A PRACTICAL MULTI-SPECTRUM HADAMARD TRANSFORM SPECTROMETER

by

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ABSTRACT

We have constructed a Hadamard Transform Spectrometer (HTS) which simultaneously obtains fifteen infrared spectra, each having 255 spectral elements. Spectra are obtained essentially in real time through use of a minicomputer with 8K words of memory and a CRT display. This permits operation of the instrument in the field.
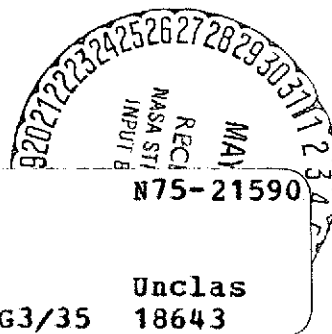
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1. Introduction

The principle and operation of Hadamard Transform Spectrometers (HTS) has by now been described in a number of papers[1,2,3,4]. The theoretical predicted multiplex advantage has been experimentally verified for one of the configurations in which these instruments can be operated[5], and astronomical observations have been carried out[6,7].

There are basically three modes in which these instruments can operate. The simplest model[1,2,5] has a single entrance slit and n exit slits. It produces a spectrum having n spectral elements. A more complex instrument[3,4] is one having m entrance slits and n exit slits, m < n. It can be operated in two different modes. If the radiation incident on the spectrometer is diffuse and homogeneous, the prime advantage of having several entrance slits is an increase in the instrument's throughput; but the final product still is a single spectrum having ~ n spectral elements. Alternately, if the radiation incident on the instrument is not homogeneous, one may wish to obtain a separate spectrum for light incident on each entrance slit position. One application for such an instrument that we hope to realize is a simultaneous analysis of the spectra of the different cloud bands of Jupiter. By aligning the instrument's entrance slits parallel to the Jovian equator, any marked differences in the infrared spectra at different Jovian latitudes should become apparent.

This last mode of operation is the one which we have

been able to make practical, in the sense that m and n
are quite large, that the equipment is moveable, and that
the operation of the instrument is essentially instantaneous.

The only previously discussed instrument of this kind
was described by Phillips and Harwit[8], who constructed a
pilot model having 19 exit slits and up to 19 entrance
slits. With this instrument these authors were able to
demonstrate the principle of operation.

The new instrument we describe has 15 entrance slits
and 255 exit slits. It obtains 15 different spectra each
having 255 spectral elements. The total number of intensity
measurements that have to be made is 15 x 255 = 3825. Data
can be gathered at a rate of up to five readings per second
so that the entire spectral run lasts only some thirteen
minutes. This rate, which is perfectly adequate for most
astronomical observations, is limited primarily by the rate
at which our minicomputer can process the incoming data
in real time. At the end of the thirteen minute run, the
spectrum is available for immediate viewing on a CRT display.
In our laboratory we photograph this display with Polaroid
film, but hard copies of other types could also be used.
In addition we can punch and store the spectra on paper
tape for further processing.

2. Instrumentation

The optical configuration, the drive of the encoding
masks that are used and the spectral capabilities of our

instrument have been described in detail by Harwit, et al.[4],
in sections VII A and B of their paper. For brevity we will
not repeat this description, but we are including a schematic
diagram of the optical layout, Fig. 1., and a table of
instrumental parameters, Table I.

The actual operation of the instrument procedes in
exactly the manner specified there. The only difference
lies in the way that the data points are handled. Harwit,
et al., used these data to obtain a single spectrum averaged
over the entire radiation collected by the fifteen entrance
slits of the instrument. Here we describe the handling of
the data to obtain fifteen different spectra, one corresponding
to each entrance slit of the spectrometer.

The main difficulties that arise in the implementation
of this mode of operation involve speed, ease of verification,
and self-containment. We wanted to construct an instrument
that could be transported to any astronomical observatory
where infrared observations can be made, and we wanted to
be able to verify in real time that adequate spectra were
being obtained. We therefore chose to center our construc-
tion around a readily transported minicomputer programmed to
produce a cathode ray display.

