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FINAL REPORT
FURTHER ANALYSIS OF THE EFFECTS OF BAFFLES ON COMBUSTION INSTABILITY

By

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ABSTRACT

A computerized analytical model, developed previously at Rocketdyne, to predict the effects of baffles on combustion instability, has been modified in an effort to improve the ability to properly predict stability effects. The model has been modified (1) to replace a single spatially-averaged response factor by separate values for each baffle compartment; (2) to calculate the axial component of the acoustic energy flux, and (3) to permit analysis of travelling waves in a thin annular chamber.

Allowance for separate average response factors in each baffle compartment was found to significantly affect the predicted results. With this modification, an optimum baffle length was predicted which gave maximum stability. For baffle lengths less than this optimum (~12 percent of chamber length), increasing baffle length improved stability, as found experimentally.

Calculations of the average acoustic energy flux showed a maximum in the axial component of this flux at the downstream end of the baffle.

Some difficulties were encountered in analysis of thin annular chambers which are believed to be due to an as yet unidentified error in the program logic. However, the acoustic analysis,which appears free from error, indicates a spinning mode quickly shifts to a standing mode in a baffled chamber. Additional work is needed to properly define these effects.

CONTENTS

ILLUSTRATIONS

NOMENCLATURE

c	=	sound velocity, in./sec
D	=	droplet diameter, in.
$G(\vec{r}\|\vec{r}_0)$	=	Green's function
I	=	acoustic energy flux, in.-lbf/in.^{2}-sec
I_{cn}	=	imaginary component of harmonic response factor for n^{th} term, defined by Eq. 3
Im ( )	=	imaginary part of the quantity enclosed
i	=	$(-1)^{1/2}$
k	=	complex wave number (complex angular frequency divided by sound velocity) in.$^{-1}$
$k^{(1)}$	=	k for first transverse mode
$k^{(2)}$	=	k for second transverse mode
ℓ	=	baffle length, inches
L	=	length of main chamber, inches
$\overline{M}$	=	steady flow Mach number
p	=	pressure, lbf/in.^{2}
P'	=	oscillatory pressure divided by time averaged pressure
p_n	=	Fourier amplitude coefficient for n^{th} pressure term
R_{cn}	=	real component of harmonic response factors for n^{th} term, defined by Eq. 3
Re ( )	=	real part of the quantity enclosed
$\mathcal{R}_{cn}$	=	complex harmonic response factors
$\vec{r}$	=	position vector, inches
S	=	surface area, sq. in.
t	=	time, sec

NOMENCLATURE (Cont.)

U	=	magnitude of velocity difference between the gas and droplet, in./sec
u	=	axial velocity, in./sec
u_n	=	Fourier amplitude coefficient for n^{th} velocity term
v	=	transverse velocity, in./sec
W	=	burning rate divided by time averaged burning rate
w	=	burning rate, lbm/sec
z	=	axial position, in.
α	=	growth coefficient, cRe {k}, sec^{-1}
β	=	specific acoustic admittance of surface, dimensionless
β_{2L1}	=	specific acoustic admittance at injector face for first transverse mode
β_{2L2}	=	specific acoustic admittance at injector face for second transverse mode
γ	=	ratio of heat capacities, constant pressure to constant volume
ϵ_m	=	1 if $m = 0$, 2 if $m \neq 0$
η	=	sector angle, radians
θ	=	angular position
μ	=	index referring to μ^{th} compartment; also, viscosity, lbm/in.-sec
ξ	=	pressure gradient, lbf/in.^{3}
π	=	3.14159
ρ	=	gas density, lbm/in.^{3}
ψ	=	phase angle defined by Eq. A-7
ϕ_n	=	phase angle between terms in pressure expansion
ϕ_2	=	phase angle between first and second pressure harmonics

NOMENCLATURE (Cont.)

SUBSCRIPTS

a	=	refers to main chamber
b	=	refers to baffle compartments
est	=	estimated value
n	=	refers to n^{th} harmonic; also, nozzle
μ	=	refers to μ^{th} baffle compartment
0	=	refers to source coordinates for Green's functions
0L1	=	refers to nozzle end condition, first harmonic
2L1	=	refers to injector end condition, first harmonic
2L2	=	refers to injector end condition, second harmonic

SUPERSCRIPTS

$\bar{}$ (over bar)	=	denotes time averaged value
1	=	denotes first harmonic quantity
2	=	denotes second harmonic quantity
μ	=	refers to μ^{th} baffle compartment
' (prime)	=	time varying quantity
*	=	complex conjugate

SUMMARY

A computerized analytical model, developed previously at Rocketdyne, to predict the effects of baffles on combustion instability, has been modified in an effort to improve the ability to properly predict stability effects. The model has been modified (1) to replace a single spatially averaged response factor by separate values for each baffle compartment; (2) to calculate the axial component of the acoustic energy flux, and (3) to permit analysis of travelling waves in a thin annular chamber.

This analytical model has been developed by coupling an acoustic analysis of the wave motion within baffled chambers with a model for the oscillatory combustion response of a propellant droplet developed by Heidmann, the response factor model. The response factor model exhibits a significant contribution from the first harmonic of the fundamental mode of oscillation. This nonlinearity could be calculated from a complicated multiorder perturbation analysis of the wave motion. To avoid this complication, a simplifying assumption was used, that the spatial distribution of pressure and velocity for the harmonic contribution are to be equal to those given by the linear acoustic analysis for the harmonic mode. Calculations with the resulting analytical model showed that the model predicted a worsening of stability with increasing baffle length rather than improving as found experimentally. Diagnostic calculations showed that the spatial average injector face boundary condition used to represent the combustion response was a very poor approximation and that the boundary condition actually varied widely with position. This was considered a potential cause for the unsatisfactory prediction of the variation of stability with baffle length.

The model was modified to allow for separate average response factors in each baffle compartment. Calculations made with the modified model showed that this change significantly affected the predicted results. With this modification an optimum baffle length was predicted which

gave maximum stability. For baffle lengths less than this optimum (~12 percent of chamber length), increasing baffle length improved stability, as found experimentally.

Calculations of the average acoustic energy flux showed a maximum in the axial component of this flux at the downstream end of the baffle.

Some difficulties were encountered in analysis of thin annular chambers which are believed to be due to an as yet unidentified error in the program logic. However, the acoustic analysis, which appears free from error, indicates a spinning mode quickly shifts to a standing mode in a baffled chamber. Additional work is needed to properly define these effects.

INTRODUCTION

This report describes results obtained under NASA Contract NAS3-17821. The purpose of the program was to modify a computerized analytical model, developed by Rocketdyne under Contract NAS3-11226 for analysis of the stability of baffled combustion chambers, in an effort to improve the ability to properly predict stability effects. Such models are needed to aid the analysis and design of baffled combustion chambers but no satisfactory model exists currently. Substantial progress has been made under the contracts noted above but additional work is needed.

