass flow continuity relationship during rapid pressure changes; the effect of such changes tends to be overestimated in the simplified ballistic model when small increment sizes are used. In References 3, 4, and 5, this difficulty was avoided during tailoff by calculating the nozzle end stagnation pressure directly from the pressure gradient. In the new program, the same method is also applied to the period involving the burning of the length of grain associated with $\mathrm{X}_{\mathrm{Ta}}$ as similar difficulties were encountered in the region when small increment sizes were used.

## Internal Ballistics (Area Subroutine)

During the present study, an error was found in the first expression for Abnc in Eqs. A4c of References 4 and 5. Specifically the first value of $R_{3}$ in the equation should be replaced by

$$
\left[\left(D_{0} / 2\right)^{2}-R_{7}^{2}\right]^{1 / 2}+\left[\left(D_{0} / 2\right)^{2}-\left(R_{7}+y\right)^{2}\right]^{1 / 2}
$$

and $R_{3}$ should be replaced by $R_{7}$ throughout the remainder of Eqs. A4c. The program listing contains the necessary changes to the computer program. It will be noted that additional changes applicable to Eq. A4c have been made to further refine the closure geometry effects for a circular perforated grain. Also, application of $4 b$ and $c$ has been modified in the computer program to avoid the possibility of negative square roots in terms such as $\left[D_{0}^{2}-\left(D_{i}+2 y\right)^{2}\right]^{\frac{1}{2}}$.

## Ovality and Malalignment (Oval Subroutine)

Ovality and lack of concentricity of the grain perforation with respect to the motor case can clearly influence the ballistic performance of SRMs during the critical tailoff period. As a first approach to the problem it is assumed that the burning surface geometry embodying the effects of ovality and malalignment may be defined by specifying three radial reference planes - one near the head end of the rocket, one at the aft end of $L_{T a}$ and one at the aft end of $z_{0}$. The reference planes must intersect the cylindrical portion of the rocket motor case to eliminate the effect of end closures on the geonetric properties to be calculated. The implied assumption here is that the central portion of the rocket dominates the influences of ovality and malalignment.

The geometry of the reference planes is illustrated by Figure II-1. To fix ideas, both the exterior and interior (bore) surfaces of the grain are assumed to be distereced inte sual shaped surfaces from nominally circular ones. However, as disicussed later, with sone restrictions and loss in accuracy, the analysis is also applicable to grains with the star-shaped perforations treated in the program.

The exterior and interior grain perimeters are next. assumed to be elliptical and the interior grain surface to remain elliptical as burning progresses. The latter assumption is not rigorous for burning normal to the surface but the error introduced is insignificant for the small degrees of ovality to be encountered in practice. Thus, the ellipse defining the burning perimeter is given by Equation (8) where $y$ is the distance burned

$$
\begin{equation*}
x_{g}^{2} /\left(a_{g}+y\right)^{2}+y_{g}^{2} /\left(b_{g}+y\right)^{2}=1 \tag{8}
\end{equation*}
$$

-8-

Figure II-1. Geometry for analysis of ovality and malalignment.
and $\quad a_{g}$ and $b_{g}$ are semi-axes of the ellipse defining the initial burning grain perimeter. The exterior grain perimeter is expressed in the coordinates $x_{g}$ and $y_{g}$ of the interior perimeter by the equation

$$
\begin{equation*}
\left(x_{g} \cos \alpha-y_{g} \sin \alpha+e_{x}\right)^{2}+\left(a_{c}^{2} / b_{c}^{2}\right)\left(x_{g} \sin \alpha+y_{g} \cos \alpha+e_{y}\right)^{2}=a_{c}^{2} \tag{9}
\end{equation*}
$$

where $a_{c}$ and $b_{c}$ are the semi-axes of the ellipse defining the grain exterior. The burning perimeter at each reference plane is determined as follows. Equations (8) and (9) are rewritten in polar coordinates $r$ and $\theta$. The perimeter is determined through numerical integration of the equation

$$
\begin{equation*}
s \simeq \int_{0}^{2 \pi} r d \theta \tag{10}
\end{equation*}
$$

At each value of $\theta$ in the integration the radial coordinate of the grain bore and exterior is computed from (8) and (9), respectively. If the exterior value exceeds the bore value at the $\theta$ position, $r$ is given the value calculated for the bore; otherwise, a zero value is assigned.

Burning perimeters for each reference plane thus vary from one SRM to another as a result of eccentricity of the grain bore and exterior, which is specified by the independent variables $e_{x}$ and $e_{y}$, orientation of the ovality of the bore with respect to the exterior, and the out-of-roundnesses of both the bore and exterior. It is possible that some correlation may exist between these variables in particular cases. However, if the case is segmented, it would appear that the head end and aft end geometric features are independent and they are treated as such in the remainder of this report. On the other hand, in general, there should be a close correspondence between the exterior grain geometries and the locations of the bore centers at the aft ends of $\mathrm{L}_{\mathrm{Ta}}$ and $z_{0}$, and it has been assumed that these are identical. The geometry of the interior grain is determined by the distributed variables $D_{i}, z_{0}$, and $x_{T a}$; i.e., these determine the initial semi-axes and the exterior geometry is determined by $D_{0}$ and $z{ }_{c}$ where $z_{c}$ has been introduced to account fọr axial variations in the outside ${ }^{c} d i a-$ meter of the grain. It is clear that the independent specifications of $e_{x}$ and ey at both ends of the rocket provides for a statistically distributed malalignment of the grain bore and exterior.

From the burning perimeter values obtained as outlined above, correction factors are next calculated and applied to the standard calculations of perimeter to account for the ovality and malalignment. For this purpose when solution of the ovality equations indicates that burnthroughs have occurred at adjacent reference planes, the burning perimeters are assumed to vary linearly between the planes. When burnthrough has occurred at only one of two adjacent reference planes, the portion of the correction factor applicable to each end is weighted in proportion to the corresponding length that has or has not experienced burnthrough to determine the overall correction factors (computer symbols: SEN, SEH, CHINAV, CHIN and CHIH).

It is recognized that the approach represents considerable idealization of the general behavior to be expected, particularly with respect to the assumed qualities of the axial distributions of the parameters between reference planes. Nevertheless, the model seems to capture the essence of the performance effects associated with ovality and malalignment. Even with highly nonlinearly distributed ovality and eccentricities between reference planes the effects on performance should be roughly the same as determined here. Also, factors such as the precise shape of the ovals would appear to play a secondary role in influencing performance variations.

Care has been taken in the above discussion to differentiate between the grain exterior and the case interior. As far as the equations go the pertinent item is the grain exterior. Its shape, however, may be influenced by the case and it is a choice of the user as to whether or not the statistical variations of ovality of the case alone should determine the variation in the grain exterior or if variation in insulation thickness should be statistically combined with the case variation to arrive at the qualities of the grain exterior.

As mentioned earlier, the analysis may also be applied to star grain configurations. In this case, the star points are in effect disregarded. The rationale for this is that by far the most important effects of malalignment and ovality occur just before and during tailoff and are dominated by the behavior of the remaining propellant web which may be approximated for this purpose by a circular perforated grain of the same web thickness. The capability to treat star grains has thus been incorporated into the computer program. However, it is applicable only when the entire grain is a star grain. When both circular perforated and star grains are present together, it is assumed that the circular perforated grains determine the performance characteristics in as far as the effects of ovality and malalignment are concerned.

An additional application for the ovality and malalignment is noteworthy. This has to do with the temperature gradient within the propellant grain which is an important variable that has not been taken directly into account in the analysis because no convenient way has yet been found to do so. Some insight into the gross effects of such gradients can be gleaned by statistically incorporating additional variations in the out-of-roundness of the grain to represent the general effects of having a biaxial burning rate which could be different in magnitude and orientation at the two main reference planes.

Statistical Analysis
This section describes the statistical analysis used in selecting values for the distributed variables. The basic computational methods as related to the logic of the statistical procedure used in the computer program are presented.

The computer routine is designed to accept several different types of data, perform the specified operations required to obtain a frequency dis-
tribution for each variable, and select a value based on its statistical frequency curve. The routine is divided into two subroutines. One subroutine, "setup," is called once and generates the required frequency curves for the statistical variables given. The other, "input," is called as required and provides specific values for each variable given. The latter subroutine utilizes the Monte Carlo technique for selection of values which will be discussed later. Figure II-2 is a general flow chart of the setup and input subroutines.

Frequency curves. The primary task of the setup subroutine is to obtain a frequency curve for each statistical variable from the data given. For each variable, the ultimate product of this subroutine, a cumulative distribution function (CDF), is obtained from its frequency curve. A CDF is a step function which jumps at regular intervals and is constant between jump points. At each jump point the magnitude of the jump is the probability that the variable will be within that interval; thus, as the name implies, the probabilities are accumulated over the range of the statistical variable. Examples of the types of CDFs produced by the setup subroutine are shown in Figure II-3. This CDF can be obtained in the present program from several types of input data, ranging from raw data points to specifying the actual CDF directly (See Section IV).

Basically, there are two classes of requests allowed for input statistical variables. The first class contains all variables which require little if any statistical analysis. This includes such requests as to hold certain variables constant or to obtain the CDF directly from a given histogram; as illustrated in Fig. II-3A. The other class requires statistical analysis to obtain the frequency curve. The analysis consists of obtaining the first four moments of the statistical variable and generating an equation which approximates the actual frequency curve for the data. The method selected for obtaining the frequency curve from the first four moments is known as Pearson's system (Ref. 7). The variables and sequence of calculations were chosen to parallel those of Pearson's system which is discussed in sufficient detail in Ref. 7 to permit direct adaptation to a computer program.

Basically, Pearson's system consists of a family of curves with a criterion value used to determine which equation of the family best describes the data. This criterion value, $K$, is evaluated as follows:

$$
\begin{equation*}
K=\beta_{1}\left(\beta_{2}+3\right)^{2} / 4\left(4 \beta_{2}-3 \beta_{1}\right)\left(2 \beta_{2}-3 \beta_{1}-6\right) \tag{11}
\end{equation*}
$$

where

$$
\begin{align*}
& \beta_{1}=\mu_{3}^{2} / \mu_{2}^{3} \\
& \beta_{2}=\mu_{4} / \mu_{2}^{2} \tag{13}
\end{align*}
$$

Here $\mu_{2}, \mu_{3}, \mu_{4}$ are the second, third and fourth moments of the variable about its mean, respectively.


$$
-13-
$$



The curve is determined to be one of the three main types as follows:


Transitional types exist, but are not incorporated in the present program.
The statistical analysis section of the subroutine initially obtains the first four moments. It then evaluates $K$ and determines the appropriate type of curve to use. From these moments and the total number of data points for each statistical variable, the appropriate parameters for the proper equation are evaluated. Thus the equation representing the frequency curve is established. This equation is evaluated over the specified range and using Simpson's rule to integrate the curve, the CDF is obtained for each statistical variable, as shown in Fig. II-6B. The CDF for each variable is then stored in an array.

Monte Carlo. When called, the Monte Carlo or input subroutine generates values for all statistical variables. These values are determined based on the frequency curve for each variable. The Monte Carlo subroutine performs this function by obtaining a random number and using the CDF array produced earlier.

The CDF consists of values from 0 . to 1 . with corresponding variable ( x ) values over the appropriate range. A random number (RANDU) with a value between 0 . and 1. is obtained from the random number generator discussed below. This value establishes the appropriate CDF value, and the corresponding $x$ value is then selected from the array to fix the proper value of the variable. Each cime the Monte Carlo routine is called, it determines new values for all the statistical variables; that is, it selects a new set of variables for each SRM whose performance is to be analyzed. This is accomplished as follows.

In the computer program, the CDF is stored in an array of 100 elements (a different number of elements may be specified by program changes). A second array ( $x$ ) also contains 100 values corresponding to the possible values which the statistical variable may assume. For any $x$ array value, the corresponding CDF array value is the accumulated area under the frequency curve to that point, expressed as a percentage of the total area:

$$
\begin{equation*}
\operatorname{CDF}(N)=\int_{x(1)}^{x(N)} f(x) d x / \int_{x(1)}^{x(100)} f(x) d x \tag{16}
\end{equation*}
$$

where $f(x)$ is the frequency distribution. The values of the CDF array, thus, will progressively increase from an initial value of zero to a final value of one at a rate depending on the given frequency curve. To randomly
select a value for a given statistical variable, the random number generator (RANDU) is invoked to obtain a random number between zero and one. This random number is compared against CDF array values until the smallest CDF value that is larger than the random number is found. The value of the corresponding element of the x array is assigned as the appropriate random value for the given statistical variable. The random numbers, obtained from a rectangular distribution from zero to one, are thus transformed by the CDF in such a way that as a series of random numbers is selected for a single variable, the corresponding variable values occur at a frequency corresponding to their probability distributions.

Random number generator. Based on its size, five statements, this subroutine would not appear to be worthy of a lengthy discussion. The subject, however, is quite critical to the statistical portion of this program. Without some understanding of the random number generator (RNG) the results may prove both surprising and less than satisfactory.

The numbers generated by this routine are not actually random numbers, but rather are referred to as pseudo-random numbers. This fact may cause a less than comfortable feeling. It turns out, however, that the characteristics of pseudo-random numbers are adequate for the present purposes. Real random numbers do not cycle. The pseudo-random numbers have a cycle, but the period is reasonably large. For IBM 370 hardware configurations, approximately $2^{30}$ selections are made. It is unlikely that this program will be used extensively enough for this to become a problem. The major consideration when using the RNG is that it requires a number to initiate the random generation. After the initial "seed" number, the routine will automatically reset this number for the next iteration as is done when a new set of variables is to be selected for an SRM. For any given seed number, the random number produced is always the same and the new seed number generated will also be equivalent. Thus, if one hundred random numbers are generated from a given initial seed number, the same, exact set of numbers will be generated given that same initial seed number. Often this is a useful characteristic; however, it is essential to understand that to obtain two different sets of random numbers, two different initial seed values must be given.

## III. DISCUSSION OF INPUT AND OUTPUT

In this section, each input parameter is defined in the order in which it is encountered in the program. The English or Greek symbol is given first followed by the computer symbol in parentheses. The English or Greek symbols are provided mainly for convenience in consulting the basic analyses of References 3,4 and 5 as only a few of these are used in the present report. Where appropriate, additional discussion of the variable and recommended or typical nominal numerical values are given. Sketches of geometric characteristics are presented for clarification. Although many of the variables are the same as used in References 3, 4 and 5, the discussion relative to these is repeated here for ready reference. In addition, the present outputs of the program are defined. It is the aim of this section to provide a guide to the user in the specification of his problem and the interpretation of the outputs.

Concise printed definitions of all input variables are also given with the computer program listing (See Section IV). The definitions appear in groups throughout the "main" program and the "area" subroutines. In general, each group is divided in the computer program and in this section into two subgroups: the first group containing the variables describing the SRM which are always fixed for the Monte Carlo investigation and the second group containing those that are subject to being statistically distributed.

The original basis for classification of the two types of variables was to be the analysis of the sensitivities of the performance of the SRM to the variable under consideration. Of course, each variable considered would also have to be an independent one or at least relatively so. Each varlable whose variation could reasonably be expected to influence the performance calculations for the SRM was examined one at a time. A few variables whose effects were obviously of very minor significance were omitted at this point and classified without further evaluation as nonstatistical (fixed value) variables. Among these were the temperature and pressure sensitivity coefficients of the propellant properties. The thrust loss coefficients were also classified as fixed, as there is no practical way of distinguishing in experimental data analysis between the variations in thrust due to the variation in the statistical variables and that due to other factors. A baseline design was then selected based on the nominal values for the sample case SRM of Section $V$ and the performance of a pair of SRMs computed. One motor in the pair had the baseline values of the variable and the other had the baseline values except for the variable under consideration which was changed a small amount. The amount of the change was somewhat arbitrary but represents a rough estimate of the maximum range of variation for a wide variety of SRMs scaled to the size of the baseline SRM; i.e., the range reflects an estimate of the maximum variation to be expected when no special attention is given to control of the variables during manufacture. In the case of
the examination of the effects of grain ovality and malalignment, several variable changes were examined together in order to reduce computational time.

Partial results are given in the Appendix in the form of the pairs of thrust-time traces obtained for each variable. These plots should be of interest to the SRM developer as they indicate possible results of deviations from manufacturing specifications and tolerances. Their utility as far as the present study goes is somewhat limited as it was decided that for the Monte Carlo investigation all but a very few of the variables would be statistically determined in order to minimize cumulative errors that might result from neglect of a large number of seemingly unfmportant variables. It will be noted that there are no plots in the Appendix for a few of the distributed variables listed in the computer program and in this section of the report. This is because either the variable has no influence for the design or option selected (e.g., ${ }^{\theta} \mathrm{cn}$ and $\theta_{\text {ch }}$ for $C_{o p}=1$ ); the effects of the variable is an obvious one ( $\theta$ ); data is not presently available for a meaningful investigation ( $\alpha$ and $\beta$ ); or the final selection of variables differed from that considered during the earlier sensitivity investigation (e.g., R RAl was substituted for $\gamma$ and $C^{*}$ and $R_{i s}$ and $R_{c}$ were both introduced as independent variables eliminating $\tau_{s}$ ).

The present section provides information only on nominal values and the units of the variables. Instructions for preparing the input for the description of the distribution of statistical (distributed) variables is given in Section IV based on the analysis of Section II. It should be noted here that the statistical variables may be held constant if desired by use of the proper code on the data cards for the variables. With minor program modifications it is also possible to accommodate additional distributed variables, the number of such variables being limited only by the core storage capacity of the computer. The listing of variables follows.

Seed Number
$I_{x i}$ (IX1) The seed number for the random number generator. An odd eight-digit integer should be used. This number initiates the generation of random numbers forming the basis for selection of the various sets of input variables for each SRM. The seed number must be changed to change the sets of variables generated when repeating an analysis of the same motor pairs (see Section II for further discussion).

User's Options - Fixed Values
$I_{\text {eo }}$ (IEO) $0 \quad$ For no consideration of grain ovality or malalignment.
1 For consideration of above. Calculations of ovality and malalignment effects approximately triples computer

|  | time requirements. These calculations should be by-passed by use of the option provided if there is a basis for assuming in a particular case that the effects are negligible. |
| :---: | :---: |
| $\mathrm{I}_{\mathrm{po}}$ (IP0) | 0 For no plots and no statistical or difference analysis of results. |
|  | 1 For plots and tabular output of motor pair differences. |
|  | 2 For tabular output of motor pair differences without plots. |
|  | 3 For plots and statistical analysis of results only. Codes 1 and 2 will also yield statistical analysis of the motor pair results. |
| N(j) (NUMPLT ( ${ }^{\text {( }}$ ) | An integer designating whether or not a specific output plot is desired: |
|  | 0 If a specific plot is desired. |
|  | 1 If a specific plot is not desired. |
|  | The order of specification of NUMPLT( $J$ ) is as follows: |
|  | 1 F versus T for the motor palr. |
|  | 2 F versus $T$ for the motor pair during talloff. |
|  | 3 Difference in $F$ between the pair of motors versus $T$. |
|  | 4 Difference in ITOT between the pair of motors versus $T$. |
|  | 5 Time integral of absolute value of difference in $F$ of motors versus $T$. |

## Propellant Characteristics - Distributed Values

$\rho_{p}$ (RHO) Density of the solid propellant (slugs/in ${ }^{3}$ )
$a_{1}$ (A1), $a_{2}$ (A2) Propellant burning rate coefficients in the equation $r=a P^{n}$ above and below the transition pressure, respectively (in/sec-psian).
$\mathrm{n}_{1}$ (N1), $\mathrm{n}_{2}$ (N2) Burning rate exponents corresponding to $\mathrm{a}_{1}$ and $\mathrm{a}_{2}$, respectively (1).
$\alpha_{e b}$ (ALPHA) Erosive burning coefficient in the Robillard-Lenoir burning rate rule (Equation III-11, Ref. 3) (in $\left.{ }^{2} .^{8}-\mathrm{ft}^{0.8} / \mathrm{sec}^{1.8} \mathrm{lbf}^{0.8}\right)$.

| B (BETA) | Erosive burning pressure coefficient in the RobillardLenoir rule (1). |
| :---: | :---: |
| $\mathrm{R}_{\mathrm{OA} \mathrm{\ell}}(\mathrm{ROAL})$ | The oxidizer to aluminum ratio (1). Variations. in this quantity determine variation in the thermochemically determined characteristic exhaust velocity CSTARN and the ratio of specific heats GAMN of the chamber gases. The latter variations are determined by a regression analysis which gives CSTARN and GANN as functions of ROAL. The resultant functional relationship must be used in the program where indicated on the program listing (Section IV). CSTARN and GAMN are determined at 1000 psi chamber pressure and $60^{\circ} \mathrm{F}$ propellant bulk temperature. If no aluminum is present in the propellant CSTARN and GANN should be set at this point to their nominal values at the reference conditions. |
| Basic Motor Dimensions - Fixed Values |  |
| L (L) | Total initial perforated grain length including gaps at slots (in.). This is used only in the erosive burning rate equation. An estimate will suffice. |
| $\mathrm{T}_{\mathrm{w}}$ (TAU) | Estimated web thickness of the controlling propellant length (in.). (See 20 below for definition of controlling length.) The actual web thickness is calculated from DI and DO (or RC and RSI, RIWS or RIWW)and includes statistical variations. If the grain is tapered, the length average value should be specified excluding lengths having additional taper not represented by 20 and segments located anywhere having a step decrease in thickness. Such step decrease nust be handled by the additive tabular input option if they introduce two significantly different web thicknesses for the same grain type; e.g., two circular perforated grains. If a circular perforated grain is used, it is assumed that it will have the approximate average web thickness of the controlling propellant length. |

## Basic Motor Dimensions - Distributed Values

$D_{e}$ (DE) Diameter of the nozzle exit (in.).
$D_{i}$ * (DTI) Initial diameter of the nozzle throat (in.).
$\theta_{\mathrm{n}}$ (THETA) Cant angle of the nozzle with respect to the motor (degrees).
$\alpha_{n}$ (ALFAN) Exit half angle of the nozzle (degrees).

| $L_{\text {Ta }}($ LTAP) | Estimated axial length of grain at the aft end at start of first talloff having an additional taper not represented by ZO or THETAG (in.). See Figure III-1. This variable permits the designer to specify an additional taper at the nozzle end of a circular perforated or star grain to produce regressivity shortly before tailoff. |
| :---: | :---: |
| $\mathrm{X}_{\mathrm{Ta}}$ (XI) | Difference in web thickness at ends of LTAP (in.). See Figure III-I. |
| $z_{0}(\mathrm{ZO})$ | Initial difference between web thicknesses due to grain bore taper at the head and aft ends of controlling grain length, not including any initial length associated with LTAP or THETAG (in.). The controlling length of the grain is the axial distance between the head end of the grain and the position of expected maximum Mach number in the port. In general this length terminates whenever there is an abrupt decrease in web thickness near the aft end of the chamber. |
| $2_{c}(\mathrm{ZC})$ | Initial difference between web thicknesses due to grain exterior taper at the head and aft ends of the controlling grain length (in.). |

## Ovality and Malalignment - Distributed Values

(Not required if $I E O=1$ ). The following variables characterize the ovality and lack of concentricity of the grain interior and exterior at two reference radial cross-sections - one near the head end of the grain and one near the nozzle end. Based on the mathematical analysis of the burning perimeters at these two planes, correction factors are calculated and applied to the burning surface calculations to account for approximate effects of ovality and malalignment. The effects of interior and exterior grain taper are taken into account through the parameters $\mathrm{ZO}, \mathrm{ZC}$, and XT. See Section II for details.
$\Delta R_{c n}(R O N D C N)$, One half the difference between the maximum and minimum diameter of the grain exterior at the nozzle and head end $\Delta R_{c h}$ (RONDCH) reference planes, respectively (in.).
$\Delta R_{g n}$ (RONDGN), One half the difference between the maximum and minimum diameter of the grain interior at the nozzle and head end $\Delta \mathrm{R}_{\mathrm{gh}}$ (RONDGH)
$e_{x n}$ (EXN)
$e_{y n}$ (EYN) reference planes, respectively (in.).

The eccentricity of the center of the grain interior with respect to the center of the grain exterior in the $x_{c}$ and $y_{c}$ directions, respectively (See Fig.. II-1) at the nozzle end reference plane (in.).

