NASA TECHNICAL NOTE

NASA TN D-7889

NASA TN D-7889

EVALUATION OF THE PRESENT THEORETICAL BASIS FOR DETERMINATION OF PLANETARY SURFACE PROPERTIES BY EARTH-BASED RADAR

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • AUGUST 1975

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-7889		
4. Title and Subtitle EVALUATION OF THE PRESENT THEORETICAL BASIS FOR DETERMINATION OF PLANETARY SURFACE PROPERTIES BY EARTH-BASED RADAR		5. Report Date August 1975
		6. Performing Organization Code
7. Author(s) Leo D. Staton		8. Performing Organization Report No. L-9889
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23665		10. Work Unit No. 879-11-36-01
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

Spaceflight programs such as the planned Viking landing on Mars often require the determination of planetary surface slopes and surface dielectric constants by Earth-based methods. Heavy reliance is often placed on radar backscattering data for estimation of these surface properties. The present report, which is primarily directed toward users of such estimates rather than toward radar workers, is an assessment of the basic theory by which the raw radar data are interpreted. It is shown that serious difficulties and internal inconsistencies are present in the available theoretical formulas. The discussion brings into serious question the reliability of the presently available results for these surface properties as obtained by Earth-based radar methods, and it points out that the theoretical difficulties are not likely to be remedied in the near future.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Radar astronomy Martian surface Rough-surface scattering Planetary astronomy	Unclassified – Unlimited Subject Category 89

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	33	$3.75

*For sale by the National Technical Information Service, Springfield, Virginia 22161

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SUMMARY

Spaceflight programs such as the planned Viking landing on Mars often require the determination of planetary surface slopes and surface dielectric constants by Earth-based methods. Heavy reliance is often placed on radar backscattering data for estimation of these surface properties. The present report, which is primarily directed toward users of such estimates rather than toward radar workers, is an assessment of the basic theory by which the raw radar data are interpreted. Certain serious difficulties and internal inconsistencies in the available theoretical formulas are reemphasized. These difficulties include questions of the fundamental applicability of Gaussian height statistics to planetary terrain, questions of the validity of certain assumptions necessary for applying results for perfectly conducting surfaces to dielectric surfaces, and questions surrounding a widely used rough-surface autocorrelation function that leads to several absurd and self-contradictory conclusions. The discussion therefore brings into serious question the reliability of the presently available results for these surface properties as obtained by Earth-based radar methods, and it points out that the theoretical difficulties are not likely to be remedied in the near future.

INTRODUCTION

The interpretation of electromagnetic signals scattered from rough surfaces has important applications in both terrestrial and astronomical studies. This report concentrates on certain aspects of the astronomical area, particularly on methods of inferring surface electrical and scattering properties from data obtained by Earth-based radar reflections from celestial bodies (for example, from Mars). Since they can reveal information about characteristic planetary surface slopes and surface particle composition and/or density, such inferences are important for the planning of planetary soft-landing missions such as the NASA Viking exploration of Mars in 1976. The principal aim of this paper, therefore, is to review and to criticize the basic theory by which radar astronomical data have been interpreted in pursuit of this practical goal. Since rough-surface scattering theory is still a highly active field of research, it is not surprising that most existing inferences of planetary surface properties, such as detailed surface slopes and dielectric constants, rest on somewhat unsatisfactory grounds. Of course, this does not demean the spectacular successes of radar astronomy over the last 20 years in other areas of lunar and planetary studies, such as large-scale topographic mapping, planetary rotation-rate determination, and interplanetary distance measurements.

In the present report the fundamental statistical notions and the electromagnetic scattering approximations which are the basis for most inferences of surface properties from radar astronomical data are discussed, and certain inconsistencies in this work are pointed out. Some of these inconsistencies have been discussed in the literature of rough-surface scattering but have not yet been taken into account in the planetary radar work.

SYMBOLS

A	covariance matrix defined by equation (37)
C	quantity defined as coefficient of $\sin^2 \varphi$ in equation (47)
D	upper radial cutoff value associated with equation (44)
E	magnitude of electric field
e	magnitude of unit polarization vector
F	integrand function of equation (9); also, function defined by equation (38)
f	physical quantity associated with a statistically described surface
H	magnitude of magnetic field
h	local height of rough surface
$\overleftrightarrow{I}$	unit dyad
J_0	zeroth-order Bessel function of the first kind
j	imaginary unit, $\sqrt{-1}$

k	wave number, $2\pi/\lambda$
L	total length and total width of scattering surface
ℓ	surface correlation length
M_{ij}	cofactor of covariance matrix
N	total number of sample functions in a statistical ensemble; also, total number of specular points on a rough surface
n	number of sample functions in ensemble having height between h and $h + dh$; also, magnitude of unit normal vector
P	angular power scattering function at angle φ
p	probability density
Q	reflection coefficient for plane-wave incidence on plane surface
q	magnitude of difference between incident and scattered propagation vectors
R	distance from radar antenna to planetary center
r	distance from point on scattering surface to field point
S	area of scattering surface
t	slope of surface in x-direction, $\frac{dh}{dx}$
u	distance between point with coordinate x and a given stationary point on rough surface
x,y,z	coordinates with respect to mean surface
α	local slope angle

γ	angle between local surface normal and overall mean surface normal
Δ	operator indicating small increment
ϵ	dielectric constant
λ	wavelength
μ	magnetic permeability
ρ	radius of curvature; also, autocorrelation function of rough surface
Σ	total radar cross section, length squared
σ	standard deviation; also, radar cross section per unit width, length; and radar cross section per unit area, dimensionless
τ	time
ϕ	phase angle of individual scattered wave in equation (21)
φ	planetary great-circle angle from sub-Earth point, equal to angle of incidence with respect to mean surface
ψ	Green's function of equation (6); also, local angle of incidence
ω	angular frequency

Superscripts:

sc	scattered field
*	complex conjugate
'	quantity evaluated on scattering surface; also, first derivative
"	second derivative

Subscripts:

a,b	two principal directions of curvature on a surface
f	property associated with the physical quantity f
h	horizontal polarization direction
i	general member of a collection; also, incident field
j,k	general member of a collection
m,p	specular point, summation index
o	total field incident on planet
R	received signal
s	scattered field
sp	quantity evaluated at specular point
v	vertical polarization direction
x	vector component in x-direction; also, derivative with respect to x in equations (24) to (28)
y	vector component in y-direction; also, derivative with respect to y in equations (24) to (28)
z	vector component in z-direction; also, general polarization direction
1,2	dummy variables of integration

Mathematical notation:

–	(over symbol) average value over a particular sample surface of an ensemble

$\rightarrow$	(over symbol) vector quantity
$\langle\ \rangle$	expectation value computed over an ensemble of sample surfaces
$\hat{}$	(over symbol) unit vector
(;)	simple probability density
(|)	conditional propability density
(||)	joint probability density
∇	vector gradient operator
| |	absolute value of a quantity

STATISTICAL NATURE OF THE PROBLEM OF ROUGH-SURFACE SCATTERING

The exact calculation of the fields scattered by an object immersed in a specified incident electromagnetic field is an intractable problem for all but the simplest geometrical shapes, and solutions for even these simple shapes often require extensive numerical computations, the complexity of which depends upon the ratio of the incident wavelength to the characteristic size of the scattering object. Thus, for scattering objects as complicated as rough planetary surface terrain, no exact solution is possible and the problem becomes a search for an appropriate approximation scheme. Furthermore, if the basic problem is to deduce the properties of the scattering object by measurements on the scattered fields, the difficulty is manifestly compounded in that no unique solution exists. That is, it is impossible, in principle, to define the scattering object solely from measurements of the scattered field taken at any finite number of points in space. For any progress at all, then, plausible models of the scattering object must be constructed, with the field measurements being used to fix the values of whatever parameters the model incorporates. For the study of scatterers having the qualities of rough planetary terrain, for which it is impossible to ascribe, a priori, any particular shape, it is clear that the model must be a statistical one, the results of which must be interpreted only in a statistical sense.

Statistical analyses of rough-surface scattering have been developed by many authors, most of them using the techniques of S. O. Rice (ref. 1). These techniques are very closely related to the methods used in statistical communication theory (ref. 2). It is only necessary to replace the time variable by a spatial variable and to perform an obvious extension from one to two independent variables in order to adapt the formalism of reference 2 to the description of the statistical quantity of interest for rough-surface scattering problems. This quantity is the variable terrain height defined over some datum surface such that this height is a random function of position on the surface. The following elementary ideas are reviewed in this section for reference.

