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4. COMBINED RADIATION, CONVECTION, AND CONDUCTION FOR A SYSTEM WITH A PARTIALLY TRANSMITTING WALL

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16. Abstract
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COMBINED RADIATION, CONVECTION, AND CONDUCTION FOR A SYSTEM WITH A PARTIALLY TRANSMITTING WALL

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SUMMARY

The net radiation method is developed for systems having both opaque and partially transparent walls. Heat convection is also included at the boundaries as well as heat conduction through the partially transparent windows. For an opaque wall, a local heat balance is formed by directly using the local incoming and outgoing radiative fluxes. At a window boundary however, the transmitted portions of the energy must not be included in the radiative fluxes contributing to the local energy balance. Specific equations are derived for a window between two parallel plates where one plate is at an elevated temperature typical of what would be encountered in an electric furnace, and the other plate is being cooled. A two-band model is used with cutoff wavelengths typical of glass or quartz. There are appreciable fractions of radiant energy in both the transparent and opaque regions of the window. Numerical results are obtained for the window temperature and the heat flow through the window. The effect on these quantities of various plate temperatures and plate emissivities is shown.

INTRODUCTION

Two methods have commonly been used in the literature to compute radiation heat transfer in enclosures: ray tracing and the net radiation method. The ray tracing method uses the procedures of geometric optics to follow the paths through the system of specific quantities of radiation. In the net radiation method a system of simultaneous equations is derived by utilizing heat balances at the enclosure boundaries. The equations are sufficient to determine all the heat fluxes within the system.

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For devices such as solar collectors and solar stills ray tracing is the method that has been used to compute the transmission through the glass cover plates (refs. 1 and 2). The net radiation method has been well developed in the literature for radiation exchange in enclosures with opaque surfaces (refs. 3 and 4). As shown in reference 5 the net radiation method provides a convenient technique for computing the radiative behavior of semitransparent layers. Little has been done in the literature to apply the net radiation method for systems involving both opaque and partially transmitting walls especially for situations where both convection and conduction are also present. The procedure is especially useful in these instances as the radiative fluxes can be readily combined with the fluxes from other heat transfer modes.

When a radiative component must be included that is partially transmitted through a wall, the application of the net radiation method is unfamiliar to most engineers. When conduction and/or convection is also present at an interface, the transmitted portion of the radiation must be subtracted as it does not contribute to the heat flux at that location. An additional complication is present when the temperatures are in the range typical of those in furnaces (1000 to 2000 K). For radiation emitted at these temperatures materials such as glass or quartz are good transmitters for a portion of the radiant energy spectrum, while for the remainder of the spectrum these materials are good absorbers (poor transmitters). The purpose of this report is to show how the net radiation method can be applied for systems having both opaque and partially transmitting walls with simultaneous convection and conduction, and to give some typical results for an application of interest.

In some metallurgical electric furnaces, a glass or quartz envelope is used to contain an inert gas or maintain a vacuum around a heated sample. In a typical geometry the sample would be surrounded by a radiation shield which is enclosed in the inert gas or vacuum envelope. The outside of the envelope is either exposed to the surrounding environment or is enclosed by a water cooled jacket.

Figure 1 shows the typical transmission characteristics of window glass as a function of wavelength. There is a region of high transmission extending across the visible region and portions of the adjacent ultraviolet and infrared regions. In the ultraviolet there is a strong cutoff and at wavelengths shorter than this cutoff the transmission is very low. For the thermal conditions that will be discussed here there is negligible energy in the ultraviolet region so the effect of this cutoff can be neglected. The cutoff of interest here is in the infrared, and for glass is typically at about 2.8 micrometers (increases to about 4.0 μm for quartz). At furnace temperatures, say 1500 K, the emitted energy is centered about the infrared cutoff wavelength as shown in figure 1 so that there are appreciable fractions of the radiant energy in both the transparent and the opaque regions of the window. There is a complicated interaction as some of the radiation is absorbed in the opaque region of the window while other portions are transmitted and reflected between the boundary walls. The absorbed portions contribute to the conduction and convection fluxes, and to reradiation by the window.

A well known application involving heat radiation through windows is the flat plate solar collector. In this instance the heat transfer interaction is not as complicated as for a furnace window. The incident solar radiation is from a very high temperature source and is essentially all at short wavelengths in the transparent region of the glass cover plates. The reemission from the solar absorber is at a much lower temperature (typically 350 K as indicated in fig. 1) and this energy is all in the region where the window is almost opaque. Hence for the solar collector, with regard to the window properties the incident and reemitted energies can be treated separately to a good approximation.

The results of interest for the furnace design are how hot the window will become under the influence of combined radiation, convection, and conduction, to what extent the window acts as a shield in reducing heat losses from the furnace, and how effective an adjacent cooled plate is in reducing the window temperature.

ANALYSIS

Although a metallurgical furnace of the type considered here is typically cylindrical, the concentric radiation shield, glass or quartz envelope, and cooled plate can be approximated as parallel plates (fig. 2). The shield and the cooled plate are assumed gray and each is isothermal at its respective temperature level. The intervening spaces are considered to be gas filled (nonabsorbing gas) to make the analysis more general, and the heat transfer coefficients can differ in each of the spaces. Some results will also be given for the spaces evacuated. Because of the low thermal conductivity of the glass or quartz and because appreciable heat fluxes are likely to be transferred, a significant temperature gradient across the window is expected; thus one-dimensional conduction through the glass or quartz envelope is also included in the analysis.

Radiation Characteristics of a Single Window by Itself

Prior to considering a window in a system with opaque walls the window is considered first by itself. When radiation is incident on a window, there is a multiple reflection process caused by the interaction with the first and second surfaces of the window. A convenient procedure in the system analysis is to separate this interaction by first analyzing the window by itself. To obtain the transmission, absorption, and reflection characteristics of a window for incident radiation by use of the net radiation method, consider a single layer as shown in figure 3. The spectral reflectivity at each of the interfaces is ρ_λ and the spectral transmittance across the interior of the layer is τ_λ. The relations between the outgoing and incoming energy fluxes at each interface can be expressed in terms of the reflection at the interface and the transmission across the interface. Using the net radiation concepts, the outgoing spectral flux is written for each side of each interface to give (see fig. 3),

$$dq_{\lambda o,1} = \rho_\lambda\, dq_{\lambda i,1} + (1 - \rho_\lambda) dq_{\lambda i,2} \tag{1a}$$

$$dq_{\lambda o,2} = (1 - \rho_\lambda) dq_{\lambda i,1} + \rho_\lambda\, dq_{\lambda i,2} \tag{1b}$$

$$dq_{\lambda o,3} = \rho_\lambda\, dq_{\lambda i,3} + (1 - \rho_\lambda) dq_{\lambda i,4} \tag{1c}$$

$$dq_{\lambda o,4} = (1 - \rho_\lambda) dq_{\lambda i,3} + \rho_\lambda\, dq_{\lambda i,4} \tag{1d}$$

The transmittance of the medium is used to relate the internal $dq_{\lambda i}$'s and $dq_{\lambda o}$'s,

$$dq_{\lambda i,2} = \tau_\lambda\, dq_{\lambda o,3} \tag{2a}$$

$$dq_{\lambda i,3} = \tau_\lambda\, dq_{\lambda o,2} \tag{2b}$$

To obtain the overall reflection and transmission characteristics, it is only necessary to consider a unit amount of radiation incident on one side of the window. Hence, let $dq_{\lambda i,1} = 1$ and $dq_{\lambda i,4} = 0$. The $dq_{\lambda i,2}$ and $dq_{\lambda i,3}$ are eliminated by substituting equations (2) into equations (1); the resulting equations are solved for the $dq_{\lambda o}$'s to obtain,

$$dq_{\lambda o,1} = \rho_\lambda \left[1 + \frac{\tau_\lambda^2 (1 - \rho_\lambda)^2}{1 - \rho_\lambda^2 \tau_\lambda^2}\right] \tag{3a}$$

$$dq_{\lambda o,2} = \frac{1 - \rho_\lambda}{1 - \rho_\lambda^2 \tau_\lambda^2} \tag{3b}$$

$$dq_{\lambda o,3} = \frac{\rho_\lambda \tau_\lambda (1 - \rho_\lambda)}{1 - \rho_\lambda^2 \tau_\lambda^2} \tag{3c}$$

$$dq_{\lambda o,4} = \frac{\tau_\lambda(1 - \rho_\lambda)^2}{1 - \rho_\lambda^2\tau_\lambda^2} \tag{3d}$$

For a unit value of $dq_{\lambda i,1}$, the fractions reflected and transmitted by the window are equal to $dq_{\lambda o,1}$ and $dq_{\lambda o,4}$ so that

$$R_\lambda = \rho_\lambda\left[1 + \frac{(1 - \rho_\lambda)^2\tau_\lambda^2}{1 - \rho_\lambda^2\tau_\lambda^2}\right] \tag{4a}$$

$$T_\lambda = \frac{\tau_\lambda(1 - \rho_\lambda)^2}{1 - \rho_\lambda^2\tau_\lambda^2} \tag{4b}$$

