AN INFORMAL PAPER ON LARGE-SCALE DYNAMIC SYSTEMS

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First, let us examine the title of this symposium. What is meant by "large scale" and "dynamic"? Figure 1 shows a system is large when it requires more than one decision maker to control it. Almost all interesting and difficult problems of a large-scale system are introduced by the fact that there is more than one controller involved; the reasons for having more than one decision maker or controller involved are several: the institutional bodies (e.g., local or regional government) often wish to retain their decision-making power against, say, the federal government; we have a natural aversion to dictatorship, which is another way of being centrally controlled; bureaucratic inertia often prevents us from controlling problems effectively in a centralized manner. These institutional constraints are quite familiar to anyone involved in politics, but they also occur naturally in any large organization. Communication difficulties arise; for example, because of the time required to transmit data from one place to another, by the time the central source receives the data it may be too late for effective control. An example would be a vehicle on Mars remotely controlled from Earth. A few seconds are required to transmit data from one place to another. The cost of transmission may make it no longer worthwhile to transmit all data to a centralized source. Finally, the computation time required to process the data at a centralized source may be too great. We have in mind particularly on-line control where the computation must be done quickly.

Figure 2 illustrates a decentralized control versus decomposition in computation. When we talk about decentralized control of large-scale dynamic systems we often have in mind on-line, real-time control (such as control of a power distribution network). There, you are talking about the time required to process the data is on the order of seconds and the response time of the systems is, at best, in minutes. On the other hand, many large-scale planning problems or decision-making problems can be done effectively off-line (such as planning of economic allocations or preliminary planning of a water resources system). For such problems, the computation time available is in hours or days and the response time of such a system may be in terms of days or years; therefore the control problems and planning problem are vastly different. Control problems are probably repetitive and day to day while planning problems are most often only one shot affairs. The system is planned and built and it lasts 20 to 50 years or maybe 20 or 50 months.

In planning, there are decomposition techniques for the computation, the best-known technique is the so-called "decomposition technique in large-scale mathematical programming." However, this is not really the thing of interest to us here. I am concerned only with decentralized control. First, there are some questions about the real usefulness of these decomposition techniques in mathematical programming. Consider a very large-scale planning problem that requires 10 hours of computer time, using the standard LP programming. With the decomposition technique, let's say you may be able to solve the problem in 1 hour. The difference between 10 hours and 1 hour of computation is really not that significant. Experience with these decomposition techniques often shows that, in terms of through-put, they offer little improvement because a special program must be written for each decomposition. A different planning problem requires its own special program. The process of collecting the data and processing it may take more than 10 hours. As a result, my experience with these decomposition techniques has shown that they are not used very often simply

because they are not economical. On the other hand, on-line, real time control is entirely different. It is not a question of whether you should use centralized or decentralized control; the institutional and communication constraints mentioned previously simply force you to use decentralized control.

Figure 3 defines what is meant by "dynamics" in a problem. Of course, most of you know intuitively what is meant by dynamics, but here I want to consider decomposition, particularly in terms of the problem it generates with decentralized control. A dynamic decision problem requires one to choose different decisions at different instances in time, based on different information available at the time. Often this is how a problem statement would appear in the language of game theory and decision theory. This is called extensive form formulation. It is the form we normally see when we first try to formulate the problem. On the other hand, for theoretical purposes, a control problem may be stated another way, namely, choose a strategy among all admissible strategies. What is a strategy? A strategy is a formula that tells you what decision you should make under all possible circumstances at all possible times. Mathematically, this means a map from the product space of the information available and time to the space of choices available. Once you have chosen a strategy, you have really indicated how you will behave under all possible circumstances. In principle, once a strategy is fixed you can always evaluate the cost of performance of a control system. And if you define the class of all possible strategies you are willing to consider, then you have essentially defined all possible performances with respect to each individual strategy. This then becomes an extremely simple-minded optimization problem, namely, pick the strategy that gives you the best performance — the normal form of formulation because, theoretically, it is a very clean statement of the problem and because the problems of dynamics and information have been suppressed in terms of a properly defined class of admissible strategies. For example, in the familiar language of control theory, suppose you want to use open-loop control only, that is, without any feedback information. Then, in normal form, we simply say the class of admissible strategy is the class of maps that are constant, that is, independent of information received in any given time. When we must choose a strategy among the class of constant strategies, it is the equivalent statement to open-loop control in extensive formulation.

