NASA CR 120851
BOEING D3-8611-3

ACOUSTIC RADIATION FROM LINED, UNFLANGED DUCTS — DUCT TERMINATION IMPEDANCE PROGRAM

BY
R. J. BECKEMEYER AND D. T. SAWDY

DECEMBER 12, 1971
THE BOEING COMPANY
WICHITA DIVISION - WICHITA, KANSAS, 67210

PREPARED FOR

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

NASA-LEWIS RESEARCH CENTER
CONTRACT NAS 3-14321
H. BLOOMER, PROJECT MANAGER

IG720284

NASA CR 120851
BOEING D3-8611-3

TOPICAL REPORT

ACOUSTIC RADIATION FROM LINED, UNFLANGED DUCTS – DUCT TERMINATION IMPEDANCE PROGRAM

by
R. J. Beckemeyer and D. T. Sawdy

THE BOEING COMPANY
3801 SOUTH OLIVER
WICHITA, KANSAS 67210

Prepared For
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
December 12, 1971

CONTRACT NASA 3-14321

NASA-LEWIS RESEARCH CENTER
CLEVELAND, OHIO
H. BLOOMER, PROJECT MANAGER
V/STOL AND NOISE DIVISION

FOREWORD

The work described herein was done by The Boeing Company, Wichita Division, under NASA contract NAS 3-14321 with Mr. H. Bloomer, V/STOL and Noise Division, NASA - Lewis Research Center, as Project Manager.

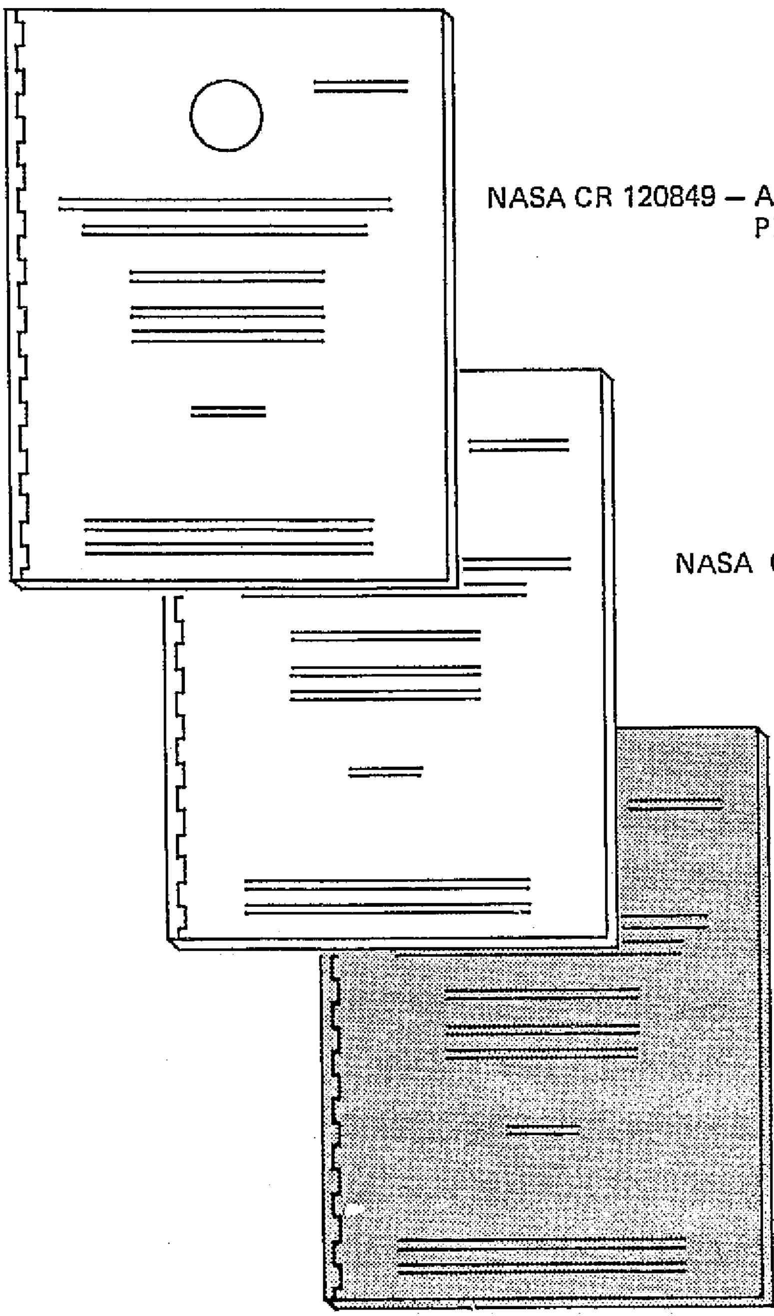
ABSTRACT

An acoustic radiation analysis has been developed to predict the far-field characteristics of fan noise radiated from an acoustically lined unflanged duct. This analysis is comprised of three modular digital computer programs which together provide a capability of accounting for the impedance mismatch at the duct exit plane. This report discusses the Duct Termination Impedance Program whose relationship with the other two modular reports of the analyses is illustrated on the following page.

Admissible duct configurations include circular or annular, with or without an extended centerbody. This variation in duct configurations provides a capability of modeling inlet and fan duct noise radiation.

DEVELOPMENT OF ACOUSTIC RADIATION ANALYSIS OF TURBOFAN NOISE FROM LINED, UNFLANGED DUCTS

OVERALL DOCUMENT ORGANIZATION



NASA CR 120849 – ACOUSTIC SOURCE DISTRIBUTION PROGRAM

NASA CR 120850 – DIRECTIVITY INDEX PROGRAM

NASA CR 120851 – DUCT TERMINATION IMPEDANCE PROGRAM

TABLE OF CONTENTS

NOMENCLATURE

Analytical Variables

a	Centerbody Radius
b	Outer Duct Radius
c	Sound Speed in Air
$\underline{I}$	Acoustic Intensity
k	Wave number, $k = \omega/c$
m	Angular Mode No.
r	Radial Coordinate
r_j	Radial Coordinate Integration Variable
R	Distance between Points
R_j	Distance between Point on Duct & Field Point
t	Time
u	Radial Velocity Component
v	Axial Velocity Component
V	Velocity Distribution on Exit Face
z	Axial Coordinate
Z	Modal Impedance
θ	Angular Coordinate
θ_j	Angular Coordinate Integration Variable
Π	Total Sound Power
ρ	Air Density
τ	Radiation Resistance
ϕ	Velocity Potential
Φ_j	Source Strength Distribution
χ	Radiation Reactance
ω	Frequency, Radians/Second

INDEX OF FIGURES

FIGURE | PAGE

1.0 INTRODUCTION

1.1 Background

This report discusses the Duct Termination Impedance Program that forms the intermediate link for the package of modular computer programs which together form a first generation capability for the analytical prediction of far-field noise being radiated from an unflanged inlet or exhaust duct.

The development of this program had its origin in the need for a method to correct the far-field directivity patterns for the discontinuity in the duct impedance that exists at the duct termination. It is known that when a propagating wave traverses the duct exit plane that part of the energy is reflected back up the duct while the remainder is transmitted through the interface. In addition, the radial variation of the termination impedance generates higher radial duct modes. Thus, both reflected and transmitted waves contain modes other than the modes comprising the incident wave. Obviously, it is necessary to understand the concept of modal coupling to determine the amount of energy radiated to the far-field and its modal content.

The subject program couples the infinite space exterior to the duct, to the interior of the duct by accounting for the impedance mismatch at the duct exit plane. This digital program utilizes the output from the Acoustic Source Distribution Program, Reference 1, to generate the pressure distribution and impedance which act on the duct exit plane. This information is then used by the Directivity Index Program, Reference 2, to determine the far-field radiation patterns for a semi-infinite length duct which are corrected for the impedance mismatch at the duct exit plane.

1.2 Technical Approach

This technical development is concerned with the effect of the impedance mismatch at the duct exit plane on the radiation of fan or compressor noise from an inlet or exhaust duct. Admissible duct configurations, shown in Figure 1, include unflanged circular or annular ducts, with or without extended centerbodies.

The approach presented here is an extension of the case of the radiation of sound from a duct with an infinite baffle on its end to the unflanged case which accounts for the diffraction of sound around the outside lip of the duct. Phased contributions of the outer and extended inner duct surfaces and the duct exit plane must be determined and then combined to determine the total pressure acting on the open face.

The duct termination impedance characteristics of the duct configuration are determined from the spatial distributions of acoustic sources over the duct walls and exit plane. The source distribution which satisfies the imposed boundary conditions is determined by the Acoustic Source Distribution Program, Reference 1. These boundary conditions require that the acoustic perturbation velocity normal to the outer duct walls must vanish. In addition, the normal velocity distribution over the duct exit plane must equal the velocity distribution associated with either a single mode or a combination of modes of discrete frequency.

The acoustic source strengths are numerically integrated over the duct walls and exit plane to yield the pressure distribution on the duct face. From the calculated pressure distribution and the known velocity distribution, two impedances are calculated. The first is a localized or point impedance which is defined as the ratio of the pressure on a given segment on the duct face to the velocity on that segment. This distributed or local impedance reflects the modal coupling effects of the impedance mismatch at the open end of the duct. The second impedance is a modal impedance which is constant across the duct face and provides a measure of the radiation efficiency of the given velocity distribution.

Several assumptions have been made in this development. These concern the transport characteristics of the model and the decomposition of the acoustic sound energy into the acoustic modes present in the duct. The acoustic sound is assumed to be radiated through an ideal fluid (nonviscous) which has no mean flow. This simplification will not significantly affect the results for noise propagation through a locally subsonic flow field of an inlet. However, radiation through an exhaust or jet flow will not be represented properly by this model.

The velocity distribution is assumed to have a Cos $(m\theta)$ type of angular dependence. This simplification is justified by the angular characteristic functions determined from the solution of the governing wave equation for sound propagation through annular ducts. This assumption does not affect the ability to couple different modes in the far-field program, Reference 2, but does restrict this program to the extent that for a velocity distribution whose angular dependence is specified by a combination of different m angular modes, a separate analysis must be made for each set of angular modes.

2.0 MATHEMATICAL DEVELOPMENT

The acoustic pressure on the open face of an unflanged duct is formulated in terms of the acoustic velocity potential. This formulation provides a mathematical model with a straightforward method of satisfying the velocity boundary conditions on the duct walls and exit plane. The phase of the pressure wave transmitted through the duct exit plane is accounted for in the phase relationship of the potential function.

Since the velocity potential is governed by a linear set of equations, superposition may be used to determine the field potential ϕ in the region exterior to the duct. The potential for the mathematical model shown in Figure 2, is determined by the superposition of three potentials ϕ_0, ϕ_1, ϕ_2, which represent the contribution of the duct face, outer duct wall and extended centerbody duct wall, respectively. This technique is expressed mathematically by

$$\phi\,(r,\theta,z,t) = \phi_0 + \phi_1 + \phi_2 \tag{1}$$

The component velocity potentials are determined by a spatial distribution of monopole acoustic sources on each reflecting surface. For example, the velocity potential due to a distribution of harmonically vibrating sources on the face of the duct is

$$\phi_0 = e^{-i\omega t} \int_0^{2\pi}\int_a^b \frac{\Phi_0(r_0)\, e^{i\omega R_0/c}}{R_0} \cos(m\theta_0) r_0 \, d\theta_0 dr_0 \tag{2}$$

The form of this integral equation is derived from the following assumptions:

- Acoustic velocity at the point P may be determined from a monopole acoustic source $e^{-i(\omega t - kR_0)}/R_0$ which represents an outward propagating wave and is a solution of the wave equation in spherical coordinates.
- The angular dependence of the potential may be described in terms of the characteristic function Cos $(m\theta)$ for a given angular mode number m.
- The radial distribution of sources is given by the spatial distribution function $\Phi_0(r)$ which is determined by the Acoustic Source Distribution Program, Reference 1.

Similarly, the velocity potential terms for the outer and extended inner duct wall surfaces, respectively, are

$$\phi_1 = e^{-i\omega t} \int_{-\infty}^{0}\int_0^{2\pi} \frac{\Phi_1(z_1)\, e^{i\omega R_1/c}}{R_1} \cos(m\theta_1)\, d\theta_1\, b\, dz_1 \tag{3}$$

$$\phi_2 = e^{-i\omega t}\int_0^{\infty}\int_0^{2\pi}\frac{\Phi_2(x_2)\, e^{i\omega R_2/c}}{R_2}\cos(m\theta_2)\, d\theta_2\, a\, dx_2 \tag{4}$$

Acoustic pressure is related to the velocity potential by

$$p = \rho \frac{\partial \phi}{\partial t}$$

The velocity potential of Equation (1) yields the following expression for pressure varying harmonically.

$$p = -i\omega\rho\left\{\phi_0 + \phi_1 + \phi_2\right\} \tag{5}$$

To determine the pressure at the point r, θ on the open face of the duct, the separation distances in Equations (2) through (4) are

$$R_j = \sqrt{(r\cos\theta - \lambda_j\cos\theta_j)^2 + (r\sin\theta - \lambda_j\cos\theta_j)^2} \tag{6}$$

where $j = 0, 1, 2$

$\lambda_0 = r_0$

$\lambda_1 = b$

$\lambda_2 = a$

A useful quantity in the study of sound radiation from ducts is the modal impedance in the axial direction. The modal impedance at the duct exit plane is defined in terms of the acoustic pressure and velocity, in any mode, which are present on the duct exit plane.

$$\bar{Z} = \frac{p}{v}.$$

The modal impedance which is expressed mathematically as a complex quantity, is written in terms of its real and imaginary parts which are normalized by the specific impedance of air ρc, by

$$\bar{Z} = \rho c(\tau - iX) \tag{7}$$

The variable τ is the radiation resistance which is a measure of acoustic energy radiated through the exit plane and the variable X is the radiation reactance which is a measure of the acoustic energy reflected back up the duct.

Acoustic pressure and velocity are related to the impedance through the sound intensity radiated through the duct exit plane. In a stationary medium, the sound intensity is defined to be the time average of pressure times velocity. Mathematically, the intensity is related to the real part of the product of the complex conjugate of the pressure and velocity by

$$I = \frac{1}{2} R_e (p^* v) \tag{8}$$

Sound intensity, normal to an area, is the rate at which the sound energy crosses a unit of that area. Thus, the total sound power radiated through the duct exit is

$$\Pi = \int_a^b \int_0^{2\pi} I \; r_0 \, dr_0 \, d\theta_0 \tag{9}$$

Sound intensity is related to the impedance by two expressions; one which involves only the radiation resistance

$$I = \frac{1}{2} \rho c \tau |v|^2$$

and the other which includes the radiation reactance.

$$I = \frac{1}{2} \frac{\tau |p|^2}{\rho c (\tau^2 + X^2)}$$

These expressions are substituted into Equation (9) and the resulting expressions for radiated energy are equated to yield the following relations for the impedance:

$$\tau = \frac{1}{\rho c} \frac{\iint Re(p^* v) \, r_0 \, dr_0 \, d\theta_0}{\iint |v|^2 \, r_0 \, dr_0 \, d\theta_0} \tag{10}$$

$$X^2 = \frac{1}{(\rho c)^2} \frac{\iint |p|^2 \, r_0 \, dr_0 \, d\theta_0}{\iint |v|^2 \, r_0 \, dr_0 \, d\theta_0} - \tau^2 \tag{11}$$

3.0 METHOD OF SOLUTION

The constraint on this program to be compatible with the Acoustic Source Distribution Program of Reference 1, required that the "Box Method" technique be used to solve Equations (2) through (4). This mathematical technique assumes that the variation of the source distribution over a small portion or box of the reflecting surface is constant, an assumption which allows the integrations to be evaluated by a summation. To facilitate the numerical solution of these equations, the duct exterior face is divided into a sequence of M annular rings, and the duct walls into N and N2 cylindrical segments as shown in Figure 3.

Application of the box method to ϕ_0, the duct face contribution to exit plane pressure, enables the integral equation to be written as

$$\phi_0 = \sum_{I=1}^{M} \Phi_{0_I} \sum_{K=1}^{L} \int_{\Delta r_I} \int_{\Delta \theta_K} \cos(m\theta_0) \frac{e^{i\frac{\omega}{c} R_0}}{R_0} r_0 \, dr_0 \, d\theta_0 \tag{12}$$

For the duct wall contributions, the equations become

$$\phi_1 = \sum_{I=1}^{N} \Phi_{1_I} \sum_{K=1}^{L} \int_{\Delta z_I} \int_{\Delta \theta_K} \cos(m\theta_1) \frac{e^{i\frac{\omega}{c} R_1}}{R_1} b \, dz_1 \, d\theta_1 \tag{13}$$

$$\phi_2 = \sum_{I=1}^{N_2} \Phi_{2_I} \sum_{K=1}^{L} \int_{\Delta z_I} \int_{\Delta \theta_K} \cos(m\theta_2) \frac{e^{i\frac{\omega}{c} R_2}}{R_2} a \, dz_2 \, d\theta_2 \tag{14}$$

Equations (12) through (14) are integrated numerically. The numerical technique used is an even order Gaussian quadrature scheme. The integration for each annular segment is performed by an iterative method. An initial even order quadrature is chosen (usually 2nd or 4th order) and the integral evaluated. The order is increased by 2 and the integration repeated.

The two values of the integral are compared. If they agree to within a specified error, the next segment is integrated. If agreement is not obtained, the order is again increased and the procedure repeated up to a specified maximum order. At this point, the last calculated value of the integral is used and the integration procedure passes to the next segment.

The quadrature method used is the Gauss formula for arbitrary intervals,

$$\int_a^b f(x)\,dx = \frac{b-a}{2} \sum_{j=1}^{NG} f(x_j)\, w_j \tag{15}$$

This relation states that the integral may be approximated by a summation of NG (the order of the quadrature) evaluations of the function at a prescribed abscissa x_j which are multiplied by the associated weight w_j. The abscissae and weights of the Gaussian Quadrature may be found in Reference 3. The referenced abscissae have the normalized range $-1 < y < 1$ and are related to the arbitrary interval abscissae by

$$x_j = \frac{(b+a)}{2} + \frac{(b-a)}{2} y_j$$

The numerical equations for the evaluation of the contributions of the duct face, outer duct wall and extended centerbody wall to the pressure, denoted by p_0, p_1 p_2, respectively, are

$$p_0 = \omega \sum_{I=1}^{M} \Phi_{0_I} \sum_{K=1}^{L} \frac{\Delta r_I\, r_I \Delta\theta_K}{4} \sum_{j}^{NG}\sum_{k}^{NG} \left\{ \frac{\cos(m\theta_k)}{R_0} \sin\left(\frac{\omega}{c} R_0\right) - i\,\frac{\cos(m\theta_k)}{R_0} \cos\left(\frac{\omega}{c} R_0\right) \right\} r_{0_j}\, w_j\, w_k \tag{16}$$

where the quadrature points are

$$r_{0_j} = \frac{\Delta r_I}{2} - \frac{\Delta r_I}{2} x_j$$

$$\theta_k = \frac{\Delta\theta_K}{2} - \frac{\Delta\theta_K}{2} x_k$$

$$p_1 = \omega \sum_{I=1}^{M} \Phi_{1_I} \sum_{K=1}^{L} \frac{b \Delta z_I \Delta\theta_K}{4} \sum_{j}^{NG}\sum_{k}^{NG} \left\{ \frac{\cos(m\theta_k)}{R_1} \sin\left(\frac{\omega}{c} R_1\right) - i\,\frac{\cos(m\theta_k)}{R_1} \cos\left(\frac{\omega}{c} R_1\right) \right\} w_j\, w_k \tag{17}$$

$$p_2 = \omega \sum_{I=1}^{M} \Phi_{2_I} \sum_{K=1}^{L} \frac{a \Delta z_I \Delta\theta_K}{4} \sum_{j}^{NG}\sum_{k}^{NG} \left\{ \frac{\cos(m\theta_k)}{R_2} \sin\left(\frac{\omega}{c} R_2\right) - i\,\frac{\cos(m\theta_k)}{R_2} \cos\left(\frac{\omega}{c} R_2\right) \right\} w_j\, w_k \tag{18}$$

The acoustic pressure distribution on the duct face is determined by performing the operations of Equation (5) for each annular ring.

$$p_I = \rho(p_0 + p_1 + P_2)_I$$

In the evaluation of p_o, the integral $\cos(\frac{\omega}{c}R_o)/R_o$ in Equation (16) is nearly singular when the integration is performed over the box containing the control point. The effect of this singularity on the numerical quadrature is a degradation in its convergence. If the error resulting from a low order quadrature is not acceptable, either a higher order quadrature or a closed form evaluation of Equation (16) must be used.