The overall spectral absorptance of the glass window is then

$$A_\lambda = 1 - R_\lambda - T_\lambda \tag{4c}$$

Window in System with Opaque Plates

To write the heat balance in the system, the window emittance must be known. Because of the diathermanous nature of the window, the emitted energy leaving one side of the window will to some extent be a mixture of the energy spectrums typical of the local temperatures throughout the window thickness. The concept of emittance for a partially transparent medium is strictly applicable only for a medium at uniform temperature. As an engineering approximation this effect can be neglected here for two reasons, and the emittance applied locally at each window surface. The first reason is that for the thin windows considered here, the temperature variation throughout the window is small relative to that required to produce a significant spectral shift of the emitted radiation (for a blackbody the wavelength at peak emission varies as $1/T$). The second reason arises from the fact discussed earlier that window materials such as glass or quartz have the characteristic of being almost completely transparent in a wavelength range, and are highly absorbing (almost opaque) at other wavelengths. Thus in the transparent region the approximation of defining a window emittance is not significant as relatively little energy is emitted in this wavelength range. In the highly absorbing region there is good emitting ability throughout the window thickness, but the approximation is again valid as little of this energy can penetrate within the window and the emission is chiefly from a region near each window surface. Thus at each surface a window spectral emittance can be utilized as obtained from Kirchhoff's law,

$$\epsilon_{\lambda,w} = A_\lambda = 1 - R_\lambda - T_\lambda \tag{5}$$

Using the relations in equations (4) and (5) for single window characteristics, the net radiation equations can now be written for the enclosure system. First consider the opaque surfaces in figure 2. From the net radiation equations in spectral form, the outgoing radiant energy is written in terms of the emitted and reflected portions as

$$dq_{\lambda o,1} = \epsilon_1 e_{\lambda b,1}\, d\lambda + (1 - \epsilon_1) dq_{\lambda i,1} \tag{6a}$$

$$dq_{\lambda o,4} = \epsilon_4 e_{\lambda b,4}\, d\lambda + (1 - \epsilon_4) dq_{\lambda i,4} \tag{6b}$$

The heat balance at each of the opaque surfaces involves the heat transfer to the adjacent gas. This is a total energy quantity, that is, it involves energy at all wavelengths, so the radiative fluxes are integrated over all λ to form a balance with the convective energy. This yields

$$q = \int_{\lambda=0}^{\infty} (dq_{\lambda o,1} - dq_{\lambda i,1}) + q_g$$

$$= \int_{\lambda=0}^{\infty} (dq_{\lambda o,1} - dq_{\lambda i,1}) + h_g(T_1 - T_g) \tag{7a}$$

$$q = -\int_{\lambda=0}^{\infty} (dq_{\lambda o,4} - dq_{\lambda i,4}) + q_a$$

$$= -\int_{\lambda=0}^{\infty} (dq_{\lambda o,4} - dq_{\lambda i,4}) + h_a(T_a - T_4) \tag{7b}$$

The q is the total heat transfer across the system; it is the external energy supplied at surface 1 and removed from surface 4.

The net radiation equations are now considered for each surface of the window. By use of the overall coefficients for the window, the equations for the outgoing fluxes, including the reflected and transmitted components, are

$$dq_{\lambda o,2} = \epsilon_{\lambda,w} e_{\lambda b,2}\, d\lambda + R_\lambda\, dq_{\lambda i,2} + T_\lambda\, dq_{\lambda i,3} \tag{8a}$$

$$dq_{\lambda o,3} = \epsilon_{\lambda,w} e_{\lambda b,3}\, d\lambda + R_\lambda\, dq_{\lambda i,3} + T_\lambda\, dq_{\lambda i,2} \tag{8b}$$

Consider a heat balance including the radiative fluxes, the convection and the conduction at each surface of the window. To obtain the interaction with conduction and convection, it is necessary to consider total heat balances, that is, balances that include energy at all wavelengths. For this energy balance care must be taken in realizing that the radiation components transmitted through the window will not contribute to the local energy balance. Hence the conservation of energy at each of the two window surfaces results in

$$\int_{\lambda=0}^{\infty} \left[(1 - T_\lambda) dq_{\lambda i,2} - (dq_{\lambda o,2} - T_\lambda\, dq_{\lambda i,3})\right] = \frac{k_w}{w}(T_2 - T_3) - h_g(T_g - T_2) \tag{9a}$$

$$\int_{\lambda=0}^{\infty} \left[(1 - T_\lambda) dq_{\lambda i,3} - (dq_{\lambda o,3} - T_\lambda\, dq_{\lambda i,2})\right] = h_a(T_3 - T_a) - \frac{k_w}{w}(T_2 - T_3) \tag{9b}$$

Across the gaps between the window and the opaque walls there are the relations for the radiative terms,

$$dq_{\lambda i,1} = dq_{\lambda o,2} \tag{10a}$$

$$dq_{\lambda i,2} = dq_{\lambda o,1} \tag{10b}$$

$$dq_{\lambda i,3} = dq_{\lambda o,4} \tag{10c}$$

$$dq_{\lambda i,4} = dq_{\lambda o,3} \tag{10d}$$

The convective heat transfer in the gaps when they are gas filled gives the following relations:

$$q_g = h_g(T_1 - T_g) = h_g(T_g - T_2) \tag{11a}$$

$$q_a = h_a(T_3 - T_a) = h_a(T_a - T_4) \tag{11b}$$

Hence from equations (11)

$$T_g = \frac{T_1 + T_2}{2} \tag{12a}$$

and

$$T_a = \frac{T_3 + T_4}{2} \tag{12b}$$

It is desired to obtain relations for the heat flow q, and the temperatures T_2 and T_3 on both sides of the window. To begin the solution, equations (12) are used to eliminate T_g and T_a from equations (7) and (9), and equations (10) are used to eliminate the $dq_{\lambda i}$ from equations (6) to (9). The result is then the following system of equations:

$$dq_{\lambda o, 1} = \epsilon_1 e_{\lambda b, 1}\, d\lambda + (1 - \epsilon_1) dq_{\lambda o, 2} \tag{13a}$$

$$dq_{\lambda o, 4} = \epsilon_4 e_{\lambda b, 4}\, d\lambda + (1 - \epsilon_4) dq_{\lambda o, 3} \tag{13b}$$

$$q = \int_{\lambda=0}^{\infty} (dq_{\lambda o, 1} - dq_{\lambda o, 2}) + \frac{h_g}{2}(T_1 - T_2) \tag{14a}$$

$$q = -\int_{\lambda=0}^{\infty} (dq_{\lambda o, 4} - dq_{\lambda o, 3}) + \frac{h_a}{2}(T_3 - T_4) \tag{14b}$$

$$dq_{\lambda o, 2} = \epsilon_{\lambda, w} e_{\lambda b, 2}\, d\lambda + R_\lambda\, dq_{\lambda o, 1} + T_\lambda\, dq_{\lambda o, 4} \tag{15a}$$

$$dq_{\lambda o, 3} = \epsilon_{\lambda, w} e_{\lambda b, 3}\, d\lambda + R_\lambda\, dq_{\lambda o, 4} + T_\lambda\, dq_{\lambda o, 1} \tag{15b}$$

$$\int_{\lambda=0}^{\infty} \left[(1 - T_\lambda) dq_{\lambda o, 1} - dq_{\lambda o, 2} + T_\lambda\, dq_{\lambda o, 4}\right] = \frac{k_w}{w}(T_2 - T_3) - \frac{h_g}{2}(T_1 - T_2) \tag{16a}$$

$$\int_{\lambda=0}^{\infty} \left[(1 - T_\lambda) dq_{\lambda o, 4} - dq_{\lambda o, 3} + T_\lambda\, dq_{\lambda o, 1}\right] = -\frac{k_w}{w}(T_2 - T_3) + \frac{h_a}{2}(T_3 - T_4) \tag{16b}$$