The term random function means a quantity, in this case the height, which takes on variable values for its various argument coordinates x and y, such that at a given point the quantity takes on a particular value within the interval $(h, h+dh)$ according to a probability $p(h;x,y)\,dh$. The situation is thus describable by a (sufficiently) large collection or ensemble, say N in number, of the physically possible realizations of specific functions $h_i(x,y)$ (for $i = 1, 2, \ldots, N$) such that at every point (x,y), each physically possible value of h, to within some arbitrarily small tolerance dh, is taken on by some one or more, say $n(h,x,y)$, of the $h_i(x,y)$. Then the probability density $p(h;x,y)$ is related to this ensemble by the relation

$$p(h;x,y)\,dh = \frac{n(h,x,y)}{N} \tag{1}$$

The totality of information in the ensemble of specific functions $h_i(x,y)$ constitutes a random process. All this information may be represented by the infinite set of joint probability densities $p_j\left(h_1,h_2,\ldots,h_j \| x_1,y_1;x_2,y_2;\ldots;x_j,y_j\right)$ defined by

$$p_j\left(h_1,h_2,\ldots,h_j \| x_1,y_1;x_2,y_2;\ldots;x_j,y_j\right)\,dh_1 dh_2 \ldots dh_j = \frac{n\left(h_1,x_1,y_1;h_2,x_2,y_2;\ldots;h_j,x_j,y_j\right)}{N} \tag{2}$$

where $j = 1, 2, 3, \ldots, \infty$, and $n\left(h_1,x_1,y_1;h_2,x_2,y_2;\ldots;h_j,x_j,y_j\right)$ is the number of functions in the ensemble which take on the specific set of values h_k at the points $\left(x_k,y_k\right)$ within the arbitrarily small tolerances dh_k, and where k takes on the values $1, 2, \ldots, j$.

With these notions, physical experiments such as the following can be analyzed. Suppose that some physical quantity, say $f(h,x,y)$, which is a function of height and position on a randomly rough surface, must be measured. Further, suppose that a collection of surfaces which have been produced by the same physical process is available for study, each surface differing in the detailed height variations upon it. The fact that the surfaces were produced by the same process ensures that these surfaces are sufficiently alike that it is sensible to perform averages over results obtained from each individual surface. This similarity is summarized in the notion that the sample surfaces all belong to a single statistical ensemble. If measurements of the quantity f are made at the corresponding points (x,y) on each surface and the results are averaged to obtain a result $\bar{f}(x,y)$, there is reason to identify $\bar{f}(x,y)$ with the quantity

$$\langle f(h,x,y) \rangle = \int_h f(h,x,y)\ p(h;x,y)\ dh \tag{3}$$

which is the expectation value of $f(h,x,y)$ at the point (x,y). The probability density $p(h;x,y)$ is that defined in equation (1) in terms of an idealized ensemble of sample surfaces. Note that $\langle f(h,x,y) \rangle$ does not necessarily agree with any particular measurement at the point (x,y) on any particular experimental surface. A logical second set of experimental measurements can be performed by finding the set of values $f(h,x,y)$ at a large number of points (x,y) on a single experimental surface, say the kth one, and subsequently averaging these results to obtain the quantity $\bar{f}_k$. Certain further stipulations about the character of the random process allow the identification of $\bar{f}_k$ with a quantity calculated from the probability density. These stipulations are summarized in the notion that the random process is ergodic (ref. 2). An example of an ergodic process is the stationary Gaussian random process (with an integrable autocorrelation function) for which all the joint probability densities of equation (2) are independent of the location of the origin of the coordinates x and y. The second-order joint probability density for this process has the exponential form[1] indicated subsequently. When the random process is ergodic, one has

$$\bar{f}_k = \langle f \rangle = \int_h f(h,x,y)\ p(h;x,y)\ dh \tag{4}$$

Although the physical quantity $f(h,x,y)$ may depend on position, the ergodic stipulation is such that the averages over the random process obliterate this dependence. Thus, the expectation value $\langle f \rangle$ is independent of position, and $\bar{f}_k$ or $\langle f \rangle$ is a global property of the particular sample surface and does not necessarily correspond to any value $f(h,x,y)$ measured at a particular point on the surface. A measure of the degree to which a single measurement at a single point may differ from $\bar{f}_k$ is the variance σ_f of the quantity $f(h,x,y)$ which is given by

---

[1]Virtually all the work done to date pertaining to rough-surface scattering theory has assumed that the random processes involved are of this kind.

$$\sigma_f^2 = \langle f^2 \rangle - \langle f \rangle^2 \tag{5}$$

These statistical ideas enjoy successful application in many areas of theoretical physics, but their application to the problem of radar scattering from planetary surfaces is not without conceptual difficulties. The goal of many of the planetary scattering applications (e.g., refs. 3 and 4) has been to study as small an area of the planetary surface as possible within the resolutional capabilities of the radar system, with the quantity $\langle f \rangle$ being related to the electromagnetic power backscattered by the area. In order for such an identification to be reliable, it is necessary first of all that the surface topography be describable as a sample function of a stationary ergodic random process. Unfortunately, terrestrial experience shows that natural terrain, in contrast to the ever-changing surface of the oceans, is fixed and over wide areas can assume large-scale features that obviously are not random in character. The fact that a measured quantity, such as backscattered power, may fluctuate from point to point on such a surface is not evidence of randomness. An example which amply illustrates this is the rapid fluctuation with aspect angle of the radar cross section of an aircraft, a target whose every contour represents careful design rather than randomness. Thus, the basic assumption that a planetary surface is describable by a stationary ergodic random process is open to serious question, and certain contrary evidence is in fact in existence. For example, Hughes (ref. 5) has emphasized that a Gaussian height distribution is not appropriate for the Moon, and, as is discussed in the section "Experimental Results for Radar Scattering," it has been shown (ref. 6) that certain lunar radar studies have led to absurd results for the surface correlation length, a crucial parameter in the statistical description of any surface.

An additional weakness in the statistical analyses surrounding planetary radar work is that apparently no one has calculated the variance of the scattered power (eq. (5)) in order to obtain an idea of the theoretical error associated with the reported results.

The next section outlines the electromagnetic theory into which these statistical ideas are inserted to yield the overall planetary scattering theory.

ELECTROMAGNETIC SCATTERING APPROXIMATIONS

If S represents a closed surface containing all sources of the electromagnetic field, then it can be shown (ref. 7) that the electric field $\vec{E}_P$ at a point P exterior to S is given by

$$\vec{E}_P = \frac{1}{4\pi} \oint_S \left[-j\omega\mu(\hat{n} \times \vec{H})\psi + (\hat{n} \times \vec{E}) \times \nabla\psi + (\hat{n} \cdot \vec{E})\nabla\psi \right] dS \tag{6}$$

where $\vec{E}$ and $\vec{H}$ are the electric and magnetic field vectors on the surface and $\hat{n}$ is the outward unit vector normal to S. Green's function $\psi = \exp(-j\vec{k} \cdot \vec{r})/r$, where r is the distance from a given point on the surface to the field point P. All fields are assumed to have an exponential time dependence $\exp(j\omega\tau)$. Equation (6) can be transformed (ref. 7) into an alternative form

$$\vec{E}_P = -\frac{1}{4\pi} \oint_S \left(\psi \frac{\partial \vec{E}}{\partial n} - \vec{E} \frac{\partial \psi}{\partial n} \right) dS \tag{7}$$

Equations (6) and (7) are the mathematical representation of the Huygens-Fresnel principle, which states that the field at a given point is produced by the sum of the fields of individual wavelets propagating from each point on the preceding wave front. Equation (7) can be written separately for each component of $\vec{E}$, and the resulting scalar equation is the basis for much of the work in the field of physical optics.