We have used this display primarily for rapid visual
inspection and for photographs that can be used as a quick
reminder about information punched out and stored on paper
tape. Since punching 3825 spectral points on paper tape
requires some 13 minutes, we plan to punch and store only

those spectra that are of genuinely high quality to warrant
further processing in the laboratory. Processing of this
type may involve various atmospheric corrections, normali-
zation to a calibration standard, and so on.

3. Algorithm for Real Time Decoding of Spectral Data

In our minicomputer assisted system we make use of the
following algorithm for reducing the spectra in real time.

Let both the entrance and the exit masks be linear
arrays encoded by Reed Muller codes[1]. Then the matrix
of spatial-spectral elements ψ is related to the matrix of
measurements η, by and postmultiplying by S^{-1}

$$s\psi S = \eta \quad . \tag{1}$$

To obtain the spatial-spectral information about the
viewed scene we solve this equation by premultiplying the
data matrix by s^{-1}

$$\psi = s^{-1}\eta S^{-1} \quad . \tag{2}$$

Now consider the element η_{11}. It is multiplied only
by elements of the first column of s^{-1}; and in turn it
multiplies only the elements of the first row of S^{-1}. To
a given spectral-spatial element ψ_{ij} it therefore contributes
an amount $s_{i1}^{-1}\eta_{11}S_{1j}^{-1}$. But the elements s_{i1}^{-1} and S_{1j}^{-1}
all have values, either +1 or -1, and the result is that
each element ψ_{ij} of the matrix ψ receives a contribution
$+\eta_{11}$ or $-\eta_{11}$ from the reading η_{11}.

This procedure is generally valid. Any reading $\eta_{k\ell}$
will make additive contributions that can only have values

$+\eta_{k\ell}$ or $-\eta_{k\ell}$, to each element ψ_{ij} of the ψ matrix.

For real time decoding we therefore need the following:

(a) A memory that consists of bins containing the contributions to the elements ψ_{ij} accumulated up to any given time, t, in the cycle of measurements. For a device that can resolve m spatial and n spectral elements, this memory requires mn bins and of the order of mn memory words.

(b) For each acquired reading $\eta_{k\ell}$ we perform a series of additions of values either $+\eta_{k\ell}$ or $-\eta_{k\ell}$, one to each of the mn memory bins ψ_{ij}. But before that can be done, we need to decide on the assignment of + or - needed for a given bin. This is done in the following way:

We store the sequence of + and - signs in one column of s^{-1} and in one row of S^{-1} and in one cycled permutation of each of these vectors. Let us designate these signs by their positions in these two vectors, as s_i^{-1} and S_j^{-1} respectively, i=1,...m; j=1,...n. (Since each of these sequences is cyclic it can respectively be brought into its kth and ℓ^{th} cycling position after a measurement $\eta_{k\ell}$.) The elements of the two vectors then are multiplied in all possible combinations, to give a matrix having mn elements.

$$\sum_{ij} = s_i^{-1} S_j^{-1} \qquad i = 1,\ldots m; \; j = 1,\ldots n.$$

Each element $\sum_{ij}$ is either + or - depending only on whether the signs s_i^{-1} and S_j^{-1} are similar or dissimilar for a particular combination of i and j values.

The additions $+\eta_{k\ell}$ or $-\eta_{k\ell}$ to the bins ψ_{ij} are made as successive elements $\sum_{ij}$ are computed, so that the elements $\sum_{ij}$ need never be stored. Fig. 3 shows the relation of $\sum_{ij}$ to a super-array containing the set of all elements that are constructed at various stages of the computation.

When only a restricted number of spectral elements is of interest, we need to compute elements ψ_{ij} representing only selected j values. This might be useful, for example, if only certain atmospheric CO_2 lines needed to be studied, and the spectral elements inbetween were of lesser interest. In that case only $\sum_{k+i-1,\ell+j-1}$ elements corresponding to a given j values need to be used, and the computing time decreases as p/n, where n is the total number of available spectral elements, and p is the actual number of interest.

4. Actual Use of the Computer

We have used Computer Automation minicomputer models L.S.I. and also Alpha-16 with the program we are about to describe.

We start out with the inverse of the codes, s^{-1} and S^{-1}, for the entrance and exit masks stored in the computer. We store 509 elements of the exit mask, i.e. the 255 elements exhibited in the first mask position, plus an additional 254 elements representing the further cycling of this mask. Similarly we store 29 elements for the entrance mask, representing the first 15 elements used, plus the further cycling of 14 elements.

