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PLANETARY SPECTRA FOR ANISOTROPIC SCATTERING

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Abstract

We examine here some of the effects on planetary spectra that would be produced by departures from isotropic scattering. The phase function $\tilde{\omega}(1 + a \cos\Theta)$ is the simplest departure to handle analytically and the only phase function, other than the isotropic one, that can be incorporated into a Chandrasekhar first approximation. This approach has the advantage of illustrating trends resulting from anisotropies while retaining the simplicity that yields physical insight.

An algebraic solution to the two sets of anisotropic H functions is developed in the appendix. It is readily adaptable to programmable desk calculators and gives emergent intensities accurate to 0.3 percent, which is sufficient even for spectroscopic analysis.

I. INTRODUCTION

A considerable literature exists on the photometric and polarization effects of anisotropic scattering by planets. But spectroscopic effects have been examined quantitatively almost entirely with isotropic-scattering models. An important exception is the work of van de Hulst and Grossman (1968), who initiated a large number of numerical solutions with the doubling method and the Henyey-Greenstein phase function.

The problem is not that spectroscopic data are more difficult to analyze with anisotropic scattering. Rather, the "trouble" is that, virtually any reasonable model of atmospheric inhomogeneity and scattering phase function can be made to duplicate the data well enough, although one's conclusions may be dependent on the adopted model. The trade-off of one parameter for another allowed van de Hulst and Grossman to develop their "similarity relations". But they noted, "As a consequence, it will be difficult, in practice, to interpret absorption lines observed in diffuse reflection in a unique manner."

The investigation of other analytic models may help to illustrate weaknesses in interpretations that are made on the basis of isotropic, homogeneous theory. The phase function $\tilde{\omega}(1 + a \cos\Theta)$ is not very representative of Mie scattering with a strong forward lobe, such as we find in Venus' atmosphere. Nevertheless, it will illustrate some of

the effects to be expected (e.g., the role that anisotropy of scattering can have in causing absorption bands to vary with position on the disk and the planetary phase).

II. REFLECTED INTENSITY FOR THE ANISOTROPIC PHASE FUNCTION: A SUMMARY

The radiative transfer problem with a phase function for single scattering of $\tilde{\omega}(1 + a\cos\Theta)$ was solved exactly more than 25 years ago (Chandrasekhar, 1950 - hereinafter referred to as "RT"). The solution for the emergent intensity, I, is divided into two parts: one, $I^{(0)}$, independent of the azimuth of the incident sunlight (relative to the observer), and the other, $I^{(1)}$, dependent on azimuth. Each of the components is solvable in terms of particular Chandrasekhar H-functions. Thus, having tabulations of these H-functions is tantamount to having numerical solutions for any given parameters, $\tilde{\omega}$ and a.

In the conventional notation (RT), the equation of transfer for $I(\tau;\mu,\psi)$ is

$$\mu \frac{dI}{d\tau} = I - \frac{1}{4\pi}\int_{-1}^{1}\int_{0}^{2\pi} I\, p(\mu,\mu';\ \psi-\psi')\,d\psi'd\mu' \qquad (1)$$

$$- \frac{F}{4}\, e^{-\tau/\mu_o}\, p(\mu,-\mu_o;\ \psi-\psi_o).$$

With a scattering phase function,

$$p(\mu,\mu';\ \psi-\psi') = \tilde{\omega}(1 + a\cos\Theta),\quad -1 \le a \le 1, \tag{2}$$

and

$$\cos\Theta = \mu\,\mu' + (1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\cos(\psi-\psi'), \tag{3}$$

we write

$$I(\tau;\mu,\psi) = I^{(0)}(\tau;\mu) + I^{(1)}(\tau;\mu)\cos(\psi-\psi_o). \tag{4}$$

Equation (1) now yields separate equations for the azimuth-independent intensity, $I^{(0)}$, and the azimuth-dependent amplitude, $I^{(1)}$. Both equations may be solved in the first approximation in a manner analogous to, but rather more messy than, the solution for isotropic scattering (RT, pp 149ff).

