General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

APPLICATION OF LOCAL LINEARIZATION
AND THE TRANSONIC EQUIVALENCE RULE
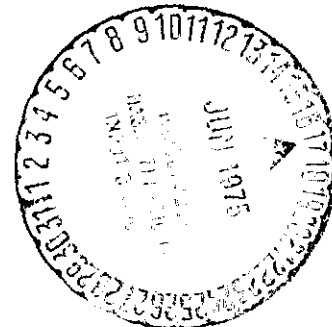TO THE FLOW ABOUT SLENDER ANALYTIC
BODIES AT MACH NUMBERS NEAR 1.0

Richard W. Tyson
Ralph J. Muraca

January 30, 1975

1. Report No. NASA TM X-72654	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Application of Local Linearization and the Transonic Equivalence Rule to the Flow about Bodies at Mach Numbers Near 1.0		5. Report Date January 30, 1975
		6. Performing Organization Code
7. Author(s) Richard W. Tyson and Ralph J. Muraca		8. Performing Organization Report No.
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665		10. Work Unit No.
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics & Space Administration Washington, DC 20546		13. Type of Report and Period Covered Technical Memorandum
		14. Sponsoring Agency Code

15. Supplementary Notes

Interim Technical information release subject to possible revision and/or later formal publication.

16. Abstract

The local linearization method for axisymmetric flow is combined with the transonic equivalence rule to calculate pressure distribution on slender bodies at free-stream Mach numbers from .8 to 1.2. This is an approximate solution to the transonic flow problem which yields results applicable during the preliminary design stages of a configuration development. The method can be used to determine the aerodynamic loads on parabolic arc bodies having either circular or elliptical cross sections. The method is particularly useful in predicting pressure distributions and normal force distributions along the body at small angles of attack. The equations discussed in this paper may be extended to include wing-body combinations.

17. Key Words (Suggested by Author(s)) (STAR category underlined) Transonic Flow, Local Linearization, Equivalence Rule, Body of Revolution	18. Distribution Statement Unclassified - Unlimited		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 50	22. Price* $3.75

APPLICATION OF LOCAL LINEARIZATION AND

THE TRANSONIC EQUIVALENCE RULE TO THE FLOW ABOUT

SLENDER ANALYTICAL BODIES AT MACH NUMBERS NEAR 1.0

RICHARD W. TYSON

RALPH J. MURACA

SUMMARY

The local linearization method for axisymmetric flow is combined with the transonic equivalence rule to calculate pressure distributions on slender bodies at free-stream Mach numbers from .8 to 1.2. The method can be used to determine the aerodynamic loads on parabolic arc bodies having either circular or elliptical cross sections. The method is particularly useful in predicting pressure distributions and normal force distributions along the body at small angles of attack. The primary application of this method would be to establish trends for the flow variables as a function of geometry, and to evaluate configurations during the preliminary design stage. The equations discussed in this paper may be extended to include wing-body combinations.

INTRODUCTION

The purpose of this paper is to describe an approximate procedure for determining the pressure distributions at free-stream Mach numbers near unity for a number of slender bodies both non-lifting and lifting. The quality of results will be demonstrated by comparison with experimental data. The analysis is based on the small disturbance theory of inviscid transonic flow, and makes use of the approximations of slender body theory, the transonic equivalence rule, and the method of local linearization.

Results are presented for several bodies having identical axial distributions of cross-sectional area and various elliptical cross sections. Angles of attack included in these comparisons range from $0°$ to $6°$. The examples chosen were selected to enable comparison with existing data obtained in wind tunnel tests.

In addition to a description of the method and a presentation of the basic equations used in the derivation of the working equations, the computer program listings and input instructions are included as an appendix.

SYMBOLS

a	Speed of sound
C_p	Pressure coefficient $\dfrac{p - p_\infty}{\dfrac{\rho_\infty}{2} U_\infty^2}$
C_N	Normal-force coefficient, F_N/qS
d	Maximum body diameter
F_N	Normal force
k	$\dfrac{M_\infty^2(\gamma + 1)}{U_\infty}$
ℓ	Length of body
M	Local Mach number
p	Local static pressure
p_∞	Free stream static pressure
r	Radius of body
S	Body cross section area
$S\dfrac{dC_N}{dx}$	Product of reference area and normal force coefficient derivative
t	Planar body thickness

U_∞ — Free stream velocity

u_1, u_2, u_3 — Velocity components

u, v, w — Perturbation velocity components

x, r, θ — Cylindrical coordinates where x extends in the direction of the free-stream velocity

$\overline{\theta}$ — Total cone angle, in radians

α — Angle of attack

γ — Ratio of specific heats

λ — Ratio of major to minor axis length for an ellipse

ρ_∞ — Free stream density of air

T — Thickness ratio ℓ/d

ϕ — Perturbation velocity potential

ϕ_x — Denotes partial derivative with respect to x; $\dfrac{\partial \phi}{\partial x}$

ϕ_{xx} — Denotes second partial derivative with respect to x; $\dfrac{\partial^2 \phi}{\partial x^2}$

Subscripts

∞ — Free stream conditions

i, j, k — Dummy subscript used in index notation

0 — Stagnation conditions

Superscript

'   Denotes total derivative with respect to x

PRESENTATION OF EQUATIONS

The equations to be presented here assume very small perturbation velocities compared to the free stream velocity U_∞ . The equations of motion are then simplified by neglecting small terms in the perturbation velocities. The Method of Local Linearization described in ref. 1 is used to further simplify the equations and combines with the transonic equivalence rule to give approximate solutions to these equations in the transonic flow regime.

Small Perturbation Equations of Transonic Flow

The equations of motion for steady frictionless flow are given in ref. 2 as:

$$u_i \, u_j \, \frac{\partial u_i}{\partial x_j} = a^2 \, \frac{\partial u_k}{\partial x_k} \, , \, i, \, j, \, k = 1, \, 2, \, 3. \qquad (1)$$

The speed of sound is related to the velocity components by

$$\frac{u_1^2 + u_2^2 + u_3^2}{2} + \frac{a^2}{\gamma - 1} = \frac{a_0^2}{\gamma - 1} \, , \qquad (2)$$

where a_0 denotes a reference condition.

Expanding and substituting the velocity field, defined by

6

$$u_1 = U_\infty + u$$

$$u_2 = v \tag{3}$$

$$u_3 = w,$$

where u, v, w are small perturbation velocity components, into equations 1 & 2 and neglecting terms containing squares of the perturbation velocities yields the equation:

$$(1 - M_\infty^2) \frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2} + \frac{\partial w}{\partial x_3} = M_\infty^2 (\gamma + 1) \frac{u}{U_\infty} \frac{\partial u}{\partial x_1} + M_\infty^2 (\gamma - 1) \frac{u}{U_\infty} (\frac{\partial v}{\partial x_2} + \frac{\partial w}{\partial x_3})$$

$$+ M_\infty^2 \frac{v}{U_\infty} (\frac{\partial u}{\partial x_2} + \frac{\partial v}{\partial x_1}) + M_\infty^2 \frac{w}{U_\infty} (\frac{\partial u}{\partial x_3} + \frac{\partial w}{\partial x_1}) \tag{4}$$

Retaining terms on the right hand side of eq. (4) containing $\partial u / \partial x_1$, and dropping all other higher order terms yields the small perturbation equation valid for subsonic, transonic and supersonic flow:

$$(1 - M_\infty^2) \frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2} \frac{\partial w}{\partial x_3} = M_\infty^2 (\gamma + 1) \frac{u}{U_\infty} \frac{\partial u}{\partial x_1} \tag{5}$$

We shall assume the flow to be irrotational and isentropic so that a perturbation velocity potential ϕ exists, where

$$u = \frac{\partial \phi}{\partial x_1} \; ; \; v = \frac{\partial \phi}{\partial x_2} \; ; \; w = \frac{\partial \phi}{\partial x_3}$$

Thus, the equation for frictionless, irrotational flow with small perturba-

tions becomes:

$$(1 - M_\infty^2) \frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_2^2} + \frac{\partial^2 \phi}{\partial x_3^2} = \frac{M_\infty^2 (\gamma + 1)}{U_\infty} \frac{\partial \phi}{\partial x_1} \frac{\partial^2 \phi}{\partial x_1^2} \tag{6}$$

Consider a body-fixed Cartesian coordinate system as shown on figure 1, where x_1, x_2, x_3 correspond to the body axis x, y, z. If the origin is located at the nose with the x-axis directed rearward and aligned with the longitudinal axis of the body, the pressure coefficient can be written (ref 1.);

$$C_p = - \frac{}{U_\infty} (\phi_x + \alpha \phi_z) - \frac{1}{U_\infty^2} (\phi_y^2 + \phi_z^2), \tag{7}$$

where the subscript notation is used to denote derivatives.

The Method of Local Linearization for Axisymmetric Flow

Approximate solutions of good accuracy for axisymmetric flow with $M_\infty \approx 1$ past a wide class of slender pointed bodies may be obtained by application of the method of local linearization. A discussion of the development of the method and the resulting equations follow.

Equation (6) can be rewritten

$$\lambda \phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{8}$$

where

$$\lambda = 1 - M_\infty^2 - k \, u_\infty$$

and

$$k = M_\infty^2 \; (\gamma + 1)/U_\infty$$

If λ varies sufficiently slowly, it can be considered as a constant in the initial stages of the analysis. The equation is then reduced to the 3 - D Laplace equation for which a solution is known. This resulting solution for the velocity component u is then improved in the following manner. By differentiating the solution for u with respect to x and replacing the previously assumed constant λ term appearing in the Laplace solution by its actual value a non-linear ordinary differential equation results. This differential equation may then be integrated numerically.

This method results in the following first order nonlinear ordinary differential equations in u/U_∞. For $M_\infty < 1$ the following equation results:

$$\frac{d(u/U_\infty)}{dx} = \frac{S'''(x)}{4\pi} \; \ell n \; (1 - M_\infty^2 - k \; u) + E'(x) \quad ,$$

for $0 < x < \ell$

where

$$E(x) = \frac{S''(x)}{4\pi} \; \ell n \; \frac{S}{4\pi x(\ell - x)} + \frac{1}{4\pi} \int_0^\ell \frac{S''(x) - S''(\xi) \; d\xi}{|X - \xi|} . \tag{10}$$

For $M_\infty > 1$ the following equation results;

$$\frac{d(u/U_\infty)}{dx} = \frac{S'''(x)}{4\pi} \; \ell n \; (M_\infty^2 - 1 + k \; u) + H'(x), \tag{11}$$

$$\text{where } H(x) = \frac{S''(x)}{4\pi} \; \ell n \; \left[\frac{S}{4\pi x^2}\right] + \frac{1}{2\pi} \int_0^x \frac{S''(x) - S''(\xi)}{x - \xi} \; d\xi \quad , \tag{12}$$

9

for $0 < x < \ell$.

For $M_\infty = 1$ the following equations result;

If $u < 0$ we have

$$\frac{d\,(u/U_\infty)}{dx} = \frac{S'''(x)}{4\pi}\,\ell n\,(-k\,u) + P'_E(x)\ ,\tag{13}$$

$$P_E(x) = \frac{S''(x)}{4\,\pi}\,\ell n\,\frac{S(x)}{4\pi x(\ell - x)} + \frac{1}{4\,\pi}\,\int_0^x \frac{S''(x) - S''(\xi)}{x - \xi}\,d\xi\ .\tag{14}$$

If $u > 0$ we have

$$\frac{d\,(u/U_\infty)}{dx} = \frac{S'''(x)}{4\,\pi}\,\ell n\,(k\,u) + P'_H(x),\tag{15}$$

and

$$P_H(x) = \frac{S''(x)}{4\,\pi}\,\ell n\,\frac{S(x)}{4\pi x^2} + \frac{1}{2\,\pi}\,\int_0^x \frac{S''(x) - S''(\xi)}{x - \xi}\,d\xi\tag{16}$$

To solve these equations it is necessary to define a value for u at some point along the body in order to evaluate the constant of integration. The initial value of u may be defined in several different ways. If there is no experimental data available for the particular body under consideration then there are two classes of cases to be distinguished. One class includes the foreparts of bodies with a convex corner, such as a cone-cylinder, for which the necessary condition is supplied by the fact that the velocity must be sonic at the corner, that is;

$$\frac{u}{U_\infty} = \frac{1 - M_\infty^2}{M_\infty^2(\gamma + 1)}\tag{17}$$

10

The other includes smooth bodies for which a value for u cannot be specified a priori. The procedure to be followed in the latter case is to find that point (x) along the body which satisfies the condition $S''(x) = 0$. The solution to Laplace's equation at this point will be input as the initial u. Numerical integration must progress in both directions from this point. The local linearization program which was developed performs this integration and provides initial conditions as required.

