

# SOME NOTIONS OF DECENTRALIZATION AND COORDINATION IN LARGE-SCALE DYNAMIC SYSTEMS

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## INTRODUCTION

In this paper, we shall discuss some notions of decentralization and coordination in the control of large-scale dynamic systems. Decentralization and coordination have always been important concepts in the study of large systems. Roughly speaking, decentralization is the process of dividing a large problem into subproblems so that it can be handled more easily. Coordination is the manipulation of the subproblem so that the original problem is solved. A great deal of literature is available dealing with these two topics, especially decentralization. In this paper, we shall discuss the various types of decentralization and coordination that have been used to control dynamic systems. Our emphasis will be to distinguish between on-line and off-line operations. This distinction is not, of course, unique. However, it helps to understand the results available by indicating the aspects of the problem which are decentralized. This discussion is informal and no attempt has been made to give precise definitions. Our main objective is to illustrate intuitively “what” is decentralized in the decision-making.

The hierarchical approach with a coordinator has been suggested as a possible way to control a large system. We propose a coordination scheme that is suitable for stochastic systems. This is discussed with respect to the various notions of decentralization.

## INFORMATION AND COMPUTATION IN DECISION MAKING

In this section, we consider the control of a dynamic system. A dynamic system is given with a set of control inputs and a set of measurements. This is to be controlled by a decision maker (controller) in real time to achieve certain objectives. The objectives may be to optimize or stabilize the overall system.

The information available to the decision maker consists of two parts:

1. *Prior information* – this includes information on the system structure, values of the parameters, constraints on the controls, and so on. If the problem is stochastic, then it may also include the statistics of the uncertain quantities. The word “prior” is used in a relative sense. Prior information available now is usually deduced from measurement collected in the past.

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2. A posteriori information – This includes measurements obtained in real time as well as controls used in the past. When several controllers are present, a posteriori information includes messages communicated among the controllers.

The decision maker has to generate the controls in real time based on his prior information and the posteriori information. Figure 1 illustrates the structure of the control problem.

The job of the decision maker can be divided into two parts, depending on whether it is carried out on-line or off-line.

Off-line operation involves generating the control law using all available prior information. This, of course, also makes use of the knowledge about the availability of measurements. On-line operation involves generating the control value at any time using the control law and the posteriori information available at that time. Usually, the off-line operations involve human beings or computers doing the computation while the on-line operations involve hardware of onboard computers. For example, if the system is linear, the noises are Gaussian and the cost functional is quadratic, then the off-line operations involve the solution of Riccati's equations and the on-line operations involve only matrix-vector multiplications and additions for discrete time problems. Figure 1 is then modified to figure 2.

## COMPLEXITY OF THE CONTROLLER

We shall discuss the complexity of the controller with respect to the information processing requirements. There are two operations which must be done in real time: transmitting the measurements to the control agents and computing the control values from these measurements. Then the complexity of the control system is reflected by the size of the communication system involved and by the amount of on-line computation required. If we want a quantitative measure, we can count the number of wires connecting the inputs to the measurements and evaluate the size of the real time computers. The off-line computation discussed in the previous section reflects the complexity involved in finding the control laws.

To build a control system, it is often necessary to put some constraints on the complexity of the controller. The complexity of the communication system can be constrained rather easily. This is the case studied in dynamic team theory on stochastic control with nonclassical information pattern (refs. 1 and 2). The on-line computation involved can also be constrained by using control laws of a particular form (e.g., linear control laws). It is difficult, however, to constrain the complexity of the off-line computation. As a matter of fact, the characterization of computational complexity is not easy (ref. 3). There is often a tradeoff between the complexities of the different operations. For example, if complete information about the system is allowed, satisfactory performance can often be obtained with very simple control laws that require simple off-line computation. On the other hand, if less information is available, the control laws are often more complicated.