Under NASA Contract NAS3-11226, Rocketdyne developed analyses of the effects of baffles on the wave motion and stability of two-dimensional (rectangular) and cylindrical chambers. The initial effort was directed primarily toward developing methods for analytically describing the wave motion or acoustic behavior of baffled chambers without consideration of combustion effects. Results from this effort were described in Ref. 1 and 2. Successful methods of analysis were developed.

Initial attempts at introducing the effects of combustion in an approximate way were made with a gain-type admittance boundary condition. Results from calculations with this admittance-type boundary condition showed a physically unsatisfactory result, increasing baffle length worsened the predicted stability. Because this result is contrary to known stability behavior, it was concluded that the model formulation, corresponding to simple pressure coupling, was inadequate and some means of including velocity coupling effects was necessary. Consequently, the more recent work has been directed toward including a better representation of the combustion effects.

To achieve this better representation, an analytical model was developed by coupling the acoustic analysis of the wave motion within baffled chambers developed earlier with a model for the oscillatory combustion response of a propellant droplet developed by Heidmann, the response factor model (Ref. 3). The response factor model includes a significant contribution from the first harmonic of the fundamental mode of oscillation, which could be calculated from a nonlinear, multi-order perturbation analysis of the wave motion. For this program, the spatial distribution of pressure and velocity for the harmonic contribution were assumed to be equal to those given by the linear acoustic analysis for the harmonic mode.

A computer program was developed for numerical solution of the coupled equations. After this program was thoroughly checked out, a series of calculations was made to investigate the variation of predicted stability with changes in model parameters. This investigation showed that the model as developed did not properly predict the variation of stability with increasing baffle length. The model predicted a worsening of stability with increasing baffle length rather than improving, as found experimentally. Therefore, diagnostic calculations were performed to determine the reasons for the improper prediction.

The diagnostic calculations showed that the spatial average injector face boundary condition used to represent the combustion response was a very poor approximation. The boundary condition varied widely with position. At the end of the program, initial attempts were made to minimize this effect by analysis of cases for more than one baffle, but the model still improperly predicted stability trends. Therefore, upon completion of that effort, it was recommended that additional analysis be done with an improved approximation to the spatially varying boundary condition. Results from this effort are described in Ref. 4, the final report for that program.

One objective of the current investigation was to modify the analytical model to allow for separate spatial average boundary condition in each baffle compartment. Although in principle a spatially varying boundary condition can be analyzed, the likelihood of computational difficulties seemed high and that level of complexity was not undertaken.

A second objective of the current investigation was to calculate the acoustic energy flux at several positions in the chamber.

A third objective was to modify the analysis method to allow description of spinning waves in a thin annular chamber. Previously, most of the work had been done for two-dimensional "slab"-type chambers with some work on cylindrical chambers. Analysis of a thin annular chamber allowed some of the analytical simplifications of a two-dimensional analysis to be retained while admitting the possibility of spinning modes, which have been predicted to be more unstable by the Heidmann response-factor analysis (Ref. 3).

ANALYTICAL APPROACH

The method of analysis involves use of a coupled analytical model developed by combining an acoustic analysis of the wave motion in the chamber with the Heidmann response factor model to represent the combustion. These models and analytical methods are described in Ref. 4. Basically, the response factor was used to specify a boundary condition for the acoustic calculation, through an approximate interrelationship, and the acoustic calculation was used to specify the oscillatory pressure and velocity environment needed for the response factor calculation. A multidimensional root-finding technique was developed to solve the coupled equations.

Descriptions of the acoustic analysis may be found in Ref. 1, 2, and 4. The response factor analysis was based on the work of Heidmann (Ref. 3). Heidmann's response factor was intended to represent the rate of oscillatory energy generation from the propellant combustion and was developed from analysis of the combustion of a single droplet, assuming vaporization controlled combustion.

The droplet combustion rate was represented as

$$W \propto \left(\frac{\rho U D}{\mu}\right)^{\frac{1}{2}} \qquad (1)$$

Oscillatory parameters were defined with reference to time-averaged (but not zero amplitude) conditions, i.e.,

$$\rho = \bar{\rho}(1 + \rho') \tag{2}$$

Heidmann defined both nonlinear and harmonic response factors (Ref. 3). For the baffle analysis, harmonic response factors have been used for the first and second harmonic modes, these being defined by

$$R_{cn} + i\ I_{cn} \equiv \frac{\int_0^{2\pi} W'\ p_n\ d\omega t}{\int_0^{2\pi} \left[\mathrm{Re}\ (p_n)\right]^2\ d\omega t} \tag{3}$$

The oscillatory pressures, velocities and densities to be used in evaluation of these integrals have been obtained from the baffled-chamber acoustic calculation. For simplicity, the integrations are performed numerically but, in addition, the Heidmann formulation has been extended to account for non-zero oscillatory growth or decay.

The response factor analysis has been coupled to the acoustic analysis through an approximate boundary condition. Based on work by Cantrell (Ref. 5), an interface condition has been chosen as

$$\gamma\bar{M}\ R_{cn} = \beta_n \tag{4}$$

where β is the wall specific acoustic admittance and $\bar{M}$ is the steady flow Mach number. In addition, for simplicity, the response factor was spatially averaged over the cross section so that a spatially uniform admittance could be used. Initially, this spatial averaging was done over the entire injector end of the chamber. During the current program, separate averages over each baffle compartment have been used.

Because Heidmann found the effect of wave distortion to be highly significant in determining the response factor, wave distortion has been included in an approximate manner for the evaluation of the response factor. The analysis of the chamber acoustics being used is linear and cannot predict distortion and extension of the analysis to a multiorder perturbation scheme to predict nonlinearities has been considered impractial. Therefore, an assumption has been used that the distortion would arise from a harmonic mode to the fundamental which would behave spatially as the harmonic (linear) chamber mode as well.

Solution of the coupled analytical modes represents a complicated root finding problem. The overall iterative solution procedure comprises a set of sequential iterative solution procedures. The coupled model is used with a four-dimensional Newton-Raphson procedure to calculate the fundamental-mode injector-end admittance (real and imaginary parts), the harmonic mode pressure amplitude (P_{02}) and the phase angle between the fundamental and harmonic modes (ϕ_2). The fundamental mode amplitude (P_{01}) and steady-state axial gas-to-droplet velocity difference ($\Delta\overline{V}$), as well as the chamber configuration, Mach number, γ, etc., are considered parameters in the calculation.

COMPARTMENT RESPONSE FACTORS

During the current program, the response factors were spatially averaged over each baffle compartment. The procedure has been set up with an iteration based on one baffle compartment with proportionate iterative changes being made for the remaining compartments. This approach is satisfactory.