Figure III-1. Basic motor dimensions. $L_{G c i}, L_{G n i}$, and $\theta_{G}$ are used only for circular perforated grains. The controlling grain length may be $L_{G c i}$ or some lesser value depending on the position of maximum port Mach number. $\tau_{w}$ is the length average web thickness over the grain length excluding the lengths associated with $\mathrm{L}_{\mathrm{T} a}, \theta_{\mathrm{G}}$ and the head end dome.
$\mathbf{e}_{\mathbf{x h}}(E X H)$,
$\mathbf{e}_{\mathbf{y h}}(E Y H)$
$a_{a n}($ ALPHAN $)$,
$a_{a h}($ ALPHAH $)$

The eccentricity of the center of the grain interior with respect to the center of the grain exterior in the $x_{c}$ and $y_{c}$ directions, respectively, at the head end reference plane (in.).

The angular orientation of the ovality of the grain interior with respect to the grain exterior at the nozzle and head end reference planes, respectively (degrees).

## Basic Performance Constants - Fixed Values

$\Delta y_{1}$ (DELTAY) Initial desired burn increment (in.). This increment will be used by the program for initial $5 \%$ of the web thickness burned, for the period shortly preceding and following tailoff and at such other times as the rate of pressure change is large. Larger increments will automatically be used at other times. An initial increment size of approximately $0.001 \tau_{\mathrm{w}}$ is recommended for the Monte Carlo program. If the increment size used is twice the recommended value, the maximum thrust imbalance calculated may be decreased by as much as 5 percent, representing a loss in program accuracy.
$I_{I}$ (II) Number of steps in the integrations of perimeters in the OVAL subroutine. Approximately 25 steps appears to offer a good compromise between accuracy and computation time, the latter being more strongly influenced than the former by the step size.
$x_{\text {out }}$ (XOUT) Distance burned at which the propellant breaks up (in.). This permits the option of specifying termination of burning resulting from possible structural breakup of propellant. If this option is not desired, XOUT should be set to some large value; e.g., 1000 in .
( $\Delta \mathrm{P} / \Delta \mathrm{y})_{\text {out }} \quad$ Rate of change of pressure with respect to distance burned at which the propellant is extinguished ( $1 \mathrm{~b} / \mathrm{in}^{3}$ ). This permits the option of specifying termination of burning when it is determined that an abrupt tailoff will not permit the computer to handle the rapid change in surface area. If a gradual tailoff is expected, DPOUT may be set to some large value; e.g., $10,000 \mathrm{psia} / \mathrm{in}$. For large motors ( 120 in . dia. and up) where the tailoff is expected to be abrupt, a value of $500 \mathrm{psia} / \mathrm{in}$. is recommended. In general, larger values may be used for smaller motors.
$\zeta_{F}$ (ZETAF) Thrust loss coefficient (1). In the absence of data to the contrary, a value of 0.98 is recommended.

| $t_{\max } \mathrm{q}^{\text {(TMAXQ) }}$ | Estimated time at which maximum dynamic pressure on the vehicle occurs (sec.). This permits the program to compute an estimate of the thrust differential of the motor pair at the time of maximum dynamic pressure. |
| :---: | :---: |
| $t_{\text {bl }}(\mathrm{TB})$ | Estimated burning time (sec.). This and HB below permit the program to calculate delivered specific impulse and thrust based on an assumed trajectory which was determined from analysis of typical large SRM applications. |
| $h_{b}$ ( $H B$ ) | Estimated burnout altitude (ft.). To obtain sea level performance characteristics, HB should be set equal to zero. |
| $\mathrm{P}_{\text {ref }}$ (PREF) | Reference average nozzle stagnation pressure used in the nozzle throat erosion equation (psia). See also ERREF below. |
| $\mathrm{D}_{\mathrm{t} \times \mathrm{ref}}$ (DTREF) | Reference nozzle throat diameter used in the nozzle throat erosion equation (in.). See also ERREF below. |
| $\left(\pi_{p}\right)_{k}($ PIPK $)$ | Temperature sensitivity coefficient of pressure at constant ratio K of burning surface to throat area ( $/{ }^{\circ} \mathrm{F}$ ). |
| $\mathrm{a}_{\mathbf{c} * \mathrm{~T}}$ (CSTART) | Temperature sensitivity of $\operatorname{CSTAR}$ at constant $\mathrm{K}\left(/^{\circ} \mathrm{F}\right)$. (See Eq. 4, Section II). A typical value is 0.000038 . |
| $a_{c * p}$ (CSTARP) | Pressure sensitivity of CSTAR (1) (See Eq. 5, Section II). |
| $\mathrm{P}_{\text {tran }}$ (PTRAN) | Transition pressure at which the burning rate coefficient and exponent change (psia). |
| $a_{\gamma P}$ (GAMP) | Pressure sensitivity of the ratio of specific heats (1) (See Eq. 6, Section II). A typical value is 0.00527. |

## Basic Performance Constants - Distributed Values

| $\mathrm{E}_{\mathrm{ref}}$ (ERREF) | Reference nozzle throat erosion rate (in/sec) (See Eq. 1 <br> Section II.) A set of typical values for a carbon tape- <br> phenolic impregnated throat is 0.00763 in/sec, 560 psia <br> and 57.285 in. for ERREF, PREF and DTREF, respectively. |
| :--- | :--- |
| $\mathrm{T}_{\mathrm{gr}}$ (TGR) | Bulk temperature of the propellant grain ( $\left.{ }^{\circ} \mathrm{F}\right)$. |
| $\mathrm{T}_{\mathrm{igr}}$ (TIGR) | Ingition delay (sec.) at a TGR of $60^{\circ} \mathrm{F} . \quad$ The time required <br> to reach ninety-five percent of initial equilibrium pressure <br> should be used. |

The Program and Basic Grain Configuration and Arrangement - Fixed Values
$I_{\text {op }}$ (INPUT) 1 For only tabular input.

2 For only equation input.
3 For a combination of 1 and 2.

| $G_{o p}$ (GRAIN) | 1 | For an entirely circular perforated grain. |
| :--- | :--- | :--- |
|  | 2 | For star grain only. |
|  | 3 | For a combination of 1 and 2. |
| $S_{o p}$ (STAR) | 0 | For an entirely circular perforated grain. |

1 For standard star (See Fig. III-2).
2 For truncated star (See Fig. III-3).
3 For wagon-wheel (See Fig. III-4).
NOTE: The different types of star grains may not be combined in a single configuration.

Number of thrust termination passageways in grain (1) NT is zero if there are no thrust termination passageways.

1 If a star grain is at head end and a circular perforated grain at aft controlling end.

2 If a circular perforated grain is at both ends. A star grain segment may still be present.

3 If a circular perforated grain is at head end and a star at the aft controlling end.

4 If a star grain is at both ends.
If grain $=1$, value of order must be 2 .
If $\operatorname{grain}=2$, value of order must be 4 .
It is important to realize that ORDER establishes the controlling port area equations to be used. Thus if the nozzle end segment is not indeed the controlling one (the one that establishes the maximum Mach number in the port), ORDER should be specified to designate the actual controlling segment as the nozzle end segment. GRAIN, STAR, NT and ORDER are not used for INPUT $=1$, but values must be assigned for continuity of computer operations.




```
Cop (COP) For extreme ends of a circular perforated grain only:
    0 If both ends are conical or flat.
    1 If head end is conical or flat and aft end is spherical.
    2 If both ends are spherical.
    3 If head end is spherical and aft end is conical or flat.
```

Tabular Burning Surface and Port Areas - Fixed Values

| $A_{b p T}(A B P K)$ | Burning area in the port $\left(i n^{2}\right)$. |
| :--- | :--- |
| $A_{b s T}(A B S K)$ | Burning area in the slots $\left(i n^{2}\right)$. |
| $A_{b n T}(A B N K)$ | Burning area at the nozzle end $\left(n^{2}\right)$. |
| $A_{p h T}(A P H K)$ | Port area at the nozzle end of the controlling grain <br> length $\left(i n^{2}\right)$. |

ApnT (APNK) Port area at the head end of the grain (in ${ }^{2}$ ). APHK and APNK are not required when INPUT is 2 or 3 ; the equation inputs must be used to provide the information in these cases. Values of all A's must be specified to completely describe the burning surface and port areas versus distance burned. The program computes intermediate values by linear interpolation. The number of values required is arbitrary and limited only by the storage capacity of the computer, but values must be specified for $y=0$. Also, as an example of the procedure for specifying terminal values which must be followed, burning surfaces must be specified as zero at burnout of the tabular surfaces and at a exceeding the highest anticipated calculated performance value. Separate input cards must be prepared for each value of $y$ and arranged as described in Section IV of this report. The use of these tabular values in conjunction with the equation inputs (INPUT = 3) increases the flexibility of the program considerably. Frequently it is easy to estimate burning surface effects which end constraints on the equation inputs neglect. In this case a table of input values can be readily prepared from estimates of the effects. Also, the program can be run first without the tabular values and outputs used as an aid in obtaining the estimate. For example, the burning perimeters of a star grain can be determined in this way and the values used to estimate the effects of a head end closure on the star grain.
$V_{c i T}$ (VCIT) Initial volume of chamber gases associated with tabular input (in ${ }^{3}$ ).

| $\mathrm{X}_{\mathrm{Tz}}(\mathrm{XTZO})$ | Difference between the initial circular perforated grain diameter at the nozzle end of LGCI and the nominal value of DI less ZO and less twice XT (in.) (See Fig. III-1). |
| :---: | :---: |
| S (S) | Number of burning flat ends of a circular perforated grain not including an extreme aft grain end (1). |
| Geometry for Circular Perforated Grain - Distributed Values |  |
| $\mathrm{D}_{0}$ (DO) | Length average outside diameter of circular perforated grain, excluding lengths extending into the closure (in.). |
| $D_{i}$ (DI) | Length average inside initial diameter of circular perforated grain (in.). Only the controlling length excluding LTAP should be considered in the averaging. |
| $\theta_{G}$ (THETAG) | Angle burning surface element of circular perforated grain located at the extreme nozzle end of chamber makes with the motor axis (degrees). See Figure III-1. THETAG must be set to zero if a star grain is located at the nozzle end (GRAIN $=3$, ORDER $=3$ ) or if aft end burning surfaces are represented by tabular values. THETAG is $90^{\circ}$ if the circular perforated grain represented by equations has a flat burning surface located at the extreme nozzle end of the chamber. If thetac is less than or equal to $5^{\circ}$, a value must be assigned (zero is satisfactory), but the effect of THETAG on burning surface area is not computed (See also LGNI and LTAP). THETAG is zero if the end surface is flat and inhibited. |
| $\mathrm{L}_{\text {Gci }}$ (LGCI) | Initial total axial length of circular perforated grain represented by equation inputs not including gaps (in.). LGCI excludes lengths associated with thetag. |
| $L_{\text {Gni }}$ (LGNI) | Initial slant length of a burning conical circular perforated grain at the nozzle end (in.). LGNI is set equal to zero if THETAG is less than or equal to $5^{\circ}$. In this case the length otherwise associated with LGNI should be added to LGCI. If the error in burning surface area thus introduced is deemed significant, a correction may be introduced by making use of tabular inputs in combination with the equation inputs. Basic effects of small THETAG on tailoff may be accounted for by specification of LTAP. If a nozzle end burning surface is flat (THETAG $=90^{\circ}$ ) LGNI equals one half the difference between inside and outside local grain diameters. |


| $\theta_{\text {cn }}$ (THETCN) | The contraction angle of a circular perforated grain bonded to the nozzle closure (degrees). See Figure III-1. Use an estimated value which yields approximately the correct volume of propellant burned. If a star shaped grain is located at the extreme nozzle end of the chamber or if tabular values are used to represent downstream burning surfaces, THETCN is $90^{\circ}$. THETCH is also $90^{\circ}$ if the extreme aft end of the grain is inhibited, but only a flat-ended, inhibited grain (THETAG $=0$ ) which does not extend into the nozzle closure may be accurately represented. THETCN is zero for a burning flat end (THETAG $=90^{\circ}$ ) which does not extend into the closure. THETCN must be assigned a value even if COP is 1 or 2 , but it will not affect the numerical results. |
| :---: | :---: |
| $\theta_{\text {ch }}$ (THETCH) | The contraction angle of a circular perforated grain bonded to the head end (degrees). See Figure III-1. Use an estimated value which yields approximately the correct volume of propellant burned. THETCH is $90^{\circ}$ if the extreme forward end of the circular perforated grain (bonded or not) represented by equations is flat. A head end flat burning surface is treated by proper specification of S. THETCH must be assigned a value even if COP is 2 or 3 , but it will not affect the numerical results. |
| Basic Geom | for Star Grains - Fixed Values (The wagon-wheel is considered a type of star grain for the purpose of this program.) |
| $\mathrm{n}_{\mathrm{s}}$ (NS) | Number of burning flat end surfaces of a star grain not located at extreme nozzle end of the chamber (1). |
| $n_{p}$ (NP) | The number of star points (1). |
| $\mathrm{n}_{\mathrm{n}}$ (NN) | Number of burning flat end surfaces (0 or 1) of a star grain located at the extreme nozzle end of the chamber (1). |
| Basic Geome | for Star Grains - Distributed Values |
| $\mathrm{L}_{\text {Gsi }}$ (LGSI) | Initial total axial length of star-shaped perforated grain represented by equations (in.). No provision comparable to the use of LGNI for circular perforated grains is made here to treat eifects resulting from THETAG greater than $5^{\circ}$. Adjustments may be made, however, by use of tabular input values in conjunction with the equation inputs. Also, effects of taper, including additional small taper at the nozzle end, on tailoff may be treated by use of the variables 20 , XT and LTAP. |
| $\mathrm{R}_{\mathrm{c}}$ (RC) | The star grain outside radius (in.). |
| f (FILL) | The fillet radius at star valleys (in.). See Figure III-2, 3, 4. |

Special Geometry for Wagon-Wheel Grain - Distributed Values

| $\mathrm{R}_{\text {iww }}$ (RIWW) | The length average initial radius of the inside of the propellant web (in.). |
| :---: | :---: |
| $\ell_{1}, \ell_{2}(\mathrm{~L} 1, \mathrm{~L} 2)$ | The lengths of the pairs of parallel sides of the first and second sets of grain points, respectively (in.). See Figure III-4). |
| $\alpha_{1}, \alpha_{2}$ <br> (ALPHA1,ALPHA2) | The angles between the slant sides and the center lines of the points of the first and second sets of grain points, respectively (degrees). The angles should not exceed 90 degrees. |
| $h_{w}(H W)$ | The half-width of the star points (in.). HW must not exceed TAUWW (HW $\leq$ TAUWW). |
| Special Geometry for Truncated Star Grain - Distributed Values |  |
| $\mathrm{R}_{\mathrm{p}}$ (RP) | The length average initial radius of the truncation (in.). See Figure III-3. |
| $\mathrm{R}_{\text {is }}$ (RIS) | The length average initial radius of the inside of the propellant web at the bottom of the slots (in.). |
| Special Geometry for Standard Star Grain - Distributed Values |  |
| $\theta_{f}$ (THETAF) | Angular location of the fillet center of standard star from the line of symmetry (degrees). |
| $\theta_{p}$ (THETAP) | The apex angle of the star point (degrees). |
| $\mathrm{R}_{\text {iws }}$ (RIWS) | The initial radius of the inside of the propellant web of standard star grain (in.). See Figure III-2. If the grain is tapered, the length average value should be used. |
| Geometry of Thrust Termination Passageways - Distributed Values |  |
| $\ell_{\text {TP }}$ (LTP) | Initial length of the termination passageway between the centers of gravity of perimeters of the bases (in.). See Figure III-1. |
| $\mathrm{D}_{\mathrm{TP}}$ (DTP) | Initial diameter of the termination passage (in.). |
| $\theta_{\text {TP }}$ (THETTP) | The acute angle between the axis of the passage and the motor axis (degrees). |
| $\tau_{\text {eff }}$ (TAUEFF) | Estimated effective web thickness at the termination port (in.). The user must judge the distance burned at which the effect of the termination passage on modification of the burning surface geometry ceases to be significant. In general, this should be between two-thirds and full web thickness. The equation used to account for the burning surface is based on a passageway in a circular perforated grain terminated at the case by a flat inclined plane. Thus only a rough estimate of the effect of the termination passage is provided. |

Special Equation Inputs - (Fixed Relationships required only at the option of the user. May be used when INPUT $=1,2$, or 3.)

| $B_{b p}$ (BBP) | Additive burning surface input as function of $y$ for port burning surface ( $\mathrm{in}^{2}$ ). |
| :---: | :---: |
| $\mathrm{B}_{\mathrm{bs}}$ (BBS) | Additive burning surface input as function of $y$ for slot burning surface ( $\mathrm{in}^{2}$ ). |
| $\mathrm{B}_{\mathrm{bn}}$ (BBN) | Additive burning surface input as function of $y$ for nozzle end burning surface ( $\mathrm{in}^{2}$ ). In order to make use of the option of specifying the $B^{\prime}$ 's, a minor program modification is required. The B's are all set equal to zero in the present program. If this option is to be used, the program statements assigning values to the B's are easily replaced with the desired equation inputs. |

## Program Outputs

The variables whose values are printed by the present program are defined below. Additional variables may also be printed with minor program modifications. In addition to the variables listed below, the present program prints out values of all input variables including those selected for each SRM from statistical distributions. The input characteristics of the statistical distributions are also printed.

Time dependent data - single motors

| $t$ ( T ) | Operating time (sec.). This is calculated from the time of initiation of ignition. |
| :---: | :---: |
| $y(Y)$ | Average distance burned (in.). |
| $\mathrm{P}_{\text {on }}$ (PONOZ) | Stagnation pressure at the nozzle end of the chamber ( $1 \mathrm{~b} / \mathrm{in}^{2}$ ). |
| $\mathrm{P}_{\mathrm{h}}$ (PHEAD) | Pressure at the head end of the chamber ( $1 \mathrm{~b} / \mathrm{in}^{2}$ ). |
| $A_{b p}+A_{b s}+A_{b n}$ <br> (SUMAB) | The total burning surface of the propellant (in ${ }^{2}$ ). |
| F (F) | The delivered thrust based on the assumed trajectory (lbf). Losses are included. |
| $\mathrm{I}_{\mathrm{T}}$ (ITOT) | Total delivered specific impulse (lbf-sec). Losses are included. |

## Time independent data - single motors

| $W_{p 1}$ (WP1) | Propellant weight calculated from mass discharge rates ( 1 bm ) . |
| :---: | :---: |
| $\mathrm{W}_{\mathrm{p} 2}$ (WP2) | Propellant weight calculated from the products of burning surfaces and incremental distances burned (1bm). |
| $W_{p}(W P)$ | Arithmetic average of WP1 and WP2 (1bm). A check on the calculation accuracy is provided by comparison of this with WP1 and WP2. |
| $\left.P_{h} \max ^{(P H M A X}\right)$ | Maximum head end chamber pressure calculated by the program ( $1 \mathrm{~b} / \mathrm{in}^{2}$ ). |
| $I_{x 1}$ (IXI) | Initial seed number for the current configuration. New seed numbers are automatically selected for each variable based upon this seed number. The initial seed number may be used as an input seed number to reproduce the calculations for the configuration. See Section II. |
| $\mathrm{I}_{\mathrm{xf}}$ (IX) | Final seed number for the current configuration. This final seed number may be used as an input seed number to continue the calculations without random number cycling before $2^{30}$ random numbers have been selected. |

Time dependent data - motor pairs. The following data are available in tabular and/or graphical form, subject to the option of the user (See IPO). To compute the imbalances between two motors the data for the motor which has the fewer computational y-steps is subtracted from the data of the other motor to determine the imbalances. No difficulty with regard to the interpretation of the results arises from this since in most instances It is the absolute value of the differences which is important. For those variables whose absolute values are not taken, changes in sign of the differences indicate points where the crossings between the two motors' traces occur.
$\triangle F$ (FDIFF) The difference in thrust between the two motors (lbf).
$\Delta I_{T}$ (IDIFF) The difference in total impulse between the two motors (lbf-sec).
$\left|\Delta I_{T}\right|$ (IADIFF) The absolute value of the difference in total impulse between the two motors ( $1 \mathrm{bf}-\mathrm{sec}$ ).

$$
\begin{equation*}
\left|\Delta I_{T}\right|=\int_{0}^{t}\left|F_{1}-F_{2}\right| d t \tag{17}
\end{equation*}
$$

## Time independent data - motor pairs

(FMAX1,TFMX1, The maximum and minimum algebraic thrust imbalance (lbf)
(DFTO1,DFTO2) The absolute value of the thrust imbalance when the first

FMIN1, TFMN1)
(FMAX2,TFMX2, FMIN2, TFMN2)
(TDFTO1,TDFTO2, DTW)
(FW1,FW2,DFW)
(DFMQ, TMAXQ)
(AFMAX, TFMAX, AFMAXT, TFMAXT) and the time (secs) at which they occur, respectively, during web action time.

The maximum and minimum algebraic thrust imbalance (lbf) and the times (secs) at which they occur, respectively, during tailoff.

The time at which tailoff begins for the first motor of a pair to begin tailoff, for the final motor to begin tailoff and the absolute value of the difference between the two times, respectively, (secs).

The thrust at the beginning of tailoff for the first motor of a pair to begin tailoff, for the final motor to begin tailoff and the absolute value of the difference between the two thrusts, respectively, (lbf). motor of a pair begins tailoff and the final motor begins tailoff, respectively.
(FDIFIG, TDIFIG) The absolute value of the thrust imbalance (lbf) which occurs during the initial portion of operation ( $t<0.02 t_{b}$ ) and the time (secs) at which it occurs, respectively.
(DIT,ADIT)
The absolute value of the thrust imbalance (lbf) which exists when the maximum dynamic pressure occurs on the vehicle and the estimated time (secs) at which this event occurs, respectively.

The absolute value of the maximum thrust imbalance (lbf) which exists and the time (secs) at which it occurs during web action time and tailoff, respectively.

The total impulse imbalance and the absolute value of the total impulse imbalance (lbf-secs) accumulated during tailoff, respectively:

$$
\begin{align*}
& \Delta I_{t}=\int_{t 1}^{t_{b}}\left(F_{1}-F_{2}\right) d t  \tag{18}\\
& \left|\Delta I_{t}\right|=\int_{t 1}^{t_{b}}\left|F_{1}-F_{2}\right| d t \tag{19}
\end{align*}
$$

where $t_{t 1}$ (TDFTO1) is the earlier time at which tailoff

## begins in the two motors and $t_{b}$ is the time at which operation of both motors ends. <br> (DFIOOK,TIOOK) The absolute value of the thrust imbalance (lbf) which exists when the last motor reaches $100,000 \mathrm{lb}$. thrust during tailoff and the time (secs) at which it occurs, respectively.

## IV. THE COMPUTER PROGRAM

This section contains the instructions for the preparation and arrangement of the data cards. Also, a complete listing of the program statements is given. The program was written for use on an IBM 370/155 computer and requires approximately 168 K storage locations on that machine. The program also is designed to be used with a CALCOMP 663 drum plotter. The plotter requires one external storage device (magnetic tape or disk). In addition to the one storage device required for the plotter, three other external storage units are required. Unit 1 is used to store the output data, pertinent to the imbalance calculations, for the first motor in each pair of motors. Unit 2 is used to store the nonstatistical data which remain constant for all of the motors. Unit 4 is used to store the values of the statistical variables for use with each motor. Only minor program modifications are required to eliminate the plotting capability of the program. Also, Unit 2 can be eliminated by using repeated sets of data cards for the nonstatistical variables. Hence, it is relatively simple to modify the program to require only 2 external storage units. Elimination of the other two external storage units would require significant program modification.

Input Data
The discussion below gives the general purpose, order and FORTRAN coding information for the input data.

Card 1 Total number of individual motors to be analyzed (42X, I2)

Co1. 1-42 NUMBER OF CONFIGURATIONS TO BE TESTED $=$
43-44 Number of rocket motors to be analyzed

## Card 2 Initial seed number (IIO)

Co1. 1-10 Initial 8-10 digit seed number
It is necessary to describe one type of statistical analysis for each statistical input variable. The method for doing this is described below using Cards 3 through 9. Note that only one type of statistical analysis may be requested for each variable. Hence, only the card or cards necessary for that particular type of statistical analysis are input for each variable. For example, to obtain a Type II analysis only Card 5 and Cards 5A would be would be used. In addition, it is necessary that the data cards for the variables to be used in a given configuration be placed in the order in which they are input into the computer program. In some cases certain variables are not required for an analysis. In such cases, the cards for those variables should be omitted. As many Cards 5 through Cards 9A as required may be used.

## Card 3 Variable name (2A4) (one card for each variable)

## Col. 1-8 Name of statistical variable.

NOTE: One Card 3 immediately precedes the Card 4 thru Card 9B used for each variable. Also, END should be used as the last variable name before using Card 9B below.