## VARIOUS TYPES OF DECENTRALIZATION

In the control of traditional small-scale systems, all measurements on the system are generally pooled together to generate the controls. Also, the control laws that govern the relationship between the measurements and the controls are usually determined in a centralized manner. In the control of large-scale systems, the centralized approach will give rise to serious problems of implementation. In addition, the decision problems themselves may be so complicated that they exceed the capacity of the fastest computers. Some kind of decentralization is therefore desirable. Based on the discussion in the previous section, two types of decentralization are distinguished. The first type deals with the real time operations and is used to reduce the complexity of implementation. The values of certain control variables will depend only on a subset of measurements. This reduces the complexity of the communication system required. We shall refer to this as *decentralized control*. The other type deals with the off-line operations and is used to reduce the complexity of finding the control laws. We shall refer to this as *decentralized off-line computation*. When decentralized off-line computation is used, the control laws can be visualized as being generated from several computers operating independently of each other.

We shall look at the various decentralized schemes considered in the literature and attempt to classify them according to the two types of decentralization discussed above.

### Case 1: Centralized Control and Centralized Off-Line Computation

This is typical of traditional small-scale systems. The generation of the control values from the measurements as well as the determination of the control laws are handled by a single decision maker. The classical linear-quadratic-Gaussian problem and the pole allocation by centralized state feedback (ref. 4) all fall into this category.

### Case 2: Decentralized Control and Centralized Off-Line Computation

This occurs when the information pattern is decentralized but the control laws are still computed in a centralized manner. Thus there is more than one control law, each of which transforms some set of measurements into a set of controls. On the other hand, the control laws are all determined by a single decision maker (fig. 3). Examples of decentralized control and centralized off-line computation include nonclassical, linear-quadratic-Gaussian problems (dynamic teams) (refs. 1, 2, 5) and stabilization of systems with decentralized feedback (refs. 6, 7). Usually, these are characterized by simple communication systems relatively complicated on-line and off-line computation. This is especially obvious in optimization problems. For the centralized control and centralized computation case of the linear-quadratic-Gaussian problem, the optimal control law consists of the optimal deterministic gain acting on the estimate of the Kalman-Bucy filter. The on-line computation involves only a finite-dimensional filter. The off-line computation involves the solution of Riccati equations. For the nonclassical, linear-quadratic-Gaussian problem, however, the on-line computation is complicated because a filter of growing dimension is needed. The off-line computation is even more complicated (the solution is not known yet). We can explain the trade-off between complexity in communication and computation as follows. Since the off-line computation is

centralized, the decision maker will try to generate the missing measurements needed in the control by use of more complicated control laws.

### Case 3: Centralized Control and Decentralized Off-Line Computation

The main interest here is to reduce the complexity of the computation of the control laws. A multilevel approach in finding the optimal control laws can be considered as part of this category (ref. 8).

### Case 4: Decentralized Control and Decentralized Off-Line Computation

This case is the same as case 2 except that the control laws are also computed in a decentralized manner. We can regard both a posteriori information and prior information as decentralized. Figure 4 illustrates such a control configuration. Examples in this category include adjustment processes in resource allocation problems (ref. 9) where the off-line decentralized computation is usually conducted in an iterative manner and the decentralized stabilization of systems with unknown global structure (refs. 10, 11). Since decentralized (prior and a posteriori) information is the rule in large-scale systems, this approach deserves further investigation.

## COORDINATION APPROACH IN LARGE-SCALE SYMAMIC SYSTEMS

The hierarchical approach has been proposed as a possible way to control large-scale systems (ref. 8). Much of the existing work deals with off-line computation of the optimal control strategies. The main function of the coordinator is to coordinate the decentralized off-line computation of the control laws. In references 12 and 13, a possible approach for the multilevel, hierarchical control of stochastic systems has been proposed. We shall relate this approach to the notions of decentralization discussed earlier. Some of the results are summarized. M. Athans also discusses this approach in another paper of this proceedings.

In a multilevel approach, there are (at least) two levels of decision makers. In the simplest case, a two-level hierarchy is considered. The local controller of each subsystem knows the local dynamics of the subsystem under his control. The local measurements of his subsystem are also available. At the higher level, the coordinator knows the global dynamics of all subsystems and their interactions. He also collects measurements periodically from the lower level. Thus we have the following information pattern.