Now use equation (13) to eliminate $dq_{\lambda o, 2}$ and $dq_{\lambda o, 3}$ from equations (14), (15), and (16). Also add equations (14a) and (14b) and divide by 2 to obtain a symmetric form. This yields

$$\left[1 - (1 - \epsilon_1) R_\lambda\right] dq_{\lambda o, 1} - (1 - \epsilon_1) T_\lambda\, dq_{\lambda o, 4} = \epsilon_1 e_{\lambda b, 1}\, d\lambda + \epsilon_{\lambda, w}(1 - \epsilon_1) e_{\lambda b, 2}\, d\lambda \tag{17a}$$

$$-(1 - \epsilon_4)T_\lambda \, dq_{\lambda o, 1} + [1 - (1 - \epsilon_4)R_\lambda]dq_{\lambda o, 4} = \epsilon_4 e_{\lambda b, 4} \, d\lambda + \epsilon_{\lambda, w}(1 - \epsilon_4)e_{\lambda b, 3} \, d\lambda \tag{17b}$$

$$q = \frac{1}{2} \int_{\lambda=0}^{\infty} \left[\frac{\epsilon_1}{1 - \epsilon_1} (e_{\lambda b, 1} \, d\lambda - dq_{\lambda o, 1}) - \frac{\epsilon_4}{1 - \epsilon_4} (e_{\lambda b, 4} \, d\lambda - dq_{\lambda o, 4})\right] + \frac{h_g}{4} (T_1 - T_2) + \frac{h_a}{4} (T_3 - T_4) \tag{18}$$

$$\frac{1}{1 - \epsilon_1} \int_{\lambda=0}^{\infty} \{- [T_\lambda (1 - \epsilon_1) + \epsilon_1]dq_{\lambda o, 1} + T_\lambda (1 - \epsilon_1)dq_{\lambda o, 4} + \epsilon_1 e_{\lambda b, 1} \, d\lambda\} = \frac{k_w}{w} (T_2 - T_3) - \frac{h_g}{2} (T_1 - T_2) \tag{19a}$$

$$\frac{1}{1 - \epsilon_4} \int_{\lambda=0}^{\infty} \{T_\lambda (1 - \epsilon_4)dq_{\lambda o, 1} - [T_\lambda (1 - \epsilon_4) + \epsilon_4]dq_{\lambda o, 4} + \epsilon_4 e_{\lambda b, 4} \, d\lambda\} = - \frac{k_w}{w} (T_2 - T_3) + \frac{h_a}{2} (T_3 - T_4) \tag{19b}$$

Equations (17a) and (17b) are solved simultaneously for $dq_{\lambda o, 1}$ and $dq_{\lambda o, 4}$ to yield

$$dq_{\lambda o, 1} = \frac{e_{\lambda b, 1} \, d\lambda \, \epsilon_1 [1 - R_\lambda (1 - \epsilon_4)] + e_{\lambda b, 4} \, d\lambda \, \epsilon_4 (1 - \epsilon_1) T_\lambda + \epsilon_{\lambda, w} (1 - \epsilon_1) \{e_{\lambda b, 2} \, d\lambda [1 - R_\lambda (1 - \epsilon_4)] + e_{\lambda b, 3} \, d\lambda \, T_\lambda (1 - \epsilon_4)\}}{[1 - R_\lambda (1 - \epsilon_1)][1 - R_\lambda (1 - \epsilon_4)] - (1 - \epsilon_1)(1 - \epsilon_4)T_\lambda^2} \tag{20a}$$

$$dq_{\lambda o, 4} = \frac{e_{\lambda b, 1} \, d\lambda \, \epsilon_1 (1 - \epsilon_4) T_\lambda + e_{\lambda b, 4} \, d\lambda \, \epsilon_4 [1 - R_\lambda (1 - \epsilon_1)] + \epsilon_{\lambda, w} (1 - \epsilon_4) \{e_{\lambda b, 3} \, d\lambda [1 - R_\lambda (1 - \epsilon_1)] + e_{\lambda b, 2} \, d\lambda \, T_\lambda (1 - \epsilon_1)\}}{[1 - R_\lambda (1 - \epsilon_1)][1 - R_\lambda (1 - \epsilon_4)] - (1 - \epsilon_1)(1 - \epsilon_4)T_\lambda^2} \tag{20b}$$

These relations are substituted into equations (18), (19a), and (19b) to give

$$q = \int_{\lambda=0}^{\infty} \frac{(A_{\lambda,41}e_{\lambda b,1} - A_{\lambda,14}e_{\lambda b,4}) + (B_{\lambda,14}e_{\lambda b,2} - B_{\lambda,41}e_{\lambda b,3})}{2D_{\lambda,14}} d\lambda$$

$$+ \frac{h_g}{4}(T_1 - T_2) + \frac{h_a}{4}(T_3 - T_4) \tag{21}$$

where

$$A_{\lambda,jk} = [1 - (R_\lambda + T_\lambda)(1 - \epsilon_j)]\epsilon_k(1 - R_\lambda + T_\lambda)$$

$$B_{\lambda,jk} = [\epsilon_k T_\lambda(1 - \epsilon_j) - \epsilon_j(1 - R_\lambda + \epsilon_k R_\lambda)]\epsilon_{\lambda,w}$$

$$D_{\lambda,jk} = [1 - R_\lambda(1 - \epsilon_j)][1 - R_\lambda(1 - \epsilon_k)] - T_\lambda^2(1 - \epsilon_j)(1 - \epsilon_k)$$

$$\int_{\lambda=0}^{\infty} \frac{1}{D_{\lambda,14}}[E_{\lambda,14}e_{\lambda b,1} + F_{\lambda,14}e_{\lambda b,2} + G_{\lambda,14}e_{\lambda b,3} + H_{\lambda,14}e_{\lambda b,4}]d\lambda$$

$$= \frac{k_w}{w}(T_2 - T_3) - \frac{h_g}{2}(T_1 - T_2) \tag{22a}$$

$$\int_{\lambda=0}^{\infty} \frac{1}{D_{\lambda,14}}[H_{\lambda,41}e_{\lambda b,1} + G_{\lambda,41}e_{\lambda b,2} + F_{\lambda,41}e_{\lambda b,3} + E_{\lambda,41}e_{\lambda b,4}]d\lambda$$

$$= -\frac{k_w}{w}(T_2 - T_3) + \frac{h_a}{2}(T_3 - T_4) \tag{22b}$$

where

$$E_{\lambda,jk} = \epsilon_{\lambda,w}\epsilon_j[1 - R_\lambda(1 - \epsilon_k)]$$

$$F_{\lambda,jk} = \epsilon_{\lambda,w}\left\{T_\lambda^2(1 - \epsilon_j)(1 - \epsilon_k) - [T_\lambda(1 - \epsilon_j) + \epsilon_j][1 - R_\lambda(1 - \epsilon_k)]\right\}$$

$$G_{\lambda,jk} = \epsilon_{\lambda,w}^2(1 - \epsilon_j)(1 - \epsilon_k)T_\lambda$$

$$H_{\lambda,jk} = \epsilon_{\lambda,w}(1 - \epsilon_j)\epsilon_k T_\lambda$$

The temperature of each surface of the window, T_2 and T_3 is found from the simultaneous solution of equations (22a) and (22b), and q can then be evaluated from equation (21). Some typical results will be obtained by means of a two-band calculation.

Relations for Two-Band Calculation

As shown in figure 1, glass or quartz has a well-defined cutoff wavelength λ_c in the infrared region at which the transmittance as a function of wavelength changes from a high value to a low value. In view of the shape of the typical transmittance curve in figure 1, a reasonable approximation that will be used for the present problem is that the transmittance is a constant on either side of λ_c. As mentioned before, for glass there is also a region of very low transmission in the ultraviolet region, but there is negligible energy in this range in the radiation spectrum of the present problem which is concerned with furnace temperatures. For a surface at temperature T_n let the fraction of blackbody radiation in the short wavelength range $0 \leq \lambda \leq \lambda_c$ be designed by $F_{\lambda_c T_n}$, that is

$$\int_{\lambda=0}^{\lambda_c} e_{\lambda b,n}\, d\lambda = F_{\lambda_c T_n} \sigma T_n^4$$

The $F_{\lambda_c T_n}$ is a function only of $\lambda_c T_n$ and is given to a very good approximation by (ref. 6)

$$F_{\lambda_c T_n} = \frac{15}{\pi^4} \sum_{m=1,2,\ldots} \frac{e^{-mv}}{m^4} \left\{\left[(mv + 3)mv + 6\right]mv + 6\right\} \qquad v \geq 2$$

$$F_{\lambda_c T_n} = 1 - \frac{15}{\pi^4} v^3 \left(\frac{1}{3} - \frac{v}{8} + \frac{v^2}{60} - \frac{v^4}{5040} + \frac{v^6}{272\,160} - \frac{v^8}{13\,305\,600}\right) \qquad v < 2$$

where $v = 1.4388\times10^4/\lambda_c T_n$ and $\lambda_c T_n$ is in $(\mu m)(K)$. Then equations (22a) and (22b) can be written as

$$\frac{\sigma}{D^{s}_{\lambda,14}D^{l}_{\lambda,14}}\left\{\left[E^{s}_{\lambda,14}D^{l}_{\lambda,14}F_{\lambda_c T_1} + E^{l}_{\lambda,14}D^{s}_{\lambda,14}\left(1 - F_{\lambda_c T_1}\right)\right]T_1^4\right.$$

$$+ \left[F^{s}_{\lambda,14}D^{l}_{\lambda,14}F_{\lambda_c T_2} + F^{l}_{\lambda,14}D^{s}_{\lambda,14}\left(1 - F_{\lambda_c T_2}\right)\right]T_2^4$$

$$+ \left[G^{s}_{\lambda,14}D^{l}_{\lambda,14}F_{\lambda_c T_3} + G^{l}_{\lambda,14}D^{s}_{\lambda,14}\left(1 - F_{\lambda_c T_3}\right)\right]T_3^4$$