In order to apply equation (6) or (7) to rough-surface scattering, it is necessary to identify the closed surface S and to relate the fields $\vec{E}$ and $\vec{H}$ on S to their sources. Usually, S is identified with the actual scattering surface, and the source of the fields $\vec{E}$ and $\vec{H}$ is taken to be an isolated point, exterior to S, but placed so that no energy from the source can reach P without being scattered by S. Even if the shape of the scattering surface is known precisely, the determination of $\vec{E}$ and $\vec{H}$ on the surface in the presence of electromagnetic radiation from a distant point source is a formidable problem. The problem is greatly simplified if the scattering surface is a perfect conductor whose surface undulations are gently varying on a lateral scale of very many wavelengths. In such a circumstance the region in the neighborhood of any given point has the characteristics of a very large, nearly planar surface. Provided that there are no paths by which energy reflected from a given point on the surface can reach another point on the surface, the electromagnetic fields at the given point are approximately those which result from reflection of a plane electromagnetic wave by a perfectly conducting plane tangent to the surface at the given point. When this approximation, called the tangent-plane approximation, is incorporated into equation (6) or (7), it forms the basis for most so-called physical-optics methods of surface scattering study.[2] For a surface satisfying the assumed conditions, the $\vec{E}$ and $\vec{H}$ fields in this approximation are easily evaluated from the incident field values and the boundary conditions for a perfect conductor. Thus

[2]Rea, Hetherington, and Mifflin (ref. 8) have even asserted that the tangent-plane approximation is universally valid for perfectly conducting surfaces whatever their shape. However, a surface shape allowing a previously reflected ray as well as the incident plane wave to reach a given point will alter the total incident field and thus invalidate the approximation.

$$\left.\begin{aligned}
\vec{E} \cdot \hat{n} &= 2\vec{E}_i \cdot \hat{n} \\
\vec{E} \times \hat{n} &= 0 \\
\vec{H} \cdot \hat{n} &= 0 \\
\vec{H} \times \hat{n} &= 2\vec{H}_i \times \hat{n}
\end{aligned}\right\} \tag{8}$$

where $\vec{E}_i$ and $\vec{H}_i$ are the components of the incident plane wave.

The physical-optics method with incorporation of the tangent-plane approximation is thus capable of describing the scattered electric and magnetic fields of a relatively smooth, perfectly conducting surface in a way that retains the coherence effects due to phase interference among field amplitudes scattered from different points on the surface. Under appropriate conditions[3] which are rigorously discussed in references 7 and 9, the physical-optics formalism reduces to that of geometrical optics. Geometrical-optics propagation is characterized by the fact that the wave amplitude and phase at a given point on the wave front are related uniquely to those points and only those points on preceding wave fronts which lie on the ray trajectory traced by the Poynting vector passing through the given point. The method of stationary phase, when applied to a wave front satisfying the conditions of geometrical-optics propagation, then leads to the results of ray optics (ref. 7) for which all diffraction phenomena are excluded.

A theory to describe scattering from rough, natural surfaces could, on the basis of the present discussion, follow one of four procedures. First, equation (6) or (7) could be used, so that the results conform to exact methods of electromagnetic theory. For rough surfaces, this approach is entirely intractable.

In the second approach the tangent-plane approximation for evaluation of the total surface fields could be applied to equation (6) or (7) so that the results contain the effects of point-to-point coherence. Unfortunately, most natural surfaces are dielectric rather than perfectly conductive. Even for plane waves incident upon locally plane regions, the reflection coefficients are complicated functions of the angle of incidence and are different for the two components of polarization. Furthermore, since a part of the incident wave energy penetrates a dielectric material, one must stipulate that none of this penetrating energy can be scattered from underlying strata to emerge subsequently from the surface.

The third approach would involve incorporation of the tangent-plane approximation, required assumptions for valid application to dielectric surfaces (as in the second

[3]These conditions are satisfied in the far-field zone of a source or scattering object for which case the field variations both along and transverse to the wave front are small on a scale of many wavelengths.

approach), and in addition, application of a method of stationary phase, so that the results take on the character of a geometrical-optics approximation. This is the approach taken, for example, in references 10 and 11 and applied in references 3 and 4. Kodis in reference 12 applies this approach in a consistent way for the special case of a perfectly conducting surface and effects the transition from equation (6) through the classical method of stationary phase to the formalism of pure ray-optics scattering.

The fourth approach would invoke ray optics from the outset and would treat the scattering surface as a collection of mirrorlike facets from which the incident rays are specularly reflected according to the usual laws of elementary geometrical optics. This approach is followed in the work of references 8, 13, and 14.

Each approach requires that the rough surface be treated in a suitable statistical manner. Details of the statistics vary among the approaches and give rise to certain difficulties in reconciling the results of the several approaches. First, that approach containing the least number of theoretically objectionable features, the method of Kodis (ref. 12) and certain extensions of his work by Barrick (ref. 15), are examined.

STATIONARY-PHASE APPROACH OF KODIS

Kodis (ref. 12) begins his work on perfectly conducting, rough surfaces by applying the tangent-plane approximation to an expression similar to equation (6) and by evaluating the surface fields by means of the equivalent of equation (8). He considers the incident plane wave to have unit amplitude when it reaches the surface, time dependence $j\omega\tau$, propagation vector $\vec{k}_i$, and electric field polarization direction $\hat{e}$. He finds for the field $\vec{E}^{sc}$ scattered in the direction of $\vec{k}_s$ the expression

$$\vec{E}^{sc}(\vec{k}_i,\vec{k}_s) = \frac{jk}{2\pi r} \exp (jkr) \int \vec{F}(\hat{n},\vec{k}_i,\vec{k}_s) \exp \left[j(\vec{k}_i - \vec{k}_s) \cdot \vec{r}'\right] dS' \tag{9}$$

where the integration is over the scattering surface, $\vec{r}'$ is the vector position of a point on the surface, $r = |\vec{r}|$ represents the distance from the origin to the field point, and

$$\vec{F}(\hat{n},\vec{k}_i,\vec{k}_s) = \left(\overleftrightarrow{I} - \vec{k}_i\vec{k}_s\right) \cdot \left(\vec{k}_i \times \hat{e} \times \hat{n}\right)$$

with $\overleftrightarrow{I}$ being the unit dyad. For backscatter, $\vec{k}_s = -\vec{k}_i$, and F reduces to $(\hat{n} \cdot \vec{k}_i)\hat{e}$. For sufficiently small wavelengths, the phase of the exponential term of the integrand in equation (9) varies very rapidly on a distance scale for which the remainder of the integrand varies only slightly. The principal part of the integral of equation (9) can thus be approximated by the classical method of stationary phase (refs. 7 and 16). Because of the rapid oscillation of the integrand, significant contributions to the integral can occur only at the end points of the interval or at those interior points at which the derivative of the exponential argument vanishes. The exponential argument can thus be represented by a Taylor's series in the neighborhood of these stationary points, and the slowly varying remainder of the integrand can be evaluated at the stationary points themselves and thus removed from the integrand. The integral then becomes a summation of contributions from each of the stationary points.

Kodis treats in detail the case of a one-dimensionally rough or corrugated surface with height in the z-direction given by $h(x)$ (ζ in Kodis' notation) and width L in the y-direction extending from $y = -L/2$ to $L/2$. He also gives the results of the straightforward extension to a two-dimensionally rough surface. In both cases the mean surface is assumed to be planar. He finds for the one-dimensional case

$$\vec{E}^{sc}\left(\vec{k}_i,\vec{k}_s\right) \propto \sum_{m=1}^{N} \frac{jk}{2\pi r} \frac{\vec{F}_m}{\hat{n}_m \cdot \hat{e}_z} \exp(jkr) \int_{S_m} \exp(jk\vec{q} \cdot \vec{r}\,')\, dy'\, dx'$$

$$= \sum_{m=1}^{N} \frac{jk}{2\pi r} \frac{\vec{F}_m}{\hat{n}_m \cdot \hat{e}_z} \exp(jkr) \int_{S_m} \exp jk\left[\vec{q} \cdot \vec{r}\,'_m + \frac{1}{2}(x' - x'_m)^2 \frac{d^2}{dx'^2}(\vec{q} \cdot \vec{r}\,')\bigg|_{x'=x'_m}\right] dy'\, dx' \tag{10}$$

where $\vec{q} = \vec{k}_i - \vec{k}_s$, dS' has been replaced by $dy'\, dx'/(\hat{n}_m \cdot \hat{e}_z)$, and the points x'_m are the points at which $\vec{q} \cdot \vec{r}$ is stationary with respect to x and y. Thus, at these points

$$\frac{dh(x')}{dx'}\bigg|_{x'=x'_m} = h'(x'_m) = -\frac{q_x}{q_z} \qquad q_y = 0 \tag{11}$$

