NASA TECHNICAL MEMORANDUM

NASA TM X-71770

NASA TM X-71770

(NASA-TM-X-71770) COST PERFORMANCE SATELLITE DESIGN USING QUEUEING THEORY (NASA) 35 p HC $3.75 CSCL 22A

N75-29155

Unclas

G3/15 31425

COST PERFORMANCE SATELLITE DESIGN USING QUEUING THEORY

by Gerald F. Hein
Lewis Research Center
Cleveland, Ohio 44135

TECHNICAL PAPER presented at
Sixth Annual Simulation Conference cosponsored by
the Institute of Electrical and Electronics Engineers and
the University of Pittsburgh
Pittsburgh, Pennsylvania, April 24-25, 1975

E-8044

COST PERFORMANCE SATELLITE DESIGN USING QUEUING THEORY

Gerald F. Hein
Space Flight Programs Directorate
NASA Lewis Research Center
Cleveland, Ohio

ABSTRACT

The use of satellites for wide area direct broadcasts results in large power requirements for the satellite. Such satellites are or will be used to provide remote health care delivery and educational services to remote regions and many other new services including warnings to the public of impending natural disasters. These new special purpose satellites are very cost sensitive to the number of broadcast channels, usually will have Poisson arrivals, fairly low utilization (less than 35%), and a very high availability requirement. To solve the problem of determining the effects of limiting C the number of channels, the Poisson arrival, infinite server queuing model will be modified to describe the many server case. The model is predicated on the reproductive property of the Poisson distribution.

For small changes in the Poisson parameter under the assumptions stated previously, the resulting distribution of states or number in the system will be Poisson. A difference equation has been developed to describe the change in the Poisson parameter. When all initially delayed arrivals reenter the system, a (C + 1) order polynomial must be solved to determine the new or effective value of the Poisson parameter. When less than 100 percent of the arrivals reenter the system, the effective value must be determined by solving a transcendental equation.

The model was used to determine the effects of limiting the number of channels for a disaster warning satellite. State probabilities and delay probabilities were estimated for several values of the number of channels C for arrival and service rates obtained from disaster warnings issued by the National Weather Service. The results predicted by the queuing model were compared with the results of a digital computer simulation.

INTRODUCTION

The use of geosynchronous orbit satellites for communications began in the 1960's with SYNCOM I. Since then a network has been established to provide international telecommunications via satellite to about 80 countries throughout the world. The use of modern telecommunications has become so commonplace that there is often a tendency to forget how recent are the technological achievements which make such usage possible.

The present network for commercial traffic is used for point-to-point transmission of telephone and television. Such a network requires large and expensive ground transmitter-receiver systems.

In late 1971 an initiative study was conducted for the Executive Office of the President primarily by NASA and included about 100 participants from various government agencies. This study, "Communications for Social Needs," was an effort to solve social problems through the use of advanced technology. One of the results of the study was the conclusion that there is an evolving need for what might be called special purpose communications satellites.

The initiative study concentrated on an area of communication satellite applications in which the ground receiver systems or terminals were very large in number and hence a driving cost parameter. Contrasting the commercial and the special purpose types of communication satellites, one finds the following differences:

Commercial: Low power, point-to-point transmission, small number of expensive terminals, large number of channels

Special purpose: High power, wide area coverage, large number of low cost terminals, small number of channels

In usage, the operation of the special purpose type generally consists of transmitting information to many receivers over a relatively large geographic area. The applications considered in the aforementioned study were the uses of communications for remote health care delivery, electronic mail handling, law enforcement, education, and as a possible means of warning the public of impending natural disasters, such as hurricanes or tornados.

The most important difference between the commercial and the special purpose satellites is that the major design objectives are radically opposed to one another. Herbert Raymond[3] suggested that the most meaningful parameter for a commercial satellite system is probably the utilization factor. It should be high in order to maintain a profitable system. The special purpose satellite must be designed using availability as the major parameter. In the remote regions of the far northern hemisphere, for example, where neighboring villages may be separated by distances of hundreds of miles, the need for emergency medical care can be met through the use of paraprofessionals communicating with doctors via satellite. However, it is not reasonable to expect anyone to wait for service in this application. Should satellites be used in the future to warn the public of impending natural disasters, it would be essential to dispatch warnings as quickly as possible. A system which warns of tornados 15 minutes after their occurrence would be of little value to the public.

The National Aeronautics and Space Administration (NASA) and the National Oceanic and Atmospheric Administration (NOAA) have been conducting joint investigations of various technologies in order to examine the feasibility of using communication satellites for one of the applications mentioned previously, namely, to provide warnings to the general public in the event of an impending natural disaster. The various candidate systems for disaster warning which have been suggested for consideration include the mass ringing of telephones, microwave transmission of radio signals, terrestrial radio networks and the use of communication satellites.

Government organizations other than NASA are conducting studies of terrestrial systems and NASA is confining its investigation to the use of satellites. When completed, the studies will be used to determine the most cost effective system. At the present time, satellites offer a very viable alternative because several meteorological functions may be combined with the communication function, and the satellite system has the desirable property of being "hardened" against natural disasters. That is, satellites are not prone to destruction from an impending natural disaster.

The functions of a natural disaster warning system as reported by Hein and Stevenson[2] are:

1. Route disaster warnings to the general public

2. Provide disaster communications among national, regional, and local weather offices and affected areas

3. Provide environmental information to the public

4. Provide a system for collecting decision information for the dissemination of warnings

The natural disasters which would be monitored by a DWS include tornados, severe thunderstorms, flash floods, tsunami, earthquakes, hurricanes, forest fires, winter storms, and a category called other.

In September 1973, the Aerospace Systems Group of the Computer Sciences Corporation in Arlington, Virginia, began work on a feasibility study of using satellites for a natural disaster warning system. This study, funded by the NOAA and managed by the NASA Lewis Research Center, provided the conceptual design of a satellite DWS.

As will be indicated in more detail, the primary problem with the design of such a satellite DWS is the determination of the minimum number of communication channels required to service the system needs. This determination requires an extensive analysis of the message traffic for the proposed system in the early 1980's.

The satellite system will require a high Effective Isotropic Radiated Power (EIRP). Present satellite power is on the order of 50 watts per channel and designs of the mid- to late seventies will be limited to less than 500 watts per channel. As reported by the Computer Sciences Corporation, DWS satellites will require more power than present designs can deliver. If the satellite is to be a viable alternative, the total power will be limited to something less than 10 kilowatts which thus precludes the addition of large safety margins for the channel requirements.

There is a genuine need to determine the minimum number of channels for the DWS requirements. The application of satellite technology for the solution of social problems such as those discussed in the study "Communications for Social Needs" may be delayed if the problem discussed above is not solved. The requirements for the DWS satellite are very similar to the requirements for a large class of satellites which may be used to solve some social problems. For lack of a better description, this class of satellites will be called special purpose communication satellites. The salient need is to determine the performance criteria required for the estimation of the channel requirements. The alternatives are to overdesign which is not feasible here, use simulation techniques which may be expensive or use an analytical technique.