Card 4 Input for Type I statistical analysis (I2, 2X, 7E10.0)
Co1. $1-2 \begin{cases}\text { Code }=10 & \begin{array}{l}\text { Raw data given; obtain CDF directly from } \\ \text { histogram. }\end{array} \\ \text { Code }=11 & \begin{array}{l}\text { Raw data given; obtain CDF from Pearson's } \\ \text { equation of the frequency curve. }\end{array}\end{cases}$
5-14 $\mathrm{Xl}=$ Number of raw data points given.
15-24 X2 = Mean value of first interval of histogram.
25-34 X3 = Histogram interval width.
35-44 $\mathrm{X4}=$ Number of intervals in histogram.
45-74 Blank
Card 4A Subsequent Type I data cards (10E8.0)
. Col. 1-8 Raw data points equivalent to the number specified in X1. Ten data points per card for 9-16 as many cards as required (e.g., 46 data points : would require 5 data cards with the last card 72-80 having the final four fields blank).

Card 5 Data input for Type II statistical analysis (I2, 2X, 7E10.0)
Col. $1-2\left\{\begin{array}{cl}\text { Code }=20 & \begin{array}{l}\text { Histogram given; obtain CDF directly from } \\ \text { histogram. }\end{array} \\ \text { Code }=21\end{array} \quad \begin{array}{l}\text { Histogram given; obtain CDF from Pearson's } \\ \text { equation of the frequency curve. }\end{array}\right.$
5-14 $\mathrm{X} 1=$ Number of intervals in histogram.
15-24 X2 = Mean value of first interval of histogram.
25-34 X3 = Interval width.
35-74 Blank

## Card 5A Subsequent Type II data cards (10E8.0)

| Co1. | $1-8$ The same number of data <br> $9-16$ points as specified in $\mathrm{X1}$, <br> $\vdots$ for as many data cards as <br> necessary  |
| :---: | :--- |
|  |  |

Card 6 Input for Type III statistical analysis (I2, 2X, 7E10.0)
Col. 1-2 Code $=31 \quad$ Four moments given; obtain CDF from Pearson's equation of the frequency curve.

5-14 $\mathrm{Xl}=$ First moment about zero.
15-24 X2 = Second moment about mean.
25-34 X3 = Third moment about mean.
35-44 $\quad \mathrm{X} 4=$ Fourth moment about mean.
45-54 $\quad \mathrm{X} 5=$ Histogram interval width.
55-64 X6 $=$ Mean value of first interval of histogram.
65-74 $\quad \mathrm{X7}=$ Total number of data points used.
NOTE: No data cards required.
Card 7 Input for Type IV statistical analysis
Col. 1-2 Code $=40 \quad$ CDF given; read in the given CDF.
5-14 $\mathrm{XI}=$ Number of intervals in CDF.
15-24 $\quad$ X2 $=$ Mean value of first interval of CDF.
25-34 X3 $=$ Interval width.
35-74 Blank
Card 7A Subsequent Type IV data cards (10E8.0)
Col. 1-8 CDF values corresponding to the cumulative
9-16 frequency up through each interval. Data
: should be provided for as many intervals as
72-80 indicated by the value given for Xl .

## Card 8 Input for Type $V$ statistical analysis (Use appropriate card below)

Card 8A Normal distribution to obtain CDF.
Col. 1-2 Code $=51$
5-14 $\mathrm{Xl}=$ Mean of normal distribution.
15-24 $\mathrm{X} 2=$ Standard deviation.
25-34 $\quad \mathrm{X} 3$ = Beginning X value of CDF (optional).
35-44 $\quad \mathrm{X} 4=$ Ending X value of CDF (optional).
45-74 Blank
NOTE: If either X 3 or X 4 is omitted, a three-sigma limit is assumed; thus, if both values are left blank, a six-sigma limit will be generated by the program. If a zero value is desired for X3 or $\mathrm{X} 4, \pm .0000001$ should be used instead.

Card 8B Rectangular distribution to obtain CDF (I2, 2X, 7E10.0)
Col. 1-2 $\quad$ Code $=52$
5-14 $\quad X 1=$ Beginning $X$ value.
15-24 $X 2=$ Ending $X$ value
25-74 B1ank
Card 8C J-Distribution to obtain CDF
Col. 1-2 Code $=53$
5-14 $\quad \mathrm{Xl}=$ Mean(beginning X value).
15-24 $\quad \mathrm{X} 2=$ Standard deviation.

Card 8C (Cont'd)
Col. 25-34 $\quad \mathrm{X} 3=$ Ending X value (optional)
35-74 Blank
NOTE: The J-distribution is defined herein as the right half of a normal frequency curve. The X1 value specified should be the mean as if the full normal curve were being specified. The X3 value is optional; if not specified, a three sigma limit will be assumed. If zero is desired for the X3 value, $\pm .0000001$ should be used instead.

Card 9 Input for Type VI statistical analysis (use appropriate card below)

Card 9A Use a constant for this value (I2, $2 \mathrm{X}, 7 \mathrm{~F} 10.0$ )
Co1. 1-2 Code $=60$ Use a constant value for this variable.
5-14 X1 = Desired constant value.
15-74 Blank
Card 9B Indicates end of data (I2)
Col. 1 -2 $\quad$ Code $=90$
Card 10 Initialization of variables (22F3.1)
Col. 1-66 Zero's or blank card
Card 11 Ovality and output options (5X, I1, 5X, I1, 9X, 5I)
Col. 1-5 IEO $=$ \%
$6 \begin{cases}0 & \text { Nò ovality analysis } \\ 1 & \because \\ \text { Ovality analysis }\end{cases}$
7-11: $\quad$ IPO $=$

12
$\begin{cases}0 & \text { No plots or statistical analysis } \\
1 & \text { Plots, statistical analysis and tabular output } \\
2 & \text { Tabular output and statistical analysis } \\
3 & \text { Plots and statistical analysis }\end{cases}$

Card 11 (Cont'd)
Col. 13-17 NUMPLT(J) $=$
$18 \begin{cases}0 & \text { Plot thrust time trace } \\ 1 & \text { Do not plot thrust time trace }\end{cases}$
19 . $\begin{cases}0 & \text { Plot talloff thrust time trace } \\ 1 & \text { Do not plot talloff thrust time trace }\end{cases}$
$20 \begin{cases}0 & \text { Plot thrust imbalance } \\ 1 & \text { Do not plot thrust imbalance }\end{cases}$
$21 \begin{cases}0 & \text { Plot impulse imbalance } \\ 1 & \text { Do not plot impulse imbalance }\end{cases}$
$22 \begin{cases}0 & \text { Plot absolute impulse imbalance } \\ 1 & \text { Do not plot absolute impulse imbalance }\end{cases}$
Card 12 Nonstatistical motor dimensions (3X, F10.2, 5X, F10.3)
Co1. 1-3 $\mathrm{L}=$
4-13 Value of $L$
12-18 TAU $=$
19-28 Value of TAU
Card 13 Nonstatistical performance constants (requires 4 data cards)
Card 13A (8X, F10.3, 4X, 14, 6X, F10.2, 7X, F10.2, 7X, F10.4)

| Col. | $1-8 ;$ | DELTAY $=$ |
| :---: | :---: | :--- |
| $8-18$ | Value of DELTAY |  |
|  | $19-22$ | II $=$ |
| $23-26$ | Value of II |  |
| $27-32$ | XOUT $=$ |  |
| $33-42$ | Value of XOUT |  |

Card 13A (Cont'd)
Col. 43-49 DPOUT $=$
50-59 Value of DPOUT
60-66 $\quad$ 2ETAF $=$
67-76 Value of ZETAF
Card 13B (4X, F10.1, 4X, F10.1, 6X, F10.2, 7X, F10.3, 6X, F10.5)
Co1. 1-4 TB =
5-14 Value of TB
15-18 $\mathrm{HB}=$
19-28 Value of HB
29-34 PREF =
35-44 Value of PREF
45-51 DTREF =
52-61 Value of DTREF
62-67 PIPK =
68-78 Value of PIPK
Card 13C (8X, F10.7, 7X, F10.2, 8X, F10.7, 6X, F10.7)
Co1. $1-8 \quad$ CSTART $=$
9-18 Value of CSTART
19-25 PTRAN $=$
26-35 Value of PTRAN
36-43 CSTARP $=$
44-53 Value of CSTARP
54-59 GAMP $=$
60-69 Value of GAMP

```
    Card 13D (7X, F10.3)
Col. 1-7 TMAXQ =
        8-17 Value of TMAXQ
    Card 14 Description of type of grain configuration (9x, I2, 9x, I2,
        8x, I2, 6X, F4.0, 9X, I2, 7X, I2)
Col. 1-9 INPUT =
    10-11 Value of INPUT (1, 2 or 3)
        12-20 GRAIN =
        21-22 Value of GRAIN (1, 2 or 3)
        23-30 STAR =
        31-32 Value of STAR (0, 1, 2 or 3)
        33-38 NT =
        39-42 Value of NT
        43-51 ORDER =
        52-53 . Value of ORDER (1, 2, 3 or 4)
        54-60 COP =
        61-62 Value of COP (0, 1, 2 or 3)
        Card 15 Tabular values for geometry at y = 0.0 (requires 2 data cards)
        (Not required if INPUT = 2)
    Card 15A (6X, F6.2, 10X, F11.2, 10X, F11.2, 8X, F11.2)
Co1. . 1-6 YT =
    7-12 0.0
    13-22 ABPK =
    23-33 Value of ABPK
    34-43 ABSK =
    44-55 Value of ABSK
```


## Card 15A (Cont'd)

```
Col. 55-62 ABNK =
    63-71 Value of ABNK
    Card 15B (22X, F11.2, 9X, F11.2, 8X, F11.2)
Col. 1-22 APHK =
    23-33 Value of APHK
    34-42 APNK =
    43-53 Value of APNK
    54-61 VCIT =
    62-72 Value of VCIT
    Card 16 Tabular inputs for y greater than 0.0 (requires 2 data cards
        for each y value) (Not required for INPUT = 2)
    Card 16A (6X, F6.2, 10X, F11.2, 10X, F11.2, 8X, F11.2)
Co1. 1-6 YT =
        7-12 Value of YT
        13-22 ABPK =
        23-33 Value of ABPK
        34-43 ABSK =
        44-54 Value of ABSK
    55-62 ABNK =
    63-73 Value of ABNK
    Card 16B (22X, F11.2, 9X, F11.2)
Col. 1-22 APHK =
    23-33 Value of APHK
    34-42 APNK =
    43-53 Value of APNK
```


# Card 17 Non-statistical c.p. grain geometry (Not required for GRAIN $=4)(6 \mathrm{X}$, F10.3, 3X, F10.0) 

Co1. 1-6 $\quad \mathrm{XTZO}=$
7-16 Value of XTZO
17-19 $\quad \mathrm{S}=$
20-29 Value of S
Card 18 Non-statistical star grain geometry (Not reguired for GRAIN $=2)(4 \mathrm{X}, \mathrm{F} 10.0,4 \mathrm{X}, \mathrm{F} 10.0,4 \mathrm{X}, \mathrm{F} 10.0)$

Co1. $1-4 \quad$ NS $=$
5-14 Value of NS
15-18 NP =

19-28 Value of NP

29-32 $\mathrm{NN}=$
33-42 Value of NN
Finally, Figure IV-1 is a schematic representation of the data deck construction, and Table IV-1 presents an example set of data. This is the same data as used in the sample case presented in Section $V$ of this report. Note that these are all the data cards which are required for this example for any number of configurations.

## Program Listing

Figure IV-2 shows a block diagram of the overall program and Table IV-2 presents the complete program listing.

As previously mentioned, the program has been designed to produce graphical presentations of the computational results. Program statements that must be removed in order to delete the plotter compilation requirements are indicated by check marks ( $>$ ) in Table IV-2. Removal of these statements is necessary if the user's computer is not equipped for CALCOMP plotting. However, if other plotters are available, generally only the plotting subroutines need be replaced.


Figure IV-1. Schematic of data deck.




## EXAMPLE DATA SHEETS (CONT'D) <br> TABLE IV-1.



Figure IV-2. Block diagram of computer program.

```
C
```



```
C *
C #
C *
*
*
*
*
*
                            R. H. SFORZINI, W. A. FOSTER, JR. AND J. S. JOHNSON, JR.
                                    AEROSPACE ENGINEERING DEPARTMENT
                                    NOVEMBER 1974*
                    PREPAREI AT AUBURN UNIVERSITY *
                    UNOER MUD. NO. I2 TO CCOPERATIVE AGREEMENT WITH
                                NASA MARSHALL SPACE FLIGHT CEIVTER
                    BY *MCNTE CARLO PERFORMANCE AVNALYSIS DF SRM PAIRS*
```

UNOER MUD. NO. 12 to CCOPERATIVE agreement With ..... *
NASA marshall space flight ceivter ..... *

```**R. H. SFORZINI, W. A. FOSTER,NOVEMBER 1974*
```

C
INTEGER 分RAIN

```
    REAL IDIFF
    REAL NGEA,NDIS,MNOL,MNI,JROCK,N,L,MEI,PE,ISP,ITOT,MU,MASS,ISPVAC
    REAL N1,N2,NSEG,K1,K2,KEH,KEN,NS,LCC,LTAP
    REAL NZ,MCBAR,ISP2,ITVAC,KA,KH,LAMHDA
    CONNON/CDNSTI/ZW,AE,AT,THETA,ALFAN
    COMNON/CONST2/CAPGAN,ME,BUI, ZETAF,TB,HE,GAM
    COMNOIN/CONST3/S,NS,GRAIN
    CONNON/CONST4/CELDI,DC,DI,LC,XT,ZO
    COMNO:Y/CONSTS/KPLT
    COMBON/VARIAI/T,DELY,DELTAT,PCNOL,PHEAD,RNOZ, PHEAD, SUMAB, PHMAX
    CONMON/VARIAZ/ABPORT, ABSLOT, ABNUZ, APHEAD, APMOZ,DADY, ABPZ, ABENZ,ABSZ
    COMMON/VARIA3/ITOT,ITVAC,JROCK,ISP,ISPVAC,FODIS,MNOZ,SG,SUMMT
    COMMON/VARIAL/RNT,RHT,SUMZ,RI,RZ,R3,RHAVE,RNAVE,RBAR,YH,KCUNT
    COMMC&/VARIAS/ABMAIO, ABTO,SUMCY,VCI,VC
    CCMNG:&/VARIAG/YDI.TE
    COMMO:V/VARIAT/Y,IHRLSI
    COMMCN/PLOTT/IPO,iDUY,IPT,IOP
    COMNCIN/PLCT?/:NUPMPL
    COMMEN/OVALA/CHIH,CHIN,SEN,SEH,CHINH,NZ,HZ
    COMMCN/OVAL&/CHINV,CHIHAV,SENN
    COMNCN/OVALC/RGNOCN,ROGDCH,RUROGN,ROIDGH, EXN, EYN, EXH, EYH,
    2ALPHAN,ALPHHIH
    COMmON/OVALN/z,ZQ
    COMmON/OVALMz/KKI,II
    CONMON/SEEC/IX
    COMMCN/PAIRI/TW1,T:Z2,DTW,FW1,FW2,OFW1,DFW2,DFW,TMAXG,DFMQ,
    2FDIFF,TDIFF.NX
    COMMCN/PAIR2/FMAXI, TFMXI,FMI:N1, IFMNI,
    2 FNAX2,TFNX2,FMI:N2, IFNN2
    COMMCN/PAIR3/AFMAX,TFMAX, AFNAXT, IFMAXT
    COMMDN/UUTI/FDIFIG,TDIFIG,DIT,ADIT
    CGMMCN/UUT2/CFIOOK,T100K
    comnon/Dataz/I
    CONMON/TOFF/LFTOL,UFIDZ,TDFTOL,TOFTO2
    DIMENSIO: FCIFF(400), TDIFF(400)
    DIME:VSIOH NUMPLT(5)
```

```
            DATA PI.G/3.14159.32.1725/
            READ(5,500) INRUNS
```



```
C &EAD IN THE NUMBFR OF CONFIGURATIONS TO EE TESTED
```



```
    500 FORMAT (42x,I2)
        NPAIRS=NRUNS/2
        IOP=0
        NPLOT=0
        KPLOT=0
        TWl=0.0
        FW1=0.0
        WRITE(G,111112)
11112 FORMAT(2OX, 'DATA FOR STATISTICAL ANALYSIS PROGRAM')
            CALL SETUP
            DO 901 I=1.NRUNS
            REWIND }
            IXI=IX
            CALL INPUT
            WRITE(6,602) I
    602 FORMAT(1H1.42X, 'CONFIGURATION NUMBER ',I2)
            IF(I-1);CCC,5COO,5001
    5000 READ(5,4)9) SUMDY,AVS,ZW,Y,T,DELTAT,RNOZ,RHEAD,SUMAB,PHMAX,SUMZ.IT
        1OT,RHT,RNT,R1,R2,R3,RHAVE,RNAVE,RBAR, ITVAC, SUIAMT
            WRITE(2,499) SUMOY,AINS, LW,Y,T,DELTAT, RNOZ, RHEAD,SUMAB, PHMAX,SUM2,I
            ITOT,RHT,KNT,R1,R2,R3,RHAVE,RVAVE,RBAR,ITVAC, SUMMT
            GO TO 5002
            5001 READ(2,497) SUNOY,ANS,IW,Y,T,DELTAT,RNOL,RHFAD,SUMAB,PHMAX,SUMZ,IT
            10T, KHT,R,\T,R1,R2,R3,RHAVE,RNAVE,RBAR,ITVAC, SUNMT
            5002 CONTINUE
```



```
C * & INITIAL VALUES OF SELECTED VARIABLES EQUAL IO ZERO *
C * ***NOTE*** THESE VALUES MUST BE ZEROED AT THE BEGINNING OF *
C EACH CCNFIGURATION RUN * *
```



```
    497 FORMAT(22F3.1)
            IF(I-1) 5003,5003,5004
    5003 READ(5,491) IEC,IPO,(NUMPLT(JP),JP=1,5)
        WRITE(2,491) IEO,IPO,(NUNPLT(JP),JP=1,5)
        GO TU 5005
    5004 READ(2.471) IEC,IPO,(NUMPLT(JP),JP=1,5)
    5003 CONTINUE
        491 FORMAT(5x,11,5x,11,9X,5I1)
```



```
C READIN THE USER'S UPTIONS *
```




```
C O FOR NO OVALITY
```


## TABLE IV-2 (CONT'D)


C * PROPELLANT GASES ..... *
C * ..... *
CSTARN $=-17.8475$ *ROAL +5239.7GAMN=ROAL*5.67357E-3+1.11707
C *WRITE 6,6031 RHO, A1, N1, A2, N2, ALPHA, BETA,ROAL, CSTARIN,GAMN
603 FORMAT $/ / / .20 X,{ }^{\prime}$ PROPELLAINT CHARACTERISTICS'./.13X, $/$ RHO $=1 . F 8.6 . / .1$



IF(IPO)4002,40C2,3999
3999 IF(I.EQ.L) CALL GSIZE(12CO.0.11.0.1121)
IF((-1) * $\ddagger$ ) $4000,4000,4001$
4000 REWINO 1
$K P L T=1$
GO TU 4002
4001 KPLT $=2$
4002 CONTINUE
$R H O=R H O / G$
IF(I-1) $3006,5006,5007$
5006 READ $(5,502) \mathrm{L}$, TAU
WRITE 2,502$) \mathrm{L}$, TAU
GO TO 5008
5007 READ 2,502$) \mathrm{L}$, TAU
5008 CONIINUE
IF(IEO) 6CCO.6COO,6 ..... 60016000 READ (4, 11111) LE,DTI,THETA,ALFAN,LTAP,XT,ZO, ZCGO TU 6002
6001 REAU(4, 11111) CE, DTI, THETA, ALFAN,LTAP,XT,ZO,ZC,
2RONDCN, RINDCH, RONDGN, RONDGH, EXN, EYY, EXH, EYH, ALPHAN, ALPHAH
6002 CONTINUE
C $*$ READ IN BASIC MOTOR DIMENSICNS ..... *
C ..... あ
l IS the total length of the grain in inches C $*$ ..... *C * tau is the estimated average web thickness of the controlling *
GRAIN LENGTH IN INCHES C **
C

* THE FOLLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL C ..... ** ANALYSIS PROGRAM
* 


C ..... *
DE IS THE DIAMETER OF THE NOLZLE EXIT IN INCHES * C * de IS the diameter of the nozzle exit in inches ..... *
C * DII IS the initial diameter of the ivollle throat in inches ..... *
C theta is the cant angle of the nozzle with respect to the ..... *

TABLE IV-2 (CONT'D)
FORMAT(3x,F10.2,5x,F10.31
IF(IEO) $5 C 03,6 C 03,6004$
6003 WRITE( 6,6040$)$ L,TAU,DE,DTI,THETA,ALFAN,LTAP,XT,ZO,ZC
GO TO 6005
6004 WRITE $(6,604)$ L,TAU,DE,DTI,THETA, ALFAN,LTAP,XT,LO,ZC,
2RONCCN, RONCCH, RONOGN, RUNDGH, EXIN, EY:N, EXH, EYH, ALPHAN, ALPHAH
600; CONTINUE
604 FORMATI//,20X,'BASIC.MOTOR DIMENSIONS', /,13X,'L=, F8.2,/.13X,
1'TAU $=$ ',F6.3,1,l $3 \times$, DE $=$ ',


$4^{\circ}$, 1PE11.4./.13x,'2C = .