$$\left. + \left[H^{s}_{\lambda,14}D^{l}_{\lambda,14}F_{\lambda_c T_4} + H^{l}_{\lambda,14}D^{s}_{\lambda,14}\left(1 - F_{\lambda_c T_4}\right)\right]T_4^4\right\}$$

$$= \frac{k_w}{w}(T_2 - T_3) - \frac{h_g}{2}(T_1 - T_2) \tag{23a}$$

$$\frac{\sigma}{D^{s}_{\lambda,14}D^{l}_{\lambda,14}}\left\{\left[H^{s}_{\lambda,41}D^{l}_{\lambda,14}F_{\lambda_c T_1} + H^{l}_{\lambda,41}D^{s}_{\lambda,14}\left(1 - F_{\lambda_c T_1}\right)\right]T_1^4\right.$$

$$+ \left[G^{s}_{\lambda,41}D^{l}_{\lambda,14}F_{\lambda_c T_2} + G^{l}_{\lambda,41}D^{s}_{\lambda,14}\left(1 - F_{\lambda_c T_2}\right)\right]T_2^4$$

$$+ \left[F^{s}_{\lambda,41}D^{l}_{\lambda,14}F_{\lambda_c T_3} + F^{l}_{\lambda,41}D^{s}_{\lambda,14}\left(1 - F_{\lambda_c T_3}\right)\right]T_3^4$$

$$\left. + \left[E^{s}_{\lambda,41}D^{l}_{\lambda,14}F_{\lambda_c T_4} + E^{l}_{\lambda,41}D^{s}_{\lambda,14}\left(1 - F_{\lambda_c T_4}\right)\right]T_4^4\right\}$$

$$= -\frac{k_w}{w}(T_2 - T_3) + \frac{h_a}{2}(T_3 - T_4) \tag{23b}$$

It is necessary to solve equations (23a) and (23b) by simultaneous iteration because T_2 is in the function $F_{\lambda_c T_2}$ and T_3 is in $F_{\lambda_c T_3}$. Once T_2 and T_3 are evaluated, then q can be determined from equation (21) as

$$q = \frac{\sigma}{2}\left\{\left[\frac{A^s_{\lambda,41}}{D^s_{\lambda,14}} F_{\lambda_c T_1} + \frac{A^l_{\lambda,41}}{D^l_{\lambda,14}}\left(1 - F_{\lambda_c T_1}\right)\right]T_1^4\right.$$

$$- \left[\frac{A^s_{\lambda,14}}{D^s_{\lambda,14}} F_{\lambda_c T_4} + \frac{A^l_{\lambda,14}}{D^l_{\lambda,14}}\left(1 - F_{\lambda_c T_4}\right)\right]T_4^4 + \left[\frac{B^s_{\lambda,14}}{D^s_{\lambda,14}} F_{\lambda_c T_2} + \frac{B^l_{\lambda,14}}{D^l_{\lambda,14}}\left(1 - F_{\lambda_c T_2}\right)\right]T_2^4$$

$$\left.- \left[\frac{B^s_{\lambda,41}}{D^s_{\lambda,14}} F_{\lambda_c T_3} + \frac{B^l_{\lambda,41}}{D^l_{\lambda,14}}\left(1 - F_{\lambda_c T_3}\right)\right]T_3^4\right\} + \frac{h_g}{4}(T_1 - T_2) + \frac{h_a}{4}(T_3 - T_4) \qquad (24)$$

Some results for specified conditions will be evaluated a little later.

Because of the complexity of equations (23) and (24) it is worthwhile to examine a simplified case. As was previously discussed, for wavelengths smaller than the cutoff wavelength, glass or quartz is typically quite transparent (neglecting the ultraviolet region), while for wavelengths longer than the cutoff, the window becomes almost opaque. For the simplified case it is assumed that the window is perfectly transparent for $0 \leq \lambda \leq \lambda_c$ and perfectly opaque for the remainder of the wavelength region. For the simplified case the surface reflectivities of the window are set equal to zero. Based on these assumptions, equations (23) and (24) reduce to the following:

$$\sigma\epsilon_1\left[\left(1 - F_{\lambda_c T_1}\right)T_1^4 - \left(1 - F_{\lambda_c T_2}\right)T_2^4\right] = \frac{k_w}{w}(T_2 - T_3) - \frac{h_g}{2}(T_1 - T_2) \qquad (25a)$$

$$\sigma\epsilon_4\left[\left(1 - F_{\lambda_c T_4}\right)T_4^4 - \left(1 - F_{\lambda_c T_3}\right)T_3^4\right] = \frac{h_a}{2}(T_3 - T_4) - \frac{k_w}{w}(T_2 - T_3) \qquad (25b)$$

$$q_{approximate} = \frac{\sigma}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_4} - 1}\left(F_{\lambda_c T_1} T_1^4 - F_{\lambda_c T_4} T_4^4\right) + \frac{k_w}{w}(T_2 - T_3) \qquad (26)$$

where equations (25a) and (25b) were used to reduce the last term on the right side of equation (26) to the form given.

Although equations (25a) and (25b) must be solved by simultaneous iteration for the unknown temperatures T_2 and T_3, the solution is much simpler than dealing with equations (23a) and (23b). The first term in the approximate heat flux equation (26) is the radiant exchange that occurs as a result of the portion of emitted energy from each opaque wall that is in the wavelength region for which the window is completely transparent ($\lambda < \lambda_c$). The second term accounts for all the other energy that is transferred; this is by radiation in the region where the window is opaque and by convection. This energy must pass through the window by conduction thus giving the form shown for the second term. The values of the temperatures T_2 and T_3 in this term are obtained from equations (25a) and (25b) which involve the radiation in the opaque region ($\lambda > \lambda_c$) and the convection.

Although equations (25) and (26) can be placed in dimensionless form, there are so many independent groups even for these simplified equations that the dimensionless form is not especially advantageous unless very extensive sets of results are to be presented. To aid in the physical interpretation, the typical results presented here will be given in dimensional form.

Another configuration of interest in addition to figure 2 is where the outside of the glass or quartz window is exposed directly to the room, that is, there is no outer cooling jacket. This is shown in figure 4. The present solution can be directly applied since the surroundings can be represented as a blackbody at the room temperature. Hence for this case let $\epsilon_4 = 1$ and T_4 be equal to the room temperature. The h_a is the convective coefficient for the outside of the window exposed to the room environment. If there is sufficient vacuum in the gap to eliminate convection and conduction between the hot surface and the window, let $h_g = 0$.

RESULTS AND DISCUSSION

As indicated by the limiting equations (25) and (26), there are several parameters in the present problem; for the actual situation in figure 2 there are even more parameters than in the limiting case. The parameters involve the wall emissivities, absorption coefficient of the glass, heat transfer coefficients in the gaps, thermal conductivity of the window, the cutoff wavelength, and so forth. It is not feasible to give results for all the ranges of variables that can be encountered. The main purpose of the present report is to demonstrate how the net radiation method can be applied to this type of system. Hence only some typical numerical results will be given; the results will also be compared with those from the approximate solution.

All the results that follow are for a fixed geometry. The gap spacings a and g are both 1.0 centimeters (0.394 in.) and the glass thickness w is 0.635 centimeter (0.25 in.). The thermal conductivity of the window k_w, and the heat transfer coefficients h_a and h_g in the gaps were kept constant throughout the calculations except for a few cases where the gaps were evacuated so that $h_a = h_g = 0$. In the range of temperatures expected in furnaces, silica glass had a thermal conductivity approximately equal to 0.05 W/(cm)(K) as given in reference 7.

To obtain h_a and h_g, the Rayleigh number between the window and opaque walls was calculated assuming nitrogen filled gaps at atmospheric pressure, a gap width of 1.0 centimeter, and temperature levels (typically 1000 K) and differences (a few hundred degrees K) typical of furnace operation. The Rayleigh number based on the gap width was generally 10^3 or less, and for only a few cases became as high as 4×10^3. The Rayleigh number for the onset of free convection in a vertical enclosed gap is about 10^3 (ref. 8), and for a Rayleigh number of 4×10^3 the conduction heat transfer is only enhanced about 25 percent by the convective effect. Thus for the present conditions the heat transfer is essentially all by heat conduction. The heat transfer coefficient used here is based on the temperature difference from a surface to the average gas temperature. For only heat conduction, with a constant thermal conductivity, the average temperature would be at the midpoint of the gap. Hence the convective heat transfer coefficient is replaced in the pure conduction case by $h_g = k/(g/2)$ and $h_a = k/(a/2)$ (the factor of 1/2 arises from the h in eq. (11) being based on the temperature difference relative to the average gas temperature in the gap). Based on typical average nitrogen temperatures of 1400 and 900 K in the gaps, the values were obtained as $h_g = 16.9$ W/(m^2)(K) and $h_a = 12.6$ W/(m^2)(K). To simplify the interpretation of the results by not changing too many variables, these values were retained throughout the calculations except when the gaps were evacuated so that $h_g = h_a = 0$.