In view of the extremely poor convergence of the quadrature in Equation (16), a closed form approximation of the integral was used. A technique was developed that consisted of removing the singularity by a change of variables and using the series expansion for the exponential function. The excellent convergence properties of this approximation required only a small number of terms. This integral equation is approximated by

$$-i\omega \iint_{\Delta r \Delta\theta} \frac{e^{i(\frac{\omega}{c}R_0)}}{R_0}\, r\, dr\, d\theta \approx$$

$$2\,\frac{(\omega A)^2}{c} \tan\alpha\, \Gamma_{R_1} + 2\,\frac{(\omega B)^2}{c} \cot\alpha\, \Gamma_{R_2}$$

$$-i\left\{ 2\,\omega A \ln\left[\frac{\sec\alpha + \tan\alpha}{\sec\alpha - \tan\alpha}\right] \Gamma_{I_1} - 2\,\frac{(\omega A)^3}{c^2} \tan\alpha \sec\alpha\, \Gamma_{I_2}\right.$$

$$\left. + 2\,\omega B \ln\left[\frac{\csc\alpha + \cot\alpha}{\csc\alpha - \cot\alpha}\right] \Gamma_{I_3} - 2\,\frac{(\omega B)^3}{c^2} \cot\alpha \csc\alpha\, \Gamma_{I_4} \right\}$$

where

$$A = \frac{r\Delta\theta}{2}$$

$$B = \frac{\Delta r}{2}$$

$$= \text{arc tan}\left[\frac{\Delta r}{r\Delta\theta}\right]$$

$$WA = \frac{\omega A}{c}$$

$$WB = \frac{\omega B}{c}$$

ln is the natural logarithm

$$\Gamma_{R_1} = 1 - (WA)^2(\sec^2\alpha + 2)/36 + (WA)^2\left[\sec^4\alpha + 4/3(\sec^2\alpha + 2)\right]/1800$$

$$- (WA)^6(\sec^6\alpha + 6/5(\sec^4\alpha + 4/3(\sec^2\alpha + 2)))\ /141120$$

$$\Gamma_{R_2} = 1 - (WB)^2(\csc^2\alpha + 2)/36 + (WB)^4\left[\csc^4\alpha + 4/3\ \csc^2\alpha + 8/3\right]/1800$$

$$- (WB)^6(\csc^6\alpha + 6/5(\csc^4\alpha + 4/3\ \csc^2\alpha + 8/3))/141120$$

$$\Gamma_{I_1} = 1 - (WA)^2/12 + (WA)^4/320 - (WA)^6/16128$$

$$\Gamma_{I_2} = 1/6 - (WA)^2(\sec^2\alpha + 3/2)/240 + (WA)^4\left(\frac{\sec^4\alpha}{3} + \frac{5}{12}\sec^2\alpha + 5/8\right)/5040$$

$$\Gamma_{I_3} = 1 - (WB)^2/12 + (WB)^4/320 - (WB)^6/16128$$

$$\Gamma_{I_4} = 1/6 - (WB)^2(\csc^2\alpha + 3/2)/240 + (WB)^4\left(\csc^4\alpha + \frac{5\csc^2\alpha}{4} + \frac{15}{8}\right)/15120$$

The local impedance for each annular ring is calculated from the acoustic velocity and pressure determined at the duct termination.

$$Z_I = \frac{p_I}{v_I}$$

The radiation resistance and the radiation reactance of the modal impedance are calculated from the numerical form of Equations (10) and (11).

$$\tau = \frac{1}{\rho c} \frac{\sum_{I=1}^{M} \mathrm{Re}\,(p^* v)_I (\Delta r_I)^2}{\sum_{I=1}^{M} |v|_I^2 (\Delta r_I)^2} \tag{19}$$

$$\chi = \frac{1}{\rho c} \sqrt{\frac{\sum_{I=1}^{M} |p|_I^2 (\Delta r_I)^2}{\sum_{I=1}^{M} |v|_I^2} - \tau^2 \rho^2 c^2} \tag{20}$$

4.0 RESULTS

4.1 Application

The numerical techniques developed in Section 3 are applied to the mathematical representation of the radiation problem developed in the Acoustic Source Distribution Program, Reference 1. Duct segmentation parameters and corresponding source distributions are input to this program to determine the termination impedance characteristics of the duct configuration. Far-field directivity patterns corrected for termination impedance effects are determined for each mode and then combined to yield the total noise signature. In the event both an inlet and fan duct are to be modeled, their respective far-field directivity patterns can be calculated and then combined to yield the total noise signature of the nacelle.

An example duct radiation problem is presented to illustrate the output of this program. A source distribution which was generated by Reference 1 for a plane wave velocity distribution on the face of a circular duct (example problem 1) was input to this program. The resulting pressure distribution for the duct exit plane is shown in Figure 4. Values of pressure which are normalized by the characteristic impedance of air, ρc are plotted against the radial position of each control box. Behavior of the pressure near the outer duct wall is due to the singular behavior of the source distributions, which is discussed in Reference 1. Since the velocity distribution for a plane wave is uniform across the duct, the resulting local impedance is proportional to the pressure. Thus, the local impedance is shown to vary across the duct cross section. The value of modal impedance that resulted from this analysis is given by its components:

$$\tau = .520$$

$$\chi = 1.93$$

4.2 Limitations

There are several limitations to this analytical program. First, the effect of mean flow within the duct on the termination impedance has been neglected. Although application of this analysis to a nacelle radiation problem does not yield an exact model, it is anticipated that this simplification will not significantly affect results for noise propagation through a locally subsonic flow field of an inlet. However, radiation through an exhaust or jet flow will not be represented properly by the present analysis.

The second limitation to the program is restriction of the specified velocity distribution on the face of the duct to a Cos $(m\theta)$ type of angular dependence. This assumption does not affect the capability of the program to combine different m angular modes in the far-field by superposition of the source distributions. It does, however, restrict this program to the extent that for a velocity distribution whose angular dependence is specified by a combination of different m angular modes a separate analysis must be made for each set of angular modes.

5.0 CONCLUSIONS AND RECOMMENDATIONS

An acoustic radiation analysis has been developed that predicts the angular distribution of acoustic energy in the far-field that is radiated from an unflanged duct. In spite of its many advantages, this acoustic radiation analysis is limited in its range of application. For this reason, this analysis has been relegated to the role of a first generation analysis in a commitment to develop an analytical package which will predict radiation characteristics of a turbofan jet engine. As discussed in Reference 1, the present version of the analysis reaches optimum efficiency when it is applied to low frequency noise radiating from an infinite duct. This and the other limitation due to a $\text{Cos}(m\theta)$ type of angular dependance of the velocity distribution are an inherent property of the "box method" numerical technique.

A second generation version of the program could avoid these limitations by utilizing a collocation technique of evaluating the integrals of Equations (2) through (4). The primary feature of this technique is that a selected analytical expression is used to represent the source distribution. The time saving feature of the collocation technique is that it requires fewer evaluations of the integrals. The collocation procedure would enable the analysis to handle any combination of angular modes, a result which would remove the angular modal coupling limitation and eliminate the need for a large number of angular boxes.

In addition to the collocation procedure, a second generation version of the analysis should contain a method of accounting for a mean flow within the duct. This method should account for the effects of velocity gradients on the distortion of the radiation field by the jet.

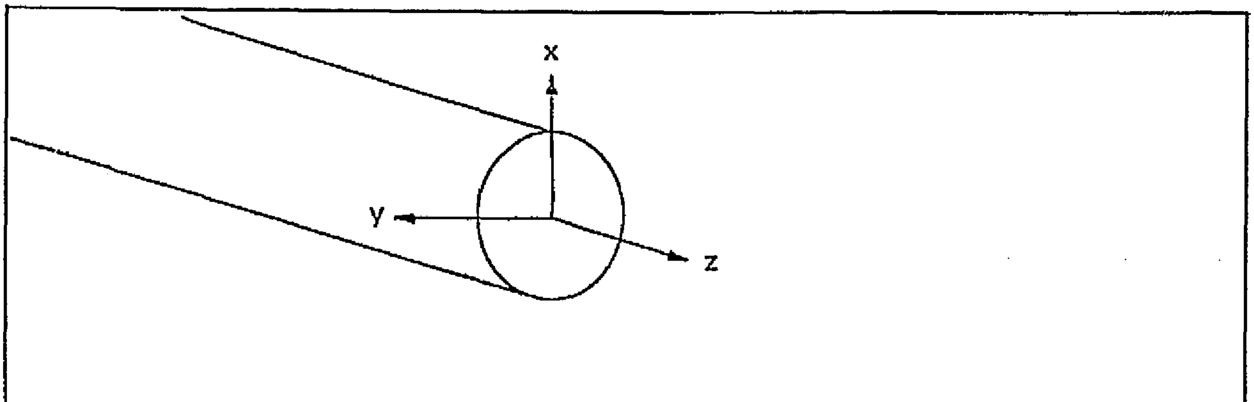
6.0 REFERENCES

1. Beckemeyer, R., and Sawdy, D., "Acoustic Radiation from Lined Unflanged Ducts — Acoustic Source Distribution Program," NASA CR 120849, December 1971

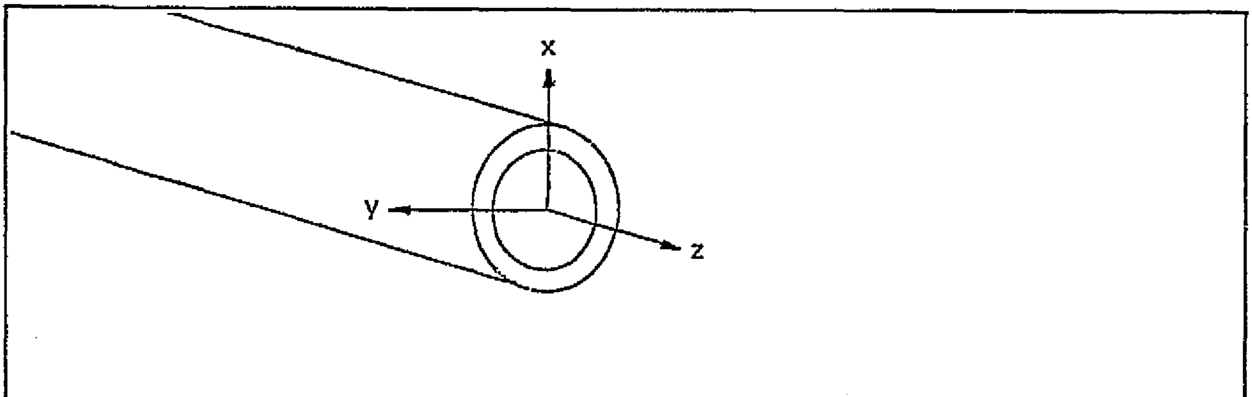
2. Beckemeyer, R., and Sawdy, D., "Acoustic Radiation from Lined Unflanged Ducts — Directivity Index Program," Nasa CR 120850, December 1971