The local linearization program solves equations 9 through 16 for u/U_∞ using trapezoidal and the Runge-Kutta integration techniques. The program then computes pressure coefficients using equation 7. A listing of this program and sample input/output are included in the appendix.

The Transonic Equivalence Rule

The transonic equivalence rule first stated by Oswatitsch (references 3 and 4) relates the flow around a slender body of arbitrary cross section to that around an "equivalent" nonlifting body of revolution having the same longitudinal distribution of cross-sectional area.

Following the development of reference 1, the general solution for the flow over a body can be written

$$\Phi (x, y, z) = U_\infty (x + \alpha z) + \phi (x, y, z) \qquad (18)$$

and the pressure coefficient is defined by equation (7).

From slender body theory:

$$\phi = \phi_2 + g(x) \qquad (19)$$

where ϕ_2 is the solution of the 2-D Laplace equation for the actual body shape, and g (x) is a function of the cross sectional area distribution.

11

Using the equivalence rule the potential can be written

$$\phi = \phi_{2,\alpha} + \phi_{2,t} - \phi_{2,B} + \phi_B \tag{20}$$

where $\phi_{2,\alpha}$ is the angle of attack solution from Laplace's equation, $\phi_{2,t}$ is the thickness solution from Laplace's equation, $\phi_{2,B}$ is the solution to the 2-D Laplace's equation for the equivalent body, ϕ_B is the equivalent body contribution.

Consider the application of this method to the case of a body having an elliptical cross section. The equivalent body will be a body of revolution (circular cross section) having an identical $S(x)$ distribution.

The equivalent body potential solution is:

$$\phi_{2,B} = \frac{U_\infty}{2\pi} S' \ln r + 2U_\infty \alpha r \sin \theta \ , \tag{21}$$

and

$$g'(x) = - \frac{U_\infty}{2} \left[(C_p)_B + \frac{S''}{2\pi} \ln \left(\frac{S}{\pi}\right) + \frac{S'^2}{4\pi S} \right] \tag{22}$$

The thickness solution is obtained from the potential solution for the case of an expanding ellipse. Physically it can be interpreted as the effect of passing a body through a plane of fluid normal to the path of motion.

From reference 5 the solution for a uniformly expanding circle in terms of a complex potential, W, is:

$$W = r V_r \log \xi \tag{23}$$

Where V_r represents the velocity at which the radius of the circle is growing, and ξ is the complex variable in the transformed plane. The

12

relationship between ξ and σ, the complex variable in the physical plane is

$$\xi = \frac{1}{2}\left[\sigma + (\sigma^2 - a^2 + b^2)^{\frac{1}{2}}\right] \tag{24}$$

where $\xi = Y + iZ$ and a and b are the length of the major and minor axis of the ellipse. The quantity $r_o V_r$ can be written

$$r \frac{dr}{dt} = r \frac{dr}{dx} \frac{dx}{dt} = r \, r' \, U_\infty \tag{25}$$

but, $S' = 2\pi r \, x'$

therefore

$$r \, V_r = \frac{S' U_\infty}{2\,\pi} \, . \tag{26}$$

The relationship between r, and a and b, is $r = \sqrt{ab}$. Now $W(\xi)$ is the complex potential defined by

$$W(\xi) = \phi + i\psi$$

where ϕ is the perturbation potential in the equation for C_p. Substituting equations 24 and 26 and into equation 23 yields the thickness solution for an ellipse

$$\phi_{2,t} = \text{R.P.} \left[\frac{U_\infty S'}{2\,\pi} \, \ln \frac{\sigma + (\sigma^2 - a^2 + b^2)^{1/2}}{2}\right] \tag{28}$$

where R.P. denotes the real part of the potential function. Similar analysis of flow over a cylinder (ref. 6) yields the expression for angle of attack effects:

13

$$\phi_{2,t} = \text{R.P.} \left[\frac{iU_\infty \alpha}{2\pi} \left[-\sigma - (\sigma^2 - a^2 + b^2)^{1/2} \right] + \frac{(a^2 + b^2)^{1/2}}{\sigma + (\sigma^2 - a^2 + b^2)^{1/2}} \right] \quad (29)$$

The constants a and b in eq. 28 may be related to the cross sectional area of the ellipse, $S = \pi ab$ and $\lambda = \frac{a}{b}$.

Differentiating eq (28) with respect to x yields:

$$\frac{\partial \phi_{2,t}}{\partial x} = \text{R.P.} \ \frac{U_\infty}{2\pi} \ \frac{\partial}{\partial x} \left[S' \ \ln \left(\frac{\sigma + (\sigma^2 - a^2 + b^2)^{1/2}}{2} \right) \right] \quad (30)$$

Substituting for σ, where $\sigma = re^{i\theta}$ yields,

$$\frac{\partial \phi_{2,t}}{\partial x} = \text{R.P.} \ \frac{U_\infty}{2\pi} \left\{ S'' \ \ln \left[\frac{r}{2} \left(e^{i\theta} + (e^{i2\theta} - \lambda + \frac{1}{\lambda})^{1/2} \right) \right] \right.$$

$$\left. + \ S' \left[\frac{r'}{r} + \ln \left(\frac{e^{i\theta} + e^{i2\theta} - \lambda + 1/\lambda}{2} \right) \right] \right\} \quad (31)$$

Differentiating $\phi_{2,t}$ with respect to y & z yields,

$$\frac{\partial \phi_{2,t}}{\partial y} = \text{R.P.} \ \frac{U_\infty}{2\pi} \ S' \left[1 + \frac{re^{i\theta}}{(r^2 e^{i2\theta} - a^2 + b^2)^{1/2}} \right] \frac{1}{re^{i\theta} + (r^2 e^{i2\theta} - a^2 + b^2)^{1/2}} \quad (32)$$

and

$$\frac{\partial \phi_{2,t}}{\partial z} = \text{R.P.} \ i\frac{U_\infty}{2\pi} \ S' \left[1 - \frac{re^{i\theta}}{(r^2 e^{i2\theta} - a^2 + b^2)^{1/2}} \right] \frac{1}{re^{i\theta} + (r^2 e^{i2\theta} - a^2 + b^2)^{1/2}} \quad (33)$$

14

In the angle of attack solution for an ellipse, $r = \dfrac{a + b}{2}$, and $\lambda = \dfrac{a}{b}$,

form the two relationships between a, b and r.

Differentiating eq 29 with respect to x yields:

$$\frac{\partial \phi_{2,\alpha}}{\partial x} = R.P. \left\{ \frac{iU_\infty \alpha}{2} \quad r' \left[-e^{i\theta} - \left(e^{i\theta} + \frac{4(1-\lambda)}{1 + \lambda}\right)^{1/2} \right. \right. + $$

$$\left. \left. \frac{4}{e^{i\theta} + \left(e^{i2\theta} + \frac{4(1-\lambda)}{1 + \lambda}\right)^{1/2}} \right] \right\} \tag{34}$$

Differentiating eq 29 with respect to y and z yields:

$$\frac{\partial \phi_{2,\alpha}}{\partial y} = R.P. \frac{iU_\infty \alpha}{2} \left\{ -1 - \frac{re^{i\theta}}{(r^2 e^{i2\theta} - a^2 + b^2)^{1/2}} - \frac{4r^2}{\left[re^{i\theta} + (r^2 e^{i2\theta} - a^2 + b^2)^{1/2}\right]} \right. $$

$$\left. - \frac{4r^3 e^{i\theta}}{\left(re^{i\theta} + (r^2 e^{i2\theta} - a^2 + b^2)^{1/2}\right)\left(r^2 e^{i2\theta} - a^2 + b^2\right)^{1/2}} \right\} \tag{35}$$

and

$$\frac{\partial \phi_{2,\alpha}}{\partial z} = R.P. \frac{iU_\infty \alpha}{2} \left\{ -i - \frac{ire^{i\theta}}{(r^2 e^{i2\theta} - a^2 + b^2)^{1/2}} - \frac{4r^2}{\left[re^{i\theta} + (r^2 e^{i2\theta} - a^2 + b^2)^{1/2}\right]} \right. $$

$$\left. \frac{- i \, 4r^3 e^{i\theta}}{\left(re^{i\theta} + (r^2 e^{i2\theta} - a^2 + b^2)^{1/2}\right)(r^2 e^{i2\theta} - a^2 + b^2)^{1/2}} \right\} \tag{36}$$

With the derivatives of ϕ defined all of the terms on the right
hand side of equation 7 can be evaluated except the pressure coefficient
distribution on the equivalent body C_{P_B}. This data must be input from
either experiment or from the local linearization program previously
discussed. Evaluation of equation 7 is performed in the Transonic
Aerodynamics Program. These results may then be integrated to obtain
values for lift, drag, and pitching moments on slender bodies. Since the
aerodynamic loading, lift and all lateral forces and moments may be
expressed in terms of differences in pressure between points at the same
longitudinal station, these quantities depend solely on ϕ_2. A listing of
the Transonic Aerodynamics Program with sample input/output appears in the
Appendix.

APPLICATIONS

The bodies chosen for analysis fall into three categories, bodies
of revolution, elliptical bodies, and planar bodies. All were previously
tested in wind tunnels and experimental data for comparison were readily
available. Data for four models tested in the Ames 14-Ft Transonic
Wind Tunnel at Mach numbers from .8 to 1.2 are presented in reference 7.
Configurations were analyzed and comparisons with experimental data
are shown in figure 2 through figure 6.

The body chosen initially is a body of revolution with axial location of maximum cross-sectional area at $x/\ell = .5$, and a fineness ratio of 12. The radius of the body is given by

$$r(x,\theta) = \frac{4\sqrt{\lambda}\; r_{MAX}\; (x - x^2)}{\sqrt{\cos^2\theta + \lambda^2 \sin^2 \theta}} \,, \tag{37}$$

where $r_{MAX} = 3.0$.

Figures 2a and 2b show the pressure coefficients computed by the local linearization program, and experimental data from reference 7. The data shown on Figure 2a at $M_\infty = 1.0$ are in good agreement up to the location $x/\ell = 0.7$, where a shock occurs. The results shown on Figure 2b at $M_\infty = 1.2$ are in good agreement over the entire body. Both cases were run with initial conditions obtained from experimental data. Figure 3 shows the differences which result when the initial condition is derived from experimental data and when it is calculated using the solution at the point where $S''(x) = 0$. The major difference between the results obtained using a computed u_o versus an experimentally determined value is a shifting of the curve along the C_p axis.

The second body chosen for analysis was a parabolic arc body of elliptical cross-section. The body radius is given by equation 37 above.

The Transonic Aerodynamics Program is used to calculate angle of attack variation over this body with $\lambda = 1$. Experimental data was used for the equivalent body at zero angle of attack. Comparisons with wind tunnel results are shown in figures 4a and 4b with Mach number equal to 1.0 and 1.2. The pressure coefficient, C_p, is plotted versus x/ℓ at an angle of attack of $6°$.

The agreement between computed and measured results is not good for the $M_\infty = 1.0$ case. The general trend of the data is correct, however, the analysis predicts lower value of C_p over most of the body. At $M_\infty = 1.2$ the agreement is better although the analysis continues to underpredict C_p.

Figure 5a shows results for a configuration with $\lambda = 3$ at Mach number equal to 1.0. The equivalent body pressure coefficient is again obtained from experimental data. The same configuration is shown in figure 5b at $6°$ angle of attack. This analysis was done using the local linearization program to predict the equivalent body pressure coefficient used in the Transonic Aerodynamic Program.

For the case at $\alpha = 0$ the agreement between computed and measured data is good. The analysis predicts slightly lower values of C_p over the forward portion of the body. Since measured data was used to obtain C_{p_B} the sudden increase in C_p on the rear portion of the body caused by a shock is reflected in the computed results.

For the case at $\alpha = 6°$ the agreement between computed and measured data is not as good. The data at $\theta = -90°$ shows better agreement than that at $\theta = +90°$. The general trend in C_p is predicted but the computed values

18

are consistently lower.

The final body chosen for comparison is the planar body shown in figure 6a. The elliptic cone has a major to minor axis ratio of 16.5 and is compared to the model tested in reference 1. Shown on figure 6a are the computed data for a circular and an elliptical cone with identical cross-sectional area distribution. Also shown is experimental data for the elliptic cone. As can be seen the agreement is quite good over the forward portion of the body; however, as the base of the cone is approached, the calculated pressures are higher than measured. This is probably due to wall interference effects as cited in reference 1.

Results for the elliptic cone at $\alpha = 4^\circ$ are presented on figure 6b. The pressure coefficient distribution along two meridonal locations on the body are shown. The data at $\theta = +90^\circ$ represents the windward meridian and the $\theta = -90^\circ$ the leeward meridian. In both instances the theory predicts higher pressures than measured. The reason for this could be the same as cited for the $\alpha = 0^\circ$ results.