The potential solution of the channel requirements problem for special purpose communication satellites was thought to be in the problem class of the GI/G/C queue. An extensive analysis was made of the types of messages sent over the teletype network of the National Weather Service. It was found that the messages could be classified into six different groups of input. It was also demonstrated that the six types of messages could each be represented by a Poisson distribution. Although such a simplification should be expected for large numbers of messages over the period of a month, the hypothesis testing was done for relatively short intervals such as four or five days because of requests by those involved with the problem at NOAA. It is very important to know the distributions of what NOAA personnel have called "spikes" or short bursts of increased traffic intensity such as that experienced during the rampage by Hurricane Agnes from June 14 to 28, 1972 or the tornados of April 3, 1974, so that the communication system can be designed to handle such traffic loads.

ANALYTICAL MODEL

To develop the framework of the model, reference will be made to the Poisson arrival model with an infinite number of servers. The infinite server case has been solved for Poisson arrivals. The basic hypothesis is that the state distribution for the many server case should be similar to that for the infinite server case. Because of the reproductive property of the Poisson distribution and the fact that the mean and variance are equal, a change in the Poisson parameter λ to $\lambda + \Delta\lambda$ results in a Poisson distribution with a mean and variance of $\lambda + \Delta\lambda$.

In the case of an infinite number of servers with Poisson arrivals, Takacs proved that with an arbitrary service discipline, the limiting distribution of the states can be represented by a Poisson distribution. Moreover, the distribution is independent of the initial state. The proof of this theorem is given in Takacs[4] and is stated here.

Let $\alpha = \int X \, dH(X)$ and $P[\xi(t) = K] = P_k(t)$ where α is the average service time, $P_k(t)$ is the probability of being in state K at time t, θ is the Poisson arrival parameter, and $\xi(t)$ is the queue size or number in the system.

Theorem 1. If $\xi(0) = 0$, then

$$P_k(t) = \exp\left\{-\theta \int_0^t [1 - H(X)]dX\right\} \frac{\left\{\theta \int_0^t [1 - H(X)]dX\right\}^K}{K!} \tag{1}$$

for $K = 0, 1, 2, \ldots$ and if $\alpha < \infty$, then

$$\lim_{t\to\infty} P_K(t) = P_K^* \qquad (K = 0, 1, 2, \ldots)$$

exists and we have

$$P_K^* = e^{-\theta\alpha} \frac{(\theta\alpha)^K}{K!} \tag{2}$$

In the case of the many server queue without storage, arrivals will have a certain probability of being rejected if there is no server available when the arrival enters the system. In the case of the special purpose communication satellite, most of the arrivals will keep trying to enter the system until a server is available. In many problems, however, a fixed portion will leave the system and not return. The fixed fraction not returning will be specified by $(1 - \gamma)$ where $0 \leq \gamma \leq 1$.

As stated previously, γ will usually be 1 in the application here.

In the theorem of Takacs, the state probabilities of the system can be described with a poisson distribution. The parameter for this distribution is the Poisson parameter for the arrival rate times the average service time. That is, $\lambda = \theta\alpha$. When the number of servers is constrained, the effect of the constraint may be described as an effective increase in the average number of arrivals. Since the service time remains constant, a difference equation can be written to describe the effects of server-customer interaction. An arbitrary time interval will be selected so that the rejected arrivals will have a very low probability of entering the system more than once during the same time interval. For the first interval of time at startup, the parameter $\lambda_1 = \theta_1\alpha$ may be described as

$$\lambda_0 = \lambda_0 + 0 \tag{3}$$

During the second time interval

$$\lambda_1 = \lambda_0 + \gamma\lambda_0\Phi(C,\lambda_0) \tag{4}$$

where γ is the portion of rejects returning; Φ is the complementary cumulative of the Poisson distribution; λ_0 is the Poisson parameter, and C is the number of servers in the system.

To simplify the notation, $\Phi(i)$ will be used to denote $\Phi(C,\lambda_i)$. In general,

$$\lambda_j = \lambda_0[1 + \gamma\Phi(j-1) + \gamma^2\Phi(j-1)\Phi(j-2) + \ldots + \gamma^j\Phi(j-1)\ldots\Phi(0)]$$

$$= \lambda_0\left\{1 + \sum_{i=0}^{j-1}\gamma^{j-1}\left[\prod_{K=1}^{j-1}\Phi(K)\right]\right\} \tag{5}$$

The proof of convergence for this sequence is given in Hein.[1]

<u>$C = n, \gamma = 1$</u>. For the general n server case

$$\lambda_0 = e^{-\lambda_*}\left[\lambda_* + \lambda_*^2 + \ldots + \frac{\lambda_*^n}{(n-1)!} + \frac{\lambda_*^{n+1}}{n!}\right] \tag{6}$$

Differentiating with respect to λ_* and equating to zero yields

$$\frac{\lambda_*^{n+1}}{n!} - \frac{\lambda_*^n}{n!} - \frac{\lambda_*^{n-1}}{(n-1)!} - \ldots - \lambda_* - 1 = 0 \tag{7}$$

According to the rule of signs developed by Descartes, this polynomial has exactly one positive root. To obtain the roots of polynomials there are many computational methods available. One which is rather easy to use is the Newton-Raphson method. According to this method the next estimate of the root of $f(x)$ is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

After the value of λ_{*MAX} is obtained λ_{0MAX} may be computed. If λ_{0MAX} is less than C, then λ_{0MAX} is the maximum λ_0 for that C; otherwise C is the limit.

APPLICATION OF MODEL

An extensive analysis of six years of NWS message traffic demonstrated that message arrivals in the five categories: hurricanes, tornados, winter storms, small craft, river and other could be described using Poisson distributions. Broadcasting times for all categories except hurricanes may be described using uniform densities. Hurricane message broadcasts may be described using a log normal density. Complete results of the statistical analysis are reported in Hein.[1]

Using time units of one minute, estimates of relative message frequencies and broadcast times for December were calculated from NWS data. December was selected because it yields the highest traffic density and the longest average broadcast times.

