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# INVISCID FLOW ABOUT BLUNTED CONES OF LARGE OPENING ANGLE AT ANGLE OF ATTACK

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## SUMMARY

Application of a general method for calculation of inviscid hypersonic flow fields to the title problem is discussed. It is concluded that the complications caused by the requirement for sonic flow at the rear corner and particularly of the uncertain position of the stagnation streamline lead to sufficient difficulties with convergence of iterations that a practical procedure is not likely to be found.

## INTRODUCTION

During the past few years, the numerical analysis of high speed inviscid flows has reached a point where numerous codes, reflecting a variety of approaches, are available. These vary widely in complexity and in the precision of their predictions. Among the more simple is a method originated by Maslen[1] which bears a strong family relationship to the familiar methods of integral relationships. This method was originally intended for analysis of hypersonic axisymmetric flow over smooth bodies and it does indeed provide simple and accurate results, quite useful for repeated engineering calculations.

Subsequently, a number[2-7] of extensions have been carried out including application to radiating gases and to nonequilibrium chemistry. Also, Schneider has presented[8] an analytic discussion of the basis of the method. Another tack[9,10] has been concerned with extension to three dimensional flows. In this case the question has not really been whether the basic idea is applicable in three dimensions. Rather it has been whether the resulting computer code would remain sufficiently efficient to be of practical value.

The present study has been concerned with application of the method to a deceptively difficult problem. This is the high speed flow over a blunted cone having a large opening angle and at angle of attack. The shape is typical of planetary entry vehicles, notably the Viking Aeroshell. The geometry is simple; the problem, of course, is that the velocity near the surface remains subsonic for interesting cone angles, reaching sonic values only at the rear corner.

# ANALYSIS

## A. General Considerations

Inasmuch as the main analytic structure of the method has been given elsewhere[9,10], the development will not be repeated here in detail, although Appendix A shows the final system of equations used. A flow diagram and identification of the program elements is given in Appendix B.

The basis of the analysis is this. Consider the equations of motion in a shock oriented system, and observe that the shock layer is thin. The variation of pressure across the layer (controlled by the lateral momentum equation) is smooth and generally moderate as compared to, say, the density variation. The latter is related to the entropy variation behind the curved shock and generally falls rapidly toward the body, so that the mass flow in the shock layer is concentrated near the shock. On the basis of these arguments, it is arbitrarily assumed that the variation in pressure across the layer is quadratic in the stream function - i.e., in the mass flow - and can be written explicitly in terms of the pressure at the shock and its normal gradient. Thus

$$P = P_{shock} + P_1 (\psi - \psi_{shock}) + P_2 \; \frac{\psi^2 - \psi^2_{shock}}{2\,\psi_{shock}}$$

The sum $(P_1 + P_2)$ is the normal gradient; the split is based on arguments[9] for simple cases. Then for a given shock, the entropy is known on a streamline and the remaining thermodynamic properties are directly calculable as is the speed for an isoenergetic condition. Thus for axisymmetric or plane flow, there remains only to invert the definition of the stream function to obtain the physical coordinates. For unsymmetric flow, an ordinary differential equation (eq. A11, Appendix A) must be solved first to determine the streamline direction (laterally). All this is quite straightforward and good results have been obtained in a number of cases.

The complications arise almost entirely from the real requirement that the body, not the shock, be specified so that one is led to an iterative process. For axisymmetry, this iteration is not difficult and can be performed rapidly. For some three-dimensional situations the same statement applies even though the requirement to solve a differential equation hurts. The main question is the number of iterations required.

For the present case of a blunt cone of large opening angle, there are two severe complications each of which separately very much increases the number of iterations required. One is the occurrence of sonic flow at the rear corner and the other deals with the position of the stagnation point. We examine these in turn.

