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## REVISION OF GEODETIC PARAMETERS

E. M. Gaposchkin and M. R. Williamson

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\section*{REVISION OF GEODETIC PARAMETERS}
E. M. Gaposchkin and M. R. Williamson

\section*{ABSTRACT}

Laser data from nine satellites and 12 stations are combined with surfacegravity data to ot in spierical harmonics representing the geopotential complete through degree and order 18. This laser-data-only solution provides a reasonable improvement to the gravity field.

\section*{INTRODUCTION}

Smithsonian Astrophysical Observatory has published gravity-field solutions utilizing both satellite-tracking and terrestrial gravity data (see, e.g., Gaposchkin, 1974), which were based primarily on precision-reduced camera data and on then-available surface-gravity data. Now, however, :e have better data, and new types of data will soon be available. The work reported here is the beginning of a general revision and extension of our knowledge of the geopotential. In this first iteration, we will experiment with new data (?aser ranges), verify new methods of data reduction, and prepare for new types of data (altimeter and satellite-to-satellite tracking data). We seek here to improve our knowledge of the gravity field so that better satellite orbits, consistent with surface-gravity data, can be calculated. This is done by using laser data only, whose accuracy (approximately 1 m ) is far greater than the ephemeris accuracy (approximately 10 m ). The situation has changed since 1971, when the bulk of the data used were 4 -arcsec camera data. Furthermore, substantial revisions have been made in the treatment of orbit perturbations, and all these
advances indicate a new attempt at improved geopotential coefficients.

\section*{OBJECTIVES}

This computation has several objectives, the primary one being to use laser data only in a determination of the earth's gravity field, with the aim of computing satellite orbits to an accuracy comparable with that of the laser data. We can obtain a realistic gravity field consistent with surface-gravity data. A secondary objective is to study the consequences of using data that provide no ties to an inertial reference frame, as was the case with camera data. For our third objective, we will investigate the effects of one satellite on another in the solution; that is, we can optimize a solution for one satellite by using only data from that satellite. The question then becomes: How much does adaing data from a second satellite degräde the orbit computed for the first? Of course, improved orbits and a more accurate geoid are necessary for analyzing satellite altimetry data for geodetic and oceanographic purposes.

\section*{REFINEMENTS AND TECHNIQUES}

Improvements in perturbation calculations have been numerous. The inclination function for tesseral harmonics, as formulated by Kaula, computationally loses accuracy for high-degree coefficients. It has bern replaced by a mathematically equivalent formula derived from group theory (Gaposcikin, 1973). The interaction terms between \(\mathrm{J}_{2}\) and resonant harmonics have also been improved. Lunar and solar perturbations and body tides and ocean tides have been computed to the necessary accuracy (Kozai, 1973). Perturbations arising from the noninertialness of the coordinate system have been corrected and improved (Kinoshita, 1975a,b),
and those due to direct solar radiation pressure (Aksnes, 19.5), albedo pressure (Lautman, 1975a,b), and infrared radiation (Lautian, 1975c) have all been included and tested (Gaposchkin et al., 1975).

The compilation of surface-gravity data used in Gar,oschkin (1974) ras since been augmented (Williamson and Gaposchkin, 1975). The surface-gravity data are summarized in Table 1, and the distribution of these data is shown in Figure 1. Table 2 compares the \(1^{\circ} \times 1^{\circ}\) gravity anomalies from the Defense Mapping Agency Aerospace Center (DMAAC, 1973) with other available compilations.

Three coordinated programs have provided laser tracking data in sufficient density to be used for a gravity-field determination: the International Satellite Geodesy Experiment (ISAGEX) held in 1971, the Earth Physics Satellite Observation Campaign (EPSOC) in 1972-1973, and the Geos 3 campaign in 1975. There has been a steady improvement in the volume, reliability, and accuracy of the data. One of the objectives of the ISAGEX program was to obtain data for determining the gravity field, and a number of orbital arcs are suitable for this purpose. The EPSOC program was directed to the study of long-period effects and polar motion. Some arcs from EPSOC are also used for determining the tesseral harmonics. Finally, during the current Geos 3 prograr routine data are being obtained and included in the analysis. Table 3 lists the participating stations for each campaign that are used in the analysis reported here. The distribution of the stations is shown in Figure 2.

Currently, nine satellites in orbit are equipped with cube-corner reflectors suitable for laser ranging; these satellites are useful for a cravity-field determination, especially as their distribution in inclination and height is reasonably good. Table 4 lists their orbital characteristics and the number of arcs used, and the distribution of satellites is given in Figure 3.

Satellite orbits are usually computed for between 8 and 16 days, the interval depending primarily on the availability of data. Also, each arc covers at least one oscillation of the resonant period. Table 5 gives the resonant periods for the satellites, while Table 6 lists the constants used to calculate the satellite orbits.

\section*{SOLUTIONS}

The normal equations for surface-gravity data have been computed complete from degree 2 through 18. The combination solution included a number of harmonics of higher degree that are resonant with one or more of the satellite orbits. To this set of surface-gravity normal equations was added a set of normal equations, satellice by satellite. The system was solved after adding one satellite (Geos 1), and the result has been compared with all the satellites and with surface gravity. For the satellite comparison, one arc from each satellite was selected: generally, the arc with the most data. The orbit was recalculated with the revised gravity field, and the orbital fit in terms of \(\sigma_{0}\) was used as the criterion. In all combinations, the satellite data were used at their a priori weight. The surface gravity was given se eral weights, and all solutions were tested in order to determine the optimum weight for the combination.