To evaluate R_λ, T_λ, and $\epsilon_{\lambda,w}$ from equation (4), the ρ_λ and τ_λ are needed. The ρ_λ was found from Snell's law using an index of refraction $n = 1.5$ and an incidence angle of 58° which was found in reference 9 to give good results for diffuse incident radiation. This gave $\rho_\lambda = 0.080$ for all λ. The $\tau_\lambda = e^{-a_\lambda L}$ where L is the path length in the window for incidence at 58°. The window thickness w was taken as 0.635 centimeter (0.25 in.). From reference 10 using $a_\lambda = 0.2$ centimeter^{-1} gave $\tau_\lambda = 0.857$ for $\lambda < \lambda_c$, and using $a_\lambda = 5.7$ centimeter^{-1} gave $\tau_\lambda = 0.0125$ for $\lambda > \lambda_c$ where $\lambda_c = 2.8$ micrometers which is typical for many types of glass (quartz has a higher cutoff wavelength ≈ 4 μm). These values give results for ϵ_w comparable to those in reference 11. These conditions will be held constant, and they also apply quite well for quartz as well as for glass with the exception of the cutoff wavelength which is 2.8 micrometers for glass and 4.0 micrometers for quartz.

The principal quantities that will be varied in the figures that follow are the temperatures and emissivities of the opaque walls. Some typical numerical results from equations (23) and (24) will be given, and the results will also be compared with those from the approximate solution in equations (25) and (26).

In figure 5 the temperature T_1 of the hot wall is 1200 K and the cooled wall is at $T_4 = 320$ K which is typical of cooling by nonpressurized water. The cutoff wavelength is representative of that for glass, $\lambda_c = 2.8$ micrometers. Figures 5(a) and (b) give the temperatures on the hot and cool sides of the window, while figure 5(c) gives the heat transferred from the hot to the cold wall by the combined modes of radiation, convection, and conduction. The results are plotted as a function of the emissivity of the hot wall with the different curves corresponding to various emissivities of the cooled wall. Two sets of curves are shown: the solid lines correspond to the complete solution (eqs. (23) and (24)), while the dashed lines are for the simplified approximate solution (eqs. (25) and (26)).

The emissivities of the hot and cool walls have opposing effects on the window temperatures since increasing ϵ_1 will increase the heat transfer to the window while increasing ϵ_4 will increase the heat transfer away from the window and to the cooled wall. Thus the lowest window temperatures will be obtained when ϵ_1 is low and ϵ_4 is high. The largest heat losses correspond to both ϵ_1 and ϵ_4 being large. The losses are substantially diminished by reducing at least one of the wall emissivities. To have both a low window temperature and a small heat loss it is the ϵ_1 that should be kept small. It may be possible to do this only to some extent if the hot surface is a metal because the emissivity increases at the high temperatures characteristic of furnace operation.

The temperatures in figure 5 are predicted quite well from the approximate solution, the values being within about 3 percent of the exact equations. The heat flux in figure 5(c) is also in reasonable agreement with the exact solution; the largest deviation is when both ϵ_1 and ϵ_4 are large and then the difference increases to about 15 percent. The approximate solution is probably adequate for most design purposes.

In figure 6 the effect is shown of letting the temperature of the cooler wall be at a higher temperature than in figure 5. This reduces the heat loss as shown by figure 6(c) but results in an increased window temperature as shown by figure 6(a). Figure 6(b) gives the temperatures on the two sides of the window; the temperature difference across the window ranges from about 10 to 25 K. This difference can be used in equation (26) to obtain an approximate idea of the amount of heat being transferred by means other than by direct transmission in the transparent region of the window.

The purpose of figure 7 is to illustrate the effect of increasing the temperature of the hot surface; the conditions are the same as for figure 5 except that the hot wall temperature is increased from 1200 to 2000 K. Figure 7(a) shows the resulting window temperatures which of course are higher than those in figure 5. The heat flux being transferred is shown in figure 7(b), and comparisons are made in figures 7(b) and (c) with the approximate solution in equations (25) and (26). At these higher temperatures the approximate solution has significant deviations from the exact solution for many conditions and is somewhat inadequate for window temperature predictions. Some of the window temperatures shown are above the melting point for glass. The results provide a guide as to what conditions are necessary to avoid melting of the window.

Figure 8 shows the effect of taking the situation in figure 5 and removing the gap conduction and/or convection heat transfer. Conduction and/or convection provides a means of heat transfer parallel to the radiative heat transfer and hence as shown in figure 8(b) the heat transfer is reduced when the gaps are evacuated. The window temperature is generally increased when $h_a = h_g = 0$ but there are some conditions where the window temperature is decreased. To explain this behavior it is realized that heat is transferred both to and from the window by conduction and/or convection. When these two portions are equal, the average window temperature $T_{w,avg}$ (neglecting the fact that the two surfaces of the window are somewhat different in temperature) would be given by

$$h_g(T_1 - T_{w,avg}) = h_a(T_{w,avg} - T_4)$$

Inserting the conditions of figure 8(a) yields $T_{w,avg} = 824$ K. Thus if the window temperature as shown by the solid line is greater than about 824 K, the window is losing more heat by convection than it is gaining; hence, in this instance removing the convection (dashed lines) causes the window temperature to be increased. If, on the other hand, the window as shown by the solid line is below about 824 K, the convection is providing a net heat gain by the window; letting the convection be zero then causes a reduction in window temperature. This accounts for the crossing of the dashed and solid curves in figure 8(a). When T_1 was raised to 2000 K, the radiative terms increase substantially in comparison with the convection terms. The calculations showed that, at this temperature level, the convection has a minor influence and the dashed and solid curves are practically the same in this instance.

The cutoff wavelength can vary with the type of optical material being used. Quartz has a cutoff of about 4 micrometers which is somewhat larger than that for glass which is at about 2.8 micrometers. Figure 9 shows the effect of increasing the cutoff wavelength from 2.8 to 4.0 micrometers on the window hot side temperature and on the heat flux being transferred. Increasing the cutoff causes a greater fraction of the energy to be in the transparent region of the window. For a blackbody surface emitting at 1200 K, the fraction of the energy in the transparent region of the glass is $F_{\lambda_c T} = F_{(2.8)(1200)} = 0.35$ while for quartz this increases to $F_{(4.0)(1200)} = 0.61$. This shift reduces energy absorption for the quartz window and reduces the window temperature. The increased region of transparency also reduces the effectiveness of the window as a radiation shield, and the heat being transferred is therefore increased.

CONCLUSIONS

An analysis has been developed to show how the net radiation method can be applied for systems involving both opaque and partially transparent walls. The analysis includes conduction and/or convection heat transfer in the gas adjacent to the walls, and heat conduction through partially transparent windows. For opaque walls the net radiative fluxes are used directly in forming local heat balances. For a window, however, care must be taken in the heat balance at a window boundary to include only the portions of the radiative terms that interact locally with the convection and conduction. The transmitted portion of the radiation is subtracted from the net radiative fluxes that would be used if the wall were opaque.

Specific results are obtained for a window between two opaque parallel plates, one plate being at much higher temperature than the other. The high temperature was in a range typical of electric furnaces. At this level the radiative energy is fairly equally divided between the wavelength regions below and above the cutoff wavelength of the window. Thus a portion of the radiation passes quite readily through the window while the remaining radiation is in a region where the window is practically opaque.

The effect on the window temperature and the amount of heat transferred is demonstrated for various wall temperatures and plate emissivities, with and without gas in the gaps on both sides of the window, and for the cutoff wavelengths typical of glass and quartz. To reduce both window temperature and heat loss, it is most desirable to reduce the emissivity of the hot surface. The quartz has a higher cutoff wavelength than glass which means that relative to glass there is greater transmitted and less absorbed energy. Thus for all other characteristics being the same, when quartz is used, the heat loss will be increased and the window temperature reduced.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, February 21, 1975,
505-04.