CONCLUSION

The approximate method presented herein represents a useful tool for establishing the variations of pressure and velocity along the surface of analytic bodies. The method will appropriately reflect the effects of geometry changes on the local pressure coefficients. The computer solutions require minimal input and short computation times consequently the method is well suited to the analysis of configurations during the preliminary design stages.

In this context the quality of agreement between computed and measured results is acceptable. The procedures by which these solutions have been obtained are not restricted to the particular examples selected for display in this paper, but possess much greater generality.
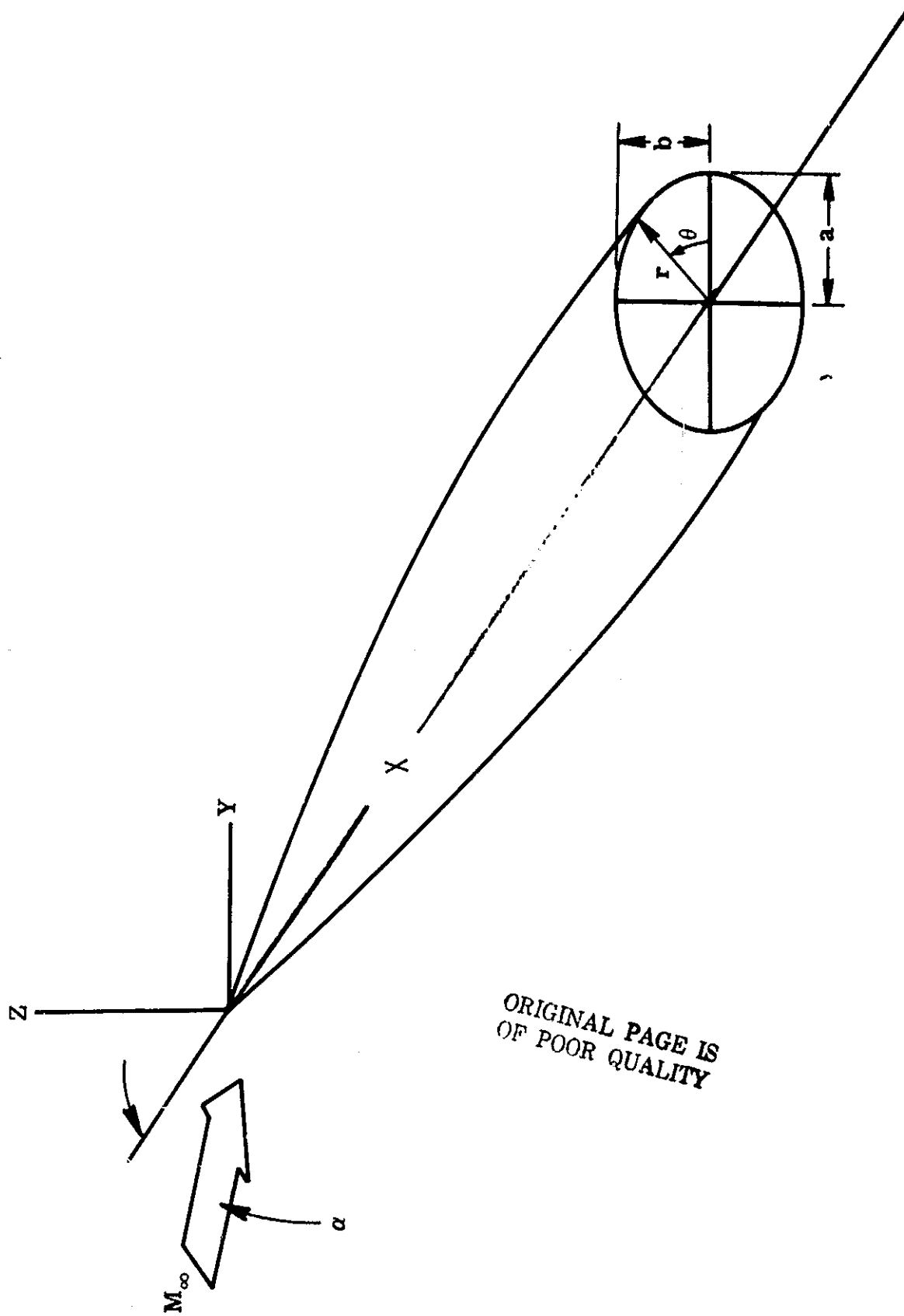
Figure 1   BODY IDENTIFICATION
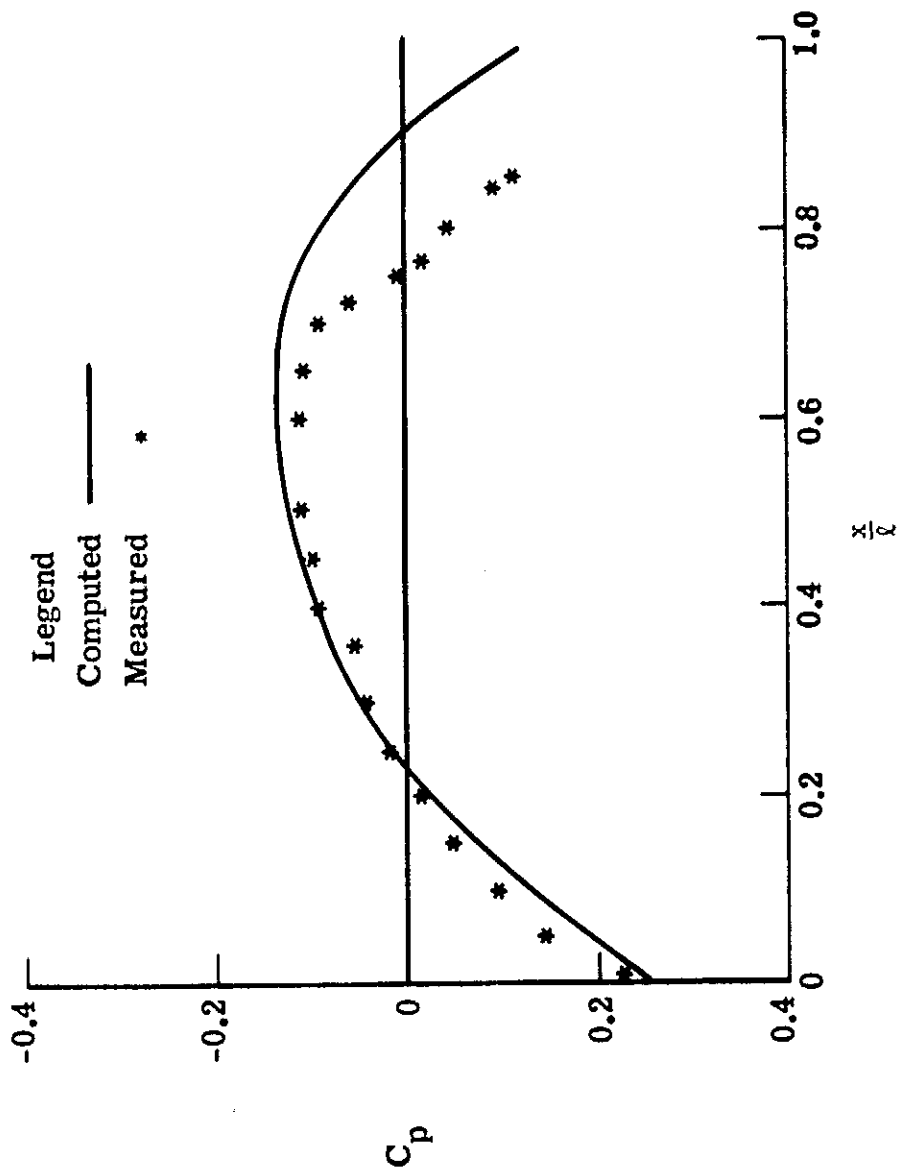
Figure 2(a).- Comparison of pressure coefficients on a parabolic arc body at $M_\infty = 1.0$, S_{max} at $x/\ell = .50$
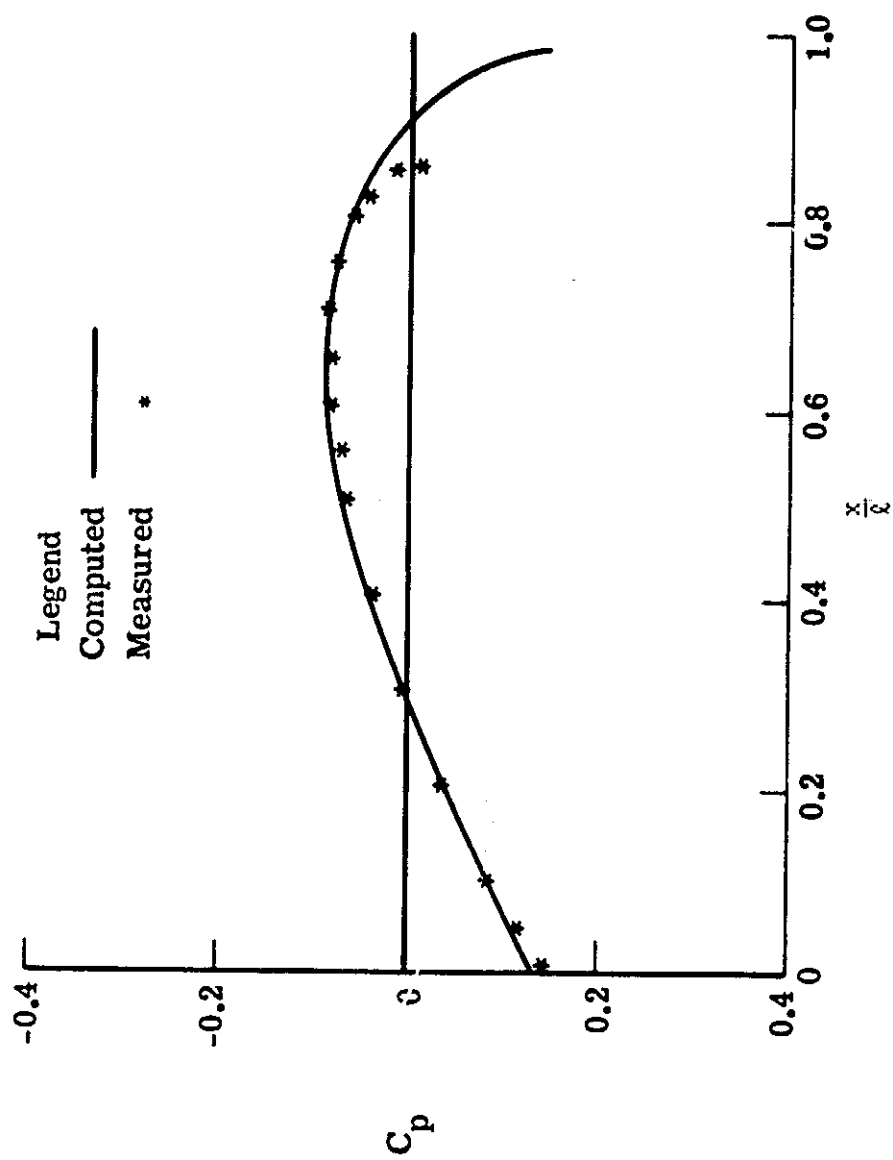
Figure 2(b).- Comparison of pressure coefficients on a parabolic arc body at $M_\infty = 1.2$. S_{max} at $x/\ell = .50$
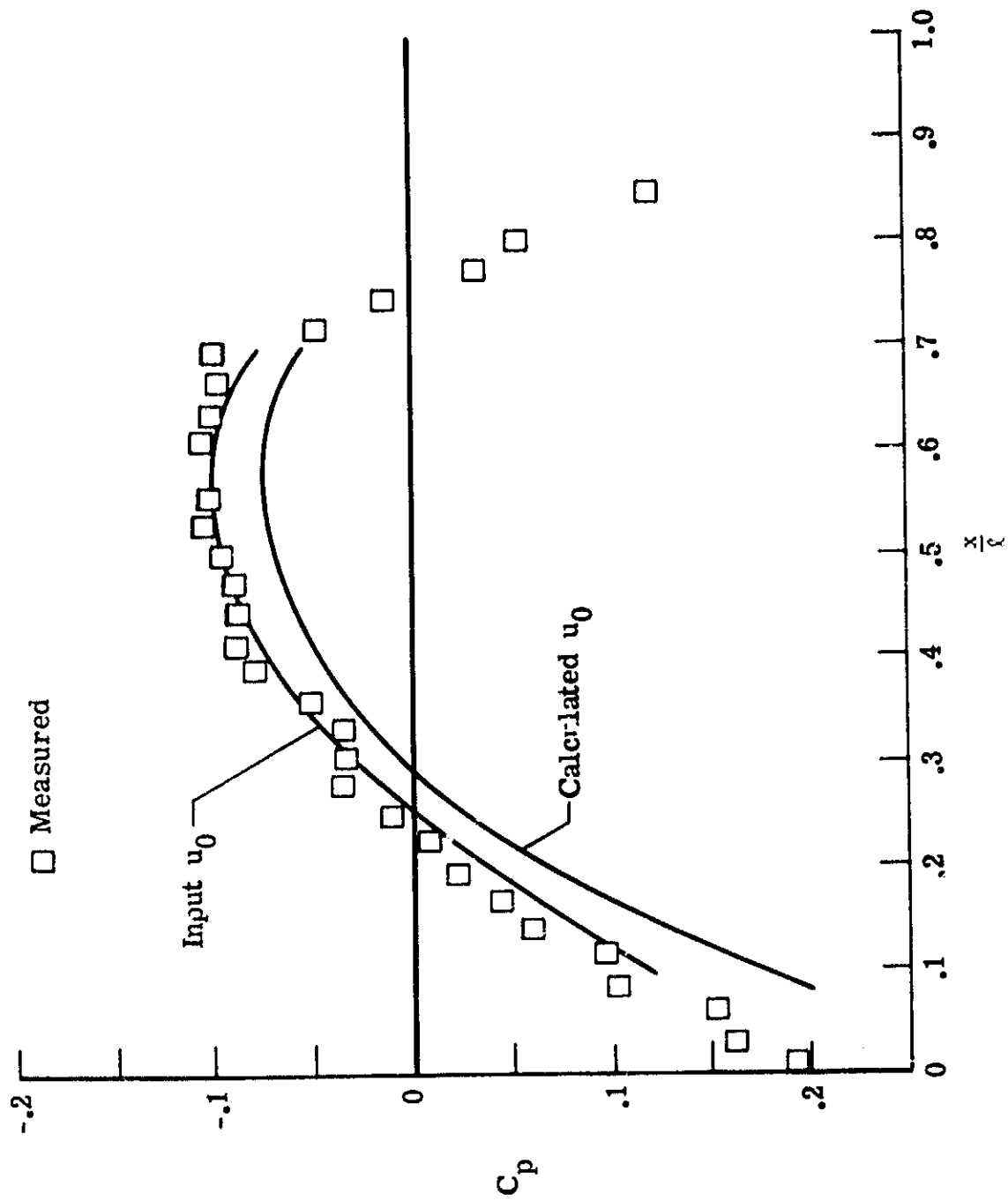
Figure 3.- Effect of initial conditions on the pressure coefficients on a parabolic arc body.