The solution procedure is:

1. Choose initial values for p_{02}, ϕ_2, $\beta_{2L1}^{(\mu)}$, k_1, $\beta_{2L2}^{(\mu)}$.
2. Solve the acoustic equations (Newton's method) for the k_1 corresponding to $\beta_{2L1}^{(\mu)}$.
3. Calculate the fundamental mode pressure and velocity distributions p_1, u_1, v_1.
4. For $k_2 = 2\ \mathrm{Re}\ (k_1) + i\ \mathrm{Im}\ (k_1)$ solve for the corresponding harmonic admittances from the acoustic equations with the assumed relationship

$$\beta_{2L2}^{(\mu)} = (\beta_{2L2}^{(\mu)})_{est} \left[\frac{\beta_{2L2}^{(1)}}{(\beta_{2L2}^{(1)})_{est}} \right] , \ \mu \geq 2 \tag{5}$$

5. Calculate harmonic mode pressure and velocity distributions
6. Adjust p_{02} and ϕ_2 (through use of Newton's method) until

$$\beta_{2L2}^{(1)} = -\gamma\ \overline{M}\ \overline{\mathcal{R}}_{c2}^{(1)} \ . \tag{6}$$

6a. Calculate local response factors (n = 1 and 2):

$$\mathcal{R}_{cn} = \left(\frac{\omega}{\pi}\right) \left[\frac{1}{\overline{W}} \int_0^{2\pi/\omega} \frac{W}{p_n\ e^{i\phi_n}\ e^{-ik_n t}}\ dt \right. \\ \left. \frac{(i - e^{-2\alpha/(\omega/\pi)})}{-ik_n p_n\ e^{i\phi_n}} \right] \tag{7}$$

6b. Calculate spatial averaged response factors

$$\overline{\mathcal{R}}_{cn}^{(\mu)} = \frac{\int_{S^{(\mu)}} \mathcal{R}_{cn} \, \mathrm{Re}\,(p_n)^2 \, dS}{\int_{S^{(\mu)}} \mathrm{Re}\,(p_n)^2 \, dS}$$

7. Replace $\beta_{2L1}^{(\mu)}$ by $-\gamma\ \overline{M}\ \overline{\mathcal{R}}_{c1}^{(\mu)}$

8. Replace $\beta_{2L2}^{(\mu)}$ by $-\gamma\ \overline{M}\ \overline{\mathcal{R}}_{c2}^{(\mu)}$

9. Increment p_{02}, ϕ_2 and $\beta_{2L1}^{(1)}$ and calculate corresponding $\overline{\mathcal{R}}_{c1}^{(1)}$

10. Use four dimensional Newton's method to solve for new estimates for p_{02}, ϕ_2, and $\beta_{2L1}^{(1)}$. Return to step 2 and continue until convergence is achieved on $\beta_{2L1}^{(1)}$

This solution procedure has been found satisfactory, although partial suppression of the predicted corrections was necessary in some cases. Results obtained with this procedure will be described in the next section.

ACOUSTIC ENERGY FLUXES

The acoustic energy flux was calculated for several regions in the chamber employing an analytical formulation developed by Rice (Ref. 6), who gives an expression for the acoustic energy flux in the presence of a uniform steady flow. Rice's expression may be written as

$$I_x = \frac{1}{2} \sum_{jk} \mathrm{Re} \left\{ p_j^{*} v_{x_k} + \frac{\overline{M}}{2} \left[\frac{p_j^{*} p_k}{\rho c} + \rho c \left(v_{x_j}^{*} v_{x_k} + v_{y_j}^{*} v_{y_k} \right) \right] \right\} \tag{9}$$

Spatial and time averaged values of this energy flux were calculated by numerically integrated over time and transverse position, with the local values of pressure and velocities being calculated from the acoustic analysis.

ANNULAR CHAMBER WITH TRAVELLING WAVES

Previous calculations had all been done for standing rather than travelling modes. Because Heidmann (Ref. 3) reported that travelling modes were predicted to be more unstable, a variation of the analysis was set up for travelling waves. Equations corresponding to those obtained previously for the two-dimensional "slab" configuration were developed for a thin annular chamber, which may exhibit either standing or travelling modes.

Equations for this annular case are given in the Appendix. By suitably redefining parameters it was possible to minimize the changes necessary to the existing two-dimensional computerized model to permit analysis of the annular case.

RESULTS

The analytical approach outlined above was used to obtain the results needed to meet the program objectives. However, the results obtained for the annular case were somewhat unsatisfactory because incorrect results were obtained when certain limiting cases were tried. This deficiency in the annular case is believed due to an error in program logic which has not yet been found. The remaining results were entirely satisfactory.

COMPARTMENT RESPONSE FACTORS

The computer program for a two-dimensional "slab" chamber was modified to employ separate spatially averaged response factors for each baffle compartment. A series of calculations was done for a chamber with a single baffle and a chamber length-to-width ratio of 1.5, $P_{01} = 0.2$, $\gamma = 1.2$, $\overline{M} = 1/3$, $\beta_n = 0.0 + i\ 0.0$, and $\Delta\overline{V} = 0.02$. Results for several baffle lengths are summarized below:

ℓ/W	$k_W^{(1)}$	$\beta_{2L1}^{(1)}$	$\beta_{2L1}^{(2)}$	P_{02} ϕ_2/π
0.02	3.234, -.0341	.3376, .3725	.3395, -.4131	.1553, -.766
0.10	3.229, -.0675	.4414, .2007	.4493, .2793	.1000, -.850
0.18	3.155, -.1467	.3128, .0248	.3340, .1756	.0525, -.908
0.20	3.115, -.1392	.2539, .0630	.2733, .1993	.0482, -.878
0.22	3.074, -.0902	.1566, .1301	.1666, .2172	.04296, -.905
0.24	3.017, -.0139	.0210, .1747	.0221, .1873	.03346, -.692
0.25	2.998, +.01894	-.0265, .1740	-.0260, .1620	.02869, -.653
0.26	2.9616, +.0527	-.0796, .1645	-.0709, .1248	.02350, -.609

These results may be compared with similar results obtained for a single spatial average response factor,