Normal form formulation has many theoretical advantages, but it does not tell you how to solve the problem. In many instances, when there is more than one decision maker involved, particularly with game theory, you want to focus your attention on certain aspects of the problem peculiar to the fact that you are playing a game, without having to worry about a detailed solution, the dynamics, and the information. Often certain aspects of a control problem can be discussed in terms of normal form without detailed information (aspects of this problem are discussed later). Such concepts are not only applicable to purely static algebraic problems, but they apply equally in dynamic systems provided you realize we are working in the normal form. On the other hand, certain aspects of the dynamic information must be treated in the extensive form manner to show the problem areas.

Having thus defined the scope of the problem in terms of large-scale dynamic systems, let me first hasten to say that we know very little about decentralized control dynamics systems. Figure 4 is a very rough attempt to classify the different types of studies on decentralized control of dynamic systems in terms of whether the technique is deterministic or stochastic (they can also be classified according to what aim they have). For control problems, four types of questions can be asked. First a structural question — what is the right kind of model for the system? Once this is understood, how do you optimize the control? As often happens, you may not be able to solve this;

then you ask for something less, namely, can I do something to the system to make it stable? Finally, you may simply ask the basic question: "Is it feasible to do something about the system?"

In terms of decentralized control in the deterministic phase of the diagram, the first block covers questions such as foundation and philosophy of hierarchial control, organization, how to distribute the payoff among different decision makers, etc. Under optimization, we have mentioned large-scale mathematical programming already, economic problems such as optimal resource allocation by one supervisor among different departments, the vector payoff question when decision makers have different payoffs, and the Pareto optimality (sometime fashionably called Paretian analysis). It is just a different way of saying "How do you reconcile or trade off different objectives such as more guns or more butter?" Under stability, a whole class of problems come under the name of adjustment process. These are really interconnected dynamic systems, with one controller for each dynamic system. They all make adjustments to improve their performance and each adjustment would affect other systems. When a controller adjusts to improve his position, will this lead to a stable process that is good for everyone or will there be cut-throat competition? Finally, under feasibility, you have questions such as decentralized controllability – is it possible to control systems from one state to another in a decentralized manner?

On the stochastic side, the basic emphasis is essentially for questions dealing with the structure of information. We want to understand basically what is meant by information in a many-person decision problem. What do we mean by information structural properties and so forth? In stochastic optimization, specifically, terms such as team theory (another way of saying decentralized decision theory) and the question of value of information, and such appear.

So far as I know, no work has been done on the stochastic stability of decentralized systems or the feasibility question in stochastic models. Most of the discussion that follows concerns the structure of optimization for the stochastic phenomenon. My colleague and former co-worker, Dr. K. C. Chu, will discuss in another paper the role of team theory in decentralized control. I would like to discuss briefly the adjustment processes and decentralized stability or feasibility. This work, which appeared recently in the Russian literature, is, I think, quite interesting. Particularly, the adjustment process relates also to the work Professor Šiljak discusses later.

The main part of this paper concerns the problem of information in general, and many-person decision problems, which, of course, includes that of decentralized control. There are really only two kinds of decision variables. First are the decisions made by human beings, $u_1,...,u_n$ during different times at different places by different decision makers. Another set of variables is nature's decisions, $\xi_1,...,\xi_m$, often taken to be uncertain. These are noise in the measurement systems, disturbances in the control systems, or the flip of the coin, anything considered uncertain but given the probability of distribution. Every event under the sun is a function of human decisions as well as nature; these are the only fundamental variables in the problem. In dynamic systems, state variables are really secondary because the state of the system at a given time is a result of all past decisions made by the controller as well as the noise or disturbance that has occurred. Therefore, human and nature's decision variables are considered fundamental. Since every event is a function of these variables, the information available to a decision maker must also be a function of these two sets of variables. The particular function that relates u and ξ is called the *information structure* of the problem and these two sets of variables and definitions are shown in figure 5(a). Figure 5(b) defines what we mean by strategy. As mentioned before briefly, strategy is a mapping from information to decision, whether by adaptive control, stochastic control, or whatever. This is really the

most general definition of a control law — a prediction of behavior on the basis of information available at a particular time. Since information is defined as a function of u and ξ, this definition of strategy further relates information to u or defines the implicit equation $u_i = \gamma_i(\eta_i[u,\xi])$. If we specify the exact information structure (specify who knows what), furthermore specify what the strategies are (what each decision maker will do under all possible circumstances, i.e., all possible information patterns), and specify nature's decision for the given probability density ξ then the equation labeled (*) in figure 5(b) defines a set of equations that, when they have a solution, gives the actual decisions the human decision makers will make. The model represented by (*) can be extended to give even more general situations in game theory.