The exact solutions for the emergent intensity have also been obtained. For $I^{(0)}$ we have

$$I^{(0)}(0,\mu) = \frac{\tilde{\omega}F}{4}\ \frac{\mu_o}{\mu+\mu_o}\ H^{(0)}(\mu)H^{(0)}(\mu_o)[1-c_o(\mu+\mu_o)-a(1-\tilde{\omega})\mu\mu_o], \tag{5}$$

where (RT, p 141)

$$c_o = \frac{\tilde{\omega}\ \alpha_1^{(0)}a(1-\tilde{\omega})}{2-\tilde{\omega}\ \alpha_o^{(0)}} \tag{6}$$

and where $\alpha_n^{(0)}$ is the nth moment of $H^{(0)}(\mu)$.

The solution for $I^{(1)}$ is

$$I^{(1)}(0,\mu) = \frac{\tilde{\omega}aF}{4} \frac{\mu_o}{\mu+\mu_o} (1-\mu^2)^{1/2} (1-\mu_o^2)^{1/2} H^{(1)}(\mu) H^{(1)}(\mu_o). \tag{7}$$

The integral equation for $H^{(j)}(\mu)$ (where j = 0 or 1) is

$$\frac{1}{H^{(j)}(\mu)} = \left[1 - 2\int_0^1 \psi^{(j)}(\mu')d\mu'\right]^{1/2} + \int_0^1 \frac{\mu'\psi^{(j)}(\mu')H^{(j)}(\mu')}{\mu + \mu'} d\mu'. \tag{8}$$

Here the "characteristic function", $\psi^{(j)}$, is related to k_j, the root of the "characteristic equation" that gives the τ dependence of the equation of transfer (1), by (RT, p 114)

$$\psi^{(j)}(\mu) = \frac{1}{2} (1-k_j^2 \mu^2)^{1/2}. \tag{9}$$

We have in the first approximation

$$k_o = \frac{1}{\mu_1} [(1-\tilde{\omega}a\mu_1^2)]^{1/2}, \tag{10}$$

and

$$k_1 = \frac{1}{\mu_1} \left[1 - \frac{\tilde{\omega}a(1-\mu_1^2)}{2}\right]^{1/2}, \tag{11}$$

where $\mu_1 = 1/\sqrt{3}$ is the first Gaussian division point. Hence if we write the characteristic function as

$$\psi^{(j)}(\mu_1) = r_j + s_j\mu_1^2, \tag{12}$$

we obtain $r_o = \tilde{\omega}/2$, $s_o = \tilde{\omega}a(1-\tilde{\omega})/2$, and $r_1 = -s_1 = \tilde{\omega}a/4$.

In Tables 1 and 2 we give solutions for $H^{(0)}$ and its moments obtained by successive evaluations of equation (8). The solution was iterated until successive values of the zeroth moment, $\alpha_o^{(0)}$, converged to 1 part in 10^4 with a seven-point Gaussian quadrature. When used with Chandrasekhar's tabulations of $H^{(0)}$ for a = 0 (RT, p 125), a = +1 (RT, p 139), their moments (RT, pp 126, 328, 141), and $H^{(1)}$ (RT, p 141), these tables provide a convenient basis for tabulating solutions to equation (4). Additional values of $H^{(0)}$ for $\tilde{\omega} = 0.975$ and several values of a between -1 and +1 were tabulated by Harris (1957).

III. PLANETARY SPECTRA

In addition to evaluating solutions to the integral equations, I have developed (cf. Appendix) an approximate analytic solution to equation (4), which is accurate to 0.3 percent at worst. With this expression I have made a number of sample comparisons of planetary spectra with strongly forward (a = +1) and backward (a = -1) scattering functions.

Let θ be the polar angle (co-latitude) of the observed point, in a coordinate system whose pole is the pole of the ecliptic. Also let ϕ be the longitude of the point measured from the sub-earth point; Φ is the planetary phase angle. Then χ and χ_o are the zenith angles (at the point of observation) of the earth and sun. That is,

$$\mu \equiv \cos\chi = \sin\theta \cos\phi , \tag{13}$$

$$\mu \equiv \cos\chi_o = \sin\theta \cos(\Phi-\phi), \tag{14}$$

and the azimuthal angle of scattering is given by

$$\cos(\psi-\psi_o) = \frac{\mu\mu_o - \cos\Phi}{(1-\mu^2)^{1/2}(1-\mu_o^2)^{1/2}} . \tag{15}$$

With this geometry we obtain the spectral absorption curves in Fig. 1 for a weak line. The corresponding curves for isotropic scattering (a = 0) are not drawn in, but they lie almost mid-way between the extreme anisotropic cases.