B. Sonic Corner

Consider first the logic used for, say, a hemisphere cylinder. The procedure[9] has been to postulate the shock curvature as a quadratic in distance along it and to iterate this form to satisfy simultaneously the nose position and curvature and the body position and slope at a point (A) downstream of sonic flow. The actual iteration proceeds easily. Subsequently the solution continues in a marching fashion with a very straightforward iterative process. So far, no problem. The result is quite insensitive to the position of point (A). On the other hand, for the present geometry, the sonic point is at the rear corner so that the above logic would force one to do the whole flow with a quadratic shock curvature. This is much too restrictive. The alternate procedure is to choose A at some forward position and try to go ahead. What happens is familiar from the method of integral relations. The solution typically diverges before reaching to the rear end. By altering the position of the match point (A) in an iterative way, the solution yielding the desired sonic corner value can be found. Such a process works well but obviously increases the computing time. When the ratio of specific heats ($\gamma$) drops, convergence becomes slower and the sonic behavior becomes a much more localized phenomenon. Parenthetically, it may be noted that McDonald[6] has given an alternate procedure which should work well, though some of his results for low $\gamma$ are not correct.

In three dimensions, it should be apparent that this sonic condition gets messy, and may indeed be a mixed condition[11], with the flow supersonic on a portion of the rear edge. One may well require that the (now) line of the match point (corresponding to point A discussed earlier) might have to vary in downstream location depending on the angular position. Such an event boggles my mind.

C. Stagnation Region

For axisymmetric flow the position and shape of the stagnation streamline are trivially given. For an unsymmetric case with a rounded geometry giving supersonic flow quickly, the position and shape are not hard to find in the spirit of this method. Particularly for high Mach number, the shock is loosely parallel to the body and it has been assumed that is exactly so and thus that the stagnation streamline is straight and normal to body and shock. Even though this may not be strictly true the assumption usually appears to introduce no important error.

Now consider the case of interest here. Figure 1 shows a spherically blunted cone of $60^{\circ}$ half angle, at $20^{\circ}$ angle of attack. At point B, the body is normal to the flow direction. If the corresponding shock is normal at the associated point (K), we are led to a shape like, say CKD.

Unfortunately, it is quite reasonable that the shock be more like EFG for which the normal point (F) is far removed from B. Furthermore there is no longer much reason to expect a simple stagnation line (FH?). The complications grow apace.

D. Effects on Calculation

So long as the match line (A) can be at a uniform axial distance downstream, the sonic corner condition causes no problem beyond that described for the axisymmetric case. Figure 2 shows calculated results from the present program run for a zero angle of attack case and compares these with those from an axisymmetric program and a result given by South[12]. The iteration proceeded without trouble.

When the same configuration is at angle of attack, as in Figure 1, disaster strikes. The program has no objection to computing the flow behind either of the shocks sketched. What it does get confused at is the idea of an iterative process converging on what is now, from the shock viewpoint, an entirely arbitrary body shape. Efforts to date to conduct such an iteration have met with no success.

Numerous calculations were attempted within the framework of shocks like CKD(fig. 1); that is, shocks whose normal point (K) lies opposite the corresponding body normal point (B) so that the stagnation line (KB) is parallel to the freestream. The results showed clearly that convergence of the iterative process cannot occur within this limited family of shocks. Next, calculations were extended by simply moving the normal point (F, fig. 1) an arbitrary distance relative to point B. The results showed considerable improvement over the previous cases. However convergence was not obtained, nor did a reasonable iteration logic appear. Recall the logic used[10] in the absence of this complication. We assume a shock whose curvature (CS) is given by

$$CS = A(\theta) + zB(\theta) + z^2 D(\theta)$$

where the most general form of A is

$$A(\theta) = \frac{a}{1 + b\,\mathrm{Cos}^2\theta}$$

Z and $\theta$ are cylindrical coordinates, with origin at the shock normal point ( K , fig. 1). Now a, b are found by matching to the body position, slope and curvature at the point opposite K. Then $B(\theta)$, $D(\theta)$ are found by matching position and slope at a Z=constant surface downstream. Even this logic requires some compromise to permit reasonably rapid convergence. Now we add a displacement of the shock to F so that the body opposite no longer need be normal.

While it seems that, in principle, the desired convergence can be obtained, the process appears so tedious as to be foolish -- remember that the whole object of this study was to provide a reasonably rapid means of computing the flow field.