ℓ/W	$k_W^{(1)}$	$\beta_{2L1} = \gamma M \overline{\mathcal{R}}_{c1}$	p_{02}	ϕ_2/π
0.020	3.2220 + 0.03123i	-0.2536 + 0.3315i	0.13906	-0.36332
0.100	3.2292 + 0.04830i	-0.3908 + 0.3592i	0.139182	-0.30644
0.150	3.2266 + 0.06858i	-0.4399 + 0.3301i	0.121892	-0.27313
0.200	3.2114 + 0.12815i	-0.4403 + 0.2621i	0.108021	-0.32056
0.250	3.1131 + 0.19331i	-0.3295 + 0.1998i	0.061798	-0.37978
0.300	2.8193 + 0.23135i	-0.2144 + 0.0791i	0.018374	-0.41505
0.330	2.6806 + 0.22877i	-0.1924 + 0.0489i	0.010915	-0.45221
0.370	2.5116 + 0.22455i	-0.1785 + 0.0200i	0.005728	-0.50542
0.375	2.4920 + 0.22439i	-0.1778 + 0.0172i	0.005300	-0.51188
0.380	2.4725 + 0.22431i	-0.1773 + 0.0145i	0.004902	-0.51840
0.500	2.0851 + 0.22211i	-0.1812 - 0.0176i	0.000874	-0.67324
0.600	1.8454 + 0.20969i	-0.1854 - 0.0231i	0.000337	-0.83374
0.700	1.6527 + 0.19364i	-0.1871 - 0.0245i	0.000227	-0.96188
0.800	1.4942 + 0.17713i	-0.1879 - 0.0251i	0.000190	-1.02869

These summaries show that the change from a single average response factor to a separate average for each baffle compartment has substantial and important effects. Most notably, with a single average response factor, a general worsening of stability with increasing baffle length is predicted whereas the opposite is observed experimentally. Conversely, with separate average response factors an improvement is predicted until a nondimensional baffle length of 0.18 is reached with a subsequent worsening with further increases in length. These effects are shown graphically in Fig. 1, where the growth coefficients have been scaled for the main combustion chamber of the Space Shuttle Main Engine.

Calculated pressure histories are shown in Fig. 2 through 4 for several nondimensional baffle lengths, again scaled for the SSME main combustion chamber. The decay rate at the optimum baffle length (ℓ/W = 0.185 is 613 sec^{-1} which corresponds to a damp time of ~4 milliseconds (from an initial overpressure of 100% to <10% zero-to-peak amplitude). Also, the diminishing contribution from the second harmonic with baffle length is