$M_\infty = 1.00$, $\lambda = 1.0$
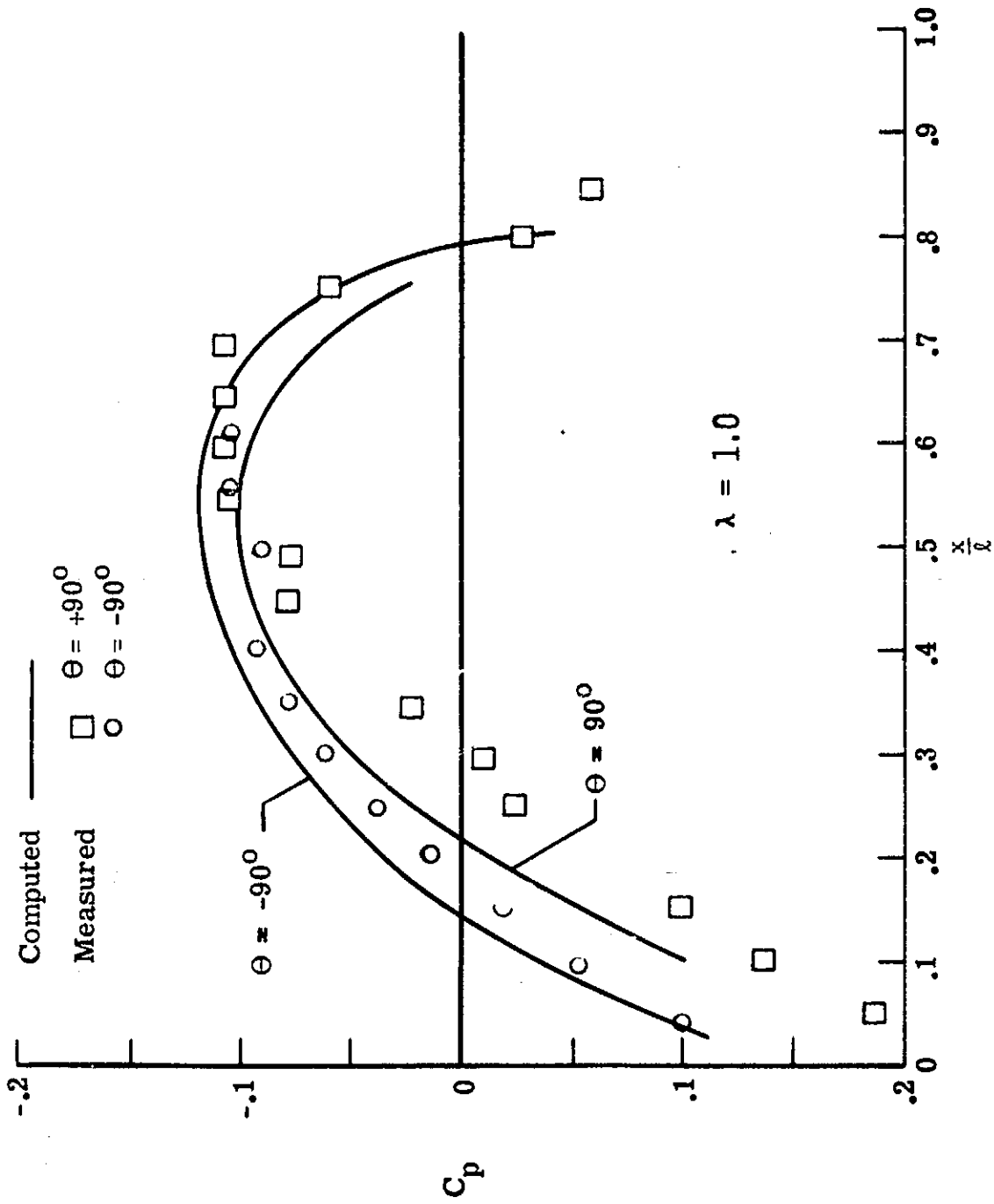
Figure 4(a).- Comparison of pressure coefficients on a parabolic arc body at $\alpha = 6^o$ and $M_\infty = 1.0$.
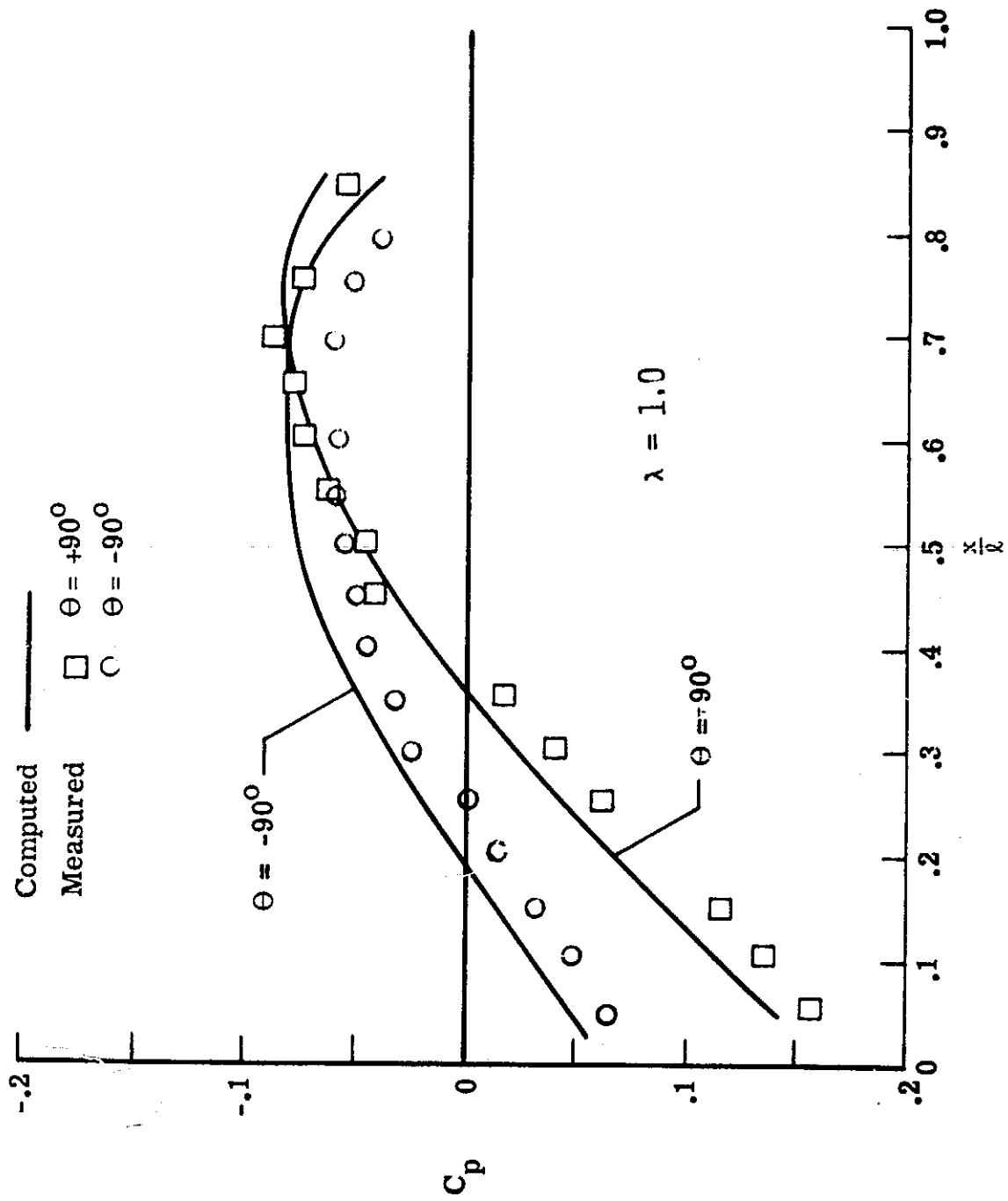
Figure 4(b).- Comparison of pressure coefficients on a parabolic arc body at $\alpha = 6^o$ and $M_\infty = 1.2$.
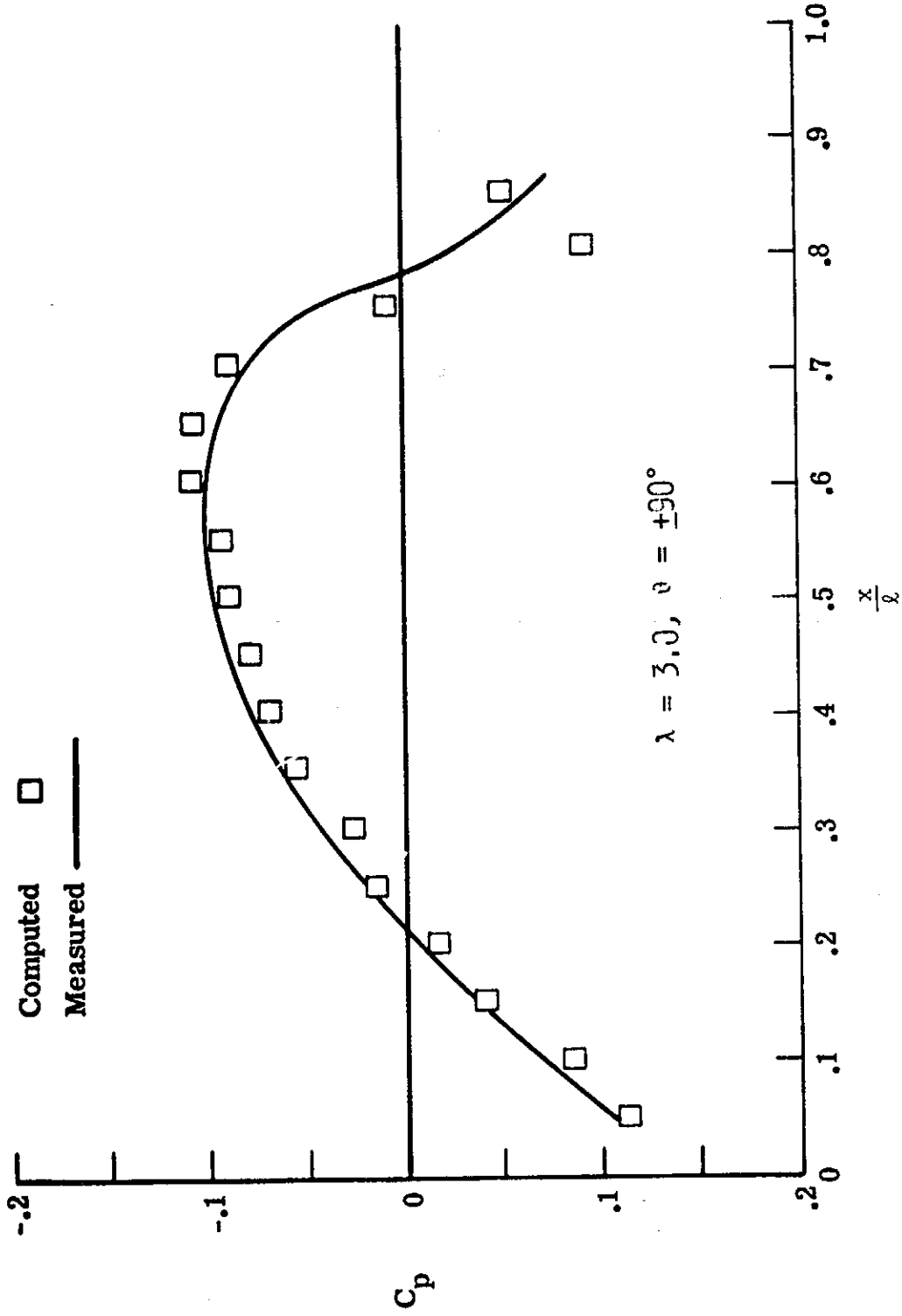
Figure 5(a).- Comparison of pressure coefficients on a parabolic arc body with elliptical cross section at $\alpha = 0^o$ and $M_\infty = 1.0$.