3. Abramowitz and Stegun, "Handbook of Mathematical Functions," Dover Publications, 1965.

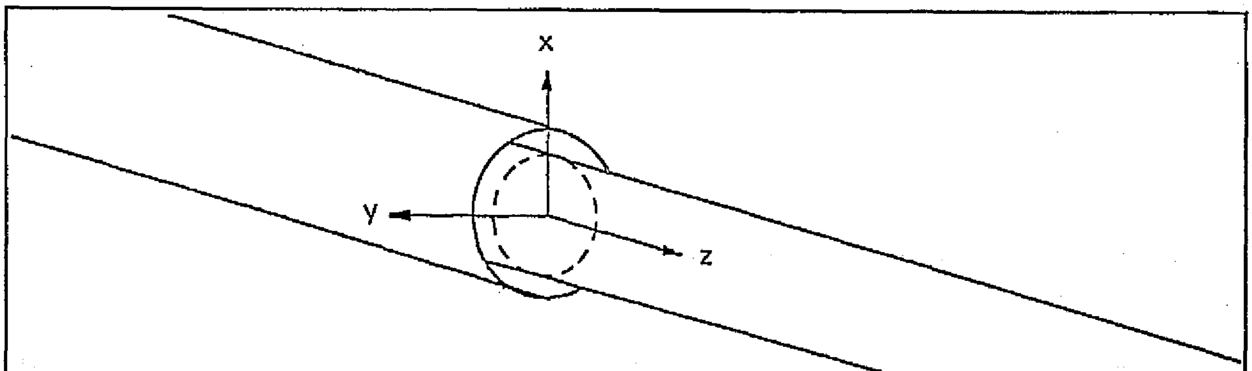
7.0 FIGURES



A. Circular Semi-Infinite Duct

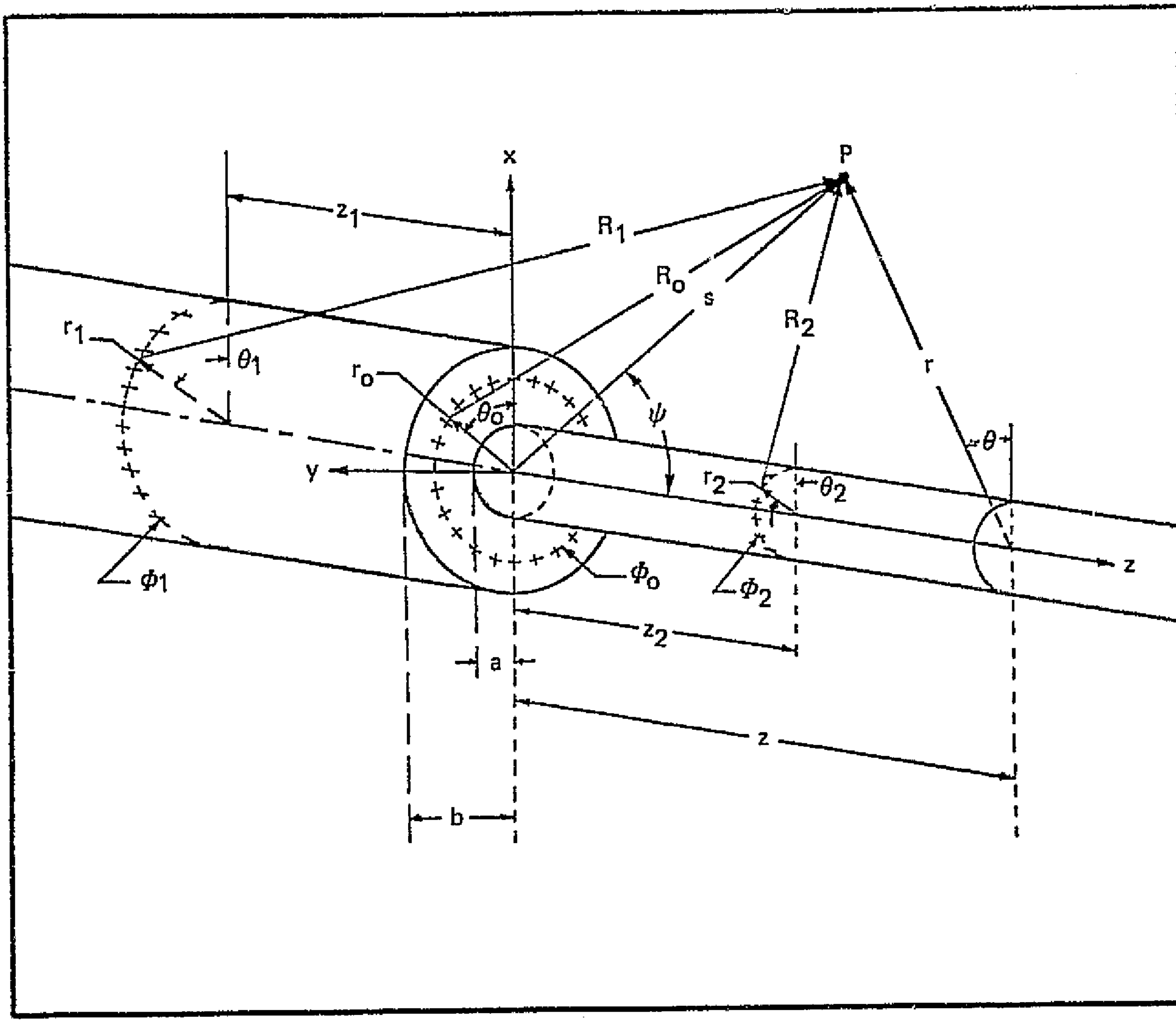


B. Annular Semi-Infinite Duct

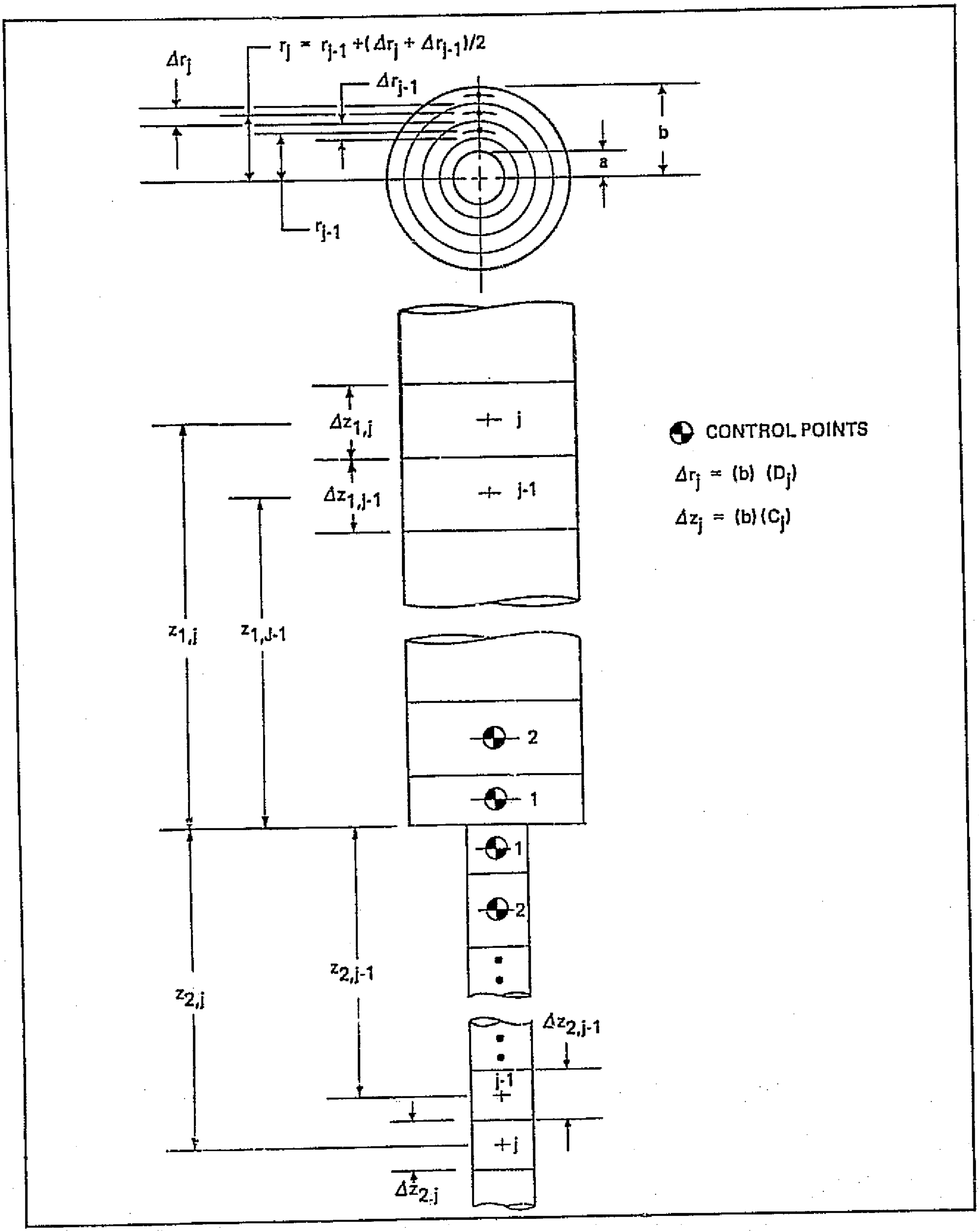


C. Annular Semi-Infinite Duct
With Infinite Centerbody

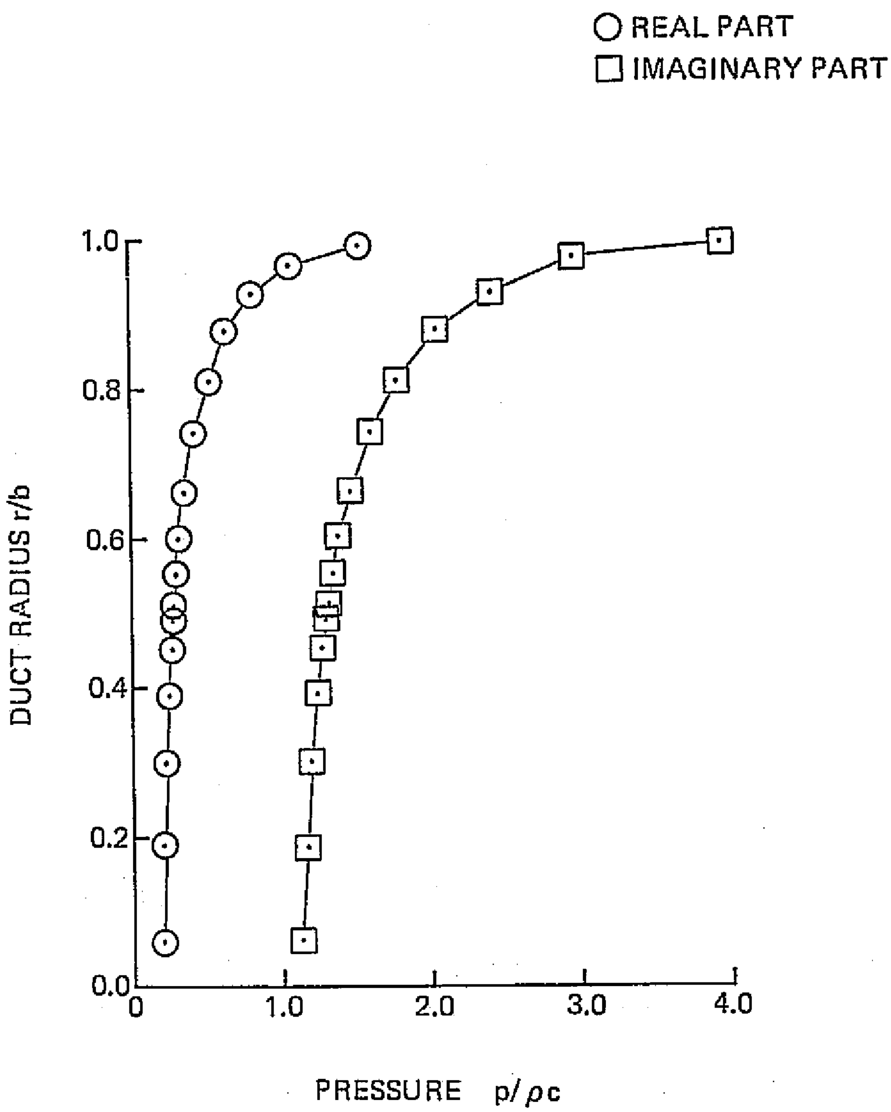
POSSIBLE DUCT CONFIGURATIONS
FIGURE 1



GEOMETRY
FIGURE 2



BOX METHOD INTEGRATION AND CONTROL BLOCKS
FIGURE 3



PRESSURE DISTRIBUTION ON DUCT EXIT PLANE
FIGURE 4

APPENDIX 1

PROGRAM DESCRIPTION

NOMENCLATURE

Computer Program Variables

A	Centerbody Radius
ALIMP	Local Impedance
B	Outer Duct Radius
C(I)	Dimensionless Size of Ith Box on Duct Outer Wall
CHI	Radiation Reactance
C2(I)	Dimensionless Size of Ith Box on Extended Centerbody Wall
CHK	Convergency Criterion
D(I)	Dimensionless Size of Ith Box on Duct Exit Face
INK	Configuration Parameter
K	No. of Polar Angles at which Far Field Data is Required
L	No. of Angular Boxes
MW	Angular Mode No.
M	No. of Angular Boxes
N	No. of Cylindrical Boxes on Duct Wall
N2	No. of Cylindrical Boxes on Extended Centerbody Wall
P(I)	Pressure Distribution of Ith Box
PHI(I)	Source Strength Distribution of Ith Box
R	Radial Coordinate
RO,RI	Distance between Point on Duct and Field Point
RHO	Density of Air
S	Sound Speed in Air
TAU	Radiation Resistance
TT	Angular Coordinate
TI	Angular Coordinate Integration Variable
V(I)	Velocity Distribution of Ith Box on Exit Face
W	Frequency Radians/Second
WDS	Wave Number W/S
XANG	Nondimensional Angular Size for Control Box

The basic operation of the Duct Termination Impedance Computer Program is described and the input/output formats are listed. A flow chart of the program is presented in Appendix 2 and listings of the main program and its subroutines are presented in Appendices 3 through 7. An example problem which illustrates the input preparation and output format of the program is presented in Appendix 8.

INPUT

The input data for the program consists of 23 variables whose symbols and card formats are described below:

CARD OR CARD SET	FORMAT		DATA
1	3612	K	Number of angular points, ψ, at which the directivity index will be calculated.
		M	Number of annular rings used to represent the face of the duct.
		N	Number of cylindrical rings used to represent the outer duct wall.
		N2	Number of cylindrical rings used to represent the extended inner duct wall (if present)
		INK	Indicator for extending the inner duct wall past the duct face. 0 does not extend the wall, 1 extends it to infinity.
		MW	Angular mode number.
		NN	Maximum number of passes to be made through the quadrature iteration loop. NN is related to the highest quadrature order NG by $NN = \frac{NG - 2}{2}$.
		L	Number of angular segments.
		LP	Number of integrations in integration routine.
2	6E12.6	A	Inner duct radius
		B	Outer duct radius
		W	Circular frequency, radians/second

		S	Speed of sound in units consistent with those of A and B
		CHK	Convergence criteria for numerical integration
		XANG	Nondimensional angular size for control box Radians = 2 π(XANG).
		RHO	Air density in units consistent with A, B, and S.
3	6E12.6	C(I), I = 1, N	Outer duct wall box lengths. These lengths are normalized by the outer duct radius, numbered with C(1) closest to the duct exit plane and I increasing in the negative z direction.
4	6E12.6	D(I), I = 1, M	Annular ring box widths. These widths are normalized by the outer duct radius, numbered D(1) at the duct centerline (or adjacent to the centerbody wall duct exit plane intersection/and I increasing to D(M) at the lip of the duct.
5*	6E12.6	C2(I) I = 1, N2	Extended centerbody wall box lengths. These lengths are normalized by the outer duct radius, numbered with C2(I) closest to the duct exit plane and I increasing in the positive z direction.
6	6E12.6	V(I), I = 1, M	Velocity distribution on the duct exit plane. This distribution is numbered with V(1) at the duct centerline (or adjacent to the centerbody wall duct exit plane intersection) and I increasing to V(M) at the lip of the duct.
7	6E12.6	PHI0 (I) = 1, M	Source distribution on duct face. Method of numbering this source distribution is analogous to D(I).
8	6E12.6	PHI1(I), I = 1, N	Source distribution on outer duct wall. Method of numbering this source distribution is analogous to C(I).
9*	6E12.6	PHI2(I) I = 1, N2	Source distribution on extended centerbody wall. Method of numbering this source distribution is analogous to C2(I).

*Card sets 5 and 9 are needed only if INK = 1.

PROGRAM FLOW

Program operation consists of two separate parts. The first part is a loop which calculates the local pressure distribution and the resulting local impedance. Equations (16) through (18) are solved to yield the exit plane and duct wall contributions p_0, p_1, p_2 to the pressure distribution on the duct exit plane. The second part calculates the value of the modal impedance. Equations (19) and (20) are solved to yield the resistive and reactive components of the modal impedance.

Four subroutines are required to calculate the pressure distribution on the duct face. PRES0, PRES1, and PRES2 calculate the contribution of exit plane and duct wall contributions to the pressure distribution. GAUSS is the subroutine that provides the weights and abscissae for the Gaussian quadrature described in Equation (15). Listings of PRES0, PRES1, PRES2, and GAUSS are presented in Appendices 4 through 7, respectively.

OUTPUT

The output format consists of three groups of data:

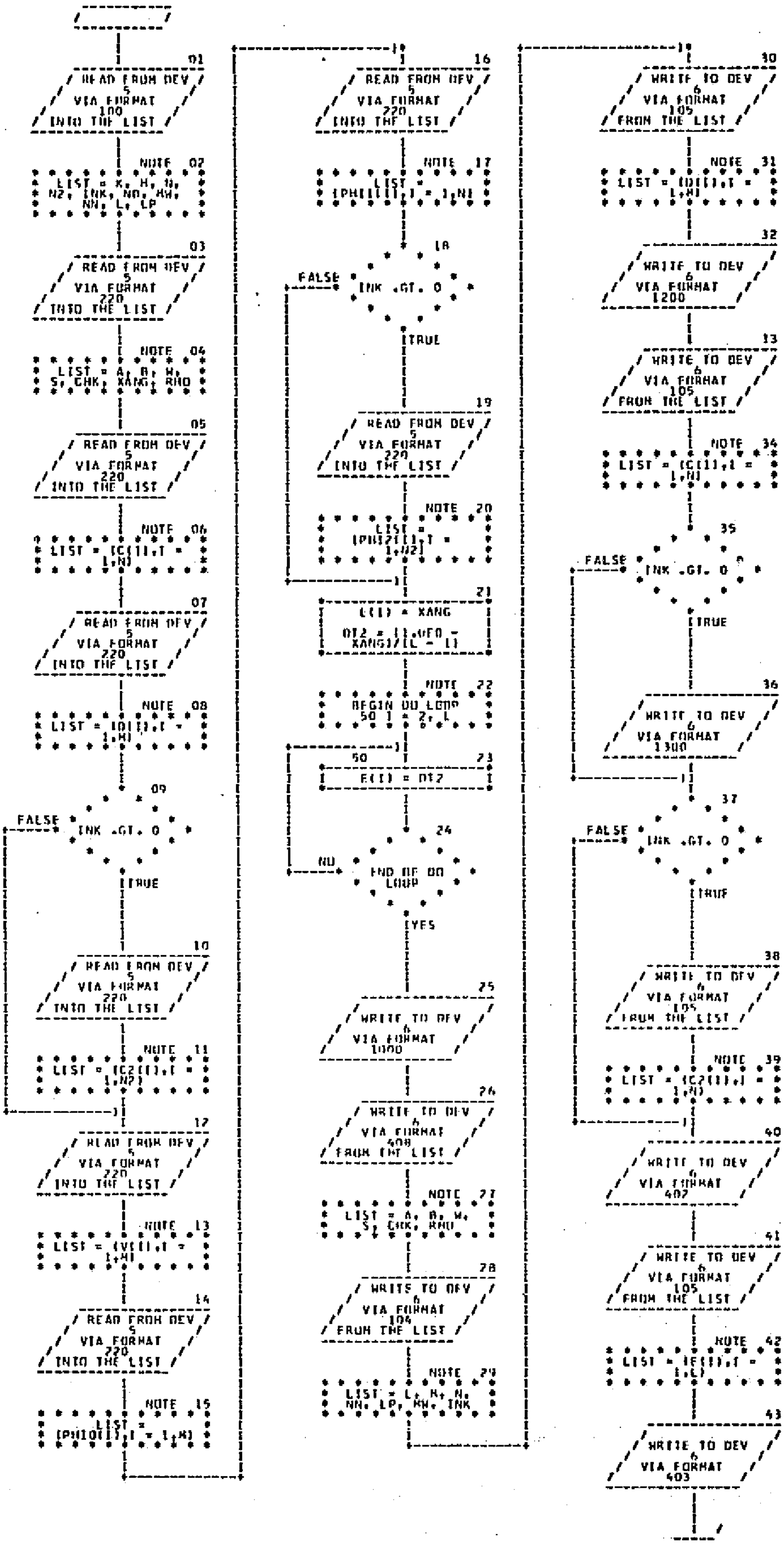
1. Input Data — All pertinent input data are printed out.

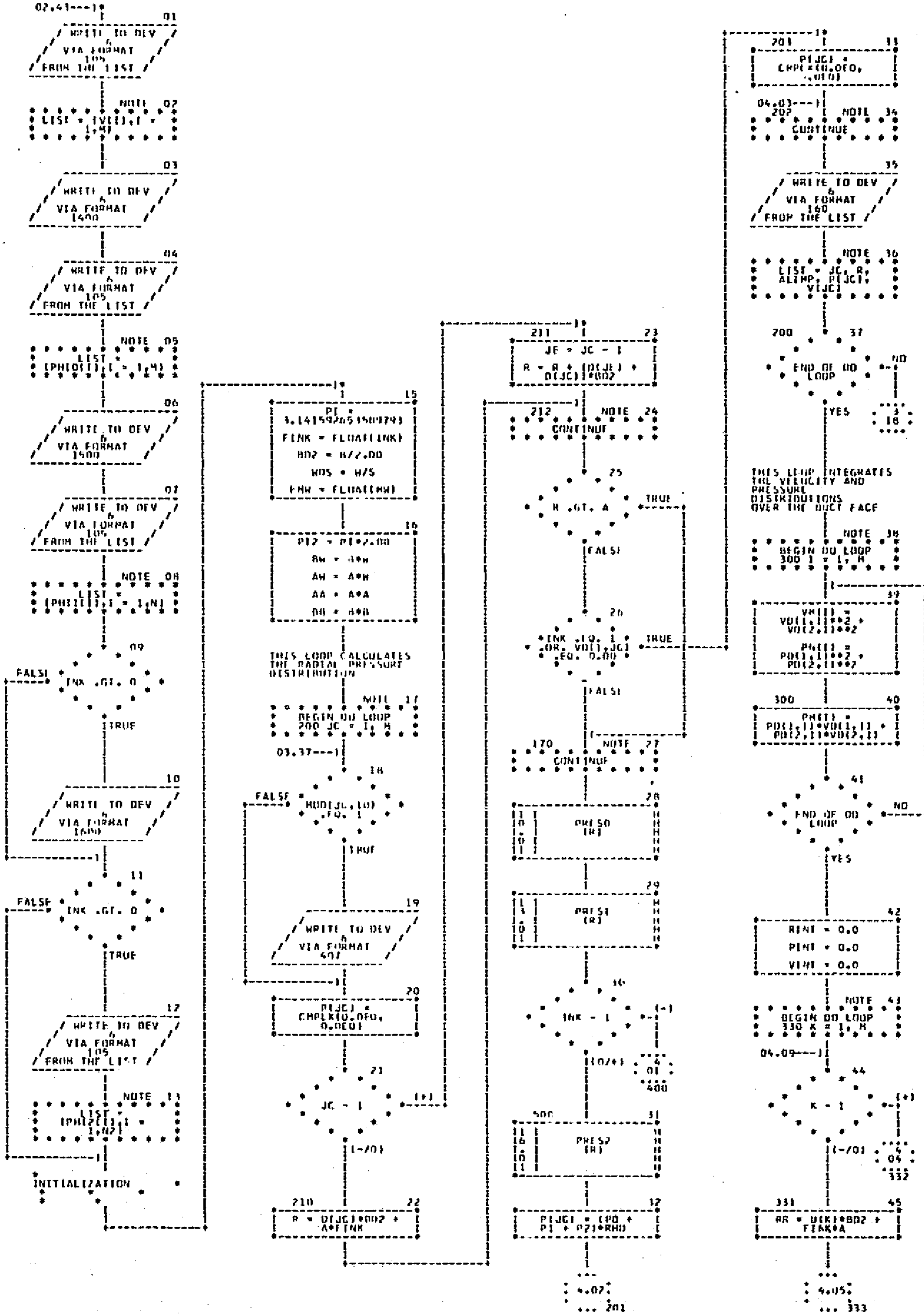
2. Distribution of pressure and impedance — Values of local impedance, pressure and velocity for each annular ring and its corresponding radial position are printed out in row format. For each angular integration that fails to meet the convergence criteria, values of angular position, radial position, differences of the real and imaginary parts of the integral are printed out.