Generally speaking, the limb/center ratios are not strongly dependent on the latitudinal zone but are sensitive to the anisotropy factor. At 60° the limb/center absorption ratio is 0.50 for backward scattering (a = -1) and 0.70 for a = +1. At the crescent, there is, of course, much less variation, but the trend is reversed: ~ 1.0 for a = -1 and 0.92 for a = +1.

Similarly, we can make some useful estimates of how the phase variation will change with a. For weak lines a simple approximation, treating the continuum reflection as weighted over the planet like a lambertian reflector, predicts that every zonal band has the same phase variation (Chamberlain 1975). This uniformity, in fact, is nearly the case in our model calculations, but again the variation is strongly

dependent on a. The phase absorption ratio, computed for ($\Phi = 60°$)/($\Phi = 150°$) is 1.5 for a = -1; 2.25 for a = 0; and 3.3 for a = +1.

That the spectral phase curve will go through a secondary minimum near full phase seems unlikely, at least with the simple anisotropic phase function examined here. For purposes of comparison with the complete phase curves for isotropic scattering (Chamberlain 1970, Fig 6), the albedos adopted in the present example ($\tilde{\omega}_\nu = 0.975$, $\tilde{\omega}_c = 0.99$) correspond to the parameters used in that paper of w = 0.0154 (very weak line) and q = 1.54 (moderately important continuum absorption).

Finally, the curve of growth for continuum albedos $\tilde{\omega} = 1$, is very nearly linear with $(1-\tilde{\omega}_\nu)^{1/2}$, for observations at any one point, as it is in the isotropic case (Chamberlain, 1965). However, the slope varies with the anisotropy factor and to some degree with the scattering azimuth.

Further discussion of the calculations will be more illuminating in the context of reliable data or comparisons with more realistic (and complicated) phase functions. It will also be necessary to consider what effects vertical inhomogeneity will have on the spectral phase variation. It is not clear that the effects of inhomogeneity and anisotropy can be distinguished. We expect to report on these matters at a later date.

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APPENDIX: ALGEBRAIC SOLUTIONS FOR H-FUNCTIONS

In this Appendix we develop practical formulae for computing the H-functions and intensities quickly and accurately, without need for interpolation. The algebraic solution here is a generalization of the Chandrasekhar "first approximation", which can handle phase functions expressed as Legendre polynomials of order N=0 and 1. To go to order N requires (RT, p 151) an nth approximation given by $n > N/2 + 1/4$. Being restricted by choice to the first approximation is equivalent to limiting $\Psi(\mu)$ to the two terms given in equation (12).

The technique developed here was previously introduced for isotropic scattering (N=0) in a paper concerned with coupled spectral lines (Chamberlain and Smith, 1975). In that problem the parameters μ and μ_0 are no longer simple direction cosines and may assume values exceeding unity. The present technique has the useful quality of providing H-functions quite accurately for large μ and μ_0, whereas linear interpolation from a table of $H(\mu)$ can in such cases be very inaccurate (Chamberlain and Wallace, 1974, Appendix B).

The first approximation, as obtained from a one-point Gaussian summation, is

$$H^{(j)}(\mu) = \frac{1 + \mu/\mu_j}{1 + k_j\mu} , \qquad (A.1)$$

where j is an index to signify either the $H^{(0)}$- or $H^{(1)}$-functions of equations (5) and (7). However, μ_1 is not an especially accurate weighting factor over the whole range of μ. Hence we could get a better representation by

$$H^{(j)}(\mu) = \frac{1 + A^{(j)}(\tilde{\omega},a)\mu}{1 + k_j\mu} \tag{A.2}$$

and evaluate $A^{(j)}$ to give the best fit to some specified set of points.