	Prior information	Measurements
Coordinator	Centralized	Centralized periodic
Local controller	Decentralized	Decentralized

This reflects the intuitive notion that the local controllers have detailed but local information and the coordinator has coarse but global information. The local controllers send their measurements and controls to the coordinator periodically. The coordinator computes new coordinating

parameters based on this information and his prior information about the structure of the system. Using these coordinating parameters, the local controllers compute their control laws for the next period in a decentralized manner (fig. 5).

## FORMULATION AND SUMMARY OF RESULTS

The linear-quadratic-Gaussian case is presented here. Consider a system consisting of  $N$  subsystems coupled together:

$$x_i(k+1) = A_{ii}x_i(k) + v_i(k) + B_i u_i(k) + \xi_i(k) \quad i = 1, \dots, N$$

where  $x_i$  is the state of the  $i$ th subsystem,  $u_i$  is the control,  $\xi_i$  is the driving noise, and  $v_i$  is the interaction from other subsystems given by

$$v_i(k) = \sum_{j \neq i} A_{ij} x_j(k)$$

The cost function is a sum of the cost functionals of the individual subsystems:

$$J = \sum_{i=1}^N J_i$$

$$J_i = E \left\{ x_i'(T) E_i x_i(T) + \sum_{k=0}^{T-1} x_i'(k) Q_i x_i(k) + u_i'(k) Q_i u_i(k) \right\}$$

The local measurement of the  $i$ th controller is

$$y_i(k) = C_i x_i(k) + \theta_i(k)$$

where  $\theta_i(k)$  is the measurement noise. Let

$$Y_i(k) = \{y_i(0), \dots, y_i(k)\}$$

$$U_i(k) = \{u_i(0), \dots, u_i(k)\}$$

$$Y(k) = Y_1(k) \cup Y_2(k), \dots, \cup Y_N(k)$$

$$U(k) = U_1(k) \cup U_2(k), \dots, \cup U_N(k)$$

where  $n$  is an integer such that  $k \geq n_Q$ . This implies that the coordinator collects measurements from the lower level every  $Q$  units of time.

For each lower-level controller to control his system based on his information, some knowledge of  $v_i$  is required. The job of the coordinator is to provide this information so that  $v_i$  is the best estimate of the interaction based on his information, that is

$$\left\{ v_i(t) - \sum_{j \neq i} A_{ij} x_j(t) | Y_0(k) \right\} = 0 \quad t \geq k .$$

At the same time he would like to minimize the cost functional.

The coordinator can transmit any portion of his stored data to the lower level except the structure of the overall system. Thus, he can transmit the value of the matrices  $A_{ij}$  but not how they interact to form the overall system. Since  $v_i(k)$  depends on  $Y_0(k)$ , we shall also let  $u_i(k)$  depend on  $Y_0(k)$ . This implies that the a posteriori information of the coordinator can be used by the lower level. We then have the following problem. Given

$$x_i(k+1) = A_{ii} x_i(k) + v_i(k) + B_i u_i(k) + \xi_i(k)$$

$$J = \sum_{i=1}^N J_i .$$

Then each lower-level controller possesses the following information.

*A priori information:*

$$x_i(k+1) = A_{ii} x_i(k) + v_i(k) + B_i u_i(k) + \xi_i(k)$$

$$y_i(k) = C_i x_i(k) + \theta_i(k)$$

$$J_i = E \left\{ x_i'(T) F_i x_i(T) + \sum_{k=0}^{T-1} x_i'(k) Q_i x_i(k) + u_i'(k) R_i u_i(k) \right\} .$$

The statistics of  $x_i(0)$ ,  $\xi_i(k)$ ,  $\theta_i(k)$ ,  $k = 0, \dots, T-1$  are all known;  $v_i(k)$  represents the interaction from the other subsystems, of which the  $i$ th controller is completely ignorant.

*A posteriori information:*

$$Y_i(k), U_i(k-1) .$$

The information of the coordinator consists of

*A priori information:*

$$x(k+1) = Ax(k) + Bu(k) + \xi(k)$$

$$y(k) = Cx(k) + \theta(k)$$

$$J = \sum_{i=1}^N J_i .$$

The statistics of all random quantities are known. Thus, the coordinator has complete a priori (structural) information of the system.