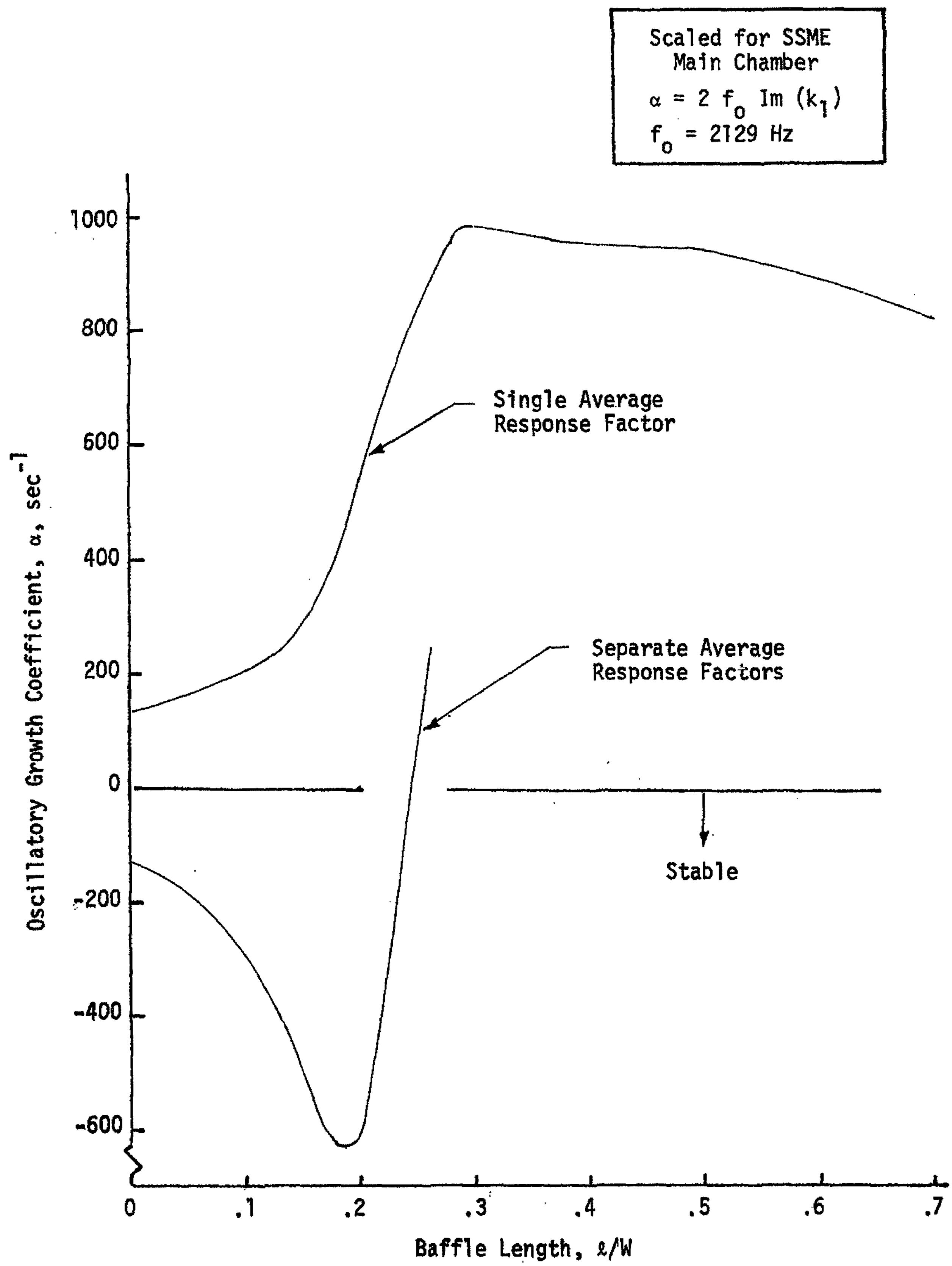

Figure 1. Variation of Predicted Growth Coefficient with Baffle Length

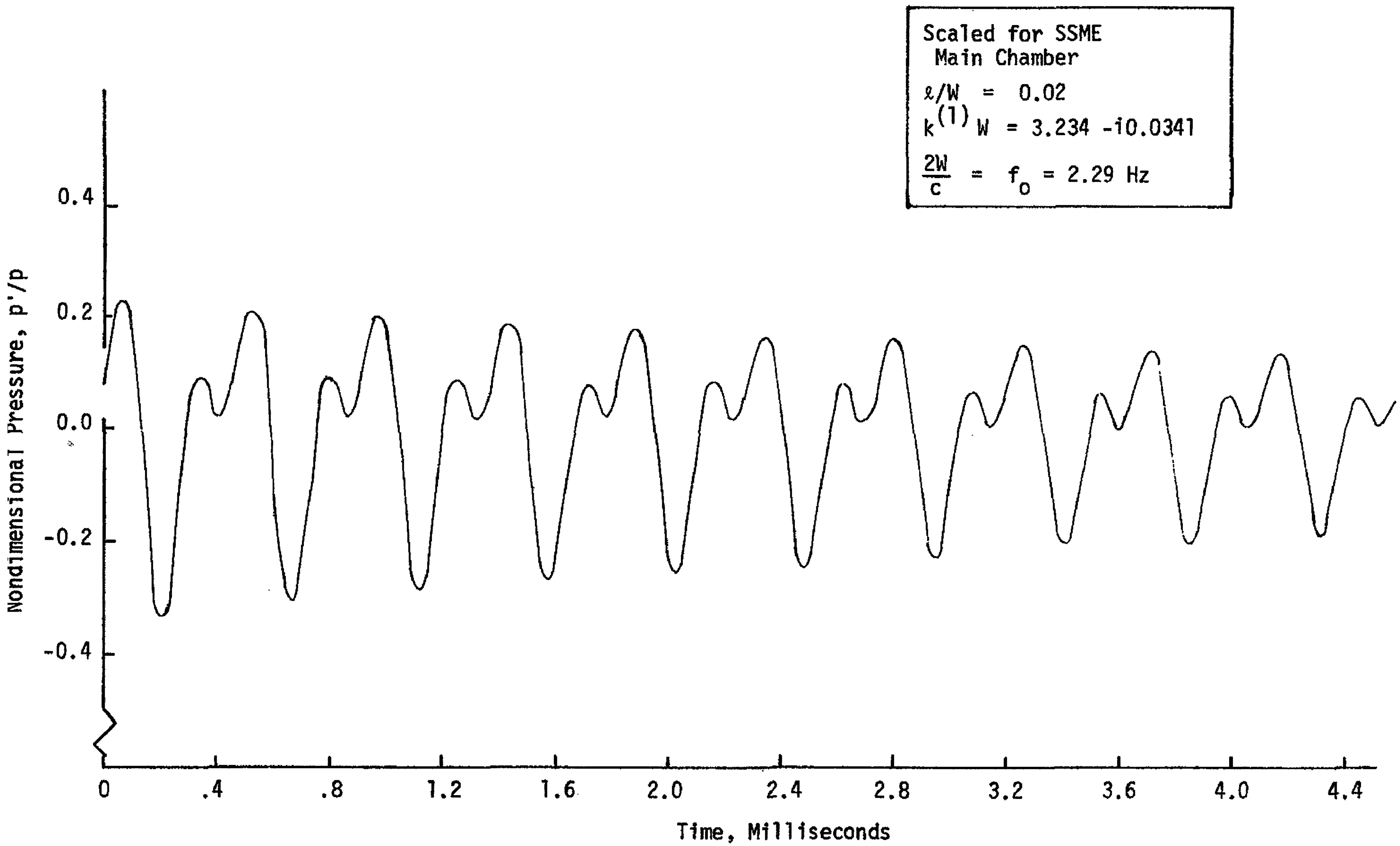

Figure 2. Predicted Pressure History, $\ell/W = 0.02$

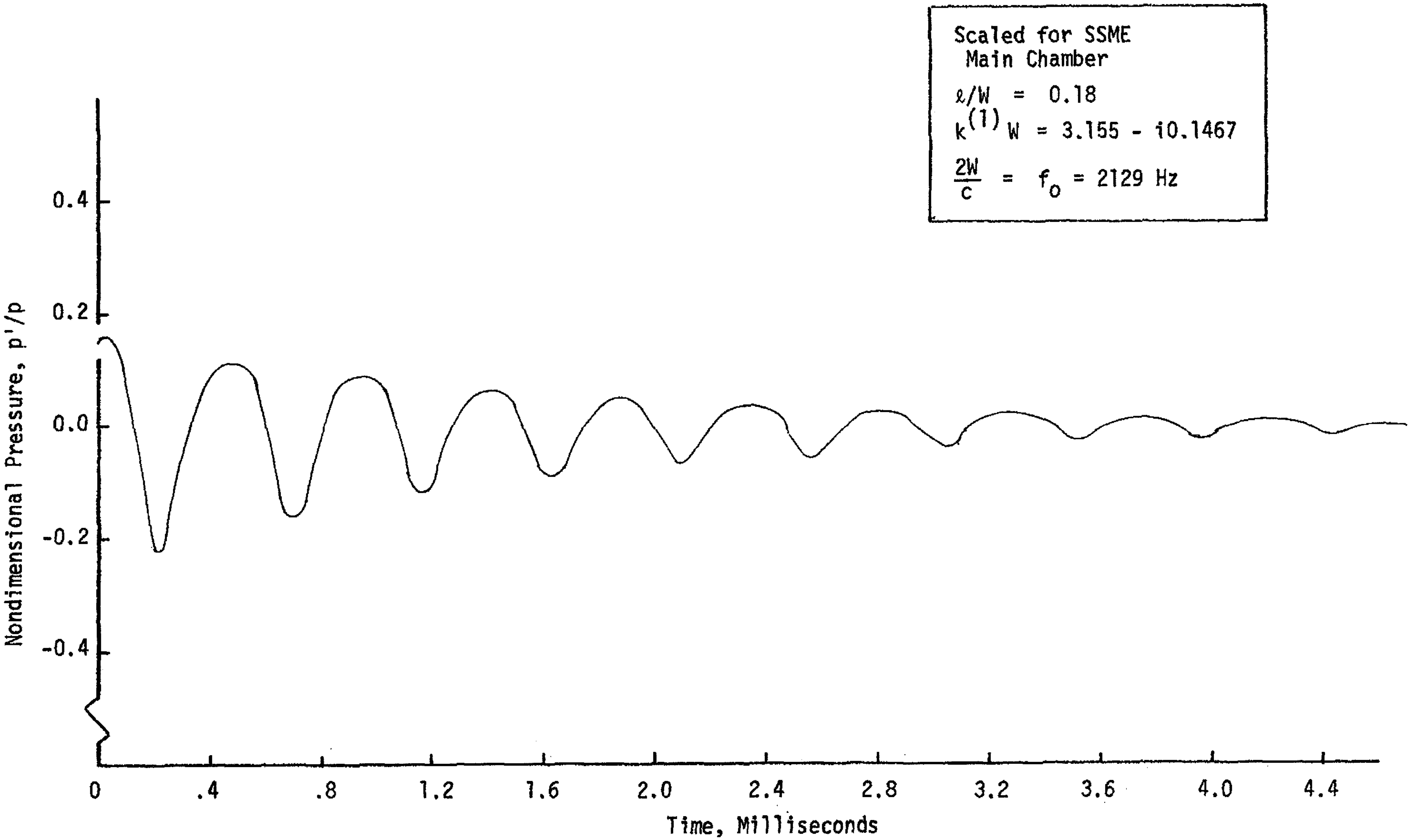

Figure 3. Predicted Pressure History, ℓ/W = 0.18

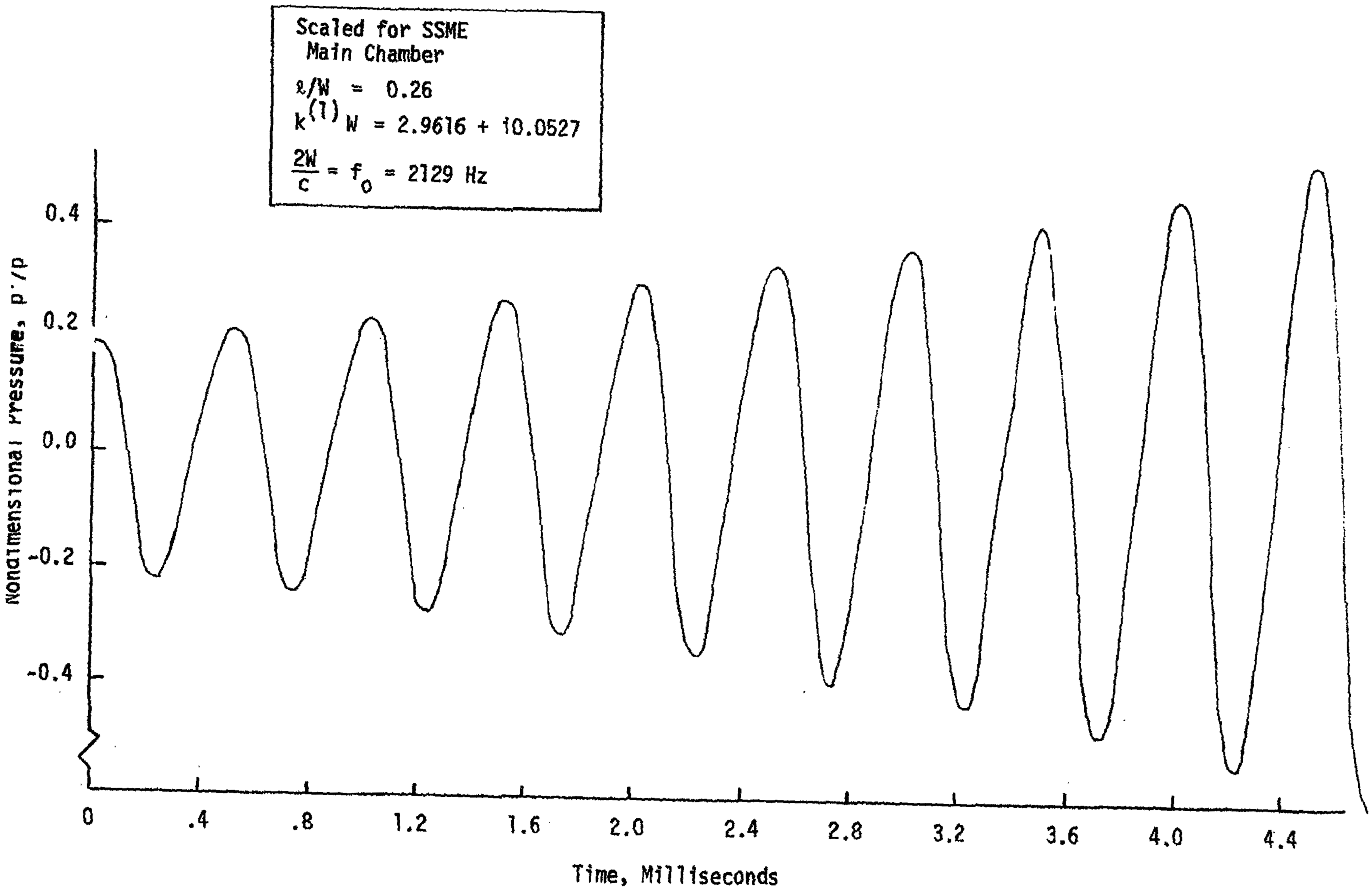

Figure 4 . Predicted Pressure History, $\ell/W = 0.26$

apparent from comparison of Fig. 2, which shows substantial distortion and second harmonic content (~78 percent) and Fig. 3 and 4, which exhibit a nearly sinusoidal wave form (~26 and ~12 percent second harmonic, respectively).

ACOUSTIC ENERGY FLUXES

The computer program was modified to calculate the time and spatial average acoustic energy flux at the injector face, at the baffle tips on the compartment side, at the baffle tip on the main chamber side and at the nozzle entrance. However, only a limited number of calculations were made. Representative results for $\ell/W = 0.18$ are:

Location	$I_x/\bar{p}c$
"Injector" face	0.0063
Baffle tip (compartment side)	0.0085
Baffle tip (chamber side)	0.0078
"Nozzle" entrance	0.0028

where I_x refers to axial component of the energy flux and is nondimensionalized by the time average chamber pressure, $\bar{p}$, and the sound velocity. The small discontinuity at the injector face may be due to the approximations which have been used to solve the acoustic equations or to truncation of the series representations for pressure and velocity.

ANNULAR CHAMBER WITH TRAVELLING WAVES

A modified version of the computer program was developed for the annular, travelling wave case as outlined above. The program was set up to accomodate initial approximations corresponding to either travelling or standing waves. Some modifications to the program were necessary to achieve