X-921-75-157
PREPRINT

NASA TM X- 70924

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(NASA-TM-X-70924) POLAR MOTION SPECTRA BASED UPON DOPPLER IPMS AND BIH DATA (NASA) 15 p HC $3.25 CSCL 03B
N75-27979
Unclas
G3/89 28807

JUNE 1975

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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ABSTRACT

Spectra based upon recent Doppler, International Polar Motion Service, and Bureau International de l'Heure data are compared. An oscillation is found at 1.3 cpy which might be due to an Eulerian motion of the solid inner core. An extended 15-year I.P.M.S. data set is filtered and analyzed to yield a Chandler peak with a period 430.8 solar days and a full width at half-maximum of 0.7 days (Q=600). The data is reanalyzed in overlapping 3-year segments and indicates that the excitation of the Chandler wobble is a discrete process and that periods as long as three years occur in which the driving mechanism is essentially quiescent.

POLAR MOTION SPECTRA

BASED UPON DOPPLER, P.M.S., AND B.I.H. DATA

Since its development in 1967 by Burg, the maximum entropy method of spectral analysis [Burg, 1967; Burg, 1968; Peacock and Treitel, 1969; Lacoss, 1971] has proved successful in treating short data records which could not be resolved with the traditional techniques utilizing Fourier transforms or autocorrelation functions. The phenomenon of polar motion is one of those which has been successfully treated. International Polar Motion Service (I.P.M.S.) [Claerbout, 1969; Currie, 1974] and Bureau International de l'Heure (B.I.H.) [Smylie et al., 1973] data sets have been analyzed elsewhere already. Here we develop the power spectrum for recently acquired Doppler satellite tracking data [Anderle, 1973] and compare it with spectra for B.I.H. and I.P.M.S. data for the same epoch. In the second part of this paper, we develop the power spectrum for a filtered segment of I.P.M.S. data.

The Doppler satellite tracking data spans the four-year period from 1971 through 1974, with a two-day sampling period. Two gaps appear in the data. The first is for one data observation for New Year's Eve 1971-1972; the second is for 29 observations near the end of 1972. The data set was made continuous by constructing the appropriate prediction filter and calculating approximate values for the missing points consistent with the statistical properties of the surrounding data. B.I.H. and I.P.M.S. data were analyzed for comparison with the Doppler data, with data lengths of six (1968-1973) and seven (1968-1974) years, respectively.

The maximum entropy technique of spectral analysis was now applied to the three sets of data. This technique as described by Burg is utilized by first generating a prediction error filter either by an iterative process which Burg used [Burg, 1968; Levinson, 1947] or by a Wiener-Hoph matrix equation. If the Wiener-Hoph equation is used, the prediction error filter length is limited to 10 per cent of the data length, since the equation uses a matrix composed of autocorrelation values which are usually limited to a 10 per cent lag. Then to extend the filter further, the iterative technique must be applied. In this investigation, it was found that application of the iterative process alone gave better spectral resolution than the combination of the Wiener-Hoph and iterative techniques.

An aspect of the maximum entropy technique, which is not yet well understood, is whether limits should be placed on the length of the prediction error filter. Given a data set with L points, the filter may be generated to a maximum length of L-1. It would seem at first that the maximum length is also the optimum length, since the spectral resolution can be estimated as the Nyquist frequency divided by some characteristic number of the calculation method, e.g., the number of autocorrelation lags, or Fourier transforms, or elements in the prediction error filter. Thus, a longer filter would be expected to improve the resolution between two oscillations such as the annual and Chandler components of polar motion.

In practice, as the filter length is increased, the spectrum starts to seriously deteriorate and often bears little resemblance to the anticipated spectral power distribution. This point has been recognized before [Currie, 1974]. Smylie et al. [1973] used a 404 point data set, but only generated a 76 element prediction error filter. With 884 data points, Currie [1974] generated 133 filter elements, or a 15 per cent filter. Using implicit double precision arithmetic and the criterion described below, we have stably extended the filter length to 62-65 per cent for three separate data sets having 138, 438, and 725 data points.

The filter length criterion is based on the fact that each data set contains a finite amount of noise power, denoted by H in Figure 1. In generating a prediction error filter, the noise power H is calculated as the noise power estimate P. With an infinite data set, as the filter length is increased the estimate P_∞ would slowly converge to the value H, as indicated in the figure [illegible] descending dashed line. However, because of the finite data length L, as the filter approaches length L-1, the noise power estimate P_L is observed to behave in the pathological manner indicated in the figure and is no longer a good estimate for H.

In this investigation, the optimum filter length was determined by observing the behavior of P_L and ending the iterative process when the noise power estimate