For each reading $\eta_{k\ell}$ we essentially make use of the matrix Fig. 3, making use of elements k to k+14 of the stored entrance code and elements ℓ to ℓ+254 of the stored exit code. This matrix consists of + and - signs. When s_{ik}^{-1} and $S_{\ell j}^{-1}$ have the same sign, both being + or both being -, then the matrix position ij is assigned a + sign and the reading $\eta_{k\ell}$ is added to the accumulatively stored value of ψ_{ij}. If the elements s_{ij}^{-1} and $S_{\ell j}^{-1}$ have dissimilar signs, then a - sign is assigned to ij and the reading $\eta_{k\ell}$ is subtracted from the stored ψ_{ij} values. This whole process takes ~ 100 millisec, and is carried out while the succeeding intensity measurement $\eta_{k, \ell+1}$ is being made.

In practice we start with the first spatial element, i = 1, and add or subtract the contributions to all the ψ_{1j} values, successively going from j = 1 to j = 255. We then repeat this procedure for i values going from 2 to 15. This whole procedure is carried out while the exit mask is moving from position ℓ to $\ell \pm 1$ depending on whether the exit mask is moving forward or back. The entire process is then repeated for the next reading $\eta_{k, \ell \pm 1}$. When the exit mask reaches its 255th position, ℓ remains unchanged, but the entrance mask moves from the position k to k+1.

For odd values of k, the exit mask moves in the direction of increasing ℓ values, and for even values of k, it moves toward decreasing values. In short, the exit mask moves back and forth as readings are taken. After the entrance mask has moved through all its 15 positions, and

the total 3825 readings have been taken and added onto or subtracted from the ψ_{ij} elements, the run is completed.

A display program allows us to display the information in a number of different ways:

a) We can call for the spectrum corresponding to any one of the entrance slit position, and display it individually.

b) We can display all fifteen of these spectra simultaneously, with the zero baseline of each spectrum vertically displaced from the next one. While this format is somewhat crowded, it does allow a quick comparison of the individual spectra.

c) Finally we can display the sum of the different spectra. In order to do this, we have to take into account that the spectrum for a given entrance slit is displaced by one spectral element from the spectrum for the next entrance slit position[8]. In other words, the wavelength for element ψ_{ij} corresponds to the wavelength for element $\psi_{i+1,j+1}$, because of the slightly displaced light paths through the spectrometer.

These three forms of display are shown in Figure 4 as a), c) and b) respectively.

We feel we have been successful in realizing an optical system that had been under discussion for some time[3,8], and that the practical problems faced in constructing such a system are not excessive.

Acknowledgements

REFERENCES

1. N. J. A. Sloane, T. Fine, P. G. Phillips, and M. O. Harwit, Appl. Opt. $\underline{8}$, 2103 (1969).

2. J. A. Decker, Jr., and M. Harwit, Appl. Opt. $\underline{8}$, 2252 (1969).

3. M. Harwit, P. G. Phillips, T. Fine and N. J. A. Sloane, Appl. Opt. $\underline{9}$, 1149 (1970).

4. M. Harwit, P. G. Phillips, L. W. King, and D. A. Briotta, Jr., Appl. Opt. $\underline{13}$, 2669 (1974).

5. J. A. Decker, Jr., Appl. Opt. $\underline{10}$, 24 (1971).

6. P. G. Phillips, D. A. Briotta, Jr., Appl. Opt. $\underline{13}$, 2233 (1974).

7. J. R. Houck, J. B. Pollack, D. Schaack, C. Sagan and J. A. Decker, Jr., Icarus $\underline{18}$, 470 (1973).

8. P. G. Phillips and M. Harwit, Appl. Opt. $\underline{10}$, 2780 (1971).

Table I.   Relevant Instrumental Parameters

Focal length = 49.5 cm	Grating ruling = 20 lines/mm
f number = 7.5	Grating order = First
Slot width = 0.1 mm	Grating blaze = 9.5μ in first order
Slot height = 3.5 mm (or 10 mm)	Range* = 8-14μ (1250-715 cm^{-1})
Readings per second = 5	
	Resolution = 0.01μ (1 cm^{-1})

*The instrument can also be tested in the 1.7μ region in

6th order.