7/,13X,'EYN= ',IPEll.4,/,13X,'EXH= 1,1PELI.4,/,13X,'EYH= 1,1PEll.4,
8/.13X,'ALPHAV = ',1PE11.4,/.13X,'ALPHAH= 1,1PE11.4)

1'TAU = ',FG.3./.13X,'CE= ',


```
    3.,1PEL1.4,/,13X,'LTAP= 0,LPE11.4./.13X,0XT= , 1PE11.4,/,13X,"ZO=
    4',1PE11.4./.13X,'ZC= ',1PEL1.4)
        THETA=THFTA/57.29578
        ALFAN=ALFAN/57.29573
        ALPHAN=ALPHAN/57.27578
        ALPHAH=ALPHAH/57.29578
        IF(I-1) 5CC9,5CO9.5010
5009 REAO\5,503) DELTAY,II,XOUT,DPQUT,ZETAF,TB,HB,PREF,DTREF,PIPK,
        2CSTART, PTRAN, CSTARP,GAMP,TMAXO
        WRITE{2,503) DELTAY,II,XOUT,DPQUI,ZETAF,TB,HB,PREF,DTREF,PIPK,
        2CSTART,PTRAN,CSTARP,GAMP TMAXG
        GO TO 5011
5010 READ{2,503) DELTAY,II,XOUT,DPOUT, ZETAF,YB,HH,PREF,DTREF,PIPK,
    2CSIART, PTRAN,CSTARP,GAMP , TMAXO
5011 CONTINUE
        READ(4,11111) ERREF,IGR,TIGR
```



```
C READ IN BASIC PERFORMANCE CONSTANTS *
C # *
C * DELTAY IS THE DESIRED BURN INCREMENT DURING TAILOFF IN INCHES
C II IS THE NUMBER OF INTEGRATICN STEPS USED IN OVAL *
C XOUT IS THE DISTAVVCE BURIVED IV INCHES AT WHICH THE PROPELLAVT *
C * BREAKS UP
C *
        DPOUT IS THE DEPRESSURIZATION RATE IN LB/IN**3 AT WHICH THE *
                PROPELLANT IS EXTIVGUISHED
        ZETAF IS THE THRUST LOSS COEFFICIENT *
        TMAXQ IS THE ESTIMATEO IIME AT hHICH THE MAXIMUM DYNAMIC *
            PRESSURE OCCURS ON THE VEHICLE *
        TB IS THE ESTIMATED RURV TIME IN SECCNDS *
        HB IS THE ESTIMATED BURNOUT ALTITUDE IN FEET *
        PREF IS THE REFERENCE JOZZLE STAGNATICN PRESSURE *
        DTREF IS THE REFERENCF. THROAT DIAMETER *
        PIPK IS THE TEMPERATURE SENSITIVITY COEFFICIENT OF PRESSURE &
                        PER DEGREE F AT CONSTAIVT K
        *
        CSTART IS THE TEMPERATURE SENSITIVITY PER DEGREE F OF CSTAR *
            AT CONSTAVT PRESSURE *
        CSTARP IS THE PRESSURE SENSITIVITY OF CSTAR
        PTRAN IS THE PRESSURE IV PSIA ABOVE WHICH THE BURNING RATE *
            EXPCNENT CHAINGES
                *
        GAMP IS THE PRESSURE SENSITIVITY OF GAM *
```


## CONIINUE



```
C \(*\) THE FOLLCWING VARIARLES ARE OBTAINED FROM THE STATISTICAL \(\quad *\)
```




```
C *
    ERREF IS THE REFERENCE THROAT ERUSION RATE IN IN/SEC
```

```
C * TGR IS THE BULK TEMPERATURE OF THE GRAIN IN DEGREES F
C * TIGR IS THE IGNITIDN DELAY IN SECONDS AT 6O DEGREES F
```



```
503 FORNAII 8X,F10.3,4X,I4,6X,F10.2,7X,F10.2,7X,F1O.4,/,4X,F10.1,4X,
    2F10.1,oX,F10.2,7X,F10.3,6X,F10.5,/, 8X,F10.7,7X,F10.2,8X,F10.7,
    36X,F10.7./.7X,F10.3)
        WRITE(6,606) UELTAY,II,XOUT,DPOUT,ZETAF,TR,HB,ERREF,PREF,DTREF
        2,TGR,PIPK,CSTART,PTRAN,CSTARP,TIGR,GANP,TMAXQ
    606 FORMAT///, 20X, 'BASIC PERFORMAINCE CONSTANTS',/,13X, DDELTAY= ',F5.3,
    1/,13x,'II= , I4.
    1/,13X,'XOUT= ',F7.2,/,13X,'DPOUT= ',F9.2,/,13X,'ZETAF= 1,F6.4,/.13
    2X,'TG= ',FS.1,/,13X,'HB= ',F7.0./.13X,.ERREF= '
    3,FR.5,/.13X, 'PREF= ,FG.2./,13X,'DTREF=, ,F7.3,/,13X, TGR= , F7.3,
    4/,13x,'PIPK= ',F7.5,/,13X,'CSTART= ',F10.7,/,13X,'PTRA:V= 0,F8.2
```



```
    6/,13X,'TMAXG= ',F7.3)
    TIG=TIGR*EXP(P{PK*(60.0-TGR))
    T=TIG
    A=Al
    N=Nl
    CSTARR=CSTARN*EXP(CSTART*(TGR-GO.))
    GAM=GAM:V
    Q=A&EXP{P{PK*(1.-N!)*(IGR-60.)!
    KKI=0
    CHIH=1.0
    CHIN=1.0
    CHIV:J=1.O
    CHINAV=1.O
    SEN=0.0
    SENN=0.0
    SEH=0.0
    CHINH=1.0
    NDUM=0
    IPT=0
    MNL=.85
    MEl=7.0
    Z=ZO+ZC
    ZQ= ZO
    S=0.0
    NS=0.0
    KOUNT =0
    KEWAT=0
    ABMAI IV=0.0
    ABTO=0.0
    TW2 =0.0
    DTW=0.0
    FW2=0.0
    OFH=0.0
```

```
    DELY=DELTAY
    TOP =GAM+1.
    BOT=GAM-1.
    ZAP=TOP/(2.*BOT)
    CAPGAM=SORT(GAM)*(2./TOP)**2AP
    AE=PI*DE*CE/4.
    1 IF(XT.LE.O.0) TE=0.0
    TCALL=(TAL-XT-ABS(Z/2.) )/1.05
    IFIIEO.EQ.I.AND.Y.GT.TCALLI CALL DVAL
    IF(XT.LE.O.O) GO TO 40
    TL=(Y-TAU+XT+Z/2.)*LTAP/XT
    IF(TL.LE.0.0) TL=0.0
    IF(TL.GE.LTAP) TL=LTAP
    TE=LTAP-LTAP*CHINAV
    IF(IEO.EO.O) TE=TL
40 IF(T-TIG) 41,41,42
41 DT=DTI
    CSTAR=CSTARR
    GO TO 43
42 RADER=ERREF*((PONOZ/PREF)**0.8)*((DTREF/DT)**0.2)
    DT=DT+(2.O*RADER*DELTAT)
43 AT=P1*UT*DT/4.
    CALL AREAS
    IF(Y.LE.O.O) VC=VCI
    IF(ABS(ZW).GT.O.O) GO TO 20
    IF(SUMAB.LE.O.O) GO TO 3l
    X=(ABPORT+AESLOT)/SUMAB
90 MNOL=AT*X/APNOZ*(2.*(1.+BOT/2.*MNL*MN1//TOP)**LAP
    IF(ABS(MNOZ-MNI).LE.0.002) GO TO 2
    MNI=MNOL
    GO TO }9
    2 VNOZ=GAM*CSTAR*HNOZ*SQRT(|(2./TCP)**(TOP/BOT/)/{1.*BOT/2.*MNOZ*MNO
    12)!
    PRAT=(1.*BOT/2.*MNO2*MNOL)**(-GAM/BOT)
    JROCK=AT/APNOL
832 SUMYA=DELY*(ABP2+ABN2+ABS2)
    IFIY.EG.0.OI SUMYA=0.0
    VC=VC+SUNVYA
    IF(Y.GT.O.O) GO TO II
    PONOZ=(Q*RHO*CSTAR*SUMAB/AT)**(1./(1.-N))*(1.*(CAPGAM*JROCK)**2/2.
    1)**(N/(1.-N))
    PON=PUNOZ
    CSTAR=CSTARR*(PONDZ/1000.1**CSTARP
    MOIS =AT*PCNOL/CSTAR
    P2 = PONOL
    PONOZ2=PUNOZ
    PNOZ=PRAT*PCNOZ
    P4=2.*MDIS*VNOZ/(APHEAD + APNOL) + PNOL
```

IF(GRAIN.EG.3) $\quad$ 4 $4=$ MOIS*VNOZ/APNOZ + PNOZ
5 PNOI=PRAT*PCNCZ
PHEAC=2. $\because N D I S * V N O Z /(A P H E A D+A P N O Z)+P N O Z$
IF(GRAIU.EQ.3) PHEAD)=MOIS*VNOZ/APNOZ +P JOZ
IF (PHEAD.LT.PTRAN):V=ill
IF(PHEAD.LT.PIRA:N) $A=A 1$
IF(PHEAD.GE.PTRAN): $=: 12$
IF(PREAD.GE.PIRAS)A=AZ
RHEAD $=0$ © $P H E A D * * i V$
ZIT=MDIS*X/APNOZ
R:NI = RHEAD
PHEAD2 = PHEAC
IF(PUNOL-LT.PTRAN)N=iNl
IF(PGNOZ.LT.PTRAN) $A=A 1$
IF (PONOZ.GE.PTRAV)N=N2
IF(PCNOZ.GE.PTRA:1) $A=A 2$
3 RNOZ=RNI-(IRNI-O*PNOZ**N-ALPHA*2IT**.8/(L**.2*EXP(BETA*RN1*RHO/ZIT
1)))ノ(1.+ALPHA*21T**.8*RETA*RHD/ZIT/(L**.2*EXP(BETA*RNI*RHO/LITl))

IF(ABSIRNI-RNOZ).LE.0.002) GO TO 4
RNI = RNU 2.
GO 103
4 AVEL=(KHEAC+RVGL)/2.
IF(Y.GT.O.O) GG TO 7
RN2 $=$ RNOZ
RH2 $=$ RHEAD
POOJ= PQ:NOZ
DPCDY $=0.0$
$A \cup E 2=A \cup E I$
7 RVAVE=(RINCZ + RUS2)/2.
RHAVE $=($ RHEAD + RH2 $) /$ ?

DRDY=(AVFL-AVE2)/DELY
$R B A R=(A V E 1+A V E 2) / 2$.
GMAX $=1.0002 \mathrm{FP}$ CIS
GMIN=0.9994*MDIS
IF(Y.GT.O.C) GO TO 12
GMAX $=1,001 *$ MDIS
GMIN=0.793*MDIS
If(NGEH.GE.GNIN.AND.MGEN.LE.GMAX) GO TO 6
MDIS = MGEN
PONOZ=MDIS*CSTAR/AT
GO 105
6 POVJ=PONOZ
$1 /$ GAM=GAMN*(PGNOZ/1000.) **GAMP
$T O P=G A M+1$.
BOI $=G A M-1$.
$2 A P=T O P /(2 . * B O T)$
CAPGAM $=\operatorname{SORT}(G A M) *(2 . / T O P) * \# 2 A P$

```
    ME=SQRT(2./BOT*(TOP/2.*|AE*ME1/AAT|**|l./2AP|-1.|!
    IF(ABS(ME-MEL).LE.O.OO2) GOTO 9
    MEL=ME
    GO TO 17
9 IF(Y.LE.O.OI CALL OUTPUT
    IF(Y.LE.U.O) GO TO 10
    DELTAT=2.*DELY/(RHAVE +RNAVE)
    Z=Z+DELTAT*(RNAVE-RHAVE)
    ZQ=ZQ+UELTAT*(RNAVE-RHAVE)
    T=T+DELTAT
    CALL OUTPUT
10 IF(Y.LE..O5*TAU) GO TO 16
    SINK1=VC/(CAPGAM*CSTAR)**2*RBAR*DPCDY/12.
    MASS=.01*MDIS
    ANS4=Y+1C.O*DELTAY
    IF(KOUNT.GT.O) GO TO 16
    IF(ABSISINK1).LE.MASS.AND.ANS4.LE.ANS-XT)GO TO 18
    GO TU 16
1B DELY=10.*DELTAY
    GO TO 55
16 DELY=OELTAY
55 YLED=Y
    Y=Y +DELY
    IFIY.GE.(TAU-XT-Z/Z.).AND.KEHAT.EQ.O\ DELY=TAU-XT-Z/2.-YLED
    IFIY.GE.(TAU-XT-2/2.I.AND.KEWAT.EQ.OI Y=TAU-XT-2/2.
    IFIY.GE.(TAU-XT-Z/2.).AND.KEWAT.EQ.OI KEWAT=1
    ANS=TAU-ABS(Z/2)
    IF(Y.GE.ANS.AND.KOUNT.EQ.O\ DELY=ANS-YLED
    IFIY.GE.ANS.AND.KOUNT.EQ.OI Y=ANS
    DELTAT=2.*DELY/(RHAVE+RNAVE)
    SUM2=SUMAB
    RN2=RNOZ
    RH2=RHEAD
    AVE2=AVE1
    GO TO 1
11 CSTAR=CSTARR* (PONO2/1000.1**CSTARP
    MDIS=AT*PONOZ/CSTAR
    GO TO 5
12DPCDY=(1./(1.-N))*((PHEAD24PONO22)/((ABP2+ABN2+ABS2)*2.)*DADY)
    IF(ABS(DPCDY).GE.DPOUT.OK.Y.GE.XOUT) GO TO 25
    SINKI=VC/(CAPGAM*CSTAR)**2*RBAR*DPCDY/12. +(PHEAD2 + PONOZ2)/2.*(RNAV
    1E+RHAVE)/2.*(ABP2+ABN2+ABS2)/(12.*(CSTAR*CAPGAM)*#2)
    STUFF=MGEN-SINKI
    MDIS=STUFF
    PONO2=MDIS*CSTAR/AT
    IF(2.*Y+DI +DELDI.GE.DO) PONOZ=PDNJ+DPCDY*DELY
    IFISTUFF.GE.GMIN.AND.STUFF.LE.GMAXI GO TO 14
    GO TO 5
```


## TABLE IV-2 (CONT'D)

```
    14 PL=PONOZ
    PONJ= PONOL
    PO\OlZ=(Pl+!2)/2.
    P2=PO.VOL
    P3=PHEAI
    PHE^D2=(P3+P4)/2.
    P4 = PHEAO
    MDIS=AT*PCNOZ/CSTAR
    IF(KEWAT.EQ.1) 50 TO 2221
    GO TO 2222
2221 WRITE(6,2223)
```





```
    KEWAT=KEVIAT+1
2222 CONTINUF
    IF(Y.LT.ANS) SO TO 17
    ZW=L
    YW=Y
    SUMBA=SUNAB
    PI=PCNOL
    RH2=RHEA1)
    RN2=RNU2
    RAVE=AVEI
    ABMAIIN= SUNAB
    ABTO=0.0
    HRITE(6,51)
```




```
    IF((-1)**I.LT.C) TWl=r
    IF((-1)**I.LT.0) Fril= THRLST
    IF((-1) &%I.GT.O) 「W2=T
    IF((-1)**I.GT.C) FWR=IHRLST
    IF(TW2.NG.O.I DTW=ABSITW2-TWI)
    IF(TW2.NE.0.1 CFW=ABS(FW2-FW1)
20 AVS2=TAU+ABS(Ln/2.)
    KOUNT=KOUNT +1
    IF(KOUNT.EG.1) GO IO 17
    DELY'N=OFLTAY
    OY2=DELYW
    IF(LW) 32.32,33
32 IF(Y.LT.ANS2.AIND.ABS(ZW).GT.DY2) GO TO 211
    SUMAB=ARMAIN
    GO TO SL
211 SUMDY = SUN[YY + OELYW
    SUMAB=(1.+SUMDY/ZW-DELYW/(2.*ZW))*^BTO-(SUMCY/ZW-DELYW/(Z.*ZW))*AB
    IMAIN
    GO TO 31
```

```
33 IF(Y.LT.ANS2.AND.ZW.GT.OY2) GO TO 21
    SUMAB=ABTD
    GO 10 31
21 SUMDY = SUPDY + DELYW
    SUMAB=(1.-SUMDY/ZW+DELYW/(2.*2W))*ABMAIN+(SUMDY/ZW-DELYW/(2.*ZW))*
    LABTO
31 IF(SUMAR.LE.O.O) PCNOZ=PONUZ/2.
    IF(SUMAB.LE.O.O) GO TO 25
    CSTAR=CSTARR*(PONOZ/1000.)**CSTARP
    MDIS=AT*PCNCZ/CSIAR
    ABAVE=(SUMAB+SUMBA)/2.
    SUMYA=DELY*ABAVE
    VC=VC+SUNYA
    DADY=(SUMAB-SUMBA)/DELY
    PBAR=(P1+PONOL)/2.
    SUMBA=SUNAH
22 DPCDY=PBAR/(1.-N)*1./ABAVE*DADY
    IF(PONOZ.LE.5.0I GO TO 25
    KNOZ=Q*PCNOZ**N
    RHEAD=RNOL
    RBAR=(RHH:AD+RAVE)/2.
    MGEI=RHO* (RNOZ + RHEAD)/2.*SUMAB
    GMAX=1.0002*MCIS
    GMIN=0.9798*MCIS
    SINK1=VC/(CAPGAM*CSTAR)**2*RBAR*DPCDY/12.*PBAR*ARAVE/|12.*(CAPGAM
    ##CSTAR)**2)*RBAR
    STUFF=MGEN-SINKI
    IF(STUFF.GE.GMIN.AND.STUFF.LE.GMAX) GO TO 23
    MDIS=STUFF
    PONOL=PGIJJ+CPCCY*DELY
    IF(PGNOZ.LE.O.01 PONOZ=0.0
    PBAR=(PI+PCNOL)/2.
    GO TO 22
23RHAVE=(RH2+RHEAD)/2.
    RNAVE=(RN2+RNOZ)/2.
    RH2=RHEAD
    RN2 = RNOL
    PHEAO=PO:1OZ
    RAVE=RHEAD
    PI=PONOZ
    PONJ=PONOZ
    MDIS = AT*PCNOZ/CSTAR
    IF(ABS(DPCOY).GE.DPOUT) GO TO 25
    IF(Y.GE.XOUT) GO TO 25
    GO TU 17
25 SUMAB=0.0
    RHEAD=0.0
    RNOZ = RHEAD
```

TABLE IV-2 (CONT'D)

```
    PHEAD=PONOZ
    MDIS=AT*PGNOZ/CSTAR
    WRITE (6,318)
```




```
        1'1
    DELTAT=2.0*DELY/(RHAVE+RNAVE)
    T=T+DELTAT
    CALL OUTPUT
    IF(PCNOZ.LE.O.O) GO TO 100
    TIME=T
    DELTAT=.5
    TIM=TIME+5.
    PHT = PHEAD
    PONT = PUNOZ
    SG=0.0
    29T=T+DELTAT
    CSTAR=CSTARR*(PONOZ/1000.)**CSTARP
    PHEAD=PHT/EXP(CAPGAN**2*AT*CSTAR/VC*(T-TIME)*12.)
    PONOL = PHEAD
    MDIS = PONOZ * AT / CSTAR
    Y=Y*.5*RHEAO
    CALL OUTPUT
    28 IF(T.LT.TIM.ANC.PHEAD.GE.5.0) GO TO 29
100 WP1 =G*SUMMT
    WP2 = RHO* (VC-VCI)*G
    WP=(WP1+WP2)/2.
    I SP = ITOT/WP
    I SPVAC= ITVAC/hP
    WRITE(6,1022)
1022 FORMATI//, 2OX,'INDIVIDUAL MOTOR DATA')
    WRITE(6,102) WPL,WP2,WP, PHMAX,IXI,IX
102 FORMAT(13X,'WPL= , LPELL.4,/, L3X,'WP2= ',IPELL.4,/,13X,'WP= %,0000
    IIPE11.4,/,13X,'PHMAX= ', IPEL1.4,/,13X,'IXI= , IIO./,13X,'IX= ',
    2110)
    NDUM=1
    IF(IPO.NE.O) CALL DUTPUT
    IF(IPO.EQ.O) GO TO 901
    IF((-1)**I.LT.O) GO TO 901
    CALL PAIR
    CALL SIGHAR(AFMAX,S 1,S2,SAFMAX,BAFMAX,I,NPAIRS,SG1,SG2)
    CALL SIGBAR(TFNAX,SA,SH,STFMAX,BTFMAX,I,NPAIRS,SG3,SG4)
    CALL SIGBAR(AFMAXT,S3,S4,SAFMXT,BAFMXT,I,NPAIRS,SG5,SGG)
    CALL SIGBAR(TFNAXT,SC,SO,STFMXT,BTFMXT,I,NPAIRS,SG7,SG8)
    CALL SIGRARIDFTO1,ST,SU,SDFTO1,BDFTO1,I,NPAIRS,SG19,SG20)
    CALL SIGBARITOFTOL,SHA,SIA,STCFTI,BIOFTI,I, INPAIRS,SG9,SGIOI
    CALL SIGBARIDFTO2,SV,SW,SUFTO2,HDFTO2,I,NPAIKS,SG21,SG221
    CALL SIGRARITDFTO2,SJ,SK,STDFT2,BTDFT2,I,NPAIRS,SGI1,SG12)
```

CALL SIGBARIDTW,SP,SQ,SOTW,BDTW,I,NPAIRS,SGI3,SG14)
CALL SIGBAR(FWI,SLA,SMA,SFWI,HFWI,I,NPAIRS,SGLS,SG16)
CALL SIGBAR(FW2,SN,SO,SFh2,BFW2,I,NPAIRS,SGI7,SG18)
CALL SIGBAR(UFi, L1, L2,SDFW,BDFW, I,NPAIRS,SG37,SG38)
CALL SIGBAR(DFMO,SX,SY,SDFMQ,BUFMQ,I,NPAIRS,SG23,SG24)
CALL SIGBAR(FDIFIG,SE,SF,SFDFIG,BFDFIG,I,NPAIRS,SG25,SG26)
CALL SIGEAR(TOIFIG,SEA,SAF,STEFIG,BTDFIG,I,NPAIRS,SG27,SG28)
CALL SIGBARIDIT,S5,S6,SOIT,HOIT,I,NPAIRS,SG29,SG30I
CALL SIGBARIADIT,S7,S8,SADIT,BACIT,I,NPAIRS,SG31,SG32)
CALL SIGBAR(DF100K,DI,D2,SF100K, EFIOOK,I,IJPAIRS,SG33,SG34)
CALL SIGBAR(T100K, [3,D4,ST100K,BFIOOK,I,NPAIRS,SG35,SG36)
901 continue
IFIIPO.EU.CI STOP
WRITEIC,887)
887 FORMATI//, $20 X,{ }^{\circ}$ STANCARD DEVIATIONS AND MEANS FOR MOTOR PAIR DATA', 2/,14X,'VAR.',6X,' STO. DEV. ',5X,' MEAN ')
WRITE(6,888) SAFMAX, BAFMAX,STFMAX,BTFMAX, SAFMXT,BAFMXT.
2STFMXT, BTFMXT,
2SDFTOL, MCFTOL, STDFT1, BTDFT1,SCFTC2,BDFTO2,STDFT2,BTDFT2,
2SDTW, BDTW,SFWI, BFW1,SFW2, BFW2, SDFW, BDFW, SDFMG, HDFMO,
2SFDFIG, BFEFIG,STDFIG̈,BTDFIG,SDIT,BDIT,SADIT, BADIT,SF100K,BF100K, 2STl00K,HTICOK
888 FORMATIISX, 'AFMAX ',5X,1PE11.4,5X,1PE11.4.1,
213 .' TFMAX $\cdot, 5 \mathrm{X}$, LPE11.4,5X,1PE11.4./,
213X,'AFMAXT',5X, 1PELI.4,5X,1PELL.4,/,
$213 \mathrm{x}, \mathrm{T}$ TFMAXT',5X,1PE11.4,5X,1PE11.4,/,
213x,'LFTOL ',5X,IPEL1.4,5X,1PE11.4,/,
213x, 'TDFIOI',5X,LPE11.4,5x,1PE11.4./.

213x,'TDF102',5X,IPE11.4,5x,1PE11.4./.
213x, DTW 1,5x,1PELI.4,5x,1PE11.4./,
213x, 'FW1 •,5X,1PELI.4,5x,1PE11.4./,
213x,"FW2 •,5x,1PE11.4,5x.1PE11.4./,
213X, 'DFW ',5X,IPEII.4,5X,1PE11.4, /,
213x, 'DFMG $, 5 x, 1 P E 11.4,5 x, 1$ PE11.4,,
213x, 'FDIFIG'.5 5 , IPE1L.4,5X,1PE11.4, /,
213x, 'TOIFIG',5x, LPE11.4,5X,1PE11.4, /,
213x, DIT $\quad, 5 x, 1 P E 11.4,5 x$, IPEII.4./,

213x, 'OF100K',5X,1PE11.4,5X,1PE11.4,/,
213X.'T1OUK ',5X,1PE11.4,5X.1PELI.41
WRITE(6,389) SG1,SG2,SG5,SG6
889 FORMATI//. 20 X, 'ALTERNATE DISPERSIGN VALUES FOR THRUST IMBALANCE OA 2TA', /, 14X,'VAR.', $6 \times, 1$ SICMA 1 ',5X,' SIGMA 2 ',/,
313X,'AFMAX ',5X,1PE11.4,5X,1PE11.4,1,13X,'AFMAXT',5X,1PE11.4.
45x,1PE11.4)

- If(IOP.NE.0) CALL PLOT(0.0.0.0.999)

STOP
END

SUBROUTIIJE AREAS

C * SUBROUTINE AREAS CALCulates buriving areas aisd port areas for *
C * CIRCULAR PERFORATED (C.P.) GRAINS AVD STAR GRAINS OR FOR A *
C * COMBIINATIUN OF C.P. AND STAR GRAIISS *

INTEGER STAR,GRAIN, ORDER,COP
REAL MGEN,MDIS,MVOZ, MINI, JROCK,N,L,MEI,ME,ISP,ITOT,MU,MASS,ISPVAC
REAL LGCI, LGiNI,NS,N:N,IN, LGSI, VT,LTP,LGC,LS,LF
REAL M2,MDBAR,ISP2,ITVAC,L1,L2,LFW,LFWSQD
COMMON/CONSTI/ZW, AE, AT, THETA,ALFAN
COMMON/CONST3/S, NS, GRAI:V
COMPON/CONST4/DELDI,DO,DI,ZC,XT, IO
COMMON/VARIAI/T, DELY,DFLTAT, PONOZ, PHEAD, RNOZ, RHEAD, SUMAB, PHMAX
COMMON/VARIAZ/ABPORT, ABSLOT, ABNOZ, APHEAD, AP VOZ, DADY, ABPZ, ABNZ, ABS 2
COMMON/VARIA3/ITOT, ITVAC, JROCK, ISP, ISPVAC,MDIS,MVOZ, SG, SUMMT
COMMON/VARIA4/RNT,RHT, SUN2,R1,R2,R3,RHAVE,RNAVE,RBAR,YB,KOUNT
COMMON/VARIA5/ABMAIN, ABTO,SUMDY,VCI,VC
COMMON/VARIAG/YDI,TE
COMMON/VARIAT/Y, THRUST
COMMGN/OVAL^/CHIH,CHIN,SEN,SEH,CHINH,AZ,BZ
commongdaraz/idata
DATA PI/3.14159/
21111 FORMAT(E16.9)
$A B P C=0.0$
$A B \cup C=0.0$
$A B S C=0.0$
$A B P S=0.0$
$A B: U S=0.0$
$\triangle B S S=0.0$
DABT $=0.0$
$S G=0.0$
VCIT=0.0
ANUM $=\mathrm{PI} / 4$.
PID2=P1/2.
RNT $=$ RNT + RNOZ * DELTAT
RHT $=$ RHT + RHEAD*DELTAT
IF(Y.LE.O.O) AGS=0.0
$K=0$
IF(ABSIZW).GT.0.0) K=1
$Y B=Y$
IF(K.EQ.I) $Y=Y B-S U M D Y / 2$.
2 IF(K.EQ.2) $Y=Y B+A B S(Z W) / 2 .-S U M O Y / 2$.
IF(IDATA-1) 5000,5000,5001
5000 IF(Y.LE.O.OI READ(5.500) INPUT,GRAIN,STAR,NT,ORDER,COP IF(Y.LE.O.0) WRITE 2,500 ) INPUT,GRAIN,STAR,NT,ORDER,COP
GO TO 5002
5001 IF(Y.LE. 0.01 READ(2,500) INPUT,GRAIIJ,STAR,NT,ORDER,COP