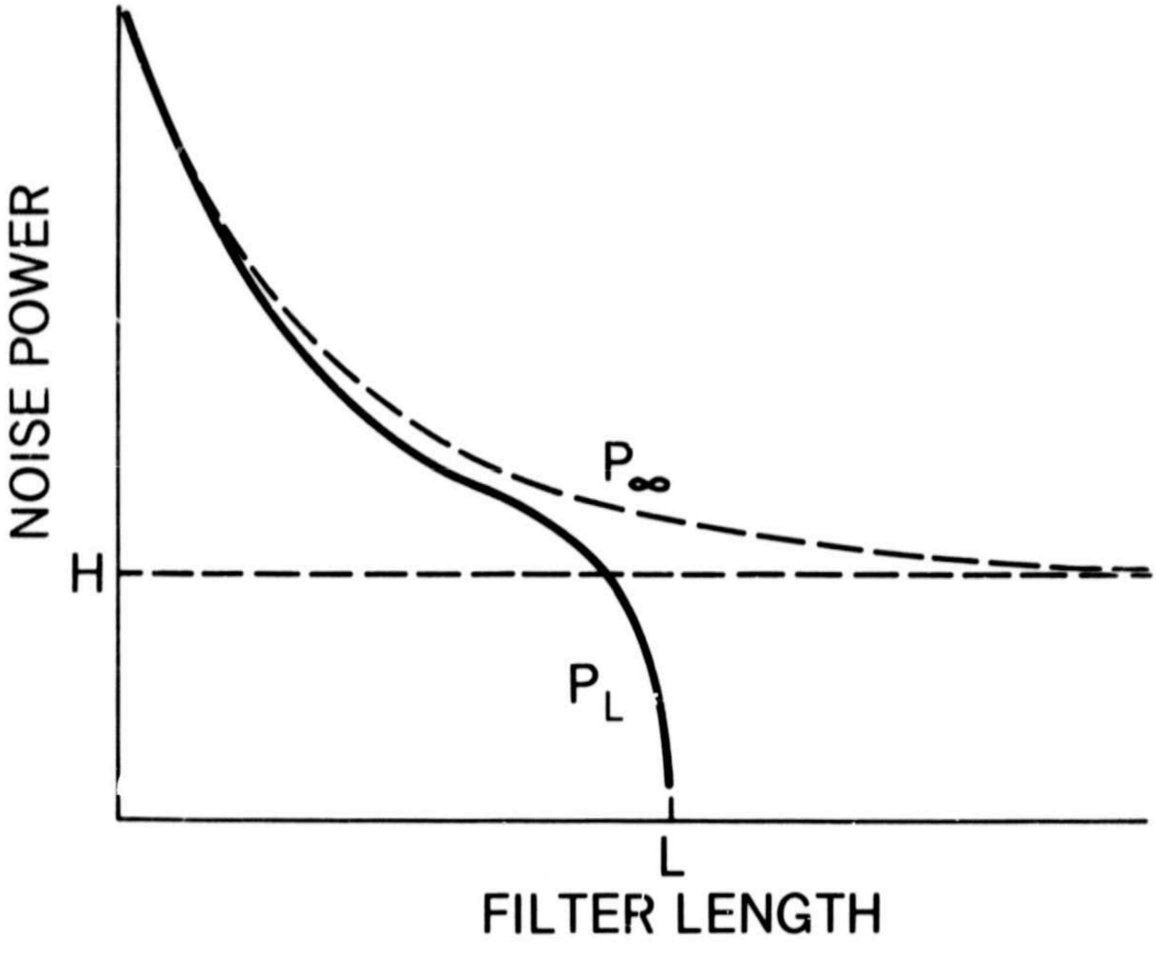

Figure 1. The anomalous behavior of the noise power estimate, P_L, establishes the optimum filter length. For the pole position data considered here, filter lengths of 55-65 per cent were utilized.

started to increase its rate of descent.* This criterion has resulted in filter lengths up to 65 per cent, allowing increased resolution while maintaining spectral stability.

The power spectra for the region -1.5 to +1.5 cycles per year (cpy) are given in Figure 2. One of the more striking features of this figure is the variation in resolution of the annual and Chandler peaks. In the Doppler spectrum, the central region between the two peaks is 32 dB down from the Chandler peak; in the I.P.M.S. spectrum, this value decreases to 25 dB; in the B.I.H., 17 dB. In addition the full width at half-maximum (FWHM) for the Chandler peak is 1-1/2 days for Doppler, 2-1/2 days for I.P.M.S., and 7 days for B.I.H.

Contrary to expectations, the largest ratio of annual-to-Chandler amplitude is in the Doppler spectrum. It had been thought that the optical data from B.I.H. and I.P.M.S. would have been more highly contaminated with annual noise. This does not now appear to be the case. The power ratio is emphasized here rather than the power estimate for each peak (area under the peak) because Lacoss [1971] has found that the power ratios are a more accurate estimate in the maximum entropy technique.

The ratio of the period of oscillation to the FWHM is found to be significantly higher for the Chandler oscillation than the typically accepted values of 20-60.

*In a recent paper Ulrych and Bishop [1975] discuss a filter length criterion based upon the mean square prediction error.

