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THE SCATTERING OF OBLIQUELY INCIDENT PLANE WAVES FROM A CORRUGATED CONDUCTING SURFACE

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ABSTRACT

A physical optics solution is presented for the scattering of plane waves from a perfectly conducting corrugated surface in the case of waves incident from an arbitrary direction and for an observer far from the surface. This solution is used to compute the radar cross section of the surface in the case of backscatter from irregular (i.e., stochastic) corrugations and is used to point out a correction to the literature on this problem.

An interesting feature of the solution is the occurrence of singularities in the scattered fields. These singularities appear to be a manifestation of focussing by the surface at its "stationary" points. Whether or not the singularities occur in the solution depends on the manner in which one restricts the analysis to the far field.

CONTENTS

INTRODUCTION

The scattering of electromagnetic radiation from irregular surfaces is a problem of relevance to a number of remote sensing applications and in particular to sensing of the ocean surface. An important special case, which can be examined for insight into the more general problem, is that of plane waves incident on corrugated surfaces (surfaces whose ordinate is a function of only one dimension). It is the purpose of this paper to present a physical optics solution for this special case for an observer in the far field of a conducting surface and for plane waves incident obliquely on the surface corrugations. This is a three dimensional problem which reduces to the two dimensional form (plane of incidence perpendicular to the corrugations) as a special case.

This work was motivated by the physical optics treatment of the scattering of plane waves from an irregular surface by Kodis (1966). This solution was formulated for the most general case, employing a three dimensional dyadic Green's function and an arbitrarily directed plane wave incident on the arbitrary surface, $z = Z(x, y)$. The two dimensional case was deduced as a special case; unfortunately, an important step seems to have been omitted in making the transition to two dimensions and as a result the physical optics solution obtained for this special case exhibits an incorrect dependence on frequency and distance. These results appear to have propagated into the literature (Barrick, 1968). The same error also occurs elsewhere in the literature on scatter from irregular surfaces, appearing in the text by Beckmann and Spizzichino (Beckmann and Spizzichino, 1966).

The solution to be presented here is for the transitional case of plane waves incident obliquely on a corrugated surface. An interesting feature of this solution are singularities which can occur depending on the manner in which the "far field" approximation is made. If this approximation is employed in a minimal way only to justify asymptotic evaluation of integrals, singularities appear in the solution for the scattered fields. The singularities disappear in the limiting case of an observer infinitely far above the surface, in which case the solutions reduce to those obtained by making the far field approximations in a conventional manner (i.e., to simplify the argument of the exponents before evaluation of the integrals). The singularities are not peculiar to the case of plane waves or to two dimensions, but rather they appear to be a manifestation of focussing by the parabolic arcs with which the asymptotic evaluation of integrals effectively represent the surface. Roughly speaking, the surface is replaced in the asymptotic limit by an infinitesimal "mirror" which, with proper curvature, can focus the incident radiation at the observation point. This is a characteristic of the asymptotic approximation to the integrals and, to the extent that the asymptotic approximation is part of the physical optics approach, the singularities are inherent to the analytical technique employed rather than a manifestation of the particular problem treated here.

SOLUTION

It is assumed that a plane wave, $\bar{e}(\bar{r}, \nu) = E_0 \hat{e} \exp(j\bar{k} \cdot \bar{r})$ is incident on the perfectly conducting surface, $z = Z(y)$, where $Z(y)$ is an arbitrary function of y and $\hat{e}$ is a unit vector in the direction of the electric field vector. A physical optics solution is to be obtained: that is, Maxwell's equations are "integrated" by means of the Kirchoff (tangent plane) approximation and then the integrals are evaluated asymptotically in the "high frequency" limit. In order to handle waves incident at oblique angles the solution will be formulated initially as a three dimensional problem. Thus, letting $\bar{\bar{G}}_0(\bar{r}/\bar{r}')$ denote the free space dyadic Green's function and $\bar{h}(\bar{r}, \nu)$ the magnetic field intensity on the surface, one obtains the following form for the (complex representation of) the scattered electric field:

$$\bar{e}_s(\bar{r}, \nu) = j\omega\mu \oiint \bar{\bar{G}}_0(\bar{r}/\bar{r}') \cdot [\hat{n} \times \bar{h}(\bar{r}, \nu)] \, ds' \tag{1}$$

where

$$\bar{\bar{G}}_0(\bar{r}/\bar{r}') = \left[\bar{\bar{I}} + \frac{1}{k^2} \nabla\nabla\right] \frac{e^{jkR}}{4\pi R} \tag{2}$$

and $R = |\bar{r} - \bar{r}'|$ is the distance from a point on the surface, $\bar{r}'$, to the observer at $\bar{r}$, and $k = 2\pi\nu/c$. Employing the Kirchoff approximation, one obtains the following form for the vector product of $\bar{h}(\bar{r}, \nu)$ with the surface normal, $\hat{n}$:

$$\hat{n} \times \bar{h}(\bar{r}, \nu) = \frac{2E_0 k}{\omega\mu} \left\{\hat{k}(\hat{n} \cdot \hat{e}) - \hat{e}(\hat{n} \cdot \hat{h})\right\} e^{jk\hat{k} \cdot \bar{r}} \tag{3}$$

Substituting Equation 2 and 3 into Equation 1, yields the following result for the scattered fields:

$$\bar{e}_s(\bar{r}, \nu) = -j2kE_o \iint \bar{F}(\bar{r}/\bar{r}') \frac{e^{jkR}}{4\pi R} e^{jk\hat{k}\cdot\bar{r}'} dx'dy' \tag{4}$$

where:

$$\bar{F}(\bar{r}/\bar{r}') = \left\{\bar{\bar{I}} - \left[1 + \frac{2j}{kR} - \frac{1}{(kR)^2}\right][\nabla R \nabla R]\right\}\left\{\hat{k}(\hat{n}\cdot\hat{e}) - \hat{e}(\hat{n}\cdot\hat{k})\right\} \tag{5}$$

At this point the results are general in the sense that no assumptions have been made on the direction of $\hat{k}$ or on the surface, z; and the results are equivalent to those presented by Kodis (1966). The omission cited above occurred in the specialization of these results to corrugated surfaces ($z = Z(y)$) and to waves incident normal to the corrugations; in particular, the x'-coordinate, which in this case no longer appears in the expression for the plane waves or the surface, was neglected, and Equation 4 was treated as if the integration was only over y'. Unfortunately, in beginning with a three dimensional formulation, one must treat the x'-integration explicitly regardless of the plane of incidence. In order to avoid this extra integration, one would have had to formulate the solution initially as a two dimensional problem (i.e., in terms of the two dimensional Green's function).

It is possible, in the process of making the transition from the three dimensional case to the strictly two dimensional case, to handle the intermediate problem of plane waves obliquely incident on a corrugated surface. This is the case to be examined here. Thus, assume a corrugated surface, $z = Z(y)$, and a plane wave incident from an arbitrary direction. Consider the x'-integration first and make the following transformation of coordinates (Senior, 1959):

$$(y' - y)^2 + (Z(y') - z)^2 = a^2$$

$$x' - x = a \sinh(\gamma)$$

one obtains:

$$R = a \cosh\gamma$$

$$dx' = a \cosh\gamma \, d\gamma$$

and the expression for the scattered field becomes:

$$\bar{e}_s(\bar{r}, \nu) = -j2kE_o \int e^{jk[k_y y' + k_z Z(y')]} \bar{I}_x(y')\, dy' \qquad (6)$$

where:

$$\bar{I}_x(y') = \frac{e^{jkk_x x}}{4\pi} \int \bar{F}(\gamma, y') \exp\{jka[\cosh\gamma + k_x \sinh\gamma]\}\, d\gamma \qquad (7)$$

