SOME ASPECTS OF CONTROL OF A LARGE-SCALE DYNAMIC SYSTEM*

Masanao Aoki

University of Illinois
Urbana, Illinois

INTRODUCTION

Problems of understanding and dealing effectively with dynamic systems of high dimension (large- scale dynamic systems) have been objects of research for some time. The need for better techniques of predicting and/or controlling dynamic behavior of large-scale systems, involving both modeling and analysis, has been felt more intensely as demands have increased rapidly for designing and managing large-scale systems (such as interconnected network of power systems, water resources, computers, warehouses, or depots). Despite varied and intensive research, many questions remain about large-scale dynamic systems for which we do not have adequate answers.

If we assume perfect and centralized information, that is, if all "relevant" data on systems and environments[1] and complete problem descriptions are available for a single decision maker, then stabilization and control of large-scale systems are usually reducible to problems of constructing algorithms for efficient information processing (such as decomposition algorithms in large-scale mathematical programming) and associated problems of designing efficient information collection and transmission structures. Existing dynamic systems with centralized information structure are usually not "too" large and may have special structures because of the increasing cost of collecting information and processing it by a single decision maker as systems become larger.

For large-scale systems, it is therefore likely that several decision makers exist who influence the dynamic behavior of large-scale systems. These systems are called decentralized systems. If we drop the assumption of centralized information pattern, we must explicitly recognize and cope with the problem of interaction of decisions and information held by several decision makers.[2] In

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[1] We make the usual distinction of risk and uncertainty. The former is what is called purely stochastic and a special case of the latter is called a parameter adaptive (ref. 1).

[2] Another aspect of research on large-scale systems becomes important if we retain the centralized information pattern but drop the assumption of perfect information. This is the aspect of learning or adaptation in decision making. In reality, neither assumption is likely to hold exactly. Most large-scale systems we encounter are likely to be imperfectly modeled and are being acted on by more than one decision maker with nonidentical knowledge of the system.

decentralized systems, no single decision maker may have enough information needed for stable operation of large-scale systems, to say nothing of "optimal" operation of systems.[3]

Thus, we characterize and somewhat limit the scope of our investigation on large-scale dynamic systems as decentralized systems and as dynamic team problems. Sharing information (such as choice of message alphabets and estimation and partial reconstruction of information content held by other decision makers) and assigning control responsibility are some of the problems we have considered (see, for example (refs. 2-4)). Questions related to reliability, security, design of good measurement and control subsystems, or situations such as oligopolistic competition are not covered here.

In the next three sections, we summarize our findings in (1) control of large-scale systems by dynamic team with delayed information sharing. (2) dynamic resource allocation problems by a team (here we assume a hierarchical structure with a coordinator (central agent) who coordinates decision making by lower-level (local) decision makers), and (3) some problems related to construction of a model of reduced dimension.

OPTIMIZATION BY A DYNAMIC TEAM WITH DELAYED INFORMATION SHARING

Consider a team composed of N decision makers. Radner (ref. 5) proved that given a quadratic performance index for a team

$$F = -u'Qu + 2\mu'u,\ u' = (u_1', \ldots, u_N'),\ \mu' = (\mu_1', \ldots, \mu_N')$$

where Q and μ may contain random variables, the (person-by-person satisfactory) optimal decisions are given as the unique solution of

$$E\left[Q_{ii} \mid \mathcal{H}_i\right]u_i + \sum_{j \neq i} E\left[Q_{ij}u_j \mid \mathcal{H}_i\right] = E\left[\mu_i \mid \mathcal{H}_i\right]; \quad i = 1, \ldots, n \tag{1}$$

where $\mathcal{H}_i$ is the information σ-field of decision maker i. This equation may be solved under a set of suitable assumptions by adapting an iterative solution technique used by Aoki (ref. 6)(see also ch. 2 of ref. 7).

To emphasize the aspect of learning or adaptation, suppose μ is imprecisely known. Suppose further decision maker i observes $\partial F/\partial u_i$ (i.e., his differential influence on F) through an additive noise. By sharing his observation with other decision makers with one period delay, the *pbps* decisions are given as the solution of a set of equations such as equation (1). The decisions consist of two parts:

$$u_t^d = u_{t-1}^c + \Delta u_t \tag{2}$$

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[3] We exclude from our consideration game situations, where decision makers may have contradictory objectives or problems of incentive for cooperation.

where u^c_{t-1} is the decision vector with the centralized information structure and Δu_t is the correction term due to individual differences in the information set at time t.

Similar results were obtained independently by Sandell and Yoshikawa (refs. 8,9). The standard stochastic linear regulator problem with quadratic cost can be reformulated as a decentralized control problem by a team with delayed information-sharing where decision makers observe different linear combinations of the state vector through noise. The structure of the decentralized control is still of the form of equation (2).

It is also possible to obtain the decentralized optimal decision when some elements of Q are not known precisely. In this case, however, it is not possible to characterize the decentralized decision rules conveniently as in equation (2). (see ref. 7).