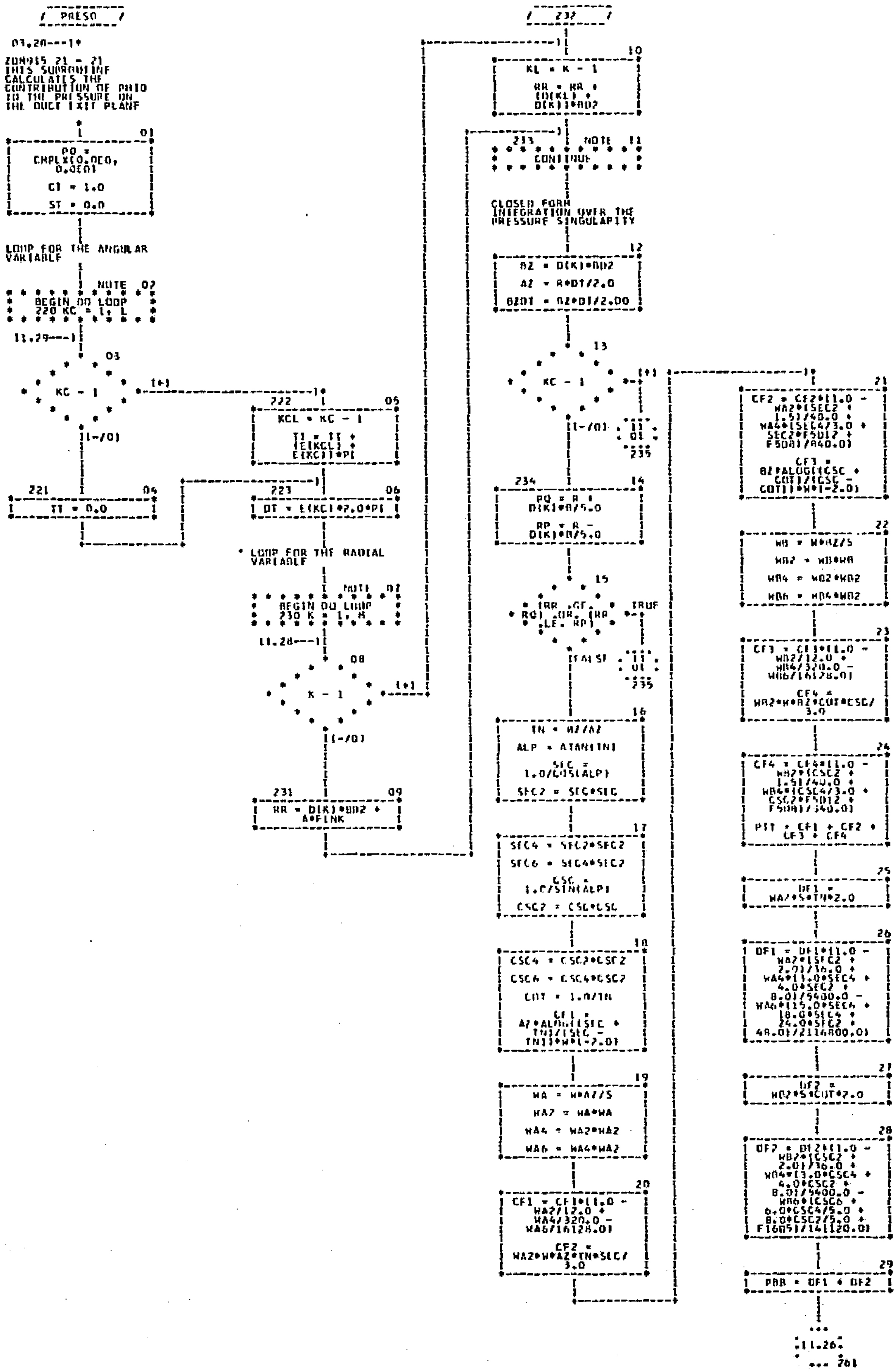
3. Modal Impedance — The resistive and reactive parts of the modal impedance are printed.

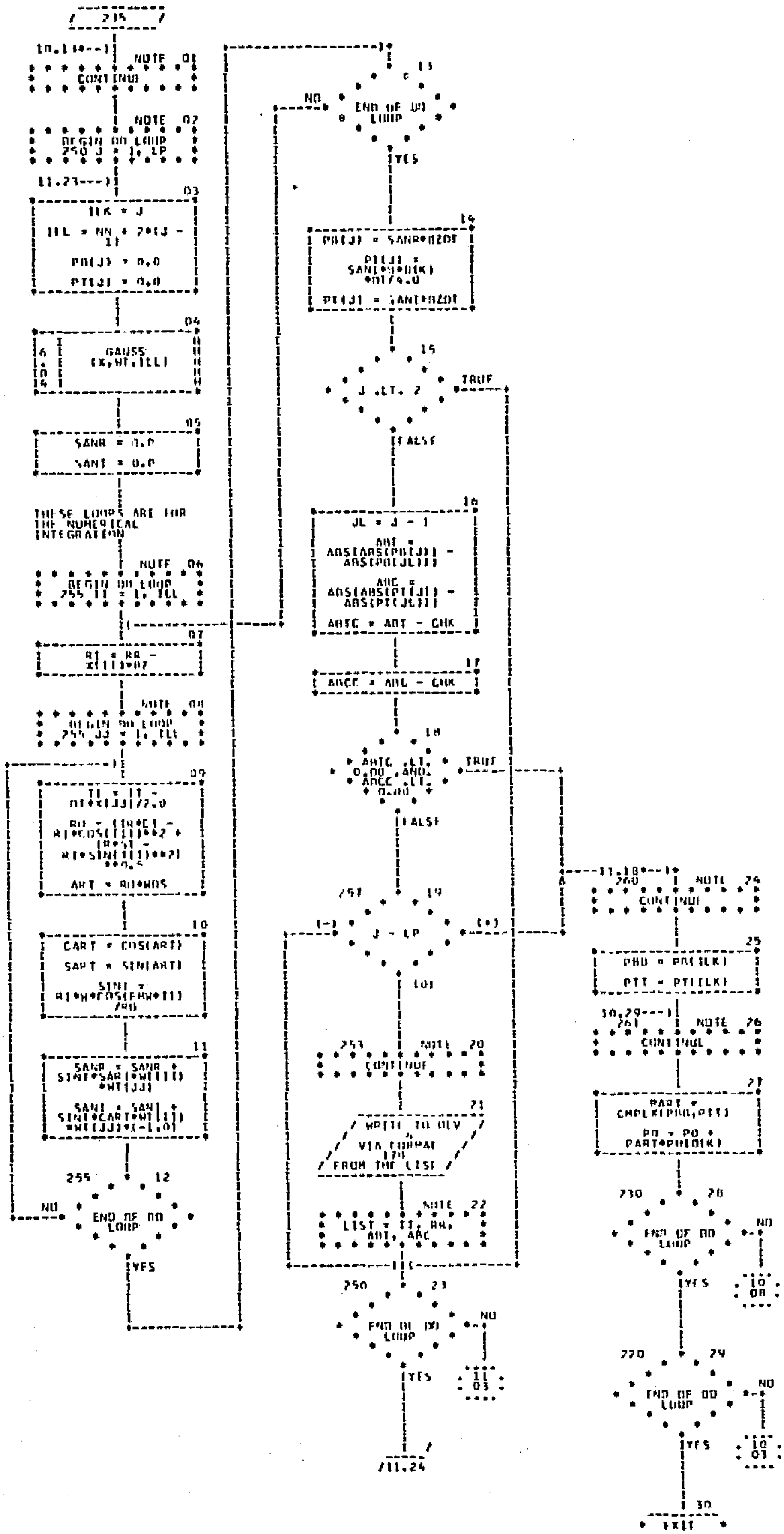
APPENDIX 2

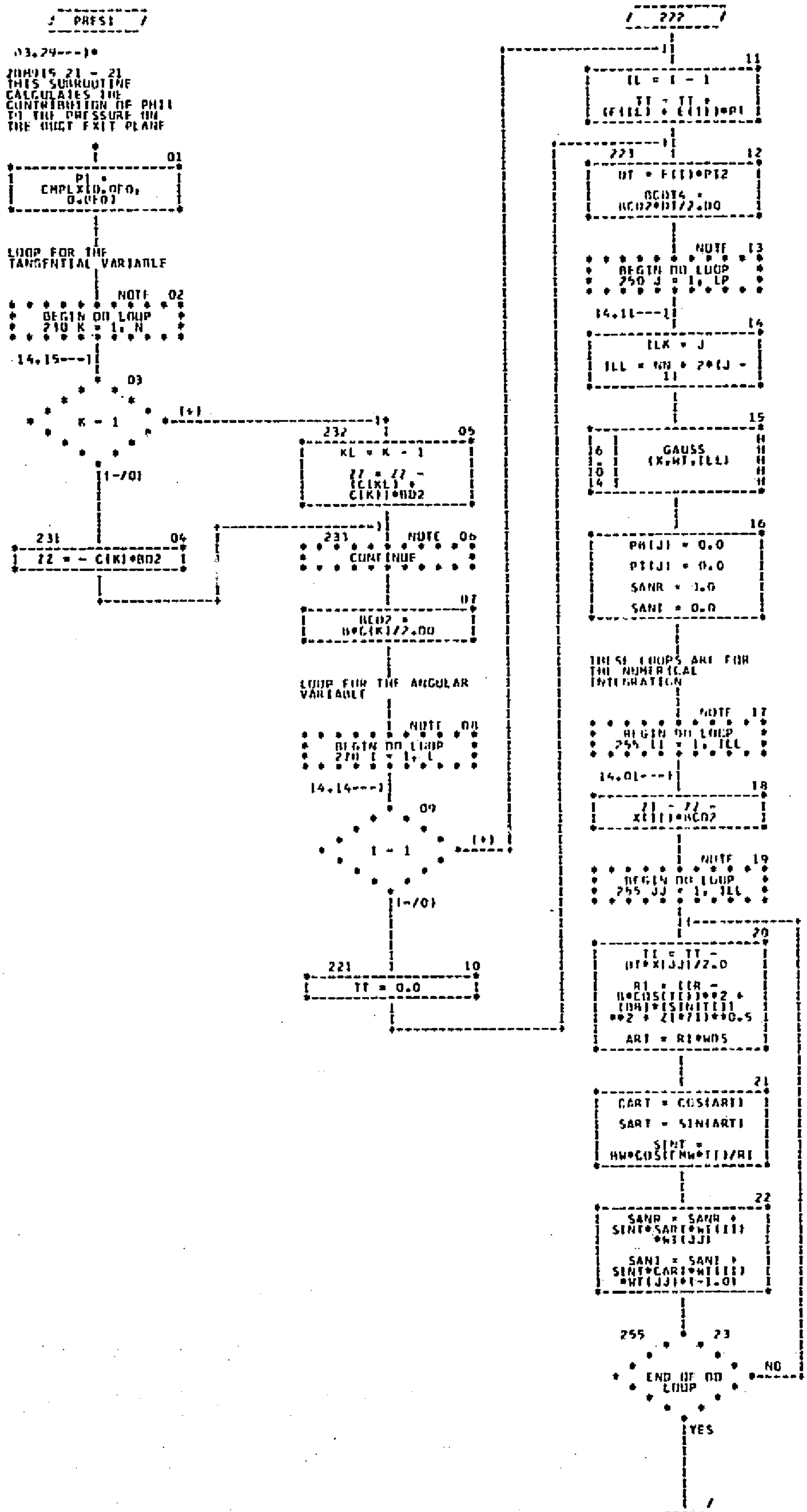
PROGRAM FLOW CHARTS

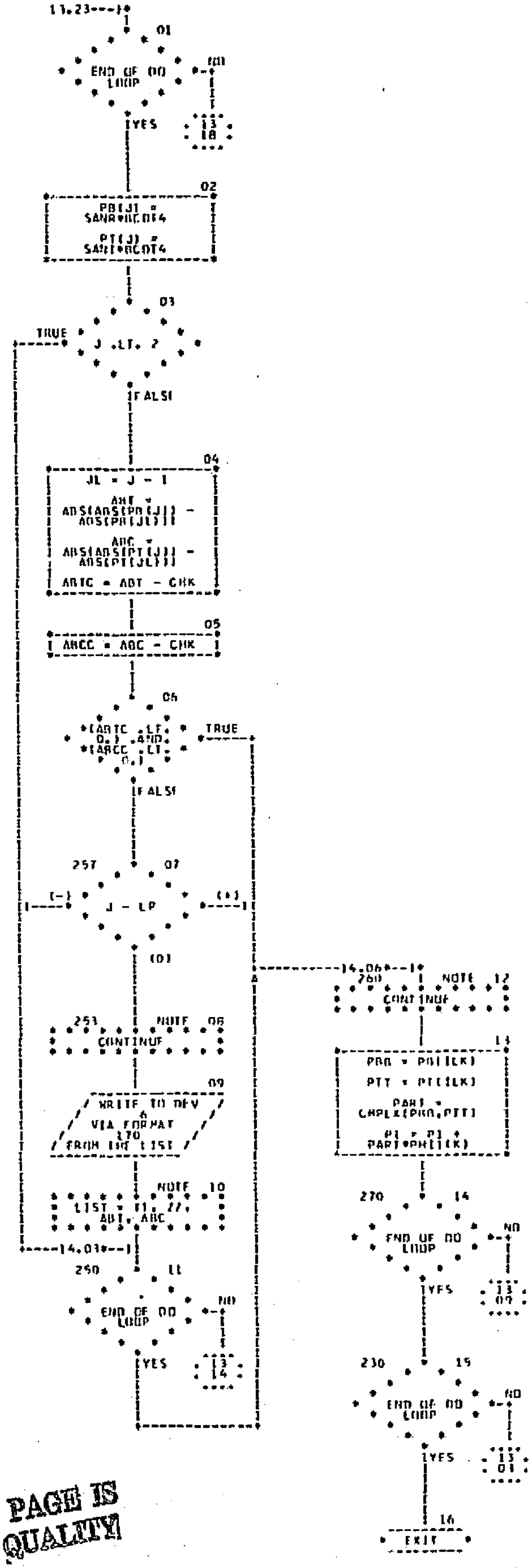

01 READ FROM DEV 5 VIA FORMAT 100 INTO THE LIST
NOTE 02 LIST = K, M, N, N2, INK, NO, MW, NN, L, LP
03 READ FROM DEV 5 VIA FORMAT 220 INTO THE LIST
NOTE 04 LIST = A, B, W, S, CHK, XANG, RHO
05 READ FROM DEV 5 VIA FORMAT 220 INTO THE LIST
NOTE 06 LIST = (C(I),I = 1,N)
07 READ FROM DEV 5 VIA FORMAT 220 INTO THE LIST
NOTE 08 LIST = (D(I),I = 1,M)
09 INK .GT. 0
FALSE
TRUE
10 READ FROM DEV 5 VIA FORMAT 220 INTO THE LIST
NOTE 11 LIST = (C2(I),I = 1,N2)
12 READ FROM DEV 5 VIA FORMAT 220 INTO THE LIST
NOTE 13 LIST = (V(I),I = 1,M)
14 READ FROM DEV 5 VIA FORMAT 220 INTO THE LIST
NOTE 15 LIST = (PHI0(I),I = 1,M)
16 READ FROM DEV 5 VIA FORMAT 220 INTO THE LIST
NOTE 17 LIST = (PHI1(I),I = 1,N)
18 INK .GT. 0
FALSE
TRUE
19 READ FROM DEV 5 VIA FORMAT 220 INTO THE LIST
NOTE 20 LIST = (PHI2(I),I = 1,N2)
21 E(1) = XANG
DT2 = (1.0E0 - XANG)/(L - 1)
NOTE 22 BEGIN DO LOOP 50 I = 2, L
50
23 E(I) = DT2
24 END OF DO LOOP
NO
YES
25 WRITE TO DEV 6 VIA FORMAT 1000
26 WRITE TO DEV 6 VIA FORMAT 408 FROM THE LIST
NOTE 27 LIST = A, B, W, S, CHK, RHO
28 WRITE TO DEV 6 VIA FORMAT 104 FROM THE LIST
NOTE 29 LIST = L, M, N, NN, LP, MW, INK
30 WRITE TO DEV 6 VIA FORMAT 105 FROM THE LIST
NOTE 31 LIST = (D(I),I = 1,M)
32 WRITE TO DEV 6 VIA FORMAT 1200
33 WRITE TO DEV 6 VIA FORMAT 105 FROM THE LIST
NOTE 34 LIST = (C(I),I = 1,N)
35 INK .GT. 0
FALSE
TRUE
36 WRITE TO DEV 6 VIA FORMAT 1300
37 INK .GT. 0
FALSE
TRUE
38 WRITE TO DEV 6 VIA FORMAT 105 FROM THE LIST
NOTE 39 LIST = (C2(I),I = 1,N)
40 WRITE TO DEV 6 VIA FORMAT 402
41 WRITE TO DEV 6 VIA FORMAT 105 FROM THE LIST
NOTE 42 LIST = (E(I),I = 1,L)
43 WRITE TO DEV 6 VIA FORMAT 403
3.01


WRITE TO DEV 6 VIA FORMAT 105 FROM THE LIST
NOTE 02 LIST = (V(I),I = 1,M)
WRITE TO DEV 6 VIA FORMAT 1400
WRITE TO DEV 6 VIA FORMAT 105 FROM THE LIST
NOTE 05 LIST = (PHI0(I),I = 1,M)
WRITE TO DEV 6 VIA FORMAT 1500
WRITE TO DEV 6 VIA FORMAT 105 FROM THE LIST
NOTE 08 LIST = (PHI1(I),I = 1,N)
INK .GT. 0
FALSE
TRUE
WRITE TO DEV 6 VIA FORMAT 1600
INK .GT. 0
WRITE TO DEV 6 VIA FORMAT 105 FROM THE LIST
NOTE 13 LIST = (PHI2(I),I = 1,N2)
INITIALIZATION
FINK = FLOAT(INK)
BD2 = B/2.D0
BOS = B/S
FNR = FLOAT(NR)
PI2 = PI*2.D0
THIS LOOP CALCULATES THE RADIAL PRESSURE DISTRIBUTION
NOTE 17 BEGIN DO LOOP 200 JC = 1, M
WRITE TO DEV 6 VIA FORMAT 407
P(JC) = CMPLX(0.0E0, 0.0E0)
JC - 1
R = D(JC)*BD2 + A*FINK
JE = JC - 1
CONTINUE
R .GT. A
INK .EQ. 1 .OR. V0(1,JC) .EQ. 0.D0
TRUE
FALSE
CONTINUE
PRES0 (R)
PRES1 (R)
INK - 1
PRES2 (R)
WRITE TO DEV 6 VIA FORMAT 160 FROM THE LIST
NOTE 36 LIST = JC, R, ALIMP, P(JC), V(JC)
END OF DO LOOP
YES
NO
THIS LOOP INTEGRATES THE VELOCITY AND PRESSURE DISTRIBUTIONS OVER THE DUCT FACE
NOTE 38 BEGIN DO LOOP 300 I = 1, M
RINT = 0.0
PINT = 0.0
VINT = 0.0
NOTE 43 BEGIN DO LOOP 330 K = 1, M
K - 1