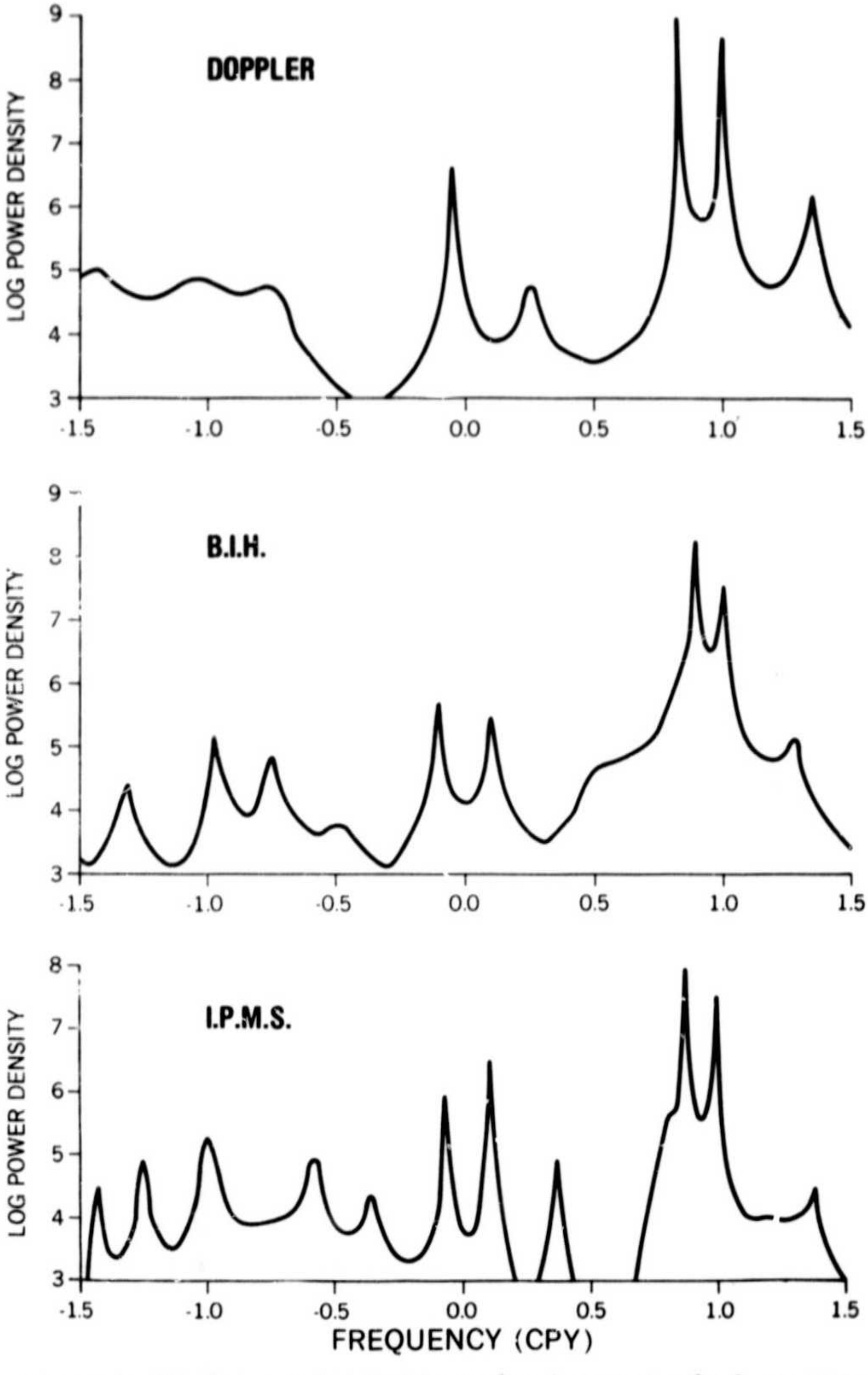

Figure 2. Maximum entropy spectra for three sets of pole position data, showing the high degree of resolution provided by the Doppler satellite data.

The values determined from these spectra are: Doppler, 290; I.P.M.S., 170; B.I.H., 60. One consequence of the high period/FWHM, or Q, for the Chandler oscillation is that the energy input needed to maintain the Chandler amplitude would not be as large as had been previously calculated. Lacoss [1971] has shown that the maximum entropy technique gives a better estimate of the spectral shape than the Blackman-Tukey autocorrelation technique. Thus, a parameter which depends upon the shape, such as Q, should be better estimated with the maximum entropy technique.

The spectra in Figure 2 all have similar structure in the region around 0.0 cpy. These peaks are probably due to the secular drift of the pole. The B.I.H. and I.P.M.S. spectra show a symmetric doublet structure, which indicates a linear motion through the pole. A variation in the rate of drift during this period caused the computer program to assign a non-zero frequency to the motion. The presence of a single strong peak in the Doppler spectrum indicates that the program assigned a finite curvature to the motion in this short data set.

All three spectra show a peak at approximately -1.0 cpy. This structure, noted by Claerbout [1969] in I.P.M.S. spectra, indicates a linear component or a non-zero ellipticity in the annual forcing function. It would be caused by a linear component in the atmospheric redistribution of mass which accompanies the annual seasons. The lack of a similar negative Chandler peak, most apparent in the I.P.M.S. spectrum, indicates that this part of the spectrum is not due to the ellipticity of the earth's equator.

An unexpected peak at approximately 1.3 cpy (275 days) appears in all three spectra. In view of the varying data collecting methods associated with the Doppler technique and the optical observations of the B.I.H. and I.P.M.S., and the different sampling periods (2 days, Doppler; 5 days, B.I.H.; 18 days, I.P.M.S.), this is probably a real oscillation. The period of the peak shows a scatter in the three spectra (266, 271, and 283 days), not considered significant since the Chandler and annual periods show a similar, though smaller, scatter. This problem of frequency shifts in the application of the maximum entropy technique to short data records has been noted before [Ulrych et al., 1973].

The power of this 1.3 cpy peak is down by two orders of magnitude from the Chandler peak in each spectrum, indicating that the amplitude of the motion is one-tenth of the Chandler, or approximately 1/2 meters. The definition of the peak indicated by the period/FWHM is relatively high, being calculated as 20-40. In the absence of other reasonable explanations for this oscillation, we have considered the possibility that it is due to an Eulerian or Chandler-type oscillation of the solid inner core. The sense of the circular oscillation is the same as the Chandler peak, and thus has the right orientation. In the earth as a whole, a 20 km equatorial bulge over a 6400 km radius is sufficient to cause a 430 day Chandler motion (from a 300 day Eulerian period). Thus, depending on the elasticity of the inner core, all that would be required to generate this type of oscillation would be a 4-5 km equatorial bulge over the measured 1230 km radius of the inner core. Since seismological analyses of the inner core boundary are uncertain on the 5 km level [Masse et al., 1974], it is possible that this bulge exists. The lack of significant density contrast at the inner-outer core boundary would, however, lead to inefficient coupling to the mantle.