Some possible questions and problems associated with this set of equations are shown in figure 6. First, since the set of equations labeled (*) is implicit, the problem arises whether a solution exists. In fact, does (*) have a unique solution for a given information structure and for each possible and admissible strategy set Γ? If there were not unique solutions, then we are in somewhat of a funny situation. If you tell what everybody knows and how everybody will behave based on what he knows, the outcome is undetermined because there is more than one possible solution. Given these two specifications γ and η (who knows what and how does every decision maker behave under all circumstances), there should be only one unique outcome. If the equation set has a unique solution, a problem will be well posed. Note that this type of question does not arise in the usual decision theory framework where dynamics does not occur (such a problem is called static or simple). In such cases, the information variable is always a function of what nature decides or uncertainty ξ, and does not depend on what other decision makers have done. For such a case, η is only a function of the nature's decision and uniqueness is trivial. If nature has acted according to some probability distribution, one's behavior is completely determined and unique. For a dynamic problem, what is known may depend on what other people have done in the past, so the information is not only a function of noise and disturbance but also of other people's controlled actions — therein the unique problem arises. An obvious question arises: what is a good information structure? This then involves the design of information systems. Whether one measurement system or one set of observations is better than another or whether you should observe one sample, three samples, and four samples, since each measurement is presumably with cost, this question of design of information system arises and the value of information or "who should know what."

A third question is the all familiar one, namely, what is a good control law? This is the usual optimization problem except here we are interested in the solution in a decentralized setting. Finally, since both the information structure and the control law are to be designed, which information structure will make the optimization problem easy to solve. (See Dr. Chu's paper for a discussion on this.)

Certain explicit results in team theory are related to this question. Figure 6 shows a particular fundamental difficulty in dynamic problems involving information structure just described. From earlier definitions, nature's decision is represented by the vector variable ξ, the basic variable in the problem, and a probability density function or distribution is introduced. But z_i and u_i are *not* the random variables for a given information structure unless the strategy is fixed. Consider a two-stage decision problem. The initial state is ξ_1; control u_1 is applied additively to yield an intermediate state x. Apply another control u_2 additively to intermediate state x to the terminal state y. A decision maker at time 1 knows the information z_1, which is simply a perfect measurement of an initial state. The second decision maker knows z_2 at some time later, which is simply a noisy measurement of the intermediate state x or $\xi_1 + u_1 + \xi_2$. Nature's decisions are random choices of

the initial state and random noise ξ_2. Note that z_2 is not a random variable at this point until the strategy of u_1 is defined. The strategy of u_1 is a function of z_1, in this case ξ_1. Once γ_1 is defined, u_1 becomes a random variable because it is a function of another well-defined basic random variable in the problem. If u_1 becomes a random variable, then z_2 also becomes a random variable because it is now a function of ξ_1,ξ_2 only. Furthermore, u_2 becomes a random variable with solution-dependent distributions. The distribution of u_2 depends on the particular γ_1 and γ_2 — the solutions we wish to obtain. Until the solution is known, γ_1 and γ_2 cannot be defined as random variables. But, until these γ terms can be defined as random variables, we cannot begin the solution process; this is the difficulty in such general information problems (fig. 8).

Assume that ξ_1,ξ_2 are random variables with very nice properties, for example, Gaussian. But this does not guarantee that z_2 is a nice random variable unless some additional restrictive assumptions are made about the control law γ_1. Since γ_1 can be arbitrary, then z_2 will be a rather arbitrary function of ξ_1,ξ_2 (fig. 8), which certainly would not generally be Gaussian so the nice property was lost. Also, for a payoff function convex in u_1,u_2 in the problem, nothing can be said about the expected value of this convex function in u_1,u_2. It is not known in fact, whether it is convex in γ_1, a fact important in proving any kind of optimality property. This can be seen fairly easily. The expected value of a convex function in u_1,u_2 is basically a function of γ_1,γ_2. In other words, once γ_1,γ_2 are fixed with the information structure (as defined earlier), the payoff is a function only of these control laws. But the control law γ_2 is a function of z_2 which, in turn, is a function of γ_1. Therefore, the dependence of the payoff function J on γ_1 is rather intricate and depends on γ_1 explicitly (where u_1 is replaced by γ_1), but it also depends on γ_1 through γ_2. Since γ_2 again is generally arbitrary, for functions originally convex in u_1, there is no guarantee that it will be convex in γ_2 unless restrictions are placed on γ_2 (such as linear or otherwise monotonic properties). Again, since γ_1,γ_2 are, in fact, the answer we are looking for and they are assumed arbitrary in the beginning, there is no prior reason to assume they should be linear and so forth. As a result, the simplest problem of this type (as shown in fig. 7) cannot be solved. This point was first brought out in Witsenhausen's paper (ref. 7), which could be regarded as the starting point of all this research. So, in general, we must impose additional restrictions on the η information structure to avoid these difficulties and this is what I meant by the question raised earlier, that is, what kind of information structure would lead to easy solutions? (See references.)

