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SPACECRAFT DETUMBLING USING MOVABLE TELESCOPING APPENDAGES

By

Peter M. Bainum
Professor of Aerospace Engineering
Department of Mechanical Engineering
Howard University
Washington, D.C. 20059, U.S.A.
and
R. Sellappan
Graduate Research Assistant
Department of Mechanical Engineering
Howard University
Washington, D.C. 20059, U.S.A.

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ABSTRACT

The dynamics of detumbling a randomly spinning spacecraft using externally mounted, movable telescoping appendages are studied both analytically and numerically. Two types of telescoping appendages are considered: (a) where an end mass is mounted at the end of an (assumed) massless boom; and (b) where the appendage is assumed to consist of a uniformly distributed homogeneous mass throughout its length. From an application of Lyapunov's second method boom extension maneuvers can be determined to approach either of two desired final states: close to a zero inertial angular velocity state, and a final spin rate about only one of the principal axes. Recovery dynamics are evaluated analytically for the case of symmetrical deployment. Numerical examination of other asymmetrical cases verifies the practicality of using movable appendages to recover a randomly tumbling spacecraft.

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**Associate Fellow, AIAA; Fellow, British Interplanetary Society; Member, American Astronautical Society.
+Student Member, AIAA; formerly with Indian Scientific Satellite Project, ISRO, Bangalore, India.

NOMENCLATURE

c	=	boom extension rate
h_1, h_2, h_3	=	components of the angular momentum vector along the principal axes
h_0	=	constant value of h_3 for symmetrical deployment maneuver
I_1, I_2, I_3	=	instantaneous values of principal moments of inertia
I_1^*, I_2^*, I_3^*	=	hub principal moments of inertia
$\bar{I}_1^*, \bar{I}_2^*, \bar{I}_3^*$	=	moments of inertia at the switching time T_{3f} in the recovery sequence to achieve final spin about the '3' axis
K	=	$2 \rho c^3$
$\ell(t)$	=	time varying length of telescopic appendages
m	=	boom end mass
P	=	$2 mc^2$
t	=	time
T	=	kinetic energy
T_{3f}	=	switching time in the recovery sequence to achieve final spin about the '3' axis
V	=	Lyapunov function
$\omega_1, \omega_2, \omega_3$	=	angular velocity components along the principal axes
Ω	=	desired final value of ω_3 (ω_{3f})
ρ	=	mass density per unit boom length
ψ_0^*, ψ_0	=	phase angles appearing in the solutions for $\omega_1(t)$, $\omega_2(t)$ and determined from conditions at $t = 0$, $t = T_{3f}$, respectively
τ	=	$\int b(t)dt$
θ	=	nutation angle
$\cdot$	=	indicates time differentiation
(0)	=	indicates initial conditions

I. INTRODUCTION

With the advent of prolonged manned missions in space and the possibility of in-orbit servicing and repairs to spacecraft there is an increased interest in studying methods that can be used to recover a spacecraft from an initial uncontrolled dynamic state. A recent paper examined methods of recovering spinning satellites to a flat-spin condition by using spin-up thrusters and multiple combinations of thrusters.[1] It was concluded that the use of such thrusters for the recovery operation are often limited by the weight and propellant capacity of the thruster system, and also the reliability problems associated with multiple thrusters in sequence. Kaplan has described an alternate recovery system which utilizes a movable-mass control device that is internal to the spacecraft and can move along a fixed direction track.[2] This device is activated upon initiation of tumble and is programmed via a control law to quickly stabilize motion about the major principal axis. In a recent related paper[3] it was concluded that the mass track should be placed as far as possible from the vehicle center of mass and be oriented parallel to the maximum inertia axis; in addition the performance of the control system can be improved through larger mass amplitudes along the track and also larger mass sizes.

It is apparent that the location and displacement amplitude of any internal control mass will be limited by the physical dimensions of the space vehicle.

Externally movable appendages could allow for a greater range of location and displacement amplitudes of such a system; however, as the size of the appendages increases the flexibility problems associated with such structures would have to be considered.

Of interest in this study will be the consideration of the detumbling dynamics of a spacecraft system with extensible boom-type appendages along the principal axes. The recovery maneuver from an initial tumble is designed to approach either of two final states: (1) close to a zero inertial angular velocity vector and (2) to approximate a final spin about a principal axis. (It is thought that small terminal residual angular rates could then be removed by temporarily activating on-board damping systems.) A key advantage of this type of system would be its potential reuse for subsequent detumbling recovery operations as the need arises.

II. ANALYSIS

A. General Considerations

The dynamics of detumbling a randomly spinning spacecraft using externally mounted, movable telescoping appendages are studied both analytically and numerically. The appendages considered are of varying length and could represent extensible booms or a tether connected to the main part of the spacecraft. Two types of telescoping appendages are condidered: (a) the case where an end mass is mounted at the end of an assumed massless member (end mass moving) as shown in Fig.1(a);and(b) where the appendage is assumed to consist of a uniformly distributed, homogeneous mass throughout its length (uniformly distributed mass moving) as shown in Fig. 1(b).

The extensible boom type appendages are assumed to originate from the center of the hub along the three principal axes. The desired final states of the system considered are: (1) zero inertial angular velocity vector and (2) a final spin about one of the principal axes. The necessary conditions for(asymptotic)stability during the detumbling sequences are determined using Lyapunov's second method.

B. End Mass Moving

1. Development of Kinetic Energy

The configuration of the system, where the end masses are assumed to be attached to the end of massless rods along all three principal axes is shown in Fig.1(a). The end masses are assumed to be identical (i.e. $m_i = m$). The rotational kinetic energy of the system can be developed in terms of the hub inertias (I_i^*) and boom lengths as:

$$T = \frac{1}{2}\,[\{I_1^* + 2m(\ell_2^2 + \ell_3^2)\}\,\omega_1^2 + \{I_2^* + 2m(\ell_3^2 + \ell_1^2)\}\omega_2^2 + \{I_3^* + 2m(\ell_1^2 + \ell_2^2)\}\,\omega_3^2 + 2m(\dot{\ell}_1^2 + \dot{\ell}_2^2 + \dot{\ell}_3^2)] \tag{1}$$

Defining,

$$\begin{aligned} I_1 &= I_1^* + 2m(\ell_2^2 + \ell_3^2) \\ I_2 &= I_2^* + 2m(\ell_3^2 + \ell_1^2) \\ I_3 &= I_3^* + 2m(\ell_1^2 + \ell_2^2)\,, \end{aligned} \tag{2}$$

Eq. (1) can be rewritten as:

$$T = \frac{1}{2}\,[I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2 + 2m(\dot{\ell}_1^2 + \dot{\ell}_2^2 + \dot{\ell}_3^2)] \tag{3}$$

If the extension rates are assumed to be constant, Eq. (3) can be expressed:

$$T = \frac{1}{2} [I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2] + \text{non-negative const.} \qquad (4)$$

Here the moments of inertia are time varying as the length of the booms varies during extension.

2. Achieve Zero Inertial Angular Rate

(a) Lyapunov Function-Kinetic Energy

The desired final state of the system is $\omega_i = 0$. A suitable Lyapunov function, in the state variables ω_1, ω_2 and ω_3, is the system rotational kinetic energy which can be written as:

$$V = T = [I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2] + \text{non-negative const.} \qquad (5)$$

The Lyapunov function, V, is positive definite in the state variables selected; for asymptotic stability $\dot{V}$ will now be examined.

Differentiating Eq. (5) with respect to time, there results:

$$\dot{V} = \frac{1}{2} (\dot{I}_1\omega_1^2 + \dot{I}_2\omega_2^2 + \dot{I}_3\omega_3^2 + 2I_1\omega_1\dot{\omega}_1 + 2I_2\omega_2\dot{\omega}_2 + 2I_3\omega_3\dot{\omega}_3) \quad (6)$$

Euler's equations of torque-free motion can be written in the following form:

$$\dot{h}_1 = \omega_3 h_2 - \omega_2 h_3 = \dot{I}_1\omega_1 + I_1\dot{\omega}_1 \qquad (7a)$$

$$\dot{h}_2 = \omega_1 h_3 - \omega_3 h_1 = \dot{I}_2\omega_2 + I_2\dot{\omega}_2 \qquad (7b)$$

$$\dot{h}_3 = \omega_2 h_1 - \omega_1 h_2 = \dot{I}_3\omega_3 + I_3\dot{\omega}_3 \qquad (7c)$$

Multiplying Eq. (7a) by ω_1, Eq. (7b) by ω_2, and Eq. (7c) by ω_3, and adding we obtain the following:

$$I_1\omega_1\dot{\omega}_1 + I_2\omega_2\dot{\omega}_2 + I_3\omega_3\dot{\omega}_3 = -(\dot{I}_1\omega_1^2 + \dot{I}_2\omega_2^2 + \dot{I}_3\omega_3^2) \qquad (8)$$

Substituting Eq. (8) into Eq. (6), there results:

$$\dot{V} = -\frac{1}{2}(\dot{I}_1\omega_1^2 + \dot{I}_2\omega_2^2 + \dot{I}_3\omega_3^2) \qquad (9)$$

From Eq. (9), we conclude that $\dot{V}$ is a negative definite function in the state variables only if $\dot{I}_1, \dot{I}_2, \dot{I}_3 > 0$.

Here it is seen that when the rotational kinetic energy is used as a Lyapunov function expressed in terms of small amplitudes of the inertial angular velocity components, that the necessary conditions for asymptotic stability are satisfied for positive constant boom extension rates and three orthogonally mounted sets of booms along the hub principal axes. This means that as time becomes extremely large (and boom lengths become infinite) it would be theoretically possible to achieve a zero inertial angular velocity state. (Of course, such a situation will, in practice, not occur due to finite length appendages and the presence of large amplitude rates for the general situation of an initial random tumble. However, it will be of interest to simulate how much of a random tumble could be removed by this process where stability is now considered in the global sense about $\omega_i = 0$.) The selection of rotational kinetic energy as a Lyapunov function has also been used by Edwards and Kaplan[3] for the system treated in Ref. 2.