5002 CUNTINUE


```
    9 DENOM=YT-YTZ
        SLOPE1 = (ABPK-ABPK2)/DENOM
        SLOPE2 = (ABSK-ABSK2)/DEINOM
        SLOPE3=(ABNK-ABNK2)/DENOM
        SLOPE4=(APHK-APHK2)/DE:NOM
        SLOPE5 = (APNK - APNK2)/UENOM
        B1=ABPK-SLOPE1*YT
        B2=ABSK-SLOPE 2 % YT
        B3=ABNKK-SLOPE 3*YT
        B4=APHK-SLOPE4*YT
        B5 = APNK SLOPES*YT
        ABPT=SLOPEl*Y+BL
        \triangleBST=SLOPE 2*Y+R2
        ABNT=SLOPE 3*Y+B3
        APHT = SLOPE4*Y +84
        APNT =SLOPE5*Y+B5
        IF(INPUT.EQ.3) GO TO 3
        GO TO 52
        6 IF(IDATA-1) 5003,5003,5004
    5003 READ(5,507) YT,ARPK,ABSK,ABNK, APHK,APNK,VCIT
        WRITE(2,507) YT,ABPK,ABSK,ABNK,APHK,APNK,VCIT
        GO TO 5005
    5004 READ(2,507) YT,ABPK,ABSK,ABNK,APHK, APNK,VCIT
    5005 CONTINUE
```



```
    507 FORMAII6X,F6.2,10X,F11.2,10X,F11.2,8X,F11.2,1,22X,F11.2,9X,F11.2,
        1 8X,F11.21
            WRITE{6,610)
        610 FORMATII3X,'TABULAR VALUES FOR YT EQUAL ZERO READ IN'I
            HRITE(6,583) ABPK,ABSK,ABNK, APHK,APNK
        583 FORMAT\13X,'ABPK=',1PE11.4,5X,'ABSK=', IPE11.4.5X, 'ABNK='.1PE11.4,
        1 5X, 'APHK=', 1PE11.4.5X,'APNK=', 1PE 11.4)
            WRITE(6,584) VCIT
        584 FORMAT(13X,'VCIT=`.1PEII.4,//)
            ABPT = ABPK
            ABST=ABSK
            ABNT=ABNK
            APHT = APHK
```

```
    APINT = APNK
    YT2=YT
    IF(INPUT.EQ.3) GO TU 3
    VCI=VCIT
    GO TO 52
    8 YT2=YT
        ABPK2 = \triangleBPK
        ABNK2 = AB:NK
        ABSK2 = ABSK
        APHK2 = APHK
        APNK2 = APNK
        IF(IDATA-1) 5006,5006,5007
    5006 READ(5,505) YT,APPK,ABSK,ABIJK,APHK,APNK
        WRITE(2,,O5) YT,ABPK,AGSK,AB:YK, APHK,APNK
        GO TU 500B
    5007 READ(2,5U5) YT,ABPK,ABSK,ABNK, APHK, APNK
    5008 CONTINUE
```



```
C * REAU IN TABULAR VALUES FOR Y=Y (VOT REQUIRED FOR IVPUT=2) *
```



```
    505 FORMAT(6x,F6.2,10x,F11.2,10x,F11.2,8X,F11.2,1,22X,F11.2.9X,F11.2)
        WRITE(6,GIL) YT
    611 FORMAT(////,13X,'TABULAR VALUES FOR YT= ',FT.3,' READ IN')
        WRITE(6,583) ABPK,ABSK,ABNK,APHK,APNK
        GO TU 9
    12 ABPT=0.0
        ABNT}=0.
        ABST=0.0
    3 IF(GRAIN.NE.2) GO TO 4
        ABPC=0.0
        ABNC=0.0
        ABSC=0.0
        GO TU 7
    4 IFIIDATA-1) 5009,5009,5010
5009 IF(Y.LE.0.0) READ(5,501) XTZO,S
        IF(Y.LE.0.0) WRITE(2.501) XTZO.S
        GO rO 5011
    5010 IF(Y.LE.0.0) READ(2.501) XTZO.S
501L CONTIINUE
    IF(Y.LE.O.O) READ(4,21LIL) DO,DI,THETAG,LGCI,LGNI,THETCN,THETCH
```



```
C * READ IN BASIC GFOMETRY FOR C.P. GRAIN (NOT REQUIRED FOR *
C &TRAIGHT STAR GRAIN) *
C * XTZO IS THE DIFFERENCE BETWEEN THE INITIAL.INTERNAL GRAIN *
C * DIAMETER AT THE NOZZLE END OF LGCI AND DI IN INCHES
    LESS TWICE XT ANO L.ESS ZO *
C * S IS THE NUMBER IIF FLAT HURNIN
    THE NOZZLE ENDJ
```

```
C 
#
```



```
C. *
C *THE FOLLUWING VARIABLES ARE OBTAINED FROM THE STATISTICAL＊
```

ANALYSIS PROGRAM ..... ＊
もも ..... ＊

```C \(*\) DO IS THE AVERAGE DUTSIDE INITIAL GRAIN DIAMETER IN INCHES
```

DI 15 THE AVERAGE INITIAL INTERNL GRAIN DIAMETER IN INCHES
DI IS THE AVERAGE INITIAL INTERNAL GRAIN DIAMETER IN INCHES ..... ＊
C

```＊
```

＊ THETAG IS THE ANGLE THE IJOLLLE END OF THE GRAIN MAKES WITH ..... ＊
THE MOTOR AXIS IN DEGREES ..... ＊
＊
LGNI IS THE INITIAL SLANT LENGTH OF THE BURNING CONICAL ..... ＊
LGCI IS THE INITIAL TOTAL LENGTH OF THE CIRGULAR PERFORATION IN INCHES ..... ＊
C
GRAIN AT THE NOZZLE END IN INCHES ..... ＊ ..... C
THETCN IS THE COJTRACTION ANGLE OF THE BONDED GRAIN IN DEGREES＊
C も THETCH IS THE CONTRACTION ANGLE AT THE HEAD END IN DEGREES ..... ＊
C

```
501 FORMAT(6X,F10.3,3X,F10.0)
            IF(Y.LE.0.0) WRITE(6,601) DO,DI,XTZO,S,THETAG,LGCI,LGNI,THETCN,TH
    IETCH
601 FORMAT(20X, 'C.P. GRAIN GEOMETRY',/. 13X,'DO= !,F7.3./.13X, DI= ',F7
```



```
    2X,'LGCI= ',F7.2,/,13X,'LGVI= ',F6.2,/,13X,'THETCN= ',F8.5,/,13X,
    3'THETCH= 0,F8.5,//1
        IF(Y.LE.O.O) TAU=(DO-DI)/2.0
        IF(Y.LE.0.0) DELDI=2.0*XT+20+XT20
        IF(Y.LE.0.0)THETAS=「HETAG/57.29578
        IF(Y.LE.0.0)THETCN=THETCN/57.29578
        IF(Y.LE.O.OITHETCH=THFTCH/57.29578
        OOSOD=00**00
        DISQD=DI*OI
        BNUM=ANUM*DOSQD
        TLL=TE
        IF(ORDER.GE.3) TLL=0.0
        YDI=2.*Y + OI
        YDISQD=YDI*YDI
        ABSC=S*ANUM*(DOSQD-YDISQD)
        IF(ABSC.LE.O.O) ABSC=0.0
        IF(YOI.GE.DOIGO TO 100
        IF(THETAG.GT.0.08727) GO TO LOL
        IF(COP.EQ.O) GO TO 700
        IF(COP.EG.1) GO TO 701
        IF(COP.EQ.2) GO TO }70
        CHCK1=DOSQO-YOISQD
        IF(CHCKL.LY.O.O) CHCKI=0.O
        LGC=LGCI - (SQRT(DOSQD-DISQD)-SORT(CHCKI))/2.-Y*CDTAN(THETCN)
        GO TO 710
    702 CHCKI=DOSQD-YDISQD
```

IFICHCKI.LT.O.O) CHCKI $=0.0$
LGC = LGCI-(SQRT(DUSOD-DISQD)-SQRT(CHCK1))
GO TO 710
701 CHCK2=DOSQD- (YDI + JELDI) $\ddagger \neq 2$
IF(CHCK2.LT.0.O) CHCK2=0.0
LGC=LUCI-(SORT(DOSOD-(DI +1)ELDI) **2)-SQRT(CHCK2))/2.
$2-Y \not \subset C O T A I S$ (THETCH)
GO TU 710
700 LGC=LGCI-Y*(COTAN(THETCV) +COTAN(THETCH))
710 ABPC=PI*YDI* (LGC-TLL-S*Y)
$A B N C=0.0$
GO 13 732.
101 CONTINUE
IF(COP.EQ.O.OR.COP.EQ.I) GO TO 720
CHCKI =DOSQO-YOISQD
IF(CHCK1.LT.O.O) CHCKI $=0.0$LGC=LGCI-(SORT(DUSOD-DISOD)-SORT(CHCK1))/2.-TLL
2-(S + TAiv(THETAS/2.)) *Y
$A B P C=P I * Y D I * L G C$
GO TO 73 C
720 LGC=LUCI-Y*COTAN(THETCH)-TLL-(S+IAN(THETAG/2.)) $\ddagger$ Y
$A B P C=P I * Y[I * L G C$
730 IFICOP.EQ.L.OR.COP.EQ.21 GOTO 731
$A B: V C=P I *(L G V I-Y * C O T \wedge N(T H E T A G+T H E T C N)-Y * T A N(T H E T A G / 2)) \neq.(D I+$
1 DELDI +Y + LGNI*SIV(THETAG) + Y*SIN(THETCN)/SIN(THETAG + THETCiN))
GO TO 732
731 IF(Y.LE.O.O) GOTO 7311
GO TO 7312
1 SQRT((DO/2.)**2-((DI+DELDI)/2.+LGVI*SIN(THETAG))**2)
7312 IF(R7+Y.LT.(OO/2.) $\%$ COS(THETAU)) GO TO 11111$A B, N C=P I *(L G V I+(1 . / S I V(T H E T A G)) \neq(1 D O / 2)-.L G V I * S I N(T H E T A G)-(D I$2-DELDI)/2.)-Y*COTAN(THETAG)-Y*COTAN(THETAG/2.) ) *( C (-DELDI)/2.$3+Y+0 U / 2.1$GO TO 22222
$11111 \mathrm{RPR}=\mathrm{SQRT}((1 \mathrm{DO} / 2) * * 2)-.R 7 \neq * 2)-S Q R T(1(D 0 / 2) * * 2)-.(R 7+Y) * * 2)$
1 2.) $* 2-(R 7+Y) * \leftarrow 2) * S I N(T H E T A G)+Y+(R T+Y) * C O S(T H E T A G))$
22222 CUNTINUE
732 JF(ABPC.LE.O.O) ABPC=0.0
IF(ABNC.LE.0.0) ABVC=0.0$100 \mathrm{ABNC}=0.0$
$A B P C=0.0$

IF(APHT.GE•ENUM) APHT=BNUM
IFIK.LT.2) APHTI=APHTAPNT = AJUM* (DI + DELDI + 2 *R:NT) ** 2


THE FIJLCWING VASIABLES ARE OBTAISED FROM THE STATISTICAL*

```
6 ANALYSIS PROGRAM ..... *
C * RIWW is the average radius of the inside of the propellant **WEB IN INCHES
```

l1 aind l2 are the lengths of the two parallel sioes of the ..... *
TWO SETS OF STAR POINTS IN INCHES ..... *
alphal and alphaz are the aingles between the slavt sides of ..... *

```THE STAR POIVTS CORRESPGNDING TO Ll ANO L2, RESPECTIVELY,*and the cevter llines of the points in degrees *
```

hW is half the width of the star poivis in inches ..... *
C
WRITE(6,422) RIWW,LI,L2, ALPHA1, ALPHA2, HW

```
422 FORMAT(20X, 'WAGON WHEEL GEOMETRY',/.13X, 'RIWW= ',F5.2./.13X.
    1 'LI= , F5.2./.13X,'L2= , F5.2,/.13X,'ALPHA1= , F7.5./.13X,
    2 'ALPHA2= ',F7.5./,13X,'HW= ',F5.2.//1
        IF(Y.LE.O.0) TAUWIW=RC-RIWW
        IF(Y.LE.C.O.ANU.GRAIN.EQ.2) DI=DO-2.O*TAUWW
    ALPHAL=ALPHA1/55.29578
    ALPHA2=ALPHA2/57.29578
    AL-P2 =ALPHAL
    XL2=L 2
    LFW=RC-IAUWW-FILL
    LFWSQD=LFW*LFW
    THETFW=ARSIN((HW+FILL)/LFW)
    SLFW=LF**SIN(THETFN)
179 KKK=0
    SG=0.0
    ENUM=(RCSQD-LFWSQD-FYSQD)/(2.*LFW*FY)
    ALPHA2=ALP2
    L2=XL2
190 YTAN=Y*TAN(ALPHA2/2.)
    COSALP=COS (ALPHA2)
    SINALP=SI:N(ALPHAC)
    IF(YTAN.GT.L2) GO TO 182
    IF(FY.GT.SLFW) GO TO 18L
    SGW=NP*(L2-2.*YTAN+(SLFW-FILL)/SINALP-Y*COTAN(ALPHA2) +FY*
    1(PID2 + IHETFW) +(LFN+FY)&(PIDNP-THETFW))
    GO TO 183
181 IF(Y.GT.TAUHW) GO TO 184
    SGW=NP*(FY*(PIDNP+ARSIN(SLFW/FYI)+(PIDNP-THETFW)*LFW)
    GO 1O 183
184 SGW=NP*FY*(THETFW+ARSIN(SLFW/FY)-ARCOS(ENUM))
    GO TO 183
182 YPO=-SLFW
```

TABLE IV-2 (CONT ${ }^{\text { }} \mathrm{D}$ )

```
    IF(ALPHA2.GE.PID2) GO TO 222
    Q=-FILL+L2*TAN(ALPHA2)-Y/COSALP
    XPI=(-Q*TAN(ALPHA2)-SQRT(-Q*Q+FYSQD/COSALP*COSALP))*COSALP*COSALP
    YPI = XPI*TAN(ALPHA2) +Q
    XPO=(YPO-Q)*COTAN(ALPHAZ)
    GO TO 223
    222 XPI=Y-L2
    YPI=-SQRT(FYSQD-XPI*XPI)
    XPD=XPI
    223 FYLS=SQRT(SLFW*SLFW+XPI*XPI)
        XPIO2 = (XPI-XPO) # (XPI -XPO)
        YPIO2 = (YPI-YPO)*(YPI-YPO)
        IF(FY.GT.FYLS) GO TO 186
        IF(Y.GE.TAUWW) GO TO 185
        SGW=NP*(SQRT(XPIO2+YPIO2) +FY*(PID2+THETFW-ARSIN(XPI/FY))+(LFW+FY)*
    1 (P[DNP-THETFW))
    GO TO 183
    185SGW=NP*(SQRT(XPIO2+YPIO2)+FY*(PIO2-ARSIN(XPI/FY)-ARCOS(ENUM)))
    GO TO 183
    186 IF(Y.GT.TAUWW) GO TO 1B7
    SGW=NP*(FY*(PIDNP*ARSIN(SLFW/FY)) +(PIDNP-THETFW)*LFW)
    GO TO 183
    187 SGW=NP&FY&(THETFW+ARSIN(SLFW/FY)-ARCOS(ENUM))
    183 IF(SGW.LE.O.O) SGW=0.0
            IF(Y.GT.0.O) GO TO 188
            AGS2=.5*(PI*RCSQD-NP*LFW*SLFW*(COS(THETFW)-SIN(THETFW)*COTAN(ALPHA
            1 2)-2.*(L2+FILL*IAN(ALPHA2/2.))/LFW)-(PI-THETFW*NP)*LFWSQD-2.*NP*F
            2ILL*(L2+SLFW/SINALP+LFW*(PIOIP-THETFW) + (PIDNP+PID2-1./SINALP)*
            .1FILL/2.1)
            AGS=AGS+AGS2
    188 CONTINUE
    SG=SG+SGW
    IF(KKK.EQ.1) GO IO 24
    L2=L1
    ALPHA2=ALPHAL
    KKK=1
    GO TO }19
    201 IF(Y.LE.O.0) REAO(4,21111)RP,RIS
```



```
C * READ IN GEOMETRY FOR IRUNCATED STAR INOT REQUIRED FOR *
C * STANDARD STAR OR WAGON WHEEL) *
C # *)
```



```
C * THE FULLOWING VARIABLES ARE OBTAIINED FROM THE STATISTICAL . *
                    THE FULLOWING VARIABLES ARE OBTAIINED FROM THE STATISTICAL . **
                    THE FULLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL . *NALYSIS PROSRAM
C *
```



```
C *
C RP IS THF INITIAL RADIUS OF THE TRUNCATION IN INCHES
```

TABLE IV-2 (CONT'D)

## *

RIS IS THE AVERAGE RADIUS OF THE INSIDE OF THE PROPELLANT \# HEB IN INCHES * OF THE SLOTS IN INCHES

IF(Y.LE.0.0) $\mathrm{HRITE}(6,603)$ RP, RIS
 1 , F7.3.//1
IF(Y.LE.O.O) TAUS=RC-RIS
IF(Y.LE.O.O.AIND.GRAIN.EQ.2) DI=DO-2.OFTAUS
THETAS = PIDNP
$R P Y=R P+Y$
$L S=R C-T A U S-F I L L-R P$
$R P L=R P+L S$
THETSI = THETAS-ARSIN(FY/RPY)
IF(THETSI.LE.O.O) GO TO 110
IF(Y.LE.TAUS) GO TO 103
THETAC=ARSIN((RCSOD-RPL*RPL-FYSQD)/(2.*FY*RPL) )
IF (THETAC.GE.0.0) GO TO 104
IF(Y.LT.RC-RP) GO TO 105
$S G=0.0$
GO TO 14
103 SG $=2$. *NP* $(R P Y * T H E T S 1+L S-(R P Y * C O S(T H E T A S-T H E T S 1)-R P)+P I D 2 * F Y)$
GO TO 14
104 SG=2.*iNP*(RPY*THETSI +LS-(RPY*COS(THETAS-THETSI)-RP) +FY*THETAC)
GO TO 14
105 SG=2.*TP*(RPY*THETS $1+$ SQRT(RCSQD-FYSQD)-SQRT(RPY*RPY-FYSQD))
14 IF (Y.LE.O.O) AGS=PI*(RCSQD-RP*RP)-NP*(PI*FILL*FILL/2.+2.*LS*FILL)
GOTO 31
110 THETAF=THETAS
THETAP = 2. \#THETAS
TAUWS = TAUS
GO TO 111
20 IF(Y.LE.O.0) READ(4,21111) THETAF,THETAP,RIWS

C * READ IN GEDMETRY FOR STAVDARD STAR (VOT REQUIRED FOR *
C * TRUNCATED STAR OR WAGON WHEEL) *
C $*$. $\quad$ *

C $\neq$ THE FULLOWIING VARIABLES ARE OBTAINED FROM THE STATISTICAL $\quad *$
C $*$ ANALYSIS PROGRAM $\quad *$

C $\ddagger$ 俍
C $*$ THETAF IS THE ANGLE LOCATION OF THE FILLET CENTER IN DEGREES *
C $*$ THETAP IS THE ANULE OF THE STAR POINT IN DEGREES *
C * RIWS IS THE AVERAGE RAOIUS OF THF IVSIDE OF THE PROPELLANT *
C $*$ WEB IN ITCHES $*$

IF(Y.LE.0.0) WRITE (6, 604) THETAF,THETAP,RIWS

```
604 FORMATI20X,'STANOARD STAR GEOMETRY',/.13X,'THETAF= ',FT.5./.13X,'T
    1HETAP = .,F7.5,/,13X,.RIWS= .,F6.3.1/1
        IF(Y.LE.O.0) TAUWS=RC-RIWS
        IF(Y.LE.O.O.AVD.GRAIN.EQ.2) DI=DO-2.0*TAUWS
        THETAF=THETAF/57.29578
        THETAP=THETAP/57.29578
        THETAS=PI/INP
        THETSI=1.00
111LF=RC-TAUWS-FILL
        CNUM=(Y+FILL)/LF
        DNUM=SIN(THETAFI/SIN(THETAP/2.)
        ENUM=(RCSQD-LF*LF-FYSQD)/(2.*LF*FY)
        FNUM=SIN(THETAF)/COS(THETAP/2.)
        IF(CNUM.LE.FVUM) GO TO l06
        IF(Y.LE.TAUWS)GO TO 107
        SG=2.**NP*FY*(THETAF+ARSIN(SIN(THETAF)/CNUM)-ARCOS(ENUM))
        GO TO 23
106 IF(Y.LE.TAUWS) SG=2.*NP*LF*(DNUN+CNUM*(PID2+THETAS-THETAP/2.
    1-COTAN(THETAP/2.))+THETAS-THETAF)
        IF(Y.LE.TAUWS) GO TO 23
        SG=2.*NP*(FY*ARSINIENUM-(THETAS-THETAP/2.))+LF*DNUM-FY*COTAN\THETA
    1P/2.)|
        GO T0 23
107 SG=2.*NP*LF*(CNUM*(THETAS+ARSIN(SIN(THETAF)/CNUM))+THETAS-THETAF)
    23 IF(THETSI.LE.0.0) GO TO 14
        IF(Y.LE.0.0) AGS=PI#RC*RC-iNP*LF*LF*(SIV(THETAF)*(COS(THETAF)-
        LSIN(THETAF)*COTAN(THETAP/2.)) +THETAS-THETAF+2.*FILL/LF*(SIN(THETAF
        2)/SIH(THETAP/2.)+THETAS-THETAF+FILL/(2.*LF)*(PID2+THETAS-THE
        3TAP/2.-COTAN(THETAP/2.)lll
24 CONTINUE
31 IF(SG.LE.O.0) SG=0.0
    IF(K.EQ.O.OR.K.EQ.2) SÓN=SG
    IF(K.LE.1) SGH=SG
    IF(Y.LE.0.0) SG2=SG
    IF(K.EQ.2) GO TO 37
    RAVEDT=RI+(SG+SG2)/2.*RBAR*DELTAT
    RNDT=R2+(SG+SG2)/2.*RNAVE*DELTAT
    RHDT=R3+(SG+SG2)/2.*RHAVE*DELTAT
    RI=RAVEDT
    R2=RINDT
    R3=RHD T
    SG2=SG
    GO TO 38
37 IF(KOUNT.NE.1) GO TO }3
    SG3=SG
    R4=R1
    R5=R2
    R6=R3
```

```
39 RAVEDT \(=24+(S G+S G 3) / 2\). FRBAR \(=\) DELTAT
    RNDT=RS+(SG+SG3)/2.*RNAVE*DELTAT
    RHDT \(=\) R6 \(+(S G+S G 3) / 2\) **RHAVE*DELTAT
    R4 = RAVEDT
    R5 = RND T
    R6 \(=\) RHD \(T\)
    SG3=SG
38 ABSS = (AGS-RAVEDT)*NS
    IFIABSS.LE.O.O.OR.SG.LE.O.0) ABSS \(=0.0\)
    \(A B N S=(A G S-R N D T) * N N\)
    IF(ABNS.LE.O.0.OR.SG.LE.O.0) ABNS \(=0.0\)
    IF(ORDER.LE.2) ABPS=(LGSI-Y*(NS+NN))*SG
    IFIORDER.LE.2) GO TO 36
    ABPS = (LGSI-TE-Y\# (idStiviN) ) \#SG
```

36. PIRCRC=PI\#RCSQD
$A P H S=P I R C R C-A G S+R H D T$
IFIAPHS.GE.PIRCRC.OR.SG.LE.O.OI APHS=PIRCRC
APNS = PIRCRC-AGS+RINDT
IF(K.LT.Z) APHSL=APHS
IF(APNS.GE.PIRCRC) APNS=PIRCRC
50 IFIMT.EQ. 0.01 GO TO 371
IF(Y.LE.0.0) READ (4,21111) LTP, DTP, THETTP,TAUEFF