Figure Captions

Fig. 1. Schematic layout of the Optical System

Fig. 2. Explanation of the significance of the different matrices and their rows and columns.

Fig. 3. Matrices generated by the computer during the reduction of the spectral data. (See text for a full explanation.)

Fig. 4. Calibration spectra obtained for the mercury vapor emission lines at 1.69 and 1.71μ. (a) The eighth of a series of fifteen individual spectra obtained; (b) an average of all 15 spectra; (c) a display of the 15 spectra, each spectrum being displaced vertically from the next. The diagonal pattern near the right hand edge represents a displacement of the peak between successive spectra. This represents the actual shift in spectral range between adjacent spatial elements.
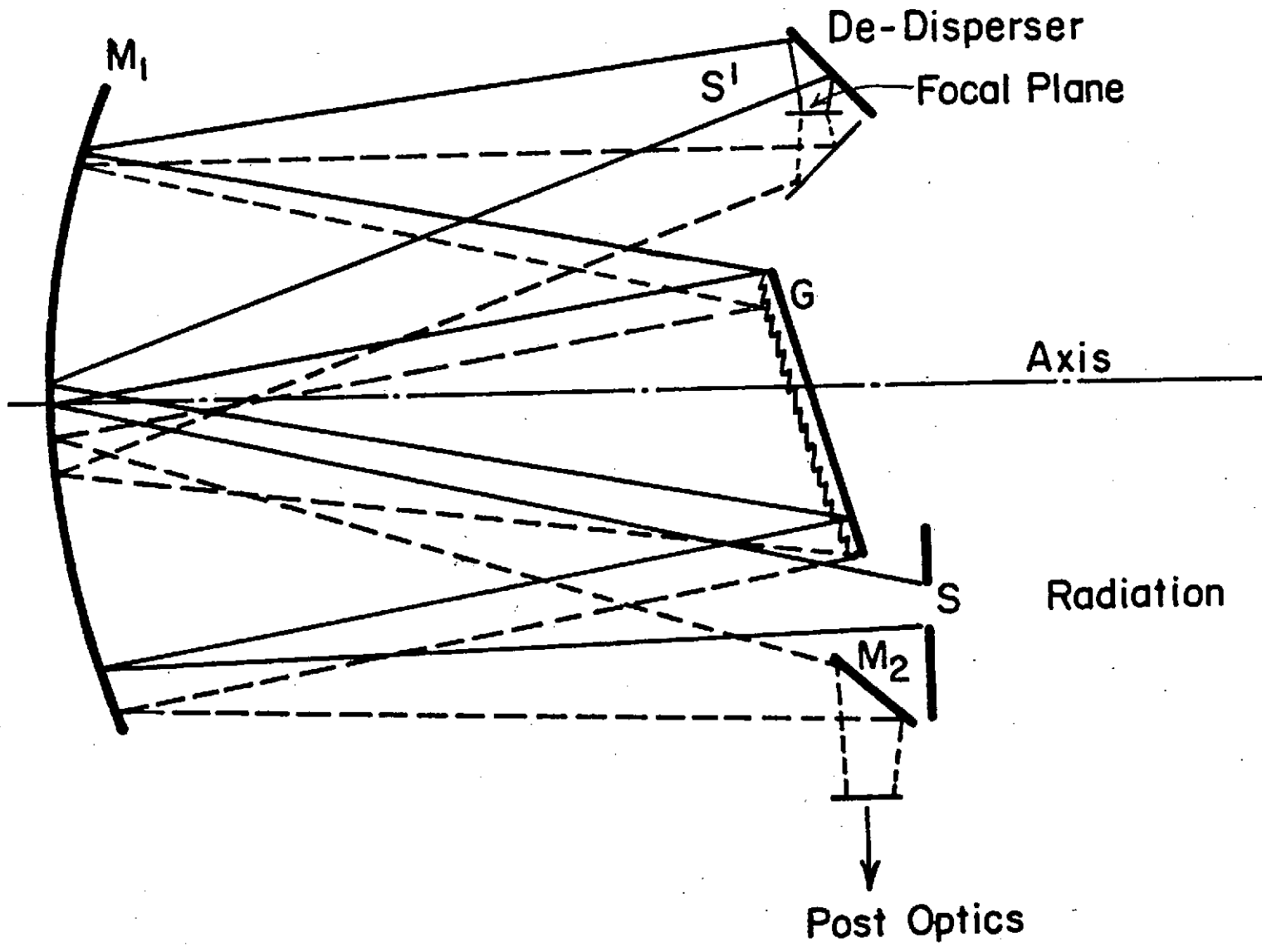
Each column
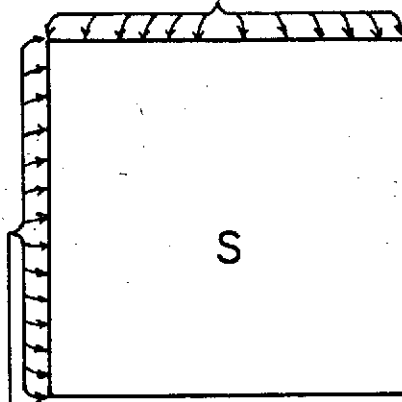represents a
different
spatial element
(entrance slit)