(b) Analytical Solution

As a special case when the spin axis is an axis of symmetry ($I_1 = I_2 = I$) during deployment, Eqs. (7) become:

$$\dot{h}_1 + b(t)\, h_2 = 0 \tag{10a}$$

$$\dot{h}_2 - b(t)\, h_1 = 0 \tag{10b}$$

$$h_3 = h_0 = \text{const} \tag{10c}$$

where

$$b(t) = \frac{I_3(t) - I(t)}{I(t)\, I_3(t)}\, h_0 \tag{11}$$

Introducing $\tau = \int b(t)dt$ Eqs. (10a) and (10b) reduce to,

$$\frac{dh_1}{d\tau} + h_2 = 0 \tag{12a}$$

$$\frac{dh_2}{d\tau} - h_1 = 0 \tag{12b}$$

which are in the standard form of the two dimensional harmonic oscillator. The solutions to Eqs. (12) can be written as [4]:

$$h_1(t) = q_0^* \cos\tau = q_0^* \cos\left(\int_0^t b(t)dt + \psi_0^*\right) \tag{13}$$

$$h_2(t) = q_0^* \sin\tau = q_0^* \sin\left(\int_0^t b(t)dt + \psi_0^*\right) \tag{14}$$

The solutions given by Eqs. (13) and (14) are identical with those previously given by Hughes as a special case of his approximate analytical solution for the motion during deployment of a spacecraft with telescoping booms where the nutation angle remains small.[5]

We apply this solution to the symmetrical configuration of Fig. 1(a) where the moments of inertia about the principal axes are expressed, for the case of a uniform extension rate, c, along all three axes, by:

$$
\begin{aligned}
I_1 &= I^* + 4m\ell^2 = I^* + 2Pt^2 \\
I_2 &= I^* + 4m\ell^2 = I^* + 2Pt^2 \qquad (15)\\
I_3 &= I_3^* + 4m\ell^2 = I_3^* + 2Pt^2
\end{aligned}
$$

and $P = 2mc^2$

Using Eqs. (15) in Eq. (11) we obtain:

$$b(t) = \frac{h_0}{2P} \left\{ \frac{1}{d_1^2 + t^2} - \frac{1}{d_2^2 + t^2} \right\} \qquad (16)$$

where $d_1 = \sqrt{I^*/2P}$ and $d_2 = \sqrt{I_3^*/2P}$ (17)

Introducing Eq. (16) in Eqs. (13) and (14), and after performing the integration, the solutions for the transverse angular velocities are obtained as:

$$\omega_1(t) = \frac{q_0^* \cos\left[\frac{h_0}{2P} \left\{ \frac{1}{d_1} \tan^{-1} \frac{t}{d_1} - \frac{1}{d_2} \tan^{-1} \frac{t}{d_2} \right\} + \psi_0^* \right]}{I^* + 2Pt^2} \qquad (18)$$

$$\omega_2(t) = \frac{q_0^* \sin\left[\frac{h_0}{2P} \left\{ \frac{1}{d_1} \tan^{-1} \frac{t}{d_1} - \frac{1}{d_2} \tan^{-1} \frac{t}{d_2} \right\} + \psi_0^* \right]}{I^* + 2Pt^2} \qquad (19)$$

where q_0^* and ψ_0^* are determined from the initial conditions.

From Eq. (10c), the angular rate about the '3' axis is written,

$$\omega_3(t) = \frac{I_3^* \, \omega_3(0)}{I_3^* + 2Pt^2} \tag{20}$$

we observe here for large values of t, the solutions for the angular velocities lead to the form:

$$\omega_i(t) = \text{const}/(I_i^* + 2Pt^2), \; i = 1,2,3 \tag{21}$$

This equation indicates that the magnitudes of the angular velocities decrease during extension of the appendages, with the square of the elapsed time.

3. Achieve Final Spin About One of the Principal Axes

(a) Lyapunov Function-Modified Kinetic Energy

The desired final state of the system is: $\omega_1 = 0$, $\omega_2 = 0$ and $\omega_3 = \omega_{3_f} = \Omega$. Using the state variables: ω_1, ω_2, and $\omega_3 - \Omega$, the Lyapunov function is defined as the modified rotational kinetic energy, which can be written as:

$$V = \frac{1}{2}\left[I_1\omega_1^2 + I_2\omega_2^2 + I_3\,(\omega_3 - \Omega)^2\right] \tag{22}$$

Here V is positive definite in the state variables selected. Differentiating Eq. (22) with respect to time, there results:

$$\dot{V} = \frac{1}{2}\left[\dot{I}_1\omega_1^2 + \dot{I}_2\omega_2^2 + \dot{I}_3(\omega_3 - \Omega)^2 + 2I_1\omega_1\dot{\omega}_1 + 2\,I_2\omega_2\dot{\omega}_2 + 2\,I_3\,(\omega_3 - \Omega)\dot{\omega}_3\right] \tag{23}$$

Using Eq. (9) in Eq. (23), we obtain,

$$\dot{V} = -\frac{1}{2}(\dot{I}_1\omega_1^2 + \dot{I}_2\omega_2^2 + \dot{I}_3\omega_3^2) + \frac{1}{2}\Omega^2\dot{I}_3 - (\dot{I}_3\omega_3 + I_3\dot{\omega}_3) \qquad (24)$$

For symmetry about the '3' axis during extension:

$$\dot{h}_3 = \dot{I}_3\omega_3 + I_3\dot{\omega}_3 = 0 \qquad (25)$$

Eq. (25) is used in Eq. (24) to obtain:

$$\dot{V} = -\frac{1}{2}[\dot{I}_1\omega_1^2 + \dot{I}_2\omega_2^2 + \dot{I}_3(\omega_3^2 - \Omega^2)] \qquad (26)$$

After rewriting Eq. (26) in terms of the state variables,

$$\dot{V} = -\frac{1}{2}[\dot{I}_1\omega_1^2 + \dot{I}_2\omega_2^2 + \dot{I}_3(\omega_3 - \Omega)^2] - \dot{I}_3\Omega(\omega_3 - \Omega) \qquad (27)$$

Also from Eq. (25), the solution for $\omega_3(t)$ is given by,

$$\omega_3(t) = I_3^*\omega_3(0)/I_3 \qquad (28)$$

We conclude from Eq. (27) that $\dot{V}$ is negative definite in the state variables only if:

$\omega_3 \geq \Omega$, $\dot{I}_1$, $\dot{I}_2 > 0$, and $\dot{I}_3 > 0$ for $\omega_3 > \Omega$

Thus for the case where a spin about one of the principal axes is a desired final condition, a modified form of the kinetic energy can be used as a Lyapunov function.

Here the final state can be achieved by extending all telescoping booms until the desired spin rate is reached and then continuing the extension of the set of booms along the nominal spin axis until the transverse components of angular velocity reach an acceptably small amplitude (within the limitations of boom length). It should be noted that if we allow $\omega_3 < \Omega$ and $\dot{I}_3 \neq 0$, there will be a difference in sign between the third and fourth terms in Eq. (27).

(b) Analytical Solution

The time at which $\omega_3 = \omega_{3f} = \Omega$ will be denoted by T_{3f}.

At $t = T_{3f}$,

$$\bar{I}_1^* = \bar{I}_2^* = I^* + 2P\,(T_{3f})^2 \tag{29}$$

$$\bar{I}_3^* = I_3^* + 2P\,(T_{3f})^2 \tag{30}$$

For $t \leq T_{3f}$, the solutions for the angular velocities can be obtained from Eqs. (18), (19) and (20).

For $t > T_{3f}$,

$$\bar{I}_3^* = \text{const} = I_3^* + 2P\,(T_{3f})^2 \tag{31}$$

$$\bar{I}^* = I^* + P(T_{3f})^2 + Pt^2 = I_f^* + Pt^2 \tag{32}$$

where
$$I_f^* = I^* + P(T_{3f})^2 \tag{33}$$

From Eq. (11), and using Eqs. (31) and (33), we obtain

$$b(t) = \omega_{3f}\left\{\frac{\bar{I}_3^*}{I_f^* + Pt^2} - 1\right\} \tag{34}$$

Introducing Eq. (34) into Eqs. (13) and (14), the solutions for the angular velocities for $t > T_{3_f}$ are,

$$\omega_1(t) = \frac{q_0 \cos[\omega_{3_f}[\frac{\bar{I}_3^*}{\sqrt{I_f^* P}} \{\tan^{-1}(\frac{t}{\sqrt{I_f^*/P}}) - \tan^{-1}(\frac{T_{3_f}}{\sqrt{I_f^*/P}})\} - t + T_{3_f}] + \psi_0]}{(I_f^* + Pt^2)} \quad (35)$$

$$\omega_2(t) = \frac{q_0 \sin[\omega_{3_f}[\frac{\bar{I}_3^*}{\sqrt{I_f^* P}} \{\tan^{-1}(\frac{t}{\sqrt{I_f^*/P}}) - \tan^{-1}(\frac{T_{3_f}}{\sqrt{I_f^*/P}})\} - t + T_{3_f}] + \psi_0]}{(I_f^* + Pt^2)} \quad (36)$$

and from Eq. (10c),

$$\omega_3(t) = \omega_{3_f} = \text{const} \quad (37)$$

Here q_0 and ψ_0 are to be determined from Eqs. (18) and (19) at $t = T_{3_f}$ and should not be confused with q_0^* and ψ_0^* which are determined at $t = 0$.

For large values of t, Eqs. (35) and (36) reduce to the form,

$$\omega_1 = \frac{q_0 \cos (\omega_{3_f} \times \text{const} \times t + \text{const})}{I_f^* + Pt^2} \quad (38)$$

$$\omega_2 = \frac{q_0 \sin (\omega_{3_f} \times \text{const} \times t + \text{const})}{I_f^* + Pt^2} \quad (39)$$

The above two equations indicate that the frequency of oscillation approaches a constant value and the magnitude of the oscillation decreases with the square of the elapsed time.

The time, $t = T_{3_f}$, at which the extension of the booms along the '1' and '2' axes are stopped can be determined from $\dot{h}_3 = 0$, yielding the result:

$$T_{3_f} = \frac{1}{2c} \sqrt{\frac{I_3^*}{m} \left(\frac{\omega_3(0) - \omega_{3f}}{\omega_{3_f}}\right)} \tag{40}$$

C. Uniformly Distributed Mass Moving

1. Achieve Zero Inertial Angular Rate

(a) Analytical Solution

The desired final state of the system is $\omega_i = 0$. The booms considered are assumed to have a uniformly distributed mass (ρ) along their lengths. The same procedure as adopted in the case of the moving end masses can be applied here to obtain the solutions for the angular velocities. Here we present only the final results.