satisfactory convergence when four baffle compartments, with individual average admittances and response factors, were analyzed. This case corresponds to two compartments in the two-dimensional analysis used previously. Some difficulty had been encountered previously with the two-dimensional analysis and with the additional degrees of freedom allowed in the annular case, the root-finding procedure failed. With these modifications, satisfactory convergence and program function was achieved. However, the results were still not satisfactory because the pressure match at the compartment-to-chamber interface was poor and, therefore, the solutions are not physically acceptable. Moreover, for the standing wave case the solutions obtained previously for the two-dimensional case could not be duplicated in the annular case. Diagnostic calculations have not revealed the source of these problems which are believed due to an error in the program logic. Therefore, additional work is needed in the area.

One interesting and significant result was obtained from the acoustic portion of the calculation, which appears to have been functioning properly. The acoustic analysis does not yield a spinning-type initial approximation for the pressure distribution is assumed. With only two iterations the pressure distribution has shifted to a standing-wave form, as shown in Fig. 5. With ten iterations a standing wave solution is clearly obtained with an excellent pressure match, as shown in Fig. 6. These results were obtained from the acoustic portion of the analysis with rigid-wall boundary conditions. Because Heidmann's results (Ref.3) indicate spinning modes (travelling waves) are more unstable than standing modes, this result implies a beneficial effect of baffles by itself, i.e., spinning modes are prevented by baffles.