The Poisson parameter and the average broadcasting time may be used to determine the initial state parameter:

$$\lambda_0 = \theta_0 \alpha$$

Once the value of λ_0 has been determined, it is necessary to determine the feasible range for the number of channels. If λ_0 is less than λ_{OMAX} for one channel, the range of feasibility is $1 \leq C \leq \infty$. The equation

$$\lambda_* = \lambda_0 + \lambda_* \left[1 - e^{-\lambda_*} \left(1 + \lambda_* + \frac{\lambda_*^2}{2!} + \ldots + \frac{\lambda_*^C}{C!} \right) \right]$$

must be solved for λ_* over the range of values of interest for C, the number of simultaneous broadcast channels in the satellite. After determining λ_* for the range of C, the state probabilities must be determined from the equation

$$P_K^* = \frac{\lambda_*^K}{K!} e^{-\lambda_*} \qquad K = 0, 1, 2, \ldots$$

These state probabilities for the effective lambda (λ_*) may be used to calculate the probability of a delay, or to determine the probability of a delay exceeding a given time value. Now the determination of the effects of limiting C becomes a relatively simple analytical procedure and does not require the use of simulation. The procedure may be used to determine the required number of channels for any queuing system with Poisson arrivals and arbitrary service. A criterion may be established in advance and then the implications of the criterion may be analyzed.

In order to determine the delay or queuing time for messages, it is necessary to introduce a time factor. The probability of a delay is the probability that W_q (queuing time) is greater than zero:

$$P(W_q > 0) = \sum_{n=C+1}^{\infty} P_n$$

where C is the number of simultaneously accessible channels in the satellite system. The average service rate per channel is $1/\alpha$. For C channels this rate is C/α. But this is true only when there are C channels available for service.

The average service rate is then

$$\frac{C}{\alpha}\left(1 - \frac{\lambda_*}{C}\right)$$

The complementary waiting time distribution is then given by

$$P\{W_q > t\} = e^{-C/\alpha(1-\lambda_*/C)t}\ P\{W_q > 0\}$$

SIMULATION RESULTS

The basic time unit for the process reported here is one minute. Because the basic time unit is small, many problems normally associated with obtaining a steady state have been avoided. The parameter of prime importance here is the number of message requests in the system at any given time. Short simulation runs of one or two days were more than adequate to demonstrate that the process cycles "infinitely often" through a single state (namely zero). When such a cycling occurs, the sample sizes required to reduce intervals of uncertainty are significantly smaller than what would normally be required because steady state is achieved upon startup. Extensive digital simulations were conducted to verify the analytic model. The simulations and the statistical analyses of the results are thoroughly documented in Hein.[1] Copies of the computer software and sample results are also included.

SUMMARY AND CONCLUSIONS

The results obtained from the model provided excellent approximations for the 6- and 4-channel cases. Statistically there was no significant difference between the predicted results and those obtained from simulations. When the traffic intensity was increased through a reduction in service channels to 2, the customer-server interaction caused a degradation in the quality of the predicted results. The interaction causes the variance of the Poisson distribution to increase through a distortion of the relative state frequencies. This distortion causes a change in the state distribution; it deviates from the predicted Poisson distribution. Although the 2-channel case may provide useful results, it also demonstrates that the model is close to the limits of usefulness.

As a result of the model described here, a more realistic appraisal of the channel requirements for a satellite DWS was obtained. There are many areas of application beyond usage of the model for communications satellites. Applications to queuing systems where low utilization/high availability is the predominant characteristic are more numerous than can be mentioned here. Moreover, the necessity of minimizing waiting time will become even more important in the future as world economies become dominated by service industries. Market strategies will include trade-off analyses of the value of minimizing waiting versus the cost of adding more service channels.

The primary value of the model is that it reduces the dependence on expensive computer simulations when availability is a prime criterion of design. Once verified, the model allows rapid calculation of system statistics and provides reliable information at relatively low cost.

REFERENCES

1. Hein, Gerald F., "An Application of Queuing Theory to the Design of Channel Requirements for Special Purpose Communications Satellites," NASA TM X-71590, 1974.

2. Hein, Gerald F. and Stevenson, Steven M., "A Digital Simulation of Message Traffic for a Natural Disaster Warning Satellite," NASA TM X-68135, 1972.

3. Raymond, Herbert G., "A Queuing Theory Approach to Communications Satellites Network Design," NTIS A71-35106, 1971.

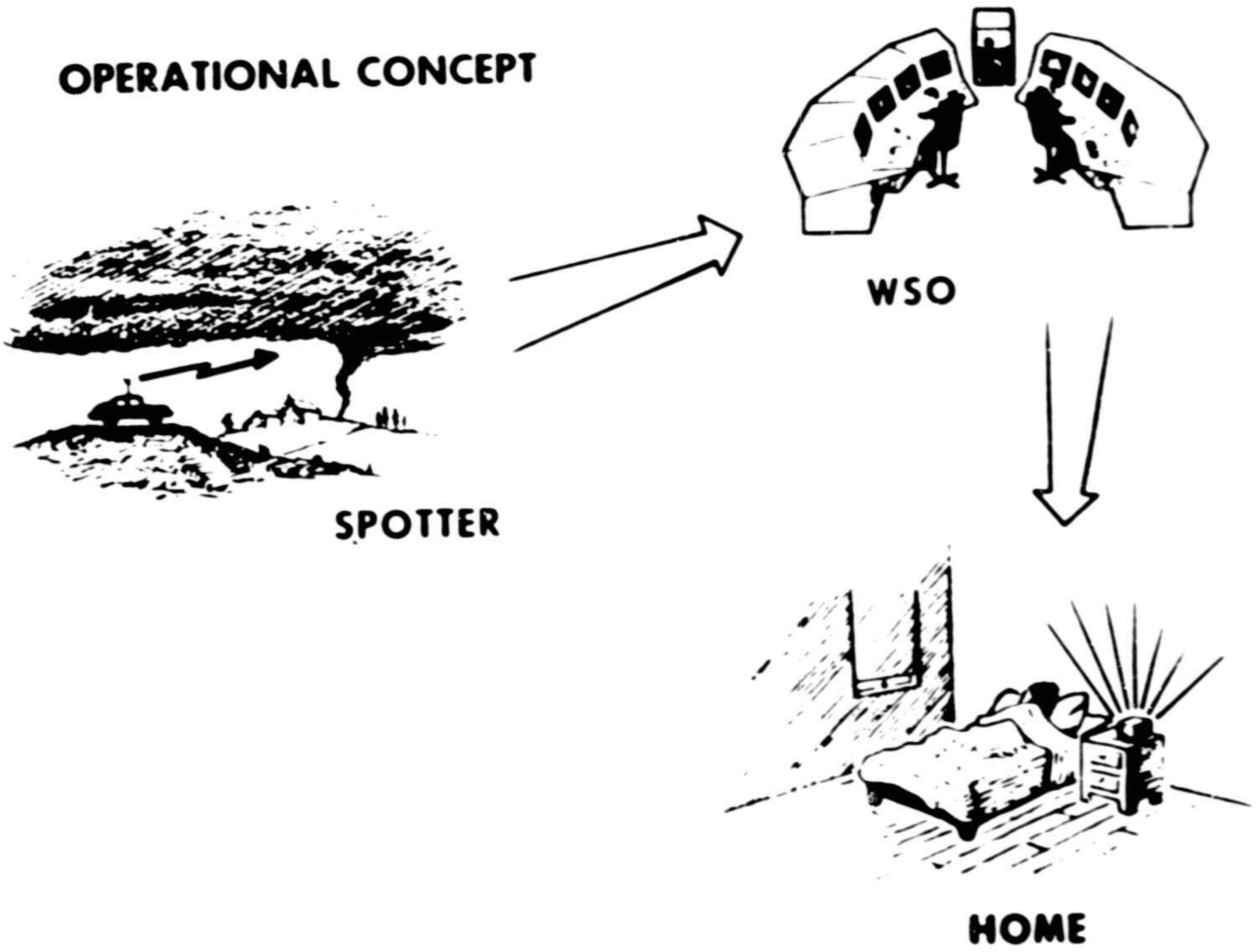
4. Takacs, Lajos, "Introduction to the Theory of Queues," Oxford Press, N.Y., 1962.