The solutions for the angular velocities are given by:

$$\omega_1(t) = \frac{q_0^* \cos \{ \int_0^t b(t)\, dt + \psi_0^* \}}{I^* + \frac{2}{3} Kt^3} \tag{41}$$

$$\omega_2(t) = \frac{q_0^* \sin \{ \int_0^t b(t)\, dt + \psi_0^* \}}{I^* + \frac{2}{3} Kt^3} \tag{42}$$

$$\omega_3(t) = \frac{I_3^* \, \omega_3(0)}{I_3^* + \frac{2}{3} Kt^3} \tag{43}$$

and $K = 2 \rho c^3$

In Eqs. (41) and (42),

$$\int_0^t b(t)\, dt = \frac{3h_0}{2K} \left[\frac{1}{6d_3^2} \ln\left\{ \frac{(d_3 + t)^2}{d_3^2 - d_3 t + t^2} \right\} + \frac{1}{d_3^2\sqrt{3}} \tan^{-1}\left\{ \frac{2t - d_3}{d_3\sqrt{3}} \right\} \right.$$

$$\left. - \frac{1}{6d_4^2} \ln\left\{ \frac{(d_4 + t)^2}{d_4^2 - d_4 t + t^2} \right\} - \frac{1}{d_4^2\sqrt{3}} \tan^{-1}\left\{ \frac{2t - d_4}{d_4\sqrt{3}} \right\} \right] \tag{44}$$

$$d_3^3 = \frac{3I^*}{2K} \quad \text{and} \quad d_4^3 = \frac{3I_3^*}{2K} \tag{45}$$

2. Achieve Final Spin About One of the Principal Axes

(a) Analytical Solution

The desired final state of the system is $\omega_1 = 0$, $\omega_2 = 0$ and $\omega_3 = \omega_{3_f} = \Omega$. For $t \leq T_{3_f}$, the solutions for the angular velocities can be obtained from Eqs. (41), (42) and (43). For $t > T_{3_f}$, the solutions for the angular velocities can be obtained as:

$$\omega_1(t) = \frac{q_0 \cos\{ \int_{T_{3_f}}^t b(t)\, dt + \psi_0 \}}{I_f^* + \frac{1}{3} K t^3} \tag{46}$$

$$\omega_2(t) = \frac{q_0 \sin\{ \int_{T_{3_f}}^t b(t)\, dt + \psi_0 \}}{I_f^* + \frac{1}{3} K t^3} \tag{47}$$

$$\omega_3(t) = \omega_{3_f} = \text{const} \tag{48}$$

where $$I_f^* = I^* + \frac{K}{3} (T_{3_f})^3 \tag{49}$$

In Eqs. (46) and (47),

$$\int_{T_{3_f}}^{t} b(t)dt = \omega_{3_f} \left[\frac{3\,\bar{I}_3^*}{K} \left\{ \frac{1}{6d_5^2} \ln \left(\frac{(d_5 + t)^2}{d_5^2 - d_5 t + t^2} \right) \right.\right.$$

$$\left.\left. + \frac{1}{d_5^2 \sqrt{3}} \tan^{-1} \left(\frac{2t - d_5}{d_5 \sqrt{3}} \right) \right\} - t + T_{3_f} \right] \tag{50}$$

$$\bar{I}_3^* = I_3^* + \frac{2}{3} K (T_{3_f})^3 \tag{51}$$

$$d_5^3 = \frac{3 I_f^*}{K} \tag{52}$$

The time T_{3_f}, at which the booms along the '1' and '2' axes are stopped, can be obtained as:

$$T_{3_f} = \left[\frac{3 I_3^*}{2K} \left(\frac{\omega_3(0) - \omega_{3_f}}{\omega_{3_f}} \right) \right]^{1/3} \tag{53}$$

III. NUMERICAL RESULTS

A. End Mass Moving

A typical detumbling maneuver for an initially uncontrolled spacecraft is illustrated in Figs. 2 and 3. In this example because of symmetry the uncontrolled torque-free nutation (Fig. 2) can be theoretically predicted. This motion is also represented by the dotted curves corresponding to a zero boom extension rate ($c_i = 0$) in Figs. 3. The effect of extension rate on the recovery is illustrated in Figs. 3(a) - (c). For small extension rates (up to 1 ft/sec) the oscillatory nature of the transverse motion is not removed until after the first cycle; the advantage of considering higher extension rates (at the expense of on-board power) for an initial fast tumbling is apparent. It should be noted that at a given time in these figures different boom lengths are represented according to the extension rate. For example, with an extension rate of 4 ft/sec after 60 ft. of extension along all three principal axes the angular velocity components have been reduced by more than a factor of 10 and, if 240 ft. of boom could be extended, by a factor of over 300. Removal of this residual angular velocity could then be achieved by temporarily activating on-board damping devices. Then the appendages could be retracted and would be ready for subsequent reuse as necessary.

Numerical examination of other cases for asymmetrical hubs also verifies the practicality of using movable appendages for the initial detumbling of randomly spinning spacecraft (Figs. 4(a) and (b)). The numerical simulation results for an asymmetrical spacecraft are compared with the closed form solution for a symmetrical extension and it is observed that the closed form solutions are only applicable when the asymmetry is small. Although the moving end mass system is effective in reducing the magnitude of all components of the angular velocities, for the masses considered here it will not effectively reduce the initial nutation angle during the response time simulated. For the symmetrical deployment shown in Fig. 4 the initial nutation angle of 27.89 degrees is maintained, whereas for the asymmetrical case, an initial nutation angle of 29.09 degrees is increased within 3.5 seconds to 36.95 degrees and maintained up to the 60 seconds simulated.

The nutation angle is defined here as the angle between the nominal symmetry ('3') axis and the total angular momentum vector.

Fig. 5 illustrates a recovery maneuver which would result in a final spin about the '3' body axis with a small transverse residual. The booms are extended so that the modified rotational kinetic energy is positive definite and its total time derivative is negative definite during the maneuver. All booms are extended until T_{3_f} at which time $\omega_3 = \omega_{3_f}$. Then, only booms along the $\pm$ '3' axis are extended to reduce the transverse residual components. At the switching time, T_{3_f}, the effect of any lag in the system has not been considered here but should be considered in the design and performance of the actual system. From the figure it can be seen that the frequency of the response of the transverse components of the angular velocity is essentially constant in the terminal part of the maneuver (consistent with the discussion in connection with Eqs. (38) and (39)).

A comparison of the recovery maneuver of an asymmetrical spacecraft with that of a symmetrical spacecraft to achieve a final spin along the '3' axis is shown in Figs. 6(a) and (b). The calculated T_{3_f} for the symmetrical spacecraft is used for stopping the booms along the '1' and '2' axes. It is observed that using this logic the final ω_{3_f} reaches a lower value (1.8 rad/sec) when compared with the desired final value (2.0 rad/sec). Also we notice from Fig. 6(a), the response of $\omega_1(t)$ for the asymmetrical case differs from that of the symmetrical case.

This is due to the increase in the order of the system equations for the asymmetrical extension (i.e. - three first order differential equations must now be considered). It should be pointed out that after T_{3_f}, for the asymmetrical case, the time response of ω_3 is not exactly a straight line as apparently indicated in Fig. 6(a) but also consists of small amplitude oscillations superimposed about this straight line solution. For larger asymmetries this oscillation would become apparent within the plotting scale shown and the difference between ω_{3_f} achieved and desired would also increase using the open loop control logic of switching the extension sequence at a pre-set T_{3_f}. It should also be noted that the nutation angle, for both cases, does not vary greatly after T_{3_f}.

B. <u>Uniformly Distributed Mass Moving</u>

For the case of a spacecraft with a uniformly distributed mass along the boom lengths a typical detumbling maneuver is illustrated in Figs. 7(a) and (b) for a symmetrical and asymmetrical hub, respectively. If 240 ft. of boom could be extended the total mass of each appendage would be about 1 slug. The effect of the increased appendage mass (when compared with the case of Figs. 3 and 4)is immediately apparent by noting the extremely small amplitude of the residual angular velocity - of the order 10^{-4} rad/sec. The nutation angle behavior for both cases of deployment is essentially the same as already described in connection with Fig. 4.

A representative recovery operation to achieve a final spin about the '3' axis with this system is shown in Figs. 8(a) and (b).

A comparison with the results of Figs. 6(a) and (b) clearly demonstrates the effectiveness of the heavier boom after the switching time. In addition, after approximately 40 seconds a drastic reduction in the nutation angle is also apparent - a result not achieved by the lighter end mass system. Despite the obvious advantages of using a more massive appendage there are, however, two practical considerations that have been ignored: (1) the increased weight of the payload package and (2) the effect of flexibility on the assumed rigid body dynamics simulated here.

IV. CONCLUDING COMMENTS

As an application for spacecraft rescue and recovery, when booms are extended along all the principal axes to detumble a symmetrical spacecraft, exact closed form analytical solutions can be obtained for all three angular velocities of the spacecraft.

Boom extension maneuvers can be determined to approach either of the two desired final states using Lyapunov's second method. The conclusions are that: (1) as time becomes extremely large (and boom lengths become infinite) it would be theoretically possible to achieve a zero inertial angular velocity state (of course, such a situation will, in practice, not occur due to finite length booms); (2) the final spin about one of the principal axes can be achieved by extending all telescoping booms until the desired spin rate is reached and then continuing the extension of the set of booms along the nominal spin axis until the transverse components of angular velocity reach an acceptably small amplitude.

Numerical examination of other cases for asymmetrical hubs also verifies the practicality of using movable appendages for the initial detumbling of randomly spinning spacecraft.

An advantage of the telescoping system as used in the recovery of tumbling spacecraft is its potential reuse. The booms can be retracted at the end of each recovery operation once the small residual angular velocity components have been removed by temporarily activating on-board damping devices. The constraints on such a system are: (1) the limitations on the extension rate, and boom lengths that are practicable; (2) the limitations on the rate of initial tumble that could be handled by the system without compromising its structural integrity.