The value of information should be discussed briefly. The value of information is simply the difference between the best performance with and without information. The difference, presumably (if it is possible to measure it in dollars), is the most one would be willing to pay for the information; this should be the basis for comparing different information system designs. This definition has several problem areas: in problems with multiple payoff, the definition of "best" requires more careful specification. Those familiar with game theory realize there are many different kinds of optimal solutions. One has to determine what is meant by best. Also, from the decision-theoretic viewpoint, one compares the expected value of the information, that is, the $E[VI]$ (fig. 9). Is this the only basis for comparison? and what is meant by more informative? This problem may appear to be fairly simple, but actually the more you look into it the more complicated it becomes. There is no uniform agreement on this definition of more informative. The expected value of information is often used as a basis for comparing whether one information system is more informative than another. Finally, perhaps a more disturbing question in the case of multiple decision making is that more information does not always lead to better payoffs, which is kind of counter intuitive when we are so used to thinking in terms of one player. The obvious answer is that if you get more information you can always ignore it and do what you did before without the information so you

could not possibly be worse off or get less payoff. The problem is that when more than one person is involved, sometimes you cannot ignore the information. This new information cannot be ignored because other people may not believe that you will ignore it and you cannot do what you did before because the other player, knowing that you have certain information, will alter his strategy. Once his strategy is altered, you can no longer dare to use your old strategy. Your strategy must be altered in the optimal manner, but his optimal payoff for the new information structure may, in fact, be worse that before. I have presented several examples (see reference list) to show that information may not always lead to a better payoff, and the question of the value of information is by no means settled.

I mentioned earlier some work in the Russian literature on decentralized control. In the last few years, they have been very interested in this line of research. Many controllers or decision makers or planners are involved in the problem, each controller having limited information. Imagine an indicator function that depends on what everybody else is doing. This function would indicate that if all values remain the same the control is moved either up or down or left or right, then you are moving away from the goal. A natural reaction at this point would be to change the parameters u_i according to the indicator function, that is, make the rate of change of u_i proportional to the indicator function or, more qualitatively, make the sign of the rate of change in u_i proportional to that of the indicator function. The value of the indicator function δ_i must be nonzero when u_i is not at the equilibrium goal. This situation is illustrated in figure 10 where all indicator functions equal zero at equilibrium. that is, where each goal is satisfied. Would such a single-minded adjustment process evolve in such a way that each goal would be satisfied. Perhaps the simplest example would be when the indicator function is a linear function of u: δ_i of u is simply in the product of u with q_i. These combined terms yield vector equation $u = Q_u$. For the time rate of change of $\|u_i\|^2$ we have the quadratic form $\langle u'(Q + Q')u \rangle$ Clearly, if this matrix $Q + Q'$ is negative definite, then $\|u\|^2$ in the limit goes to zero because of the well-known Liapunov theorem. The system would then be stable, for then $u = 0$ and the indicator function Qu is zero also. Equivalently, $Q + Q'$ is negative definite if certain sufficient conditions are satisfied. The simplest condition is that the diagonal element of Q be larger than the sum of the absolute value of its row or its column (conditions 2 and 3), usually known as the Gersgorin circle theorem. When there is diagonal dominance, $Q < 0$.

The three conditions can be put into a different form. Instead of differential changes in u and δ consider finite changes Δu_i and $\Delta \delta_i$, for which conditions 1, 2, and 3 imply conditions I, II, and III in figure 11. Conditions I, II, and III are, in fact, conditions of stability for those adjustments when the indicator functions are not differentiable and nonlinear. Often conditions I, II, and III are much more applicable than condition (i), (ii), and (iii). The reference list offers a whole set of examples drawing from electric circuits, resource allocation, game theory, etc., which are formulated to show the generality of conditions I, II, and III.