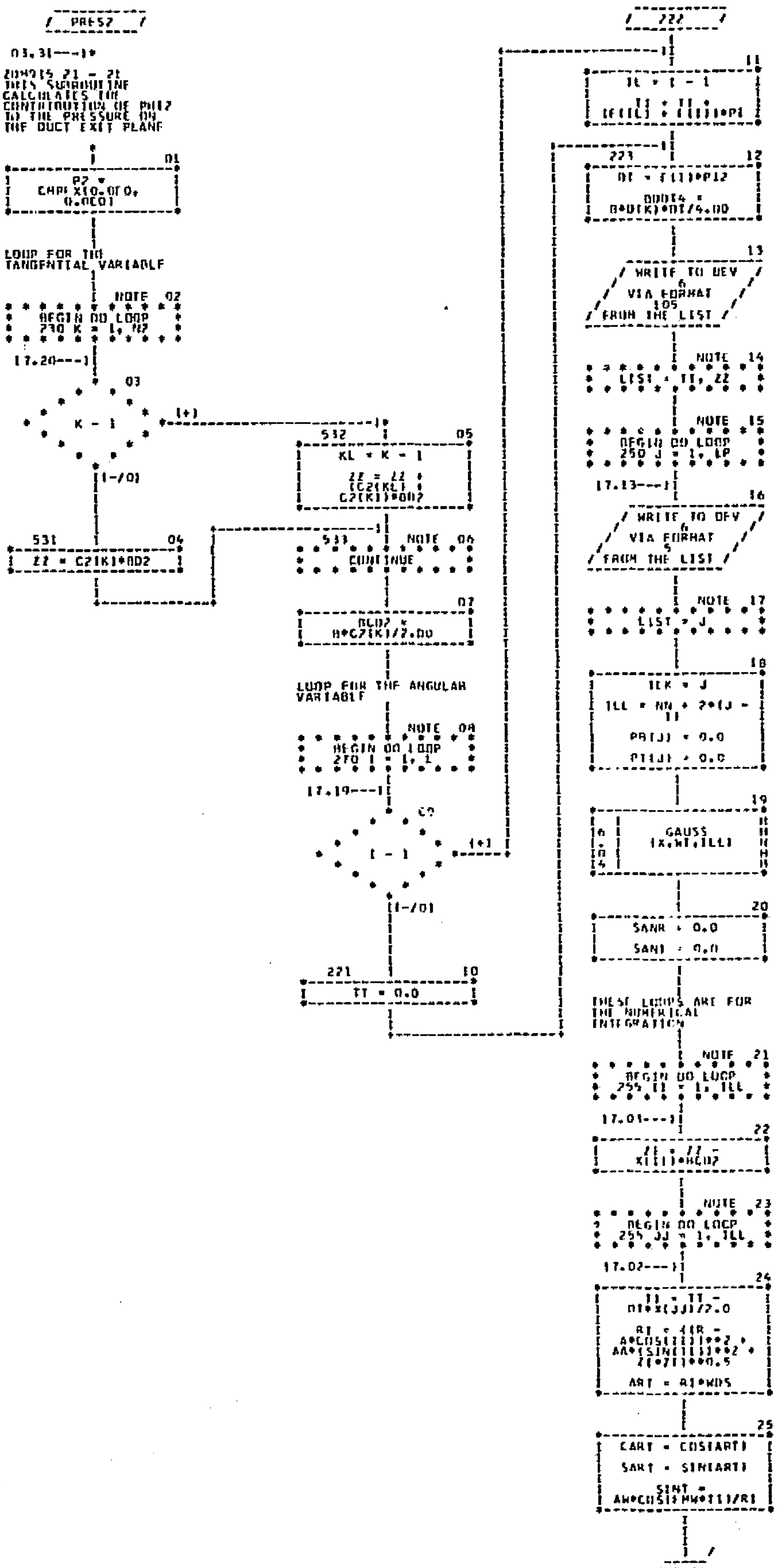

PRESO
03.20---1*
ZONOISS 21 - 21
THIS SUBROUTINE CALCULATES THE CONTRIBUTION OF PHID TO THE PRESSURE ON THE DUCT EXIT PLANE
01
PO = CMPLX(0.0E0, 0.0E0)
CT = 1.0
ST = 0.0
LOOP FOR THE ANGULAR VARIABLE
NOTE 02
BEGIN DO LOOP 220 KC = 1, L
11.29---1
03
KC - 1
(+)
(-/0)
221
04
TT = 0.0
222
05
KCL = KC - 1
TT = TT + (E(KCL) + E(KC))*PI
223
06
DT = E(KC)*2.0*PI
LOOP FOR THE RADIAL VARIABLE
NOTE 07
BEGIN DO LOOP 230 K = 1, N
11.28---1
08
K - 1
(+)
(-/0)
231
09
RR = D(K)*BD2 + A*FINK
232
10
KL = K - 1
RR = RR + (D(KL) + D(K))*BD2
233
NOTE 11
CONTINUE
CLOSED FORM INTEGRATION OVER THE PRESSURE SINGULARITY
12
BZ = D(K)*BD2
AZ = R*DT/2.0
BZDT = BZ*DT/2.00
13
KC - 1
(+)
(-/0)
11. 01 235
234
14
RQ = R + D(K)*B/5.0
RP = R - D(K)*B/5.0
15
(RR .GT. RQ) .OR. (RP .LE. RP)
TRUE
FALSE
11. 01 235
16
TN = BZ/AZ
ALP = ATAN(TN)
SEC = 1.0/COS(ALP)
SEC2 = SEC*SEC
17
SEC4 = SEC2*SEC2
SEC6 = SEC4*SEC2
CSC = 1.0/SIN(ALP)
CSC2 = CSC*CSC
18
CSC4 = CSC2*CSC2
CSC6 = CSC4*CSC2
COT = 1.0/TN
CF1 = AZ*ALOG((SEC + TN)/(SEC - TN))*W*(-2.0)
19
WA = W*AZ/5
WA2 = WA*WA
WA4 = WA2*WA2
WA6 = WA4*WA2
20
CF1 = CF1*(1.0 - WA2/12.0 + WA4/320.0 - WA6/16128.0)
CF2 = WA2*W*AZ*TN*SEC/3.0
21
CF2 = CF2*(1.0 - WA2*(SEC2 + 1.5)/40.0 + WA4*(SEC4/3.0 + SEC2*F5D12 + F5D8)/840.0)
CF3 = BZ*ALOG((CSC + COT)/(CSC - COT))*W*(-2.0)
22
WB = W*BZ/5
WB2 = WB*WB
WB4 = WB2*WB2
WB6 = WB4*WB2
23
CF3 = CF3*(1.0 - WB2/12.0 + WB4/320.0 - WB6/16128.0)
CF4 = WB2*W*BZ*COT*CSC/3.0
24
CF4 = CF4*(1.0 - WB2*(CSC2 + 1.5)/40.0 + WB4*(CSC4/3.0 + CSC2*F5D12 + F5D8)/840.0)
PTT = CF1 + CF2 + CF3 + CF4
25
DF1 = WA2*S*TN*2.0
26
DF1 = DF1*(1.0 - WA2*(SEC2 + 2.0)/36.0 + WA4*(3.0*SEC4 + 4.0*SEC2 + 8.0)/5400.0 - WA6*(15.0*SEC6 + 18.0*SEC4 + 24.0*SEC2 + 48.0)/2116800.0)
27
DF2 = WB2*S*COT*2.0
28
DF2 = DF2*(1.0 - WB2*(CSC2 + 2.0)/36.0 + WB4*(3.0*CSC4 + 4.0*CSC2 + 8.0)/5400.0 - WB6*(CSC6 + 6.0*CSC4/5.0 + 8.0*CSC2/5.0 + F16D5)/141120.0)
29
PBB = DF1 + DF2
11.26 261


CONTINUE
BEGIN DO LOOP 250 J = 1, LP
IIK = J
PR(J) = 0.0
PT(J) = 0.0
GAUSS (X,WT,ILL)
SANR = 0.0
SANT = 0.0
THESE LOOPS ARE FOR THE NUMERICAL INTEGRATION
BEGIN DO LOOP 255 II = 1, ILL
BEGIN DO LOOP 255 JJ = 1, ILL
ART = RO*ROS
CART = COS(ART)
SART = SIN(ART)
END OF DO LOOP
YES
NO
END OF DO LOOP
PT(J) = SANT*DZDT
TRUE
FALSE
JL = J - 1
ABTC = ABT - CHK
ABCC = ABC - CHK
J - LP
CONTINUE
WRITE TO DEV
FROM THE LIST
LIST = II, RR, ABT, ABC
END OF DO LOOP
CONTINUE
PRO = PR(ILK)
PTI = PT(ILK)
CONTINUE
END OF DO LOOP
END OF DO LOOP
EXIT

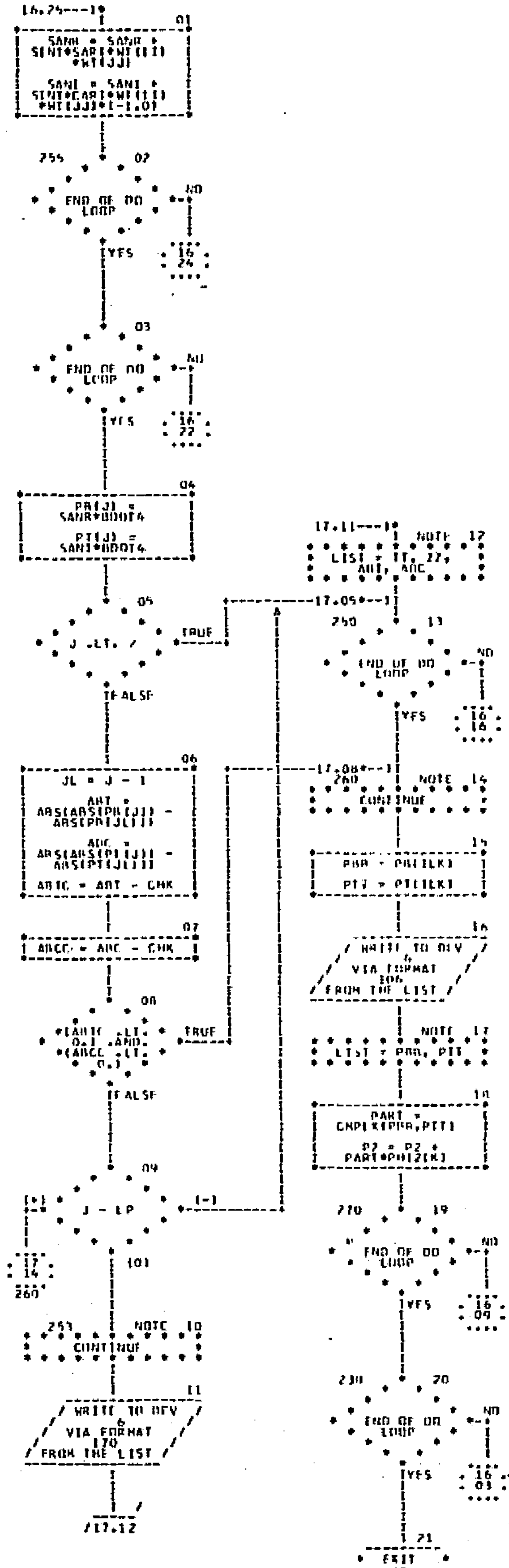

PRES1
THIS SUBROUTINE CALCULATES THE CONTRIBUTION OF PHI1 TO THE PRESSURE ON THE DUCT EXIT PLANE
LOOP FOR THE TANGENTIAL VARIABLE
BEGIN DO LOOP 210 K = 1, N
K - 1
KL = K - 1
ZZ = - C(K)*BD2
CONTINUE
LOOP FOR THE ANGULAR VARIABLE
BEGIN DO LOOP 220 I = 1, L
I - 1
TT = 0.0
222
IL = I - 1
GAUSS
PHI(J) = 0.0
PI(J) = 0.0
SANR = 0.0
SANI = 0.0
THESE LOOPS ARE FOR THE NUMERICAL INTEGRATION
ART = RI*WDS
CART = COS(ART)
SART = SIN(ART)
END OF DO LOOP
NO
YES
14.01

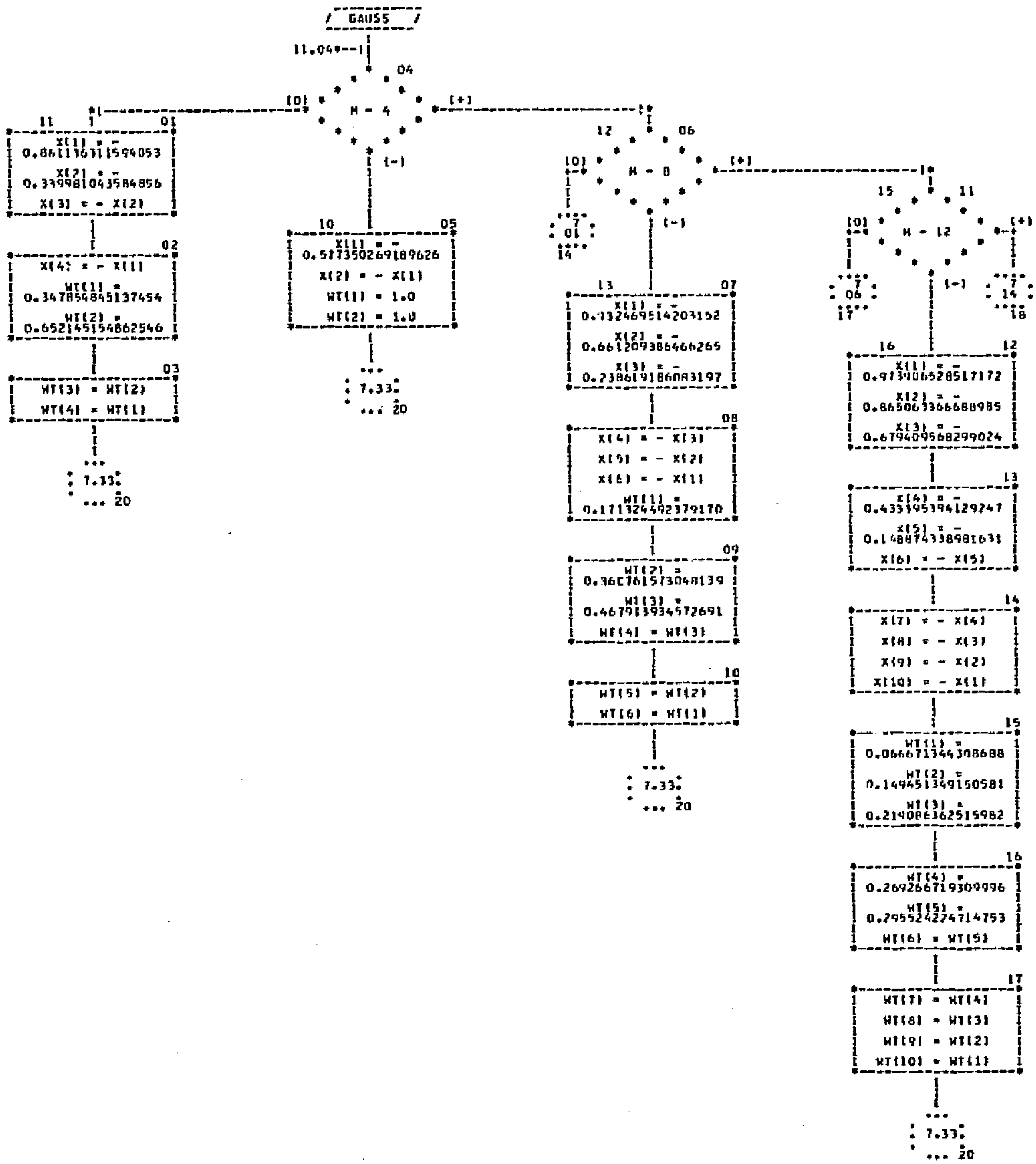


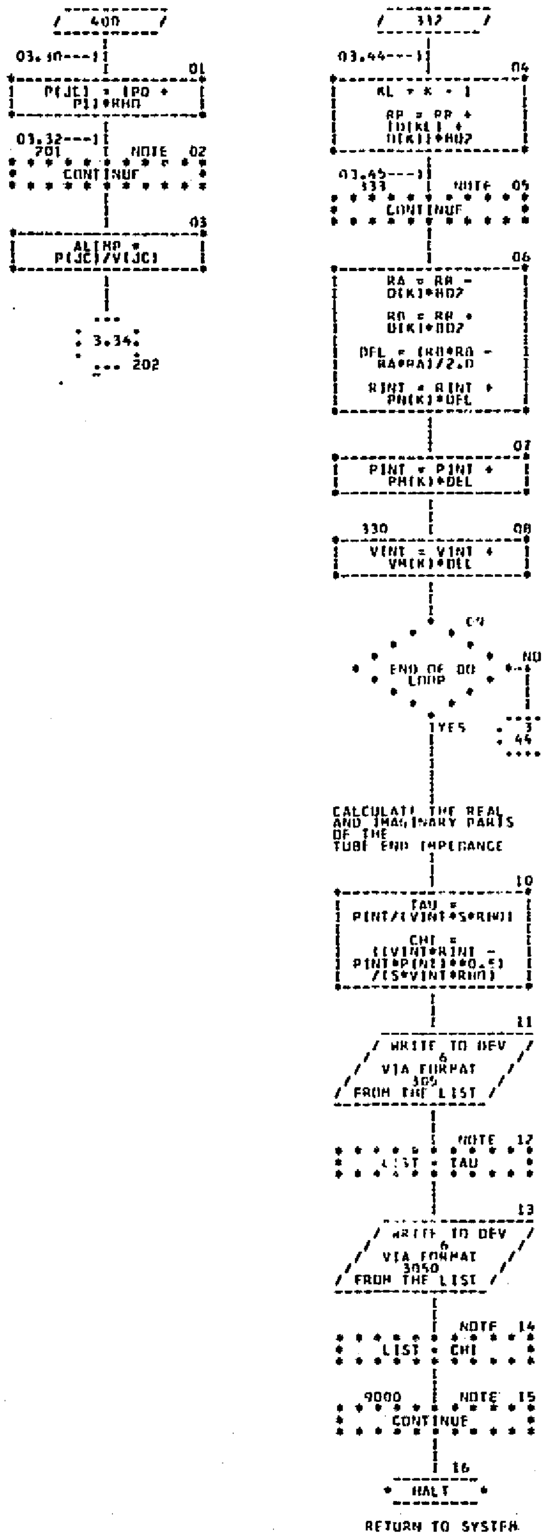
END OF DO LOOP
NO
YES
PB(J) = SANR*RCDT4
PT(J) = SANI*RCDT4
TRUE
J .LT. 2
FALSE
JL = J - 1
ABT = ABS(ABS(PB(J)) - ABS(PB(JL)))
ABC = ABS(ABS(PT(J)) - ABS(PT(JL)))
ABTC = ABT - CHK
ABCC = ABC - CHK
(ABTC .LT. 0.) .AND. (ABCC .LT. 0.)
TRUE
FALSE
J - LP
(-)
(+)
(0)
CONTINUE
NOTE
WRITE TO DEV 6 VIA FORMAT 170 FROM THE LIST
LIST = I1, I2, ABT, ABC
END OF DO LOOP
NO
YES
CONTINUE
NOTE
PBB = PB(LK)
PTT = PT(LK)
PART = CMPLX(PBB,PTT)
P1 = P1 + PART*PH1(K)
END OF DO LOOP
NO
YES
END OF DO LOOP
NO
YES
EXIT