In view of the excellent results achieved with the I.P.M.S. data for 1968.00-1974.85, it was decided to incorporate additional data and reanalyze the I.P.M.S. polar motion spectra for the 15 year period 1960.00-1974.85. The results of this analysis using the maximum entropy technique are shown in Figure 3, labeled the unfiltered spectrum. As would be expected, the results were not a significant improvement over the previous analysis of seven years of data. The Chandler peak is at 437 days with a FWHM of 3 days.

At this point a new technique was utilized which is a variation of the predictive filtering methods described by Ulrych et al. [1973] and by Smylie et al. [1973]. The data which is composed of 298 complex values is predicted in a forward and backward direction by a prediction filter of length 160 until $2048 = 2^{11}$ points have been listed. These points are then analyzed with the Sande-Tukey fast Fourier transform algorithm. In the technique of Smylie et al., the Fourier amplitudes of the annual peak were reduced to the background level and an inverse fast Fourier transform was performed to obtain a new data set with which to generate a filtered power density spectrum. We felt that a better representation of the power structure of the Chandler peak would be achieved if the Fourier amplitudes of the annual peak were multiplied with the appropriate sine or cosine

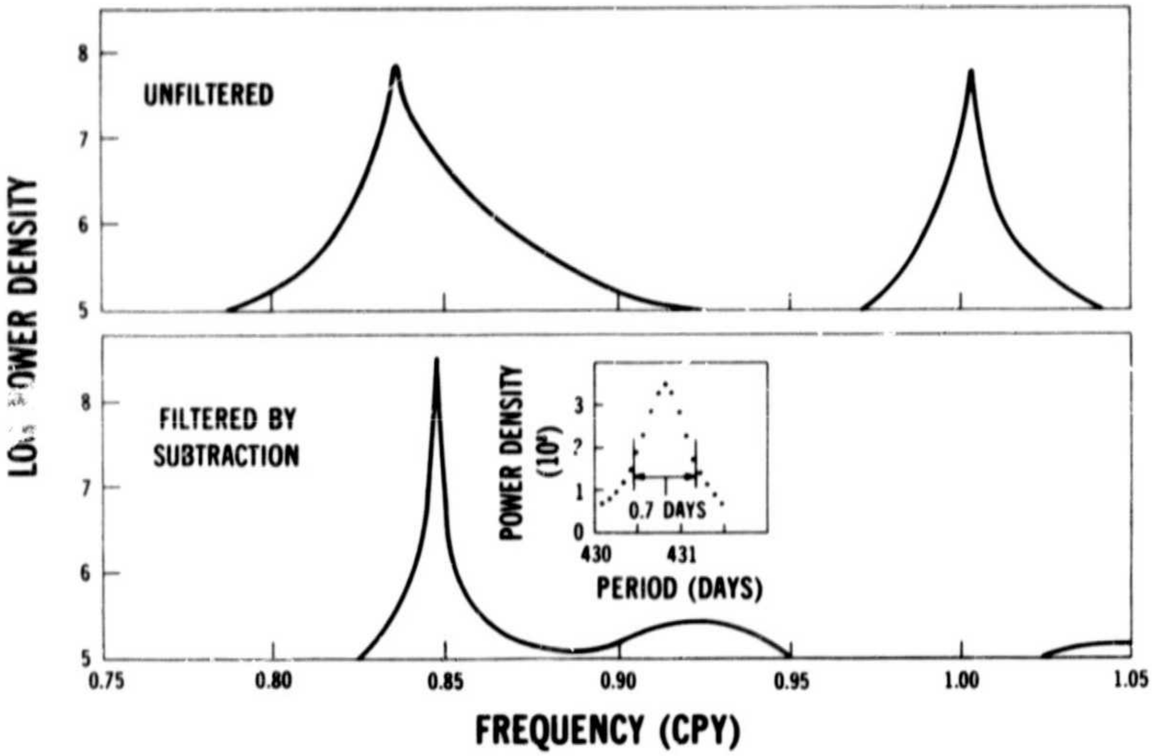

Figure 3. Filtered and unfiltered spectra for I. P. M. S. data (1960.00-1974.85). The insert shows the smooth, symmetric structure of the Chandler peak with a full width at half maximum of 0.7 days.

function, and these time series were then subtracted from the original data. Any discontinuities in the Chandler motion would be better represented by this technique. The resulting power spectrum is shown in the lower half of Figure 5. Some noise resulting from the subtraction can be observed at 30 dB below the Chandler peak, but the addition of double-precision sine and cosine time series with frequencies 1.005 cpy and 0.995 cpy can be expected to have very little effect at the Chandler frequency 0.85 cpy in the maximum entropy technique. The improvement observed is due to the removal of the interference effects from two nearly equal amplitude oscillations. The beat period of the annual and Chandler oscillations is 13 years. Thus with 15 years of data the removal of one of the oscillations should greatly enhance the resulting spectrum for the remaining term.

As can be seen from Figure 3, the resulting spectrum does show a significant improvement over the unfiltered spectrum. The peak appears at 430.8 solar days with a FWHM of 0.7 days. This corresponds to a period/FWHM (=Q) =600. The insert in Figure 3 shows the smooth symmetric structure of the Chandler peak.

The value for Q associated with this filtered spectrum is significantly higher than commonly accepted values for Q, 20-60. A Q of 600 would allow the damping of the Chandler motion to be accomplished by the mantle without requiring damping by the oceans. According to Stacey [1970], if the Chandler motion is damped by the mantle alone, then the Q of the Chandler oscillation should be 7.5 times greater than the Q of the mantle. Q(Chandler) =600 would imply Q(mantle) =80 at 0.85 cpy. Analyses of seismic oscillations with frequencies around one cycle per minute yield values for the average Q(mantle) between 200-250 [Stacey, 1969].