## CONCLUSIONS

The assumptions made to simplify the system of equations of motion do indeed lead to a flow model capable of computing the flow behind a quite general smooth shock in three dimensions. However, the practical problem of finding the shock for a specified body leads to an iterative process which can greatly increase the computing time. For the geometry studied in the present work, the very large cone angle leads, at appreciable angle of attack, to a non-negligible displacement of the stagnation point. This unfortunate circumstance introduces another dimension to the iterative process making it so extensive as to preclude its practical use.

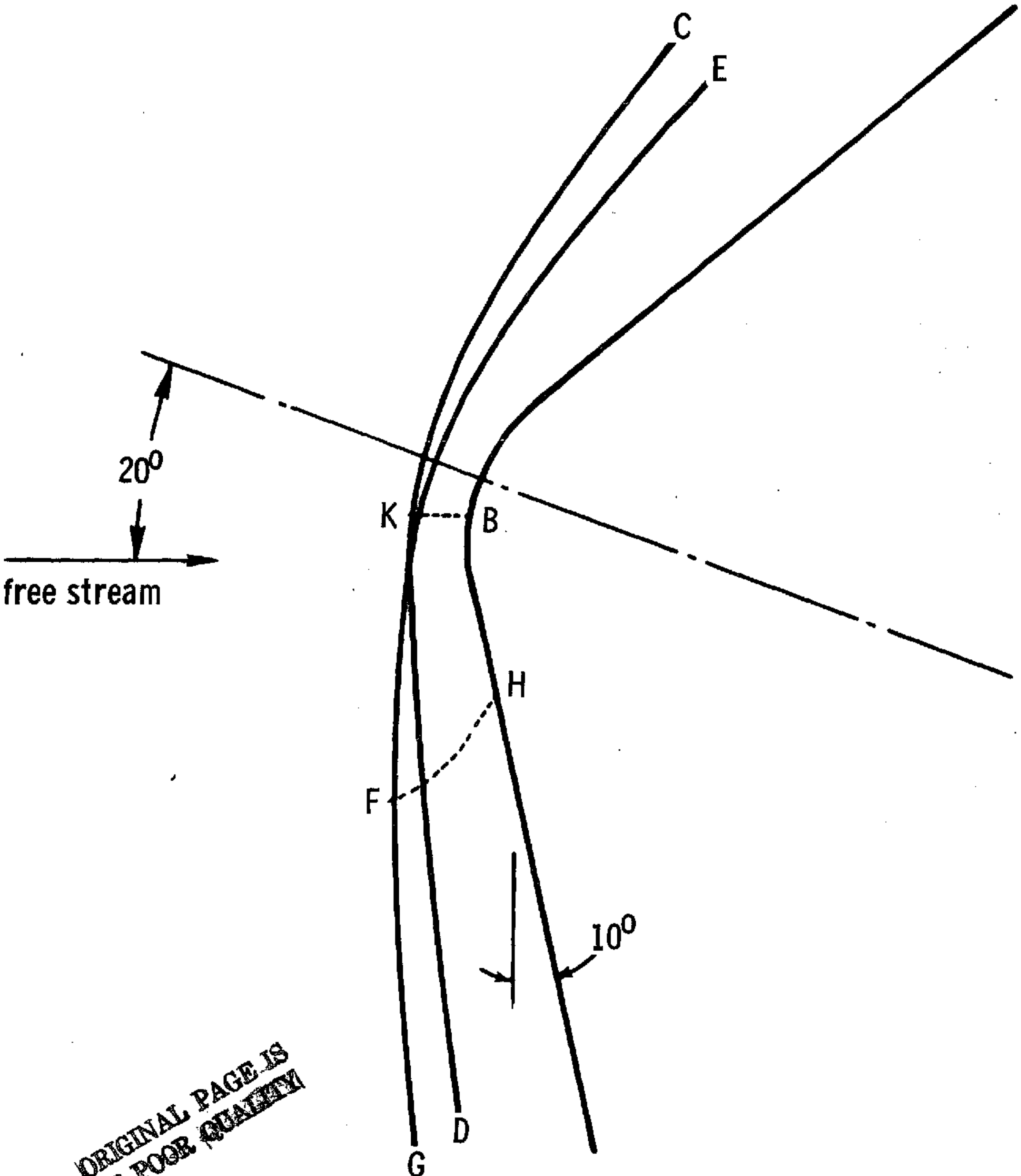





Fig. 1 - Sketch of shock geometry

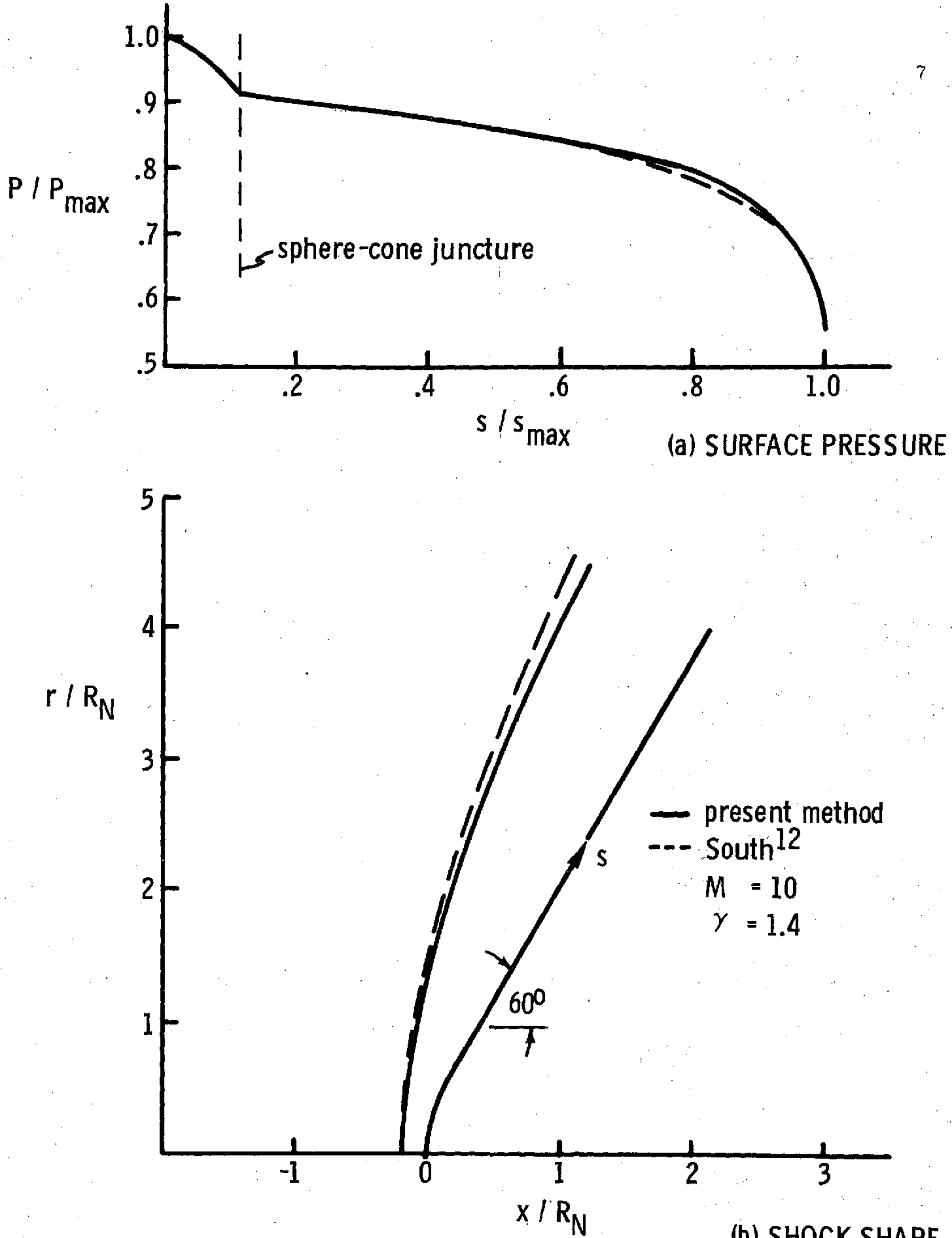



Fig. 2 Surface Pressure and Shock Shape for Sphere Cone $M_{\infty}=10$, $\theta_c=60^{\circ}$, base to nose radius ratio = 4.0.