Equation (11) implies that $\vec{q}$ is collinear with $\hat{n}_m$, the unit normal at the stationary-phase points x'_m. This is the usual condition for specular reflection. The integrals in equation (10) are performed over small neighborhoods S_m about each of the points r'_m and can be written as

$$
\left.
\begin{aligned}
I_m &= \exp\left(jk\vec{q}\cdot\vec{r}'_m\right)\int_{S_m} \exp jk\left[\frac{(x - x'_m)^2}{2} q_z\, h''(x'_m)\right] dy'\, dx' \\
&= L \exp\left(jk\vec{q}\cdot\vec{r}'_m\right)\int_{\Delta x_m} \exp jk\left[\frac{(x - x'_m)^2}{2} q_z\, h''(x'_m)\right] dx' \\
&= L \exp\left(jk\vec{q}\cdot\vec{r}'_m\right)\int_{\Delta u} \exp jk\left[\frac{u^2}{2} q_z\, h''(x'_m)\right] du
\end{aligned}
\right\} \qquad (12)
$$

where $u = x' - x'_m$, $\Delta u \doteq \Delta x_m$ is a small interval around x'_m, and

$$
h''(x'_m) = \frac{d^2}{dx'^2} h(x')\bigg|_{x'=x'_m}
$$

In the last integral in equation (12), the increasingly rapid oscillation of the integral with increasing u renders the integral insensitive to the exact interval Δu, so that the stationary-phase approximation allows Δu to be replaced by the interval $(-\infty,\infty)$. Then use of the formula from reference 16

$$
\int_{-\infty}^{\infty} \exp\left(jm^2u^2\right) du = \frac{\sqrt{\pi}}{m} \exp\left(j\,\frac{\pi}{4}\right)
$$

reduces equation (12) to

$$
I_m = L \exp\left(j\,\frac{\pi}{4}\right) \exp\left(jk\vec{q}\cdot\vec{r}'_m\right)\left[\frac{2\pi}{kq_z\, h''(x'_m)}\right]^{1/2} \qquad (13)
$$

Substituting equation (13) into equation (10) gives for the absolute square of the scattered electric field the value

$$
\left|\vec{E}^{sc}\right|^2 = \vec{E}^{sc}\cdot\vec{E}^{sc*} \propto \frac{L^2}{2\pi r^2}\frac{kq^2}{q_z^{\,3}}\left|\vec{F}\right|^2 \sum_{m=1}^{N}\sum_{p=1}^{N}\left[h''(x'_m)\, h''(x'_p)\right]^{-1/2} \exp\left[jk\vec{q}\cdot\left(\vec{r}'_m - \vec{r}'_p\right)\right]
$$

where use has been made of the fact that at the specular points $\hat{n}_m = -\frac{\vec{q}}{q}$. The total radar cross section Σ is then related to the scattered field by the expression

$$\Sigma(\vec{k}_i,\vec{k}_s) = 4\pi r^2 \left|\vec{E}^{sc}\right|^2$$

and the cross section σ per unit width L of the corrugated surface is then

$$\sigma(\vec{k}_i,\vec{k}_s) = L\,\frac{2kq^2}{q_z^{\,3}}|\vec{F}|^2 \left\langle \sum_{m=1}^{N}\sum_{p=1}^{N} \left[h''(x'_m)\,h''(x'_p)\right]^{-1/2} \exp\left[jk\vec{q}\cdot\left(\vec{r}\,'_m - \vec{r}\,'_p\right)\right]\right\rangle \tag{14}$$

where the ensemble average has been inserted and is taken over the physically possible realizations of random surfaces. The number of specular points N, the positions $\vec{r}\,'_m$ of the specular points, and the quantities $h''(x'_m)$ evaluated at those points differ for each realization.

It is straightforward to show that for the surface under discussion,

$$\left[h''(x'_m)\right]^{-1} = \rho_m\left[1 + h'^2(x'_m)\right]^{-3/2} \tag{15}$$

where ρ_m is the radius of curvature at the point x'_m. Also, use of equation (11) gives

$$1 + h'^2(x'_m) = \left(\frac{q}{q_z}\right)^2 \tag{16}$$

Substitution of equation (16) into equation (15) and of that result into equation (14) gives

$$\sigma(\vec{k}_i,\vec{k}_s) = L\,\frac{2k}{q}|\vec{F}|^2\left\{\left\langle \sum_{m=1}^{N}\rho_m\right\rangle + \left\langle \sum_{m=1}^{N}\sum_{\substack{p=1\\p\neq m}}^{N}\left(\rho_m\rho_p\right)^{1/2}\exp\left[jk\vec{q}\cdot\left(\vec{r}\,'_m - \vec{r}\,'_p\right)\right]\right\rangle\right\} \tag{17}$$

The first ensemble average on the right side of equation (17) can be written

$$\left\langle \sum_{m=1}^{N}\rho_m\right\rangle = \left\langle N\bar{\rho}_m\right\rangle \tag{18}$$

where $\bar{\rho}_m$ is the average radius of curvature at the N specular points of a single realization of the ensemble of rough surfaces. Kodis makes the assumption that although the number N of specular points on a given surface realization may vary from one realization to another in the ensemble, the average radius of curvature at the specular points $\bar{\rho}_m$ is the same for all realizations of the ensemble. Since the angle of incidence of the radiation, and hence the locations of the specular points, is arbitrary, this assumption is equivalent to assuming that the random process generating the radius of curvature at any point on the surface is an ergodic process. If this assumption is true, then

$$\left\langle N\bar{\rho}_m \right\rangle = \langle N \rangle \bar{\rho}_m \tag{19}$$

An equation analogous to equation (19) can be written for the second ensemble average of equation (17) as

$$\left\langle \sum_{m=1}^{N} \sum_{\substack{p=1 \\ p \neq m}}^{N} \left(\rho_m \rho_p\right)^{1/2} \exp\left[jk\vec{q} \cdot \left(\vec{r}\,'_m - \vec{r}\,'_p\right)\right] \right\rangle = \langle N(N-1) \rangle \overline{\left(\rho_m \rho_p\right)^{1/2} \exp\left[jk\vec{q} \cdot \left(\vec{r}\,'_m - \vec{r}\,'_p\right)\right]} \qquad (m \neq p) \tag{20}$$

Kodis uses equations (18), (19), and (20) in equation (17). The physically plausible assumption that the location of a specular point and the radius of curvature at that point are independent random variables over each surface realization allows Kodis to write

$$\overline{\left(\rho_m \rho_p\right)^{1/2} \exp j\left(\phi_m - \phi_p\right)} = \overline{\left(\rho_m \rho_p\right)^{1/2}}\; \overline{\exp j\left(\phi_m - \phi_p\right)} \qquad (m \neq p) \tag{21}$$

where $\phi_t = k\vec{q} \cdot \vec{r}\,'_t$ with $t = m$ and p. For a large number of specular points N and with $\phi_m - \phi_p$ distributed uniformly over all angles (as is the case for any given surface if the wavelength is short enough), $\overline{\exp j\left(\phi_m - \phi_p\right)}$ on the right side of equation (21) tends to zero. Thus, equation (17) reduces to

$$\sigma(\vec{k}_i, \vec{k}_s) = L \frac{2k}{q} \left|\vec{F}\right|^2 \langle N \rangle \bar{\rho}_m \tag{22}$$

For the backscattering case $\vec{k}_s = -\vec{k}_i$, $\left|\vec{F}\right|^2 = 1$, and $q = 2$. Kodis extends his calculations from one-dimensionally rough to two-dimensionally rough surfaces and obtains for the total backscattering radar cross section

$$\Sigma(\vec{k}_i,\vec{k}_s) = \pi\overline{\rho_a\rho_b}\langle N\rangle \tag{23}$$

where $\langle N\rangle$ is the ensemble average of the total number of specular points and $\overline{\rho_a\rho_b}$ is the average value over a surface realization of the product of the two principal radii of curvature at each specular point.

Equation (23) is analogous to a result given in reference 7 for scattering from a smoothly curving object in the stationary-phase or geometrical-optics approximation. It is also analogous to the geometrical-optics cross section of a smoothly curving body, such as an ellipsoid or sphere, given in reference 17. The theory leading to equation (23) predicts no depolarization of the reflected wave, so that if depolarization of the signal is actually observed, there is reason to believe that the underlying assumptions for validity of the theory are invalid. Depolarization in actual planetary radar reflections is discussed further in a subsequent section.