However, with a little additional effort we may obtain a "second approximation" by putting equation (A.2) under the integral of equation (8). It is then advantageous to evaluate $A^{(j)}$ after the integration, in order to optimize a fit with the second approximation.

Although a bit messy, the integration is straightforward. We have

$$\frac{1}{H^{(j)}(\mu)} = \left[1-2r_j - \frac{2s_j}{3}\right]^{1/2} + \zeta_o + \frac{\zeta_2}{k_j} + \left(\frac{\zeta_2\mu^2-\zeta_1\mu}{1-k_j\mu}\right) \ln\frac{1+\mu}{\mu} + \left(\frac{\zeta_1/k_j-\zeta_2/k_j^2}{1-k_j\mu}\right) \ln(1-k_j), \tag{A.3}$$

where

$$\zeta_o = \frac{s_j}{2k_j} + \frac{s_jA^{(j)}(\tilde{\omega},a)}{3k_j} - \frac{s_jA^{(j)}(\tilde{\omega},a)}{2k_j^2}(1 + k_j\mu) \tag{A.4}$$

$$\zeta_1 = s_j - \frac{s_j\mu}{k_j} + \frac{s_jA^{(j)}(\tilde{\omega},a)\mu}{k_j^2}(1 + k_j\mu), \tag{A.5}$$

and

$$\zeta_2 = r_j A^{(j)}(\tilde{\omega},a) - \frac{s_j}{k_j}(1 + k_j\mu) - \frac{s_j A^{(j)}(\tilde{\omega},a)\mu}{k_j}\left[1 - \frac{(1+k_j\mu)^2}{k_j\mu}\right]^{-1}. \qquad (A.6)$$

The r_j and s_j are given below equation (12).

Optimum values of $A^{(0)}(\tilde{\omega},a)$ have been obtained by a trial-and-error adjustment of $A^{(0)}$ and a fit of the resulting moments $\alpha_o^{(0)}(\tilde{\omega},a)$ to the exact values for a=0 and +1 and $\tilde{\omega}$= 0, 0.4, 0.8, and 1.0. Writing

$$A^{(0)}(\tilde{\omega},a) = A_o[1 + B_o a), \qquad (A.7)$$

we obtain

$$A_o \equiv A^{(0)}(\tilde{\omega},a) = \sqrt{3} + 0.1853\ \tilde{\omega} - 0.1251\ \tilde{\omega}^2 + 0.1851\ \tilde{\omega}^3 \qquad (A.8)$$

and

$$B_o \equiv \frac{A^{(0)}(\tilde{\omega},1)}{A^{(0)}(\tilde{\omega},0)} - 1 = -0.0210\ \tilde{\omega} + 0.0041\ \tilde{\omega}^2 + 0.0169\ \tilde{\omega}^3. \qquad (A.9)$$

The above three equations imply that $A^{(0)}(\tilde{\omega},a)$ is symmetrical in ± a and, indeed, to the accuracy of this approximation, that is the case. However, since a appears intrinsically in k_o, neither $H^{(0)}(\mu)$ nor $\alpha_o^{(0)}$ is symmetrical in a.

To summarize, the algebraic solution for $I^{(0)}$ consists of $A^{(0)}(\tilde{\omega},a)$, computed from the above three equations, and $H^{(0)}(\mu)$ from equations (A.3) through (A.6). Here k_o is given by equation (10) with $\mu_1 = 1/\sqrt{3}$, and r_o and s_o are

defined by equation (12). Then c_o is given by equation (6) with $\alpha_o^{(0)}$ and $\alpha_1^{(0)}$ obtained by Gaussian quadratures of $H^{(0)}(\mu)$. Finally $I^{(0)}(0,\mu)$ is computed from equation (5). A useful check on the accuracy of the calculations is provided by the relation (RT, p.109)

$$1 - \alpha_o^{(0)} + \frac{1}{2}\left(r_o\ \alpha_o^{(0)\,2} + s_o\ \alpha_o^{(0)}\right) = 0. \qquad (A.10)$$

We obtain $A^{(1)}(\tilde{\omega}a)$ by fitting the zeroth moments $\alpha_o^{(1)}$ to the moments of the $H^{(1)}(\mu)$-functions computed from equation (8). Values of $H^{(1)}(\mu)$ have been tabulated by Chandrasekhar (RT, p 141). Additional values of $H^{(1)}(\mu)$ and its moments have been computed by the author. We find, over the range $-1 \le \tilde{\omega}a \le 1$,

$$A^{(1)}(\tilde{\omega}a) = \sqrt{3} + 0.1055\ (\tilde{\omega}a) + 0.0143\ (\tilde{\omega}a)^2. \qquad (A.11)$$

Thus $I^{(1)}(0,\mu)$ is found from equations (7),(11),(A.3) to (A.6), and (A.11).