C * READ IN GEOMETRY ASSOCIATED WITH TERMINATION PORTS INOT *
C. REOUIRED IF NT=OI *
C *

C * the fullowing variables are obialined from the statistical *
C * ANALYSIS PROGRAM *

C *
C * LTP IS THE INITIAL LENGTH OF THE TERMINATIOV PASSAGES *
C *
IN INCHES *
otp is the initial diameter of the termination passage *
IN INCHES
thettp is the acute angle between the axis of the passage *
AND THE MOTOR AXIS IN DEGREES *
taueff is the estimated effective web thickness at the *
C * TERMINATION PORT IN INCHES *

IF(Y.LE. O.0) WRITE (6,606) LTP,DIP,THETTP,TAUEFF


THETTP = THETTP/57.29578
DABT=NT*3.14159*((OTP + 2.*Y) *(LTP-Y/SIN(THETTP))-(OTP*2.*Y)**2/4.*
1(Y+DTP/2.)*(DTP/2.)*(1.-1.1SIN(THETTP)))
IF(Y.GE.TAUEFF) DABT $=0.0$
371 IF(Y.GT.O.O) GO TO 52

TABLE IV-2 (CONT'D)

```
    IF(NT.NE.O.O) GO TO 45
    LTP=0.0
    DTP=0.0
45 IF(GRAIN.NE.2) GO TO 49
    LGC 1=0.0
    LGNI=0.0
    DISOD=0.0
    DOSQD=4.*RCSQD
    47 [F(GRAIN.EQ.1) LGSI=0.0
    VCI=1.1*(ANUM*DISSD*(LGCI +LGNI) +(ANUM*DOSQD-AGS)*LGSI +NT*LTP*ANUM*
    1 DTP*OTP)+VCIT
    52 BBP=0.0
    BBS=0.0
    BBN=0.0
    IF(K.NE.O) GO TO 52I
    IF(KKL.EQ.O.AND.KKM.EQ.O) GO TO 521
    IF(KKL.EQ.O) ABPC=ABPC*(BZ+AZ*CHIN)
    IF(KKM.EQ.O) \triangleBPC=ABPC*(AL#CHIH+BZ)
    IF(KKL.EQ.O.AND.GRAIN.EQ.2) ABPS=ABPS*(BZ+AZ*CHIN)
    IF(KKM.EQ.O.AND.GRAIV.EQ.2) ABPS=ABPS*(AZ*CHIH+8Z)
521 ABPORT = AHPT + ABPC + ABPS + OABT + BBP
    ABSLOT =ABST+ABSC+ABSS+BBS
    ABNOL=AB:VT +ABNC+ABNS +BBN
    IF(K.GE.2) GO TO 55555
    SUMAR=ABPORT +ABSLOT +ABNOZ
55555 CONTINUE
    IF(K.EQ.O) GO TO 99
    IF(2W) 322,323,323
    322 IF(K.EQ.1) ABPORT=ABPORT*CHIN
    IF(K.EQ.2.AND.GRAIN.NE.2) ABPORT=ABPORT+SEH#LGC
    IF(K.EQ.2.A.VD.GRAIN.EQ.2) ABPORT=ABPORT+SEH*(LGSI-Y*(NS+NN))
    GO TO 33333
323 IF(K.EQ.I) ABPORT=ABPORT*CHIH
    IF(K.EQ.2.AND.GRAIN.NE.2I ABPORT=ABPORT+SEN*LGC
    IF(K.EQ.2.AND.GRAIN.EQ.2) ABPORT=ABPORT+SEN*(LGSI-Y*(NS+NN))
33333 IF(K.EQ.l) ABMAIN=ABPORT +ABSLOT+ABNOZ
    K=K+1
    IF(K.GT.2) GO TO 69
    GO TO 2
69 ABTO=ABPORT+ABSLOT + ABNOZ
99 CONTINUE
    IFIY.GT.O.OI GO TO 70
    ABP1=ABPORT
    ABNL=ABNOL
    ABS1=ABSLOT
70 ABP2=(ABP1+ABPORT)/2.
    ABN2=(AB:U1 + ABNOZ )/2.
    ABS2=(ABS1+ABSLUT)/2.
```

IFIINPUT.EQ.1) GO TO 76
GO TO (71,72.73.74),ORDER
71 APHEAD=APHSI
APNOZ=APNT
$S G=S G H$
GO 1075
72 APMEAD=APHTI
$A P N O Z=A P N T$
$S G=0.0$
IF(GRAIN.EQ.3) SG=(SGH+SGN)/2.
GO 1075
73 APHEAD=APHTI
$A P: N O Z=A P$ IVS
$S G=S G N$
GO TO 75
74 APHEAD=APHS 1
APNOL=APNS
$S G=S G N$
GU TO 75
76 APHEAD=APHT
APNOL = APINT
$75 \mathrm{Y}=\mathrm{Y} \mathrm{B}$
DIFF $=$ SUM. $1 B$ B-SUM2
DADY=DIFF/DELY
$A B P 1=A B P O R T$
$A B \cap 1=A B N U Z$
$A B S 1=A B S L O T$
IF(ZW.GE.0.0) GO 1077
$A B M I=A B M A I N$
ABMAIN=ABTO
$A B T O=A B M I$
77 RETURN
END


```
    IF(F.LE.U.O) F=0.0
    IF(Y.LE.0.0) F2=F
    FBAR=(F+F2)/2.
    ITOT=ITOT+FBAR*DELTAT
    M2 = MDIS
    F2=F
    IF(PHEAD.GT.PHMAX) PHMAX=PHEAD
    GO 1O 47
45 F=0.0
47 WRITE(6,1) T,YB,PONOL,PHEAD,SUMAB,F,ITOT
    l FORMATI//,13X,'TIME = ,F7.3,12X,'Y= ',F6.3,/,13X,
```



```
    3. F= ',IPE11.4,0 ITOT= ',1PE11.4)
    IFIIPO.EQ.O) RETURN
    TPLOT(NP)=T
    FPLOT(NP)=F
    ITPLOT(NP)=ITOT
    IF(TPLOT(NP).LT.100.) GO TO 50
    NTO=NTO+1
    TOTPLT(NTO)=T
    TOFPLT(:VTO)=F
5 0 ~ R E T U R N
    2 NP=NP+2
        NTO=NTO+2
        1OP=1
        IF(KPLT-1) 4000,4000,4001
4 0 0 0 ~ N P 2 = N P - 2 ~
        NTO2=NTO-2
        WRITE(1,40021 NP2
4002 FORMAT(I4)
    WRITE(1,4003) (FPLOT(I),ITPLOT(I),TPLOT(I),I=1,NP2)
4 0 0 3 ~ F O R M A T ( E L 2 . 5 ) ~
    WRITE(1,4002) NTO2
    WRITE(1,4003) (TOFPLT(I),TOTPLT(I),I=1,NTO2)
    GO TO 1004
4001 REWIND I
    IF(IPO.VE.3) WRITE(6,9998)
9998 FORMAT(//, 20X,'TABULATED IMBALAVCE DATA',/,
    213X,' IIME !,IOX,' FDIFF ',10X.' IDIFF ',
    210X.' IADIFF '1
    READ(1,4002) NP21
    REAO(1,4003) (FPLOT1(I),ITPLTI(I),TPLOTI(I),I=1,NP21)
    READ(1,4002) NTO1
    READ(1,4003) (TOFPLI(I),TOTPLI(I),I=1,NTO1)
    NP1=NP21+2
- IF(IPO.EC.2) GO TO 8888
- IFIICOUNT.EQ.2I YSFT=1.5
- IFINUMPLT(I).NE.OI GO TO 7001
```

```
            CALL PLOTITIFPLOT1,TPLOT1,NPI,FPLOT,TPLOT,NP,'THRUST (LBSI',12g
            2'TIME (SECS)',-11.0.0.400000.0.0.0.10.0.9.0.YSFT)
-7001 'TIME ISECS
- IF(:NUMPLT(1).NE.0) XSFT=9.0
- NTL=iNTOL+2
- IFINUMPLT(2).NE.OI GO TO }700
- IF(NUMPLT(1).EQ.O) YSFT=0.0
- CALL PLOIITITOFPLI,TOTPLI,ITL,TOFPLT,TOTPLT,NTO,'THRUST (LESS:, 12.
    2'TIME (SECS)',-11,0.0.400000.0.100.0,2.0.XSFT,YSFY).
-7002 XSFT=18.0
    IF(NUMPLT(1).VE.O.AND.NUNPLT(2).NE.O) XSFT=9.0
-8888 CONTINUE
    IF(NPL-NP) 2000,2000,2001
    2000 NX=NP-2
    NY=NP1-2
    CALL INTERP{TPLOT,FPLOT,NX,TPLOTL,FPLOTL,NY,FDIFF,O\
    CALL INTERP(TPLOT,ITPLOT,NX,TPLOTI,ITPLTI,NY,IDIFF,I)
    TDIFIG=TPLOT(I)
    FDIFIG=ABS(FDIFF(I))
    DO 3000 J=2,NX
    IF(TPLOT(J).GT..O2*T8) GO TO 3001
    IF(ABS(FUIFF(J)).LT.ABS(FDIFF(J-1))) GO TO 3000
    FDIFIG=ABS(FDIFF(J))
    TDIFIG=TPLOT(J)
3000 CONTINUE
3001 CONTINUE
    00 2004 I= 1,NX
2004 TDIFF(!)=TPLOT(I)
    LUMI=U.O
    IADIFF(1)=ABS(FDIFF(1)/2.)*TPLOT(1)
    DO 2003 I=2,.vX
    FBARI=(FDIFF(I)+FDIFF(I-1))/2.
    DUMI =ABS(FBARI)*(TPLOT(I)-TPLOT(I-1))
2003 IADIFF({)=IADIFF({-I)+DUMI
    IF(IPO.NE.3) WRITE(6,9999) (TPLOT(I),FDIFF(I),IDIFF(I),IADIFF(I).
        21=1,NX)
9999 FORMAT(13X,IPE11.4,IOX,IPEL1.4,10X,1PE11.4,1OX,IPE11.4)
    TI=AMIN1(TW1,TW2)
    CALL INTRPI(IUIFF,TPLOT,NX,TI,DITL,O)
    DIT=IDIFF(NX)-DIT1
    CALL INTRPI(IADIFF,TPLOT,NX,TI,ADIT1,0)
    ADIT=IADIFF(NX)-ADITI
    CALL INTRPIIFDIFF,TPLUT,NX,TMAXQ,DFMQ,O)
    CALL INTRPI(FOIFF,TPLOT,NX,TWL,OFWl,O)
    CALL INTRPI(FUIFF,TPLOT,NX, TN2,DFW2,0)
    CALL INTRPI(TPLOT,FPLOT,NX,100000.,T100K2,1)
    CALL INTKPI(TPLOTL,FPLOTL,NY,10C000.,T10OK1,1)
    T100K=AMAX1(T100K1,T100K2)
```

CALL INTRPI(FDIFF,TPLOT,NX,T100K,DF100K,0)

| - | IFIIPO.EQ.2) GO TO 8887 |
| :---: | :---: |
| - | CALL SCALEIIADIFF,8.0, ix, 11 |
| - | YSCALI $=-4$ SS (8.0*1ADIFF(NX +2$)$ ) |
| - | YSCAL2 $=$ ABS (2.0*1ADIFF ( $N X+2$ ) |
| - | $N \mathrm{~N}=\mathrm{NX}+2$ |
| - | IF(NUMPLT(3).NE.0) G0 ro 7003 |
| - | IFINUMPLT(1).EQ.O.OR.NUMPLT(2).EQ.0) YSFT=0.0 |
| - |  |
|  | 2'TIME (SECS)',-11,-400000.,100000.0.0,26.0.4.0,XSFT,YSFTI |
| -7003 | XSFT=9.0 |
| - | IFINUMPLT(3).VE.0) XSFT=18.0 |
| - | IF(NUMPLT(4).NE.0) GU TO 7004 |
| - | IF(NUMPLT(1).EQ.O.OR.NUMPLT(2).EQ.O.OR.NUMPLT(3).EQ.0) YSFT=0.0 |
| - | CALL PLOTI(TPLOT, IDIFF, NX, 'ImPULSE [MBALANCE (LB-SECS)',27, |
|  | 2'TIME (SECS)',-11,YSCAL1,YSCAL2,0.0,26.0,4.0,XSFT,YSFT) |
| -7004 | XSFT=9.0 |
|  | IF(NUMPLT(3).NE.O.ANU.NUMPLT(4).NE.0) XSFT=18.0 |
|  | 1F(NUMPLT(5).NE.0) GO TO 7005 |
|  | IFINUMPLT(1).EQ.O.OR.NUMPLT(2).EQ.O.OR.NUMPLT(3).EQ.O.OR.NUMPLT(4) |
|  | 2.EQ.01 YSFT $=0.0$ |
|  | Call ploti (tplot, iadiffonx, 4 BS. Impulse imbalance (lb-SECS)',32. |
| - 2 | 2'TIME (SECS)',-11,IADIFF(NX-1), IADIFF(NX),0.0,26.0,0.0,XSFT,YSFT) |
| - 7005 | contivue |
|  | $n X=N X-5$ |
| -8887 | contidue |
|  | GO TO 1004 |
| 2001 | NX='VP1-2 |
|  | $N Y=N P-2$ |
|  | CALL INTERP(TPLOT1,FPLOT1, VX, TPLOT,FPLOT, NY,FDIFF,0) |
|  | CALL INTERP(TPLOTI, ITPLTI, NX, TPLOT, ITPLOT, NY, IDIFF, 11 |
|  | TDIFIG=TPLOTl(1) |
|  | FDIFIG=AHS(FDIFF(1) |
|  | DO $3002 \mathrm{~J}=2$, NX |
|  | IF(TPLOT(J).GT..O2*TR) G0 T0 3003 |
|  | IFIABS(FSIFF(J)).LT.ABSIFDIFF(J-1)) GO TO 3002 |
|  | FDIFIG=AHS(FDIFF(J)) |
|  | FDIFIG=FDIFF(J) |
|  | TOIFIG=TPLOTI(J) . |
| 3002 | Continue |
| 3003 | contivue |
|  | $0020051=1, N X$ |
| 2005 | IDIFF(I) = TPLOTI(I) |
|  | DUM1 $=0.0$ |
|  | 1ADIFF(1) $=$ ABS(FDIFF(1)/2.)*TPLOT1(1) |
|  | DO $2002 \mathrm{I}=2$, NX |
|  | FBARI $=($ FDIFF (I) + FDIFF(I-1) $/ 2$. |
|  | DUML $=$ ABS (FBARI)*(TPLOTI(I)-TPLOTI(I-1) |

```
    2002
    IADIFF(I)=IADIFF(I-1)+OUMI
        IF(IPO.NE.3) WRITE(6,9999) (TPLOTL(I),FDIFF(II;IDIFFIII,IADIFFIII.
        21=1,NX)
            TI=AMIN1(TWL,TW2)
            CALL INTRPI(IDIFF,TPLOT1,NX,TI,DITI,O)
            DIT=IDIFF(NX)-OITI
            CALL INTRPI(INDIFF,TPLOTI,:\X,TI,ADITI,O)
            ADIT=IADIFF(:NX)-ADITL
            CALL INTKPI(FDIFF,TPLDTI,NX,TMAXQ,DFMO,O)
            CALL IUTRPI(FOIFF,TOLUT1,NX,TNL,DFWL,O)
            CALL INTRPI(FDIFF,TPLOTL,NX,TH2,DFW2,O)
            CALL INTKPL(TPLOT,FPLOT,NX,100000.,T1OOK2,1)
            CALL IVTRPI(TPLOTL,FPLOTL,ivY,10C000.,T100KL,1)
            TIOOK=AMAX1(T1OOKL,T1OOK2)
            CALL INTRPI(FOIFF,TPLOTL,NX,T1OOK,DFIOOK,O)
            IF(IPO.EO.2) GU TD 1004
                            CALL SCALE(IADIFF,8.0,iNX,1)
                            YSCALI=-nBS(8.O*IAOIFF(NX+2))
                            YSCAL2=ABS(2.0*IADIFF(iNX+2))
    NX=NX+2
    IF(NUMPLT(3).NE.O) GO TO }700
    IF(INUMPLT(1).EQ.O.OR.NUMPLT(2).EQ.O) YSFT=0.O
    CALL PLOTI(TPLOTL,FOIFF,NX, 'THRUST IMBALANCE (LBS)',22.
    2'TIME (SECS)',-11,-400000.,100000.,0.0.26.0.4.0.XSFT,YSFT)
7006 XSFT=9.0
            IF(NUMPLT(3).NE.0) XSFT=18.0
            IF(INUMPLT(4).NE.O) GO TO 7007
            IF(NUMPLT(1).EQ.O.OR.NUMPLT(2).EG.O.OR.NUMPLI(3).EQ.O) YSFT=0.O
            CALL PLOTI(TPLOTl,IDIFF,NX,'IMPULSE IMBALANCE (LA-SECS)',27.
    2'TIME (SECS)',-11,YSCAL1,YSCAL2,0.0.26.0.4.0.XSFT,YSFT)
7007 XSFT=9.0
            IF(NUMPLT(3).NE.O.ANO.NUMPLT(4).NE.0) XSFT=18.0
            IF(NUMPLT(5).NE.O) GO TO 7008
                            IF(NUMPLT(1).EQ.O.OR.NUMPLT(2).EO.O.OR.NUMPLT(3).EQ.O.OR.NUMPLT(4)
                        2.EQ.0) YSFT=0.0
                            CALL PLOTI(TPLOTl,IADIFF,HX,'ABS. IMPULSE IMBALANCE (LB-SECS)',32,
                            2'TIME (SECS)',-11.IADIFF(NX-1),IADIFF(NX),0.0.26.0,0.0,XSFT,YSFT)
7003 CONTINUE
            NX=?NX-2
    1004 CONTINUE
            RETURIV
            END
```

```
- SUBROUTI NE PLOTIT YY1, X1,NP1,Y2,X2,NP2,YHDR,NY,XHDR,NX,SY1,SYZ,
- 2 SXI, SX2, XSFT,YSFTI
- DIMENSIDI XHDR(8), YHOR(8), X1(NP1),Y1(NP1),X2(NP2),Y2(NP2)
- NL=iNPL-2
- NSI =NPI-I
- N2 = NP2-2
- NS2 = :NP2-1
- X1(NSI) \(=5 \times 1\)
- \(\times 1(N P 1)=S \times 2\)
- \(\quad\) x2(NS2) \(=5 \times 1\)
- \(\mathrm{X2}(\mathrm{NP} 2)=5 \times 2\)
- YL(NSI)=SYL
- Y1(VPl) \(=\) SY2
- Y2(:JS2)=SY1
- Y2(NP2)=SY2
- CALL PLOT(XSFT,YSFT,-3)
- CALL AXISIO.0.0.0, YHDR, NY, 8.0.90.0, SY1,SY2)
CALL AXIS(0.0,0.0,XHDR,NX,14.0.0.0, SX1, SX2)
CALL LINE (X1,Y1, \(11,1,0,1)\)
CALL LINE (X2,Y2, iv2, \(1,0,21\)
- NPLOT = :VPLOT +1
- RETURN
- END
```

```
    SUBROUTINE OVAL
    REAL MI.NI
    COMMON/CONST4/DELDI,DO,DI,ZC,XT,20
    COMMON/VARIAT/Y
    COMMON/OVALM/Z,ZQ
    COMMON/OVALM2/KKI,II
    COMMON/OVALA/CHIH,CHIN,SEN,SEH,CHINH,AZ,BZ
    COMMON/OVALB/CHINN, CHI:NAV,SENN
    COMMON/OVALC/RCNOCN,RONDCH,RONDGN,RONDGH, EXN,EYN,EXH,EYH,
2ALPHAN,ALPHAH
    DATA Pl/3.14159/
    KKI=KKI +1
    IF(KKI.GT.l) GO. TO 8
    AGN=(RUNDGN+SQRT(RONDGN**2+DI**2)}/2
    BGN=AGV-RONDGN
    AGH=(RONDGH+SQRT (RONDGH**2+DI**2))/2.
    BGH=AGH-RONDGH
    DTH=2.*PI/II
    KKM=0
    KKL=0
    KKJ=0
    KKXT=0
    KKP=0
    AX=0.
    AZ=0.
    BZ=0.
    ACN=(RONOCN+(RGNDCN**2+(DO-2C)**2)**.5)/2.
    BC:N=ACiv-RONDCiN
    ACH=(RO:NDCH+(RCNDCH**2+(DOO+2C)**2)**.5)/2.
    BCH=ACH-RONDCH
    A1N=(COS(ALPHAN))**2+(ACN/BCN)**2*(SIN(ALPHAN))**2
    A1H=(COS(ALPHAH))**2+(ACH/BCH)*#2*(SIN(ALPHAH))**2
    B1N=((ACN/BCN)**2-1.)*SIN(2.*ALPHAN)
    B1H=((ACH/BCH)**2-1.)*SIN(2.*ALPHAH)
    C1:V=2.*(EXN*COS(ALPHAN)-(ACN/BCH)**2*EYN*SIV(ALPHABN))
    C1H=2.*(EXH*COS(ALPHAH)-(ACH/BCH)**2*EYH*SIN(ALPHAH))
    D1:N=2.*((ACN/BCN)**2*EYN*COS(ALPHAN)-EXN*SIN(ALPHAN))
    D1H=2.*((ACH/ECH)**2*EYH*COS(ALPHAH)-EXH*SIN(ALPHAH))
    EIV=(SIN(ALPHAN))*#2+(ACN/BCN) %%2*(COS(ALPHAN))**2
    ElH=(SIN(ALPHAH))**2+(ACH/HCH)**2*(COS(ALPHAH))**2
    F1N=ACN**2-EX.V**2-( (ACN/BCN)*EYV)**2
    F1H=ACH**2-EXH**2- ((ACH/RCH)*EYH)**2
    SENNO=PI*(DO-ZC)
    SENO= SEN:NO
    SEHO=PI*(DO+ZC)
8 KK=0
    YO=Y
3 IFIKK.EQ.1) Y=YO+ZQ/2.
```


## TABLE IV-2 (CONT'D)

```
    IF(KK.EO.1) GO TO 5
2 IF(KK.EQ.2) Y=YO-ZQ/2.
    IF(KK.EQ.2) SO TO 6
    IFIKK.EQ.O.AND.XT.GT.O.) Y=YO+XT+2Q/2.
    IFIKK.EQ.O.AND.XT.GT.O.I GO TO }
    KK=1
    GO 10 3
5 THE TA=0.0
    SUMO=0.
    DO 12 I= 1,II
    THETA=THETA +DTH
    MI=AIN*(COS(THETA))**2+BIN*SIN(THETA)*COS(THETA) +
    2E1N*(SIN(THETA))**2
    NI=CLN*CCS(THETA)+DIN*SIN(THETA)
    RC=(-N1+SQRTINI**2+4.*MI*F1:N))/(2.*M1)
    IF(RC.LT.0.) RC=(-NI-SQRT(NI##2+4.*M1*F1N))/(2.*M1)
    RG=SQRT(1./((COS(THETA)/(AG:N+Y))**2+(SIN(THETA)/(BGN+Y))**2))
    IF(RG.GE.RC) KKM=1
    IF(RG.GE.RC) RG=0.
    SUMO=SUMSHRG*DTH
12 CONTIIUE
    IFIKKM.EQ.1) SEN=SUMO
    IFISUMO.LE.O.I SEN=0.
    IF(KKM.EO.O) GO TO 9
    CHIN=SEN/SE:NO
    CHINAV=(1.+CHIHN)/2.
    IF(XT.LE.O.0) CHIVAV=1.0
    IF(KKJ.EQ.1) CHINAV=(1.-AX)*CHIN+AX*CHINN
    CHINH=(1.+CHIN)/2.
9 KK=2
    IFIZ.GE.O.O.AVD.KKM.EQ.OI GO TO 62
    GO 102
6 THETA=0.0
    SUMO=0.0
    DO 13 I=1,1I
    THETA = THETA +DTH
    Ml=A1H*(COS(THETA))**2+B1H*SIN(THETA)*COS(THETA)*
    2EIH*(SIO(THETA)|**2
    NL=CIH*C(JS(THETA) +DIH*SIIN(THETA)
    RC=(-NI+SGRT(N1**2+4.*MI*FIH))/(2.*M1)
    IF(RC.LT.O.) KC=(-N1-SQRT(N1**2+4.*M1*FIH))/(2.*M1)
    RG=SQRT(1./((COS(THETA)/(AGH+Y))**2+(SIN(THETA)/(BGH+Y))*&2))
    IF(RG.GE.RC) KKL=1
    IF(RG.GE.RC) KC=O.
    SUMO=SUMOITRG*DTH
13 CONTINUE
    IF(KKL.EQ.1) SEH=SUMO
    IFISUMO.LE.O.I SEH=O.
```

```
    CHIH=SEH/SEHO
    IF(KKL.EQ.O) CHIH=1.O
    CHINH=(1.+CHIH)/2.
    IF(KKM.EQ.1) CHINH=(SEN+SEH)/\SENO+SEHO)
    GO TO 62
    7 THETA=0.0
    SUMO=0.
    DD 11 I= 1,II
    THETA=THETA + DTH
    M1=A1N*(COS(THETA))**2+B1N*SIN(THETA)*COS(THETA)*
    2E1N*(SIN(IHETA))##2
    NL=ClN*COS(THETAI +0liN*SIN(THETA)
    RC=(-iNl+SQRT(VL**2+4.*ML*FLN))/(2.*Ml)
    IF(RC.LT.O.) RC=(-N1-SQRT(NL**2+4.*M1*F1N))/(2.*M1)
    RG=SQRT(1./((COS(THETA)/(AGV+Y))**2+(SIN(THETA)/(BGN+Y))**2))
    IF(RG.GE.RC) KKJ=1
    IF(RG.GE.RC) RG=0.
    SUMO=SUMO+RG*DIH
11 CONTINUE
    IF(KKJ.EQ.1) SENN=SUMO
    IF(SUMO.LE.O.) SENIN=0.O
    IF(KKJ.EQ.O) GO TO 9
    CHININ=SENN/SENNO
    KKXT=KKXT+1
    IF(KKXT.EQ.1) YXIP=Y
    AX=(Y-YXIP)/(XT+DO/2.-DI/2.-YXIP)
    IF(AX.LE.O.) AX=0.
    IF(AX.GE.1.0) AX=1.0
    CHIVAV=1.-AX+AX*CHINN
    KK=1
    IF(AX.LE.O.5.AND.XT.GE.0.02097#DO) GO TO 9
    GO TO 3
62Y=Y0
    IFIKKL.EQ.O.AND.KKM.EQ.O) GO TO 63
    KKP=KKPP+1
    IF(KKP.EQ.1) YZIP=Y
    AZ=(Y-YZIP)/(ABS(Z)/2.+DO/2.-DI/2.-YZIP)
    IF(AZ.LE.O.) }AL=0
    BZ=1.-AZ
6 3 \text { CONTINUE}
    RETURN
    END
```