Each column
represents a
different color
(spectral element)
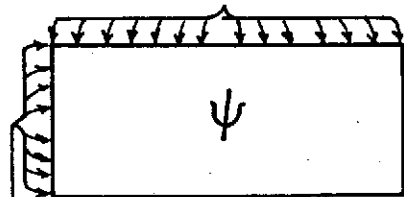
Each column
represents the
code of one of the
successive spectral
encoding mask

Each column represents
the reading obtained in
one of the successive
spectral mask positions

$$S \qquad \psi \qquad S \qquad = \qquad \eta$$

Each row represents
the code of one of
the successive spatial
encoding masks

Each row represents a
different spatial element

Each row represents a
different spectral element
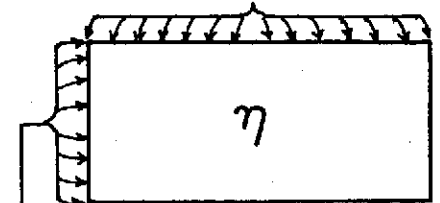
Each row represents the
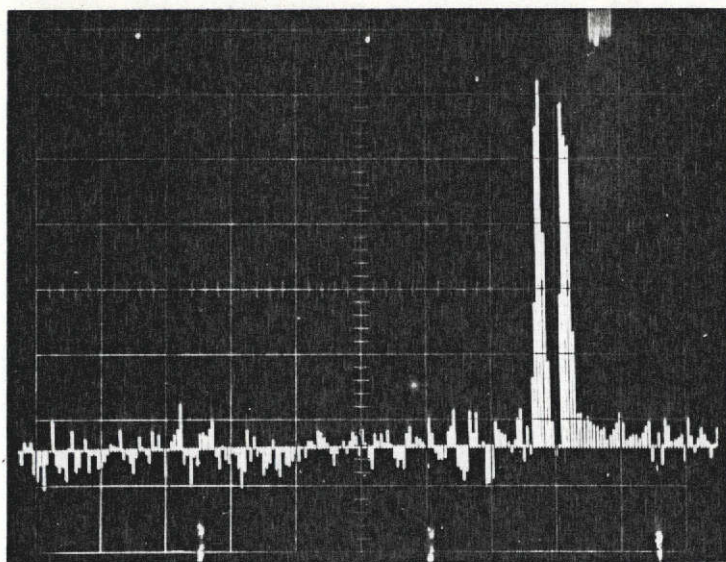readings obtained in one
of the successive spatial
mask positions