In the worst cases $H^{(0)}$ has absolute accuracy to about 3 units in the third decimal. This error is due to excursions of $H^{(0)}(\mu)$ about the precise values for a given $\tilde{\omega}$ and a. The values of the moment α_o are good to 1 or 2 units in the fourth decimal. Thus the principal errors arise from the algebraic representation of equation (A.3) and not from the formulae for $A(\tilde{\omega},a)$, equations (A.7)-(A.9). The $H^{(1)}$ functions are even slightly more accurate, so that the limiting accuracy

of $I(\mu,\mu_0)$ depends on $H^{(0)}$.

As noted at the beginning of the Appendix, in problems involving spectral lines that are coupled by having a common upper level, we concounter functions $H(x)$, where x is often greater than unity. The solution given here is especially useful for this situation. In fact, in the limit $x \to \infty$, we find

$$H(x) \to (1 - 2r - \tfrac{2}{3} s)^{-1/2}. \qquad (A.12)$$

This limit is obvious from equation (8), but it is equally valid for equation (A.3). [To prove the limit in the latter case it is necessary to expand both $\ln(1 + 1/\mu)$ and $(1 - 1/k\mu)^{-1}$ to order μ^{-2}.] The accuracy of the algebraic approximation for large arguments is very good, as shown for isotropic scattering by the Figure in the Appendix of Chamberlain and Smith (1975).

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TABLE 1

$H^{(0)}(\mu)$ FUNCTIONS COMPUTED FROM THE EXACT

INTEGRAL EQUATION

(a = -1.0)

μ \ ω	0.2	0.4	0.6	0.8	0.9	0.925	0.95	0.975
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.0217	1.0482	1.0819	1.1291	1.1638	1.1752	1.1888	1.2063
.2	1.0318	1.0726	1.1273	1.2092	1.2742	1.2965	1.3235	1.3598
.3	1.0387	1.0896	1.1605	1.2718	1.3650	1.3979	1.4387	1.4947
.4	1.0437	1.1025	1.1865	1.3235	1.4431	1.4864	1.5409	1.6175
.5	1.0477	1.1128	1.2077	1.3673	1.5116	1.5650	1.6333	1.7309
.6	1.0508	1.1212	1.2255	1.4053	1.5727	1.6359	1.7176	1.8365
.7	1.0535	1.1282	1.2406	1.4385	1.6276	1.7002	1.7952	1.9355
.8	1.0557	1.1342	1.2537	1.4680	1.6773	1.7591	1.8670	2.0287
.9	1.0575	1.1394	1.2651	1.4943	1.7227	1.8133	1.9338	2.1167
0.95	1.0584	1.1417	1.2703	1.5064	1.7441	1.8387	1.9654	2.1589
1.00	1.0592	1.1439	1.2752	1.5180	1.7644	1.8632	1.9960	2.2001

(a = -0.5)

0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.0236	1.0517	1.0865	1.1337	1.1679	1.1788	1.1919	1.2086
.2	1.0354	1.0790	1.1360	1.2185	1.2825	1.3041	1.3301	1.3648
.3	1.0434	1.0985	1.1727	1.2856	1.3776	1.4097	1.4492	1.5029
.4	1.0495	1.1136	1.2020	1.3414	1.4600	1.5023	1.5553	1.6289