APPENDIX - SYMBOLS

$A_\lambda, R_\lambda, T_\lambda$	overall spectral absorptance, reflectance, and transmittance of window
$A_{\lambda,jk}, B_{\lambda,jk}, \ldots, H_{\lambda,jk}$	spectral coefficients in derived equations
a_λ	spectral absorption coefficient
dq_λ	differential heat flux in wavelength interval $d\lambda$
$e_{\lambda b}$	blackbody spectral emissive power
$F_{\lambda T}$	fraction of blackbody radiation in range $0 - \lambda T$
h	heat transfer coefficient
k	thermal conductivity
L	path length within window
n	index of refraction
q	total heat flux being transferred
q_a, q_g	convection (or conduction) in gas filled gaps
T	absolute temperature
w	thickness of window
ϵ	emissivity
$\epsilon_\lambda, \rho_\lambda, \tau_\lambda$	spectral emissivity, reflectivity, and transmittance
λ	wavelength
λ_c	cutoff wavelength
σ	Stefan-Boltzmann constant

Subscripts:

a	gap between window and cool wall
avg	average
g	gap between hot wall and window
i	incoming
o	outgoing
w	window
λ	spectral quantity

1, 2, 3, 4 hot wall, hot side of window, cool side of window, cool wall; or 1st, 2nd, 3rd, and 4th interfaces for single layer

Superscripts:

l, s long and short wavelength radiation

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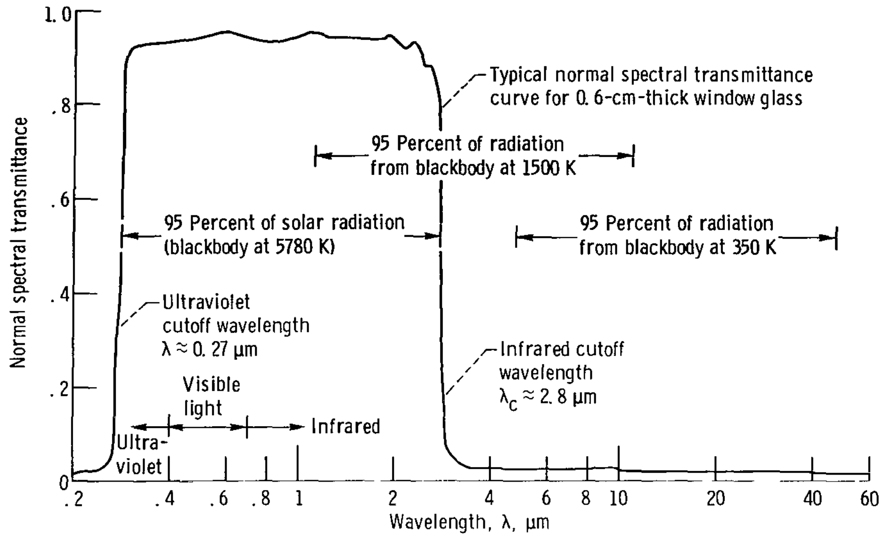


Figure 1. - Typical spectral transmission characteristics of window glass showing regions of high and low transmittance.

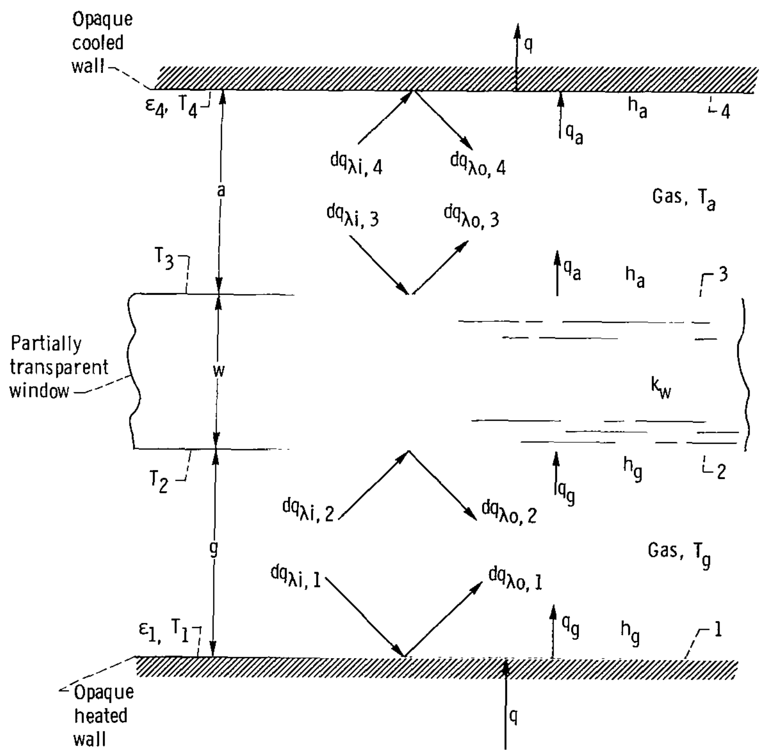


Figure 2. - Partially transparent heat conducting window between opaque plates ($T_1 > T_4$).

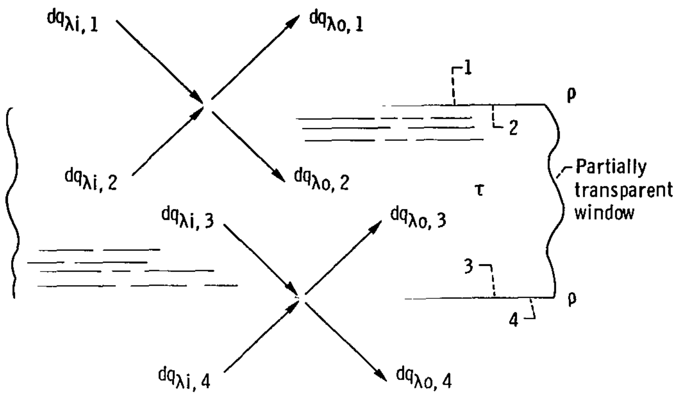


Figure 3. - Net radiation quantities for analyzing overall performance of partially transparent single layer.

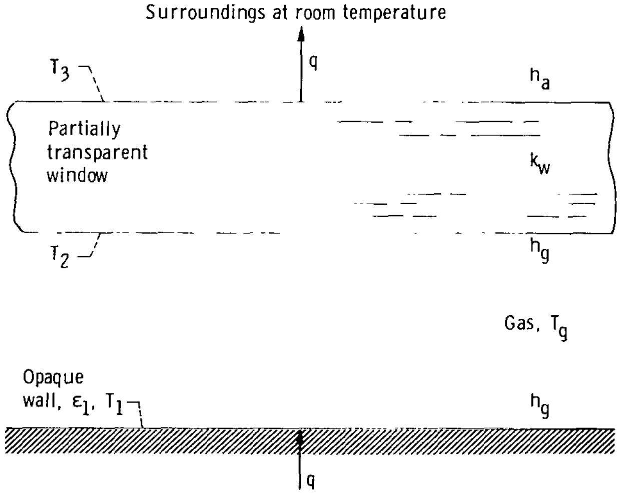
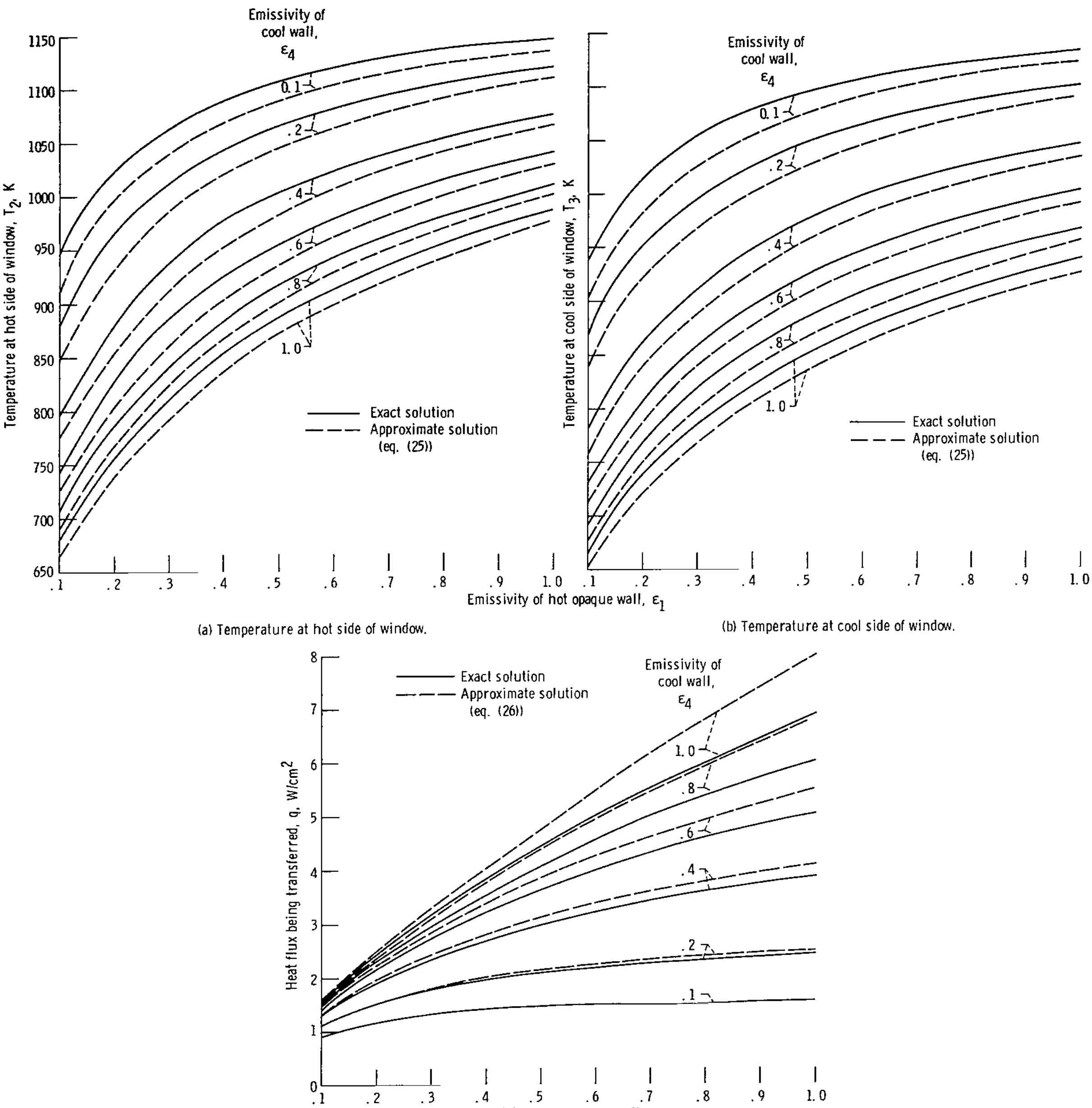


Figure 4. - Partially transparent heat conducting window exposed directly to room on one side.