Figure 4 shows the dependence of various parameters of the Chandler peak for the filtered spectrum of Figure 3 on the length of the prediction error filter utilized. The length considered optimum is indicated with the vertical line. The important points are the general stability of the parameters near the cut-off and the change in the behavior of the parameters after the cut-off is exceeded.

With the annual oscillation filtered out of the data, adequate resolution of the Chandler oscillation should be possible with the maximum entropy technique using approximately two cycles of data. The filtered data was reanalyzed in nine overlapping sets of 2.9 years duration. The resulting spectra are shown in Figure 5. An interesting result of this computation is the absence of an annual peak from all nine spectra. The filtering was done using the Fourier amplitudes for the entire 15 year set of data. Variations in the annual oscillation would appear as small annual peaks in the nine spectra in Figure 5. The absence of any annual peaks indicates a high degree of stability in the annual forcing function.

The Chandler peak is seen to vary in frequency and FWHM from spectrum to spectrum. Figure 6 shows the variation in peak period. Note that the average

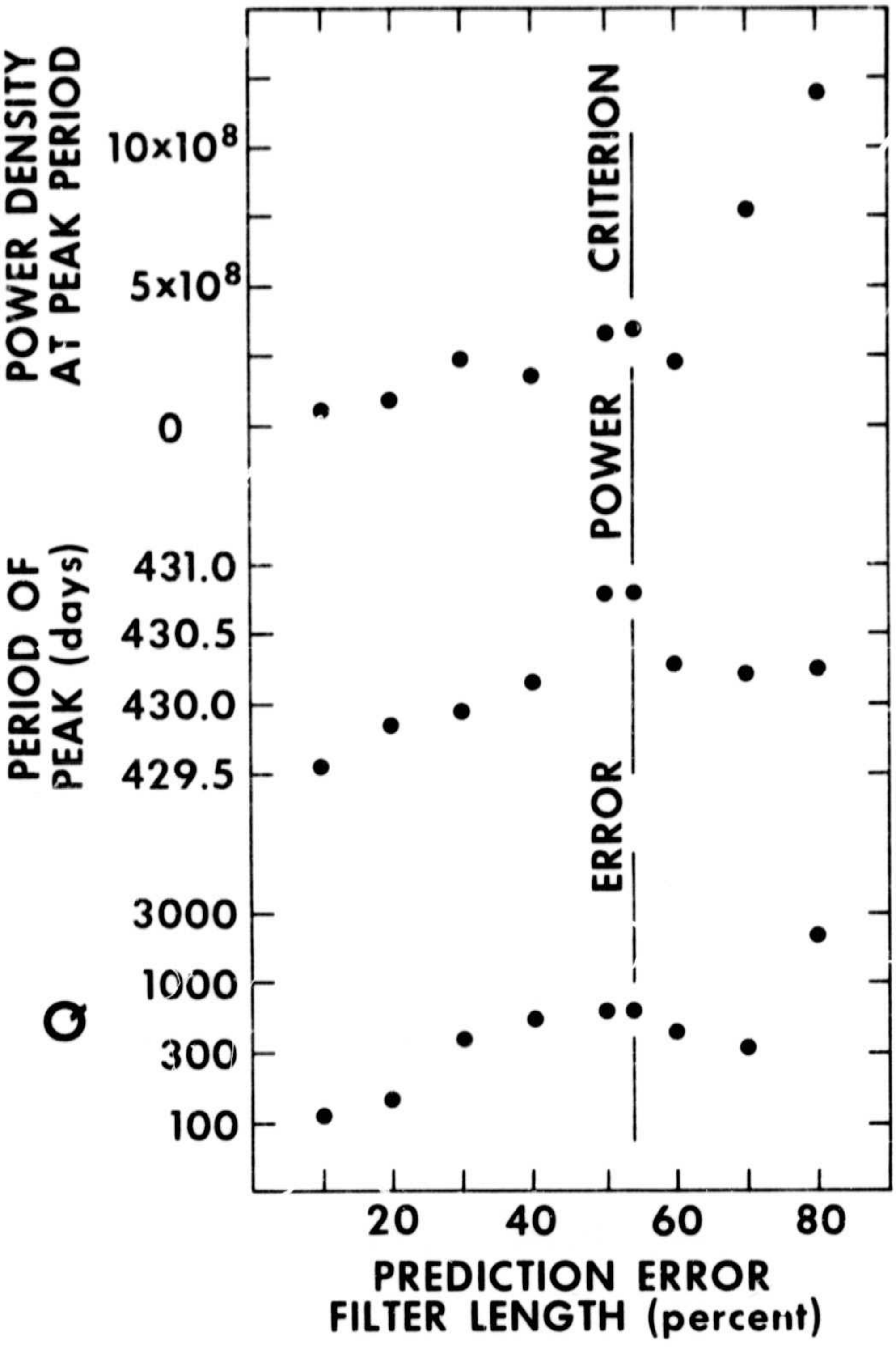

Figure 4. Dependence of Chandler peak parameters on the length of the prediction error filter. The parameter values are stable as they approach the filter length determined by the error power criterion.