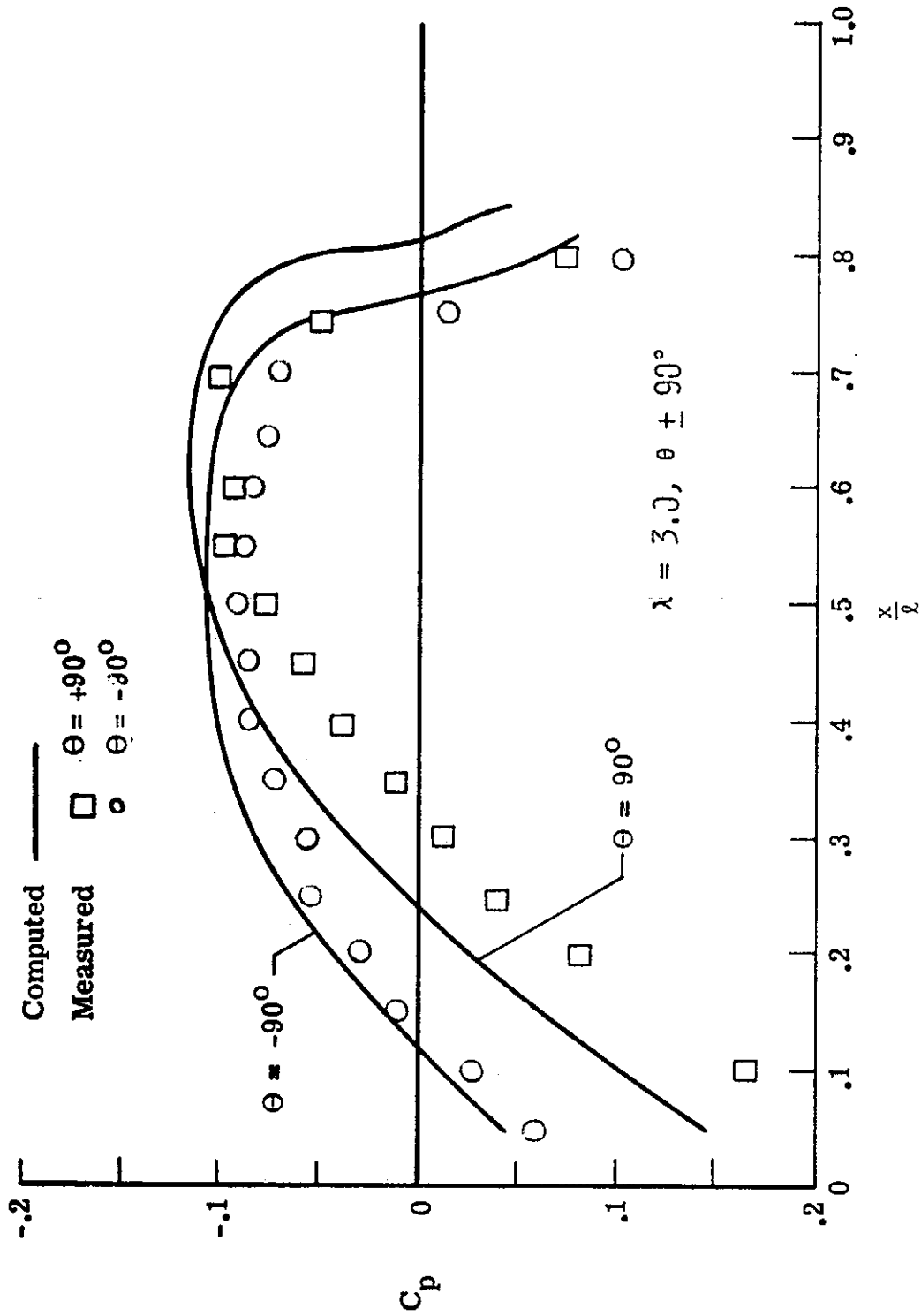
Figure 5(b).- Comparison of pressure coefficients on a parabolic arc body with elliptical cross section at $\alpha = 6^o$ and $M_\infty = 1.0$.
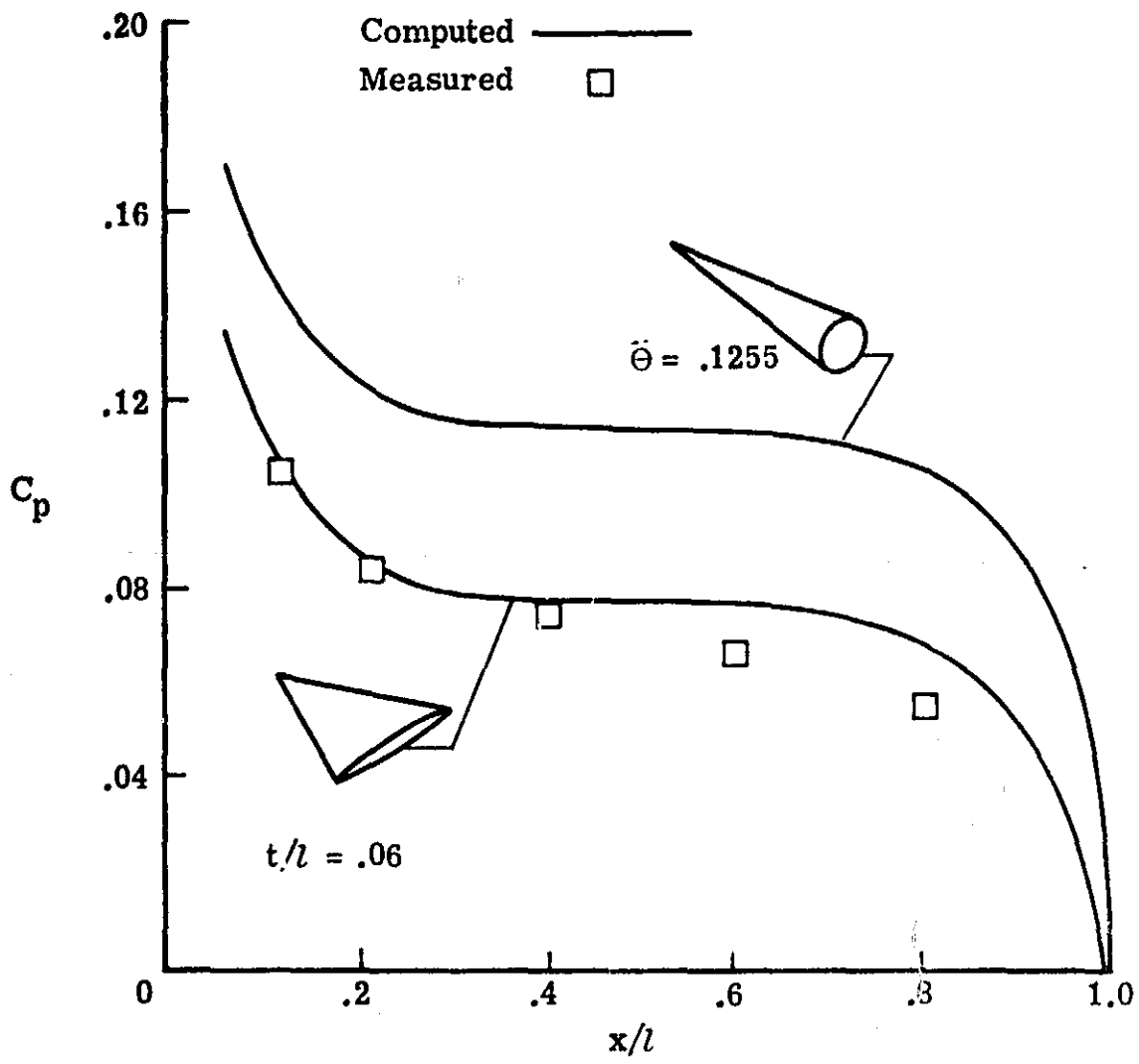
Figure 6(a).- Comparison of pressure coefficients on a planar body at $\alpha = 0^0$ and $M_\infty = 1.0$.
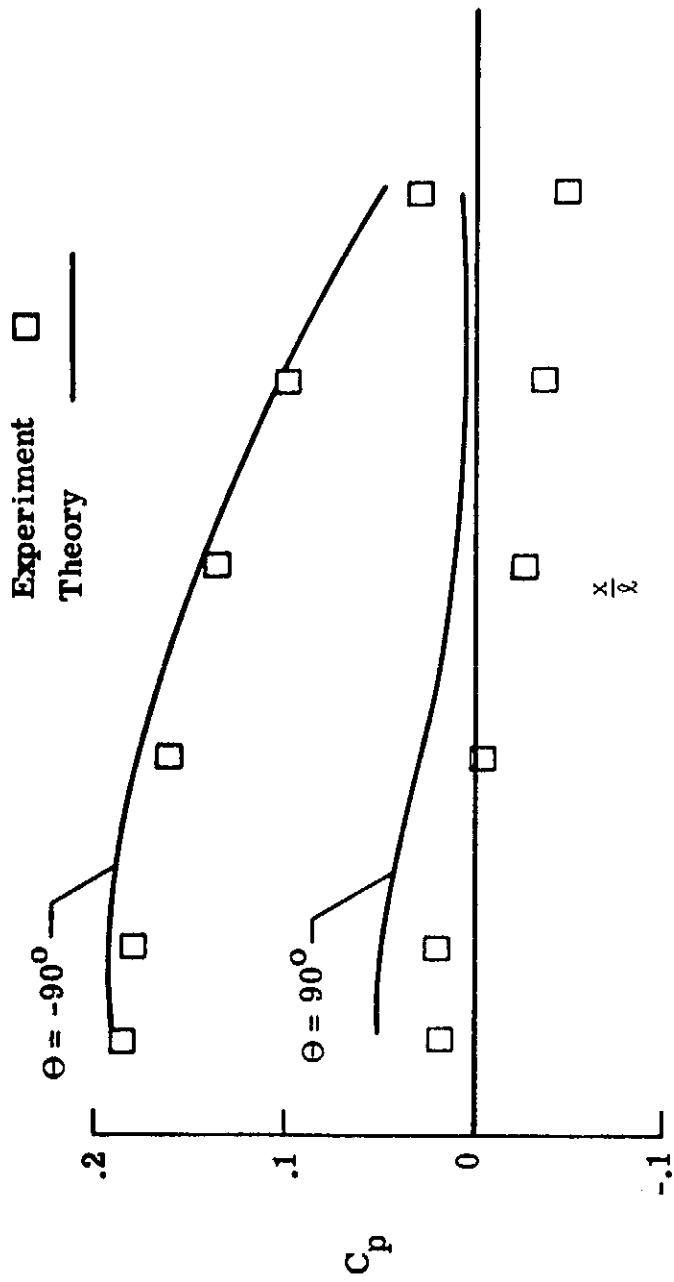
Figure 6(b).- Comparison of pressure coefficients on a planar body at $\alpha = 4^{\mathrm{o}}$ and $M_{\infty} = 1.0$.