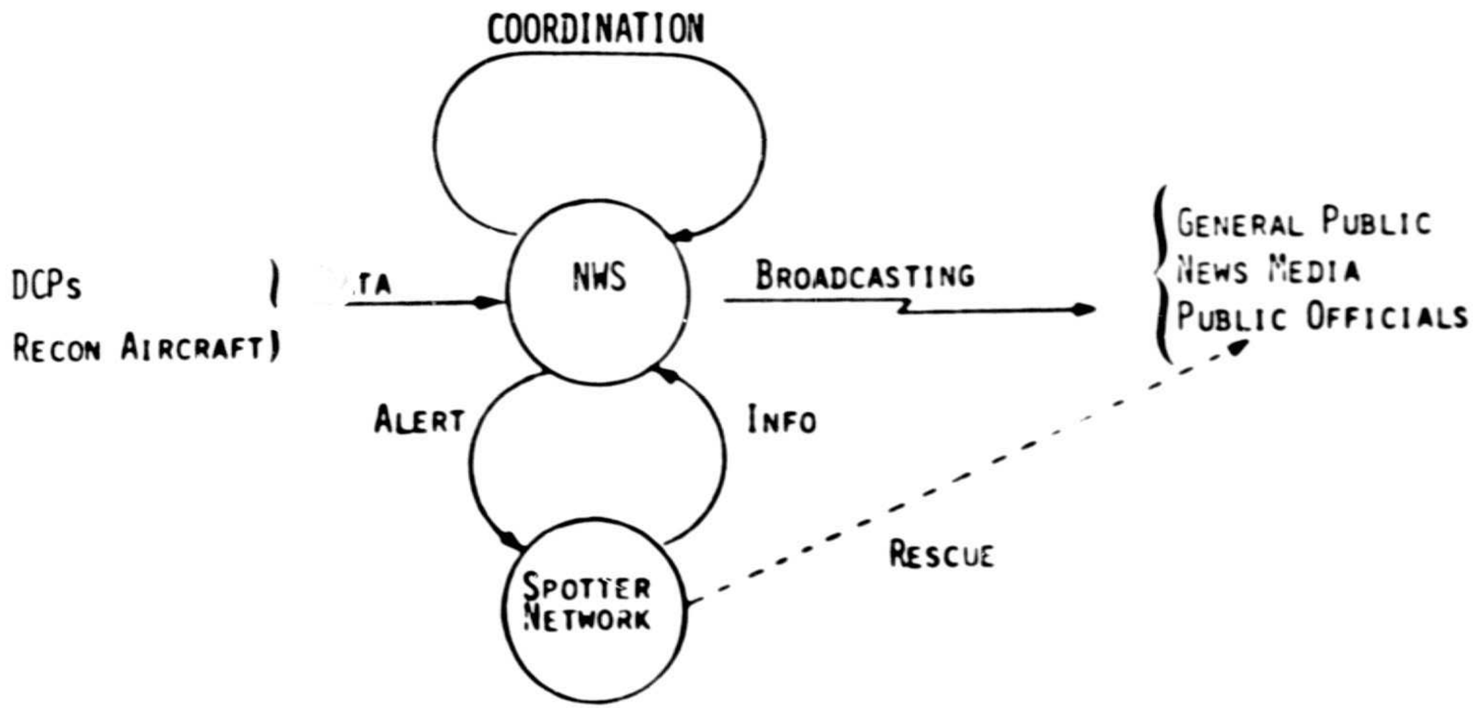
COST PERFORMANCE SATELLITE

DESIGN USING QUEUEING THEORY

Gerald F. Hein
Space Flight Programs Directorate
NASA Lewis Research Center
Cleveland, Ohio