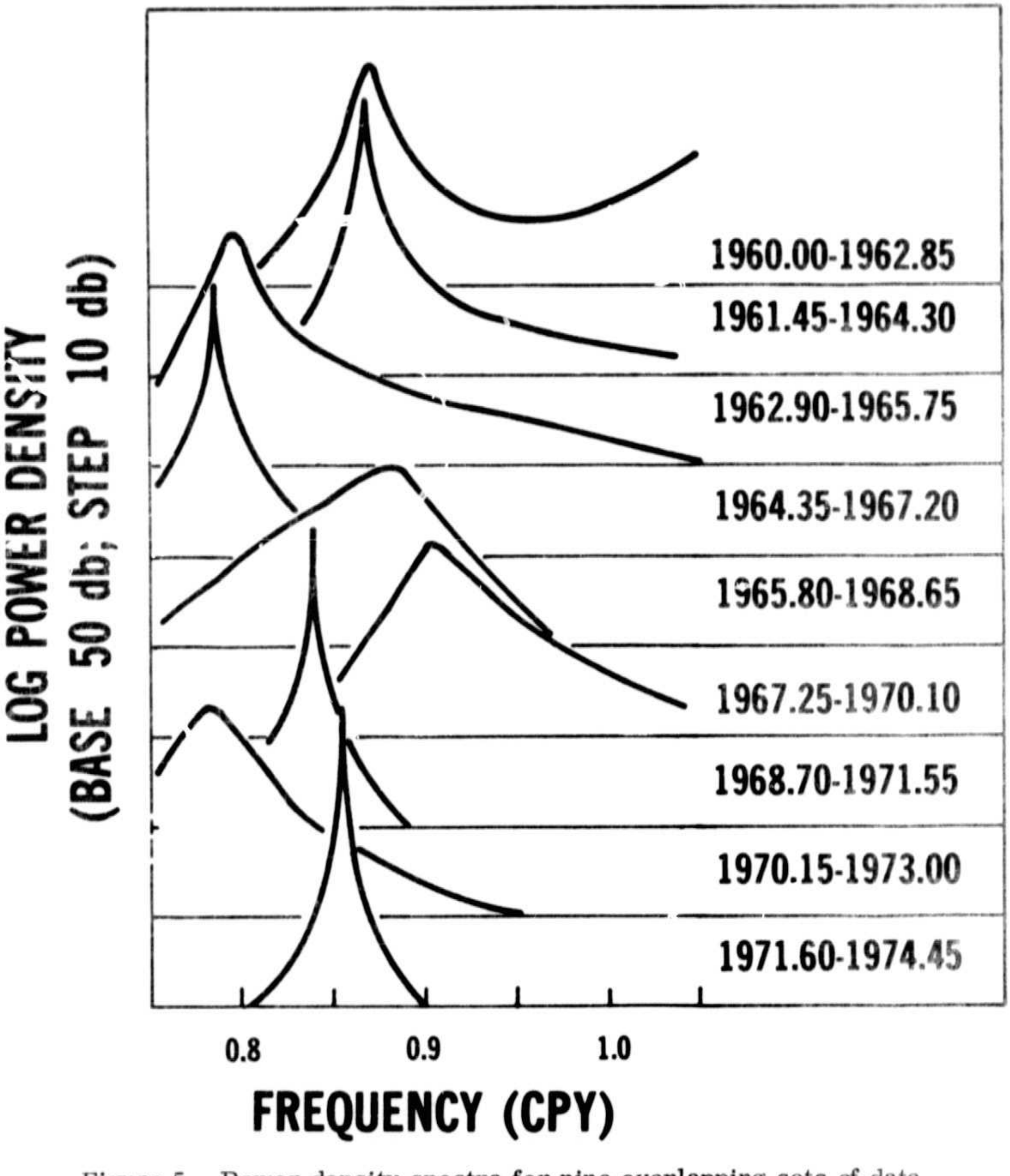

Figure 5. Power density spectra for nine overlapping sets of data.