TABLE 1

μ \ ω	0.2	0.4	0.6	0.8	0.9	0.925	0.95	0.975
.5	1.0543	1.1256	1.2260	1.3892	1.5328	1.5852	1.6517	1.7458
.6	1.0583	1.1356	1.2464	1.4308	1.5980	1.6602	1.7400	1.8550
.7	1.0615	1.1440	1.2637	1.4674	1.6569	1.7286	1.8217	1.9577
.8	1.0643	1.1512	1.2789	1.5001	1.7105	1.7914	1.8975	2.0547
.9	1.0667	1.1575	1.2922	1.5294	1.7596	1.8495	1.9682	2.1465
.95	1.0677	1.1603	1.2982	1.5429	1.7827	1.8768	2.0018	2.1906
1.00	1.0688	1.1630	1.3039	1.5558	1.8048	1.9032	2.0344	2.2336

(a = +0.5)

0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.0276	1.0592	1.0965	1.1442	1.1768	1.1870	1.1989	1.2138
.2	1.0426	1.0929	1.1550	1.2396	1.3012	1.3213	1.3452	1.3763
.3	1.0533	1.1177	1.2000	1.3170	1.4064	1.4365	1.4729	1.5214
.4	1.0616	1.1374	1.2366	1.3827	1.4990	1.5391	1.5883	1.6551
.5	1.0682	1.1536	1.2673	1.4398	1.5819	1.6320	1.6942	1.7802
.6	1.0737	1.1671	1.2936	1.4903	1.6571	1.7170	1.7922	1.8979
.7	1.0784	1.1787	1.3164	1.5354	1.7258	1.7953	1.8836	2.0094
.8	1.0824	1.1887	1.3365	1.5759	1.7890	1.8680	1.9692	2.1154
.9	1.0859	1.1975	1.3543	1.6126	1.8473	1.9356	2.0497	2.2164
0.95	1.0874	1.2015	1.3624	1.6297	1.8749	1.9677	2.0881	2.2652
1.0	1.0889	1.2052	1.3702	1.6461	1.9015	1.9987	2.1255	2.3128

TABLE 2

Moments of $H^{(0)}(\mu)$ and the Constant c_o

($a = -1.0$)

$\tilde{\omega}$	$\alpha_o^{(o)}$	$\alpha_1^{(o)}$	$\alpha_2^{(o)}$	c_o
0.2	1.0434	0.5255	0.3512	-0.0469
.4	1.1027	0.5690	0.3764	-0.0864
.6	1.1897	0.6140	0.4144	-0.1146
.8	1.3381	0.7072	0.4820	-0.1217
.9	1.4758	0.7962	0.5476	-0.1067
0.925	1.5279	0.8305	0.5732	-0.0982
0.95	1.5955	0.8755	0.6068	-0.0859
0.975	1.6943	0.9421	0.6571	-0.0660

($a = -0.5$)

0.2	1.0495	0.5293	0.3540	-0.0237
.4	1.1145	0.5683	0.3818	-0.0439
.6	1.2067	0.6248	0.4223	-0.0588
.8	1.3591	0.7208	0.4921	-0.0632
.9	1.4965	0.8100	0.5580	-0.0558
.925	1.5479	0.8439	0.5833	-0.0515
.95	1.6140	0.8880	0.6163	-0.0452
0.975	1.7099	0.9528	0.6652	-0.0349