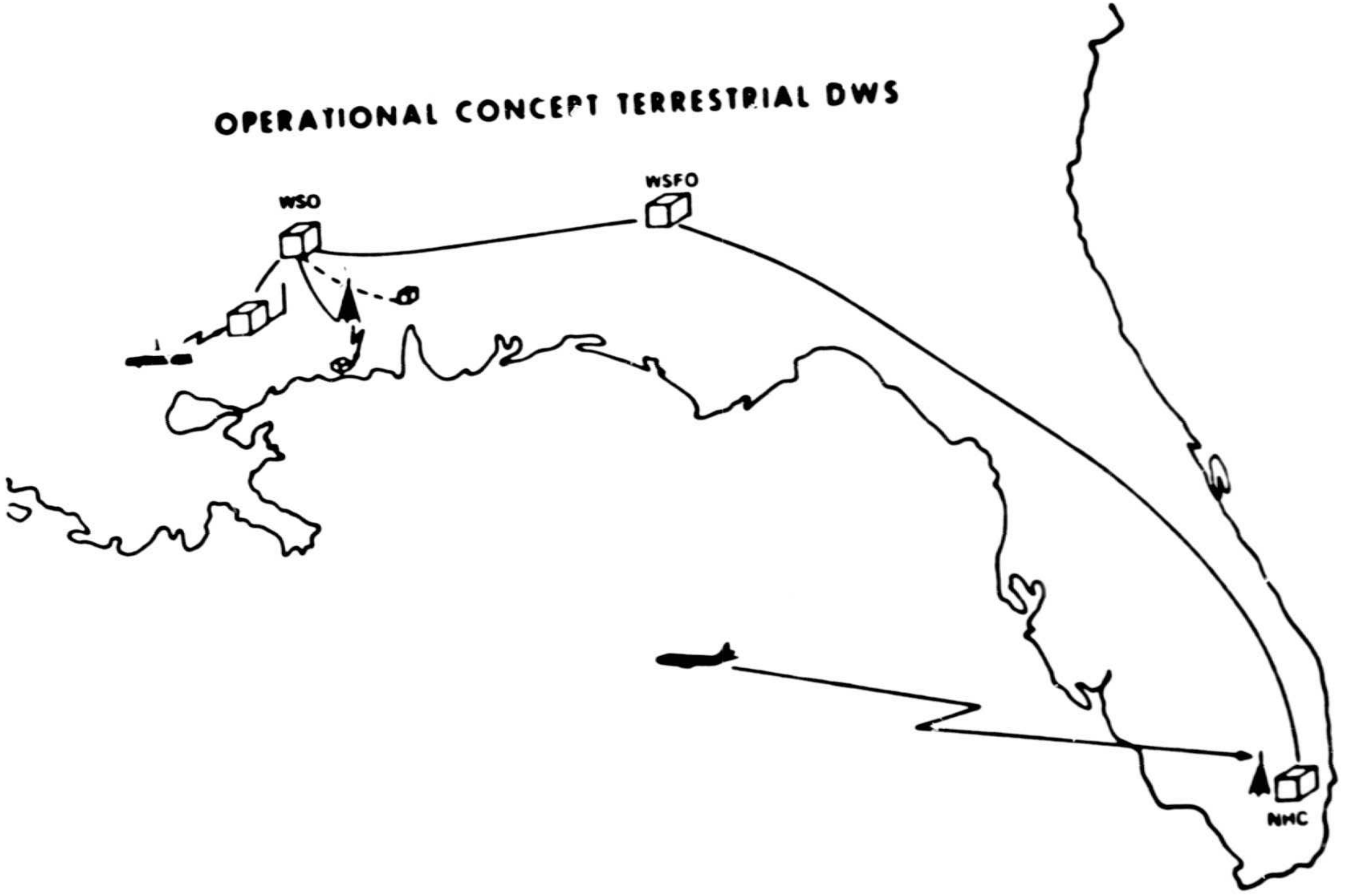
OPERATIONAL CONCEPT



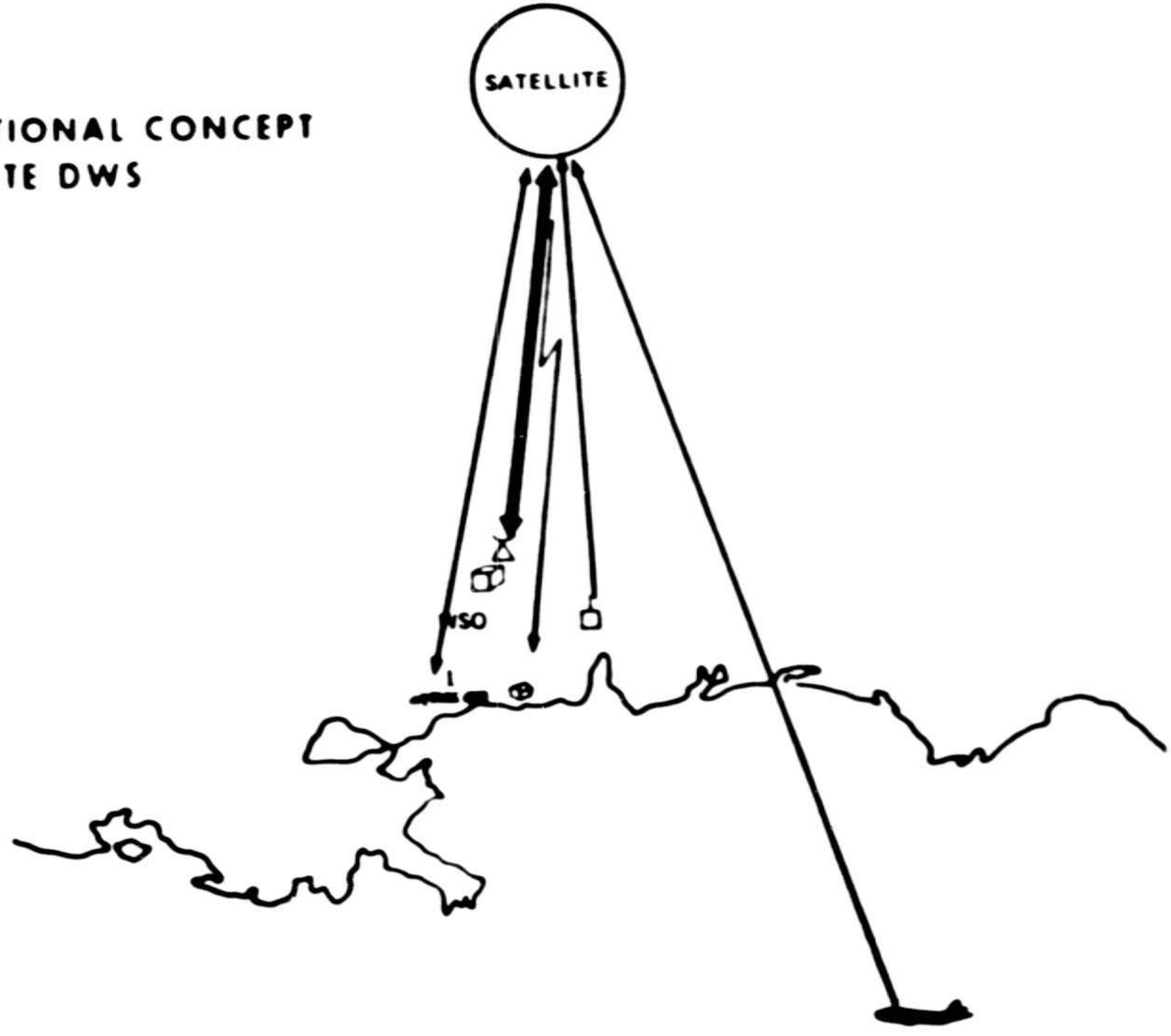


DWS REQUIRED CONNECTIVITY

OPERATIONAL CONCEPT TERRESTRIAL DWS



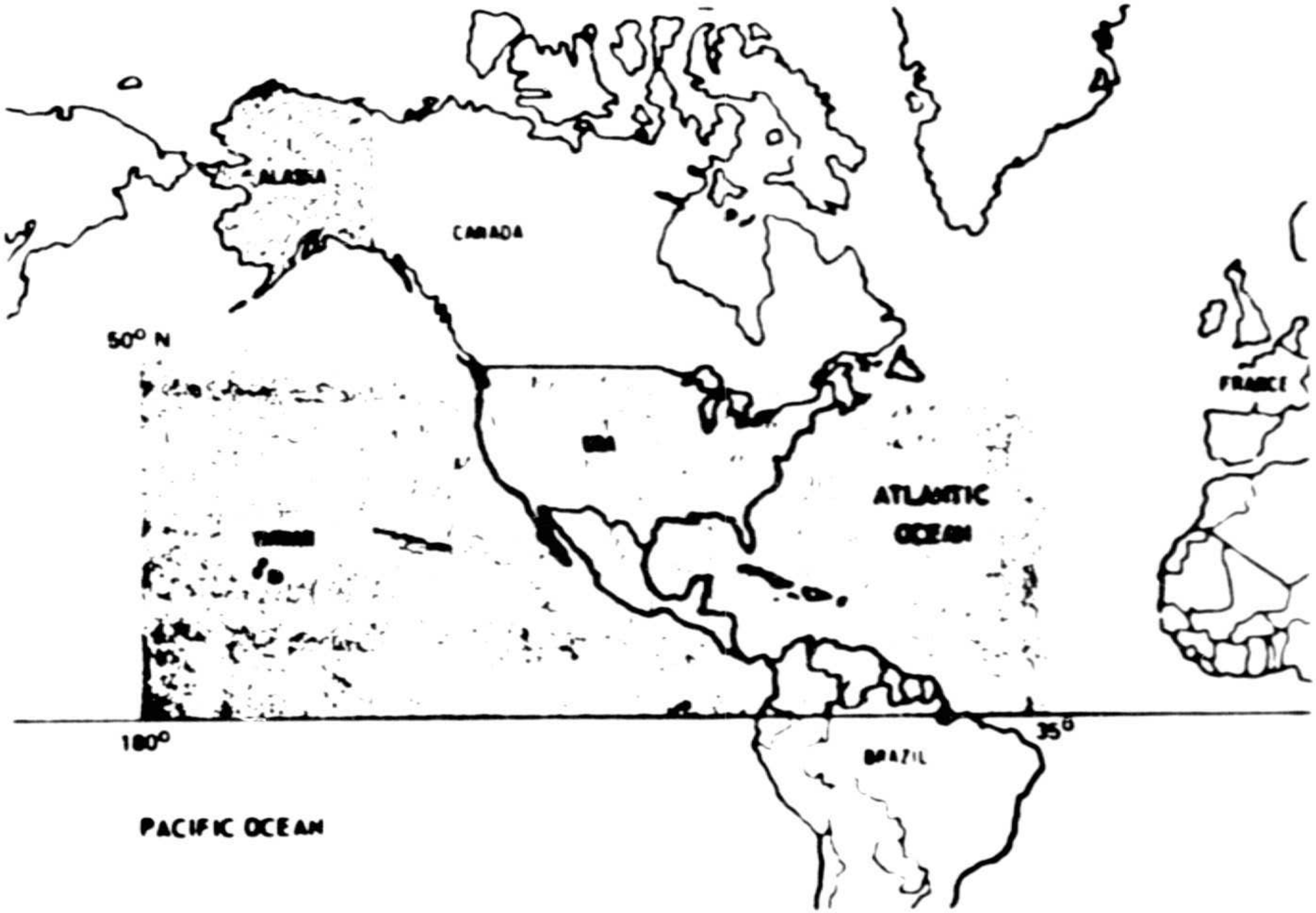
OPERATIONAL CONCEPT
SATELLITE DWS



REQUIRED RESPONSE TO DISASTER TYPES

DISASTER TYPE	SMALLEST AREA WARNED	MESSAGE ON-LINE UPPER BOUND
TORNADO OR SEVERE STORM	PART OF COUNTY	1-5 MIN
HURRICANE	PART OF COAST	1-15 MIN
RIVER FLOOD	PART OF STATE	15 MIN-1HR
SMALL CRAFT	PART OF COAST (LAKE)	15 MIN-1HR
WINTER STORM	PART OF STATE	15 MIN-1HR
OTHERS	PART OF COUNTY	1 MIN-1HR

GEOGRAPHICAL COVERAGE REQUIREMENT



STATEMENT OF PROBLEM

A NEED EXISTS FOR DETERMINING THE "EFFECTS" OF LIMITING THE NUMBER OF COMMUNICATIONS CHANNELS IN A "SPECIAL PURPOSE" COMMUNICATIONS SATELLITE

STATEMENT OF PROBLEM

A NEED EXISTS FOR DETERMINING THE "EFFECTS" OF LIMITING THE NUMBER OF COMMUNICATIONS CHANNELS IN A "SPECIAL PURPOSE" COMMUNICATIONS SATELLITE

ALTERNATIVES

1. USE SIMULATION TECHNIQUES

2. USE CURRENTLY AVAILABLE ANALYTICAL TECHNIQUES

3. EXTEND CURRENT ANALYTICAL TECHNIQUES

QUEUEING THEORY FOR TRAFFIC MODEL

- THEORY OF TAKÁCS REQUIRES INFINITE NUMBER OF CHANNELS.
- NEW MODEL DEVELOPED AT LERC ALLOWS CALCULATION OF QUEUEING TIMES FOR ANY NUMBER OF CHANNELS.
- NEW MODEL REQUIRES POISSON ARRIVALS AND AVERAGE MESSAGE SIZE.

QUEUEING THEORY FOR TRAFFIC MODEL

- THEORY OF TAKÁCS REQUIRES INFINITE NUMBER OF CHANNELS.