BARRICK'S EXTENSION OF KODIS' RESULTS

In order to complete the calculation of the radar cross section (eq. (23)) it is necessary to evaluate the quantities $\langle N\rangle$ and $\overline{\rho_a\rho_b}$. Some work of Barrick (ref. 15) that attacks this problem directly is discussed here. Barrick notes that the problem of counting the number of points on a corrugated surface at which the slope attains a specified value (i.e., the number of specular points on the surface) is formally similar to the problem of counting the average number of zero crossings per unit time of a random function of time with specified statistical properties. This problem is solved by S. O. Rice (ref. 1), and its solution is adapted by Barrick to rough surfaces and extended to a two-dimensionally rough surface. For the two-dimensional surface, Barrick finds for the expected number $n_A = \langle N\rangle/\text{Total surface area}$ of specular points per unit area

$$n_A = p\left(h_{x\,sp},h_{y\,sp}\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|h_{xx}h_{yy} - h_{xy}^2\right| p\left(h_{xx},h_{xy},h_{yy}\middle|h_{x\,sp},h_{y\,sp}\right)\,dh_{xx}\,dh_{xy}\,dh_{yy} \tag{24}$$

where

$$h_{x\,sp} = \left.\frac{\partial h}{\partial x}\right|_{x=x_m'} \qquad h_{y\,sp} = \left.\frac{\partial h}{\partial y}\right|_{y=y_m'}$$

and where $p\left(h_{x\,sp},h_{y\,sp}\right)$ is the probability density for the surface slopes evaluated at the specular values and $p\left(h_{xx},h_{xy},h_{yy}\middle|h_{x\,sp},h_{y\,sp}\right)$ is the conditional probability density for the three second derivatives of the surface height given that the first derivatives have their specular values. The double subscripted quantities h_{xx}, h_{xy}, and h_{yy} correspond to the respective second partial derivatives of the surface height $h(x,y)$. The functional notation for h has been dropped for convenience.

Barrick then develops an expression for $\overline{\rho_a \rho_b}$, or $\langle |r_1 r_2| \rangle$ in his notation. (This notation implies that he is replacing the average over a single surface with an ensemble average, a permissible substitution under a suitable ergodic hypothesis.) In his development, he adopts the rather unsatisfactory tactic of replacing $\left\langle \frac{1}{|J|} \right\rangle$ with $\frac{1}{\langle |J| \rangle}$ where $|J| = \left| h_{xx} h_{yy} - h_{xy}^2 \right|$. Barrick asserts that no quantitative measure of the error involved in this replacement is available, but that for large values of $|J|$, the error should not be excessive. It is shown here by an alternative method that Barrick's final result for $\langle |r_1 r_2| \rangle$ is in fact exact, so that his unsatisfactory assumptions regarding $\left\langle \frac{1}{J} \right\rangle$ are not essential.

From equation (24), the ensemble-averaged number dn_A of specular points per unit area such that at the specular points the second derivatives of the surface have values within dh_{xx} of h_{xx}, within dh_{xy} of h_{xy}, and within dh_{yy} of h_{yy}, is

$$dn_A = p\left(h_{x\,sp}, h_{y\,sp}\right) \left| h_{xx} h_{yy} - h_{xy}^2 \right| p\left(h_{xx}, h_{xy}, h_{yy} \middle| h_{x\,sp}, h_{y\,sp}\right) dh_{xx}\, dh_{xy}\, dh_{yy} \tag{25}$$

Assume that dn_A also represents the number of specular points with the above prescribed values of h_{xx}, h_{xy}, and h_{yy} for a single surface realization from the ensemble. This will be a plausible ergodic assumption if the number dn_A of such specular points is large. At these specular points the quantity $|r_1 r_2|$ can be written

$$|r_1 r_2| = \frac{\left(1 + h_{x\,sp}^2 + h_{y\,sp}^2\right)^2}{\left| h_{xx} h_{yy} - h_{xy}^2 \right|} = \frac{\sec^4 \gamma}{\left| h_{xx} h_{yy} - h_{xy}^2 \right|} \tag{26}$$

where γ is the angle between the mean surface normal and the local surface normal. To find the average of $|r_1 r_2|$ over a single surface, it is necessary merely to add up the quantities represented by equation (26), each quantity being multiplied by the number of times it occurs (from eq. (25)), and then to divide by the total number (from eq. (24)). Thus

$$\overline{|r_1 r_2|} = \frac{\int |r_1 r_2|\, dn_A}{n_A} = \frac{\sec^4 \gamma \iiint p\left(h_{xx}, h_{xy}, h_{yy} \middle| h_{x\,sp}, h_{y\,sp}\right) dh_{xx}\, dh_{xy}\, dh_{yy}}{\iiint \left| h_{xx} h_{yy} - h_{xy}^2 \right| p\left(h_{xx}, h_{xy}, h_{yy} \middle| h_{x\,sp}, h_{y\,sp}\right) dh_{xx}\, dh_{xy}\, dh_{yy}} \tag{27}$$

Since the triple integral in the numerator of the right member of equation (27) is unity, equation (27) is identical with the final result of Barrick. Dividing equation (23) by the total surface area and substituting equations (24) and (27) give for the radar cross section per unit area

$$\sigma = \pi \sec^4 \gamma \, p\left(h_{x\,sp}, h_{y\,sp}\right) = \pi \sec^4 \gamma \, p(\gamma) \tag{28}$$

where $p(\gamma) \equiv p\left(h_{x\,sp}, h_{y\,sp}\right)$. At each specular point contributing to the result in equation (28), the direction of incidence is parallel to the local surface normal. It is plausible therefore that equation (28) can be extended from a perfectly conducting surface to a dielectric surface by insertion of the reflection coefficient for normal incidence on a dielectric plane. Note that such a procedure is justifiable only after the reduction of the problem to that of a summation of scattering contributions from the set of specular points. An attempt to insert into equation (9) a dielectric-surface reflection coefficient, which is a complicated function of the local angle of incidence at each point on the surface, would have resulted in additional complicated factors in the integrals over S_m of equation (10). These factors would have altered the subsequent development substantially and would have led to problems of interpretation similar to those discussed in the section "Correlation-Function Method."

Barrick quite plausibly inserts the dielectric reflection coefficient into equation (28) to get

$$\sigma_{\xi\eta} = \pi \sec^4 \gamma \, p(\gamma) \left|R_{\xi\eta}\right|^2 \tag{29}$$

as the backscattering cross section per unit area for a dielectric body, where ξ and η refer to the polarization states (horizontal or vertical) of the incident and scattered waves, respectively, and the absolute values of the Fresnel reflection coefficients for normal incidence of a plane wave on a plane surface are denoted by $\left|R_{\xi\eta}\right|$. These coefficients are such that

$$\left.\begin{aligned} \left|R_{hh}\right|^2 &= \left|R_{vv}\right|^2 = \left(\frac{\epsilon - \sqrt{\epsilon\mu}}{\epsilon + \sqrt{\epsilon\mu}}\right)^2 \\ \left|R_{hv}\right|^2 &= \left|R_{vh}\right|^2 = 0 \end{aligned}\right\} \tag{30}$$

where h and v refer to horizontal or vertical polarization. Equations (29) and (30) show that for a dielectric as well as a perfect conductor, there is no depolarization of the radar signal upon reflection in this approximation. If depolarization is actually observed, these formulas must be used with extreme caution.

Equation (28) – which agrees with the results of Muhleman (ref. 13), who based his work from the outset on a pure geometrical-optics (ray-optics) method – has thus been traced back to first principles, so that its domain of validity is reasonably clear. Although equation (28) or (29) indicates that the angular scattering dependence is the same as the distribution of surface slopes, this may not be true for arbitrary slope distributions. In particular, the slope distribution must fall off with γ sufficiently rapidly so that neither surface self-shadowing nor multiple scattering is important. Surface self-shadowing would tend to reduce the number of contributing specular points per unit area and would be important for large angles of incidence or large surface slopes. Simple geometrical considerations show that multiple scattering could be important if there are a significant number of surface slope angles α such that $\alpha \geqq \frac{\pi}{4} - \frac{\gamma}{2}$. For slopes with such angles, incident rays can reach a given specular point by intermediate reflection from nonspecular points located on those slopes.

An alternate route to equation (29) has occupied a principal place in the literature of rough-surface scattering. That route is through the so-called correlation-function method, whose intermediate results have provided many of the formulas used in planetary radar data reduction methods.

CORRELATION-FUNCTION METHOD

The theoretical work which has had the most impact on radar studies of planetary surface properties has probably been that of Hagfors (refs. 10, 11, and 14). This work, which falls into the third category discussed previously in the section "Electromagnetic Scattering Approximation," unfortunately contains many contradictory features, and applications of parts of it to some lunar and planetary radar data analyses (refs. 3, 4, and 18) have led to unreasonable results. Some of these results are discussed in the following section. The purely theoretical aspects of Hagfors' work are discussed in this section.