REFERENCES

1. Heaslet, M. A. and Spreiter, Jr.: Three-Dimensional Transonic Flow Theory Applied to Slender Wings and Bodies. NACA Rep. 1318, 1957.

2. Leipmann, H. W. and Roshko, A.: Elements of Gas Dynamics. John Wiley and Sons, Inc. 1967.

3. Oswatitsch, K.: Die Theoretischem Arbeiten...Flugtechnischen Institut der Loniglich Technischen Hochschle, Stockholm. Berich Nr. 66

4. Oswatitsch, K.: The Area Rule. Applied Mechanics Reviews. Vol. 10, No. 12.

5. Nielson, Jack N.: Missile Aerodynamics, McGraw-Hill Book Company, Incs., 1960.

6. McDevitt, John B. and Taylor, Robert A.: Force and Pressure Measurements at Transonic Speeds for Several Bodies Having Elliptical Cross-Sections, NACA TR 4362, 1958.

7. Page, William A.: Experimental Study of the Equivalence of Transonic Flow About Slender Cone-Cylinders of Circular and Elliptic Cross-Section. NACA TN-4233.

APPENDIX

Local Linearization Program

Computer Program Input/Output - All input data are input according to the
NAMELIST type format. In columns 2 through 7 the name $ DATA is punched.
Column 8 is left blank. Beginning in column 9 the variable names which are
shown below and their values are punched. Each variable must be separated
by a comma and the value of the final variable must be followed by a $
sign. If optional variables are not known, they need not be punched. The
variables may be punched out to column 80 in any order.

	Variable	Definition
Optional	XO	Initial value for X along body
	CP	Pressure coefficient at XO
	A	Geometry Variables
	B	Appearing in body defining
	C	expression $R = A(BX + CX^D)$
	D	
	G	Ratio of specific heats
	M	Mach number

EXAMPLE:

$ DATA M = 1.0, XO = .48, CP = 0., A = .071, B = 1,

C = -1, D = 6.03, G = 1.4, $

If more than one case is to be run, then the input data for the second
case may be placed immediately after the data for the first.

22

The output consists of a listing of the input parameters, Mach number, Initial station and corresponding velocity, and ratio of specific heats for the medium. In addition, the equation defining the body shape is shown. The variation of velocity v, pressure coefficient CP, and body radius R, are also shown versus the longitudinal coordinate X.

```
      PROGRAM TAP 1 (INPUT,OUTPUT,TAPE5=INPUT,PUNCH)
C     THIS PROGRAM DETERMINES TRANSONIC AERODYNAMIC CHARACTERISTICS ON
C     BODIES OF REVOLUTION USING THE
C     METHOD OF LOCAL LINEARIZATION.
      NAMELIST /DATA/M,X0,U0,A,B,C,D,G,CP
      REAL M,K
      DOUBLE VAR(2),CUVAR(2)
      DIMENSION U(52),X(52),DER(2)
      COMMON /DER/CUVAR,VAR,DER,M,X0,U0
      COMMON /FC/ A,B,C,D,G,K,PI
      PI=3.14159265
    1 X0=100.
      READ(5,DATA)
      IF(EOF,5)2,3
    2 STOP
    3 IF(X0.NE.100.)PRINT 4
      IF(X0.EQ.100.)PRINT 5
    4 FORMAT(1H1,* X0 IS INPUT TO THIS PROGRAM.  U0 IS CALCULATED FROM C
     1P INPUT*)
    5 FORMAT(1H1,*X0 AND U0 ARE CALCULATED BY THIS PROGRAM*)
      IF(X0.EQ.100.) GO TO 10
      U0=(-CP-R1(X0)**2)/2.
      GO TO 11
C
C     CALCULATE X0 AND U0 IF NECESSARY
C
   10 CALL XU0(X0,U0,M)
   11 PRINT 6, M,X0,U0,G,A,B,C,D
    6 FORMAT(1H ,*MACH NUMBER =*,F6.4,5X,*X0 =*,E16.8,5X,*U0 =*,E16.8/
     1        1H ,*GAMMA =*,E16.8//
     2        1H ,*R(X) = *,F5.2,4H * (,F5.2,11H * (1-X) + ,F5.2,10H * (1
     3-X)**,F5.2,1H))
C
C     CALCULATE X AND U VALUES ALONG THE BODY
C
      CALL XU(U,X,N)
C
C     PRINT OUT   X,R,R1,S,S1,S2,U,CP
C
      PRINT 7
    7 FORMAT(///1H ,15X,*X*,15X,*U*,14X,*CP*,15X,*R*,14X,*R1*,14X,*S *,
     114X,*S1*,14X,*S2*)
      DO 8 I=1,N
      E=X(I)$ RD=R(E)$ R1D=R1(E)$ CPD=CP(U(I),E)
      SD=PI*R(E)**2
      S1D=2.*PI*R(E)*R1(E)
      S2D=2.*PI*(R(E)*R2(E) + R1(E)**2)
    8 PRINT 9,E,U(I),CPD,RD,R1D,SD,S1D,S2D
    9 FORMAT(1H ,8E16.8)
C
C     PLOT   S,S1,S2 VS. X   AND   U VS. X   AND   CP VS. X
C
      CALL PLOT(X,U,N)
```

```fortran
      GO TO 1
      END
      SUBROUTINE XU0(X0,U0,M)
      REAL M,K
      DIMENSION XI(52),YI(52)
      COMMON /FC/ A,B,C,D,G,K,PI
      EPS=1.E-15
C
C     CALCULATE X0           X0=X  SUCH THAT S2(X)=0
C     X0 LIES BETWEEN  .02  AND  .98
C     BISECTION
C
      X=0.
      S2X=0.
    1 S2P=S2X
      X=X+.01
      S2X =2.*PI*(R(X )*R2(X ) + R1(X )**2)
      IF(ABS(S2X).LT.EPS)GO TO 6
      IF(S2X*S2P.GE.0.) GO TO 1
      X1=X-.010
      X2=X-.005
      X3=X
    2 S2X1=2.*PI*(R(X1)*R2(X1) + R1(X1)**2)
      S2X2=2.*PI*(R(X2)*R2(X2) + R1(X2)**2)
      IF(ABS(S2X).LT.EPS)GO TO 6
      IF(S2 X2 .LT.0.)GO TO 4
      IF(S2X1  .LT.0.)GO TO 5
    3 X1=X2
      X2=(X3+X2)/2.
      GO TO2
    4 IF(S2 X1 .LT.0.)GO TO 3
    5 X3=X2
      X2=(X1+X2)/2.
      GO TO 2
    6 X0=X2
      IF(X0.GE.02.AND.X0.LE..98)GO TO 8
      PRINT 7,X0
    7 FORMAT(1H ,23H*** X0 OUT OF RANGE ***/
     1        1H ,*X0 =   *,E16.8)
      STOP
C
C     CALCULATE U0
C
    8 IF(M.GE.1.)GO TO 13
      DO 9 I=1,51
      XI(I)=(I-1.)*.02
      IF(X0.LE.XI(I)-.01.OR.X0.GT.XI(I)+.01) GO TO 10
      XI(I)=X0
      YI(I)=-2.*PI*(R(X0)*R3(X0) + 3.*R1(X0)*R2(X0))
      GO TO 9
   10 YI(I)=2.*PI*(R(XI(I))*R2(XI(I)) + R1(XI(I))**2)/AHS(X0-XI(I))
    9 CONTINUE
      CALL TRAP(YI,XI,N,U0)
      GO TO 16
   13 XI(1)=0.
      DO 14 I=2,51
   14 XI(I)=XI(I-1)+X0/50.
      DO 15 I=1,50
   15 YI(I)=2.*PI*(R(XI(I))*R2(XI(I)) + R1(XI(I))**2)/(X0-XI(I))
      YI(51)=-2.*PI*(R(X0)*R3(X0) + 3.*R1(X0)*R2(X0))
      N=51
      CALL TRAP(YI,XI,N,U0)
   16 U0=U0/(4.*3.14159265)
```

```
          RETURN
          END
          SUBROUTINE XU(U,X,N)
          EXTERNAL DERSUB,CHSUB
          REAL M,K
          DOUBLE CUVAR(2),VAR(2)
          DIMENSION DER(2),ELE1(2),ELE2(2),X(52),U(52)
          COMMON /DER/CUVAR,VAR,DER,M,X0,U0
          COMMON /FC/ A,B,C,D,G,K,PI
          ELE1(1)=.001
          ELE2(1)=.001
          K=M**2*(1. +G)
C
C         INTEGRATE FORWARD
C

          I=0
          II=0
          VAR(1)=DBLE(X0)
          VAR(2)=DBLE(U0)
          CI=.015625
   1      I=I+1
          CALL INT1(II,1,0,CI  ,.02,0,IERR,VAR,CUVAR,DER,ELE1,ELE2,ELT,ERRVAL
         1,DERSUB,CHSUB,0)
          X(I)=SNGL(VAR(1))
          U(I)=SNGL(VAR(2))
          IF(DABS(VAR(1)-.98D).GT..0001D)GO TO 1
          DO 2 J=1,I
          JJ=J-1
          X(50-JJ)=X(I-JJ)
   2      U(50-JJ)=U(I-JJ)
C
C         INTEGRATE BACKWARDS
C

          I=50-I+2
          II=0
          VAR(1)=DBLE(X0)
          VAR(2)=DBLE(U0)
          CI=-.015625
   3      I=I-1
          CALL INT1(II,1,0,CI   ,.02,0.,IERR,VAR,CUVAR,DER,ELE1,ELE2,ELT,ERRV
         1AL,DERSUB,CHSUB,0)
          X(I)=SNGL(VAR(1))
          U(I)=SNGL(VAR(2))
          IF(DABS(VAR(1)-.02D).GT..0001D)GO TO 3
          N=50
          IF(I.EQ.1) GO TO 7
          KK=0
          DO 6 J=I,50
          KK=KK+1
          X(KK)=X(J)
   6      U(KK)=U(J)
          N=KK
   7      CONTINUE
          RETURN
          END
          SUBROUTINE DERSUB
          REAL M,K
          DOUBLE CUVAR(2),VAR(2)
          DIMENSION DER(2),XI(52),YI(52)
          COMMON /DER/ CUVAR,VAR,DER,M,X0,U0
          COMMON /FC/ A,B,C,D,G,K,PI
          N=51
          X=SNGL(CUVAR(1))
```

```fortran
          U=SNGL(CUVAR(2))
          S  = PI*R(X)**2
          S1 = 2.*PI*R(X)*R1(X)
          S2 = 2.*PI*(R(X)*R2(X) + R1(X)**2)
          S3 = 2.*PI*(R(X)*R3(X) + 3.*R1(X)*R2(X))
          S4=2.*PI*(R(X)*R4(X) + 4.*R1(X)*R3(X) + 3.*R2(X)**2)
          IF(M-1.) 2,1,7
        1 IF(U.GE.0.) GO TO 7
C
C   M EQUAL TO 1 AND U LESS THAN 0
C
C         CALCULATE INTEGRAL FOR USE IN DERIVATIVE EXPRESSION
C         DELTA=X/50.
C         EXTEND LIMITS OF INTEGRATION FOR U .LT.0
          IF(X.GT.X0) GO TO 11
          DELTA=X0/50.
          GO TO 13
       11 DELTA=X/50.
       13 CONTINUE
          DO 4 I=1,51
          XI(I)=(I-1.)*DELTA
          IF(X.GT.XI(I)-.01.AND.X.LT.XI(I)+.01) GO TO 5
          S2I=2.*PI*(R(XI(I))*R2(XI(I)) + R1(XI(I))**2)
          YI(I)=((X-XI(I))*S3 - S2 + S2I)/(X-XI(I))**2
          GO TO 4
        5 YI(I)=S4
          XI(I)=X
        4 CONTINUE
          CALL TRAP(YI,XI,N,SUM)
C         CALCULATE DERIVATIVE
          DER(2)=(S3*ALOG(-K*U) + S3 + S3*ALOG(S/(4.*PI*X*(1.-X))) + S2*S1/S
         1 - S2*(1.-2.*X)/(X*(1.-X)) + SUM)/(4.*PI)
          RETURN
C
C   M ESS THAN 1
C
C         CALCULATE INTEGRAL FOR USE IN DERIVATIVE EXPRESSION
        2 DO 3 I=1,51
          XI(I)=(I-1.)*.02
          IF(X .LE.XI(I)-.01.OR.X .GT.XI(I)+.01) GO TO 6
          XI(I)=X
          YI(I)=S4
          GO TO 3
        6 S2I= 2.*PI*(R(XI(I))*R2(XI(I)) + R1(XI(I))**2)
          YI(I)=(ABS(X-XI(I))*S3 - S2 + S2I)/(X-XI(I))**2
C         YI(I)=(    (X-XI(I))*S3 - S2 + S2I)/(X-XI(I))**2
        3 CONTINUE
          CALL TRAP(YI,XI,N,SUM)
C         CALCULATE DERIVATIVE
          ABA=1. - M**2 - K*U
          IF(ABA.LT.0.) PRINT 20,ABA
       20 FORMAT(1H ,*ABA = *,E16.8)
          IF(ABA.LT.0.) ABA=-ABA
          DER(2)=(S3*ALOG( ABA)      + S3*ALOG(S/(4.*PI*X*(1.-X))) + S2*S1/S
         1 - S2*(1.-2.*X)/(X*(1.-X)) + SUM)/(4.*PI)
C        1 - S2*(1.-2.*X)/(X*(1.-X)) - SUM)/(4.*PI)
          Q1=S3*ALOG(ABA)
          Q2=S3*ALOG(S/(4.*PI*X*(1.-X)))
          Q3=S2*S1/S
          Q4=-S2*(1.-2.*X)/(X*(1.-X))
C         PRINT 129,X,Q1,Q2,Q3,Q4,SUM
      129 FORMAT(1H ,6E16.8)
          RETURN
```

```
C
C M GREATER THAN 1
C   OR M EQUAL TO 1 AND U GREATER THAN OR EQUAL TO 0
C
C       CALCULATE INTEGRAL FOR USE IN DERIVATIVE EXPRESSION
    7 XI(1)=0.
      DELTA=X/50.
      DO 8 I=2,51
    8 XI(I)=XI(I-1) + DELTA
      DO 9 I=1,50
      S21= 2.*PI*(R(XI(I))*R2(XI(I)) + R1(XI(I))**2)
    9 YI(1)=((X-XI(I))*S3 - S2 + S21)/(X-XI(I))**2
      YI(51)=S4
      CALL TRAP(YI,XI,N,SUM)
C       CALCULATE DERIVATIVE
      ABA=M**2 - 1. + K*U
      IF(ABA.LT.0.) PRINT 20,ABA
      IF(ABA.LT.0.) ABA=-ABA
      DER(2)=S3      *ALOG(      ABA      )/(4.*PI) +
     1        S3     *ALOG(S   /(4.*PI*X**2))/(4.*PI)   +
     2        S2     *(X*S1   -2.*S   )/(4.*PI*X*S   )  +
     3        S3     /(2.*PI) +
     4        SUM/(2.*PI)
      RETURN
      END
      SUBROUTINE TRAP(Y,X,N,SUM)
      DIMENSION X(N),Y(N)
      SUM=0.
      DO 1 I=2,N
    1 SUM=SUM+(Y(I)+Y(I-1))*(X(I)-X(I-1))/2.
      RETURN
      END
      FUNCTION R(X)
      COMMON /FC/ A,B,C,D,G,K,PI
C  EXPONENT CHECK
      IF(D.NE.0.) GO TO 1
      R =A*(B*(1.-X) + C)
      GO TO 2
    1    R=A*(B*X+C*X**D)
C    1 R=A*(B*(1.-X) + C*(1.-X)**D)
    2 CONTINUE
      RETURN
      END
      FUNCTION R1(X)
      COMMON /FC/ A,B,C,D,G,K,PI
C  EXPONENT CHECK
      IF(D.NE.0..AND.D.NE.1.) GO TO 1
      R1=-A*B - A*C*D
      GO TO 2
C    1 R1=-A*B-A*C*D*(1.-X)**(D-1.)
    1    R1=A*(B+C*D*X**(D-1.))
    2 CONTINUE
      RETURN
      END
      FUNCTION R2(X)
      COMMON /FC/ A,B,C,D,G,K,PI
C  EXPONENT CHECK
      IF(D.NE.0..AND.D.NE.1..AND.D.NE.2.) GO TO 1
      R2=A*C*D*(D-1.)
      GO TO 2
C    1 R2=A*C*D*(D-1.)*(1.-X)**(D-2.)
    1    R2=A*C*D*(D-1.)*X**(D-2.)
    2 CONTINUE
```

```
      RETURN
      END
      FUNCTION R3(X)
      COMMON /FC/ A,B,C,D,G,K,PI
C   EXPONENT CHECK
      IF (D.NE.0..AND.D.NE.1..AND.D.NE.2..AND.D.NE.3.) GO TO 1
      R3=-A*C*D*(D-1.)*(D-2.)
      GO TO 2
    1 R3=A*C*D*(D-1.)*(D-2.)*X**(D-3.)
C   1 R3=-A*C*D*(D-1.)*(D-2.)*(1.-X)**(D-3.)
    2 CONTINUE
      RETURN
      END
      FUNCTION R4(X)
      COMMON /FC/ A,B,C,D,G,K,PI
C   EXPONENT CHECK
      IF (D.NE.0..AND.D.NE.1..AND.D.NE.2..AND.D.NE.3..AND.D.NE.4.) GO TO
     11
      R4=A*C*D*(D-1.)*(D-2.)*(D-3.)
      GO TO 2
    1 R4=A*C*D*(D-1.)*(D-2.)*(D-3.)*X**(D-4.)
C   1 R4=A*C*D*(D-1.)*(D-2.)*(D-3.)*(1.-X)**(D-4.)
    2 CONTINUE
      RETURN
      END
      FUNCTION CP(U,X)
      COMMON /FC/ A,B,C,D,G,K,PI
      CP=-2.*U-R1(X)**2
      RETURN
      END
      SUBROUTINE CHSUB
      RETURN
      END
      SUBROUTINE PLOT(X,U,N)
      DIMENSION X(N),U(N)
      RETURN
      END
 $DATA M=1.0,X0=.48,CP=0.,A=.07125,B=1,C=-1,D=6.03,G=1.4,     $      .7
 $DATA M=1.2,X0=.56,CP=0.,A=.07125,B=1,C=-1,D=6.03,G=1.4,     $      .7
```