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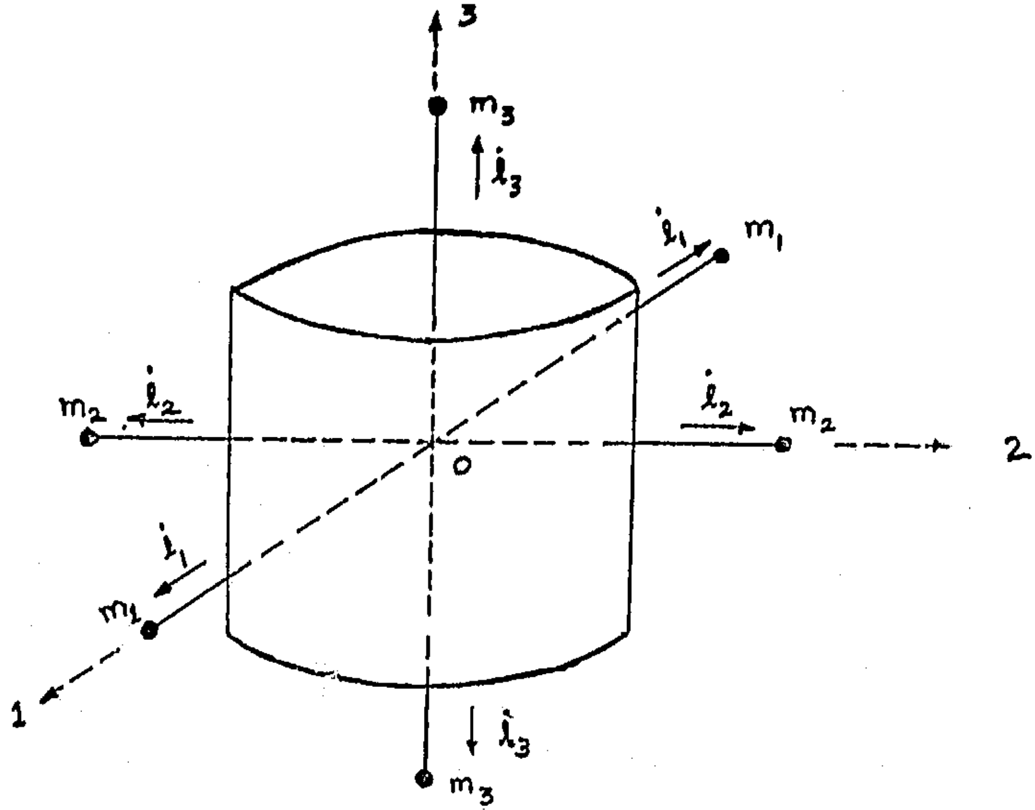
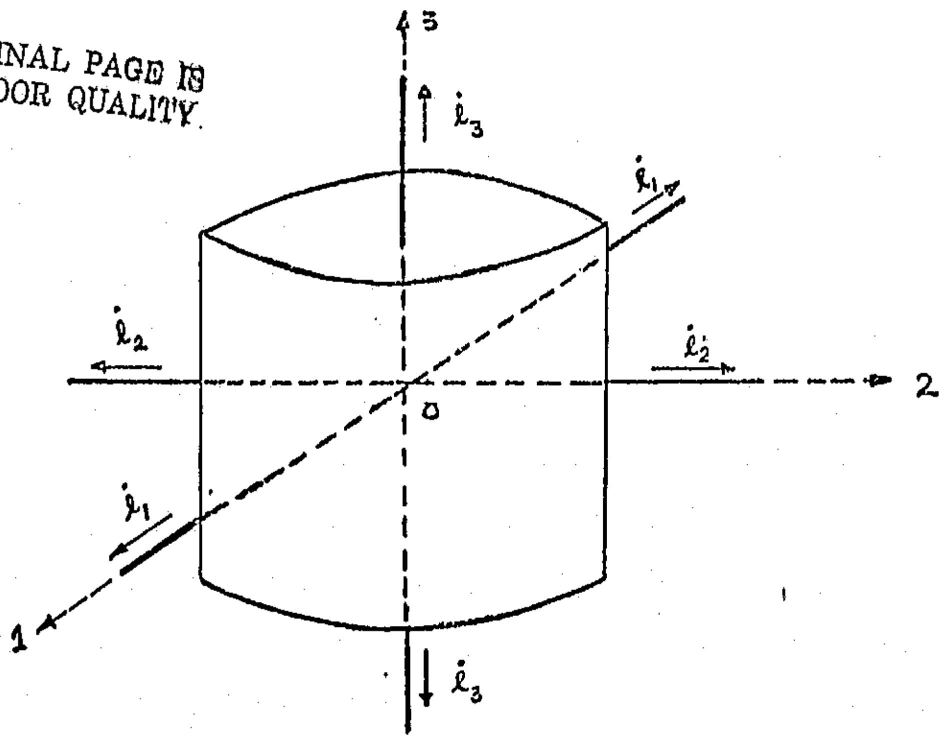