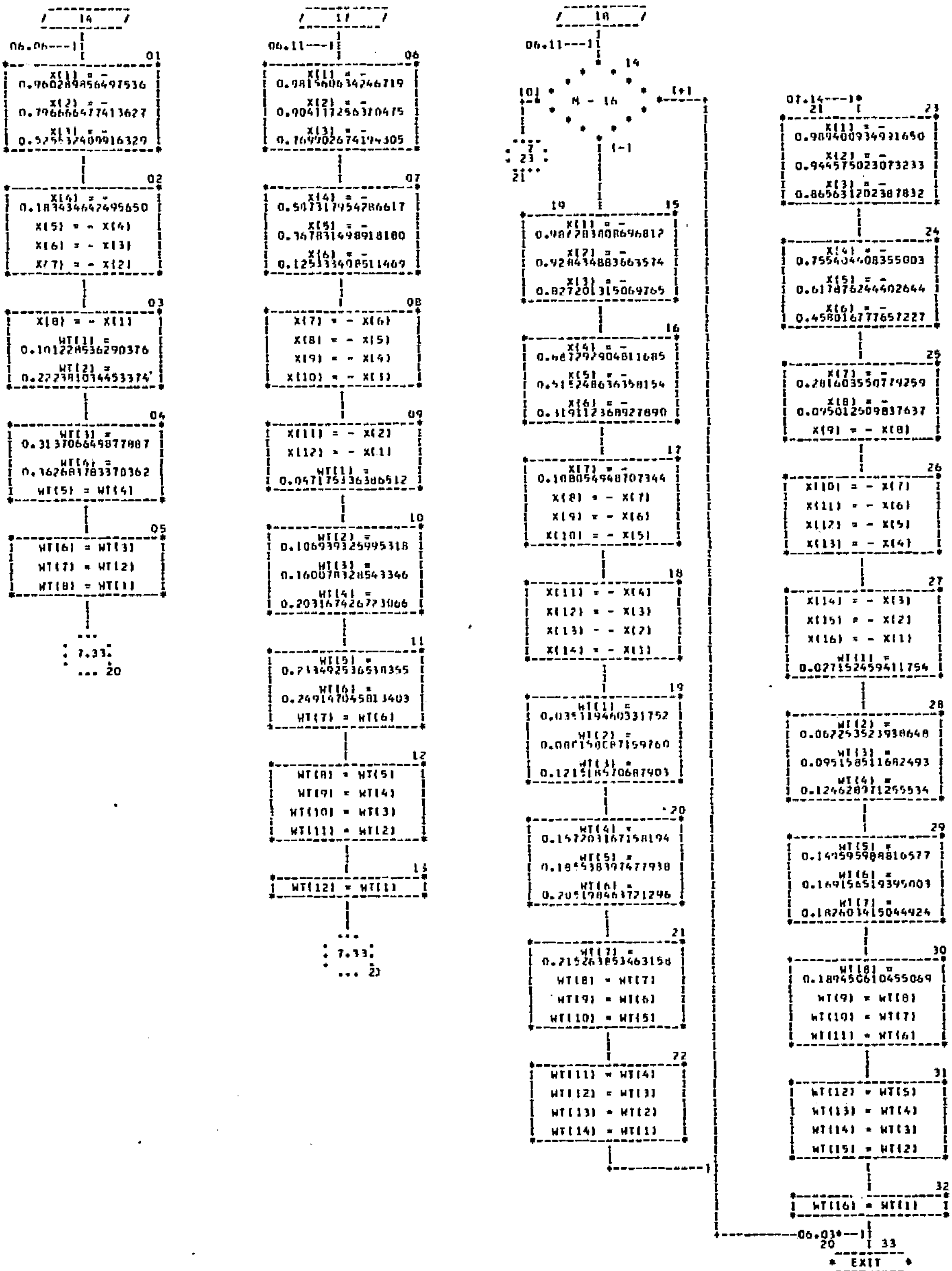


PRESZ
03.31---I*
ZDHPTS 21 - 21
THIS SUBROUTINE CALCULATES THE CONTRIBUTION OF PHI2 TO THE PRESSURE ON THE DUCT EXIT PLANE
01
P2 = CMPLX(0.0D0, 0.0D0)
LOOP FOR THE TANGENTIAL VARIABLE
NOTE 02
BEGIN DO LOOP 230 K = 1, N2
17.20---I
03
K - 1
(+)
(-/0)
531
04
ZZ = C2(K)*BD2
532
05
KL = K - 1
ZZ = ZZ + (C2(KL) + C2(K))*BD2
533
NOTE 06
CONTINUE
07
BCD2 = B*C2(K)/2.D0
LOOP FOR THE ANGULAR VARIABLE
NOTE 08
BEGIN DO LOOP 270 I = 1, L
17.19---I
09
I - 1
(+)
(-/0)
221
10
TT = 0.0
222
11
IL = I - 1
TT = TT + (F(IL) + F(I))*P1
223
12
DT = F(I)*P12
DDDT4 = B*D(K)*DT/4.D0
13
WRITE TO DEV 6 VIA FORMAT 105 FROM THE LIST
NOTE 14
LIST = TT, ZZ
NOTE 15
BEGIN DO LOOP 250 J = 1, LP
17.13---I
16
WRITE TO DEV 6 VIA FORMAT 5 FROM THE LIST
NOTE 17
LIST = J
18
IEK = J
ILL = NN + 2*(J - 1)
PR(J) = 0.0
PI(J) = 0.0
19
GAUSS (X,WT,ILL)
20
SANR = 0.0
SANI = 0.0
THESE LOOPS ARE FOR THE NUMERICAL INTEGRATION
NOTE 21
BEGIN DO LOOP 255 II = 1, ILL
17.01---I
22
ZI = ZZ - X(II)*BCD2
NOTE 23
BEGIN DO LOOP 255 JJ = 1, ILL
17.02---I
24
TI = TT - DI*X(JJ)/2.0
RI = ((R - A*COS(TI))**2 + AA*(SIN(TI))**2 + ZI*ZI)**0.5
ART = RI*WDS
25
CART = COS(ART)
SART = SIN(ART)
SINT = AW*COS(HW*TI)/RI
17.01


16.24---19
SANR = SANR +
SINT*SARI*WI(II)
*WI(JJ)
SANI = SANI +
SINT*CARI*WI(II)
WI(JJ)(-1.0)
END OF DO
LOOP
YES
NO
END OF DO
LOOP
PR(J) =
SANR*BOOT4
PI(J) =
SANI*BOOT4
J .LT. 2
TRUE
FALSE
JL = J - 1
ABT =
ABS(ABS(PR(J)) -
ABS(PR(JL)))
ABC =
ABS(ABS(PI(J)) -
ABS(PI(JL)))
ABTC = ABT - CHK
ABCC = ABC - CHK
J - LP
CONTINUE
WRITE TO DEV
6
VIA FORMAT
170
FROM THE LIST
17.12
LIST = TT, ZZ,
ABT, ABC
END OF DO
LOOP
CONTINUE
PRR = PR(LK)
PII = PI(LK)
WRITE TO DEV
6
VIA FORMAT
106
FROM THE LIST
LIST = PRR, PII
PART =
CMPLX(PRR,PII)
P2 = P2 +
PART*PHI2(K)
END OF DO
LOOP
EXIT


GAUSS
11.04
N = 4
(0)
(+)
(-)
X(1) = - 0.861136311594053
X(2) = - 0.339981043584856
X(3) = - X(2)
X(4) = - X(1)
WT(1) = 0.347854845137454
WT(2) = 0.652145154862546
WT(3) = WT(2)
WT(4) = WT(1)
X(1) = - 0.577350269189626
X(2) = - X(1)
WT(1) = 1.0
WT(2) = 1.0
7.33
20
N = 8
X(1) = - 0.932469514203152
X(2) = - 0.661209386466265
X(3) = - 0.238619186083197
X(4) = - X(3)
X(5) = - X(2)
X(6) = - X(1)
WT(1) = 0.171324492379170
WT(2) = 0.360761573048139
WT(3) = 0.467913934572691
WT(4) = WT(3)
WT(5) = WT(2)
WT(6) = WT(1)
N = 12
X(1) = - 0.973906528517172
X(2) = - 0.865063366688985
X(3) = - 0.679409568299024
X(4) = - 0.433395394129247
X(5) = - 0.148874338981631
X(6) = - X(5)
X(7) = - X(4)
X(8) = - X(3)
X(9) = - X(2)
X(10) = - X(1)
WT(1) = 0.066671344308688
WT(2) = 0.149451349150581
WT(3) = 0.219086362515982
WT(4) = 0.269266719309996
WT(5) = 0.295524224714753
WT(6) = WT(5)
WT(7) = WT(4)
WT(8) = WT(3)
WT(9) = WT(2)
WT(10) = WT(1)


400
03.30
P(JC) = (PO + P1)*RHO
03.32
201
NOTE
CONTINUE
AL(NO) = P(JC)/V(JC)
3.34
202
312
03.44
KL = K - 1
RB = RB + (D(KL) + D(K))*HO2
03.45
333
NOTE
CONTINUE
RA = RB - D(K)*HO2
RB = RB + D(K)*HO2
DEL = (RB*RB - RA*RA)/2.0
RINT = RINT + PH(K)*DEL
PINT = PINT + PH(K)*DEL
330
VINT = VINT + VH(K)*DEL
END OF DO LOOP
NO
YES
3
44
CALCULATE THE REAL AND IMAGINARY PARTS OF THE TUBE END IMPEDANCE
TAU = PINT/(VINT*S*RHO)
CHI = ((VINT*RINT - PINT*PINT)**0.5)/(S*VINT*RHO)
WRITE TO DEV 6 VIA FORMAT 305 FROM THE LIST
NOTE
LIST : TAU
WRITE TO DEV 6 VIA FORMAT 3050 FROM THE LIST
NOTE
LIST : CHI
9000
NOTE
CONTINUE
HALT
RETURN TO SYSTEM



14
06.06
01
X(1) = -0.960289856497536
X(2) = -0.796666477413627
X(3) = -0.525532409916329
02
X(4) = -0.183434642495650
X(5) = - X(4)
X(6) = - X(3)
X(7) = - X(2)
03
X(8) = - X(1)
WT(1) = 0.101228536290376
WT(2) = 0.222381034453374
04
WT(3) = 0.313706645877887
WT(4) = 0.362683783378362
WT(5) = WT(4)
05
WT(6) = WT(3)
WT(7) = WT(2)
WT(8) = WT(1)
7.33
20
17
06.11
06
X(1) = -0.981560634246719
X(2) = -0.904117256370475
X(3) = -0.769902674194305
07
X(4) = -0.587317954286617
X(5) = -0.367831498918180
X(6) = -0.125233408511469
08
X(7) = - X(6)
X(8) = - X(5)
X(9) = - X(4)
X(10) = - X(3)
09
X(11) = - X(2)
X(12) = - X(1)
WT(1) = 0.047175336386512
10
WT(2) = 0.106939325995318
WT(3) = 0.160078328543346
WT(4) = 0.203167426723066
11
WT(5) = 0.233492536538355
WT(6) = 0.249147045813403
WT(7) = WT(6)
12
WT(8) = WT(5)
WT(9) = WT(4)
WT(10) = WT(3)
WT(11) = WT(2)
13
WT(12) = WT(1)
7.33
20
18
06.11
14
N - 16
(0)
(+)
(-)
7
23
21
15
X(1) = -0.987283808696812
X(2) = -0.928434883663574
X(3) = -0.827201315069765
16
X(4) = -0.687292904811685
X(5) = -0.515248636358154
X(6) = -0.319112368927890
17
X(7) = -0.108054948707344
X(8) = - X(7)
X(9) = - X(6)
X(10) = - X(5)
18
X(11) = - X(4)
X(12) = - X(3)
X(13) = - X(2)
X(14) = - X(1)
19
WT(1) = 0.035119460331752
WT(2) = 0.080158087159760
WT(3) = 0.121518570687903
20
WT(4) = 0.157203167158194
WT(5) = 0.185538397477938
WT(6) = 0.205198463721296
21
WT(7) = 0.215263853463158
WT(8) = WT(7)
WT(9) = WT(6)
WT(10) = WT(5)
22
WT(11) = WT(4)
WT(12) = WT(3)
WT(13) = WT(2)
WT(14) = WT(1)
07.14
21
23
X(1) = -0.989400934991650
X(2) = -0.944575023073233
X(3) = -0.865631202387832
24
X(4) = -0.755404408355003
X(5) = -0.617876244402644
X(6) = -0.458016777657227
25
X(7) = -0.281603550779259
X(8) = -0.095012509837637
X(9) = - X(8)
26
X(10) = - X(7)
X(11) = - X(6)
X(12) = - X(5)
X(13) = - X(4)
27
X(14) = - X(3)
X(15) = - X(2)
X(16) = - X(1)
WT(1) = 0.027152459411754
28
WT(2) = 0.062253523938648
WT(3) = 0.095158511682493
WT(4) = 0.124628971255534
29
WT(5) = 0.149595988816577
WT(6) = 0.169156519395003
WT(7) = 0.182603415044924
30
WT(8) = 0.189450610455069
WT(9) = WT(8)
WT(10) = WT(7)
WT(11) = WT(6)
31
WT(12) = WT(5)
WT(13) = WT(4)
WT(14) = WT(3)
WT(15) = WT(2)
32
WT(16) = WT(1)
06.03
20
33
EXIT

APPENDIX 3

MAIN PROGRAM LISTING

```
C         THIS PROGRAM CALCULATES THE ACOUSTIC IMPEDANCE OF
C         THE DUCT EXIT PLANE
C                                                        7DM915 20 - 20
      INTEGER *4    DRECTP/02/
      COMPLEX *8   CMPLX
     1                       ,P0,P1,P2,P
     2                ,PHI0(40), PHI1(40),PHI2(40) ,V(40),ALIMP
      DIMENSION
     1            PN(40),PD(2,1)   ,VD(2,1)   ,VM(40),PM(40)
     2          ,D(40),C(40),P(80)    ,C2(40)     ,E(40)
      EQUIVALENCE (PD(1,1),P(1))  ,(VD(1,1),V(1))
      COMMON/DATA/L,M,N,NN,LP,MW,A,B,W,S,CHK,D,C,E,PI
      COMMON/PHI/PHI0,PHI1,PHI2,P0,P1,P2
      COMMON  /C2N2/  C2,N2
      COMMON/ANG/       FINK, BD2, WDS, FMW, PI2, BW, AW ,BB,AA
  100 FORMAT (36I2)
  220 FORMAT (6E12.5)
 1000 FORMAT ('1 TUBE END IMPEDANCE PROGRAM FOR UNFLANGED ANNULAR DUCT
     1.'// 25X, 'I N P U T   D A T A')
  402 FORMAT ('0 THETA BOX WIDTH ON DUCT FACE'/)
  403 FORMAT ('1 VELOCITY ON DUCT FACE'/)
  407 FORMAT ('1')
  408 FORMAT ('0 A =', 1PE13.6, '  B =', E13.6, '  W =', E13.6, '  S =',
     1          E13.6/ '0CHK=', E13.6, ' RHO=', E13.6)
 1100 FORMAT ('0 ANNULAR RING BOX WIDTHS'        /)
 1200 FORMAT ('0 OUTER DUCT WALL BOX LENGTHS'  /)
 1300 FORMAT ('0 INNER DUCT WALL BOX LENGTHS'  /)
 1400 FORMAT ('0 SOURCE DISTRIBUTION ON DUCT FACE.'      /)
 1500 FORMAT ('0 SOURCE DISTRIBUTION ON OUTER DUCT WALL' /)
 1600 FORMAT ('0 SOURCE DISTRIBUTION ON INNER DUCT WALL' /)
C   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
      READ  (5,100)   K, M, N, N2, INK,      MW, NN, L,LP
      READ  (5,220)   A, B, W, S, CHK, XANG   , RHO
      READ  (5,220)   (C(I), I=1,N)
      READ  (5,220)   (D(I), I=1,M)
        IF (INK .GT. 0)        READ (5,220)  (C2(I),I=1,N2)
      READ  (5,220)   (V(I), I=1,M)
      READ  (5,220)   (PHI0(I),I=1,M)
      READ  (5,220)   (PHI1(I),I=1,N)
        IF (INK .GT. C)        READ (5,220)  (PHI2(I),I=1,N2)
      E(1) = XANG
      DT2= (1.0E0 - XANG) / (L-1)
      DO 50 I=2,L
   50 E(I) = DT2
  105 FORMAT (1PE16.6, 3E18.6)
  104 FORMAT ('C L =', I4, '  M =', I4, '  N =', I4, '  NN =', I4,
     1        '  LP =', I4, '  MW =', I5, '  INK =', I5/)
      WRITE (6,1000)
      WRITE(6,408) A,B,W,S,CHK, RHO
      WRITE(6,104) L,M,N,NN,LP,MW,INK
      WRITE(6,105)  (D(I),I=1,M)
```

```
      WRITE (6, 1200)
      WRITE(6,105)  (C(I),I=1,N)
      IF (INK .GT. 0) WRITE (6, 1300)
      IF ( INK .GT. 0) WRITE (6,  105) (C2(I), I = 1,N)
      WRITE (6, 402)
      WRITE(6,105)  (E(I),I=1,L)
      WRITE (6, 403)
      WRITE(6,105) (V(I),I=1,M)
      WRITE (6, 1400)
      WRITE(6,105) (PHIO(I),I=1,M)
      WRITE (6, 1500)
      WRITE(6,105) (PHI1(I),I=1,N)
        IF (INK .GT. 0)      WRITE (6, 1600)
        IF (INK .GT. 0)      WRITE (6,105)     (PHI2(I),I=1,N2)
C**             INITIALIZATION       *      *       *       *         *         *
      PI=3.14159265358979З
      FINK =  FLOAT(INK)
      BD2 = B / 2.D0
      WDS = W / S
      FMW =  FLOAT(MW)
      PI2 = PI * 2.D0
      BW  = B * W
      AW  = A * W
      AA  = A * A
      BB  = B * B
C
C        THIS LOOP CALCULATES THE RADIAL PRESSURE DISTRIBUTION
C
      DO 200 JC=1,M
      P(JC)= CMPLX(0.CE0,0.0E0)
      IF(JC-1)210,210,211
  210 R=D(JC)*BD2  +A* FINK
      GO TO 212
  211 JE=JC-1
      R=R+(D(JE)+D(JC))*BD2
  212 CONTINUE
      IF (R.GT.A)                                   GO TO 170
        IF (INK.EQ.1 .OR. VD(1,JC).EQ.0.D0)           GO TO 203
  151 FORMAT(1HC,5X,I3,6X,D25.14)
  170 CONTINUE
      CALL PRESO(R)
      CALL PRES1(R)
      IF(INK-1) 400,500,500
  500 CALL PRES2(R)
      P(JC)= (PO + P1 + P2) * RHO
      GO TO 201
  400 P(JC) = (PO + P1) * RHO
  201 CONTINUE
      ALIMP = P(JC) / V(JC)
      GO TO 202
  203 P(JC)= CMPLX(0.CE0,0.0E0)
```

```
  202 CONTINUE
      WRITE (6, 160) JC, R, ALIMP, P(JC), V(JC)
  160 FORMAT ('    ANNULAR RING NO', I3, '    RADIAL POSITION =',1PE14.6/
     1          25X, 'LOCAL IMPEDANCE =', 2E14.6/
     2          32X, 'PRESSURE =', 2E14.6/
     3          32X, 'VELOCITY =', 2E14.6/)
  200 CONTINUE
C
C        THIS LOOP INTEGRATES THE VELOCITY AND PRESSURE DISTRIBUTIONS
C        OVER THE DUCT FACE
C
      DO 300 I=1,M
      VM(I)=VD(1,I)**2+VD(2,I)**2
      PN(I)=PD(1,I)**2+PD(2,I)**2
  300 PM(I)=PD(1,I)*VD(1,I) + PD(2,I) * VD(2,I)
      RINT=0.C
      PINT=0.C
      VINT=0.C
      DO 330 K=1,M
      IF (K-1) 331,331,332
  331 RR=D(K)*BD2  + FINK  * A
      GO TO 333
  332 KL=K-1
      RR=RR+(D(KL)+D(K))*BD2
  333 CONTINUE
      RA=RR-D(K)*BD2
      RB=RR+D(K)*BD2
      DEL=(RB*RB-RA*RA)/2.0
      RINT=RINT+PN(K)*DEL
      PINT=PINT+PM(K)*DEL
  330 VINT=VINT+VM(K)*DEL
C
C        CALCULATE THE REAL AND IMAGINARY PARTS OF THE
C        TUBE END IMPEDANCE
C
      TAU=PINT/(VINT*S*RHO)
      CHI=((VINT*RINT-PINT*PINT)**0.5)/(S*VINT*RHO)
      WRITE(6,305)  TAU
  305 FORMAT(1H ,10X,'RESISTANCE RATIO, TAU = ',E15.8)
      WRITE(6,3050) CHI
 3050 FORMAT(1H ,10X,'RFACTANCE RATIO,  CHI = ',E15.8)
 9000 CONTINUE
      STOP
      END
```