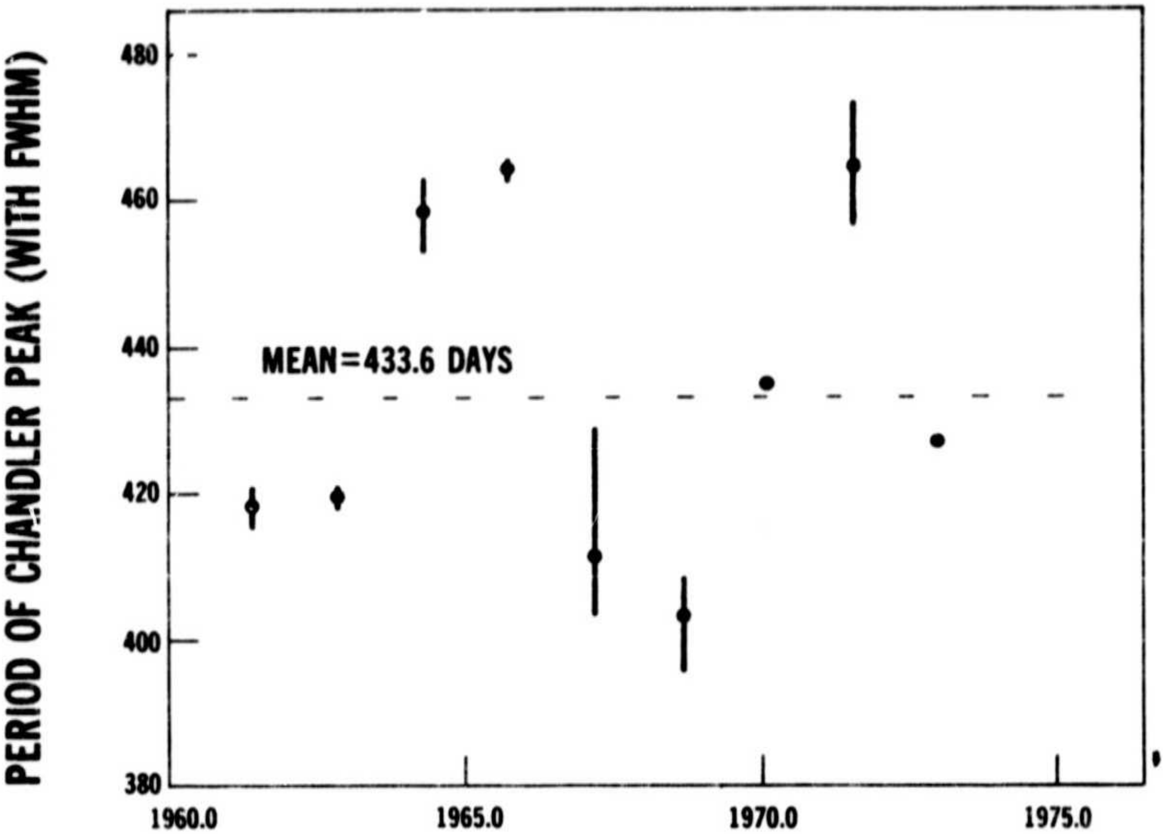

Figure 6. Variation of the peak period and full width at half maximum. Note the non-random grouping of the points.

of these nine Chandler periods is 433.6 solar days, which is very close to 430.8 days calculated from the filtered spectrum in Figure 3. This indicates that these peaks are all associated with the Chandler oscillation but are perturbed by some statistical fluctuation centered on the Chandler frequency. Figure 7 shows the variation of FWHM with the shift of the peak from 430.8 days. Seven of the points in this figure seem to show a clear functional dependence of the FWHM on the distance from 430.8.

To determine the extent of this functional dependence, the least squares criterion was used to generate a quadratic line of regression. As was expected, when all nine points are used the correlation is poor. When the two wild points are dropped ((shift=33.6, FWHM=2.5), (19.0, 25.1)), the following line of regression is established

Y = FWHM (days)

X = period shift (days), greater than zero

$Y = 0.711 + 0.129\,X + 0.010\,X^2$

Correlation Coeff. = 0.97

Note that this line of regression estimates the FWHM at zero period shift (430.8 days) to be 0.71 days. The FWHM calculated from Figure 3 is 0.7 days. The regression analysis recovers the FWHM exactly to the accuracy of the initial value.

Recall that the mean of the nine peaks, 433.6 days, recovered the Chandler period to less than 1 per cent. If the two values dropped in the regression analysis

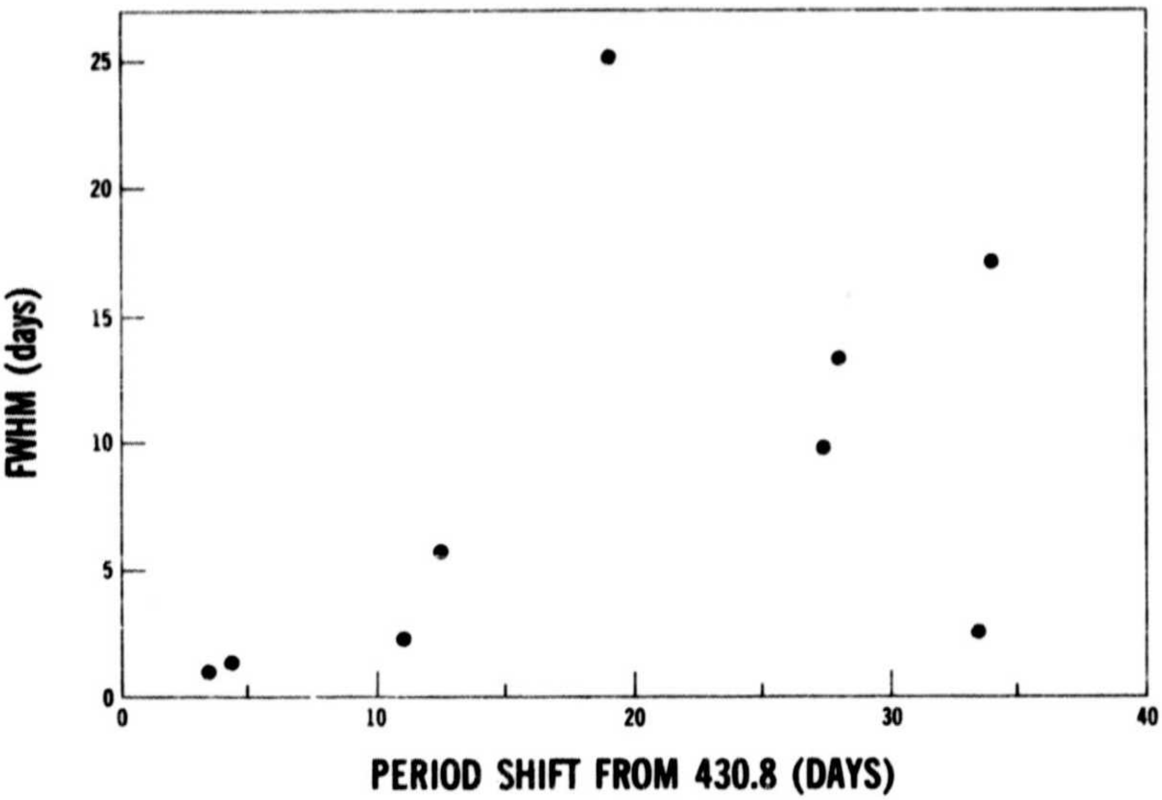

Figure 7. A possible functional dependence between the full width at half maximum and the period shift. Seven of the nine points establish a quadratic line of regression with a correlation coefficient of 0.97. The two omitted points are based on two adjacent overlapping data sets.

are removed from the mean, the new mean is 432.3 days which misses the 15 year Chandler period by 3.5 parts in a thousand.