Now consider the case in which $ka >> 1$. (a will be large if the observer is far enough above the mean surface.) Then, neglecting terms in $\bar{F}(\gamma, y')$ which are inversely proportional to ka and representing $\bar{I}_x(y')$ by the first term in its asymptotic expansion in ka, one obtains:

$$\bar{I}_x(y') = \bar{F}(\gamma_o, y') \frac{j}{4} \left\{ \sqrt{\frac{2}{\pi k \tilde{a}}}\, e^{j[k\tilde{a} - \pi/4]} \right\} e^{jkk_x x} \qquad (8)$$

where $\tilde{a} = \sqrt{k_z^2 + k_y^2}\, a(y')$ and $\gamma_o = \tanh^{-1}(-k_x)$. One will recognize the expression in braces in Equation 8 to be the asymptotic form of $H_o^{(1)}(k\tilde{a})$. Denoting this asymptotic form symbolically by a tilda over the Hankel function, $\tilde{H}_o^{(1)}(k\tilde{a})$, one obtains upon substituting Equation 8 into Equation 6:

$$\bar{e}_s(\bar{r}, \nu) = -j2kE_o\, e^{jkk_x x} \int \bar{F}(\gamma_o, y')\left[\frac{j}{4}\, \tilde{H}_0^{(1)}(k\tilde{a})\right] e^{jk[k_y y' + k_z Z(y')]}\, dy' \qquad (9)$$

Notice that when $k_x = 0$ (i.e., the plane of incidence is the y-z plane), $\gamma_o = 0$ and $\tilde{a}(y) = a(y)$. This case is just the two dimensional problem and in this case Equation 9 reduces to the result that one would have obtained by beginning initially with a two dimensional formulation in terms of the Green's function, $j/4\; H_0^{(1)}(kR)$. The implications of the asymptotic evaluation of Equation 7 in the more general case, $k_x \neq 0$, is that scattering takes place at such points that the distance from the scatter points to the observer is $\sqrt{(y' - y)^2 + (Z(y) - z)^2} \bullet \cosh(\gamma_o)$. This implies that the spherical (angular) coordinates of the line from observer to scatter point are such that: $\sin\theta_o \cos\phi_o = -k_x$.

Assuming that k is large, one can now perform the integration over y' asymptotically. Doing so, one obtains the following result for the asymptotic value of Equation 9:

$$\bar{e}_s(\bar{r}, \nu) = E_o \sum_{\text{all } y_n} \bar{F}_n(\gamma_o)\, \frac{\cos\alpha(y_n)}{\cos(\alpha+\beta_o)}\, \frac{e^{jk\varphi(y_n)}}{\sqrt{a(y_n)/\delta_o}} \sqrt{\frac{R_c(y_n)}{1 + \delta_o R_c(y_n)/[a(y_n)\,\epsilon(y_n)]}} \qquad (10)$$

where:

$$\varphi(y_n) = k_y\, y_n + k_z\, Z(y_n) + \delta_o a(y_n) + k_x x \qquad (11a)$$

$$\epsilon(y_n) = \frac{\cos\theta_i + \delta_o \cos\beta_o}{\cos\alpha(y_n)\cos^2(\alpha+\beta_o)} \qquad (11b)$$

$$\bar{F}_n(\gamma_o) = [\bar{\bar{I}} - (\nabla R)(\nabla R)] \cdot [\hat{k}(\bar{n}\cdot\hat{e}) - \hat{e}(\bar{n}\cdot\hat{k})] \qquad (11c)$$

$$R = \sin\theta_i \cos\phi_i + \delta_o \, a(y) \qquad (11d)$$

$$\delta_o = \sqrt{k_y^2 - k_z^2} \qquad (11e)$$

$$\alpha(y) = \tan^{-1}\left[\frac{\partial Z}{\partial y}\right] \qquad (11f)$$

$$\beta_o = \tan^{-1}\left[\frac{y' - y}{z - Z(y)}\right] \qquad (11g)$$

$$k_x = \sin\theta_i \cos\varphi_i \qquad (11h)$$

$$k_y = \sin\theta_i \sin\varphi_i \qquad (11i)$$

$$k_z = \cos\theta_i \qquad (11j)$$

Notice that $a(y_n)/\delta_o$ is just the distance from the observer to the n-th scatter point. When this distance is large in comparison to $R_c(y_n)/\epsilon(y_n)$, Equation 10 indicates that the field scattered from each stationary point decreases as the squareroot of the distance from the scatter point to the observer as is characteristic of scattering from a two dimensional object. The field measured by the observer as predicted by Equation 10 appears to come from many separate scattering centers, one for each y_n, and the magnitude of the radiation scattered from each such point to the observer depends on the squareroot of the distance from the scattering point to the observer, $a(y_n)/\delta_o$, on the squareroot of the radius of curvature, $R_c(y_n)$, at the scatter point and on the relative orientation of the incident ray, the observer, and the slope of the surface at the scattering point (i.e., on θ_i, β_o and $\alpha(y_n)$).