APPENDIX 4

PRESO SUBROUTINE LISTING

```
      SUBROUTINE PRESC(R)
C                                                          ZDM915 21 - 21
C        THIS SUBROUTINE CALCULATES THE CONTRIBUTION OF PHIO
C        TO THE PRESSURE ON THE DUCT EXIT PLANE
      REAL *4  F5D8/.625D0/,  F5D12/.41666666666666D0/
     1  ,F16D5 /3.2D0/ ,F6D5/1.2D0/
      COMPLEX *8  CMPLX
     1              ,PHIO(40), PHI1(40),PHI2(40),PO,P1,P2,PART
      DIMENSION X(20),WT(20),D(40),C(40),E(40),PB(40),PT(40)
      COMMON/PHI/PHIO,PHI1,PHI2,PO,P1,P2
      COMMON/DATA/L,M,N,NN,LP,MW,A,B,W,S,CHK,D,C,E,PI
      COMMON/ANG/     FINK, BD2, WDS, FMW, PI2, BW, AW,BB, AA
      PO= CMPLX(0.0E0,0.0E0)
      CT=1.0
      ST=0.0
C
C        LOOP FOR THE ANGULAR VARIABLE
C
      DO 220 KC=1,L
      IF(KC-1)221,221,222
  221 TT=0.0
      GO TO 223
  222 KCL=KC-1
      TT=TT+(E(KCL)+E(KC))*PI
  223 DT=E(KC)*2.0*PI
C
C        LOOP FOR THE RADIAL VARIABLE
C
      DO 230 K=1,M
      IF(K-1)231,231,232
  231 RR=D(K)*BD2 +A* FINK
      GO TO 233
  232 KL=K-1
      RR=RR+(D(KL)+D(K))*BD2
  233 CONTINUE
C
C        CLOSED FORM INTEGRATION OVER THE PRESSURE SINGULARITY
C
      BZ=D(K)*BD2
      AZ=R*DT/2.C
      BZDT = BZ * DT / 2.D0
      IF(KC-1)234,234,235
  234 RQ=R+D(K)*B/5.0
      RP=R-D(K)*B/5.0
      IF((RR.GE.RQ).OR.(RR.LE.RP)) GO TO 235
      TN=BZ/AZ
      ALP= ATAN(TN)
      SEC=1.0/ COS(ALP)
      SEC2 = SEC  * SFC
      SEC4 = SEC2 * SEC2
      SEC6 = SEC4 * SEC2
```

```
      CSC=1.0/ SIN(ALP)
      CSC2 = CSC  * CSC
      CSC4 = CSC2 * CSC2
      CSC6 = CSC4 * CSC2
      COT=1.0/TN
      CF1=AZ*ALOG((SEC+TN)/(SEC-TN))*W*(-2.0)
      WA=W*AZ/S
      WA2 = WA  * WA
      WA4 = WA2 * WA2
      WA6 = WA4 * WA2
      CF1=CF1*(1.0-WA2  /12.0+WA4  /320.0- WA6   /16128.0)
      CF2=WA2  *W*AZ*TN*SEC/3.0
      CF2=CF2*(1.0-WA2  *(SEC2   +1.5)/40.0+WA4  *(SEC4  /3.0+    SEC2
     1*F5D12+ F5D8 )/840.0)
      CF3=BZ*ALOG((CSC+COT)/(CSC-COT))*W*(-2.0)
      WB=W*BZ/S
      WB2 = WB  * WB
      WB4 = WB2 * WB2
      WB6 = WB4 * WB2
      CF3=CF3*(1.0-WB2  /12.0+WB4  /320.0-WB6  /16128.0)
      CF4=WB2  *W*BZ*COT*CSC/3.0
      CF4=CF4*(1.0-WB2  *(CSC2   +1.5)/40.0+WB4  *(CSC4  /3.0+    CSC2
     1*F5D12 + F5D8)/840.0)
      PTT=CF1+CF2+CF3+CF4
      DF1=WA2  *S*TN*2.0
      DF1=CF1*(1.0-WA2  *(SEC2   +2.0)/36.0+WA4  *(3.0*SEC4  +4.0*SEC2
     1+8.0)/5400.0-WA6  *(15.0*SEC6  +18.0*SEC4  +24.0*SEC2  +48.0)/2116
     2800.0)
      DF2=WB2  *S*COT*2.0
      DF2=DF2*(1.0-WB2  *(CSC2  +2.0)/36.0+WB4  *(3.0*CSC4  +4.0*CSC2  +
     18.0)/5400.0-WB6  *(CSC6  +6.0*CSC4  /5.0+8.0*CSC2  /5.0+F16D5   )/
     2141120.0)
      PBB=DF1+DF2
      GO TO 261
  235 CONTINUE
      DO 250 J=1,LP
      ILK=J
      ILL=NN+2*(J-1)
      PB(J)=0.0
      PT(J)=0.0
      CALL  GAUSS(X,WT,ILL)
      SANR=0.0
      SANI=0.0
C
C     THESE LOOPS ARE FOR THE NUMERICAL INTEGRATION
C
      DO 255 II=1,ILL
      RI=RR-        X(II) * BZ
      DO 255 JJ=1,ILL
      TI=TT-DT*X(JJ)/2.0
      RJ=((R*CT-RI* COS(TI))**2+(R*ST-RI* SIN(TI))**2)**0.5
```

```
      ART=RO*WDS
      CART= COS(ART)
      SART= SIN(ART)
      SINT=RI*W* COS( FMW      *TI)/RO
      SANR=SANR+SINT*SART*WT(II)*WT(JJ)
      SANI=SANI+SINT*CART*WT(II)*WT(JJ)*(-1.0)
  255 CONTINUE
      PB(J)=SANR*BZDT
      PT(J)=SANI*B*D(K)*DT/4.0
      PT(J)=SANI*BZDT
        IF (J .LT. 2)                                       GO TO 250
      JL=J-1
      ABT= ABS( ABS(PB(J))- ABS(PB(JL)))
      ABC= ABS( ABS(PT(J))- ABS(PT(JL)))
      ABTC=ABT-CHK
      ABCC=ABC-CHK
        IF (ABTC.LT.0.D0 .AND. ABCC.LT.0.D0)              GO TO 260
  257 IF(J-LP) 250,253,260
  253 CONTINUE
      WRITE (6,170) TT,RR,ABT,ABC
  170 FORMAT(1H0,     'INTEGRATION FOR BLOCK CENTERED AT THETA=',1PE13.6,
     1'  R=',  E13.6 ,/,19X,'DID NOT CONVERGE',/,19X,'REAL DIFF =',E13.6
     2  ,'  IMAG DIFF = ',E13.6)
  250 CONTINUE
  260 CONTINUE
      PBB=PB(ILK)
      PTT=PT(ILK)
  261 CONTINUE
      PART= CMPLX(PBB,PTT)
      PO=PO+PART*PHIO(K)
  230 CONTINUE
  220 CONTINUE
      RETURN
      END
```

APPENDIX 5

PRES1 SUBROUTINE LISTING

```
      SUBROUTINE PRES1(R)
C                                                      ZDM915 21 - 21
C        THIS SUBROUTINE CALCULATES THE CONTRIBUTION OF PHI1
C        TO THE PRESSURE ON THE DUCT EXIT PLANE
      COMPLEX *8  CMPLX
     1             ,PHI0(40), PHI1(40),PHI2(40),P0,P1,P2,PART
      DIMENSION X(20),WT(20),D(40),C(40),E(40),PB(40),PT(40)
      COMMON/DATA/L,M,N,NN,LP,MW,A,B,W,S,CHK,D,C,E,PI
      COMMON/PHI/PHI0,PHI1,PHI2,P0,P1,P2
      COMMON/ANG/     FINK, BD2, WDS, FMW, PI2, BW, AW,BB, AA
      P1= CMPLX(0.0E0,0.0E0)
C
C        LOOP FOR THE TANGENTIAL VARIABLE
C
      DO 230 K=1,N
      IF(K-1)231,231,232
  231 ZZ=-C(K)*BD2
      GO TO 233
  232 KL=K-1
      ZZ=ZZ-(C(KL)+C(K))*BD2
  233 CONTINUE
      BCD2   = B * C(K)  / 2.D0
C
C        LOOP FOR THE ANGULAR VARIABLE
C
      DO 270 I=1,L
      IF(I-1) 221,221,222
  221 TT=0.0
      GO TO 223
  222 IL=I-1
      TT=TT+(E(IL)+E(I))*PI
  223 DT=E(I)* PI2
      BCDT4 = BCD2 * DT / 2.D0
      DO 250 J=1,LP
      ILK=J
      ILL=NN+2*(J-1)
      CALL  GAUSS(X,WT,ILL)
      PB(J)=0.C
      PT(J)=C.C
      SANR=0.C
      SANI=0.C
C
C     THESE LOOPS ARE FOR THE NUMERICAL INTEGRATION
C
      DO 255 II=1,ILL
      ZI=ZZ-         X(II) * BCD2
      DO 255 JJ=1,ILL
      TI=TT-DT*X(JJ)/2.0
      RI=((R-B* COS(TI))**2+(BB )*( SIN(TI))**2+ZI*ZI)**0.5
      ART=RI*WDS
      CART= COS(ART)
```

```
      SART= SIN(ART)
      SINT=BW * COS( FMW       *TI)/RI
      SANR=SANR+SINT*SART*WT(II)*WT(JJ)
      SANI=SANI+SINT*CART*WT(II)*WT(JJ)*(-1.0)
  255 CONTINUE
      PB (J)=SANR*BCDT4
      PT (J)=SANI*BCDT4
        IF (J .LT. 2)                                   GO TO 250
      JL=J-1
      ABT= ABS( ABS(PB(J))- ABS(PB(JL)))
      ABC= ABS( ABS(PT(J))- ABS(PT(JL)))
      ABTC=ABT-CHK
      ABCC=ABC-CHK
      IF((ABTC.LT.0.).AND.(ABCC.LT.0.)) GO TO 260
  257 IF(J-LP) 250,253,260
  253 CONTINUE
      WRITE (6,170) TT,ZZ,ABT,ABC
  170 FORMAT(1HO,      'INTEGRATION FOR BLOCK CENTERED AT THETA=',1PE13.6,
     1'  Z=',  E13.6 ,/,19X,'DID NOT CONVERGE',/,19X,'REAL DIFF =',E13.6
     2  ,'  IMAG DIFF = ',E13.6)
  250 CONTINUE
  260 CONTINUE
      PBB=PB(ILK)
      PTT=PT(ILK)
      PART= CMPLX(PBB,PTT)
      P1=P1+PART*PHI1(K)
  270 CONTINUE
  230 CONTINUE
      RETURN
      END
```

APPENDIX 6

PRES2 SUBROUTINE LISTING

```
      SUBROUTINE PRES2(R)
C                                                           ZDM915 21 - 21
C        THIS SUBROUTINE CALCULATES THE CONTRIBUTION OF PHI2
C        TO THE PRESSURE ON THE DUCT EXIT PLANE
      COMPLEX *8  CMPLX
      COMPLEX *8   PHI0(40), PHI1(40),PHI2(40),P0,P1,P2,PART
      DIMENSION X(20),WT(20),D(40),C(40),E(40),PB(40),PT(40),C2(40)
      COMMON/PHI/PHI0,PHI1,PHI2,P0,P1,P2
      COMMON/DATA/L,M,N,NN,LP,MW,A,B,W,S,CHK,D,C,E,PI
      COMMON/C2N2/C2,N2
      COMMON/ANG/      FINK, BD2, WDS, FMW, PI2, BW, AW,BB, AA
      P2= CMPLX(0.0E0,0.0E0)
  105 FORMAT( 1H ,2X,4D25.14)
C
C        LOOP FOR THE TANGENTIAL VARIABLE
C
      DO 230 K=1,N2
      IF(K-1)531,531,532
  531 ZZ=C2(K)*BD2
      GO TO 533
  532 KL=K-1
      ZZ=ZZ+(C2(KL)+C2(K))*BD2
  533 CONTINUE
C
      BCD2 = B *C2(K) / 2.D0
C        LOOP FOR THE ANGULAR VARIABLE
C
      DO 270 I=1,L
      IF(I-1) 221,221,222
  221 TT=0.0
      GO TO 223
  222 IL=I-1
      TT=TT+(E(IL)+E(I))*PI
  223 DT=E(I)* PI2
      BDDT4 = B * D(K) * DT / 4.D0
      WRITE(6,105) TT,ZZ
      DO 250 J=1,LP
      WRITE(6,5) J
    5 FORMAT( 1H ,10X,I5)
      ILK=J
      ILL=NN+2*(J-1)
      PB(J)=0.0
      PT(J)=0.0
      CALL  GAUSS(X,WT,ILL)
      SANR=0.0
      SANI=0.0
C
C     THESE LOOPS ARE FOR THE NUMERICAL INTEGRATION
C
      DO 255 II=1,ILL
      ZI=ZZ-         X(II) * BCD2
```

```
      DO 255 JJ=1,ILL
      TI=TT-DT*X(JJ)/2.0
      RI=((R-A* COS(TI))**2+ AA  *( SIN(TI))**2+ZI*ZI)**0.5
      ART=RI*WCS
      CART= COS(ART)
      SART= SIN(ART)
      SINT=AW * COS( FMW      *TI)/RI
      SANR=SANR+SINT*SART*WT(II)*WT(JJ)
      SANI=SANI+SINT*CART*WT(II)*WT(JJ)*(-1.0)
  255 CONTINUE
      PB(J)=SANR*BDDT4
      PT(J)=SANI*BDDT4
        IF (J .LT. 2)                                   GO TO 250
      JL=J-1
      ABT= ABS( ABS(PB(J))- ABS(PB(JL)))
      ABC= ABS( ABS(PT(J))- ABS(PT(JL)))
      ABTC=ABT-CHK
      ABCC=ABC-CHK
      IF((ABTC.LT.0.).AND.(ABCC.LT.0.)) GO TO 260
      IF(J-LP) 250,253,260
  253 CONTINUE
      WRITE (6,170) TT,ZZ,ABT,ABC
  170 FORMAT( 1H0,     'INTEGRATION FOR BLOCK CENTERED AT THETA=',1PE13.6,
     1'  Z=',  E13.6 ,/,19X,'DID NOT CONVERGE',/,19X,'REAL DIFF =',E13.6
     2  ,'  IMAG DIFF = ',E13.6)
  250 CONTINUE
  260 CONTINUE
      PBB=PB(ILK)
      PTT=PT(ILK)
      WRITE(6,106) PBB,PTT
  106 FORMAT( 1H ,10X,'P2 REAL =',E25.7 ,' P2 IMAG=',E25.7 /)
      PART= CMPLX(PBB,PTT)
      P2=P2+PART*PHI2(K)
  270 CONTINUE
  230 CONTINUE
      RETURN
      END
```

APPENDIX 7

GAUSS SUBROUTINE LISTING

```
      SUBROUTINE GAUSS(X,WT,M)
      DIMENSION X(12),WT(12)
      IF(M-4)10,11,12
   10 X(1)=-0.577350269189626
      X(2)= -X(1)
      WT(1)=1.0
      WT(2)=1.0
      GO TO 20
   11 X(1)=-0.861136311594053
      X(2)=-0.339981043584856
      X(3)=-     X(2)
      X(4)=-     X(1)
      WT(1)=0.347854845137454
      WT(2)=0.652145154862546
      WT(3)=WT(2)
      WT(4)=WT(1)
      GO TO 20
   12 IF(M-8)13,14,15
   13 X(1)=-0.932469514203152
      X(2)=-0.661209386466265
      X(3)=-0.238619186083197
      X(4)=-     X(3)
      X(5)=-     X(2)
      X(6)=-     X(1)
      WT(1)=0.171324492379170
      WT(2)=C.360761573048139
      WT(3)=0.467913934572691
      WT(4)=WT(3)
      WT(5)=WT(2)
      WT(6)=WT(1)
      GO TO 20
   14 X(1)=-0.960289856497536
      X(2)=-C.796666477413627
      X(3)=-0.525532409916329
      X(4)=-0.183434642495650
      X(5)=-     X(4)
      X(6)=-     X(3)
      X(7)=-     X(2)
      X(8)=-     X(1)
      WT(1)=0.101228536290376
      WT(2)=0.222381034453374
      WT(3)=0.313706645877887
      WT(4)=0.362683783378362
      WT(5)=WT(4)
      WT(6)=WT(3)
      WT(7)=WT(2)
      WT(8)=WT(1)
      GO TO 20
   15 IF(M-12)16,17,18
   16 X(1)=-0.973906528517172
      X(2)=-0.865063366688985
```

```
      X(3)=-0.679409568299024
      X(4)=-0.433395394129247
      X(5)=-0.148874338981631
      X(6)=-     X(5)
      X(7)=-     X(4)
      X(8)=-     X(3)
      X(9)=-     X(2)
      X(10)=-     X(1)
      WT(1)=0.066671344308688
      WT(2)=0.149451349150581
      WT(3)=0.219086362515982
      WT(4)=0.269266719309996
      WT(5)=0.295524224714753
      WT(6)=WT(5)
      WT(7)=WT(4)
      WT(8)=WT(3)
      WT(9)=WT(2)
      WT(10)=WT(1)
      GO TO 20
   17 X(1)=-0.981560634246719
      X(2)=-0.904117256370475
      X(3)=-0.769902674194305
      X(4)=-0.587317954286617
      X(5)=-0.367831498918180
      X(6)=-0.125333408511469
      X(7)=-     X(6)
      X(8)=-     X(5)
      X(9)=-     X(4)
      X(10)=-     X(3)
      X(11)=-     X(2)
      X(12)=-     X(1)
      WT(1)=0.047175336386512
      WT(2)=0.106939325995318
      WT(3)=0.160078328543346
      WT(4)=0.203167426723066
      WT(5)=0.233492536538355
      WT(6)=0.249147045813403
      WT(7)=WT(6)
      WT(8)=WT(5)
      WT(9)=WT(4)
      WT(10)=WT(3)
      WT(11)=WT(2)
      WT(12)=WT(1)
                                                      GO TO 20
   18   IF (M-16)                      19, 21, 20
   19 X(1)=-0.986283808696812
      X(2)=-0.928434883663574
      X(3)=-0.827201315069765
      X(4)=-0.687292904811685
      X(5)=-0.515248636358154
      X(6)=-0.319112368927890
```

```
      X(7)=-0.108054948707344
      X(8)=-     X(7)
      X(9)=-     X(6)
      X(10)=    -X(5)
      X(11)=-     X(4)
      X(12)=-     X(3)
      X(13)=-     X(2)
      X(14)=-     X(1)
      WT(1)=0.035119460331752
      WT(2)=0.080158087159760
      WT(3)=0.121518570687903
      WT(4)=0.157203167158194
      WT(5)=0.185538397477938
      WT(6)=0.205198463721296
      WT(7)=0.215263853463158
      WT(8)=WT(7)
      WT(9)=WT(6)
      WT(10)=WT(5)
      WT(11)=WT(4)
      WT(12)=WT(3)
      WT(13)=WT(2)
      WT(14)=WT(1)
                                                  GO TO 20
   21 X(1)=-0.989400934991650
      X(2)=-0.944575023073233
      X(3)=-0.865631202387832
      X(4)=-0.755404408355003
      X(5)=-0.617876244402644
      X(6)=-0.458016777657227
      X(7)=-0.281603550779259
      X(8)=-0.095012509837637
      X(9)=-     X(8)
      X(10)=    -X(7)
      X(11)=-     X(6)
      X(12)=-     X(5)
      X(13)=-     X(4)
      X(14)=-     X(3)
      X(15)=-     X(2)
      X(16)=-     X(1)
      WT(1)=0.027152459411754
      WT(2)=0.062253523938648
      WT(3)=0.095158511682493
      WT(4)=0.124628971255534
      WT(5)=0.149595988816577
      WT(6)=0.169156519395003
      WT(7)=0.182603415044924
      WT(8)=0.189450610455069
      WT(9)=WT(8)
      WT(10)=WT(7)
      WT(11)=WT(6)
      WT(12)=WT(5)
```

```
      WT(13)=WT(4)
      WT(14)=WT(3)
      WT(15)=WT(2)
      WT(16)=WT(1)
   20 RETURN
      END
```