a. END MASS MOVING



b. UNIFORMLY DISTRIBUTED MASS MOVING

FIG. 1. SYSTEM GEOMETRY FOR DETUMBLING MANEUVER

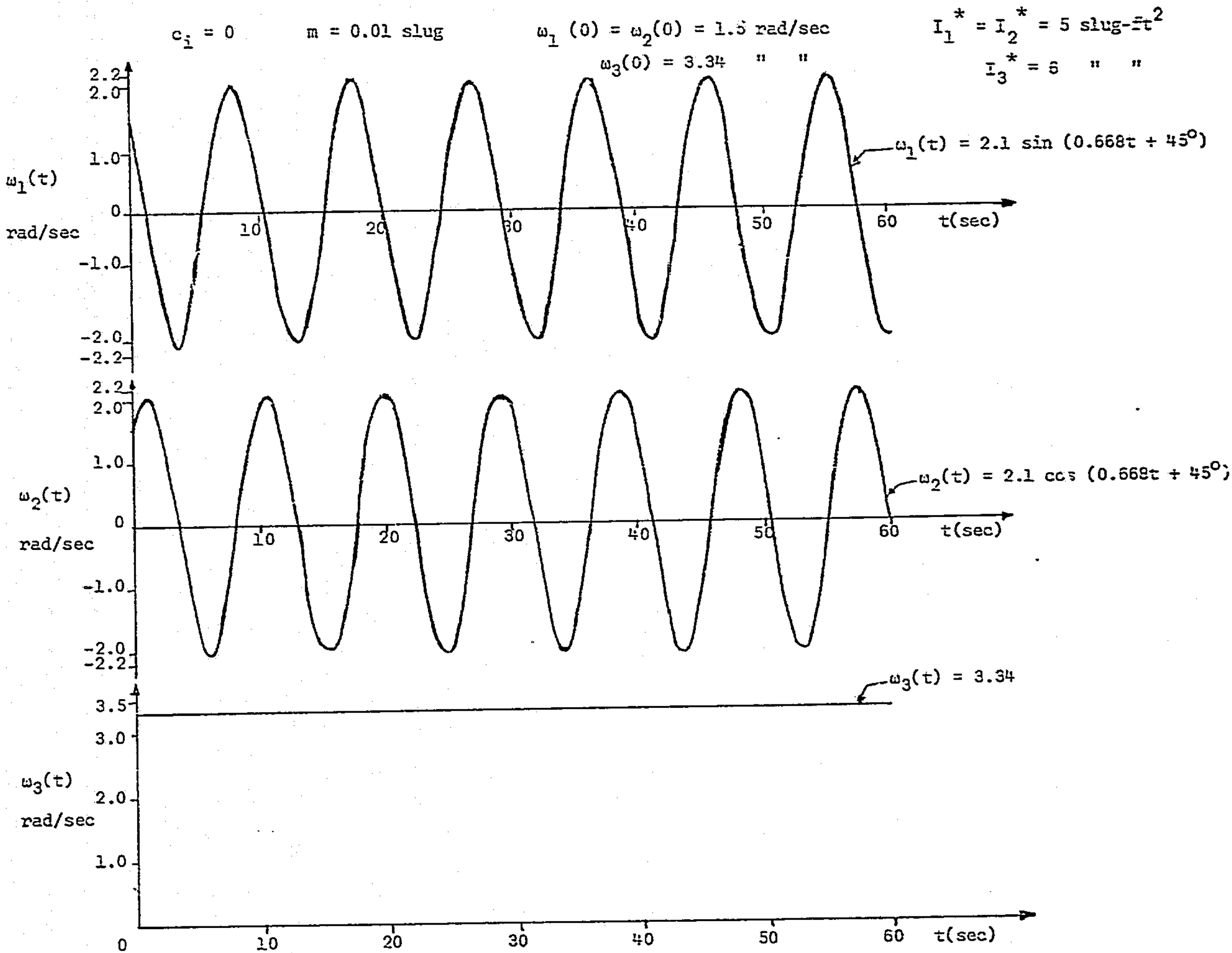


FIG. 2. DYNAMICS OF UNCONTROLLED MOTION

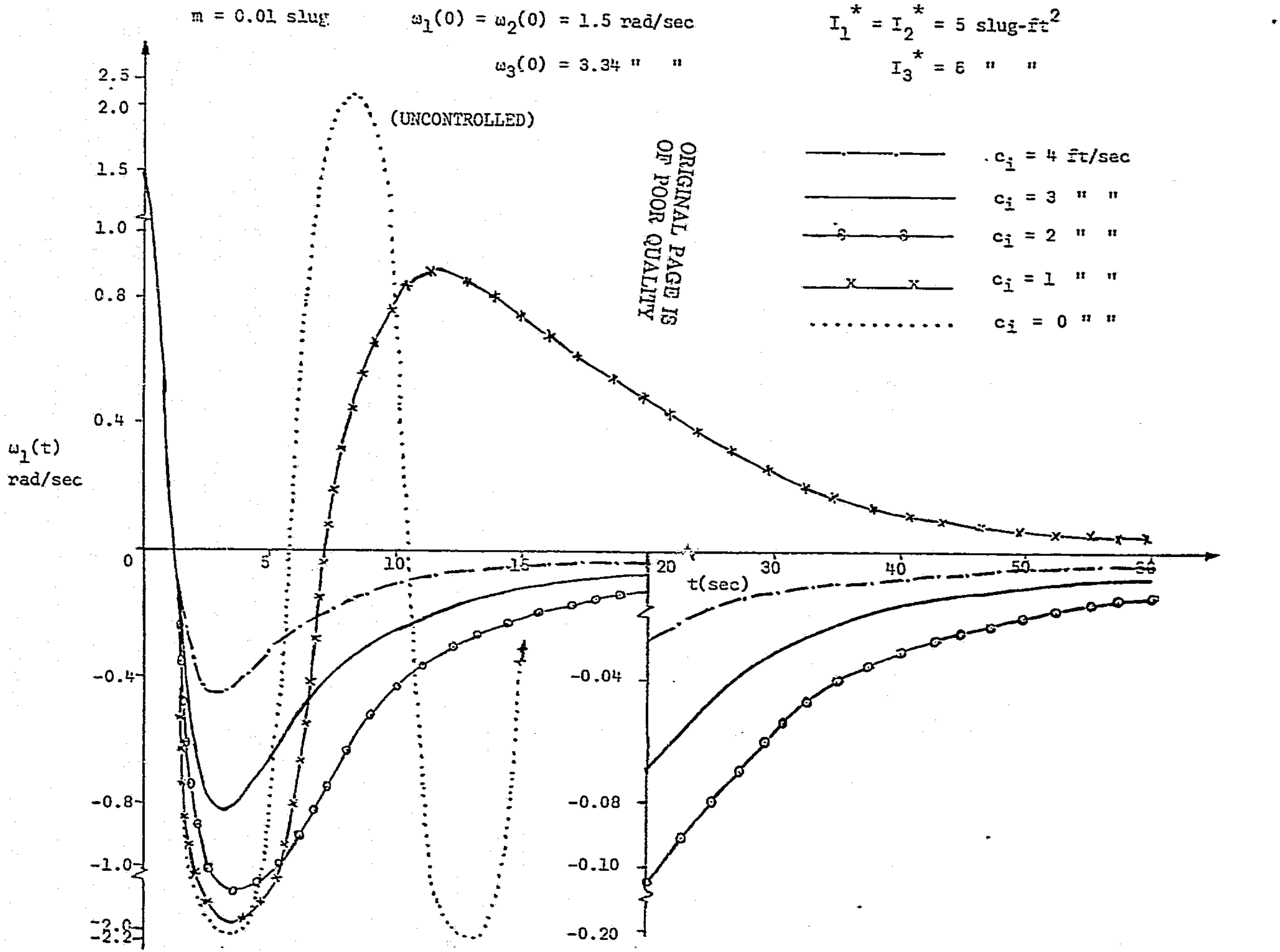


FIG. 3(a). EFFECT OF BOOM EXTENSION RATE ON RECOVERY

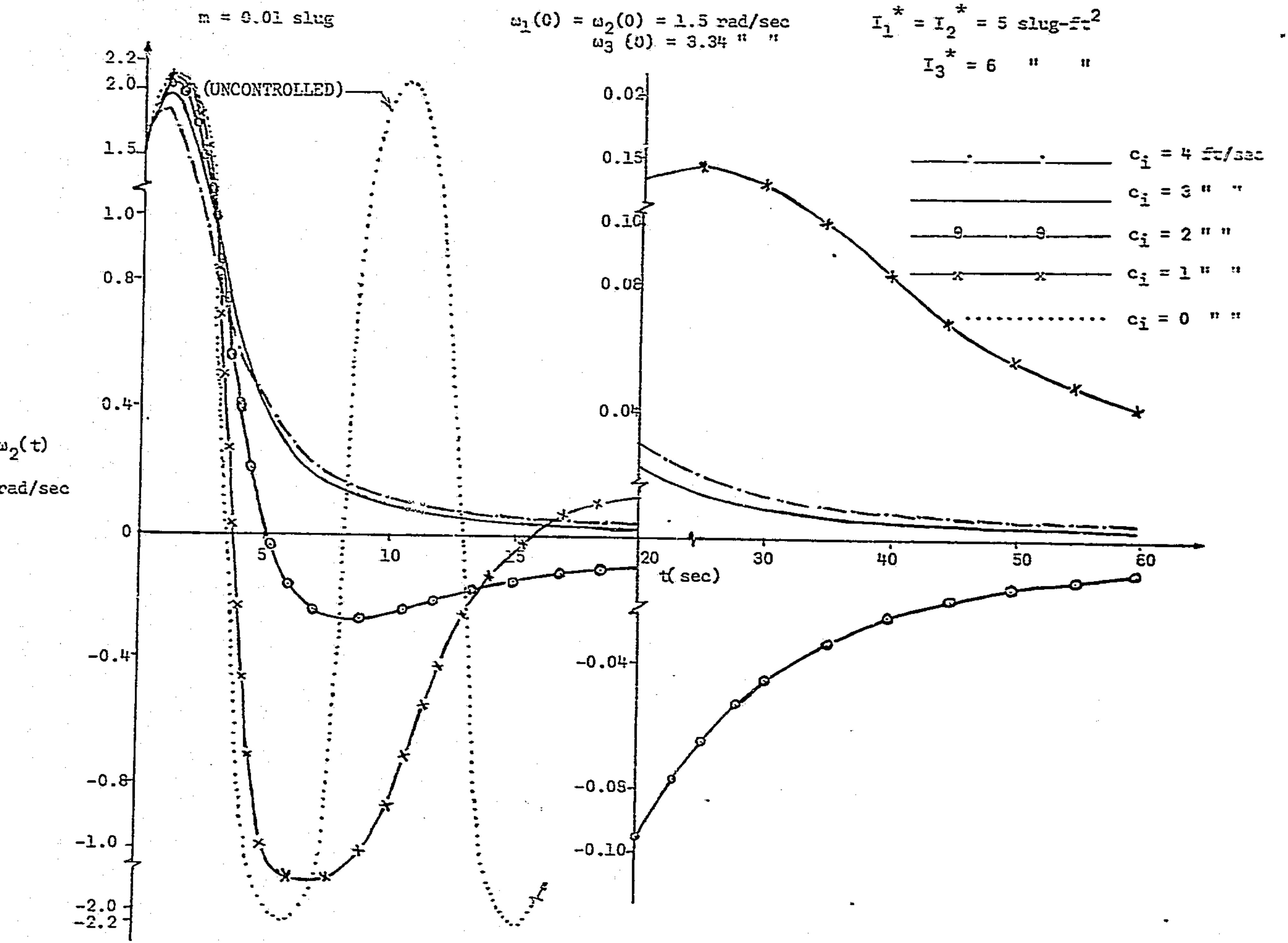


FIG.3 (b). EFFECT OF BOOM EXTENSION RATE ON RECOVERY