- NEW MODEL DEVELOPED AT LERC ALLOWS CALCULATION OF QUEUEING TIMES FOR ANY NUMBER OF CHANNELS.
- NEW MODEL REQUIRES POISSON ARRIVALS AND AVERAGE MESSAGE SIZE.

Let $\alpha = \int_0^\infty X\, dH(X)$ and $P[\xi(t) = K] = P_k(t)$

where α is the average service time, $P_k(t)$ is the probability of being in state K at time t, θ is the Poisson arrival parameter, and $\xi(t)$ is the queue size or number in the system.

<u>Theorem 1</u>. If $\xi(0) = 0$, then

$$P_k(t) = \exp\left\{-\theta \int_0^t [1 - M(X)]dX\right\} \frac{\left\{\theta \int_0^t [1 - H(X)]dX\right\}^K}{K!}$$

for $K = 0, 1, 2, \ldots$

and if $\alpha < \infty$, then

$$\lim_{t\to\infty} P_k(t) = P_K^* \qquad (K = 0, 1, 2, \ldots)$$

exists and we have

$$P_K^* = e^{-\theta\alpha} \frac{(\theta\alpha)^K}{K!}$$

$$\lambda_i = \theta_i \alpha$$

$$\lambda_0 = \lambda_0 + 0$$

$$\lambda_1 = \lambda_0 + \gamma\lambda_0 \Phi(C, \lambda_0)$$

In general,

$$\lambda_j = \lambda_0[1 + \gamma\Phi(j - 1) + \gamma^2\Phi(j - 1)\Phi(j - 2) + \ldots + \gamma^j\Phi(j - 1) \ldots \Phi(0)]$$

$$= \lambda_0\left\{1 + \sum_{i=0}^{j=1} \gamma^{j-i}\left[\prod_{K=i}^{j-1} \Phi(K)\right]\right\}$$

$$\lambda_* = \lambda_0 + \lambda_*\left[1 - e^{-\lambda_*}\left(1 + \lambda_* + \frac{\lambda_*^2}{2!} + \cdot \cdot \cdot + \frac{\lambda_*^C}{C!}\right)\right]$$

$$P_K^* = \frac{\lambda_*^K}{K!} e^{-\lambda_*} \qquad K = 0, 1, 2, \cdot \cdot \cdot$$