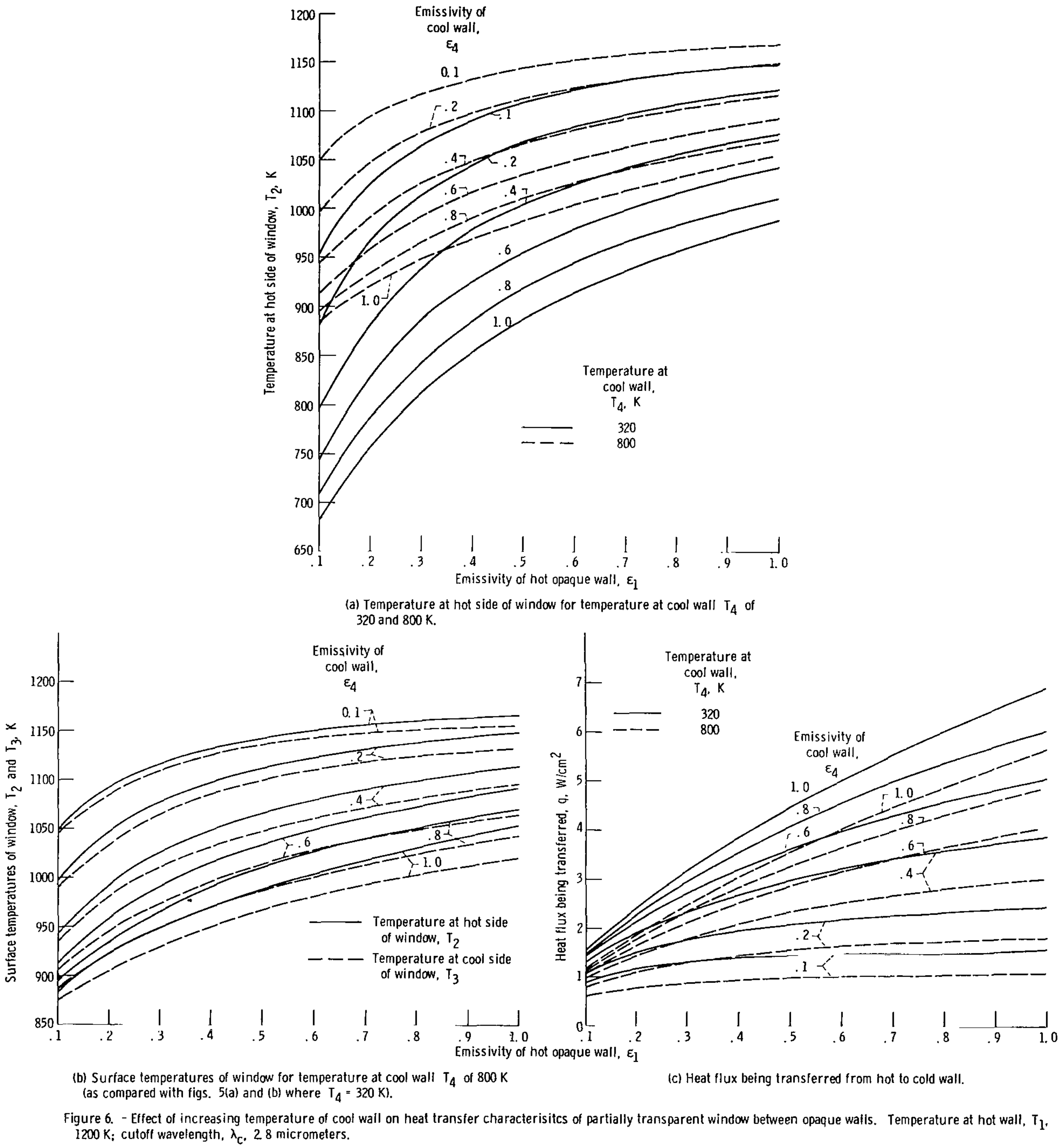


(a) Temperature at hot side of window.

(b) Temperature at cool side of window.

(c) Heat flux being transferred from hot to cold wall.

Figure 5. - Effect of wall emissivities on heat transfer characterisitcs of partially transparent window between opaque walls. Temperature at hot wall, T_1, 1200 K; temperature at cool wall, T_4, 320 K; cutoff wavelength, λ_c, 2.8 micrometers.

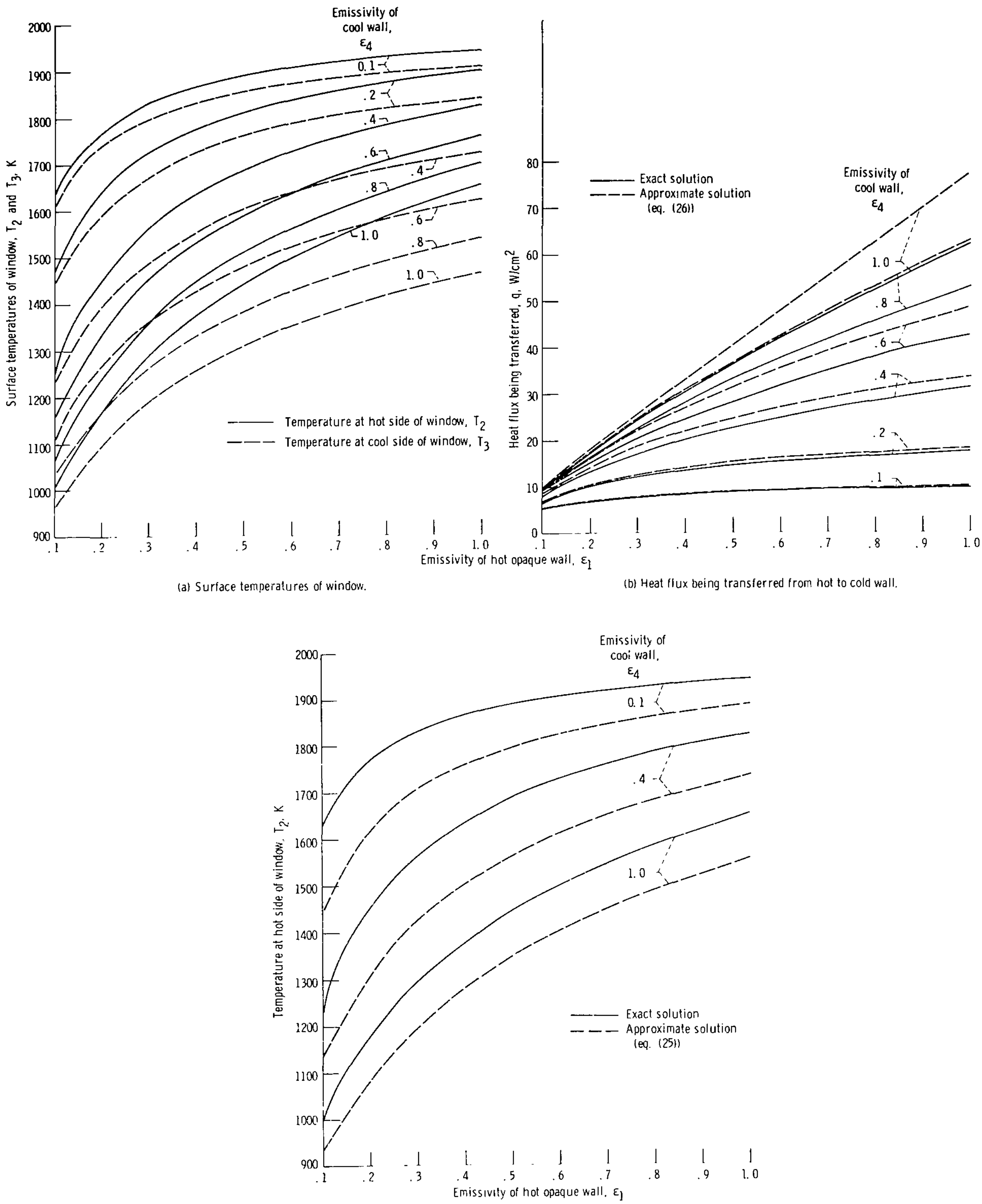


(a) Temperature at hot side of window for temperature at cool wall T_4 of 320 and 800 K.