The integration over the x'- and y'-coordinates have resulted in a pair of restrictions on the coordinates of the scatter points possible for a given incident wave and fixed observer. The x'-integration leads to the restriction, $\sin \theta_0 \cdot \cos \phi_0 = -k_x$, where θ_0 and ϕ_0 are the spherical coordinates (measured at the observer) of the line from observer to the scatter point. It follows that for each θ_0 there are two possible configurations: $\phi_0 = \pm\cos^{-1}(-k_x/\sin \theta_0)$. These two possibilities are illustrated in Figures 1 and 2. They correspond to the case of "forward" scatter (k_y in the reflected ray has the same sign as in the incident ray) as shown in Figure 1 and the case of "back" scatter as illustrated in Figure 2. In the case of back scatter, reflection takes place at points on the surface at which $\alpha(y) > 0$ whereas forward scatter occurs at points for which $\alpha(y) \lesssim 0$. When $k_x = 0$, then $\phi_0 = \pm\pi/2$ and in this case both incident and reflected waves are perpendicular to the surface corrugations. This is the degenerate case corresponding to the two dimensional problem. There also are a forward and back scatter possibility in this case. (See Figure 3.) In forward scatter the reflected and incident waves both propagate in the same direction with respect to the y-axis, and in the case of back scatter they propagate in opposite directions with respect to the y-axis.

The condition imposed by the y'-integration is that $\frac{\partial\phi}{\partial y} = 0 = k_y + k_z \tan\alpha(y) + \sqrt{k_x^2 + k_z^2}\,\frac{\partial a}{\partial y}$. This condition is equivalent to the requirement that the local angle of incidence and reflection are equal. In order to see that this is so, it is convenient to define angles $\tilde{\beta}_0$ and $\tilde{\beta}_i$ to be the (acute) angles that the projections of the reflected and incident rays onto the y-z plane make with the vertical (i.e., the z-axis). Then, in terms of $\widetilde{\beta}_0$, $\widetilde{\beta}_1$ and α, the condition $\frac{\partial\varphi}{\partial y} = 0$ becomes $\sin(\widetilde{\beta}_0 - \alpha) = \sin(\alpha - \widetilde{\beta}_1)$. This implies that the angle of incidence equals the angle of reflection as measured in the plane of the projection. Since the plane of incidence also contains the local normal (and the reflected ray) it is also true that the angle of incidence equals the angle of reflection as measured in the plane of incidence. That is, the points at which reflection takes place are "specular" points. (The fact that the normal to the surface, the incident ray and the reflected ray all lie in the same plane—the plane of incidence—is a consequence of the boundary conditions imposed by the tangent plane approximation.)