In reference 10, Hagfors attacks the problem of plane-wave scattering from a one-dimensionally rough (corrugated) surface with random height deviations from a mean plane. He extends the final result to the two-dimensionally rough, mean-planar surface. His work for this special surface is presented in detail, since it will help to clarify his work (ref. 11) on a rough spherical surface which is the basis for certain data analysis techniques. In Hagfors' development, the height of the rough surface in the z-direction h is independent of the y-direction which extends from $-\infty$ to $+\infty$. The slope of the surface $\frac{dh}{dx}$ in the x-direction is denoted by t, so that the slope angle α is given by $\alpha = \tan^{-1} t$. The angle of incidence of the impinging wave with respect to the mean surface is denoted by φ. The local angle of incidence ψ is then given by $\psi = \varphi - \alpha$. As before, k is the wave number $2\pi/\lambda$. The scalar value of an electric field component of the incident wave at point x on the surface is then of the form

$$E = E_0 \exp\left[-jk(x \sin\varphi - h \cos\varphi)\right] \tag{31}$$

By means of the tangent-plane approximation with the necessary stipulations for application to dielectric bodies, the scattered electric field amplitude on the (dielectric) surface is given by $E\,Q(\psi)$, where $Q(\psi)$ is the plane-wave reflection coefficient. After straightforward manipulation of the scalar form of equation (7), Hagfors writes for the backscattered field amplitude E_R at the radar antenna

$$E_R = \frac{E_0 k}{2\pi j R} \exp(-jkR) \int Q(\psi) \cos\psi \exp\left[-2jk(x \sin\varphi - h\cos\varphi)\right] ds \tag{32}$$

where R is the distance from the origin to the radar antenna and ds is the line element along the rough surface. Introducing $dx = \cos\alpha\, ds$ and eliminating ψ in favor of t and φ, Hagfors writes

$$E_R = K \int Q(t)(\cos\varphi + t \sin\varphi) \exp\left[-2jk(x\sin\varphi - h\cos\varphi)\right] dx \tag{33}$$

where K represents the constant factor outside the integral in equation (32). The power received at the radar antenna is then proportional to $E_R E_R^*$, and the expectation value of this quantity over a suitable ensemble of surfaces describing the random height h is proportional to the ensemble-averaged received power. Thus

$$\langle E_R E_R^* \rangle = \left\langle |E_R|^2 \right\rangle = |K|^2 \iint dx_1\, dx_2 \left\{ \left\langle Q(t_1)\, Q^*(t_2)\, (\cos\varphi + t_1 \sin\varphi)(\cos\varphi + t_2 \sin\varphi) \right. \right.$$

$$\left. \left. \exp\left[2jk(h_1 - h_2)\cos\varphi\right] \right\rangle \exp\left[-2jk(x_1 - x_2)\sin\varphi\right] \right\} \tag{34}$$

where the subscripts 1 and 2 denote dummy variables of integration. Introduction of the new variables $\Delta h = h_1 - h_2$ and $\Delta x = x_1 - x_2$ gives

$$\langle |E|^2 \rangle = |K|^2 \int dx_1 \int d(\Delta x) \Big[\Big\langle Q(t_1)\, Q^*(t_2)\, (\cos\varphi + t_1 \sin\varphi)(\cos\varphi + t_2 \sin\varphi)$$

$$\exp(2jk\, \Delta h \cos\varphi) \Big\rangle \exp(-2jk\, \Delta x \sin\varphi) \Big] \tag{35}$$

In order to evaluate the expectation value within the integral in equation (35), a joint probability density function corresponding to equation (2) must be introduced for the height difference Δh and the slopes t_1 and t_2. Hagfors assumes this joint probability density to be that of a Gaussian process and writes explicitly

$$p(\Delta h, t_1 t_2) = (2\pi)^{-3/2} (\det A)^{-1/2} \exp\left\{ -\frac{1}{2(\det A)} \Big[M_{11} (\Delta h)^2 + M_{22} t_1{}^2 + M_{33} t_2{}^2 \right.$$

$$\left. + 2M_{12}\, \Delta h (t_1 - t_2) + 2M_{23} t_1 t_2 \Big] \right\} \tag{36}$$

where the M_{ij} are the cofactors of the symmetric matrix

$$A = h_{rms}^2 \begin{bmatrix} 2\left[1 - \rho(\Delta x)\right] & -\dfrac{\partial \rho(\Delta x)}{\partial(\Delta x)} & \dfrac{\partial \rho(\Delta x)}{\partial(\Delta x)} \\[2ex] -\dfrac{\partial \rho(\Delta x)}{\partial(\Delta x)} & -\left.\dfrac{\partial^2 \rho(\Delta x)}{\partial(\Delta x)^2}\right|_{\Delta x = 0} & -\dfrac{\partial^2 \rho(\Delta x)}{\partial(\Delta x)^2} \\[2ex] \dfrac{\partial \rho(\Delta x)}{\partial(\Delta x)} & -\dfrac{\partial^2 \rho(\Delta x)}{\partial(\Delta x)^2} & -\left.\dfrac{\partial^2 \rho(\Delta x)}{\partial(\Delta x)^2}\right|_{\Delta x = 0} \end{bmatrix} \tag{37}$$

and where $\det A$ is the determinant of A. The quantity $\rho(\Delta x)$ is the autocorrelation function of the random surface height and h_{rms}^2 is the root-mean-square height. Reference 1 or 2 cover the theory surrounding equations (36) and (37). To use this formalism, it would be necessary to identify the expectation value in the integral of equation (35) with the quantity $\langle f \rangle$ of equation (3) and to substitute equation (36) into equation (3), where the single integral had been replaced by a triple integral over Δh, t_1, and t_2.

At this point a hard fact comes to the surface, namely, that $Q(t)$ is a very complicated function of t which has a different functional form for each of the two possible local polarizations of the incident wave. Since the local polarization changes with position according to the detailed variation of the rough surface, it is completely unclear how to proceed from this point. Hagfors' approach is to seek a method for effectively removing the t dependence of Q. His argument is as follows: Suppose that certain slope values $\bar{t}_1$ and $\bar{t}_2$ can be found such that the quantity $F(t_1,t_2)$ defined by

$$F(t_1,t_2) = Q(t_1)\, Q^*(t_2)\, (\cos\varphi + t_1 \sin\varphi)(\cos\varphi + t_2 \sin\varphi) \tag{38}$$

can be expanded in a Taylor's series about the points $\bar{t}_1$ and $\bar{t}_2$ with the first derivative terms vanishing identically. This condition is met if

$$\int dx_1 \int d(\Delta x)\, \exp(-2jk\,\Delta x \sin\varphi) \left[\iiint d(\Delta h)\, dt_i\, dt_j \left.\frac{\partial F}{\partial t_i}\right|_{t_i=\bar{t}_i} (t_i - \bar{t}_i)\, \exp(2jk\,\Delta h \cos\varphi)\, p(\Delta h, t_i, t_j) \right] = 0 \tag{39}$$

where $i,j = 1,2$ with $i \neq j$ and p is defined by equation (36).

Hagfors asserts that equation (39) yields

$$\bar{t}_1 = \bar{t}_2 = \tan\varphi \tag{40}$$

If equation (40) holds, and if the second and higher derivatives are arbitrarily dropped from the Taylor's series for F, then F reduces to

$$F(t_1,t_2) = F(\bar{t}_1,\bar{t}_2) = Q(\tan\varphi)\, Q^*(\tan\varphi)\, \cos^{-2}\varphi \tag{41}$$

The value $Q(\tan\varphi)$ then corresponds exactly to the Fresnel reflection coefficient for plane-wave normal incidence on a plane surface, which is independent of polarization. Thus, $Q(\tan\varphi)\, Q^*(\tan\varphi)$ is given by equation (30) (with μ set equal to unity). Use of equations (30), (40), (36), and (37) in equation (35) then gives

$$\left\langle |E_R|^2 \right\rangle = |K|^2 \left(\frac{\sqrt{\epsilon} - 1}{\sqrt{\epsilon} + 1}\right)^2 \frac{1}{\cos^2\varphi} \int_{-L/2}^{L/2} dx_1 \int_{-\infty}^{\infty} d(\Delta x)\, \exp(-2jk\,\Delta x \sin\varphi) \int dt_1 \int dt_2 \int d(\Delta h) \left[\exp(2jk\,\Delta h \cos\varphi)\, p(\Delta h, t_1, t_2)\right] \tag{42}$$