(b) Surface temperatures of window for temperature at cool wall T_4 of 800 K (as compared with figs. 5(a) and (b) where T_4 = 320 K).

(c) Heat flux being transferred from hot to cold wall.

Figure 6. - Effect of increasing temperature of cool wall on heat transfer characterisitcs of partially transparent window between opaque walls. Temperature at hot wall, T_1, 1200 K; cutoff wavelength, λ_c, 2.8 micrometers.

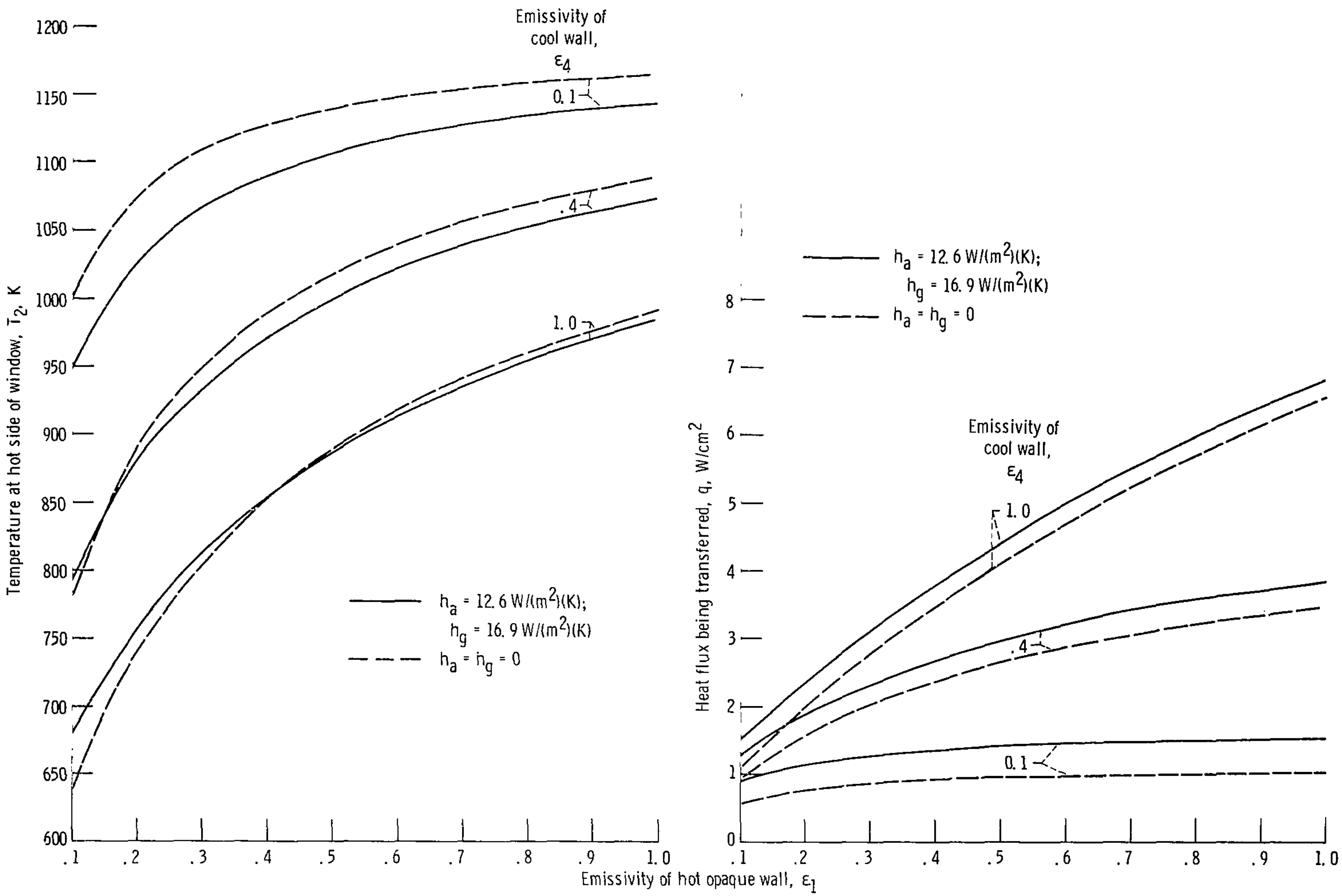


(a) Surface temperatures of window.

(b) Heat flux being transferred from hot to cold wall.

(c) Comparison of exact and approximate solutions for temperature at hot side of window.

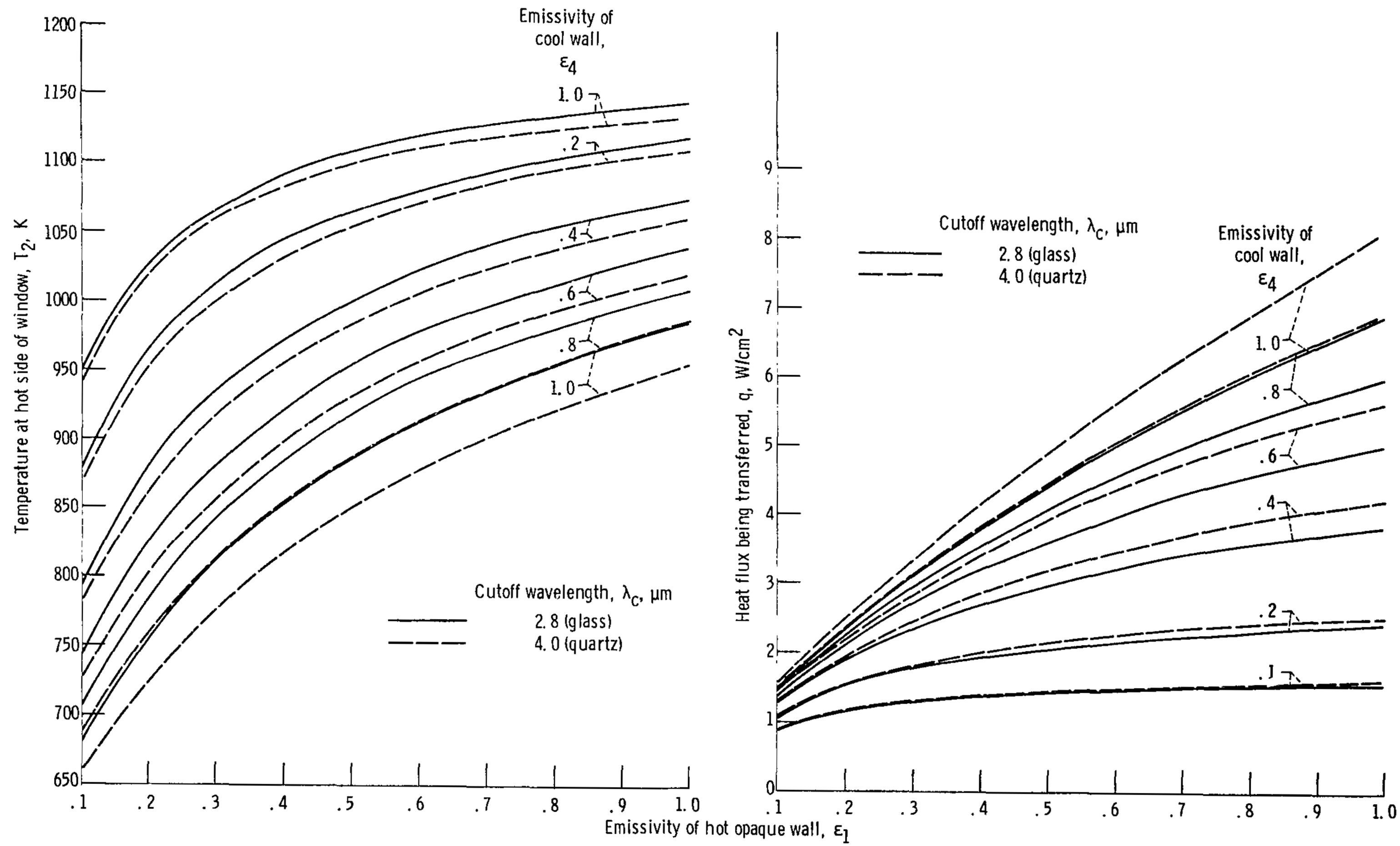
Figure 7. - Heat transfer characteristics for temperature at hot wall T_1 increased to 2000 K. Temperature at cool wall, T_4, 320 K; cutoff wavelength, λ_c, 2.8 micrometers.



(a) Temperature at hot side of window.

(b) Heat flux being transferred from hot to cold wall.

Figure 8. - Effect of vacuum in gaps between walls and window. Temperature at hot wall, T_1, 1200 K; temperature at cool wall, T_4, 320 K; cutoff wavelength, λ_c, 2.8 micrometers.



(a) Temperature at hot side of window.

(b) Heat flux being transferred from hot to cold wall.

Figure 9. - Effect of cutoff wavelength on heat transfer characteristics. Temperature at hot wall, T_1, 1200 K; temperature at cool wall, T_4, 320 K.

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