We believe that there is a strong possibility that the variations in peak frequency and FWHM shown in Figure 5 are due to small discontinuous phase shifts in the Chandler oscillation. These phase shifts are caused by changes in the moment-of-inertia tensor of the earth (earthquakes, etc.) which result in a change in the location of the figure axis. They would change the Chandler oscillation from Figure 8a to either Figure 8b or 8c. It can be seen that a phase shift greater than zero in 8b would cause a spectral analysis program transforming a short data set to assign the spectral power to a higher frequency than is in fact occurring in the oscillation; and in 8c, a phase shift less than zero would cause the analysis program to compute a peak frequency lower than the actual frequency. In both cases with non-zero phase shifts, the break in the time series would cause a broadening of the peak. This association of frequency shift and line broadening is the type of behavior shown in Figure 7.

If this postulated phase-shift process takes place very frequently ($f > 1$ cpy) and in a random manner, then the correlation observed in Figure 7 for the 2.9 year analysis would not exist. The multiple random phase shifts would cause a period shift of zero over the 2.9 year analysis. In the 15 year analysis this randomness does nullify the period shift. The high correlation observed in Figure 7 indicates that in the seven short data sets used in the regression analysis

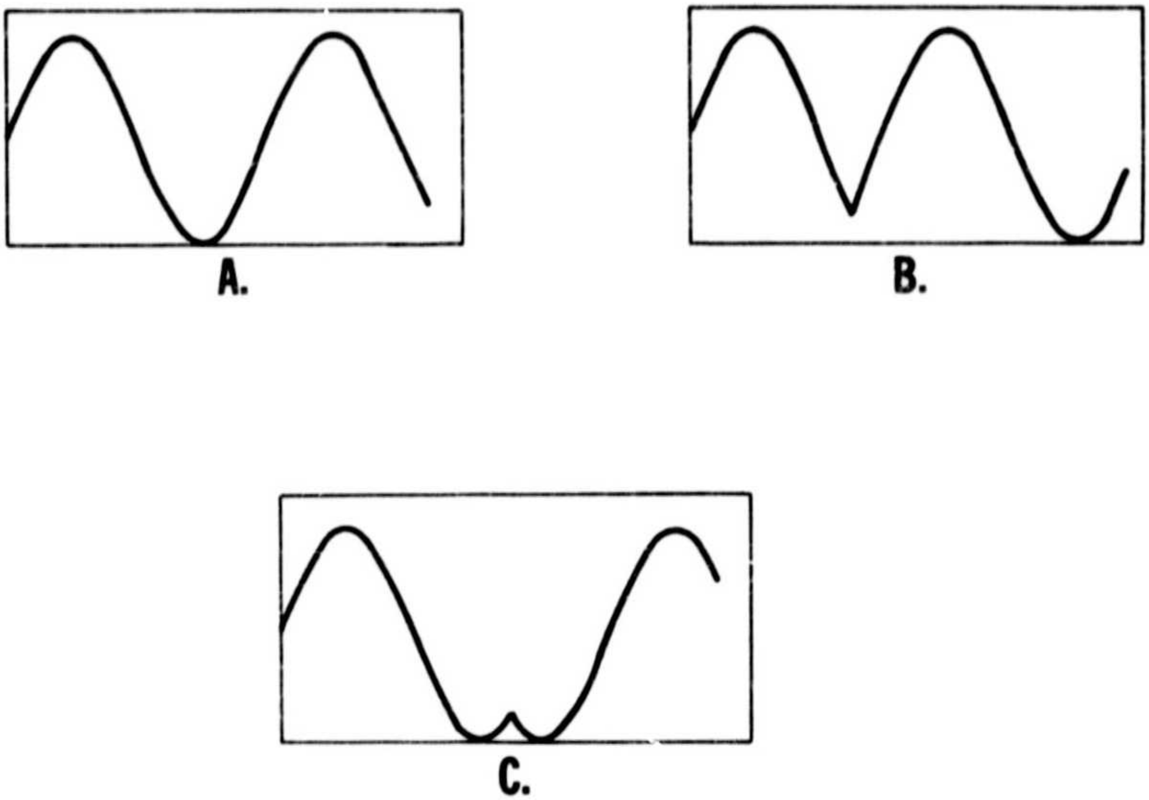

Figure 8. A possible mechanism for the period shifts observed in the spectra. (a) X-component of unperturbed Chandler motion. (b) and (c) A shift in the figure axis could result in the X-component undergoing this type of motion. Computer analysis would yield period shifts and line broadening.

probably only one significant phase shift is occurring in each period. In the two wild periods, 1964.35-1967.20 and 1965-80-1968.65, probably more than one phase shift has occurred. And it is possible that during the periods 1968.70-1971.55 and 1971.60-1974.45 the driving mechanism was quiescent or the phase shifts occurred near the ends of the period. This second possibility is very likely for at least one of these two periods since the overlapping period 1970.15-1973.00 exhibits a significant shift and broadening.

This general type of behavior in which the driving force for the wobble is applied discretely and randomly once every two or three years was postulated by Dahlen [1971].

An attempt was made to correlate the FWHM with numbers of large earthquakes using lists provided by the National Earthquake Information Service. No correlation was found. Probably the absolute number of earthquakes is too simple a quantity to correlate with the FWHM of the Chandler oscillation.

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FIGURE CAPTIONS

Figure 1. The anomalous behavior of the noise power estimate, P_L, establishes the optimum filter length. For the pole position data considered here, filter lengths of 55-65 per cent were utilized.

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