Figure 12 poses the question of feasibility of adjustment control. Let us define x_i as the amount of the ith commodity in an economy. The production of each commodity requires input from other commodities, for example, producing machine parts requires input of steel, fuel, labor, etc. We shall define a matrix A with elements α_{ij}, the amounts of ith commodity needed to produce one unit of the jth commodity. Then the net amount of ith commodity produced is defined as y_i, which is simply the gross amount x_i minus the amount needed to produce other commodities $\sum_j \alpha_{ij} x_j$. In matrix form, $y = (I-A)x$ is the well-known Leontiff input/output economy. Now the question you may raise at this point is whether the economy is productive, that is, is $y < 0$ for some $x > 0$? This

question is of interest in terms of decentralized control because if this is possible (i.e., if every positive y_i has a corresponding x_i), then decentralized control is not impossible. Whenever a positive y_i is required that can be accomplished by a positive x_i. (So if production units only have to worry locally about its product and its quotas that you don't need some sort of coordinating structure.) This condition for productive economy has specific answers for certain conditions on the matrix A, which affects the possibility of decentralized control. This is a simple example of the feasibility of adjustment, which can be generalized in many ways. Furthermore, x and y need not be interpreted as productions. For example, the component vector y_1 of y is quality of education and y_2 is the cost of education for vector x; x_1 is payment to the teacher and x_2 is the load the teachers take on. Does there exist a combination teacher pay and teacher load that will simultaneously increase the quality of education as well as lower the cost or at least maintain the cost? This question depends on the matrix A and can be generalized if y relates nonlinearly to x (see reference list).

Although this paper has been a rather rambling and somewhat disorganized survey of the subject of decentralized, large-scale dynamic systems, I think the field of large-scale systems control is very important. However, the results are very scattered at this point and certainly we are unable to claim a very unified picture of the whole field. I apologize for not being able to present a more coherent talk or discussion on this matter as a whole, but I hope the rest of the conference will try to make up for this. Thank you very much.

FOR FURTHER READING

1. Ho, Y. C.; and Chu, K. C.: Information Structure in Dynamic Multi-Person Control Problems. Automatica, July 1974.

2. Ho, Yu-Chi; and Sun, Fang-Kuo: Value of Information in Two-Person Zero Sum Problems. Journal of Optimization Theory and Applications, to appear.

3. Basa, Tamer; and Ho, Yu-Chi: Informational Properties of the Nash Solutions to Two Stochastic Nonzero-Sum Games. Journal of Economic Theory, vol. 7, no. 4, April 1974.

4. Ho, Y. C.; Blau, I.; and Basar, T.: A Tale of Four Information Structures. Proceedings of IRIA Symposium on Control Theory, June 1974; Springer-Verlag Notes on Mathematical Systems and Economics, October 1974 to appear.

5. Malishevskii, A. V.: Natural Systems. Automation and Remote Control, May 1974.

6. Malishevskii, A. V.: Models at Joint Operation of Many Goal Oriented Elements, 1 and II. Automation and Remote Control, May-June 1973.

7. Witsenhausen, H. S.: A Counter-example in Stochastic Optimum Control. SIAM J. Control, vol. 6, no. 1, 1968.

A SYSTEM IS "LARGE" WHEN IT REQUIRES MORE THAN ONE DECISION MAKER TO CONTROL IT.

I) INSTITUTIONAL CONSTRAINTS:

- LOCAL OR REGIONAL AUTONOMY VS. CENTRALIZED CONTROL

- AVERSION TO DICTATORSHIP

- BUREAUCRATIC INERTIA AGENT CENTRALIZED CONTROL

II) COMMUNICATION DIFFICULTIES:

- TIME REQUIRED FOR TRANSMISSION (EARTH — MARS)

- COST OF TRANSMISSION

- COST OF CENTRALIZED PROCESSING IN AVAILABLE TIME (ON-LINE CONTROL)

Figure 1.— Large scale dynamic systems.

ON-LINE, REAL TIME CONTROL

↑

SECONDS OF COMPUTATION TIME

MINUTES OF RESPONSE TIME

VS.

OFF-LINE PLANNING AND DECISION MAKING

↑

HOURS OR DAYS OF COMPUTATION TIME

MONTHS OR YEARS OF RESPONSE TIME

CONTROL PROBLEMS ARE REPETITIVE FROM DAY TO DAY

VS.