## TABLE IV-2 (CONT'D)

```
    SURROUTINE SETUP
    INTEGER TEMPCO.CODE
    REAL T(200)
    DEAL ANS(60)
    REAL TEMPA(10),CONST(60)
    INTEGFR OROER(6JI,CNSTNM
    - EAL fxarfa(2,100)
    REAL XX(105),YY(105)
    REAL PSEUNO(105)
    PEAL X(40,105),Y(105),FX(40,105)
```



```
C. * IF THE DIMENSION IF X AND FX ARE CHANGED M AND N SHOULD *
```

C * ALSO RE RESET ..... *

REAL MODE,MEAN,MI,MZ,K,INC
COMMON/SEED/IX
INPTNM=0
C.NSTNM=0

```
    N=105
    VI=100
    VSI=10
    M=40
    MM=0
    NII=NI+1
    NSII=NST+1
    REAO(5,100)IX
    30 CDNTINUE
    READ(5,106) NAM1,NAM2
10G FORMAT(2A4)
    REAO(5,102)CNDE, X1, X2, X3, X4, X5, X6, \times7
    WRITE(6,107) NAM1,NA42,COOE,X1,X2,X3,X4, X5,X6,X7
107 FORMAT(1X,2A4,5X,12,5X,7(1PE11.4,3X))
    IFICDDE.ED.90I GU TO 399
    INPTNM= INPTNM+1
    IF(COOF.EQ.60IGO TO }35
    MM=MM+1
    ORDER(INPTNM)=MM
    TEMDCD=CONE/10
    GO TO (31,32,33,3+,35),TEMPCD
    31 CJNTINUE
    NOI=X4
    VOII=NDI +1
    X(MM,1)= X2
    DO 311 I=2,NOI
    X(MM,I)=X(MM,I-1)+X3
    3Il CJNTINIJE
    DO 312 I=l,NOI
    Y(I)=0.
```

TABLE IV-2 (CONT'D)

```
312 CONTINUE
    H=X3
    STARTR=X2-X3/2.
    SUM=0.
    vOV=x1
    NOC=(xl+9.)/10.
    D] 313 JJ=1,NOC
    READ(5,104)(TEMPA(I),I=1,10)
    WRITE(6,109) (TFMPA(I),I=1,10)
    D# 314 J=1,10
    IF(JJ*10+J.GT.NOV)GO TO 317
    DO 315 1=1,NOI
    IF(TEMPA(J).LT.X(MM,I)+X3/2.1GO TO 316
315 continue
    GO TO 314
316 CONTINUE
    Y(I)=Y(I)+1.
    SUM=SUM+1.
314 CONTINUF -
313 CONTINUF
317 CONTINUE
    IF(CODF.EQ.11)GO TO 99
    FX(MY, l)=0.
    DO 318 I=2,NOIl
    FX(MM,I)=FX(MM,I-1)+Y(I-1)/SUM
318 CONTINUE
    GO T0 30
    32 CONTINUE
    NOI=XI
    X(MM,1)=X2
    DO 220 I=2,NOI
    X(MM,I) = X(MM,I-1) +X3
220 CONTINUF
    READ(5,104)(Y(I),I=1,NOI)
    WRITE(6,109) (Y(I),I=1,NCI)
    H=X3
    STARTR=X2-X3/2.
    IFICODE.EQ.21IGO TO 99
    SUM=0.
    OO 222 I=1,NOI
    SUM=SUM+Y(I)
222 CDNTINUE
    NOIl=NOI+I
    FX(MM,1)=0.
    DO 221 I=2,NOII
    FX(MM,I)=FX(MM,I-1)+Y(I)/SUM
221 CONTINUE
    GJ TO 30
```

```
    33 CONTINUF
    MEAN=X1
    S2=x1
    U2=X2
    U3 = X3
    U4=X4
    H=X5
    STARTR=X6
    SUMX=XT
    GO T0 331
    34 CDNTINUE
    NOI=xI
    X(MM, 1)=X2
    DO 341 I=2,NOI
    X(MM, I) = X(MM, [-1)+X3
341 CONTINUF.
    READ(5,104)(FX(MM,1),I=1,NOI)
    WRITE(6,109) (FX(M:4,I),I=1,NOI)
109 FORMAT(5X,1PIOEII.41
    GO TO 30
    35 CONTINUE
    CDDE=CODE-50
    GO TO(351,352,353,354,3551,CODE
351 CONTINUE
    MEAN=Xl
    SIGMA=x2
    IFIXG.FO.O.1X6=MEAN-3.*SIGMA
    IF(X7.EO.O.)X7=MEAN+3.*SIGMA
    x0=x6
    XV=X7
1351 CONTINUE
    H=(XN-XO)/FLONT!NI)
    D=H/FLOAT(NSI)
    X(MM, l)=X0
    INC=(XN-XO)/FLOAT(NI)
    DO 201 I=2,NTI
    X(MM,I) = X(MM,I-I)+H
201 CONTINUE
    00 202 J=2,NII
    T(1)=X(:AM,J-1)
    DN 203 KK=2.NSII
    T(KK)=T(KK-1)+D
203 CONTINUF
    0O 204 L= L,NSII
    Y(L)=(1./(SORT(6.2832)*SIGMA)|*(EXP(-.5*(IT(L)-MEAN)/SIGMA)**2))
204 CONTINUE
    CALL CAREAIY,FX,M,N,MM,NSI,J,DI
202 CONTINISE
```

```
    DD 205 I=2,NII
    FX(MM,I)=FX(MM,I)/FX(MM,NII)
205 CONTINUF
    GO TO 30
352 CONTINUE
    1VC=( x2-x1)/FLOAT(NI)
    X(MM,1)=X1
    DO 3521 I=2,NI1
    X(MM,I)=X(MM,I-1)+INC
3521 CONTINUE
    INC=1./FLOAT(NI)
    FX(MM, l)=0.
    DO 3522 I=2,NII
    FX(MM,I)=FX(MM,I-1)+INC
3522 CONTINUE
    GO TO 30
353 CONTINUE
    MEAN=X1
    SIGMA = X2
    XO=MEAN
    IFIX7.EO.O.1 X7=MEAN+3.*SIGMA
    XN=X7
    GO TO 1351
354 CONTINUF
355 CONTINUE
    gO TO 30
356 CONTINUE
    C.NSTNM=C.NSTNM+1
    ORDER(INPTNM) = 100+CNSTNM
    CONST(CNSTNM)=X1
    GO TO 30
    99 MEAN=0.
    SUMX=0.
    S1=0.
    S2=0.
    S 3=0.
    S4=0.
    S5=0.
    DO 200 L=1,NOI
    I=NClI-L+1
    SUMX=SUMX+Y(L)
    SI=SI+Y(I)
    S2=S2+S1
    S 3 = S 3+S2
    S4=$4+53
    S5 =55+54
200 CONTINUE
    MEAN=S2/SUMX
```

```
    S2=S2/SUMX
    S3=S3/SUMX
    S4= 54/SIJMX
    S5=S5/SUMX
    U2=2.*S3-S2*(1.+52)
    U3=6.*S4-3.*U2*(1.+S2)-S2*(1.+S2)*(2.+S 2)
    U4=24.*S5-2.*(13*(2.*(1.+52)+1.)-(12*(6.*(1.+5 2)*(2.+52)-1.)
    9 -S2*(1.+S2)*(2.+S2)*(3.+S2)
331 CONTINUE
    Bl=U3**2/U2**3
    R2=114/012**2
    K=(Bl*(82+3.)**2)/(4.*(2.*B2-3.**&1-6.)*(4.*B2-3.*B1)
    IF(K)1,98,94
    1 O=(6.*(B2-Bl-1.1)/(6.+3.*BL-2.**2)
    COM=B1*(R+2.)**2+16.*(R+1.)
    A1^2=.5*SORT(12)*SOPT(CUM)
    COM12=R*(R+2.)*SQRT(S1/COM)
    IFIU3.LT.0.1COM12=-COM12
    M2=.5*(R-2.+COM12)
    Ml=.5*(R-2.-C.JM12)
    YO=(SUMX/A1A2)*(N1**M1*M2**M2)/(M1+M2)**(M1+M2)*GAMMA(M1+M2+2.)/
    9(GAMMA(M1+1.)*GAMMA(M2+1.))
    AZ=A1AZ/(M1/MZ+1.)
    A1=A1A2-42
    MOOF=MEAN-.5*J3/U2*(R+2.)/(R-2.)
    MODE=4ODE*H+STAETR
    INC=A1A2/FLOAT(N)
    X(:MN, 1)=MODF+(-A) * %H
    X(MM.NIl)=MODt +\Delta2**+
    H=(X(MM,NII)-X(MM,I))/FLПAT(NI)
    X(MM, 2)=STARTR
    DO }706\quadI=3,N
    X(MM,I)=X(MM,I-1) +H
706 CONTINUE
    PSEUDO(1)=-A1
    PSEUDO(NILI)=A2
    H=1142/NI
    D] 701 I=2,NI
    PSEUDD(I)= PSEUDO(I-1)+H
701 CONTINUF
    D=H/FLOAT(NSI)
    DO 702 J=2.NIL
    T(1)=PSEUOO(.1-1)
    DO 703 KK=2,NSIL
    T(KK)=T(KK-1)+D
703 CONTINUE
    D] }704\textrm{L}=1.NSI
    Y(L)=Y0*(1.+T(L)/A1)**Ml*(1.-T(L)/A2)**M2
```

```
704 RONTINUE
    [ALL RAREA(Y,FX,M,N,MM,NSI,J,D)
702 CONTINUE
    DO 705 I=2,NT1
    FX(MM,I)=FX(MM,I)/FX(MM,NII)
705 EONTINUE
    GO TO 30
    94 IF(K-1)4,96,6
    4 \text { CONTINUE}
        R={6.*(B2-81-1.) //(2.*B2-3.*B1-6.)
        Ml=.5*(R+2.)
        COM=SQRT(16.*(R-1.)-R1*(R-2.)**2)
        V=(-R*(R-2.)*SORT(BL))/COM
        IF(U3.GE.O.)T, TO 44
        V=ABS (V)
    44 CONTINIE
        Al=SORT(U2/16.1*COM
        M.]DE=4EAN-(113*(R-2.))/((2.+U2)*(R+2.1)
        THETA=ATAN(V/R)
        IF(R.LE.lO.)GO TO 48
        Al=Al*H
        YO=SUMX/Al*SDRT(R/6.2832)*(EXP(COS(THETA)**2/(3.*R)-1.1
        9(12.*2)-THETA*V))/(COS(THETA))**(R+1)
    4 8 \text { CONTINUE}
        QRIGIN=MEAN+V*AI/R
        H=2.*ORIGIN/FIGAT(NI)
        D=H/FLIAT(NSI)
        X(MM,1)=-כRIGIN
        no 711 I=2,NI1
        X(AM,I)=X(MM,I-1)+H
711 CONTINUF
    DO 712 J=2,NII
    T(1)=X(MM,J-1)
    DO 713 KK=2,NSII
    T(KK)=T(KK-1)+D
713 CONTINUF.
    DO 71+L=1,NSII
    Y(L)=Y0*(1.+T(L)**2/Al**2)**(-M1)*EXP(-V*ATAN(T(L)/Al))
714 CONTINUE
    CALL CAREA(Y,FX,M,N,MM,NSI,J,D)
712 CONTINUE
    DO 715 I=2,NII
    FX(MM,I)=FX(MM,I)/FX(MM,NII)
715 CONTINUF
    DO 716 I=1,N11
    X(MM,I) = X(MM,I I ORIGIN
716 CONTINUE
    GO TO 30
```

```
    6 CONTINUE
    IMEAN = MEAN
    MEANI=MEAN-IMEAN
    Z=(6.*(R2-R1-1.) )/(6.+3.*BL-2.*B2)
    COM=H1*(2+2.)**2+16.*(R+1.)
    AL=.5*SOP.T(U2)*SORT(COA)
    IF(U3.LT.O.)AL=-(ABS(Al))
    COML2=(R*(R+2.I)/2.*SORT(BL/COM)
    M1=-((R-2.)/2.-COM12)
    M2 = (R-2.1/2. +C חM12
    YO=(A1*#(M1-M2-1.|/GAPMA(M1-M2-1.|)*(GAMMA(M1)/GAMMA(M2+1.|)*SUMX
    .72IGIN=MEAN-(A1*(M1-1.))/(M1-M2-2.)
    MO\capE= YF\triangleN-. 5*!)3/U2*(R+2.)/(R-2.)
    XV=AL+XN/H
    SAVEH=H
    H=(XN-AI)/FLOAT(NI)
    D=H/FLOAT(NSI)
    X(MM,1)=A1
    D? 721 I=2,NII
    X(MM,I)=X(MM,I-1)+H
    721 C.ONTINUF
    D7 722 J=2,NII
    T(1)=X(MM,J-1)
    DO 723 KK=2,NSII
    T(KK)=r(KK-1)+D
723 CONT INIJE
    DO 72.4 L=1,NSIL
    Y(L)=Y0*(T(L)-Al)**M2*T(L)**(-ML)
724 CONTINIJE
    CALL CAQEA(Y,FX,M,N,MM,NSI,J,D)
722 C.ONTINUE
    DO 725 I=1,NIL
    FX(MM,I)=FX(MM,I)/FX(MM,NII)
725 CONTINUF
    DO 726 I=1,NJ1
    X(MM,I)=IX(MM,I)-AI|*SAVEH
726 CONTINUF
    GO TO 30
    98 WRITE\6,1031
    GO TO 399
96 CONTINUE
    WRITE(6,105)
399 CONTINUE
    RETIJRN
```



```
    ENTRY INPUT
    REWIND 4
    DO 500 J=L,INPTNM
```

```
    IF(ORDER(J).GT.100)GO TO 501
    RND=RANDU(IX)
    D7 502 1=1,NIL
    IF(RND.LT.FX(ORDER(J),I))GO TO 503
502 CNNTINUE
503 CONTINUE
    ANS(J)=X(ORDER(J),I)
    GO TO 500
501 CONTINUE
    ANS(J)=C.ONST(ORDER(J)-100)
500 CONTINUE
    WRITE(4,101)(ANSII),I=1,INPTNMI
    ENDFILE 4
    REWIND 4
100 FORMAT(I10)
101 FORMAT {E16.9)
102 FORMAT(I2,2X,7E10.0)
103 FORMAT(" ', 'K=0')
104 FORMAT(10F8.0)
105 FORMAT(' ','K= 1.')
    RETURN
    END
```

```
    SUBROUTINE CAREA(Y,FX,M,N,MM,NSI,J,D)
    REAL FX(N,N),Y(N)
    NSII=NSI+1
    NSIO=NSI-1
    FX(MM,1)=0.
    SUM=0.
    DO 201 I=3,NSIO,2
    SUM= SUM+4.*Y(I-1)+2.*Y(I)
201 CONTI NUE
    AREA=0/3.*(Y(L)+SUM+Y(NSIL))
    FX(NM,J) = FX(NM,J-1) +AREA
    RETUR:V
    ENO
    FUNCTION RANDU(IX)
    IX=IX*65541
    IF(IX)5,6,6
    5 IX =IX +2147483647+1
    6 RANDU=1X
    RA:NDU=RA.VOU*.4656613E-9
    RETURN
    END
```

```
    SURROUTINE INTERP(X1,Y1,N1,X2,Y2,N2,YOIFF,ICHK)
    DIMENSION X1(NI),YI(N1), X2(N2),Y2(N2),YDIFF(N1)
    DO \(100 \quad \mathrm{I}=1\), N 1
    \(\mathrm{N} 3=\mathrm{N} 2-1\)
    CO \(200 \mathrm{~J}=1\),:13
    IF(I.GT. V2.AND.ICHK.EQ.0) YDIFF(I)=Y1(I)
    IF(I.GT. V2.AND.ICHK.EQ.1) YDIFF(I)=YI(I)-Y2(N2)
    IFII.GT.N2) GO TO 100
    IF(ABSIXI(I)-X2(J)).GT.L.E-5) GO TO 1
    YDIFF(I) \(=\) Y1(I)-Y2(J)
        GO TO 100
```



```
        YDIFF(1)=Y1(I)-((Y2(J+1)-Y2(J))/(X2(J+1)-X2(J)))*(X1(I)-X2(J))
        2-Y2(J)
        GO TO 100
    2 IF(X1(I).GE. 2 (J+1).ANO.J+l.LT.N2) GO TO 200
        IF(J.EQ.1) GO TD 3
        YDIFF(I)=Y1(I)-((Y2(J)-Y2(J-1))/(X2(J)-X2(J-1)))*(X1(I)-X2(J-1))
        2-Y2(J-1)
            GO TO 100
    3 YDIFF(I)=Y1(I)-(Y2(J)/X2(J))*X1(I)-Y2(J)
200 cointinue
\(100 \operatorname{IF}(A B S(Y D I F F(I)) . L T . A B S(Y 1(I) * 1 . E-5))\) YDIFF(I)=0.0
    IF (N1.EQ.N2.AND.ABS(XI(VI)-X2(N2)).LT.1.E-5) YDIFF(N1)=Y1(N1)
    2-Y2(N2)
    IF(ABS(YOIFF(iN1)).LT.ABS(YI(N1)*1.E-5)) YDIFF(NN) \(=0.0\)
    RETURN
    END
```

```
- SUBROUTINE PLOTI(X,Y,N,YHDR,NY,XHOR,NX,SYI,SYZ,SXI,SX2,XY,
```

- SUBROUTINE PLOTI(X,Y,N,YHDR,NY,XHOR,NX,SYI,SYZ,SXI,SX2,XY,
- 2XSFT,YSFII
- 2XSFT,YSFII
- DIMENSION X(N),Y(N)
- DIMENSION X(N),Y(N)
- DIMENSIO\& XHDR(8),YHDR(8)
- DIMENSIO\& XHDR(8),YHDR(8)
- X(N-1) $=$ SXI
- X(N-1) $=$ SXI
- $X(N)=S X 2$
- $X(N)=S X 2$
- Y(N-I) $=$ SY1
- Y(N-I) $=$ SY1
- $Y(N)=S Y 2$
- $Y(N)=S Y 2$
- Call plotixsfigySFt,-3)
- Call plotixsfigySFt,-3)
- CALL AXIS10.0,0.0,YHDR, VY, 8.0,90.0, SY1,SY2)
- CALL AXIS10.0,0.0,YHDR, VY, 8.0,90.0, SY1,SY2)
- CALL AXIS(0.0,XY,XHUR,HX,5.0,0.0,SXI,SX2)
- CALL AXIS(0.0,XY,XHUR,HX,5.0,0.0,SXI,SX2)
- NI=N-2
- NI=N-2
- Call LINe (X,Y,N1,1,0,1)
- Call LINe (X,Y,N1,1,0,1)
- KPLOT=KPLOT+1
- KPLOT=KPLOT+1
- RETURN
- RETURN
- END

```
- END
```

```
    SURROUTIINE PAIR
    REAL IDIFF,IMAXI,IMIN1,IMAX2,IMIN2,IMAX,IMIN
    БПMM\N/PAIPI/TW1,T&2,OTW,FW1,FW2,DFW1, DFW2,DFW,T:MAXO;DFMO.
    2FDIFF,TDIFF,N
    COMMON/PAIR2/FMAXI,TFMX1,FMIN1,TFMN1,
    2 FMAX2,TFMX2,FMIN2,TFMN2
    COMMON/PAIR3/AFNAX, TFMAX,AFMAXT,TFMAXT
    COMMON/OUTI/FOIFIG,TDIFIG,DIT,ADIT
    DIMENSISN FOIFF(40O), TOIFF(400)
    COMMOIN/OUT 2/DF10OK,T10OK
    COMMON/TOFF/DFTO1, DFTO!2,TDFTOL, TOFTO2
    FMAX=FDIFF(1)
    FMIN=FOIFF(1)
    FMAXI=FOIFF(1)
    FMINL=FD[FF(1)
    TFMXI=TDIFF(1)
    TFMN1=TDIFF(1)
    T=AMINI(TWL,TW2)
    D] 6 I=2,N
    K=1
    IF(TOIFF(I)-T) 7,7,8
7 FMAX=AMAXI(FOIFF(I),FMAX)
    IF(FMAX.GT,FMAXI) TFMXI=TDIFF(I)
    FMAXL=F:AAX
    FMIN=\triangleMINIIFOIFFIII,FMINI
    IF(FMIN.LT.FMINII TFMNI=TDIFF(II
    FMINI=FYIN
6 CONTINJE
8 FMAX=FPIFF(K)
    FMIN=FDIFF{K\
    FMAX2=FDIFF(K)
    FMIN2=FDIFF(K)
    TFMX2=TDIFF(K)
    TFMN2 = TDIFF(K)
    OO 9 I=K,N
    FMAX=AMAXI(FOIFF(I),FMAX)
    IF(FMAX.GT.FMAXZ) TFMXZ=TDIFF|||
    FMAX2 = FMAX
    FMIN=AMINI(FDIFF(I),FNIN)
    IFIFMIN.LT.FMIN2I TFMN2=TOIFF(I)
    FMIN2=FMIN
9 CONTINUF
    AFMAXI= ABS(FMAXI)
    \triangleFMINI=\triangleBS{FMINI)
    IF(AFYAXI.GF.AFNINI) TFMAX=TFMXI
    IF{AFYIVI.GT.AFMAXI) TFMAX=TFMNL
    AFMAX=AMAXI(AFMAXI, AFMINI)
    AFMAX2=ABS(FMAX2)
```

```
AFMIN2=ABS(FMIN2)
IF(AFMAX2.GE.AFMIN2) TFMAXT=TFMX2
IF(AFMIN2,GT.AFMAX2) TFMAXT=TFMN2
AFMAXT=AMAXI(AFMAX2,AFMIN2)
DTW=ABS(DTW)
DFW=ABS(DFW)
DFW1= ABS{DFW1)
DFW2=ABS{DFW2)
DFMQ = ABS(DFM())
FDIFIF=ABS(FDIFIF)
DF LOOK=ABS(DF10OK)
```