*A posteriori information:*

Let  $Y_0(k)$  be the information available to the coordinator at time  $k$ :

$$Y_0(k) = Y(n_\varrho) \cup U(n_\varrho - 1),$$

$$E \left\{ v_i(t) - \sum_{j \neq i} A_{ij} x_j(t) \mid Y_0(k) \right\} = \theta \quad t \geq k,$$

$$u_i(k) = \gamma_i^k [Y_i(k), U_i(k-1), Y_0(k)],$$

$$v_i(k) = \eta_i^k [Y_0(k)].$$

Find the optimal strategies  $\gamma_i^k$  and  $\eta_i^k$ ,  $i = 1, \dots, N$ ,  $k = 0, \dots, T-1$  so that  $J$  is minimized. Therefore,

- (1) The minimization is done with respect to both  $\gamma_i^k$  and  $\eta_i^k$ , for which  $\gamma_i^k$  gives the actual control and  $\eta_i^k$  generates estimate of the interaction needed in the control.
- (2) The term  $\gamma_i^k$  will not use the structure of the system since a decoupled model is given.
- (3) The coupled nature of the system is taken care of by constraint.

So far the coordinator shares almost all of his information, except the model of the entire system, with the lower-level controllers. However, by solving the preceding stochastic control problem (with nonclassical information pattern), it can be shown that only certain types of parameters need to be transmitted; specifically, (1) new global estimates  $\bar{x}_i(r\varrho/n\varrho)$  that enable the local controllers to correct his local estimates and (2) parameters that change the objectives of the local controllers.

Although the results are intuitively attractive, more work remains to be done in the hierarchical control of stochastic systems. For example, in the approach taken, the coordinator knows the detailed structure of the entire system. It would be more realistic to assume that he is interested in an aggregated model. Also, other coordinating criteria for the coordination are possible.

## SUMMARY

In this paper, we have presented various notions of decentralization and how these are related to the computation and information involved in the control of large-scale dynamic systems. Other attributes of decentralization such as reliability have been purposely omitted. Some attempt is also made to relate the various notions of decentralization to a proposed approach for the coordination of stochastic systems.

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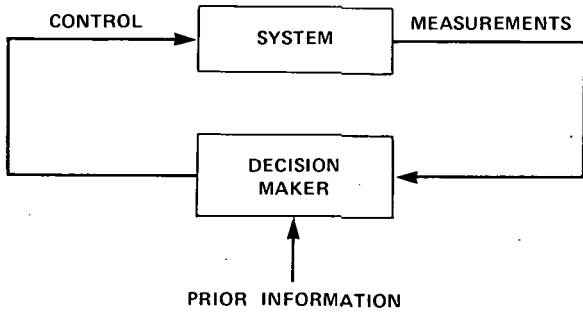


Figure 1.—Control problem I.

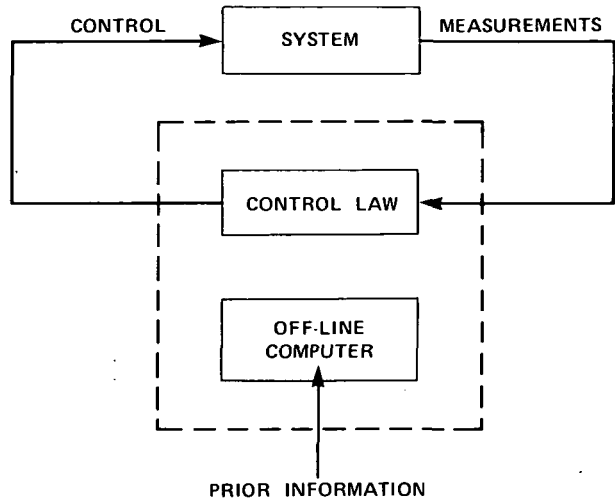


Figure 2.—Control problem II.