$$P(W_q > 0) = \sum_{n=C+1}^{n} P_n$$

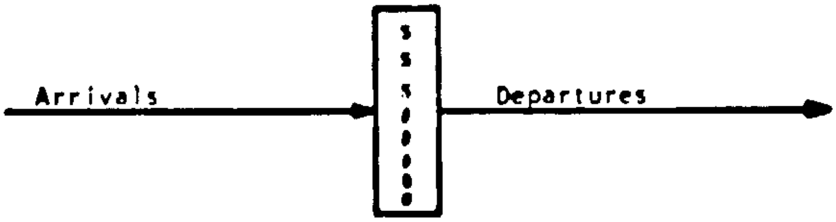
$$P(W_q > 0) = 1 - \sum_{n=0}^{C} P_n$$

$$P(W_q > t) = \sum_{n=0}^{n} P_n P(W_q > t | E_n)$$

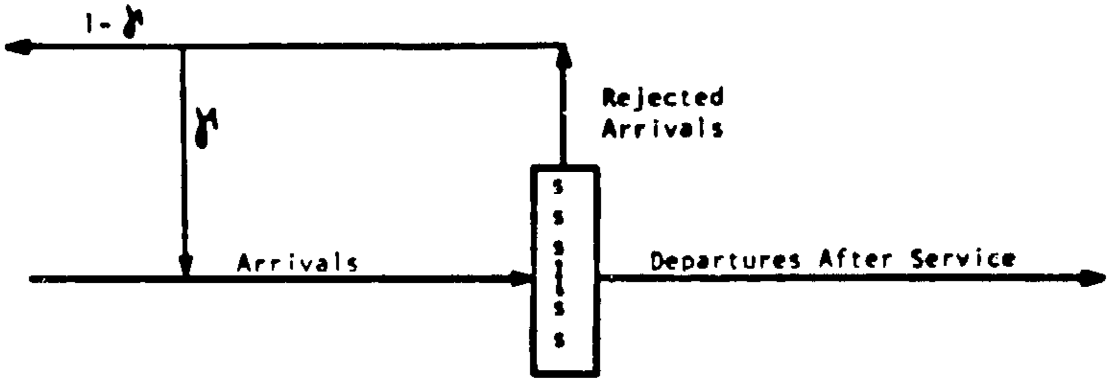
$$P(W_q > t) = e^{-C/\alpha(1-\lambda_*/C)t} P(W_q > 0)$$

and

$$P(W_q \leq t) = 1 - P(W_q > t)$$



Infinite Server Model.



Reentry Queueing Model.

DATA BASE

WARNING CATEGORIES

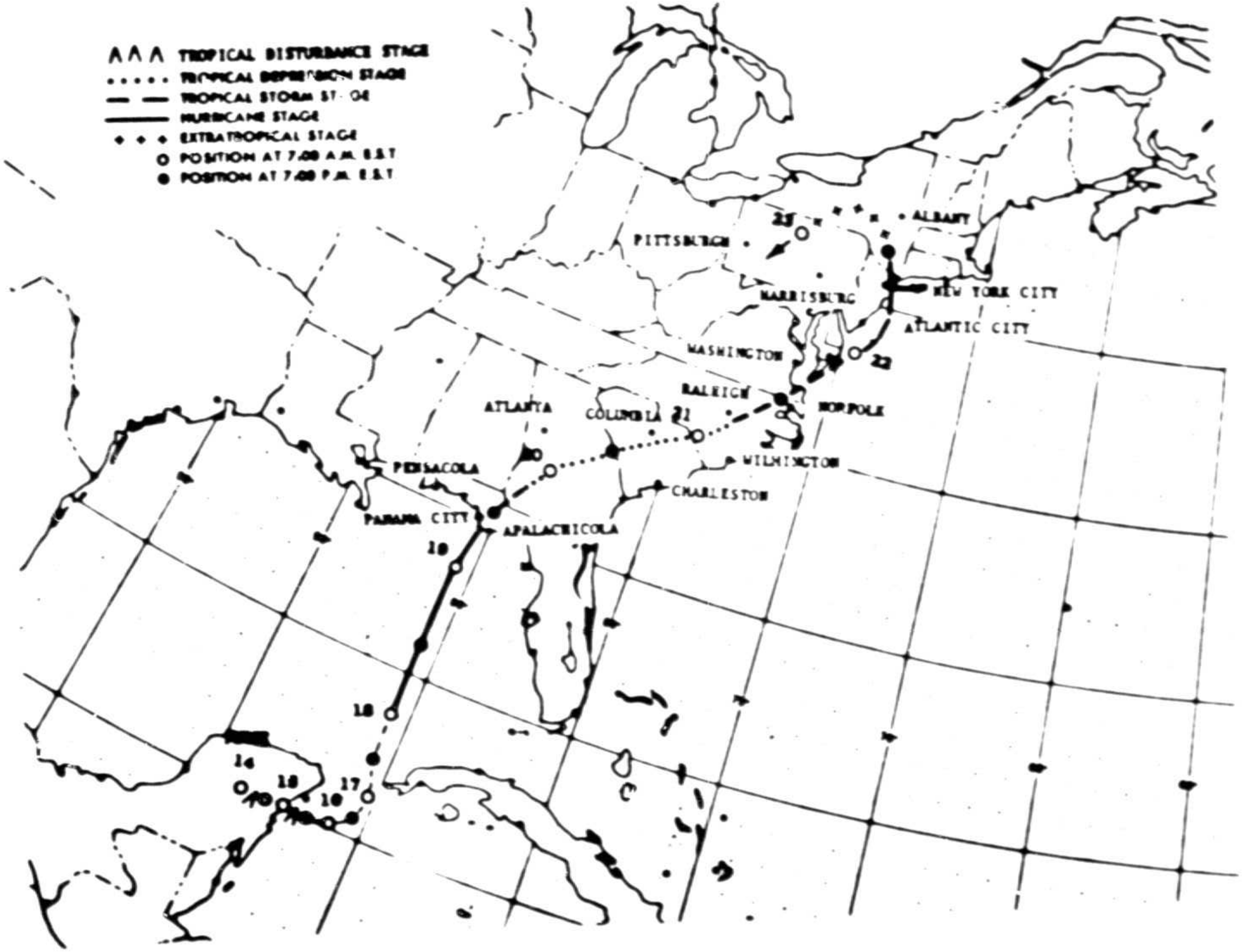
- TORNADO AND SEVERE STORMS
- HURRICANES
- WINTER STORMS
- SMALL CRAFT
- RIVER
- OTHER