The amplitude of the radiation scattered from each stationary point and measured at the observer is given by Equation 10. An interesting feature of this amplitude is the singularity which occurs at $\epsilon(y_n)\, a(y_n)/\delta_0 = -R_c(y_n)$. Since $\epsilon(y_n) < 0$ for physically meaningful geometry (i.e., waves incident toward the surface so that $k_z < 0$), it follows that $R_c(y_n) < 0$ at scatter points associated with the singularity. Thus, at these points the surface is concave toward the observer. It appears that the singularities are the manifestation of focussing which can take place at the surface due to the combination of tangent plane approximation (which preserves the phase structure of the incident plane wave) and the asymptotic evaluation of the integrals (which effectively represent the surface by an arc with radius of curvature, $R_c(y_n)$). Since the surface is perfectly conducting, the result is that each scatter point behaves as if a plane wave were incident on a concave or convex mirror, depending on the sign of the radius of curvature. When the mirror is concave, focussing can occur. For example, consider the case of two dimensions, $k_x = 0$. In this case, $\delta_o = 1$ and $\beta_o = \theta_o$ and $\beta_i = \theta_i$ where β_o and β_i are the obtuse angles associated with $\widetilde{\beta}_o$ and $\widetilde{\beta}_i$, respectively. In this case, the requirement that $\frac{\partial \varphi}{\partial y} = 0$ yields the relationship, $\theta_o = 2\alpha - \theta_i$, and it follows that $\varepsilon(y_n) = 2 \sec(\alpha - \theta_o)$. Consequently, the singularity occurs whenever $a(y_n) = 2 \sec(\alpha - \theta_o)/R_c(y_n)$, or in terms of the acute angle, $\widetilde{\theta}_o$, at $a(y_n) = -2 \sec(\alpha - \widetilde{\theta}_o)/R_c(y_n)$. Now $a(y_n)$ is just the distance along the scattered ray from the scatter point to the observer; and if one imagines a concave spherical mirror to be located at the scatter point with radius of curvature, $-R_c(y_n)$, and an axis which coincides with the normal to the surface, then $-2 \sec(\alpha - \widetilde{\theta}_o)/R_c(y_n)$ is just the distance along a reflected ray from the mirror to the focal plane. (See Jenkins and White, 1957.)

The results shown in Equation 10 were obtained by making a somewhat modified "far field" approximation. In particular, the factor, $R = |\overline{r} - \overline{r}'|$, was kept in an arbitrary form in the exponential, exp (jkR), but was treated as a large number with comensurate simplifications being made whenever this was expeditious only in multiplicative factors. This is in contrast to the more conventional application of the far field approximation in which R is approximated by the constant and linear terms in its binomial expansion for use in the exponential prior to evaluation of the integrals. (For example, see Kodis, 1966.) If one follows this latter approach, no singularities occur and the form obtained is just Equation 10 in the limiting case of $e(y_n)a(y_n)/\delta_o >> R_c(y_n)$ for all $R_c(y_n)$. That is:

$$e_s(\bar{r}, \nu) = E_o \sum_{\text{all } y_n} \bar{F}_n(\gamma_o) \cos\alpha(y_n) \sqrt{\frac{\cos\alpha(y_n)}{\cos\theta_i + \delta_o \cos\beta_o}} \frac{e^{jk\varphi(y_n)}}{\sqrt{a(y_n)/\delta_o}} \sqrt{R_c(y_n)} \quad (12)$$

Apparently the focussing is overlooked in making the far field approximation in this conventional form because this effectively removes the observer to infinity, beyond all possible focal points. It is interesting to note that the singularities also occur in the treatment of finite sources (Le Vine, 1974) and therefore are not peculiarities of plane waves.

RADAR CROSS SECTION OF A STOCHASTIC SURFACE

As a final point, a comparison will be made of the radar cross section in two dimensions of a stochastic surface based on the results derived here (Equation 12) and those based on the computation of Kodis (Kodis, 1966). In keeping with the treatment of Kodis, it is assumed that the receiving antenna is sufficiently narrow and properly oriented so that scatter only from points for which the incident and scattered ray are colinear need be considered. Furthermore, it is assumed that the $k\phi(y_n)$ are uniformly distributed over 2π and that they are independent of $R_c(y_n)$. (This assumption amounts to assuming that the incident radiation scatters incoherently from the surface.) With these assumptions, the radar cross section, σ^0, for a unit length of surface and based on Equation 12 is:

$$\sigma^0 = \frac{1}{L} \left\langle \lim_{\rho \to \infty} 2\pi\rho \left[\frac{\bar{E}_s \cdot \bar{E}_s^*}{E_o^2} \right] \right\rangle \quad (13a)$$

$$= \pi N \langle R_c(y_n) \rangle \quad (13b)$$

where L is the length of surface illuminated, the pointed brackets denote an ensemble average, N is the number of scatter points per unit length of the illuminated portion of the surface, and $\rho = a(y)/\delta_0$ is the distance from the observer to the surface. Equation 13a is just the two dimensional adaptation of the usual formula for radar cross section (Kerr, 1951; Skolnik, 1970). In comparing Equations 12 and 13 with those in Kodis (Kodis, 1966, Equations 14 and 26) notice that there is no frequency dependence in the results derived here, whereas Kodis' results are proportional to k. Notice also that power, in the two dimensional case as presented here, decays as $1/\rho$ (i.e., as a cylindrical wave) as opposed to inversely as the square of distance as occurs in the three dimensional case and in Kodis' results (Kodis, 1966, Equations 14 and 15). The three dimensional character of Kodis' results are the consequence of the omission discussed above.