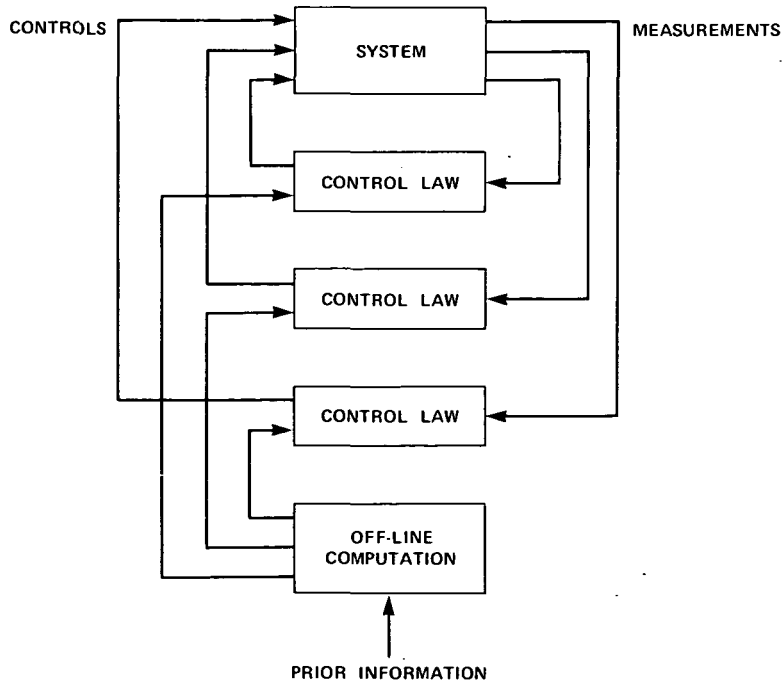


Figure 3.—Decentralized control and centralized off-line computation.

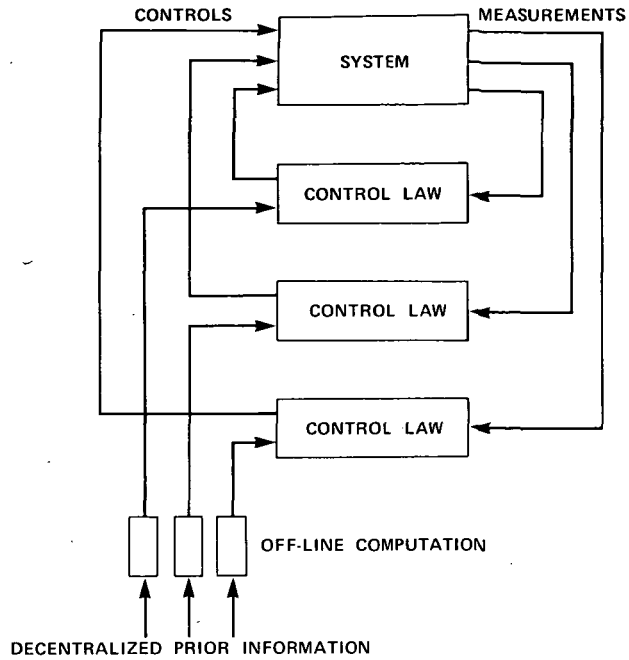


Figure 4.—Decentralized control and decentralized off-line computation.

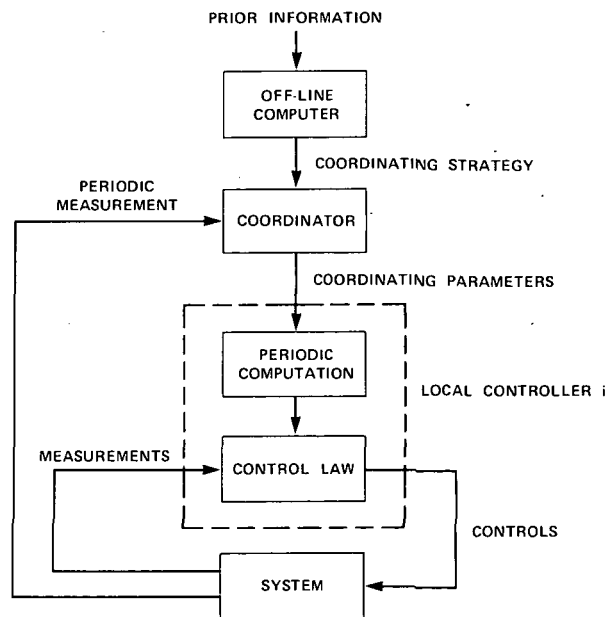


Figure 5.—Coordination.