Equation (42) can now be integrated over t_1, t_2, and Δh to give

$$\left\langle \left| E_R \right|^2 \right\rangle = |K|^2 \left(\frac{\sqrt{\epsilon} - 1}{\sqrt{\epsilon} + 1} \right)^2 \frac{1}{\cos^2 \varphi} \int_{-L/2}^{L/2} dx_1 \int_{-\infty}^{\infty} d(\Delta x) \exp(-2jk \Delta x \sin \varphi)$$

$$\exp \left\{ -4k^2 h_{rms}^2 \cos^2 \varphi \left[1 - \rho(\Delta x) \right] \right\} \tag{43}$$

where L is the extent of the surface in the x-direction. Equation (43) can be extended to a two-dimensionally rough surface for which $\rho(\Delta x)$ can be identified with $\rho(\Delta r)$, where Δr is the separation distance between two points on the surface. The one-dimensional Fourier transform indicated by the integration over Δx in equation (43) can be replaced by a two-dimensional Fourier transform such that

$$\left\langle \left| E_R \right|^2 \right\rangle = |K|^2 \left(\frac{\sqrt{\epsilon} - 1}{\sqrt{\epsilon} + 1} \right)^2 \frac{1}{\cos^2 \varphi} \int_{-L/2}^{L/2} dx \int_{-L/2}^{L/2} dy \int_{0}^{\infty} d(\Delta r) \Delta r$$

$$\exp \left\{ -4k^2 h_{rms}^2 \cos^2 \varphi \left[1 - \rho(\Delta r) \right] \right\} J_0(2k \Delta r \sin \varphi) \tag{44}$$

What Hagfors' method actually represents is a form of stationary-phase approximation similar to Kodis' method but different in detail from this classical method. The previous discussion shows that the particular choice $\bar{t}_i = \tan \varphi$ has nothing to do with the ensemble average indicated in equations (34) or (35). If the ensemble-average brackets of equation (34) are moved outside the integrals, the two exponentials of the integrand may be combined to give $\frac{dh_1}{dx_1} = \frac{dh_2}{dx_2} = t_1 = t_2 = \tan \varphi$ as a point of stationary phase of the integrand. The method of stationary phase permits the remainder of the integrand to be evaluated at this point and removed from the integrals. Up to this point, the procedure is the same as that of Kodis. Instead of evaluating the remaining integrals only in the neighborhood of the points of stationary phase, however, Hagfors' method retains the entire integration range. When the ensemble-average brackets which were placed outside the integrals of equation (34) are moved back within the integrals over x_1 and Δx, the result is equation (42).

This view of equation (43) shows it to be a combination of the method of Kodis and of an attempt to retain the effects of point-to-point coherence of the scattering process. The coherence retention is not complete, however, since the reflection coefficient allows only for locally normal incidence. Therefore, coherence can be retained, at best, only among the isolated specular points, but the integral on Δx of equation (43) contains the phase variations of a continuum of points over the entire surface. The physical meaning of equation (43) is thus somewhat obscure and it is unclear whether it is capable of describing to any reliable extent the effects of surface correlation or coherence.

ANGULAR SCATTERING LAWS

In order to proceed from equation (44) to a practical result for real surfaces, a form for the autocorrelation function $\rho(\Delta r)$ must be assumed. For reasons of mathematical simplicity, attention in the literature has been devoted almost exclusively to two forms: the exponential correlation function

$$\rho(\Delta r) = \exp\left(-\frac{\Delta r}{\ell}\right) \tag{45}$$

and the Gaussian correlation function

$$\rho(\Delta r) = \exp\left[-\frac{(\Delta r)^2}{\ell^2}\right] \tag{46}$$

where ℓ is the correlation length of the statistical process representing the rough surface. Actually, equations (45) and (46) are almost never used as they are written. For large values of $kh_{rms} \cos \varphi$, the so-called deep-phase modulation case, the exponential factor containing $\rho(\Delta r)$ in equation (44) drops rapidly to zero with the slightest departure of $\rho(\Delta r)$ from unity. It is generally assumed that equations (45) and (46) can be approximated by the first two terms of a Taylor's series expansion about a point at or near the origin. A controversy has appeared in the literature over whether the point of expansion should be the origin or another point near it and whether an expansion of equation (45), which has no Taylor's expansion about the origin, can represent a sensible correlation function at all. This controversy is discussed in detail by Barrick in reference 6 and to some extent by Fung and Chan in reference 19.

If equation (45) is approximated by $\rho(\Delta r) = 1 - \Delta r/\ell$ and if this form is substituted into equation (44), the latter equation can be integrated exactly to give for the power reflected per unit surface area

$$\langle P_R(\varphi)\rangle \propto \left(\frac{\sqrt{\epsilon} - 1}{\sqrt{\epsilon} + 1}\right)^2 \left[\cos^4 \varphi + \frac{1}{(2kh_{rms})^2}\left(\frac{\ell}{h_{rms}}\right)^2 \sin^2 \varphi\right]^{-3/2} \tag{47}$$

If equation (46) is approximated by $\rho(\Delta r) = 1 - (\Delta r)^2/\ell^2$, the analogous result is

$$\langle P_R(\varphi)\rangle \propto \left(\frac{\sqrt{\epsilon} - 1}{\sqrt{\epsilon} + 1}\right)^2 \frac{1}{\cos^4 \varphi} \exp\left(-\frac{\ell^2}{4h_{rms}^2} \tan^2 \varphi\right) \tag{48}$$

Hagfors (ref. 10) obtains these results for a mean-planar surface and in reference 11 he ostensibly obtains the same results for the case for which the mean surface is sperical as is needed in planetary studies. The development of reference 11 is somewhat unsatisfactory, however, since Hagfors is forced to drop the distinction between the subscripted dummy variables in the analog of equation (34). Furthermore, he makes some approximations in his integrand functions that are valid only if $\varphi = 0$. The effect of these arbitrary steps is to suppress the spherical nature of the surface. Notwithstanding these incongruities, he asserts that equations (47) and (48) represent the expected variation with incidence angle of the power backscattered from a spherical planetary surface, and these expressions are used in actual data reduction of radar scattering from Mars in references 3 and 4.

Note that the use of either equation (45) or (46) in equation (44) produces an integral over Δr that does not exist because of the infinite upper limit of that integral. It is only the two-term approximations which lead to a finite result. Fung and Chan (ref. 19) obtained numerical results for the integral in equation (44) by using an unspecified upper cutoff D for that integral. Although they did not comment on the dependence of their result on the value of D, it can be readily shown that their numerical results do, in fact, depend on D. Their plotted curves result from a specifically chosen value of D that bears no apparent relationship to any other physical parameter of the problem, and their results would have diverged as $\sqrt{D}$ for larger values of D.

Extensive difficulties with the exponential correlation function of equation (45) have been discussed by Barrick (ref. 6). He points out that this surface correlation function violates the fundamental assumption of the theory that the surface is of a gently varying nature. Equation (45) corresponds to a surface with many abrupt, vertical discontinuities. Furthermore, Barrick shows that in the limit of small radar wavelength, which is the only limit in which the developed theory can claim validity, the total power scattered in all directions by a perfectly conducting, rough surface described by equation (45) is zero, even if the surface slopes are small. Equation (46), on the other hand, describes an admissibly smooth surface and leads to a plausible value for the scattered power, if the slopes are small.

The angular scattering law of equation (48), being based on equation (46), does not lead to the absurdities pointed out by Barrick, although it is still subject to the inconsistencies discussed in the previous section. As $\epsilon \to \infty$, the dielectric-surface reflection coefficients for all angles of incidence reduce to those appropriate for a perfectly conducting surface. Similarly, if the limit $\epsilon \to \infty$ is taken in equation (48), the resulting angular scattering law agrees with the angular scattering law deduced by Beckmann (ref. 20) for a perfectly conducting, rough surface having the same statistical properties. Although no satisfactory derivation of equation (48) for a dielectric surface has ever been given, the agreement of that equation as $\epsilon \to \infty$ with a correctly derived result for the perfectly conducting surface makes it conceivable that there does exist some undiscovered basis for its validity. From the standpoint of Martian radar studies, however, this conjecture is merely academic, since the experimental results are not describable by equation (48).

EXPERIMENTAL RESULTS FOR MARTIAN SURFACE SCATTERING

The Haystack Microwave Facility of the Massachusetts Institute of Technology Lincoln Laboratory has been extensively used to study radar signals reflected from the surface of Mars (refs. 3 and 4). In these studies, determination of the dielectric constant and surface slopes of Mars has relied upon the previously discussed theoretical work of Hagfors. It is unfortunate that, within the context of this theory, only the theoretically unacceptable correlation function of equation (45) has been found to fit the experimental data.