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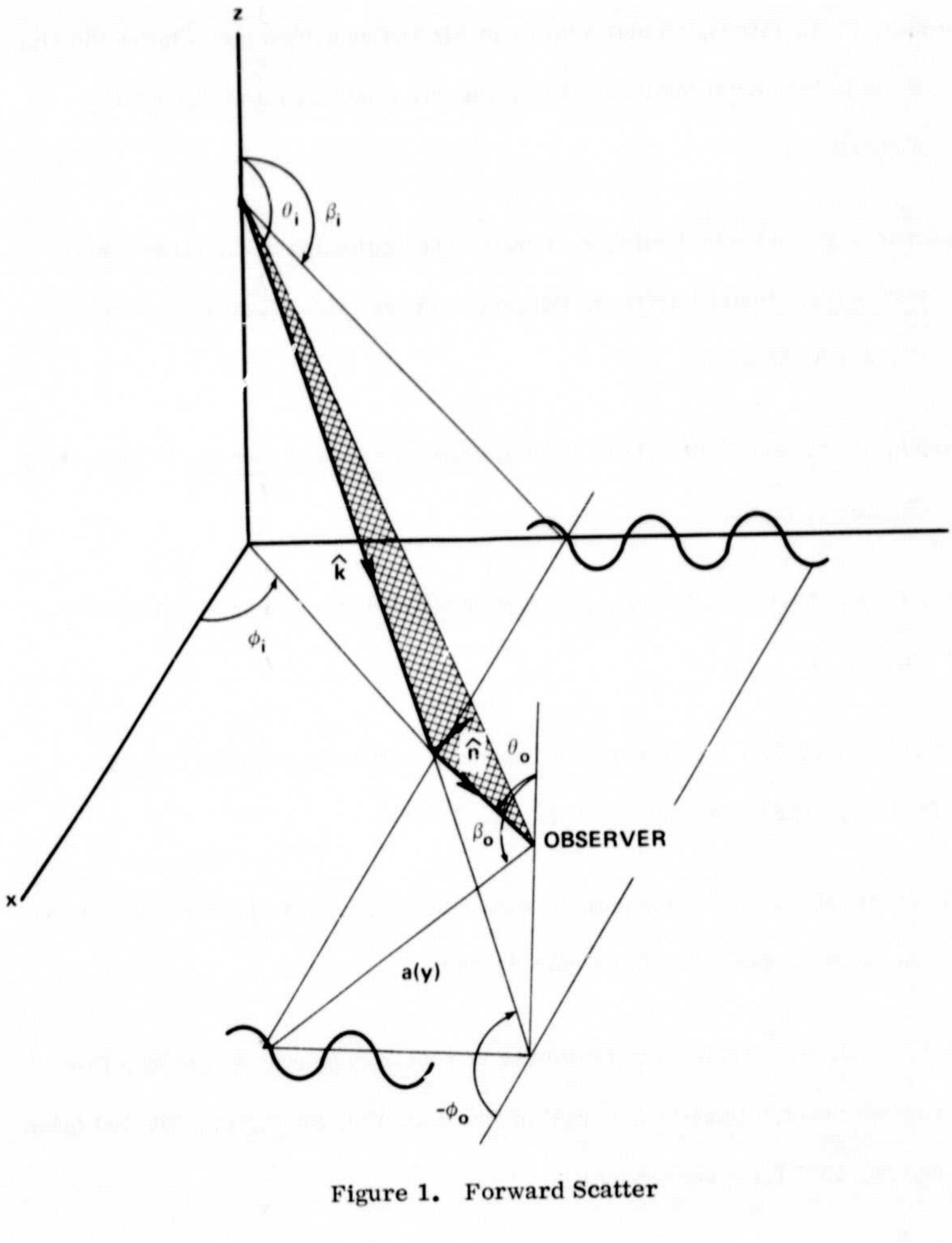