OUTPUT MOTRR PAIR OATA

FMAXI，FMINL，TFYXI AND TFMNI ARE THE MAXIMUM AND MINIMU：1 values df thrust imbalance difring einat and the times at which they occur respectivelyFMAX2，FMIN2，TFMX2 AND TFMN2 $\triangle R E$ THE HAXIMUM AND MINIMUM＊values of thrust imbalance during tail－off and the times＊AT WHICH THEY DCCUR RESPECTIVELY＊
TDFTOI，TOFTO2 AND DTW ARE THE WER TIMES FOR THE FIRST AND ..... ＊
SECOND mot Of S to begin tailgff and the absolute value ..... ＊
DF THF IIFFERENCE IN WER TIMES PFSPECTIVELY ..... 女
 ..... 立
ano seconi）motnrs to begin talloff and the absolute ..... ＊
VALUE UF THE DIFFERENCF IN THPUSTS AT WER TIME ..... あ
RESPFSTIVELY
DFTח】 $\triangle N D$ DFTR2 $\triangle P E$ THE ABSILUTE VALUES JF THF THFUST ..... ＊
IMBAIANCES WHICH EXIST WHFN THE FIRST AND SECUND MOTORS ..... 4
BEGIN TAILIJF PFSPECTIVFLY ..... ＊
DFMO AND TYAXN ARE THE ABSOLITE VALUE．OF THE THRUST ..... 4
IMBALANCF GHEN THF MAXIMIUM IYNAMIC PPESSURE OCCURS ON ..... 女thf vehicle anu the time at which it occurs respectively＊
afmax and tfmax arf the absolute value uf the maximum thrust＊
imbalance ourin ewat and the time at which it occurs ..... ＊RESPECTIVEIY
afmaxt and tfmaxt are the abSolute value df the maximum ..... ＊THRIJST imbalance during tail－off and the time at whichIt OCCUPS PESPFRTIVFLY
FDIfig．$\triangle$ ND tDIFIG ARE THE ABSOLUTE VALUE OF THE maXImUM ..... $\nmid$and the time at which it occurs respectively$\star$absolute valiJe of the total impillse imbalance duringTAIL－DFFdflook and tlook are the absolute value of the thpust\＃
THRUST IMRALANGE DURING THE INITIAL PART DF OPERATION ..... ＊＊
dit and adit are the the total impulse imbalance ahd the ..... ＊

```
C *
                    IMBALAMCE WHEN THE LAST MDTOR REACHES lOOK AND THE
                #
C *
    TIME AT WHICH IT OCCURS RESPECTIVELY
*
```



```
IF (TWl-TW2) 700,700,701
700 DFTO1 = OFW1
DFTO2= DFW2
GO TO 702
701 DFTO1 = DFW2
DFTП2=DFW1
FW1=FW2
\(F W 2=F W 1\)
702 C.ONTINUF
TDFTOL = AMIN1(Thl, TW2)
TDFTO2 = ^MAX1(TW1,TW2)
WRITE (6,1)
1 FORMAT(//, 2OX, 'MOTCR PAIF OATA')
WRITE (6, 21 FYAXL,TFMXI,FMINI,TFMNI,
2FMAX2, TFMX2,FMIN2, TFMN2, DFTO1,DFTO2,
3TDFTOL,TDFTO2, OTW,FW1,FW2, DFW, OFMQ, TMAXO,
3AFMAX, TFAAX, AFMAXT, TFMAXT, FDIFIT, TDIFIG,DIT, ADIT, DF100K, T100K
```




```
\(213 X_{1}^{\prime,}\) FMAX2 =, 1 PE11.4,13X,'TFMX2 = \(, 1 P E 11.4,1\),
```



```
\(213 x\), DFTOL = ',1PELL.4,13x, DFTO2= \(, 1 P F 11.4,1\),
```




```
2.1PE11.4,
\(213 X_{\text {. DFMT }}=\quad\).1PE11.4.13X. TMAXO \(=1.1 P E 11.4 .1\).
```



```
\(213 X\), 'AFMAXT = ', 1PF11.4.13X.'TFMAXT= 0,1PE11.4.1.
```





```
RETURN
ENO
```

```
    SUBROUTI VE INTRPI(Y,T,N,TT,DY,ICHK)
    DIMENSIO:N Y(N),T(N)
    NL=N-1
    CY=0.0
    IF(ICHK) 2,2,3
2 DO I I= 1,N1
    IF(TT.GE.T(I).AND.TT.LT.T(I+1)) DY=((Y(I+I)-Y(I))/(T(I+I)-T(I)))
2*(TT-T(I))+Y(1)
    IF(DY.INE.O.O) RETURN
l CONTINUE
3 DO 4 I=1, 眎
    IF(TT.LE.T(I).AND.TT.GT.T(I+I)) DY=((Y(I+I)-Y(I))/(T(I+I)-T(I)))
2*(TT-r(I))+Y(I)
    IF(DY.NE.O.O) RETURN
4 CONTINUE
    RETURN
    END
    SUBROUTINE SIGBAR(X,XI,XI2,SIGX,BX,ICOUNT,N,SIGI,SIG2)
    XN=FLOAT(N)
    IF(ICUUVT.GT.2) GO TO I
    XI2=0.0
    XI = 0.0
1 x I 2=x I 2+x***2
    XI=XI+X
    B X = X I / XN
    XIS=XI**2
    SIGX=SURT((XI 2/XN)-{XIS/XN**2)}
    SIGI=SGRT(XI2/XN)
    SIG2=SQRT(XI2/(2.*XN))
    RETURN
    END
```


## V. SAMPLE STUDY

In keeping with the present interest in very large SRMs the case selected for the sample study is a 146 -inch diameter motor of the type being considered for use on the Space Shuttle. However, the study does not constitute a prediction of the imbalance characteristics of the Space Shuttle. No effort has been made to select the most recent design considered for the Shuttle or to minimize or maximize imbalance. Although the statistical characteristics of the input variables should be reasonable for the most part, in some cases their selection has been somewhat arbitrary since the purpose of this study is merely to demonstrate the setup procedures, format and computational capabilities of the computer program.

The SRM for the sample case has three center segments consisting of circular perforated grains, an aft segment with a circular perforated grain and a forward segment with a truncated (slotted tube) nine point star grain. The fixed values for the sample case are given in Table V-1. It will be noted that both the head end and aft end domes are represented by hemispherical closures which is seemingly inconsistent with the fact that the head end of the foremost circular perforated grain is flat. This is an attempt to artificially correct for the curvature of the star grain segment located at the extreme head end. Experience with the simplified computer program indicates that this is a satisfactory alternative procedure to specification of an effective length of star grain. Of course, the effects of the head end closure burning surface geometry could be represented more precisely by specifying tabular input values, but it should be kept in mind that only the burning surface defined by equations is subject to statistical variation. In the present case, because the entire head end segment is consumed far in advance of tailoff, only minor errors in the predicted behavior during the critical tailoff portion of the traces should be encountered as a result of the approximation used.

Table V-2 gives the input data for the statistical variables of the sample case. Included in Table V-2 are brief comments on the sources of the data and the method in which it is applied to the present study. It will be noted that in a number of cases the convention is adopted of taking the drawing tolerances as representing $\pm 3$ standard deviations in a normally distributed population of a variable. Also, where more than one dimension controls a variable input dimension, the standard deviation of the variable is taken as the square root of the sum of the squares of the standard deviations of the controlled variables (assumed to be normally distributed and uncorrelated.) An example of this is the standard devlation of the average outside diameter of the circular perforated grain which is calculated based on the standard deviations of the outside diameter of the case, and the thicknesses of the case wall, liner and insulation.

TABLE V-1. FIXED INPUT VALUES OF SAMPLE CASE

## Options <br> IEO $=1$ <br> $I P O=1$ <br> NUMPLT(J) 00000

## Propellant Characteristics

$$
L=1367.23
$$

TAU $=39.740$
Grain Configuration
C. P. Grain Geometry $X T Z 0=0.120$
$S=1$

Basic Star Geometry
$\mathrm{NS}=1$
$N P=9$
$\mathrm{NN}=0$

```
INPUT \(=2\)
GRAIN \(=3\)
STAR \(=2\)
\(\mathrm{NT}=0\)
ORDER \(=1\)
\(C O P=2\)
    INPUT = 2
    GRAIN = 3
    STAR = 2
    T = 0
    ORDER = 1
    COP = 2
```

        \(X I Z O=0.120\)
            \(=1\)
        XOUT \(=1000.00\)
        DPOUT \(=10,000.00\)
        ZETAF \(=0.9600\)
        TMAXQ \(=60.0\)
        \(T B=122.2\)
        \(\mathrm{HB}=130,000\)
        PREF \(=560.00\)
        DTREE \(=57.285\)
        \(\mathrm{PIPK}=0.00150\)
        DELTAY \(=0.040\)
        \(I I=26\)
    Basic Performance Constants

CSTART $=0.0000380$
CSTARP $=0.0057000$
PTRAN $=0.0$
GAMP $=0.00527000$
GAMP $=0.00527000$
table v-2. INPUT DATA FOR STATISTICAL VARIABLES OF SAMPLE CASE

table v-2. INPUT data for statistical variables of sample case (CONT'D)

| Variable | Code | XI | $\underline{\mathrm{x} 2}$ | Source or Comment |
| :---: | :---: | :---: | :---: | :---: |
| EXH | 51 | 0.0 | 0.05 | $3 \sigma=0.150$ |
| EYH | 51 | 0.0 | 0.05 | $3 \sigma=0.150$ |
| ALPHAN | 52 | 0.0 | 360 | Random orientation of mandrel and case. |
| ALPHAH | 52 | 0.0 | 360 | Random orientation of mandrel case. |
| ERREF | 51 | 0.00763 | 0.00032 | POSEIDON data per Ref. 8, p. 4.5-26, $c_{v}=4.2 \%$. |
| TGR | 51 | 60.0 | 0.2333 | Ref. 8, p. 4.5-26, $6 \sigma=1.4^{\circ} \mathrm{F}$. |
| TIGR | 11 | 40.0 | 0.3740 | $\mathrm{X} 3=0.0040, \mathrm{X} 4=15$ |
| Data card for TIGR: $0.3777,0.3811,0.4030,0.3980,0.3744,0.3795$, $\begin{aligned} & 0.4266,0.4300,0.4334,0.4300,0.3980,0.3997,0.3862,0.3845, \\ & 0.3895,0.3963,0.4013,0.3929,0.4097,0.4081,0.3980,0.4030, \\ & 0.3827,0.3963,0.3980,0.3996,0.3895,0.3929,0.4081,0.4132, \\ & 0.4215,0.4148,0.3946,0.3744,0.3845,0.4136,0.4148,0.4030, \\ & 0.4013 . \end{aligned}$ <br> Reference values: $\overline{\mathrm{T}}_{\mathrm{igr}} \simeq 0.40, \mathrm{C}_{\mathrm{v}}=3.83 \%$. <br> Source: Artificial data based on $C_{v}$ of large number of motors. |  |  |  |  |
| D0 | 51 | 143.080 | 0.01462 | $3 \sigma=\sqrt{0.032^{2}+2 \times 0.02^{2}+0.01^{2}}$ |
| DI | 51 | 63.590 | 0.033333 | $3 \sigma=0.10$ |
| thetag | 60 | 10.1990 |  |  |
| LGCI | 51 | 1135.58 | 0.577 | $3 \sigma=1.0 \sqrt{3 \text { segments }}$ |
| LGNI | 51 | 51.20 | 0.33333 | $3 \sigma=1.0$ |
| THETCN | 60 | 0.0 |  |  |
| THETCH | 60 | 90.0 |  |  |
| LGSI | 51 | 189.15 | 0.3333 | $30=1.0$ |
| RC | 51 | 71.540 | 0.00731 | $3 \sigma=\sqrt{0.032^{2}+2 \times 0.02^{2}+0.01^{2}} / 2$ |
| FILL | 51 | 2.010 | 0.011111 | $3 \sigma=(1 \sqrt{9 \text { points }} \times 0.1)$ |
| RP | 51 | 12.000 | 0.01667 | $3 \sigma=0.05$ |
| RIS | 51 | 63.540 | 0.01667 | $3 \sigma=0.05$ |

Not only must the procedures used in manufacture and quality control of the motor production be recognized when specifying the input characteristics, but also the way a particular variable is used in the program. Thus, when a dimension (or other characteristic) of a variable is subject to random variation and the effect of the variation is averaged in the program, the standard deviation in the variable should be reduced. For example, the standard deviation in the fillet radii of the star points is reduced by the $\sqrt{9}$ because the nine star points each have equal effects on the burning surface. Similarly, the propellant average burning rate variation between pairs may be reduced substantially if propellant from the same mixer batch is divided between the pair of motors as was assumed in this sample case.

In treatment of characteristics such as burning rate, propellant density, and grain temperature, it is assumed that the primary concern is the variation between two motors of a pair and variation between pairs is not considered. Thus, for example, the standard deviation in these characteristics is only that within a pair and excludes any between pair variation as experienced from change of a lot of propellant between pairs or from different ambient temperature histories.

An actual printout of the statistical input data is shown in Table V-3 to demonstrate the computer program format. Table V-4 gives the complete set of variables (fixed and distributed) selected by the Monte Carlo program for the first motor of the first pair. Table V-5 illustrates the format for the printout of transient values for one SRM of the sample case and also shows the propellant weight and initial and final seed numbers for the configuration. The initial seed number may be used to repeat the calculations for the configuration. The tabular output of motor pair imbalance data shown at the bottom of Table V-5 may be omitted by proper specification of IPO.

Figure V-1 through V-5 illustrate the graphical data which may be obtained using the CalComp plotter. Any or all of the plots may be omitted by proper choice of the NUMPLT( $J$ ) input array (See also IPO). The figures are derived from the same initial seed number given in Table v-5.

Tables V-6 and V-7 illustrate the statistical analysis of the output data that may be obtained from the program by use of the appropriate value of IPO. Table $V-6$ is a facsimile of the computer output for 25 motor pairs of the sample study and Table $\mathrm{V}-7$ is a compilation of selected statistical characteristics of 50 motor pairs. The latter was obtained by separate calculations from three groups of data.

A histogram (Fig. V-6) of maximum thrust imbalances during tailoff for the 50 motor pairs demonstrates that the program results are generally consistent with the type of behavior which would be expected.
TABLE V-3. PRINTOUT OF STATISTICAL INPUT DATA FROM SAMPLE CASE


TABLE V-4. PRINTOUT OF INPUT VARIABLES FOR ONE SRM FROM SAMPLE CASE

## CONFIGURATION NUMBER 1

UPTIONS
IED= 1
$1 P O=1$
NUMPLI(J) $=00000$

PROPELLANT CHARACTERISTICS
RHO $=0.063507$
$A L=0.03665$
$\mathrm{Nl}=0.350$
$A 2=0.03667$
$\mathrm{N} 2=0.350$
ALPHA $=0.0$
BETA $=0.0$
RUAL $=4.2852$
CSTARN $=5.1632 \mathrm{E} \mathrm{O3}$
GAMN $=1.1414 E 00$

BASIC MOTOR DIMENSIONS
$t=1367.23$
TAU $=39.740$
$D E=1.4575 E 02$
DTI $=5.4437 E 01$
THETA $=0.0$
ALFAN= 1.1250 E 01
LTAP $=1.7650 \mathrm{E} 02$
$X T=3.0470 E$ CO
$20=2.4142 \mathrm{E} \mathrm{OO}$
IC= 5.6571E-03
RUNDCN $=1.9500 E-01$
RONDCH $=2.5000 \mathrm{E}-02$
RONDGN $=3.6000 E-02$
RONDGH $=2.0000 \mathrm{E}-02$
EXN $=4.2001 E-02$
EYN $=2.1001 E-02$
$E X H=3.9001 E-02$
EYH $=-2.0999 E-02$
ALPHAN= $2.5200 E 02$
ALPMAH= 3.3120E 02

BASIC PERFQRMANCE CONSTANTS
DELTAY: 0.040
$11=26$
XUUT $=1000.00$
DPUUT $=10000.00$
LETAF $=0.9600$
$T B=122.2$
$H B=130000$.
ERREF= 0.00782
PREF $\quad 560.00$
DTREF $=57.285$
TGR= 59.916
PIPK $=0.00150$
CSTART= 0.0C00380
PTRAN $=0.0$
CSTARP $=0.0057000$
TIG= 0.3906
GAMP $=0.0052700$
TMAXQ $=60.000$

GRAIN CONFIGURATION
INPUT= 2
GRAIIN $=3$
STAR = 2
$N T=0$.
DRDER=1
COP= 2
C.P. GRAIN GEOMETRY
$D 0=143.067$
$D I=63.574$
XTIO $=0.120$
$S=1$.
IHETAG $=10.19900$
LGCI $=1135.91$
LGNI $=51.14$
THETCN $=0.0$
IHETCH $=90.00000$

BASIC STAR CEOMETRY
NS
.
LGSI= 189.21
NPa 9.
RC= 71.527
FILL= 2.006
NN= 0 .
truncated star geometry
$R P=12.005$
RIS: 63.544




Figure V-3. Thrust imbalance versus time for one pair of SRMs of sample case (CalComp plot).



Figure V-5. Absolute impulse imbalance versus time for one pair of SRMs of sample case (CalComp plot).

TABLE V-6. OUTPUT VALUES FOR FINAL MOTOR PAIR AND STATISTICAL CHARACTERISTICS OF TWENTY-FIVE MOTOR PAIRS FROM THE SAMPLE STUDY.

MOTOR PAIR DATA

| FMAX1= | 3.5491 E 04 | TFMX1= | 1.0226 E 02 |
| :---: | :---: | :---: | :---: |
| FMIN1= | -1.7347E 04 | TFMN1= | 8.0128E 00 |
| FMAX2 $=$ | 2.0555 E 05 | TFMX2 $=$ | 1.1119 E 02 |
| FMIN2= | 1.5032 E 03 | TFMN2= | 1.2134 E 02 |
| TDFTO1= | 1.1066 E 02 | TDFTO2= | 1.1099 E 0 |
| FW1= | 2.0811 E 06 | FW2= | 2.0809 E 06 |
| DFTO1= | 2.2095 E 04 | DFTO2= | 1.1096 E 05 |
| DFMQ $=$ | 6.6945 E 03 | TMAXQ $=$ | 6.0000 E 02 |
| AFMAX $=$ | 3.5419 E 04 | TFMAX $=$ | 1.0226 E 02 |
| AFMAXT $=$ | 2.0555 E 05 | TFMAXT= | 1.1119 E 02 |
| FDIFIG= | 1.6448 E 04 | TDIFIG= | 2.2855E 00 |
| DIT $=$ | 8.6436 E 05 | ADIT $=$ | 8.7547E 05 |
| DF100K= | 2.8583E 04 | T100K= | 1.1869 E 02 |

STANDARD DEVIATIONS AND MEANS FOR MOTOR PAIR DATA

|  | STD. DEV. | MEAN |  |
| :---: | :---: | :---: | :---: |
| AFMAX | 0.1016 E 05 | 0.1952 E | 05 |
| TFMAX | 0.3787 E 02 | 0.8279 E | 02 |
| AFMAXT | 0.6072 E 05 | 0.9811 E | 05 |
| TFMAXT | 0.5590 E 00 | 0.1114 E | 03 |
| DTW | 0.1282 E 00 | 0.1761 E | 00 |
| FW1 | 0.4726 E 04 | 0.2079 E | 06 |
| FW2 | 0.5726 E 04 | 0.2076 E | 06 |
| DFW | 0.2551 E 04 | 0.6369E | 04 |
| DFT01 | 0.6512 E 04 | 0.6369 E | 04 |
| TDFTO1 | 0.1291 E 00 | 0.1107 E | 03 |
| DTF02 | 0.2757 E 05 | 0.2961 E | 05 |
| TDFT02 | $0.194 \% \mathrm{E} 0$ | 0.1109 E | 03 |
| DFMQ | 0.2461 E 04 | 0.3679 E | 04 |
| FDIFIG | 0.4574 E 04 | 0.6234 E | 04 |
| TDIFIG | 0.1553 E 00 | 0.2233 E | 01 |
| DIT | 0.4186 E 06 | 0.5927 E | 05 |
| ADIT | 0.2386 E 06 | 0.3585 E | 06 |
| DF100K | 0.8192 E 04 | 0.1228 E | 05 |
| Tl00K | 0.1531 E 00 | 0.1186 E | 03 |

## ALTERNATE DISPERSION VALUES FOR THRUST IMBALANCE DATA

SIGMA 1
SIGMA 2
AFMAX 0.2200E 05 0.1556E 05
AFMAXT 0.1154 E 06
0.8159 E 05

TABLE $\nabla$-7. SELECTED STATISTICAL CHARACTERISTICS OF FIFTY MOTOR PAIRS FROM THE SAMPLE STUDY

| PARAMETER | MEAN | STANDARD DEVIATION |
| :---: | :---: | :---: |
| Absolute value of maximum thrust imbalance during web action time (AFMAX) lbf. | 19,620.40 | 9250.22 |
| Time of AFMAX (TFMAX) sec. | 83.89 | 36.59 |
| Absolute value of maximum thrust imbalance during tailoff (AFMAXT) lbf. | 110,346.00 | 61,130.86 |
| Time of AFMAXT (TFMAXT) sec. | 111.60 | 0.93 |
| Absolute value of the difference in time at which the two motors of a pair begin tailoff (DTW) sec. | 0.20 | 0.14 |
| Absolute value of the thrust imbalance at input time of maximum dynamic pressure (DFMQ) lbf. | 2954.46 | 3965.88 |
| Algebraic value of the impulse imbalance during tailoff (DIT) lbf-sec. | -51,059.09 | 461,769.56 |
| Absolute value of the area between the thrust-time traces of the pair during talloff (ADIT) lbf-sec. | 406,400.00 | 237,49794 |
| Absolute value of thrust imbalance when last motor of pair reaches $100,000 \mathrm{lb}$. thrust during tailoff (DF100K) 1bf-sec. | 8554.64 | 13,469.31 |
| Time of DF100K (T100K) sec. | 118.66 | 0.29 |



## VI. CONCLUDING REMARKS

A technique has been established for statistically investigating the thrust imbalance of pairs of SRMs firing in parallel. The computer program based upon the analysis permits the imbalance characteristics of a large number of SRM pairs to be evaluated in a reasonably short time.

It remains to demonstrate the accuracy of the program by comparisons of theoretical imbalance results with those from real SRM populations. Preliminary investigations of this type were conducted during the program. The results, although encouraging, are too incomplete to warrant reporting. This is largely due to the difficulty encountered in obtaining specific data to define confidently the statistical distributions of input variables for past rocket motors. The necessary data is of ten incomplete or not readily accessible.

Additional areas for extended effort include investigation of methods for accounting for effects of radial, axial and circumferential temperature gradient differences between motors of a pair and incorporation of between pair variations of propellant characteristics into the analysis. Ability to treat the between pair variations would improve the accuracy of the program as a device for predicting the absolute performance characteristics of SRMs.

Finally, it is noted that a large number of improvements in the basic accuracy of the simplified computer program presented in References 3, 4 and 5 have been incorporated into the program presented in this report. Those using the earlier program may wish to adopt these improvements for the purpose of design and performance analysis of single rocket motors.

1. Reckmeyer, V. U., "Impulse Reproducibility," Huntsville Division, Thiokol Chemical Corporation, Huntsville, Alabama, Special Report No. 40-68, Control No. U-68-40A, October 1968,
2. Mize, J. H., and Cox, J. G., Essentials of Simulation, PrenticeHall, Englewood Cliffs, New Jersey, 1968, pp. 73, 74, 87-92, 228.
3. Sforzini, R. H., "Design and Performance Analysis of Solid-Propellant Rocket Motors Using a Simplified Computer Program," Final Report, NASA Contractor Report NASA CR-129025, Auburn University, October 1972.
4. Sforzini, R. H., "Extension of a Simplified Computer Program for Analysis of Solid-Prope1lant Rocket Motors," Final Report, NASA Contractor Report NASA CR 129024, Auburn University, April 1973.
5. Sforzini, R. H., "Analysis of Solid-Propellant Rocket Motor Using a Simplified Computer Program," Engineering Experiment Station Bulletin No. 59, Auburn University, December 1973.
6. "Solid Rocket Motor Performance Analysis and Prediction," NASA Space Vehicle Design Criteria (Chemical Propulsion) NASA SP-8039, National Aeronautics and Space Administration, May 1971, pp. 62-63.
7. Elderton, W. P., and Johnson, J. L., Systems of Frequency Curves, The University Press, London, 1969, pp. 35-70.
8. "Proposal for Solid Rocket Motor Project for the Space Shuttle Program," Vol. III, Design, Development and Verification Proposal, TWP 077326, Submitted to NASA George C. Marshall Space Flight Center by Wasatch Division Thiokol Chemical Corporation, 27 August, 1973, pp. 4.5-25 through 27.

## APPENDIX

PERFORMANCE SENSITIVITIES
-120-





























Figure A-30. Thrust versus time for two SRMs with average initial radius


मU.S. GOVERMENT PRINTING OFFICE 1975-640-453/259 REGION NO. 4