PLANNINGS ARE ONE-SHOT PROBLEMS

(DECOMPOSITION TECHNIQUE IN LARGE MATH PROGRAMMING OFTEN DOES NOT YIELD IMPROVEMENT IN "THROUGHPUT" COMPARED TO STANDARD LP PACKAGES.)
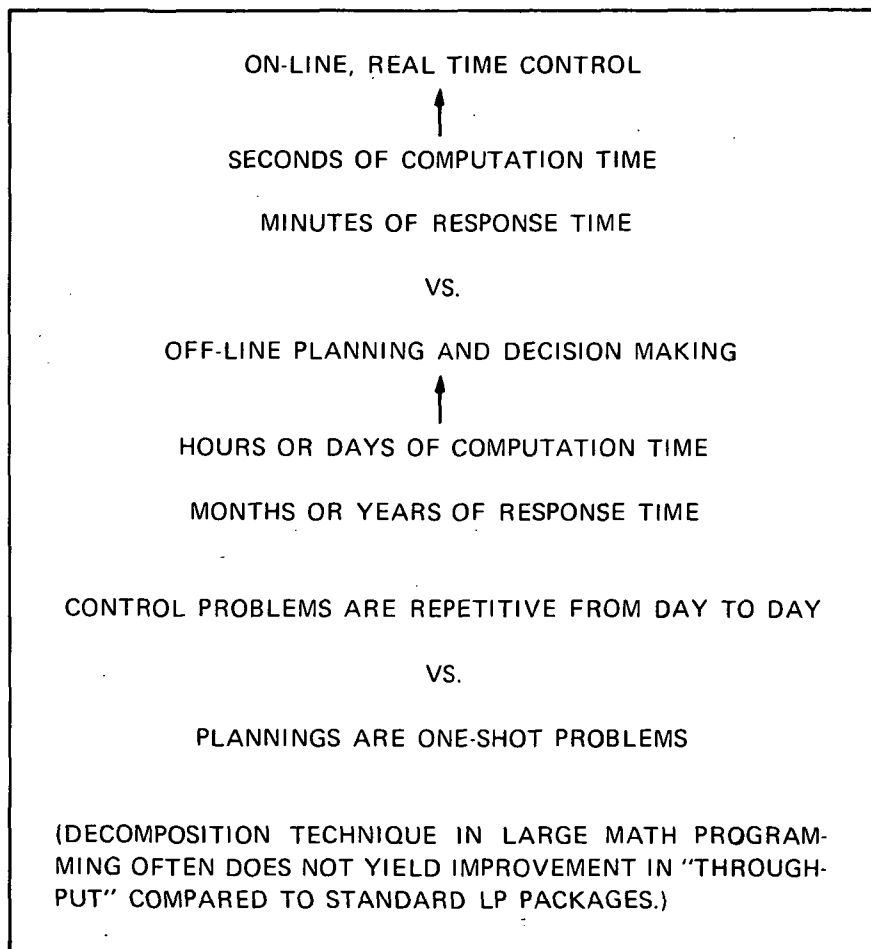
Figure 2.— Decentralized control vs. decomposition in computation.

Choosing many decisions at different instants of time based on different information available

――― EXTENSIVE form formulation (usual statement of a problem)

Choosing a strategy (which is a recipe: information x time   choice) among all admissible strategies.
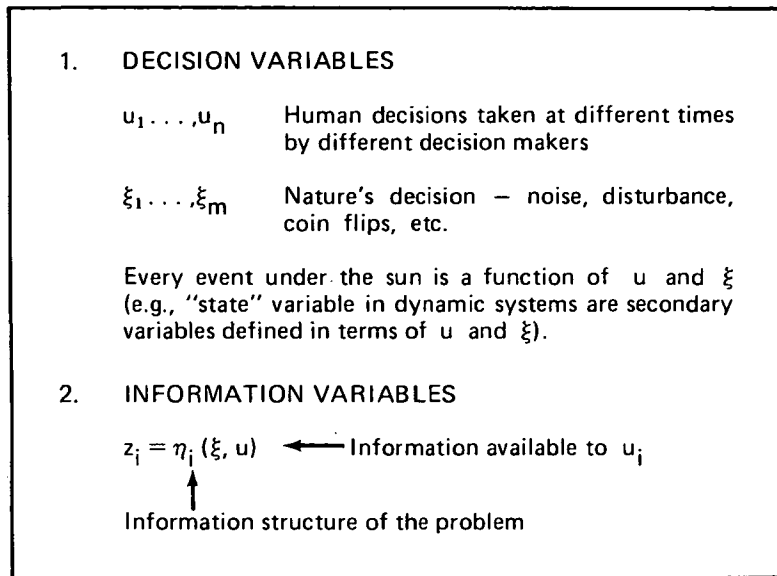――― NORMAL form formulation has the advantage (theoretically) of suppressing the detail difficulties involving dynamics and information in the word "admissible" class of strategies. Relationship between performance vs. strategy choices are clear cut and can be analyzed without being able to solve the problem.