## APPENDIX A

The Euler equations of motion are to be solved for a compressible, isoenergetic flow. Consider a set of coordinates $(z, \eta, \theta)$ related to the flow geometry (Fig. A1). We base them on the shock where z, $\theta$ are the axial and azimuthal coordinates of a point on the shock while $\eta$ is the (normal) distance from shock to field point. These coordinates are related to cylindrical ones $(x, r, \omega)$ by

$$x = z + \eta \operatorname{Sin} \tau$$

$$r^2 = r_s^2 + \eta^2 \operatorname{Cos}^2 \tau - 2\eta r_s \operatorname{Cos} \tau \operatorname{Cos} \lambda \qquad (1)$$

$$r \operatorname{Sin} (\omega - \theta) = \eta \operatorname{Cos} \tau \operatorname{Sin} \lambda$$

where $r_s = r_s(z, \theta)$ is the equation of the shock and $\nu$, $\lambda$, and $\tau$ are angles (Fig. -A1) defined by

$$\operatorname{Tan} \nu = \frac{\partial r_s}{\partial z}$$

$$\operatorname{Tan} \lambda = \frac{1}{r} \frac{\partial r_s}{\partial \theta}$$

$$\operatorname{Tan} \tau = \operatorname{Tan} \nu \operatorname{Cos} \lambda$$

Now introduce three mutually perpendicular velocity components, U, $\bar{V} \operatorname{Cos} \xi$, $\bar{V} \operatorname{Sin} \xi$ where U is normal to the shock and $\xi$ is otherwise arbitrarily chosen. Then

$$U = u_1 \operatorname{Sin} \tau - v_1 \operatorname{Cos} \tau \operatorname{Cos} (\omega - \theta + \lambda) + w_1 \operatorname{Cos} \tau \operatorname{Sin} (\omega - \theta + \lambda)$$

$$\bar{V} \operatorname{Cos} \xi = u_1 \operatorname{Cos} \tau + v_1 \operatorname{Sin} \tau \operatorname{Cos} (\omega - \theta + \lambda) - w_1 \operatorname{Sin} \tau \operatorname{Sin} (\omega - \theta + \lambda) \qquad (2)$$

$$\bar{V} \operatorname{Sin} \xi = -v_1 \operatorname{Sin} (\omega - \theta + \lambda) - w_1 \operatorname{Cos} (\omega - \theta + \lambda)$$

We define a pair of stream functions, $\psi$ and $\phi$, such that the velocity vector U is

$$\rho\vec{U} = \vec{\nabla}\phi \times \vec{\nabla}\psi \tag{3}$$

and let $\zeta$, $\sigma$, $\psi$ be the final independent variables related to the physical coordinates by

$$\begin{aligned} \zeta &= z \\ \sigma &= \theta \\ \psi &= \psi(\eta, z, \theta) \end{aligned} \tag{4}$$

Then Eq. (3) yields

$$L(\phi) = 0$$

$$\rho \bar{V}\eta_{\psi} = \frac{-\phi_{\sigma}}{e(A \text{ Cos } \xi + B \text{ Sin } \xi)} \tag{5}$$

where the operator L is

$$L = -\frac{(D \text{ Cos}\xi + E \text{ Sin } \xi)}{r} \frac{\partial}{\partial \sigma} + (A \text{ Cos } \xi + B \text{ Sin } \xi)\frac{\partial}{\partial \zeta} \tag{6}$$

where A, B, D, and E are geometric factors given by

$$A = [r_s + \eta \text{ Cos } \tau \text{ Cos } \lambda\ (\lambda_{\theta} - 1)] / r \text{ Cos } \lambda$$