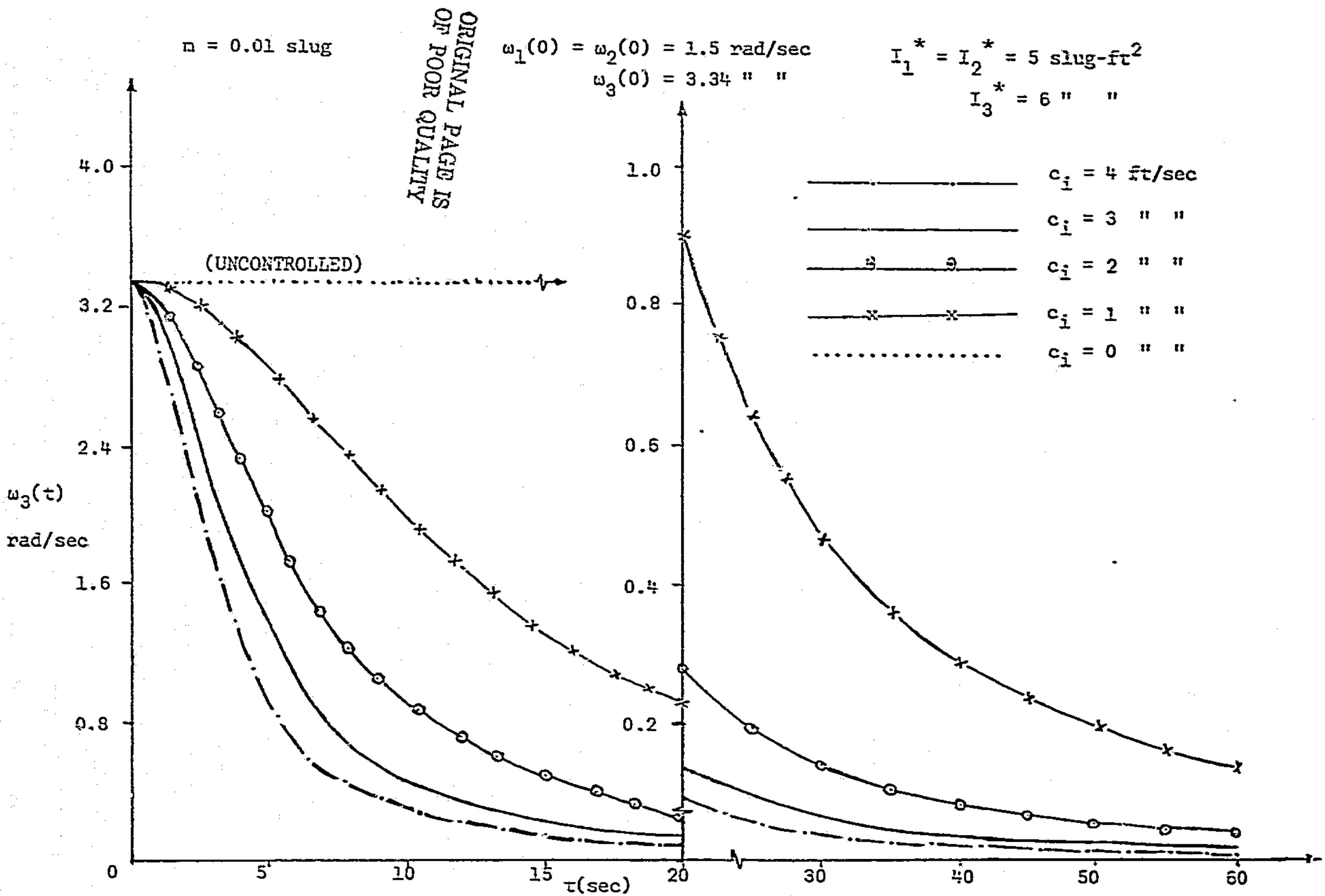


FIG. 3(c). EFFECT OF BOOM EXTENSION RATE ON RECOVERY

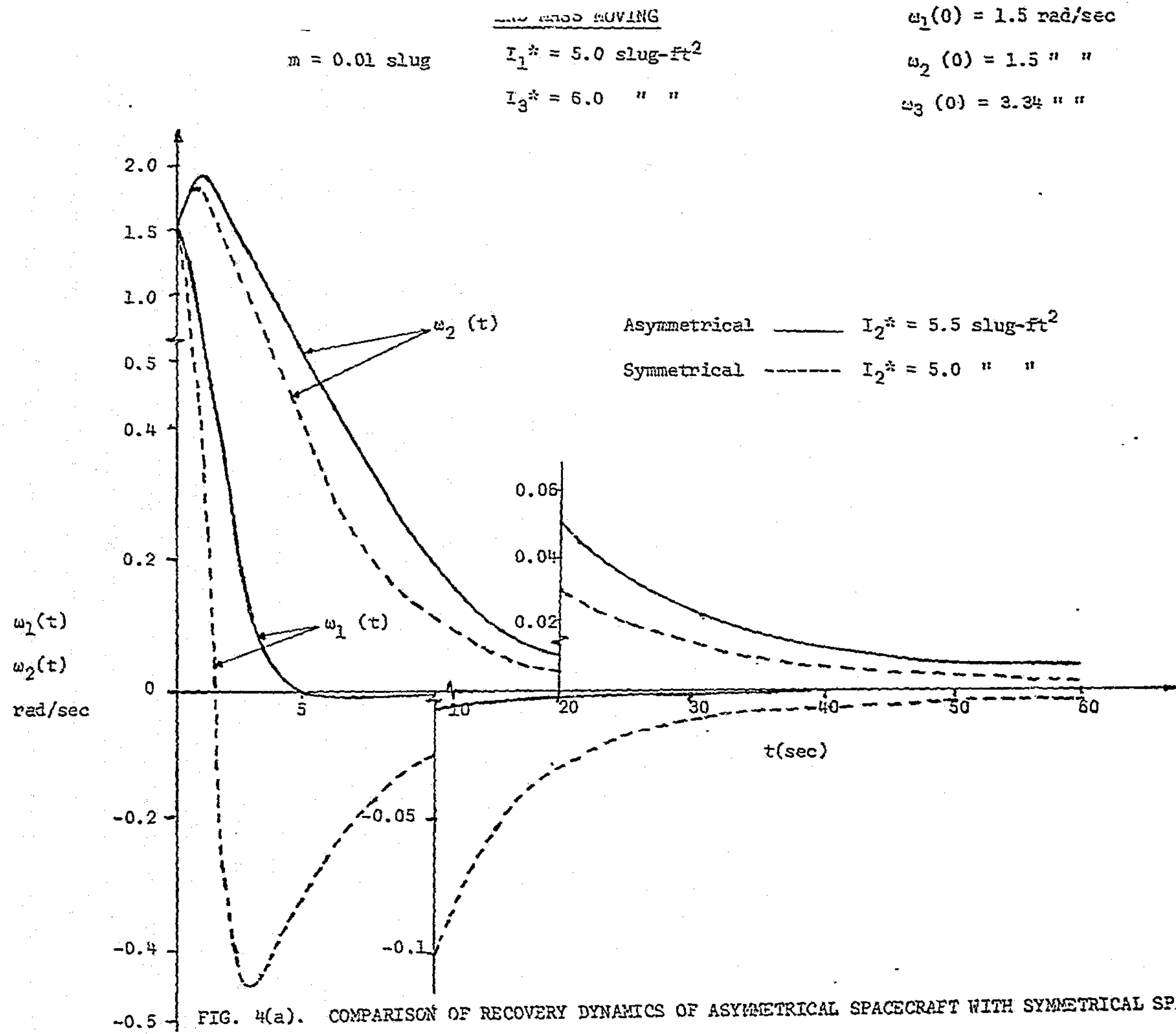


FIG. 4(a). COMPARISON OF RECOVERY DYNAMICS OF ASYMMETRICAL SPACECRAFT WITH SYMMETRICAL SPACECRAFT

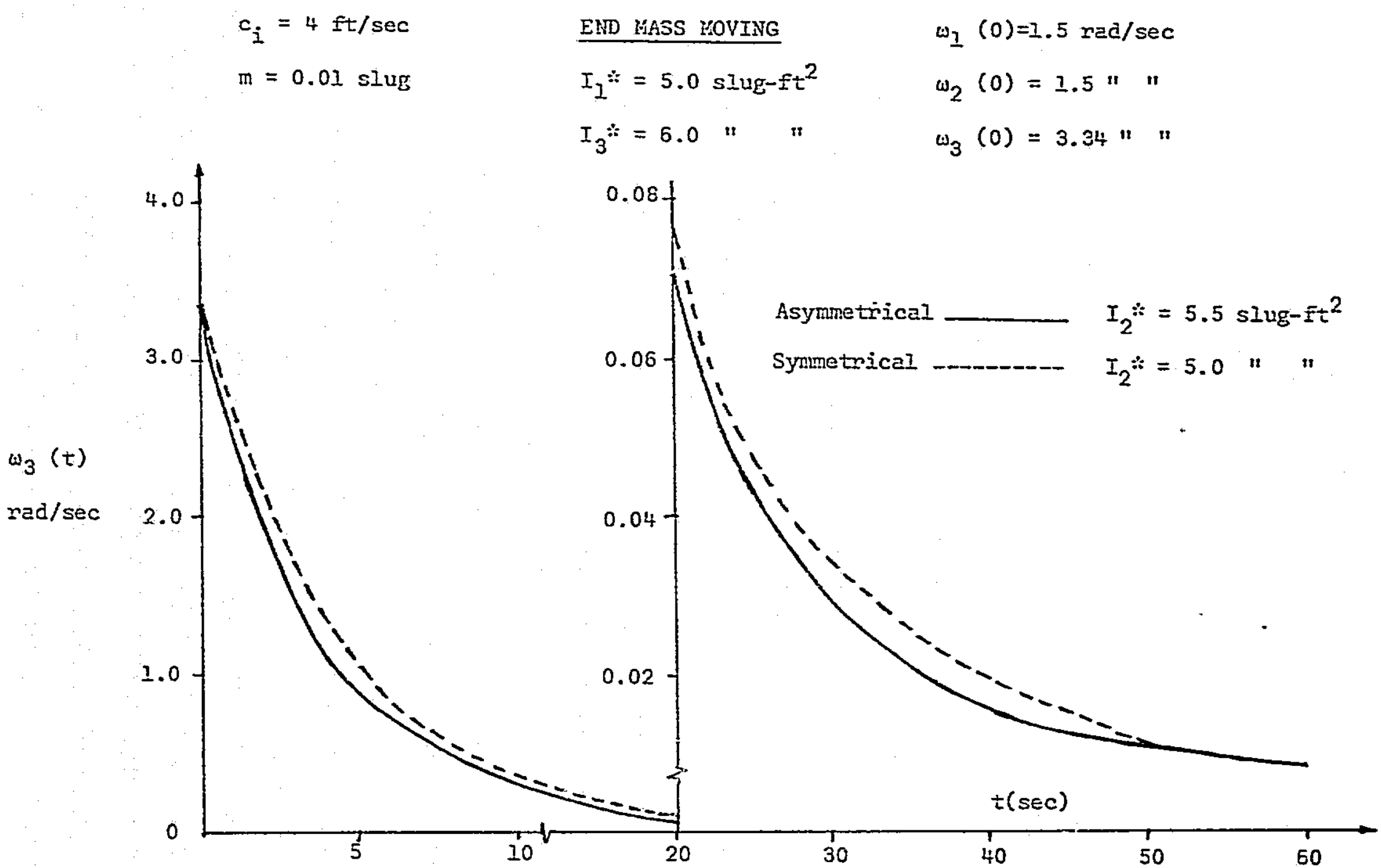


FIG. 4(b). COMPARISON OF RECOVERY DYNAMICS OF ASYMMETRICAL SPACECRAFT WITH SYMMETRICAL SPACECRAFT.