XO IS INPUT TO THIS PROGRAM. UO IS CALCULATED FROM CP INPUT
MACH NUMBER =1.∩∩∩∩   XO = -.∩∩∩∩∩∩∩E-∩1   UO = -1.832e112
GAMMA = 1.4∩0∩∩∩∩E+0∩

R(X) =   .07 * (1.0∩ * (1-X) + -1.5∩ * (1-X)** 6.∩3)

OUTPUT

X	U	CP	F
2.CC000000E-02	-3.02076530E-02	5.53387436E-02	1.42500000E-03
4.CCC00000E-02	-2.84283198E-02	5.17800828E-02	2.84999974E-03
6.00000000E-02	-2.73521334E-02	4.96877430F-02	4.27499694F-03
8.CC00C000E-02	-2.66374970E-02	4.81981176E-02	5.69958269E-03
1.00000000E-01	-2.60580230E-02	4.70400668E-02	7.12493351E-03
1.20000000E-01	-2.55410520E-02	4.60869709E-02	8.54980036E-03
1.40000000E-01	-2.51702433E-02	4.52670378E-02	9.97149425E-03
1.60000000E-01	-2.48002378E-02	4.45299876E-02	1.13988686E-02
1.80000000E-01	-2.44503231E-02	4.38350661E-02	1.28226931E-02
2.00000000E-01	-2.41010811E-02	4.31442506E-02	1.42456549E-02
2.20000000E-01	-2.37326340E-02	4.24188115E-02	1.56672804E-02
2.40000000E-01	-2.33236176E-02	4.16172720E-02	1.70865546E-02
2.60000000E-01	-2.28562684E-02	4.06935947E-02	1.85038615E-02
2.80000000E-01	-2.22351930E-02	3.95947165E-02	1.99165518E-02
3.00000000E-01	-2.15982465E-02	3.82624105E-02	2.13249013E-02
3.20000000E-01	-2.07509559E-02	3.66299362E-02	2.27260668E-02
3.40000000E-01	-1.97161675E-02	3.46215129E-02	2.41194379E-02
3.60000000E-01	-1.84339198E-02	3.21539494E-02	2.54995858E-02
3.80000000E-01	-1.68708541E-02	2.91254358E-02	2.68666085E-02
4.00000000E-01	-1.49509726E-02	2.54169910E-02	2.82160731E-02
4.20000000E-01	-1.26101166E-02	2.08933222E-02	2.95439534E-02
4.40000000E-01	-9.75807600E-03	1.53769077E-02	3.08455658E-02
4.60000000E-01	-6.26312019E-03	8.80687693E-03	3.21155003E-02
4.80000000E-01	-1.33261129E-03	0.	3.33475483E-02
5.00000000E-01	3.16456805E-03	-9.70476175E-03	3.45346267E-02
5.20000000E-01	7.45259204E-03	-1.79559957E-02	3.56687095E-02
5.40000000E-01	1.19335713E-02	-2.65590788E-02	3.67407199E-02
5.60000000E-01	1.67238511E-02	-3.57513661E-02	3.77404728E-02
5.80000000E-01	2.18705270E-02	-4.56459599E-02	3.86565709E-02
6.00000000E-01	2.74225246E-02	-5.63157306E-02	3.94763148E-02
6.20000000E-01	3.35500047E-02	-6.78130082E-02	4.01856076E-02
6.40000000E-01	3.97566945E-02	-8.01755017E-02	4.07898546E-02
6.60000000E-01	4.65495402E-02	-9.34271118E-02	4.12083607E-02
6.80000000E-01	5.37426653E-02	-1.07575621E-01	4.14867240E-02
7.00000000E-01	6.13036470E-02	-1.22607330E-01	4.15817253E-02
7.20000000E-01	6.91771392E-02	-1.38477736E-01	4.14712153E-02
7.40000000E-01	7.72784122E-02	-1.55096436E-01	4.11304970E-02
7.60000000E-01	8.54747724E-02	-1.72303231E-01	4.05327055E-02
7.80000000E-01	9.35698631E-02	-1.89330500E-01	3.96486842E-02
8.00000000E-01	1.01269231E-01	-2.07243643E-01	3.84468571E-02
8.20000000E-01	1.08130655E-01	-2.23845674F-01	3.68930978E-02
8.40000000E-01	1.13483233E-01	-2.38521029F-01	3.49505953E-02
8.60000000E-01	1.16292045E-01	-2.49471262F-01	3.25797153E-02
8.80000000E-01	1.14719330E-01	-2.53744673E-01	2.97378614E-02
9.00000000E-01	1.07606765E-01	-2.46331595F-01	2.63793246E-02
9.20000000E-01	8.67759595E-02	-2.18164506E-01	2.24551407E-02
9.40000000E-01	4.54789213E-02	-1.50239346F-01	1.79129347F-02
9.60000000E-01	-4.14407896E-02	5.24359565E-03	1.26961648E-02
9.80000000E-01	-1.49135633E-01	1.97962784E-01	6.74697236E-03

TRANSONIC AERODYNAMICS PROGRAM

Computer Program Input/Output - All input data are input according to
NAMELIST type format. In columns 2 through 7 the name and INPUT is punched.
Column 8 is left blank. Beginning in column 9 the variable names which
are shown below and their values are punched. Each variable must be
separated by a comma and the value of the final variable must be followed
by a $ sign. The variables may be punched out to column 80 in any order.

Variable	Definition
MACH	Mach number
CPB(1)	The array of pressure coefficients for the equivalent body of revolution. These values are required from $X = 0.05$ to $X = 0.95$ in increments of .05. Dimension of the array should be 17.
ITRIG = 0 = 1	If $\lambda \neq 1$, elliptical cross section. If $\lambda = 1$ and body of revolution to be run at $\alpha \neq 0$.
LAM	Ratio of major to minor axis of ellipse $\lambda = a/b$.
ALPHA	Angle of attack in radians.
THET (1)	Values of θ around body at which pressure coefficients are desired in degrees.
NTHETA	Dimension of THET(1) array.

EXAMPLE:

    $INPUT MACH = 1., CPB(1) = .15, .1, .05, .02, -.01,
    -.04, -.06, -.085, -.095, -.1, -.1, -.1, -.1, -.1, -.01,
    .05, .01, ITRIG = 0, LAM = 3, ALPHA = 1.047198,
    THET(1) = -90., -40., -0., 40., 90., NTHETA = 5, $

If more than one case is to be run, then the input data for the second
case may be placed immediately after the data for the first.

24

All parameters used in this program are complex and in the form A + iB. The real or imaginary parts or both may be zero. This reserves space for the values when they are not zero. The built-in functions for handling floating point are integer computations which have a complex counterpart.

