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NUMERICAL SOLUTION OF THE NAVIER-STOKES EQUATIONS FOR ARBITRARY TWO-DIMENSIONAL MULTI-ELEMENT AIRFOILS

by

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During the reporting period the following results have been accomplished:

(1) The incompressible Navier-Stokes solution in the vorticity-stream function formulation has been applied successfully to a number of single airfoils at Reynolds numbers up to 2000. Initial progress to higher Reynolds numbers has also been made. Results were presented in the reports (1), (2), and (3) below. A Ph.D. dissertation on this work was accepted as noted below and the degree was granted.

(2) The incompressible Navier-Stokes solution in the primitive variable formulation (velocity-pressure) has been programmed for multiple airfoils with general segment arrangements and is now in operation and under analysis. Initial results were presented in the report (3) below.

(3) The coordinate system program has been gerneralized to allow any segment arrangement for multiple bodies, and a number of arrangements have been investigated. Some results were presented in the report (3) below.

(4) The potential flow solution for multiple bodies has been tested by comparison with the analytic solution for a circular cylinder pair and found to be quite good. Some results were included in report (3) below.

(5) Programming of the compressible Navier-Stokes solution for single bodies using the primitive variable formulation has begun.

During the reporting period the following conference presentations were made (abstracts attached):

(1) "Numerical Solution of the Navier-Stokes Equations for Arbitrary Two-Dimensional Airfoils," F. C. Thames, Joe F. Thompson, and

C. W. Mastin, Proceedings of NASA Conference on Aerodynamic Analyses Requiring Advanced Computers, Langley Research Center, Hampton, Va., March 4-6, 1975. To be published as NASA-SP 347.

(2) "Numerical Solutions of the Unsteady Navier-Stokes Equations for Arbitrary Bodies Using Boundary-Fitted Curvilinear Coordinates," J. F. Thompson, F. C. Thames, R. L. Walker, and S. P. Shanks, Proceedings of Arizona/AFOSR Symposium on Unsteady Aerodynamics, Univ. of Arizona, Tucson, Ariz., March 18-20, 1975.

(3) "Use of Numerically Generated Body-Fitted Coordinate Systems for Solutions of the Navier-Stokes Equations," J. F. Thompson, F. C. Thames, C. W. Mastin, and S. P. Shanks, Proceedings of AIAA 2nd Computational Fluid Dynamics Conference, Hartford, Conn., June 19-20, 1975.

During the reporting period the following Ph.D. dissertation was accepted (abstract attached):

"Numerical Solution of the Incompressible Navier-Stokes Equations about Arbitrary Two-Dimensional Bodies," Frank C. Thames, Ph.D. Dissertation, Mississippi State University, May 1975.

NUMERICAL SOLUTION OF THE NAVIER-STOKES EQUATIONS FOR ARBITRARY TWO-DIMENSIONAL AIRFOILS

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Proceedings of NASA Conference on Aerodynamic Analyses Requiring Advanced Computers, Langley Research Center, Hampton, Va., March 4-6, 1975

ABSTRACT

A method of numerical solution of the Navier-Stokes equations for the flow about arbitrary airfoils, or other bodies, is presented. This method utilizes a numerically generated curvilinear coordinate system having a coordinate line coincident with the body contour. Streamline, velocity profiles, and pressure and force coefficients for several airfoils and an arbitrary rock are given. Potential flow solutions are also presented. The procedure is also capable of treating multi-element airfoils, and potential flow results are presented therefor.

NUMERICAL SOLUTIONS OF THE UNSTEADY NAVIER-STOKES EQUATIONS
FOR ARBITRARY BODIES USING
BOUNDARY-FITTED CURVILINEAR COORDINATES

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Proceedings of Arizona/AFOSR Symposium on Unsteady Aerodynamics, Univ. of
Arizona, Tucson, Ariz., March 18-20, 1975

ABSTRACT

A numerical solution of the time-dependent, two-dimensional incompressible Navier-Stokes equations that can treat the unsteady laminar flow about bodies of arbitrary shape, such as airfoils, as naturally as simple bodies has been developed. Unsteady boundaries, such as deforming bodies or free surfaces, can also be treated. This solution is based on a method of automatic numerical generation of a general curvilinear coordinate system with coordinate lines coincident with all boundaries of a general multi-connected region containing any number of arbitrarily shaped bodies. The curvilinear coordinates are generated as the solution of two elliptic partial differential equations with Dirichlet boundary conditions, one coordinate being specified to be constant on each of the boundaries, and a distribution of the other being specified along the boundaries. No restrictions are placed on the shape of the boundaries, which may even be time-dependent, and the method is not restricted to two dimensions or single bodies. Coordinate lines may be concentrated as desired along the boundaries. Spacing of the coordinate lines encircling the body may be controlled by adjusting parameters in the partial differential equations for the coordinates.