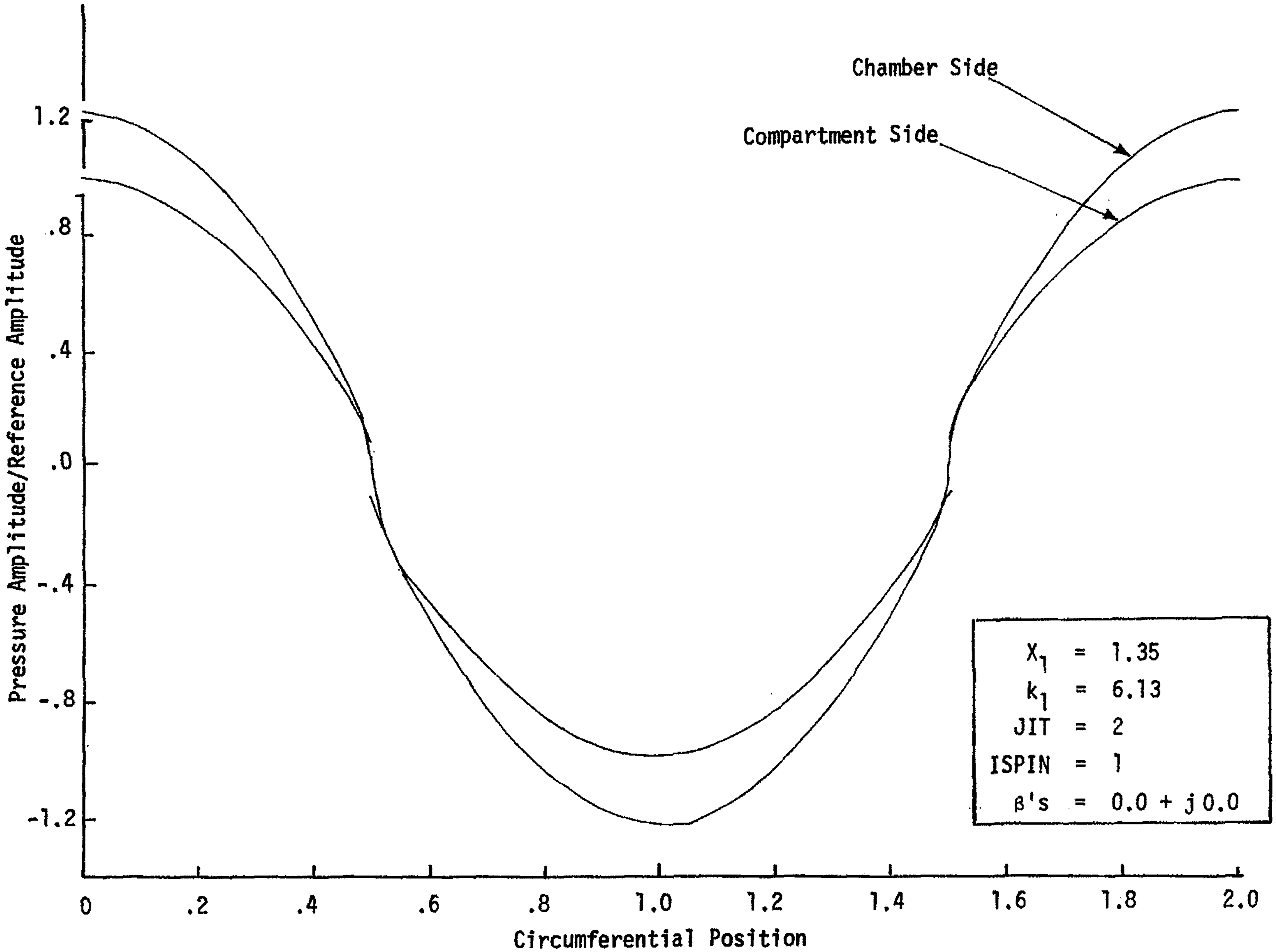

Figure 5. Predicted Pressure Distributions from Spinning Wave Initial Approximation and Two Iterations