――― Main-stream game theory approach.

Figure 3.― DYNAMICS in a problem.

	Deterministic	Stochastic
Structural	Hierarchy, organizational, payoff distribution, etc.	Information: structure and properties
Optimization	Large-scale multipayoff, resource allocation, vector payoff, Paretian analysis	Team theory value of information
Stability	Adjustment processes	?
Feasibility	Decentralized controllability, etc.	?

Figure 4.― Decentralized control of dynamic systems.

1. DECISION VARIABLES

$u_1 \ldots, u_n$ Human decisions taken at different times by different decision makers

$\xi_1 \ldots, \xi_m$ Nature's decision — noise, disturbance, coin flips, etc.

Every event under the sun is a function of u and ξ (e.g., "state" variable in dynamic systems are secondary variables defined in terms of u and ξ).

2. INFORMATION VARIABLES

$z_i = \eta_i(\xi, u)$ ◄——— Information available to u_i

↑

Information structure of the problem

(a) Design variables and information variables.

3. STRATEGIES

$$u_i = \gamma_i(z_i) = \gamma_i(\eta_i[\xi, u]) \qquad i = 1, \ldots, n$$ (*)

For given information structure η_i (i.e., WHO KNOWS WHAT), given strategies γ_i (i.e., what each decision maker should do under all possible situations), and given probability density on ξ, $p(\xi)$ (i.e., nature's strategy), then (*) defines an implicit set of equations for u_i, $i = 1, \ldots, n$ which are the ACTUAL decisions taken.

This model can be extended to cover even more general situations in game theory. (See references)

(b) Strategies.

Figure 5.— General problem of many person decision making.

1. Does (*) have a unique solution for given η and every admissible γ? Is the problem <u>well posed</u>?

2. What is a "good" η?

   $---$ Design of information system. Who should know what?

3. What is a "good" γ?

   $---$ Usual optimization question in decentralized setting.

4. What choice of η will make good γ easy to solve?

Figure 6.— Questions and problems for $u_i = \gamma_i(\eta_i[\xi,u])$.

ξ (nature's decision) is the <u>basic</u> random variable with $p(\xi)$

$z_i = \eta_i(\xi,u)$ and $u_i = \gamma_i(\eta_i[\xi,u]$ are not random variables for a given η unless γ is fixed.

Example:

intermediate state $\nearrow$ $x = u_1 + \xi_1$ $\longleftarrow$ initial state

$y = u_2 + x$

final state $\nearrow$ $\searrow$ controls at $t = 1,2$

u_1 knows $z_1 = \xi_1$ $\longleftarrow$ measurements at $t = 1,2$

u_2 knows $z_2 = x + \xi_2$

$\quad = \xi_1 + u_1 + \xi_2$

$\quad = \xi_1 + \gamma_1(\xi_1) + \xi_2$

u_2 is a random variable only when γ_1 is fixed.

z_2 is a random variable only when γ_1 is fixed.

$\therefore$ information and decisions are random variables with solution-dependent distributions!

Figure 7.— Difficulties in dynamic information problems.

∴   Even if ξ_1 and ξ_2 are random variables with nice properties (e.g., Gaussian), z_2 is not "nice" since γ_1 can be arbitrary. Also, a convex $L(u_1,u_2) \not\Rightarrow E\{L(u_1,u_2)\}$ is convex in γ_1.

∵ $E\{L(u_1,u_2)\} = J(\gamma_1,\gamma_2)$

$= J(\gamma_1,\gamma_2 \ [\xi_1 + \gamma_1(\xi_1) + \xi_2])$

not convex in γ_1 unless γ_2 is linear or otherwise possesses <u>nice</u> properties.

"Must impose additional restrictions on η, the information structure, to avoid these difficulties!"

All information is centralized. See references for other examples.

Figure 8.— General information problems.

The "best" you can do with the information

— the "best" you can do without the information

= VI

Basis for comparing information system design.

- In problems with multipayoff, definition of the "best" requires more careful specification.

- Is $\epsilon[VI]$ the only basis for comparison? Definition of "more informative?"