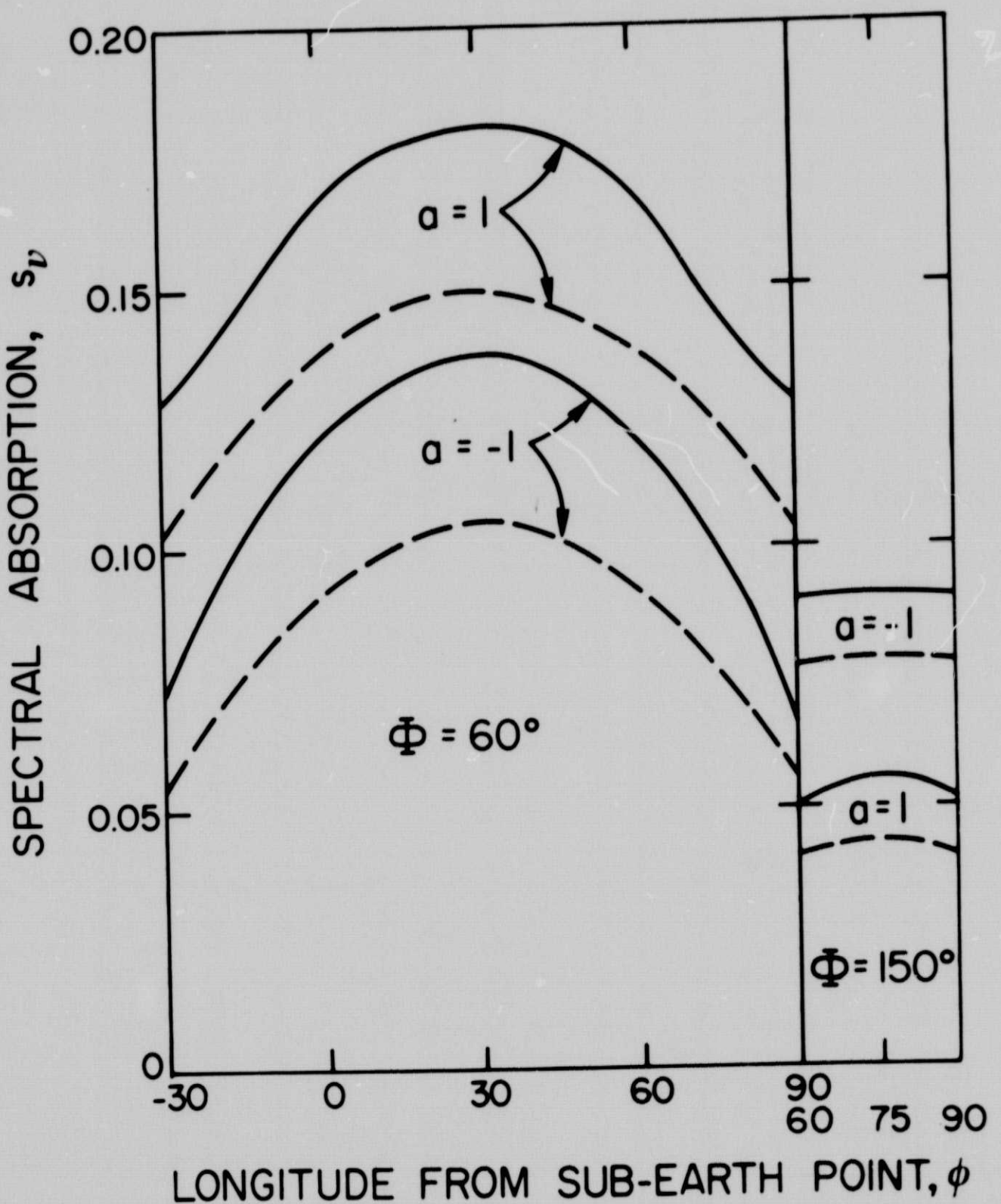
TABLE 2

(a = +0.5)

$\tilde{\omega}$	$\alpha_0^{(o)}$	$\alpha_1^{(o)}$	$\alpha_2^{(o)}$	c_o
0.2	1.0622	0.5372	0.3597	0.0240
.4	1.1403	0.5846	0.3936	0.0454
.6	1.2453	0.6495	0.4404	0.0622
.8	1.4076	0.7526	0.5157	0.0689
.9	1.5452	0.8425	0.5824	0.0622
.925	1.5948	0.8755	0.6070	0.0579
.95	1.6574	0.9175	0.6386	0.0512
0.975	1.7459	0.9775	0.6841	0.0400

LEGEND

Fig. 1 - Variation of absorption across the visible disk of a planet at two phases. The "mid-latitude" curves apply to a zonal band at latitude 45°. The absorption $s_\nu = 1-I_\nu/I_c$, is for a weak line: specifically, $\tilde{\omega}_\nu = 0.95$ with weakly absorbing continuum, $\tilde{\omega}_c = 0.99$.


0.20
0.15
0.10
0.05
0
SPECTRAL ABSORPTION, s_ν
a = 1
a = -1
Φ = 60°
a = -1
a = 1
Φ = 150°
-30
0
30
60
90
60
75
90
LONGITUDE FROM SUB-EARTH POINT, ϕ