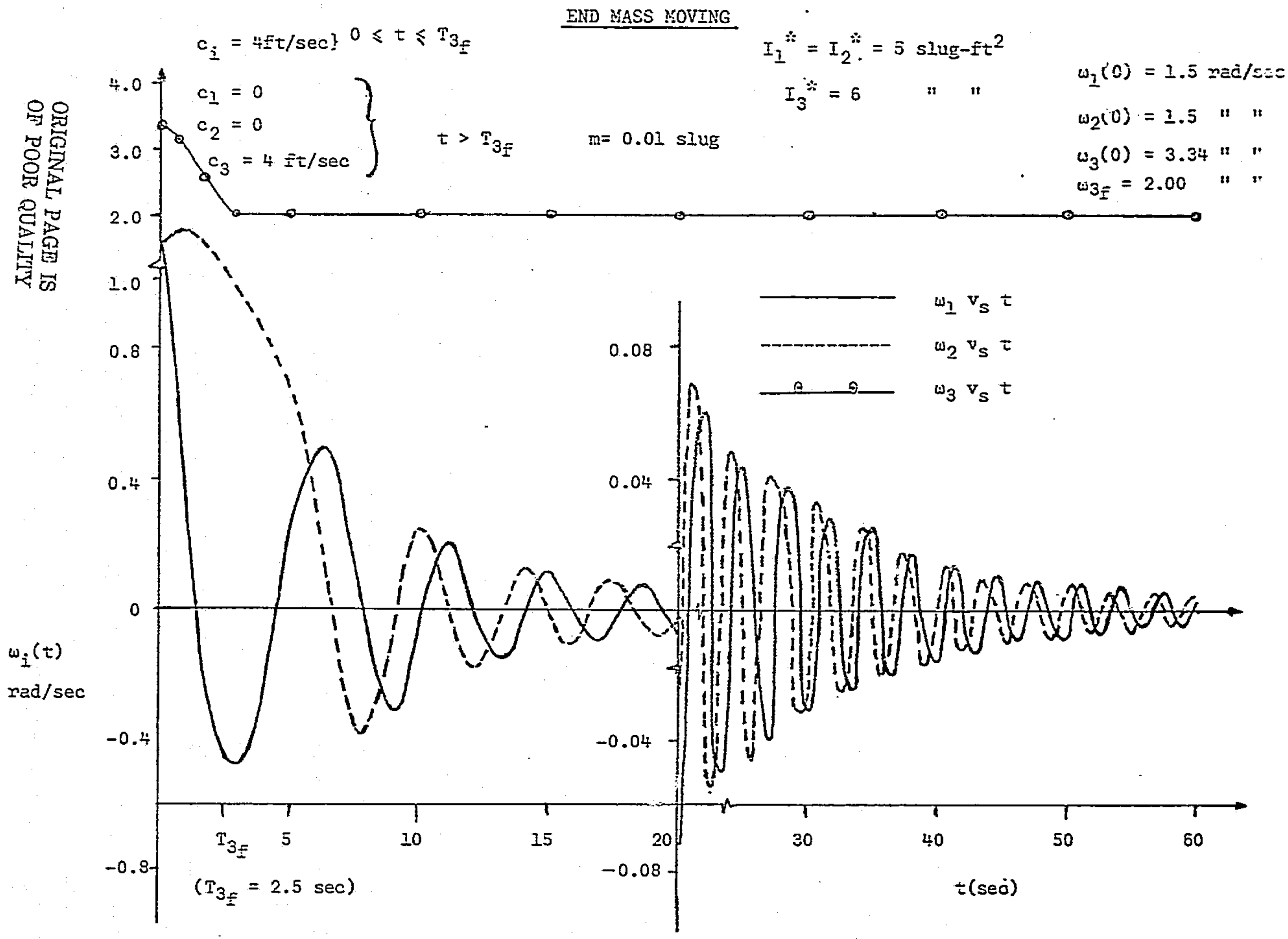


FIG 5. RECOVERY MANEUVER TO ACHIEVE FINAL SPIN ABOUT '3' AXIS

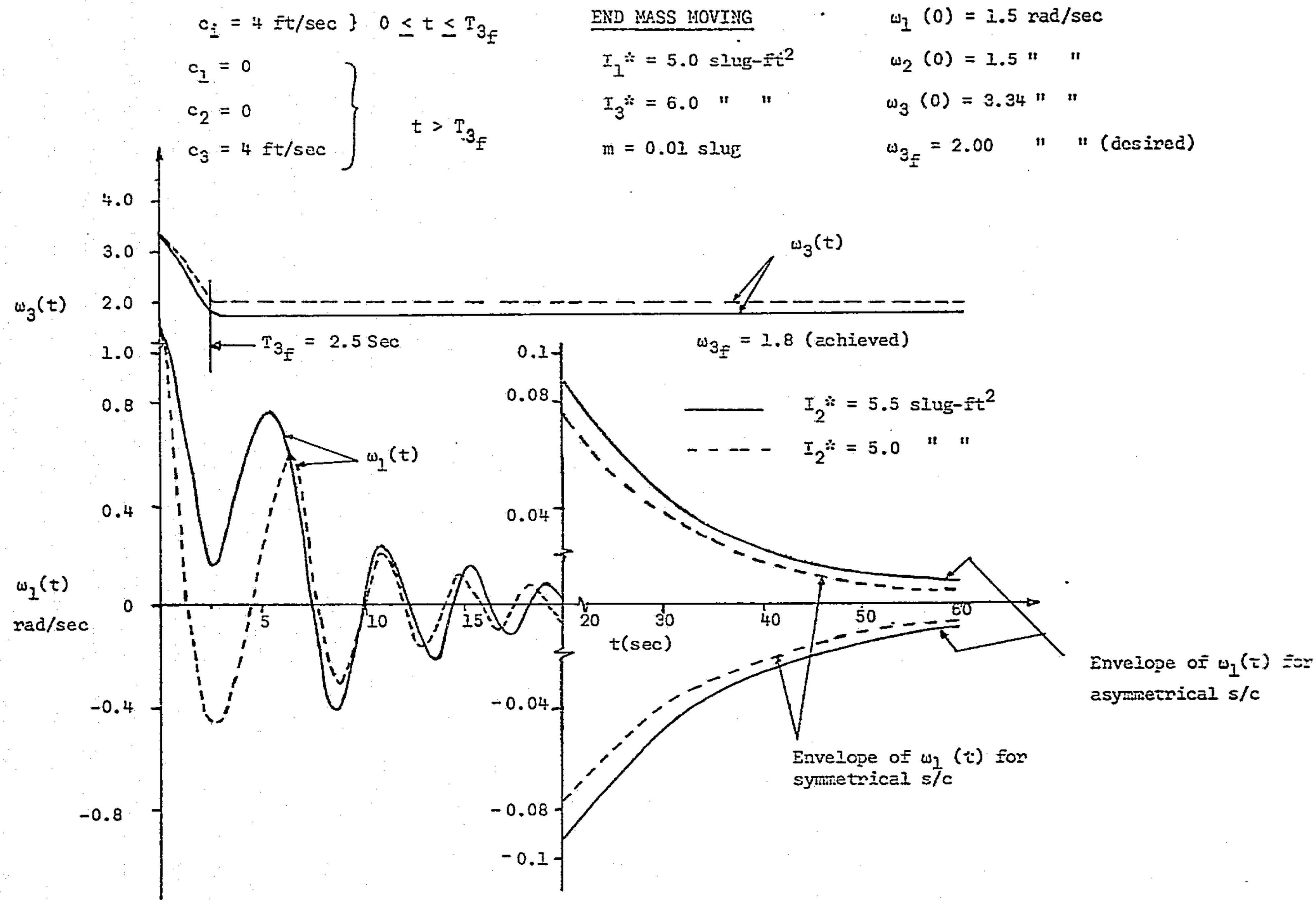


FIG. 6(a). COMPARISON OF RECOVERY MANEUVER OF ASYMMETRICAL SPACECRAFT WITH SYMMETRICAL SPACECRAFT TO ACHIEVE FINAL SPIN ABOUT '3' AXIS

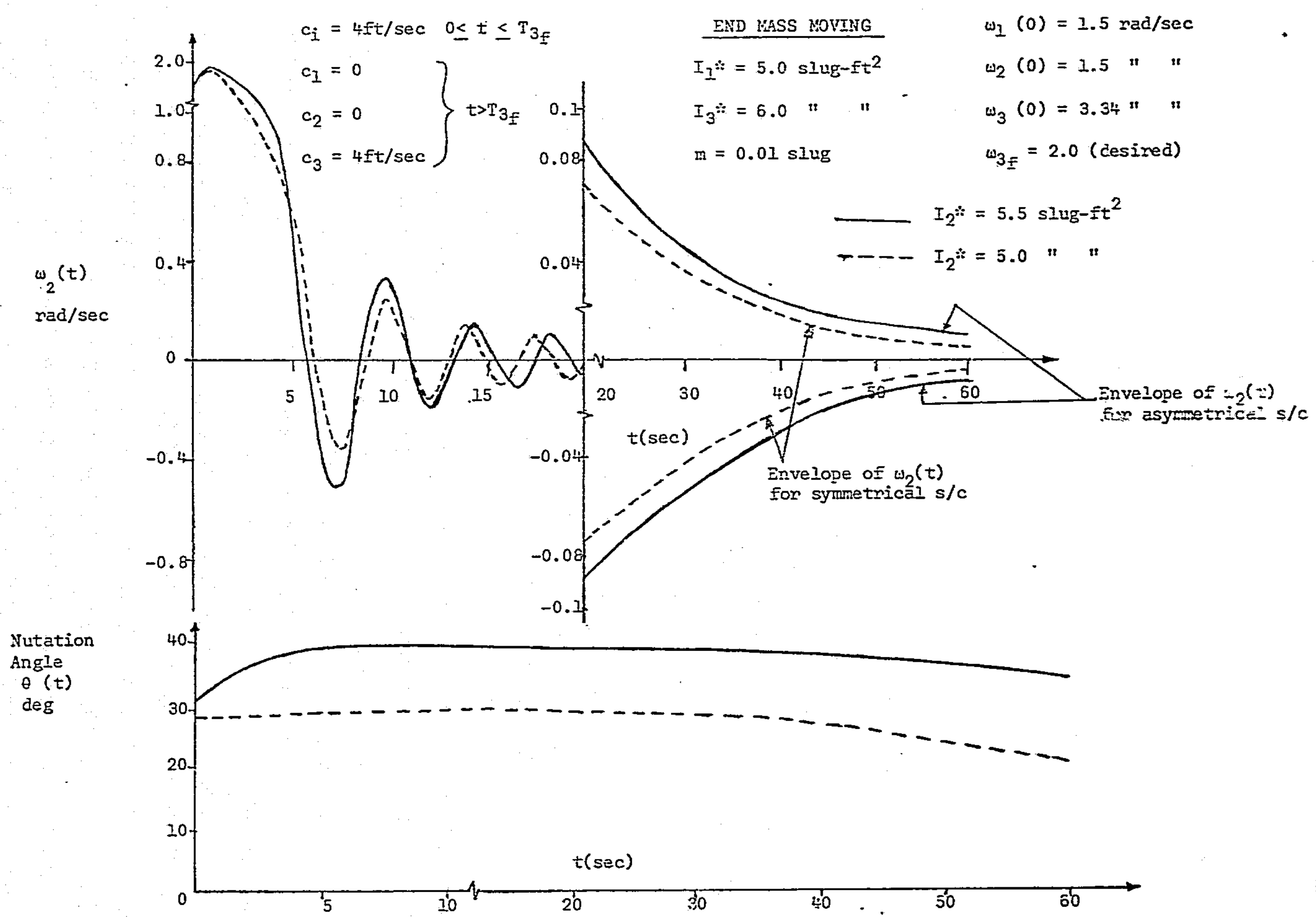


FIG. 6(b). COMPARISON OF RECOVERY MANEUVER OF ASYMMETRICAL SPACECRAFT WITH SYMMETRICAL SPACECRAFT TO ACHIEVE FINAL SPIN ABOUT '3' AXIS