- Does more information always lead to better payoffs?
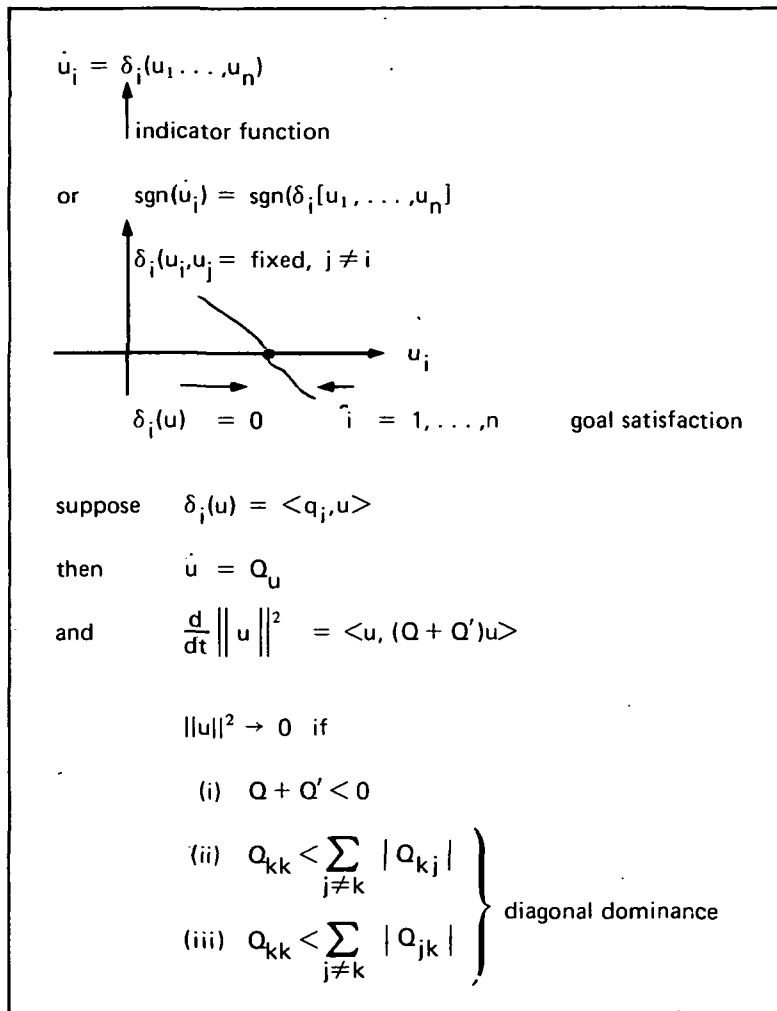
Figure 9.— Value of Information.

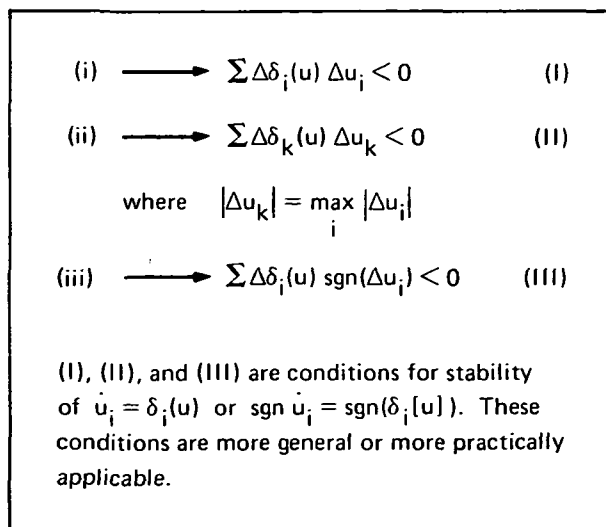Figure 10.– Stability of goal-oriented adjustment processes.

Figure 11.–Finite changes vs. differential changes.

Ex. x_i = levels of production of ith commodity

   α_{ij} = amount of ith commodity needed to produce one unit of jth commodity

$\sum_{j=1}^{n} \alpha_{ij}x_j$, total amount of ith commodity needed

$y_i \equiv x_i - \sum_{j=1}^{n} \alpha_{ij}x_j$, net production of ith commodity

or

$y = (I - A)x$, the Leonliff in/out model

Question: Does an $x > 0$ exist for every $y > 0$? If so, decentralized control is possible — each unit has to increase its own production level.

Figure 12.— Feasibility of adjustment or control.