DYNAMIC RESOURCE ALLOCATION BY A TEAM

We now briefly describe our work which deals with a dynamic version of a stochastic resource allocation problem discussion by Groves and Radner (refs. 10, 11). The problem is to allocate a finite amount of resources to n subsystems to maximize some objective function cost for the whole team. The amount available for allowcation depends on past decisions. Some parameters in the objective functions are imprecisely known.[4] This brings in the interactions between control and information, the so-called "dual control effects."

Some approximations must be made to approximately evaluate contributions to the objective function (cost) from the future (cost-to-go). The conflict arises between control and information since the larger the control is, the smaller is the estimation error variance of the parameter value, while the total amount of resource available for control is finite. Open-loop feedback, certainty equivalent, and other schemes are compared (see ref. 2 for details).

AGGREGATION UNDER PERFECT INFORMATION

Another active area of research on large-scale systems is the following: construction of models of similar structure and/or with smaller dimension and use of these models to stabilize and control large-scale systems. Several reduced-order models may be used by a single decision maker or by several decision makers in decentralized large-scale problems (see, e.g., refs. 13 and 14).

In this section, we consider the feasibility of aggregating a collection of systems (called microsystems or subsystems) into larger systems (called macrosystems). One part of the problem may be called the aggregation problem of optimal systems (or optimal configurations). It relates to the gross characterization of optimal system performance characteristics. The other class relates to the possibility of constructing a model, called a macromodel, or aggregates of microsystems and optimizing such aggregates of systems via the macromodel.

[4] This problem has never been formulated in this manner so far as we can ascertain.

In many control problems of physical systems, systems are naturally divided into several distinct subsystems because of the physical makeup of the problem. Here the word "physical" should be interpreted broadly to include socioeconomic systems as well as the usual, truly physical control systems. Loosely speaking, the concept of aggregation implies that two or more such subsystems are combined to make a larger subsystem. One of the objectives of such aggregation is usually to reduce the number of variables in the state space description of the problem. More often the problem is already stated in an aggregated fashion because of the sheer necessity of limiting the number of variables. The latter is true, for example, in describing the economic behavior of a country by a dynamic macroeconomic model.

The concept of aggregation therefore complements that of decomposition. These two concepts, taken together, form two sides of a coin. In applying the concept of decomposition to a system to form a two-level structure, the subsystems in the lower level may be aggregated to a smaller number of subsystems with some advantages, for example. The concept of decomposition, with the resultant multilevel structure, is fairly well known and some computational algorithms have been proposed. The results available to date, however, are of a mathematical nature and somewhat of the nature of further developments of Lagrange multipliers and of generalized Kuhn-Tucker conditions.

Consider a collection of subsystems whose structures are alike. They differ only in their parameter values that specify the systems completely. The problem is to allocate a limited amount of resources so that the overall performance (return) from these subsystems is optimized (maximized, to be definite). The interactions among these subsystems come therefore from the common sources of resources. No interactions through dynamic interactions among the subsystems are considered.

Consider, for example, a network of power-generating stations. Optimal performance characteristics of each power station may have the same functional dependency on the key parameter, such as the amount of fuel available, load of the station, capacity, etc. (see refs. 15, 16). More precisely, we consider a macrosystem consisting of N subsystems so that the total return from this system F is related to the individual return of the ith subsystem g_i by

$$F(x^1, x^2, \ldots, x^N) = \sum_1^N g_i(x^i)$$

where F and g_i are scalar valued and where x^i is an m vector. The term F is to be maximized over the domain D_N defined by

$$\sum_1^N x^i = x, \quad x^i \geqslant 0.$$

Among the class of problems with such separable criterion functions, we are interested particularly in the special case

$$g_i(x^i) = g(x^i, a^i), \; 1 \leqslant i \leqslant N$$

namely, we are interested in problems where subsystems are all alike and the return from each subsystem differs from each other only in its parameter so that

$$\max \left\{ g(x^1,a^1) + g(x^2,a^2) \right\} = g\left\{ x,h(a^1,a^2) \right\} \tag{3}$$

$$x^1 + x^2 = x$$

$$x^1 \geqslant 0, \quad x^2 \geqslant 0$$

where $h(a^1,a^2)$ is a known function of a^1 and a^2. This means that the functional form g is reproduced after aggregating two subsystems, a very desirable feature indeed. Equation (3) can be interpreted as follows: A controller is associated with each subsystem i, and its performance is optimized subject to the constraint that the given amount of resource be x^i. The optimal performance is denoted by $g(x^i,a^i)$, which are called the optimal performance characteristics by Kulikowski (ref. 15). Then the optimal performance of the aggregate of N such subsystems $g\{x,h(a^1,\ldots, a^N)\}$ is given by[5]

$$g\left\{ x,h(a^1,\ldots,a^N) \right\} = \max \left[\sum_1^N g_i(x_i,a^i) \right] x \epsilon D_N .$$

Conditions for aggregating microvariables pertaining to dynamic behavior of subunits to obtain macrovariables for describing a set of subunits are usually rather restrictive. Since only a small class of problems permits perfect aggregation, investigating the effects of approximate aggregation is important. However, it is useful to know when microstate variables can be aggregated without error. In the dynamic resource allocation of section 2, this type of aggregation is possible for a class of problems. An example of perfect aggregation follows (ref. 14).