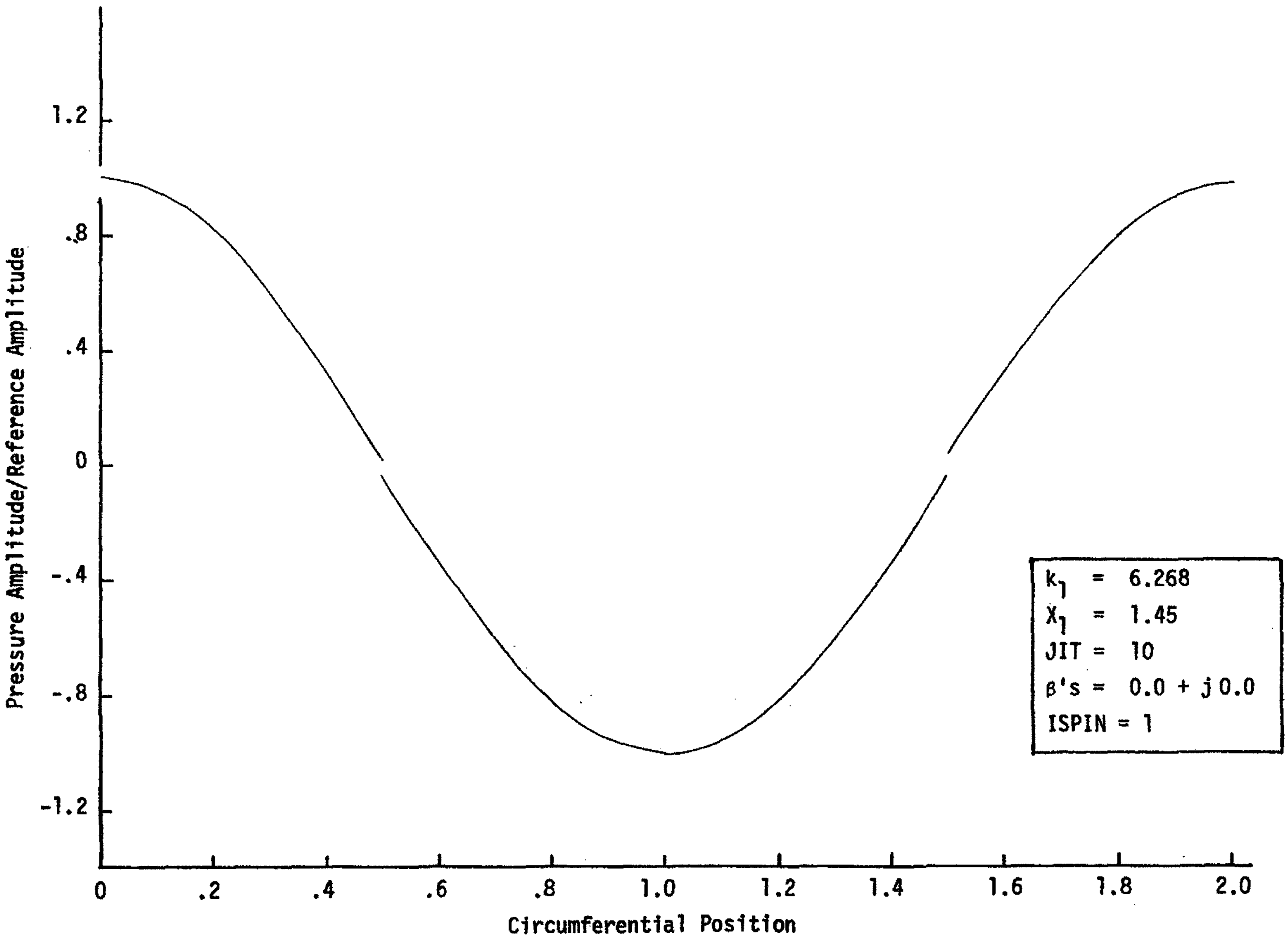

Figure 6. Predicted Pressure Distributions from Spinning Wave Initial Approximations and Ten Iterations

CONCLUDING REMARKS

Results obtained during this program are considered sufficiently encouraging to justify further work to solve existing problems in the analysis. The modification to the analytical model to allow for separate average response factors in each baffle compartment has led to predicted stability variations which are in reasonable agreement with measured behavior. Also, the additional possibilities and capability permitted by the variation of the model for thin annular chambers encourages further work to systematically eliminate the problems being encountered currently and evaluate the predicted behavior under these conditions.

In addition, problems have been encountered on several occasions with a solution to the acoustic analysis which is not a physically acceptable solution and which arises because of the approximate solution method being used. Therefore, additional effort to recast the equations to minimize or eliminate this problem is recommended.

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APPENDIX
EQUATIONS FOR THIN ANNULAR CHAMBER

Equations were developed for the acoustic behavior of a thin-annular chamber with baffles on one end which correspond to those used previously for two-dimensional "slab" case. No changes were necessary to the response factor analysis.

The acoustic analysis involves an iterative variational solution to the wave equation for the baffled chamber. The frequencies (and stability) of the allowed modes are obtained from solution of a characteristic equation which may be written as

$$\int \vec{N} \cdot \nabla p \, (p_a - p_{b_\mu}) \, dS \qquad \text{(A-1)}$$

where

$$p_a = \int G_a \, \xi dS_o \qquad \text{(A-2)}$$

$$p_{b_\mu} = -\int G_{b_\mu} \, \xi dS_o \qquad \text{(A-3)}$$

$$\xi = \vec{N} \cdot \nabla p \qquad \text{(A-4)}$$

The Green's functions for the thin annular regions may be written as

$$G_a = -\sum_m \frac{\epsilon_m}{2\pi} \frac{\cos m(\theta - \theta_o)}{k_m \sin (k_m L + \psi_m + \overline{\psi}_m)} F_m(z|z_o) \qquad \text{(A-5)}$$

$$F_m(z|z_o) = \begin{cases} \cos (k_m z + \psi_m) \cos \left[k_m (L - z_o) + \overline{\psi}_m \right] & z < z_o \\ \cos \left[k_m (L - z) + \overline{\psi}_m \right] \cos (k_m z_o + \psi_m) & z > z_o \end{cases} \qquad \text{(A-6)}$$

$$\tan \psi_m = \frac{jk\,\beta_n}{k_m} \qquad \text{(A-9)}$$

$$\tan \overline{\psi}_m = -\frac{jk\,\beta_L}{k_m}$$

$$k_m = \sqrt{k^2 - m^2/\overline{r}^2}$$

and

$$G_{b_\mu} = -\sum_q \frac{\varepsilon_q}{\eta} \frac{\cos q\pi\,\theta/\eta\; F_q(z|z_o)}{k_q \sin(k_q L + \psi_q + \overline{\psi}_q)} \qquad \text{(A-10)}$$

η = the sector angle

$$p_{b_\mu} = \sum_q \frac{\varepsilon_q}{\eta} \frac{\cos\left[q\pi\,(\theta-\mu\eta)/\eta\right]\,\cos\left[k_q(L_t - z) + \psi_q\right]}{k_q \sin(k_q L + \overline{\psi}_q)}\, I_q \qquad \text{(A-11)}$$

$$I_q = \int \xi \cos\left[q\pi\,(\theta_o-\mu\eta)/\eta\right]\,\overline{r}\,d\,\theta_o \qquad \text{(A-12)}$$

$$\xi = \left.\frac{\partial p_{b_\mu}}{\partial z}\right|_{z=L} = \sum \frac{\varepsilon_q}{\eta}\, I_q \cos\left[q\pi\,(\theta-\mu\eta)/\eta\right] \qquad \text{(A-13)}$$

$$p_a = -\sum_m \frac{\varepsilon_m}{2\pi} \frac{\int \xi \cos m(\theta-\theta_o)\,\overline{r}\,d\theta_o}{k_m \tan(k_m L + \psi_m)} \qquad \text{(A-14)}$$

Iteration coefficients may be defined from

$$p_{b\mu}^{(i)}(L) = \sum_q \frac{\varepsilon_q}{n} a_{\mu q}^{(i)} \cos\left[q\pi(\theta-\mu n)/n\right] \tag{A-15}$$

$$a_{\mu q}^{(i)} = \frac{\int \xi^{(i)} \cos\left[q\pi(\theta_o-\mu n)/n\right] \bar{r}\, d\theta_o}{k_q \tan\left[k_q \ell + \psi_q\right]} \tag{A-16}$$

$$p_{b\mu}^{(i+1)}(L) = p_a^{(i)}(L) \tag{A-17}$$

$$a_{\mu q}^{(i+1)} = -\sum_m \frac{\varepsilon_m}{2\pi} \frac{\bar{r} \int_{\mu n}^{(\mu+1)n} \int_o^{2\pi} \xi^{(i)} \cos m(\theta-\theta_o)\, d\theta_o\, d\theta}{k_m \tan(k_m L + \psi_m)} \tag{A-18}$$

The spinning or travelling wave character is introduced through the initial pressure approximation. Therefore, the initial approximation for pressure in the main chamber is assumed to be proportional to $e^{jm\theta}$, then

$$\sum \frac{\varepsilon_q}{n} a_{\mu q}^{(o)} \cos\left[q\pi(\theta-\mu n)/n\right] = e^{j\bar{m}\theta} \tag{A-19}$$

$$a_{\mu q}^{(o)} = \int e^{j\bar{m}\theta} \cos\left[q\pi(\theta-\mu n)/n\right] d\theta \tag{A-20}$$

These equations were used as the basis for the annular version of the computerized model.