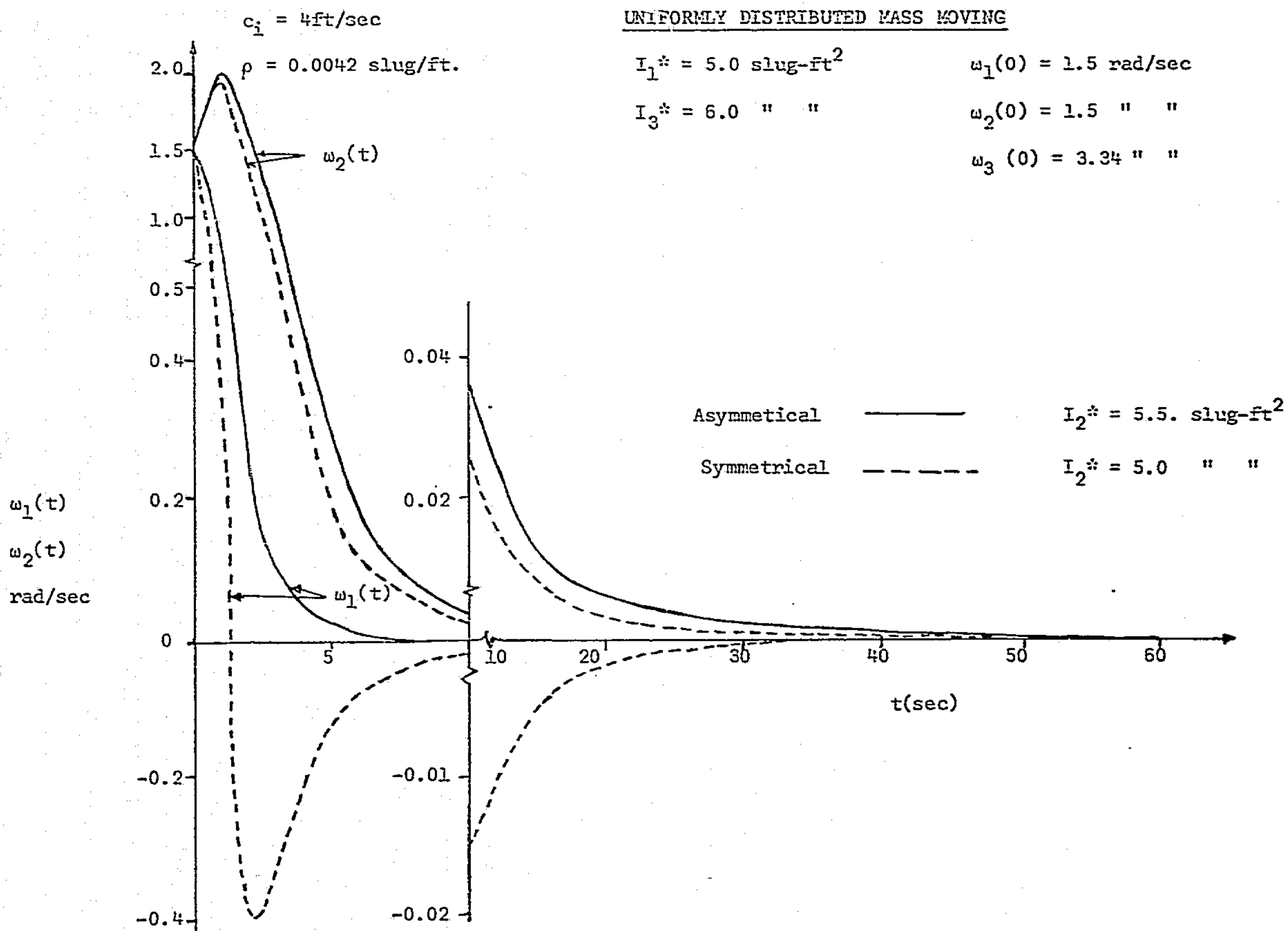


FIG. 7 (a). COMPARISON OF RECOVERY DYNAMICS OF ASYMMETRICAL SPACECRAFT WITH SYMMETRICAL SPACECRAFT

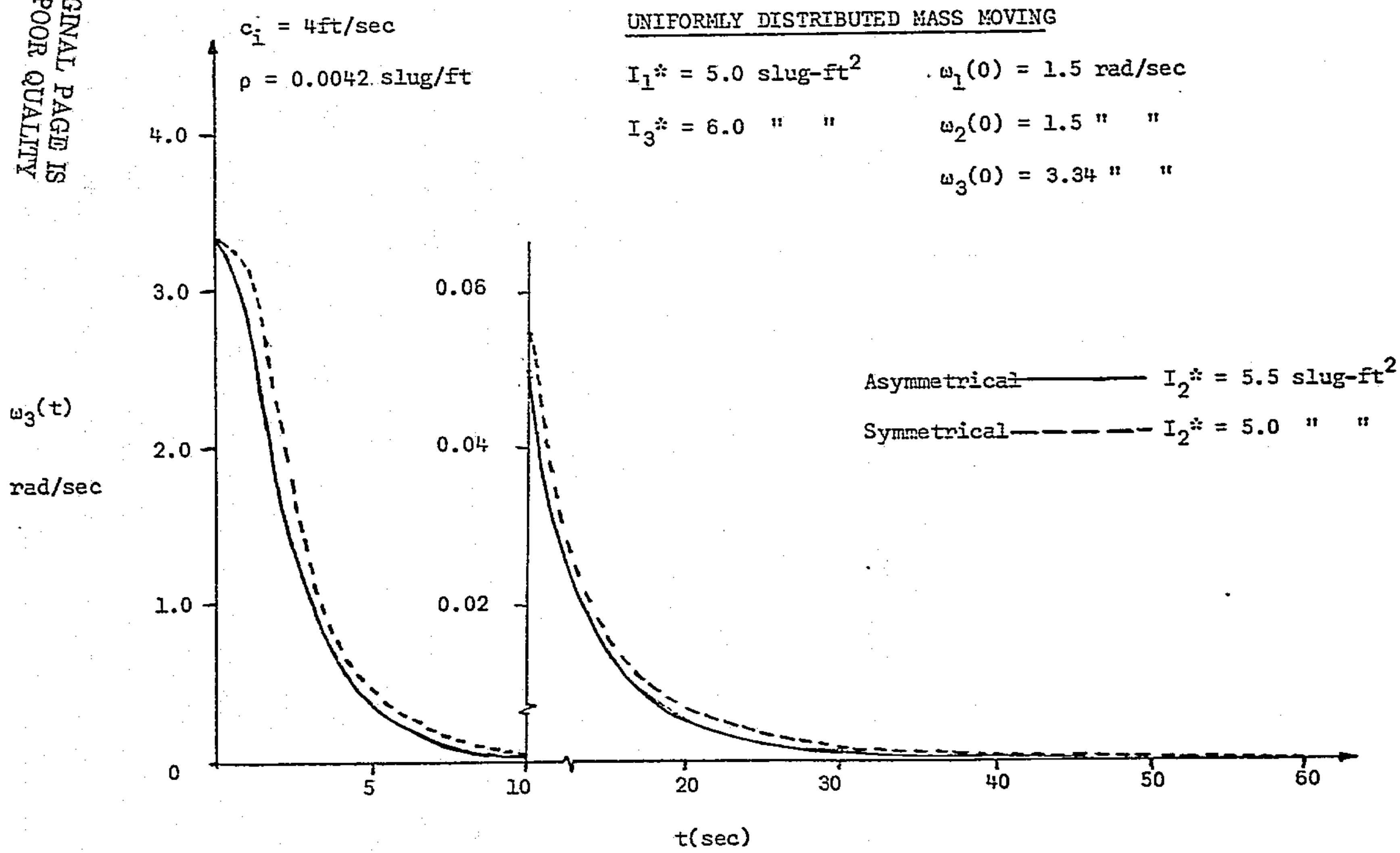


FIG. 7(b). COMPARISON OF RECOVERY DYNAMICS OF ASYMMETRICAL SPACECRAFT WITH SYMMETRICAL SPACECRAFT

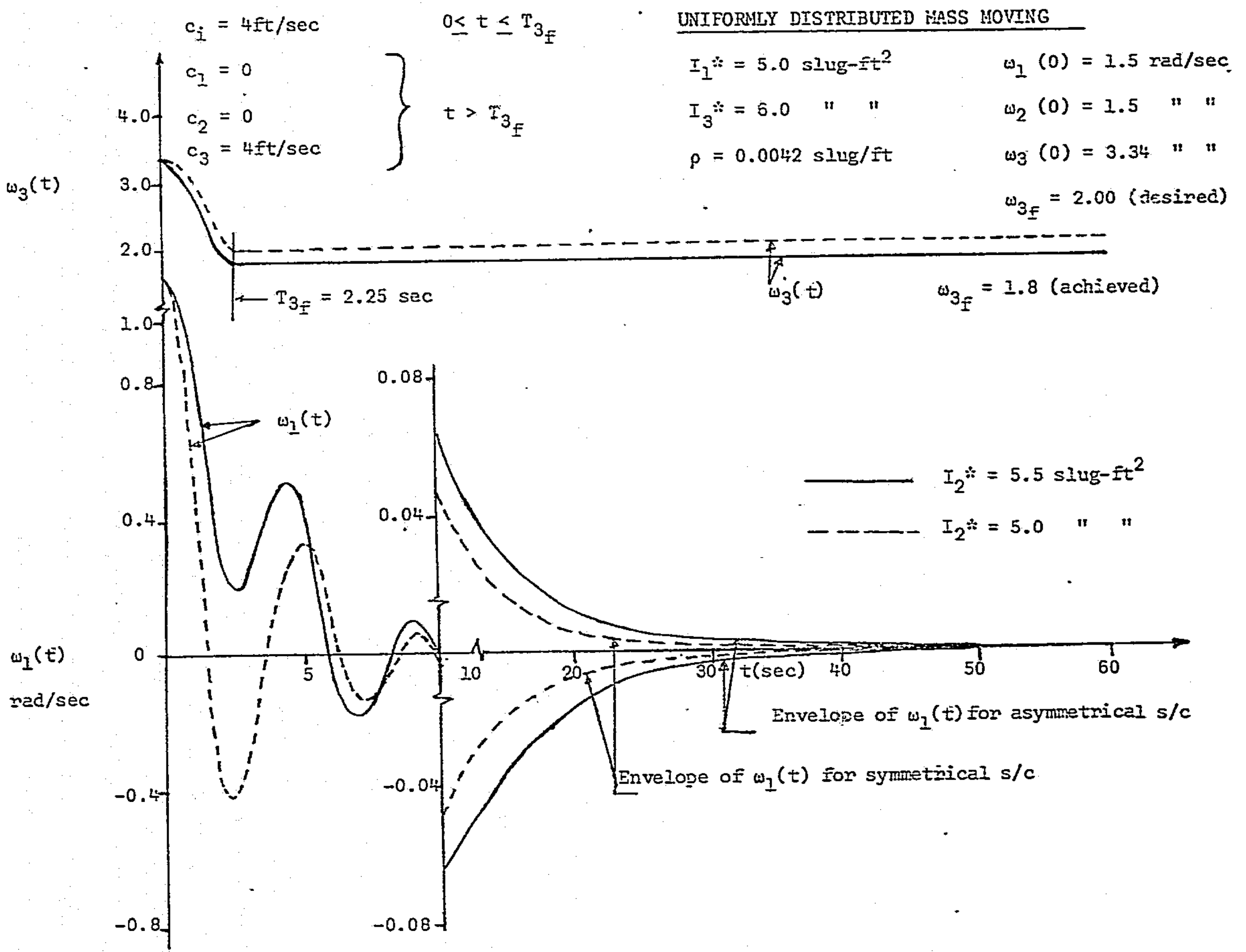


FIG. 8(a). COMPARISON OF RECOVERY MANEUVER OF ASYMMETRICAL SPACECRAFT WITH SYMMETRICAL SPACECRAFT TO ACHIEVE FINAL SPIN ALONG '3' AXIS.

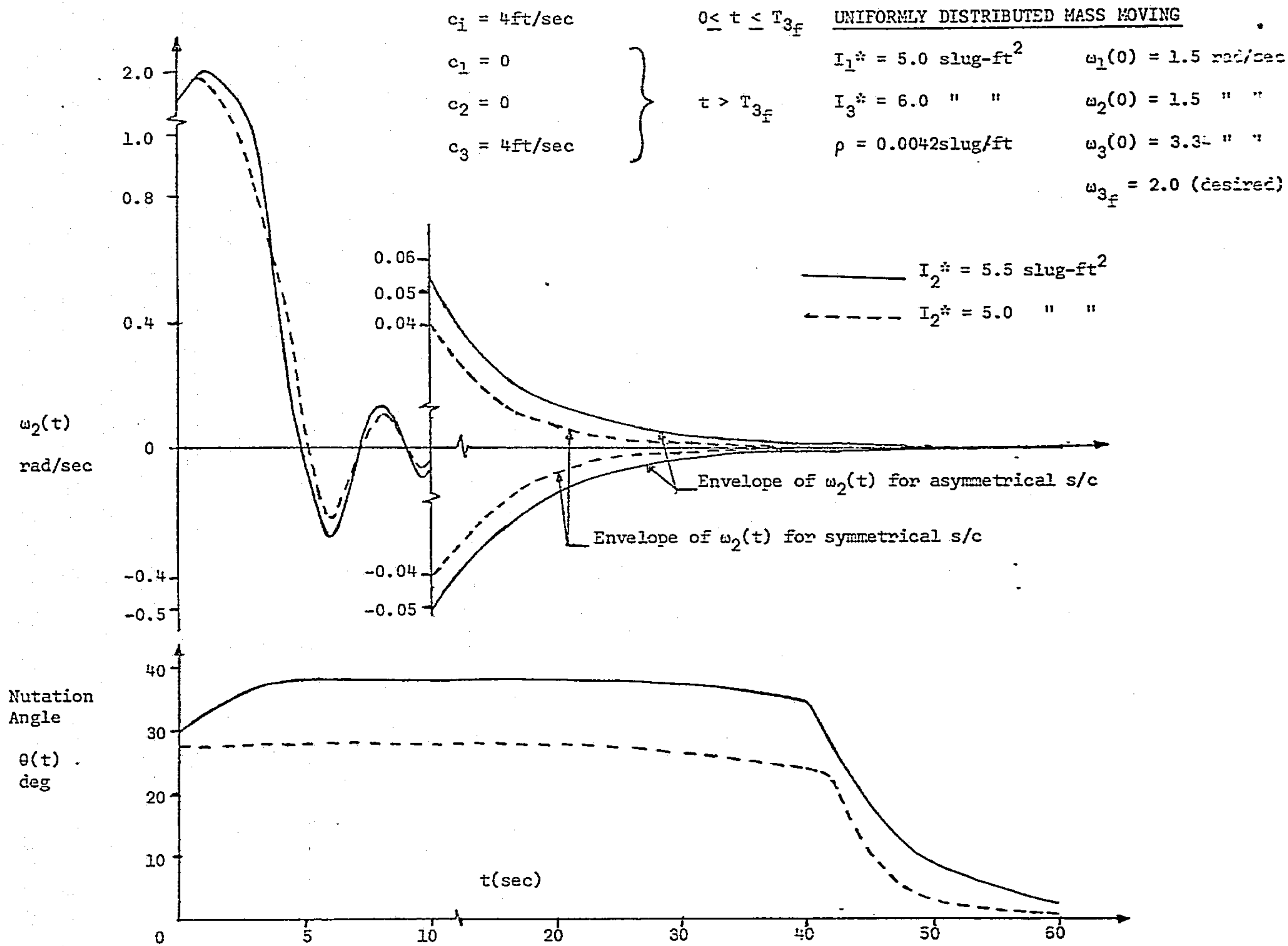


FIG. 8(b). COMPARISON OF RECOVERY MANEUVER OF ASYMMETRICAL SPACECRAFT WITH SYMMETRICAL SPACECRAFT TO ACHIEVE FINAL SPIN ALONG '3' AXIS