$$B = \frac{\eta}{r} \tau_{\theta}$$

$$D = \text{Tan } \tau \text{ Tan } \lambda + \eta \lambda_z \text{ Cos } \tau$$

$$E = (1 + n\tau_z \text{Cos } \tau)/\text{Cos } \tau$$

The entropy (S) equation is

$$L(S) = 0 \qquad (7)$$

while the isoenergetic equation relates the entropy (H) to $\overline{V}$, neglecting the small normal velocity (u, eq. 2) by

$$h + \frac{1}{2}\,\overline{V}^2 = \text{constant} \qquad (8)$$

The approximate normal momentum equation is, on integrating,

$$P(\psi, \zeta, \sigma) = P_s(\zeta, \sigma) + P_1(\zeta. \sigma)(\psi - \psi_s) - P_2(\zeta, \sigma)\left(\frac{\psi^2 - \psi^2_s}{2\psi_s}\right) \qquad (9)$$

where

$$P_1 = \left\{\eta_\psi\, \rho\, \overline{V}^2 \,\mathrm{Cos}\,\tau\, \mathrm{Cos}\lambda[\mathrm{Cos}\,\xi\, L(\tau) - \mathrm{Sin}\,\xi\, \mathrm{Cos}\,\tau\, L(\lambda - \theta)]\right\}_s$$

$$(10)$$

and

$$P_2 = \left\{n_\psi\, \rho\, \frac{\rho}{\rho_\infty}\, U^2[\mathrm{Cos}\tau\, \tau_z - \mathrm{Cos}\,\tau\, \mathrm{Cos}\,\lambda(1-\lambda_\theta)/r - \mathrm{Sin}\,\tau \mathrm{Sin}\,\lambda\, \tau_\theta/r]\right\}_s$$

The remaining momentum equation describes the turning ($\xi$, Eqs. (2)) of the streamlines. One has

$$\frac{d\,\mathrm{Sin}\,\xi}{d\zeta} = \frac{1}{\rho\bar{v}^2}\left\{\mathrm{Sin}\,\xi\left(p_\zeta - \frac{\mathrm{Tan}\,\tau\,\mathrm{Sin}\,\lambda}{r}p_\sigma\right) + \frac{\mathrm{Cos}\,\xi\,\mathrm{Cos}\,\lambda}{r\,\mathrm{Cos}\,\tau}\,p_\sigma\right\}$$

$$(11)$$

$$+\ \mathrm{Sin}\,\xi\left[\frac{\mathrm{Cos}\,\lambda\,\mathrm{Tan}\,\zeta(\lambda_\theta - 1)}{r}\right]\frac{\mathrm{Cos}\,\xi\;\mathrm{Cos}\,\lambda\;\mathrm{Tan}\,\tau}{r\,\mathrm{Cos}\,\tau}\,\tau_\theta$$

Equations (4) and (7) - (11) plus a state equation form a determinate system. The one differential operation is L which is the ordinary derivative along a streamline. After solving, one returns to physical space, finding $\eta$ by Eq. (5).