DATA

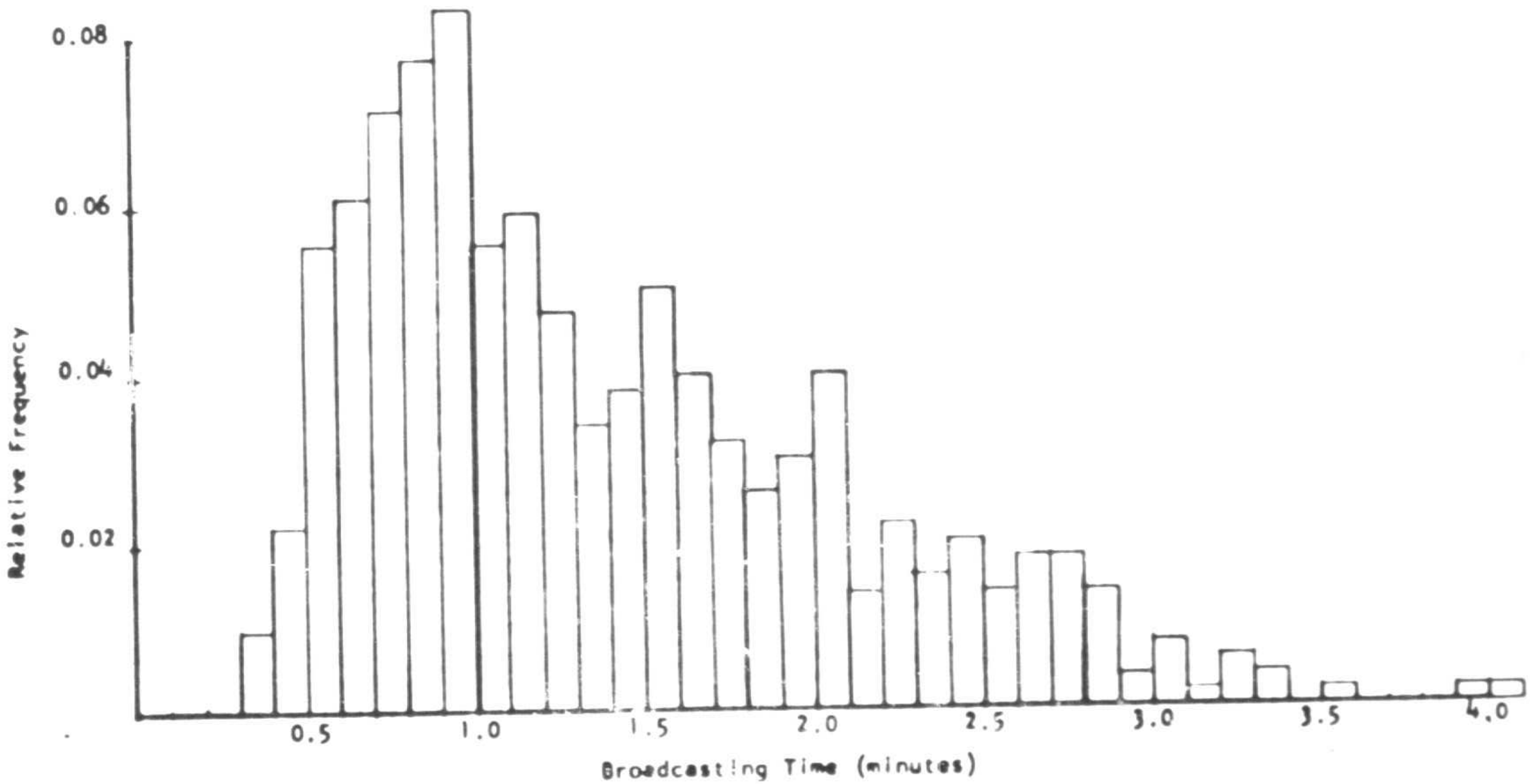
- 6 YEARS OF MONTHLY WARNING DATA (1966-71).
- ALL WARNINGS SENT DURING AGNES (1972).
- SEVERAL THOUSAND MESSAGES FROM TELETYPES (DECEMBER 1973).
- TORNADOS OF APRIL 3, 1974.

Message Category	Arrival Pattern	Broadcasting Time Density	Average Value
Hurricanes	Poisson	Log Normal	1.15 Mins.
Tornados	Poisson	Uniform	0.85 Mins.
Winter Storms	Poisson	Uniform	1.60 Mins.
Small Craft	Poisson	Uniform	1.00 Mins.
River & Other	Poisson	Uniform	1.10 Mins.

Statistical Patterns for Message Categories and Average Broadcasting Times.



Path of hurricane Agnes.



Frequency Polygon for
Hurricane Message Broadcast
Times.

RESULTS OF DATA ANALYSIS

- ARRIVALS ARE POISSON IN ALL CASES.
- AVERAGE MESSAGE BROADCASTING TIME - 1.18 MINUTES.

1985 TRAFFIC ESTIMATES

- PRESENT DATA EXHIBITS LINEAR GROWTH TREND.
- MAXIMUM RATE IN DATA BASE IS 10,000 PER MONTH.
- LINEAR EXTRAPOLATION OF UPPER BOUND OF 95% CONFIDENCE INTERVAL - 21,000 PER MONTH.
- ADDITIONAL SAFETY MARGIN FROM ADDITION OF AGNES RAISES RATE TO 23,000 PER MONTH.

BASELINE CASES

- 4, 6, 10 SIMULTANEOUS CHANNELS.
- RATES OF 23,000, 18,000 and 15,000 PER MONTH.

CASE 1: 23,000 PER MONTH

CHANNELS	4	6	10
UTILIZATION %	15.6	10.4	6.3
FREQUENCY OF DELAYS			
>30 SEC	1/WEEK	4.4 YRS	NEVER
> 1 MIN	1/MONTH	41 YRS	NEVER

CASE 2: 18,000 PER MONTH

CHANNELS	4	6	10
UTILIZATION %	12.5	8.3	5.0
FREQUENCY OF DELAYS			
>30 SEC	3 WEEKS	20 YRS	NEVER
> 1 MIN	3 MONTHS	NEVER	NEVER

CASE 3: 15,000 PER MONTH

CHANNELS	4	6	10
UTILIZATION %	10	6.7	4
FREQUENCY OF DELAYS			
>30 SEC	2 MONTHS	NEVER	NEVER
> 1 MIN	8 MONTHS	NEVER	NEVER

Delays \ λ_0	0.4	0.5	0.6254
> 5 seconds	13 years	3 years	0.7 years
>10 seconds	19 years	4 years	1.0 years
>15 seconds	27 years	6 years	1.4 years
>30 seconds	95 years	21 years	4.3 years

Mean Time Between Delays Exceeding Certain Durations for λ_0 Equal to 0.4, 0.5, and 0.6254 for 6 Channels.

SUMMARY OF RESULTS

ANALYTICAL TOOL IS RELIABLE AND SIMPLE TO USE.

COST OF OBTAINING INFORMATION SIGNIFICANTLY REDUCED.

COST OF SATELLITE SYSTEM SIGNIFICANTLY REDUCED.