USE OF NUMERICALLY GENERATED BODY-FITTED COORDINATE SYSTEMS FOR SOLUTION OF THE NAVIER-STOKES EQUATIONS

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Research sponsored by Langley Research Center, NASA, Grant NGR 25-001-055 and Office of Naval Research, USN, Contract N00014-74-C-0373-P0001

Proceedings of AIAA 2nd Computational Fluid Dynamics Conference, Hartford, Conn., June 19-20, 1975

ABSTRACT

These coordinate systems remove geometric considerations from numerical solutions of the Navier-Stokes equations so that the flow about bodies of arbitrary shape can be treated. Fields with multiple bodies or with time-dependent boundaries may also be treated. Several types of such systems and methods of control of the coordinate line spacing and orientation are discussed. All computations are done on a fixed rectangular field with square mesh without any interpolation regardless of the shape or movement of the physical boundaries and regardless of the spacing of the coordinate lines. Potential and viscous flow solutions are presented for airfoils with attached and separated flaps and for hydrofoils near a free surface.

ABSTRACT

Frank C. Thames, Doctor of Philosophy, 1975

Major: Engineering, Department of Aerophysics and Aerospace Engineering

Title of Dissertation: Numerical Solution of the Incompressible Navier-Stokes Equations About Arbitrary Two-Dimensional Bodies

Directed By: Dr. Joe F. Thompson

Pages in Dissertation: 205 Words in Abstract: 515

ABSTRACT

The purpose of this research was to develop methods capable of producing numerical solutions of the two-dimensional, incompressible, time-dependent Navier-Stokes equations about arbitrary two-dimensional bodies. As a preliminary step a method of automatic numerical generation of a general curvilinear coordinate system having a constant coordinate line coincident with each boundary of a general multi-connected physical flow region was developed. These curvilinear coordinates are generated as the solution of two elliptic partial differential equations with Dirichlet boundary conditions. One natural coordinate is specified to be constant along each boundary of the physical region while a distribution of the other is specified along the boundary contours. The transformation is initiated by the selection of a generating elliptic partial differential system of which the natural coordinates are a solution in the physical flow plane. This generating system is transformed by inter-

changing the dependent and independent variables producing a set of two coupled, quasi-linear elliptic equations for the cartesian coordinates as functions of the natural curvilinear coordinates. If the original generating elliptic system obeys a maximum principle and if the range of the natural coordinate which varies along the physical contours is made the same on each boundary, then the transformed plane will be rectangular. The quasi-linear transformation equations are solved for the physical coordinates in the transformed plane using SOR techniques. The boundaries of the physical region appear simply as input boundary conditions for the numerical solution in the transformed plane. No restrictions are placed on the shape of the boundaries in the physical plane which may even be time dependent. Further, the method is not restricted to two dimensions. Coordinate lines may be concentrated as desired along the physical contours while control over the spacing of the radial coordinate is adjusted by varying the generating elliptic system.

Once the natural coordinates are generated for a given physical domain, the vorticity-stream function formulation of the Navier-Stokes equations are transformed to the rectangular transformed plane. The equations are then approximated using central differences for the space derivatives and a backward first-order time difference for the vorticity time derivative. The flow solution is carried out by an implicit method with SOR iteration being used to converge the elliptic space variation at each time step. The vorticity distribution on the body surface is calculated at each time step utilizing a modified multi-dimensional false position iteration designed to force the tangential velocity component on the body surface to zero. Dirichlet conditions corresponding to irrotational flow were used at the remote boundary. Successful solutions about five different bodies were generated. These included a circular cylinder, cambered and flapped Karman-Trefftz airfoils, a Göttingen 625 airfoil, and a general arbitrary body-denoted the cambered rock. Reynolds numbers for the solutions varied from 200 to 2,000 at angles of attack from zero to fifteen degrees. Data defining stream function and vorticity contours, boundary layer velocity profiles, pressure distributions, and force coefficients are presented for all solutions. A mildly successful solution about a NACA 0018 airfoil at a Reynolds number of 10,000 was also accomplished.