The data reduction procedure used in references 3 and 4 consisted of fitting the angular scattering law of equation (47) to the experimental angular scattering function and matching the theoretical and the experimental powers at $\varphi = 0$ and at another value of φ. The fit at the two points gives a value for the dielectric constant ϵ and an experimental number for the coefficient of $\sin^2 \varphi$.[4] This coefficient, denoted by C, is such that $C^{-1/2}$ is approximately equal to the tangent of the angle φ for which equation (47) falls to $1/e$ of its value at $\varphi = 0$. Although this is called the root-mean-square surface slope in reference 4, this designation is clearly a misnomer, since for a surface described by equation (45), which contains arbitrarily large slopes with many vertical discontinuities (ref. 6), the root-mean-square slope is infinite. Arguments for the interpretation of $C^{-1/2}$ as an "effective" slope are given in references 3, 11, and 14. These arguments center around the assertion (ref. 14) that the effective slope seen by the radar depends upon ". . . structural detail in the correlation function near the origin which will not appreciably influence the value of the integral [equation (44)] determining the backscattered power." Unfortunately, it is precisely the form of the correlation function near the origin which gives rise to the angular scattering law used (eq. (47)). A change in the behavior near the origin from a linear variation in Δr to a quadratic variation is just

[4]In addition to radar system parameters, the constants of proportionality of equations (47) and (48) also contain the unknown quantities ℓ and h_{rms}, so that ϵ cannot be obtained by a single measurement at $\varphi = 0$.

the change necessary to achieve equation (48), which does not fit the data, rather than equation (47). Ignoring this point, Hagfors argues that the finite radar wavelength imposes a "filtering action" on the wave number spectrum generating the rough surface, so that the higher spatial frequencies of this spectrum are truncated or attenuated. Thus the infinite actual slopes of the model are of no consequence and interpretation of $C^{-1/2}$ as the "effective" root-mean-square surface slope is justified. This argument is also unsatisfactory, however, since such a filtered spatial frequency spectrum does not give rise in the first place to the angular scattering law containing the C parameter.

Because of these basic theoretical difficulties, the reliability of the Martian dielectric-constant and slope values quoted in references 3 and 4 is unclear. It may be thought that an indicator of the reliability of the method for Mars might be application of the same method to lunar radar data. Such is not the case however, because of the following considerations. Earth-based lunar data have given results for the dielectric constant in the range from 2.5 to 3, which agree well with actual values. These good values for the Moon are clouded, however, by other difficulties. Barrick (ref. 6) has pointed out that the lunar radar data imply the absurd result that the surface correlation length ℓ is as large as one-half of the distance from the Earth to the Moon. Notwithstanding this intrinsic contradiction, one might argue that a theory can give a right answer for the wrong reason, so that some confidence in the dielectric-constant values for Mars could still be retained by analogy with the lunar case. Such an argument could have merit only if the Martian dielectric constants and surface roughness were qualitatively similar to those of the Moon. The spread in radar-obtained values of ϵ for Mars ranges from 1.5 to more than 6, and the radar data indicate that Mars is ostensibly much smoother than the Moon. This marked qualitative difference in the radar return from the two bodies severely weakens the probability that a defective method, which happens to be successful for ϵ for the Moon will be successful for ϵ for Mars.

Other radar studies of Mars, using the radar facility at the Goldstone Tracking Station, has been reported in reference 21. These studies produced radar cross-section data and an empirical angular scattering function, but since no a priori theoretical model was used, it was impossible to produce any local values of dielectric constant. It is notable that the empirical angular scattering function resulting from these studies does not agree with equation (47), the angular scattering function of references 3 and 4.

DISCUSSION

The difficulties of the correlation-function method tend to obscure the relationship of this method to other methods of analysis of rough-surface scattering. Hagfors has concluded (ref. 14) that for the Gaussian autocorrelation function, the correlation-function method is equivalent to that of equation (29). The status of the exponential correlation function with regard to this equivalence remains unclear. To the extent that this equivalence holds for the Gaussian case, only one type of acceptable theory for radar scattering from rough, dielectric planetary surfaces, namely, the theory underlying equation (29), is left. This theory is constrained to small slopes and gently varying terrain on the scale of a radar wavelength. Therefore experimental data to which the theory could be reasonably applied must necessarily fall off rather sharply with the incidence angle of the signal (φ of eqs. (47) and (48) and γ of eq. (29)). The Martian radar data fall off rather slowly with incidence angle, however, so that the theory of equation (29) is still inadequate.

Refinements to the specular-point theory include the introduction of a shadowing function into equation (29) so that obscuration of specular points near the planetary limbs can be taken into account. Beckmann (ref. 22) has developed such shadowing functions and has applied them with some success to lunar scattering data, although Brockelman and Hagfors (ref. 23) contest the success of his method. In any event, the introduction of shadowing would tend to drive down the level of the return with increasing angle of incidence, an effect which is opposite to that desired for explaining the Martian data. Also, inclusion of shadowing does not alter the limitation that the basic tangent-plane-approximation theory, to which the shadowing is appended, cannot take into account multiple scattering or small-scale structure. These factors would be more important for regions near the planetary limbs, since their contribution to the total scattered power would not decrease with increasing incidence angle to the extent that the specular-point contribution does. The presence of these factors on the planetary surface would be characterized by larger-than-expected backscatter levels and increasing depolarization of the backscattered signal in the data returned from the planetary limbs. Both these features are invariably present in the radar data, giving further evidence that any theory based upon the tangent-plane approximation is incapable of describing backscattering for large incidence angles.

Other theoretical models for planetary radar data analysis are composite surface models containing two or more correlation-length scales. Some of these methods (e.g., ref. 24) are straightforward extensions of the basic theory described in the present report, whereas others (refs. 25 and 26) attempt to combine a large-scale roughness theory based on these same methods with a small-scale result based on a perturbation solution to an exactly soluble scattering problem. Recent work by Burrows (ref. 27) has indicated that the small-perturbation solution is, in fact, equivalent in many respects to the solution obtained from the tangent-plane approximation, if this approximation is arbitrarily applied to a surface with structure much smaller than the radar wavelength. Since in this case the physical basis for the validity of the tangent-plane approximation is completely lacking, the meaning of Burrows' result is unclear, but it does cast some doubt on composite models using a small-perturbation approach. None of these more elaborate approximations have yet influenced the interpretation of Martian radar data. More disheartening, however, is the fact that not one of the theories available at present is likely to bring order out of the confusing and sometimes self-contradictory radar results.

The selection of references which have been discussed in detail has been made in order to present work that has had considerable impact upon planetary radar studies. In general, the best theoretical work has been limited to perfectly conducting surfaces. For example, Hoffman (ref. 28) gave in 1955 an excellent treatment of scattering from a rough, perfectly conducting surface in the tangent-plane approximation. He obtained formal results for the scattered Poynting vector, including the cross-coupling of the polarization components (depolarization) produced by local variations in the surface normal. Probably because of the formal nature of his results, Hoffman's work has had no impact upon practical applications. An obvious extension of Hoffman's work in the direction of the theories discussed in this paper could be incorporation of the reflection coefficients for arbitrary incidence angles in order to cover the dielectric scattering case. Even if successful, such a course would no doubt produce complex formal results not readily amenable to data analysis. In addition, the fundamental difficulty (discussed previously in the section "Statistical Nature of Rough-Surface Scattering") remains that planetary surfaces may not be describable in terms of the theory of random processes, so that attempts to extract meaningful results from such theories may be hazardous at best.

CONCLUDING REMARKS

The theoretical basis for the most widely accepted and used models for describing planetary rough-surface scattering has been examined. These models are basically more capable of describing rough, perfectly conducting surfaces than the dielectric surfaces of real planets. In order to obtain models which can describe dielectrics, it has been necessary to ignore all points of the planetary surface for which the local angle of incidence of the radar wave is not zero. Such a procedure cannot be justified unless the scattering surface is gently undulating with no structure on the scale of a radar wavelength. The presence of depolarization of the returned radar signal near the planetary limbs may be evidence that multiple scattering or small-scale structure actually exists, so that the theory, which predicts no depolarization, may not even be applicable to the sub-Earth areas of the planet that show no significant depolarization.

In addition to the difficulty in extending the theory to dielectric scatterers, there are numerous questions concerning the theoretical development of the statistical scattering properties. The theoretical formulation underlying the most often quoted experimental results for the planet Mars has been found to be internally inconsistent and to lead to an absurd result for the total scattered power in the limit of small wavelength for which the theory has its only clear basis for validity.

The presently available approaches to the theory are incapable of accounting for the slow falloff of scattered power with angle of incidence for the planet Mars, and introducing shadowing effects into the theory produces a correction in the wrong direction. In view of the difficulties involved with the basic theory, it is not apparent that more elaborate methods, such as the use of composite roughness scales, would produce better results.

It is concluded that a fundamental breakthrough in the theoretical description of scattering from rough terrain is required before dependable results can be expected for such detailed properties as dielectric constant and surface slopes when even the statistical properties for the scattering body remain unknown. Such a breakthrough represents a formidable problem that is not likely to be solved in the near future.

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