APPENDIX 8

EXAMPLE PROBLEM

```
0016400000000C 23C 8
             1.C          343.0         343.0          .0001         .03
1.21
 2.50000D-02 2.50000D-02 5.00000D-02 5.00000D-02 5.00000D-02 5.00000D-02
 7.50000D-02 7.50000D-02 7.50000D-02 7.50000D-02 7.50000D-02 7.50000D-02
 1.00000D-01 1.00000D-01 1.00000D-01 1.00000D-01 1.00000D-01 1.00000D-01
 1.00000D-01 1.00000D-01 1.00000D-01 1.25000D-01 1.25000D-01 1.25000D-01
 1.25000D-01 1.25000D-01 1.25000D-01 1.25000D-01 1.25000D-01 1.25000D-01
 1.25000D-01 1.25000D-01 1.25000D-01 1.25000D-01 1.25000D-01 1.50000D-01
 1.50000D-01 1.50000D-01 1.50000D-01 1.50000D-01
 1.25000D-01 1.25000D-01 1.00000D-01 7.50000D-02 5.00000D-02 2.50000D-02
 2.50000D-02 5.00000D-02 5.00000D-02 7.50000D-02 7.50000D-02 7.50000D-02
 5.00000D-02 5.00000D-02 2.50000D-02 2.50000D-02
1.0                      1.0                      1.0
1.0                      1.0                      1.0
1.0                      1.0                      1.0
1.0                      1.0                      1.0
1.0                      1.0                      1.0
1.0
-6.05115D-02 4.39628D-02-7.07061D-02 4.86296D-02-9.07488D-02 5.77973D-02
-1.16465D-01 6.95511D-02-1.42718D-01 8.15449D-02-1.62778D-01 9.07081D-02
-1.78412D-01 9.78492D-02-2.06010D-01 1.10457D-01-2.52712D-01 1.31796D-01
-3.33809D-01 1.68875D-01-4.85594D-01 2.38349D-01-7.45917D-01 3.57679D-01
-1.10192D 00 5.20969D-01-1.36075D 00 6.38583D-01-1.68462D-01 8.12709D-02
 9.23232D 00-4.29208D 00
-5.24629D 01 2.43952D 01 3.30565D 00-1.52548D 00 9.59800D 00-4.44085D 00
 6.30965D 00-2.90579D 00 3.51748D 00-1.60596D 00 1.94227D 00-8.73900D-01
 9.67564D-01-4.22178D-01 4.56117D-01-1.86600D-01 2.33575D-01-8.53307D-02
 1.24716D-01-3.67737D-02 6.55499D-02-1.11698D-02 3.09838D-02 3.15938D-03
 7.02919D-03 1.24469D-02-8.37244D-03 1.76862D-02-1.63348D-02 1.96714D-02
-2.00660D-02 1.98723D-02-2.13445D-02 1.90820D-02-2.11972D-02 1.77486D-02
-2.02456D-02 1.61572D-02-1.89014D-02 1.44672D-02-1.74092D-02 1.28081D-02
-1.57047D-02 1.10102D-02-1.39293D-02 9.15983D-03-1.23611D-02 7.50691D-03
-1.10126D-02 6.04770D-03-9.86773D-03 4.76274D-03-8.89518D-03 3.63074D-03
-8.06117D-03 2.64105D-03-7.33882D-03 1.77059D-03-6.69152D-03 1.00417D-03
-6.10744D-03 3.31180D-04-5.56072D-03-2.68730D-04-5.03784D-03-7.98581D-04
-4.53175D-03-1.27128D-03-4.03101D-03-1.70169D-03-3.46819D-03-2.14888D-03
-2.82596D-03-2.64320D-03-2.13003D-03-3.23068D-03-1.27398D-03-4.26831D-03
 3.54277D-04-8.18834D-03
```

TUBE END IMPEDANCE PROGRAM FOR UNFLANGED ANNULAR DUCT .

I N P U T   D A T A

A = 0.0   B = 1.000000E 00   W = 3.430000E 02   S = 3.430000E 02

CHK= 9.999999E-05 RHO= 1.209999E 00

L = 30   M = 16   N = 40   NN = 2   LP = 8   MW = 0   INK = 0

1.250000E-01	1.250000E-01	9.999996E-02	7.499999E-02
5.000000E-02	2.500000E-02	2.500000E-02	5.000000E-02
5.000000E-02	7.499999E-02	7.499999E-02	7.499999E-02
5.000000E-02	5.000000E-02	2.500000E-02	2.500000E-02

OUTER DUCT WALL BOX LENGTHS

2.500000E-02	2.500000E-02	5.000000E-02	5.000000E-02
5.000000E-02	5.000000E-02	7.499999E-02	7.499999E-02
7.499999E-02	7.499999E-02	7.499999E-02	7.499999E-02
9.999996E-02	9.999996E-02	9.999996E-02	9.999996E-02
9.999996E-02	9.999996E-02	9.999996E-02	9.999996E-02
9.999996E-02	1.250000E-01	1.250000E-01	1.250000E-01
1.250000E-01	1.250000E-01	1.250000E-01	1.250000E-01
1.250000E-01	1.250000E-01	1.250000E-01	1.250000E-01
1.250000E-01	1.250000E-01	1.250000E-01	1.500000E-01
1.500000E-01	1.500000E-01	1.500000E-01	1.500000E-01

THETA BOX WIDTH ON DUCT FACE

3.000000E-02	3.344828E-02	3.344828E-02	3.344828E-02
3.344828E-02	3.344828E-02	3.344828E-02	3.344828E-02
3.344828E-02	3.344828E-02	3.344828E-02	3.344828E-02
3.344828E-02	3.344828E-02	3.344828E-02	3.344828E-02
3.344828E-02	3.344828E-02	3.344828E-02	3.344828E-02
3.344828E-02	3.344828E-02	3.344828E-02	3.344828E-02
3.344828E-02	3.344828E-02	3.344828E-02	3.344828E-02
3.344828E-02	3.344828E-02		

VELOCITY ON DUCT FACE

1.000000E 00	0.0	1.000000E 00	0.0
1.000000E 00	0.0	1.000000E 00	0.0
1.000000E 00	0.0	1.000000E 00	0.0
1.000000E 00	0.0	1.000000E 00	0.0
1.000000E 00	0.0	1.000000E 00	0.0
1.000000E 00	0.0	1.000000E 00	0.0
1.000000E 00	0.0	1.000000E 00	0.0
1.000000E 00	0.0	1.000000E 00	0.0

SOURCE DISTRIBUTION ON DUCT FACE.

-6.051150E-02	4.396280E-02	-7.070607E-02	4.862960E-02
-9.074879E-02	5.779730E-02	-1.164650E-01	6.955105E-02
-1.427180E-01	8.154488E-02	-1.627780E-01	9.070808E-02
-1.784120E-01	9.784919E-02	-2.060100E-01	1.104569E-01
-2.527120E-01	1.317959E-01	-3.338090E-01	1.688750E-01
-4.855940E-01	2.383490E-01	-7.459170E-01	3.576789E-01
-1.101919E 00	5.209690E-01	-1.360749E 00	6.385829E-01
-1.684620E-01	8.127087E-02	9.232320E 00	-4.292080E 00

SOURCE DISTRIBUTION ON OUTER DUCT WALL

-5.246289E 01	2.439519E 01	3.305650E 00	-1.525479E 00
9.598000E 00	-4.440849E 00	6.309649E 00	-2.905789E 00
3.517480E 00	-1.605960E 00	1.942269E 00	-8.739000E-01
9.675640E-01	-4.221780E-01	4.561170E-01	-1.866000E-01
2.335750E-01	-8.533067E-02	1.247160E-01	-3.677370E-02
6.554985E-02	-1.116980E-02	3.098380E-02	3.159380E-03
7.029187E-03	1.244690E-02	-8.372437E-03	1.768620E-02
-1.633480E-02	1.967140E-02	-2.006600E-02	1.987230E-02
-2.134450E-02	1.908200E-02	-2.119720E-02	1.774860E-02
-2.024560E-02	1.615220E-02	-1.890140E-02	1.446720E-02
-1.740920E-02	1.280810E-02	-1.570470E-02	1.101020E-02
-1.392930E-02	9.159829E-03	-1.236110E-02	7.506907E-03
-1.101260E-02	6.047700E-03	-9.867728E-03	4.762739E-03
-8.895177E-03	3.630740E-03	-8.061167E-03	2.641050E-03
-7.338818E-03	1.770590E-03	-6.691519E-03	1.004170E-03
-6.107438E-03	3.311799E-04	-5.560718E-03	-2.687299E-04
-5.037837E-03	-7.985809E-04	-4.531749E-03	-1.271280E-03
-4.031010E-03	-1.701690E-03	-3.468190E-03	-2.148880E-03
-2.825960E-03	-2.643200E-03	-2.130030E-03	-3.230680E-03
-1.273980E-03	-4.268307E-03	3.542770E-04	-8.188337E-03

```
INTEGRATION FOR BLOCK CENTERED AT THETA= 1.993285E-01   R= 6.250000E-02
                     DID NOT CONVERGE
                     REAL DIFF = 2.205372E-06   IMAG DIFF =   5.175781E-02

INTEGRATION FOR BLOCK CENTERED AT THETA= 6.083843E 00   R= 6.250000E-02
                     DID NOT CONVERGE
                     REAL DIFF = 2.324581E-06   IMAG DIFF =   5.171204E-02
   ANNULAR RING NO  1    RADIAL POSITION =  6.250000E-02
                         LOCAL IMPEDANCE =  8.533759E 01   4.734263E 02
                                PRESSURE =  8.533759E 01   4.734263E 02
                                VELOCITY =  1.000000E 00   0.0


INTEGRATION FOR BLOCK CENTERED AT THETA= 6.083843E 00   R= 1.875000E-01
                     DID NOT CONVERGE
                     REAL DIFF = 3.814697E-06   IMAG DIFF =   1.831055E-04
   ANNULAR RING NO  2    RADIAL POSITION =  1.875000E-01
                         LOCAL IMPEDANCE =  8.916052E 01   4.804661E 02
                                PRESSURE =  8.916052E 01   4.804661E 02
                                VELOCITY =  1.000000E 00   0.0

   ANNULAR RING NO  3    RADIAL POSITION =  3.000000E-01
                         LOCAL IMPEDANCE =  9.621284E 01   4.936169E 02
                                PRESSURE =  9.621284E 01   4.936169E 02
                                VELOCITY =  1.000000E 00   0.0

   ANNULAR RING NO  4    RADIAL POSITION =  3.874999E-01
                         LOCAL IMPEDANCE =  1.045224E 02   5.093547E 02
                                PRESSURE =  1.045224E 02   5.093547E 02
                                VELOCITY =  1.000000E 00   0.0

   ANNULAR RING NO  5    RADIAL POSITION =  4.499999E-01
                         LOCAL IMPEDANCE =  1.123271E 02   5.243489E 02
                                PRESSURE =  1.123271E 02   5.243489E 02
                                VELOCITY =  1.000000E 00   0.0


INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 5.124998E-01
                     DID NOT CONVERGE
                     REAL DIFF = 6.616116E-06   IMAG DIFF =   1.983643E-04
   ANNULAR RING NO  6    RADIAL POSITION =  4.874998E-01
                         LOCAL IMPEDANCE =  1.179450E 02   5.352490E 02
                                PRESSURE =  1.179450E 02   5.352490E 02
                                VELOCITY =  1.000000E 00   0.0
```

```
INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 4.874998E-01
                DID NOT CONVERGE
                REAL DIFF = 6.079674E-06  IMAG DIFF =  2.746582E-04

INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 5.499998E-01
                DID NOT CONVERGE
                REAL DIFF = 5.722046E-06  IMAG DIFF =  3.356934E-04
   ANNULAR RING NO  7   RADIAL POSITION =  5.124998E-01
                        LOCAL IMPEDANCE =  1.221649E 02  5.435139E 02
                               PRESSURE =  1.221649E 02  5.435139E 02
                               VELOCITY =  1.000000E 00  0.0


INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 6.624997E-01
                DID NOT CONVERGE
                REAL DIFF = 1.621246E-05  IMAG DIFF =  1.678467E-04
   ANNULAR RING NO  8   RADIAL POSITION =  5.499998E-01
                        LOCAL IMPEDANCE =  1.290967E 02  5.571472E 02
                               PRESSURE =  1.290967E 02  5.571472E 02
                               VELOCITY =  1.000000E 00  0.0


INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 5.499998E-01
                DID NOT CONVERGE
                REAL DIFF = 1.144409E-05  IMAG DIFF =  1.220703E-04

INTEGRATION FOR BLOCK CENTERED AT THETA= 6.083843E 00  R= 6.624997E-01
                DID NOT CONVERGE
                REAL DIFF = 1.811981E-05  IMAG DIFF =  2.136230E-04
   ANNULAR RING NO  9   RADIAL POSITION =  5.999997E-01
                        LOCAL IMPEDANCE =  1.399527E 02  5.787158E 02
                               PRESSURE =  1.399527E 02  5.787158E 02
                               VELOCITY =  1.000000E 00  0.0


INTEGRATION FOR BLOCK CENTERED AT THETA= 6.083843E 00  R= 6.624997E-01
                DID NOT CONVERGE
                REAL DIFF = 1.525879E-05  IMAG DIFF =  1.983643E-04
   ANNULAR RING NO 10   RADIAL POSITION =  6.624997E-01
                        LOCAL IMPEDANCE =  1.565026E 02  6.119973E 02
                               PRESSURE =  1.565026E 02  6.119973E 02
                               VELOCITY =  1.000000E 00  0.0
```

```
INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 6.624997E-01
               DID NOT CONVERGE
               REAL DIFF = 1.430511E-05  IMAG DIFF =  1.525879E-04

INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 8.124996E-01
               DID NOT CONVERGE
               REAL DIFF = 2.574921E-05  IMAG DIFF =  4.272461E-04

INTEGRATION FOR BLOCK CENTERED AT THETA= 6.083843E 00  R= 7.374997E-01
               DID NOT CONVERGE
               REAL DIFF = 2.670288E-05  IMAG DIFF =  2.136230E-04

INTEGRATION FOR BLOCK CENTERED AT THETA= 6.083843E 00  R= 8.124996E-01
               DID NOT CONVERGE
               REAL DIFF = 2.384186E-05  IMAG DIFF =  2.288818E-04
  ANNULAR RING NO 11   RADIAL POSITION =  7.374997E-01
                       LOCAL IMPEDANCE =  1.830660E 02  6.662146E 02
                              PRESSURE =  1.830660E 02  6.662146E 02
                              VELOCITY =  1.000000E 00  0.0


INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 8.749996E-01
               DID NOT CONVERGE
               REAL DIFF = 6.675720E-06  IMAG DIFF =  2.136230E-04

INTEGRATION FOR BLOCK CENTERED AT THETA= 6.083843E 00  R= 7.374997E-01
               DID NOT CONVERGE
               REAL DIFF = 1.049042E-05  IMAG DIFF =  2.288818E-04
  ANNULAR RING NO 12   RADIAL POSITION =  8.124996E-01
                       LOCAL IMPEDANCE =  2.218372E 02  7.467739E 02
                              PRESSURE =  2.218372E 02  7.467739E 02
                              VELOCITY =  1.000000E 00  0.0


INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 8.124996E-01
               DID NOT CONVERGE
               REAL DIFF = 1.335144E-05  IMAG DIFF =  1.220703E-04

INTEGRATION FOR BLOCK CENTERED AT THETA= 1.993285E-01  R= 8.124996E-01
               DID NOT CONVERGE
               REAL DIFF = 1.621246E-05  IMAG DIFF =  1.983643E-04
  ANNULAR RING NO 13   RADIAL POSITION =  8.749996E-01
                       LOCAL IMPEDANCE =  2.736311E 02  8.560786E 02
                              PRESSURE =  2.736311E 02  8.560786E 02
                              VELOCITY =  1.000000E 00  0.0
```

```
INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 8.749996E-01
                DID NOT CONVERGE
                REAL DIFF = 1.335144E-05  IMAG DIFF =  1.831055E-04

INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 9.624995E-01
                DID NOT CONVERGE
                REAL DIFF = 9.536743E-06  IMAG DIFF =  2.899170E-04
   ANNULAR RING NO 14   RADIAL POSITION =  9.249995E-01
                        LOCAL IMPEDANCE =  3.457480E 02  1.010026E 03
                               PRESSURE =  3.457480E 02  1.010026E 03
                               VELOCITY =  1.000000E 00  0.0


INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 9.249995E-01
                DID NOT CONVERGE
                REAL DIFF = 1.144409E-05  IMAG DIFF =  2.186584E-02

INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 9.874995E-01
                DID NOT CONVERGE
                REAL DIFF = 7.629395E-06  IMAG DIFF =  2.427673E-02
   ANNULAR RING NO 15   RADIAL POSITION =  9.624995E-01
                        LOCAL IMPEDANCE =  4.480442E 02  1.229231E 03
                               PRESSURE =  4.480442E 02  1.229231E 03
                               VELOCITY =  1.000000E 00  0.0


INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            R= 9.624995E-01
                DID NOT CONVERGE
                REAL DIFF = 6.675720E-06  IMAG DIFF =  2.485657E-02

INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            Z=-1.250000E-02
                DID NOT CONVERGE
                REAL DIFF = 4.768372E-06  IMAG DIFF =  7.092285E-02

INTEGRATION FOR BLOCK CENTERED AT THETA= 0.0            Z=-3.750000E-02
                DID NOT CONVERGE
                REAL DIFF = 5.722046E-06  IMAG DIFF =  1.831055E-04
   ANNULAR RING NO 16   RADIAL POSITION =  9.874995E-01
                        LOCAL IMPEDANCE =  6.368638E 02  1.635084E 03
                               PRESSURE =  6.368638E 02  1.635084E 03
                               VELOCITY =  1.000000E 00  0.0

        RESISTANCE RATIO, TAU =  0.51969945E 00
        REACTANCE RATIO,  CHI =  0.19329624E 01
```