Figure 1. Forward Scatter

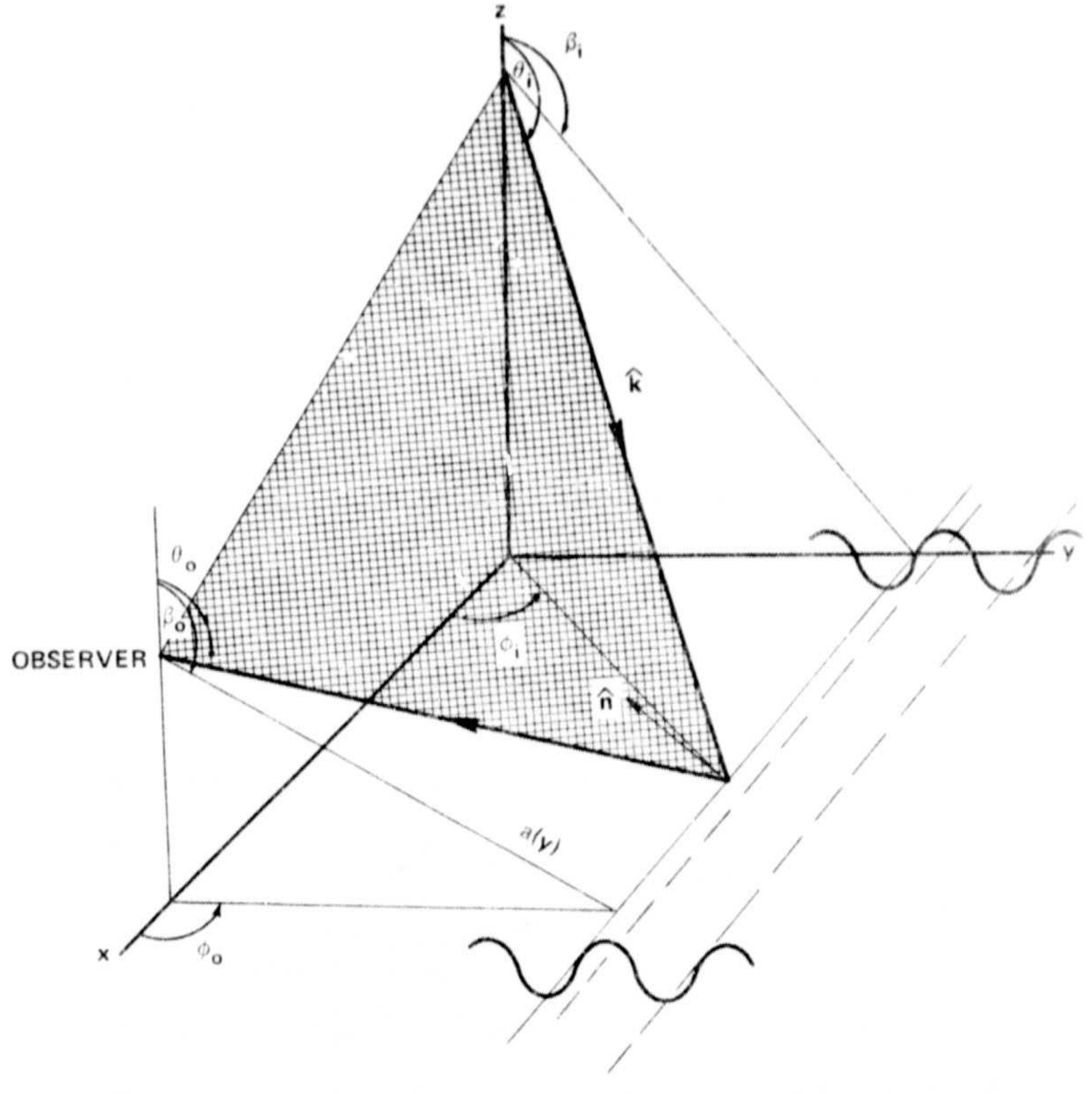

Figure 2. Back Scatter