Consider a system composed of N subsystems with the optimal performance characteristic functions of a common type given as

$$A_i^{\alpha} B_i^{\beta} \ldots Z_i^{\omega} = k_i \quad 1 \leqslant i \leqslant N \tag{4}$$

where $A_i,\ldots,Z_i$, α, $\beta,\ldots$, ω and $k_i > 0$ are given real numbers. Assume that these N subsystems are coupled by the conditions

$$A = \sum_1^N A_i, \ B = \sum_1^N B_i, \ldots, Z = \sum_1^N z_i$$

where

$$0 \leqslant \underline{B} \leqslant B \leqslant \overline{B}, \ldots, 0 \leqslant \underline{Z} \leqslant Z \leqslant \overline{Z}.$$

Interpret A as the total cost of control $B, C,\ldots,Z$ are the total amount of resources allocated to the N subsystems. They are constrained from below and above by $\underline{B}$. $\overline{B}$, ... $\underline{Z}$ and $\overline{Z}$. The problem is to minimize A.

[5] In the multilevel approach (decomposition approach), the subsystems are on the first-level and the second-level controller assigns certain x^i to the ith subsystem.

The optimal performance characteristics of the total system can be shown to be of the same type as

$$A^{\alpha}B^{\beta}\ldots Z^{\gamma} = k$$

where k is given by

$$k = \left(\sum_{1}^{N} k_i^q\right)^{1/q}, \quad \frac{1}{q} = \alpha + \beta + \ldots + \omega$$

and the optimal parameter value settings are given by

$$B_i^*/B = \ldots = Z_i^*/Z = (k_i/k)^q$$

where the starred variables indicate optimal values.

Actually, any number of subsystems can be combined or aggregated to form a larger subsystem without destroying the common functional form of the optimal performance characteristics.

For example, subsystems 1 and 2 can be combined to give

$$A_{12}^{\alpha}B_{12}^{\beta}\ldots Z_{12}^{\omega} = k_{12}$$

where

$$A_{12} \triangleq A_1 + A_2$$

$$B_{12} \triangleq B_1 + B_2$$

$$Z_{12} = Z_1 + Z_2$$

$$k_{12} = (k_1^q + k_2^q)^{1/q}$$

and where

$$A_{12}^* = \min \quad (A_1 + A_2)$$

$$B_1 + B_2 = B_{12}$$

$$\vdots$$

$$Z_1 + Z_2 = Z_{12}$$

$$A_1^*/A_{12}^* = B_1^*/B_{12}^* = \ldots = Z_1^*/Z_{12}^* = (k_1/k_{12})^q.$$

Thus, for this class of problems, instead of optimizing the system with respect to variables $B_i, C_i, \ldots Z_i$, $1 \leqslant i \leqslant N$, some or all of the N subsystems can be aggregated and optimized separately to form a system with fewer subsystems without incurring any loss in the overall system performance. This fact will be referred to as a perfect aggregation. Unfortunately, the class of functions that permits this perfect aggregation is not large and must be essentially of the type given by equation (4). Therefore, some sort of approximate aggregation procedure must be developed. One such approximation may be to approximate optimal operating characteristics with exponential-type functions.

Note also that the subsystems of the class of systems considered by Kulikowski interact only indirectly through the allocation of common resources. No direct couplings of the dynamics of the subsystem through their inputs and outputs are considered. A network of power-generating stations has been mentioned as a possible system providing a motivation for such a model.

Another important example is a network of computing centers. A computer center may be taken to be the basic subsystem. A local network consisting of several such centers may be considered next. The allocation of total computing load has been considered for a computer center consisting of several central processing units with perhaps a common memory. What is being considered is not the detailed job assignment schedule but a gross characterization of optimal operating condition. Some job assignment schedule is assumed to have been adopted and the operating characteristics involving such variables as the description of the composition of the types of computing jobs, average times of executions, characterization of computing speed, etc., are assumed to be known.

What is being proposed is to characterize the operating characteristic of a network consisting of two or more computing centers with similar operating characteristics in terms of similar macrovariables when these computing centers obtain jobs for a common source. Optimal job allocation is considered in terms of the given operating characteristics of the individual computer centers. Of course, in applying the theory to be developed in the proposed work, a major problem is to identify the important variables and obtain adequate characterizations.

When the assumption of perfect information is relaxed, we can no longer perform perfect aggregation because of interactions between control decisions and information of various decision makers.

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