Output

The output consists of a listing of the input parameters, X, theta, and the real and imaginary parts of C_p.

```
      PROGRAM CCP(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C
      REAL LAM,MACH,LAM1,LAM2,LAM3,LAM4,LAM7,LAM9
C
C
      COMPLEX A,B,C,D,E,F,G,H,AA,BB,CP,EIT(25,2),EIT2(25,2),ALPHI,CI,
     1CCPB,FLAM,SLAM,ELAM,TLAM,CRSQ1,CTHSQ1,O,P,Q,U,S,T,V
C
      DIMENSION CPB(25),THET(25),SINT(25),COST(25),W(2)
      EQUIVALENCE (W(1),CP)
C
      NAMELIST/INPUT/MACH,CPB,LAM,ITRIG,ALPHA,THET,NTHETA
C
C     CP DATA WITH ALPHA=0 FOR EXACT BODY
C
C     ITRIG=1   YES
C     ITRIG=0   NO
C
      CI=CMPLX(0.,1.)
 1    READ (5,INPUT)
      IF(EOF,5) 100,3
C
 3    WRITE(6,INPUT)
      ALPHA=ALPHA*.0174533
      ALPHI=CMPLX(0.,ALPHA)
C
      PRINT 7,MACH,LAM,ALPHA
 7    FORMAT(1H1,15X,*X*,11X,*THETA*,9X,*CP REAL*,9X,*CP IMAG*,20X,*MACH
     1 NO = *,E16.8/1H ,84X,*LAM = *,E16.8/1H ,84X,*ALPHA = *,E16.8//)
      IF(ITRIG.EQ.1) PRINT 10
 10   FORMAT(1H ,84X,*CP DATA WITH ALPHA=0 FOR EXACT BODY*//)
C
      LAM1=-LAM+1./LAM
      LAM2=LAM/(LAM+1.)
      LAM3=1./(LAM+1.)
      LAM4=4.0*(1.-LAM)/(1.+LAM)
      FLAM=CMPLX(LAM4,0.)
      SLAM=CMPLX(LAM1,0.)
C
C     EVALUATE E**ITHETA AND E**I2THETA
      DO 20 J=1,NTHETA
      THET(J)=THET(J)*.0174533
      THETA=THET(J)
      TOTHET=2.0*THET(J)
      EIT(J)=CMPLX(COS(THETA),SIN(THETA))
      EIT2(J)=CMPLX(COS(TOTHET),SIN(TOTHET))
      SINT(J)=SIN(THET(J))
      COST(J)=COS(THET(J))
 20   CONTINUE
C     X VARIES FROM .05 TO .85 IN STEPS OF .05    I = COUNTER FOR X
      X=0.
      I=0
 40   X=X+.05
      I=I+1
      IF(X.GT.(.89)) GO TO 1
```

```
C
C       CALCULATE R,R1,R2
C
C       R1=R(PRIME)
C       R2=R(DOUBLE PRIME)
C
        R=.1226*X
        R1=.1226
        R2=0.
        ALRSQ=ALOG(R*R)
        RSQ1=R1*R1
        TRSQ1=2.*RSQ1
        CRSQ1=CMPLX(RSQ1,0.)
        CTRSQ1=CMPLX(TRSQ1,0.)
        LAM7=-(2.*R*LAM2)**2+(2.*C*LAM3)**2
        ELAM=CMPLX(LAM7,0.)
        LAM9=-R*R*LAM+R*R/LAM
        TLAM=CMPLX(LAM9,0.)
C
C
        DO 50 J=1,NTHETA
C
        A=EIT(J)+CSQRT(EIT2(J)+FLAM)
C
        B=.5*(EIT(J)+CSQRT(EIT2(J)+SLAM))
C
        C1=-R*SINT(J)
        C2=R*COST(J)
        C=CMPLX(C1,C2)
C
        D=CSQRT(R*R*EIT2(J)+LLAM)
C
        E1=(2.*R*(LAM2+LAM3))**2
        E=CMPLX(E1,0.)
C
        F=CSQRT(R*R*EIT2(J)+TLAM)
C
        G1=R*COST(J)
        G2=R*SINT(J)
        G=CMPLX(G1,G2)
C
        H1=R*R2+R1*R1
        H=CMPLX(H1,0.)
C
        AA=.5*ALPHI*(-C1-C/D-E/(R*EIT(J)+D)-E*C/(D*(R*EIT(J)+D)))
C
        BB=R*R1*(1./(R*EIT(J)+F))+(C1+C/F)
        CCPB=CMPLX(CPB(I),0.)
C
        CP=ALPHI*R1*(4./A-A)-2.*H*CLOG(R*R)-CTRSQ1-2.*R*R1*CLOG(R)+H*ALRSQ
       1+CRSQ1-2.*ALPHA*(AA+BB)-(-.5*ALPHI-.5*ALPHI*(G/D-(E*D)+E*G)/(D*(R*
       1EIT(J)+D)))+R*R1*((F+G)/F+1./(R*EIT(J)+F)))**2-(AA+BB)**2
        THETA=THET(J)
        CP=CP+CCPB
C
        PRINT 47,X,THET(J),W(1),W(2)
     47 FORMAT(1H ,4E16.8)
     50 CONTINUE
C
C
C
        IF(MOD(I,7))40,60,40
     60 PRINT 7,MACH,LAM,ALPHA
        GO TO 40
```

```
$INPUT
MACH    =   0.1E+01,
CPB     =   0.15E+00,   0.1E+00,   0.5E-01,   0.2E-01,  -0.1E-01,  -0.4E-01,
           -0.6E-01,  -0.85E-01,  -0.9E-01,  -0.1E+00,  -0.1E+00,  -0.1E+00,
           -0.1E+00,  -0.1E+00,  -0.1E-01,   0.5E-01,   0.1E+00,   0.0,   0.0,   0.0,
            0.0,   0.0,   0.0,   0.0,   0.0,
LAM     =   0.3E+01,
ITRIG   =   0,
ALPHA   =   0.104E+00,
THET    =  -0.157079E+01,  -0.69813E+00,   0.0,   0.69813E+00,   0.157079E+01,   0.0,
            0.0,   0.0,   0.0,   0.0,   0.0,   0.0,   0.0,   0.0,   0.0,   0.0,   0.0,
            0.0,   0.0,   0.0,   0.0,   0.0,   0.0,   0.0,
NTHETA  =   5,
$END
```

X	THETA	CP REAL	CP IMAG
5.00000000E-02	-1.57079000E+00	4.52852426E-02	6.60435196E-02
5.00000000E-02	-6.98130000E-01	6.46910259E-02	3.48324100E-02
5.00000000E-02	0.	1.49047267E-01	-1.06920110E-02
5.00000000E-02	6.98130000E-01	1.38252865E-01	-5.03003677E-02
5.00000000E-02	1.57079000E+00	1.44066533E-01	-6.56376282E-02
1.00000000E-01	-1.57079000E+00	1.00554960E-02	4.62259170E-02
1.00000000E-01	-6.98130000E-01	2.61974663E-02	2.15020683E-02
1.00000000E-01	0.	9.97730779E-02	-2.13649660E-05
1.00000000E-01	6.98130000E-01	9.19023727E-02	-3.46094196E-02
1.00000000E-01	1.57079000E+00	9.73125155E-02	-4.55437495E-02
1.50000000E-01	-1.57079000E+00	-2.01165543E-02	2.79516092E-02
1.50000000E-01	-6.98130000E-01	-1.31071104E-02	9.27273938E-03
1.50000000E-01	0.	4.99057424E-02	9.46308299E-03
1.50000000E-01	6.98130000E-01	4.47727756E-02	-2.01977400E-02
1.50000000E-01	1.57079000E+00	5.15741795E-02	-2.71082514E-02
2.00000000E-01	-1.57079000E+00	-4.32464150E-02	1.13588437E-02
2.00000000E-01	-6.98130000E-01	-3.32335396E-02	-1.75676481E-03
2.00000000E-01	0.	1.95179334E-02	1.77149115E-02
2.00000000E-01	6.98130000E-01	1.68163891E-02	-7.14560310E-03
2.00000000E-01	1.57079000E+00	2.38253256E-02	-1.04546849E-02
2.50000000E-01	-1.57079000E+00	-6.13616663E-02	-3.41415963E-03
2.50000000E-01	-6.98130000E-01	-5.41927624E-02	-1.14853220E-02
2.50000000E-01	0.	-1.13618566E-02	2.46873384E-02
2.50000000E-01	6.98130000E-01	-1.19845535E-02	4.46660513E-03
2.50000000E-01	1.57079000E+00	-4.95212637E-03	4.29337085E-03
3.00000000E-01	-1.57079000E+00	-8.05019652E-02	-1.62292090E-02
3.00000000E-01	-6.98130000E-01	-7.59948134E-02	-1.98123021E-02
3.00000000E-01	0.	-4.26993154E-02	3.03332213E-02
3.00000000E-01	6.98130000E-01	-4.16478605E-02	1.45563797E-02
3.00000000E-01	1.57079000E+00	-3.48158342E-02	1.70123090E-02
3.50000000E-01	-1.57079000E+00	-9.06820449E-02	-2.69481402E-02
3.50000000E-01	-6.98130000E-01	-8.06538231E-02	-2.66370611E-02
3.50000000E-01	0.	-6.44643110E-02	3.46050567E-02
3.50000000E-01	6.98130000E-01	-6.21914622E-02	2.30491000E-02
3.50000000E-01	1.57079000E+00	-5.57635310E-02	2.75784948E-02

A	THET	CP REAL	CP IMAG
4.00000000E-01	-1.57079000E+00	-1.06950715E-01	-3.54328174E-02
4.00000000E-01	-6.98130000E-01	-1.07178018E-01	-3.18589270E-02
4.00000000E-01	0.	-9.16294920E-02	3.74549817E-02
4.00000000E-01	6.98130000E-01	-8.86333104E-02	2.98590290E-02
4.00000000E-01	1.57079000E+00	-8.28330260E-02	3.58082655E-02
4.50000000E-01	-1.57079000E+00	-1.04314801E-01	-4.15451321E-02
4.50000000E-01	-6.98130000E-01	-1.06579720E-01	-3.53771973E-02
4.50000000E-01	0.	-9.91532800E-02	3.08347147E-02
4.50000000E-01	6.98130000E-01	-9.59913707E-02	3.49043130E-02
4.50000000E-01	1.57079000E+00	-9.10522053E-02	4.17573300E-02
5.00000000E-01	-1.57079000E+00	-1.07809444E-01	-4.51470041E-02
5.00000000E-01	-6.98130000E-01	-1.11870347E-01	-3.70911388E-02
5.00000000E-01	0.	-1.12015904E-01	3.86950414E-02
5.00000000E-01	6.98130000E-01	-1.09243703E-01	3.51659843E-02
5.00000000E-01	1.57079000E+00	-1.05449030E-01	4.51237717E-02
5.50000000E-01	-1.57079000E+00	-1.02442503E-01	-4.61003809E-02
5.50000000E-01	-6.98130000E-01	-1.08061413E-01	-3.68999883E-02
5.50000000E-01	0.	-1.15193334E-01	3.69892465E-02
5.50000000E-01	6.98130000E-01	-1.13526599E-01	3.93849555E-02
5.50000000E-01	1.57079000E+00	-1.11051537E-01	4.58420423E-02
6.00000000E-01	-1.57079000E+00	-9.83021523E-02	-4.42672379E-02
6.00000000E-01	-6.98130000E-01	-1.05164529E-01	-3.47029523E-02
6.00000000E-01	0.	-1.18625344E-01	3.36568007E-02
6.00000000E-01	6.98130000E-01	-1.18744007E-01	3.86570253E-02
6.00000000E-01	1.57079000E+00	-1.17847847E-01	4.37889683E-02
6.50000000E-01	-1.57079000E+00	-9.53565815E-02	-3.95095785E-02
6.50000000E-01	-6.98130000E-01	-1.03141399E-01	-3.03992067E-02
6.50000000E-01	0.	-1.22311800E-01	2.86754780E-02
6.50000000E-01	6.98130000E-01	-1.24948407E-01	3.58410755E-02
6.50000000E-01	1.57079000E+00	-1.25916142E-01	3.88407475E-02
7.00000000E-01	-1.57079000E+00	-9.36540575E-02	-3.16894338E-02
7.00000000E-01	-6.98130000E-01	-1.02143026E-01	-2.38878980E-02
7.00000000E-01	0.	-1.26213089E-01	2.19680113E-02
7.00000000E-01	6.98130000E-01	-1.32160121E-01	3.08580715E-02
7.00000000E-01	1.57079000E+00	-1.35374673E-01	3.58735500E-02

X	THETA	CP REAL	CP IMAG
7.50000000E-01	-1.57079000E+00	-3.22292296E-03	-2.06686629E-02
7.50000000E-01	-6.98130000E-01	-1.20437069E-02	-1.50681414E-02
7.50000000E-01	0.	-4.02932622E-02	1.34946934E-02
7.50000000E-01	6.98130000E-01	-5.03973740E-02	2.36240620E-02
7.50000000E-01	1.57079000E+00	-5.60418423E-02	1.97635170E-02
8.00000000E-01	-1.57079000E+00	5.59084031E-02	-5.30995277E-03
8.00000000E-01	-6.98130000E-01	4.70669650E-02	-3.63902230E-03
8.00000000E-01	0.	1.54591025E-02	3.20217630E-03
8.00000000E-01	6.98130000E-01	3.21304107E-04	1.40561823E-02
8.00000000E-01	1.57079000E+00	-8.13608211E-03	5.36676517E-03
8.50000000E-01	-1.57079000E+00	1.03711426E-01	1.15251817E-02
8.50000000E-01	-6.98130000E-01	9.52241004E-02	9.50040395E-03
8.50000000E-01	0.	6.10913348E-02	-6.95764841E-03
8.50000000E-01	6.98130000E-01	3.99774204E-02	2.07854693E-03
8.50000000E-01	1.57079000E+00	2.84343138E-02	-1.23890215E-02