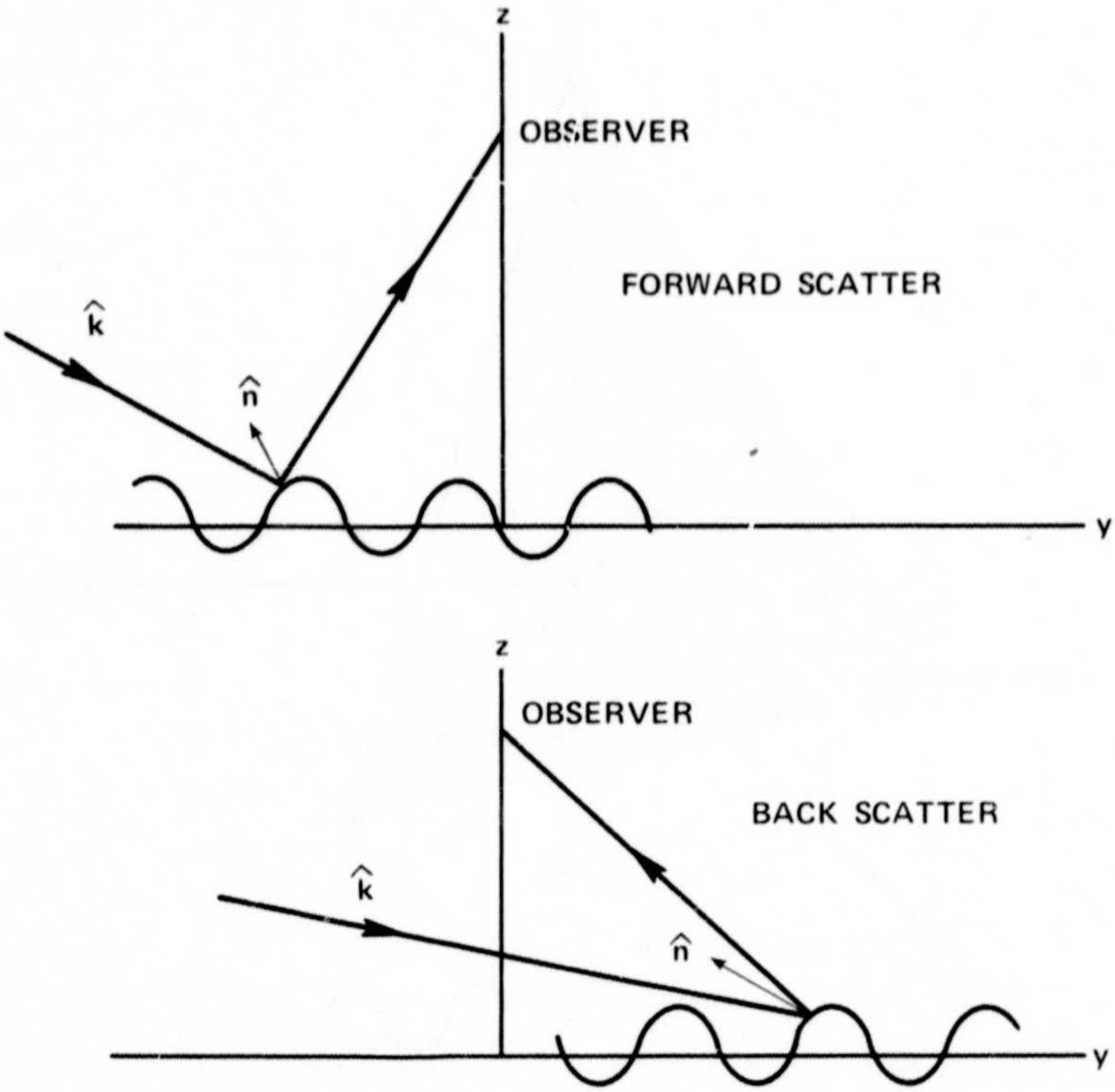

Figure 3. Scatter in the Two Dimensional Case: a) Forward Scatter; b) Back Scatter