column ℓ

row k

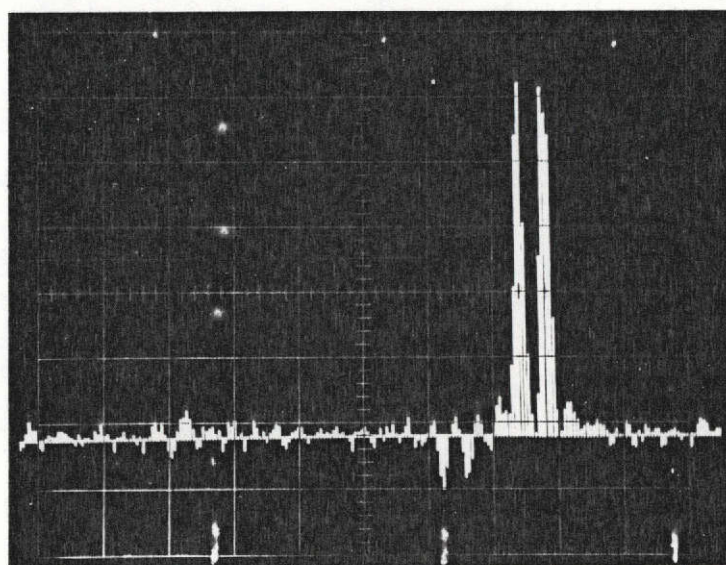Matrix of $\sum_{ij}$ values used with measurement $\eta_{k\ell}$

$2m-1$

$2n-1$
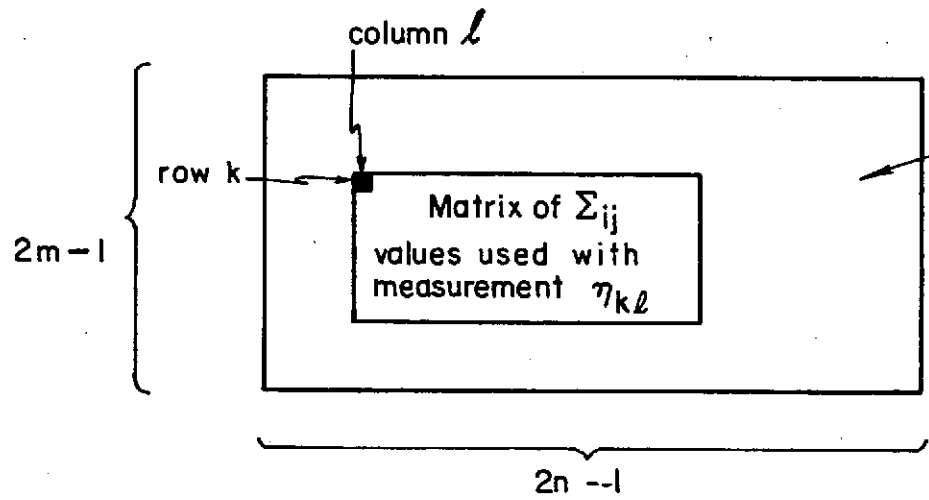
Super-array of + and − signs. Row $m+1$ to $2m-1$ are a repetition of rows 1 to $m-1$. Columns $n+1$ to $2n-1$ are a repetition of columns 1 to $n-1$
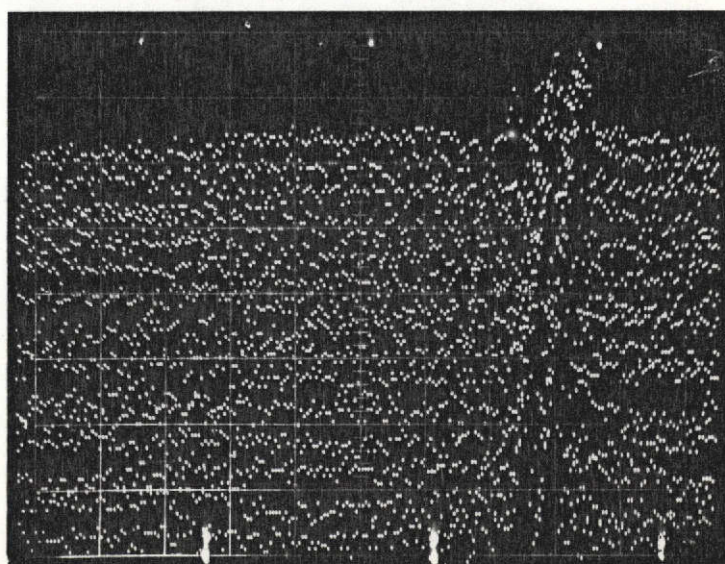
(a)

(b)

(c)