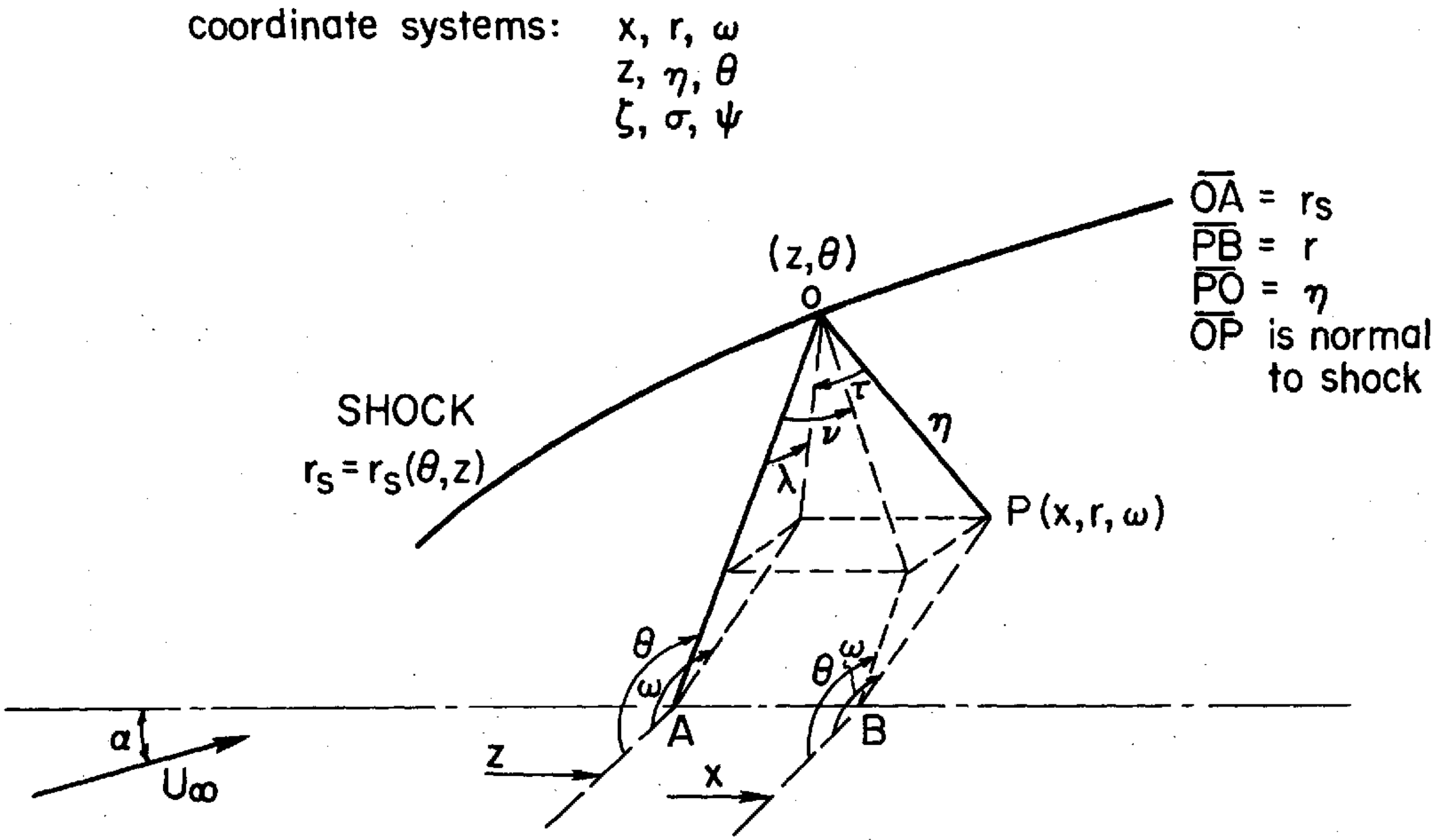



Fig. A 1. Coordinate Systems

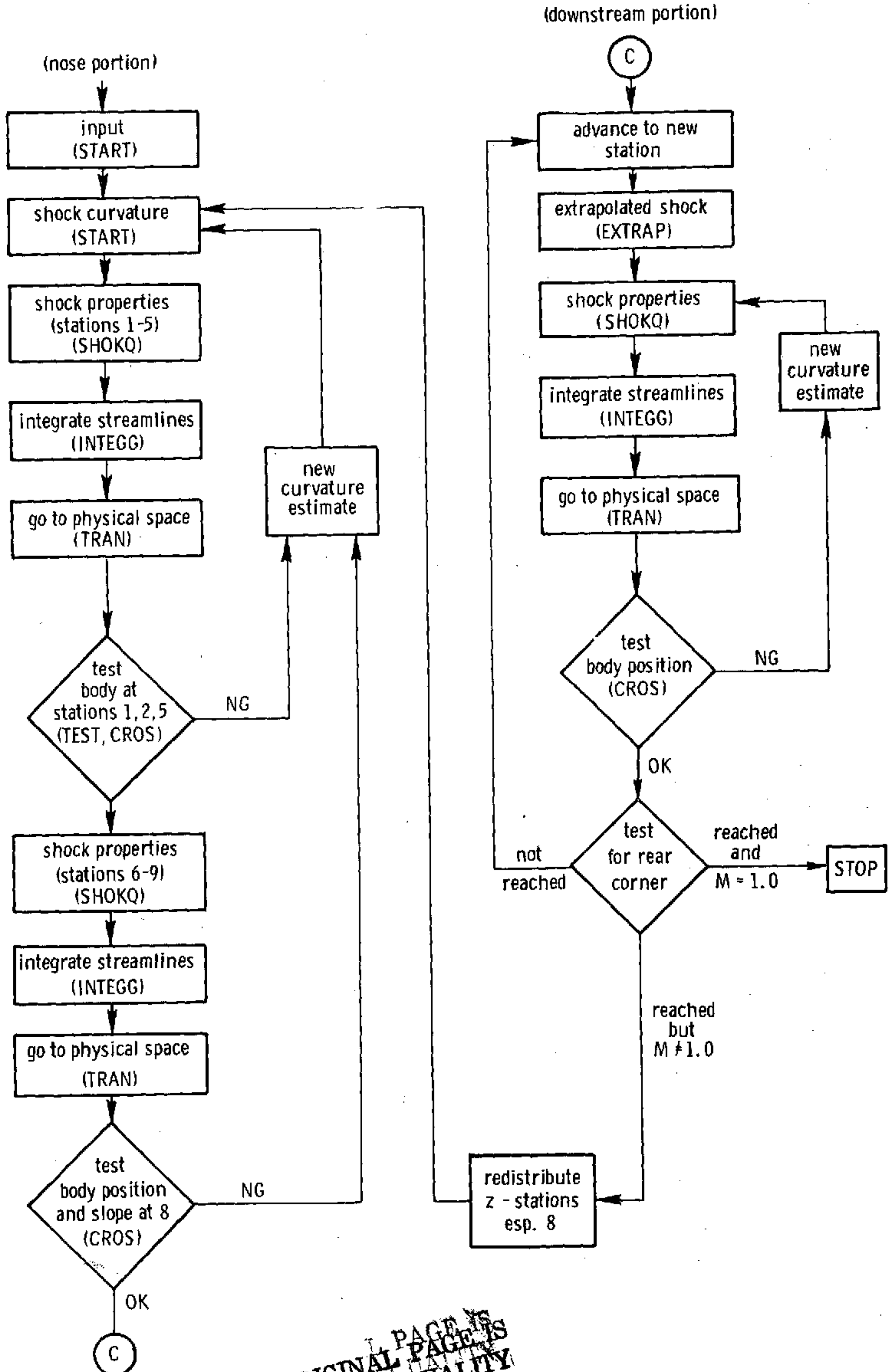
## APPENDIX B

The computer program consists of a main program plus a number of subroutines, of which the important ones are:

| | |
|---|---|
| CROS (I) | Tests the position of the body from TRAN against the actual geometric coordinates and provides an iteration scheme. |
| EXTRAP | Extrapolates the solution one step down stream when the results in CROS converge. |
| INTEGG (I) | A third order Runge-Kutta routine to integrate the equation for the streamline. |
| NONAX | Main logic program controlling the sequence of use of the subroutines. |
| SHOKQ (I) | Computes quantities at the shock for specified radius, slope and curvature distribution. |
| START | Provides initial values at the nose as well as starting estimates of the flow near it. |
| TEST | Routine to test (and provide an iteration to improve) the starting solution. |
| TRAN (I) | Transfer the results of INTEGG from the $\zeta$, $\sigma$, $\psi$ coordinates to z, $\theta$, $\eta$ ones and integrates to find $\eta$. |

The general logic (shown in the flow diagram) is to make an initial estimate of the shock slope and position (START), use that to find shock values for 5 stations (SHOKQ), integrate the streamline equations (INTEGG) and transform to physical space (TRAN) and test the result (TEST, CROS) to determine an acceptable nose curvature distribution. Then one goes through INTEGG, and TRAN to station 8, uses CROS (8) to test the body position and slope at station 8. Once that converges the solution is extrapolated (EXTRAP) and one goes thru SHOKQ, INTEGG, TRAN and CROS, iterating as necessary.

# FLOW DIAGRAM



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