The weight finally adopted is
\[
<A>\frac{27}{n A} \text { mgal },
\]
where \(n\) is the number of \(1^{\circ} \times 1^{\circ}\) squares in each \(5^{\circ} \times 5^{\circ}\) mean, \(A\) is the area of the anomaly, and <A> is the average area. This weight is twice that used in the 1973 Smithsonian Standard Earth (III) (SE III) (Gaposchkin, 1973). The ISAGEX laser data were given a \(5-\mathrm{m}\) weight, and all other laser data, a \(2-\mathrm{m}\) weight.

These combination solutions are summarized in Tab' 1. The orbit for a satellite not used in the solution is really very poor, generally because of relatively small changes in a few coefficients resonant with the gravity field. We note that

Peole, which has no resonance, degrades marginally. Tre surface gravity \(<\left(\Delta g_{t}-\Delta 3_{s}\right)^{2}>\) is only relative. A large part of those residuals is information in the higher harmonics. The best fit is obtained with only one satellite; however, when satellite data are added, the degradation is not large considering the overall accuracy of surface-gravity data.

Generally, adding satellite data also degrades the satellite orbit fit, but not very much. This overall improvement is considered quite satisfactory for one iteration of a very complex nonlinear process. All satellite orbits improved by at least \(1 \mathrm{~m}^{2}\) in the orbital fit. In percentage terms, Geos 1, BE-C, Geos 2, Starlette, Peole, and Geos 3 each improved in orbital fit by 17.9, 22.?, 43.9, \(69.2,26.2\), and \(13.6 \%\), respectively, for an average \(32 \%\) improvement!

The final adopted \(52-\operatorname{arc}\) solution (SE IV.1) can be compared with the surface-gravity data in more detail. Assuming they are statistically independent, the following quantities defined by Kaula (1966) can be computed and use to compare a geopotential model \(\left(g_{s}\right)\) with observed values of surface gravity \(\left(g_{t}\right)\) : \(\left\langle\mathrm{g}_{\mathrm{t}}^{2}\right\rangle \quad\) The mean value of \(\mathrm{g}_{\mathrm{t}}^{2}\), where \(\mathrm{g}_{\mathrm{t}}\) is the mean free-air gravity anomaly based on surface gravity, indicating the amount of information contained in the surface-gravity anomalies.
The mean value of \(g_{s}^{2}\), where \(g_{s}\) is the mean free-air gravity anomaly computed from the geopotential model, indicating the amount of information in the computed gravity anomalies.

An estimate of \(g_{h}-i . e\). , the true value of the contribution to the gravity anomaly of the geopotential wodel and the amount of information common to both \(g_{t}\) and \(g_{s}\).
The mean-square difference of \(g_{t}\) and \(g_{s}\).
\(E\left(\varepsilon_{s}^{2}\right)\)
\(\mathrm{E}\left(\varepsilon_{\mathrm{t}}^{2}\right)\)
E ( \(\delta \mathrm{g}^{2}\) )
The mean-square error in the geopotential model.
The mean-square error of the observed gravity.
The mean square of the error of omission - that is, the difference between true gravity and \(\mathbf{g}_{\mathrm{h}}\); this term is then the model error.

If the geopotential model were perfect, then \(\left\langle g_{\mathrm{s}}^{2}\right\rangle=\left\langle\mathrm{g}_{\mathrm{h}}^{2}\right\rangle\), which in turn would equal \(\left\langle g_{t} g_{s}>\right.\) if \(g_{t}\) were free from error and known everywhere. Then, \(\varepsilon_{s}^{2}\) would be zero even though \(g_{s}\) would not contain all the information necessary to describe the total field. The information not contained in the model field i.e., the error of omission, \(\delta \mathrm{g}\) - then consists of the higier order coefficients. The quantity \(<\left(g_{t}-g_{s}\right)^{2}>\) is a measure of the agreement between the two estimates \(g_{\mathrm{t}}\) and \(\mathrm{g}_{\mathrm{s}}\) and is equal to
\[
\left\langle\left(\mathrm{g}_{\mathrm{t}}-\mathrm{g}_{\mathrm{s}}\right)^{2}\right\rangle=\mathrm{E}\left(\varepsilon_{\mathrm{s}}^{2}\right)+\mathrm{E}\left(\varepsilon_{\mathrm{t}}^{2}\right)+\mathrm{E}\left(\delta \mathrm{~g}^{2}\right)
\]

Another estimate of \(g_{h}\) can be obtained from the gravimetric estimates of degree variance \(\sigma_{\ell}^{2}\) (Kaula, 1965):
\[
\mathrm{E}\left(\mathrm{~g}_{\mathrm{h}}^{2}\right)=\mathrm{D}=\sum_{\ell} \frac{\mathrm{n}_{\ell}}{2 \ell+1} \sigma_{\ell}^{2}
\]
where \(n_{\ell}\) is the number of coefficients of degree \(\ell\) included in \(g_{h}\), and
\[
\sigma_{\ell}^{2}=\gamma^{2}(\ell-1)^{2} \sum_{\mathrm{m}}\left(\overline{\mathrm{C}}_{\ell \mathrm{m}}^{2}+\overline{\mathrm{S}}_{\ell \mathrm{m}}^{2}\right)
\]

We also have
\[
E\left(\varepsilon_{s}^{2}\right)=\left\langle g_{s}^{2}\right\rangle-\left\langle g_{s} g_{t}\right\rangle
\]
and
\[
\mathrm{E}\left(\varepsilon_{\mathrm{t}}^{2}\right)=\left\langle\mathrm{g}_{\mathrm{t}}^{2}\right\rangle /\langle\mathrm{n}\rangle .
\]

These values are given in Table 8 for SE III and for this solution.

The information in the surface-gravity data solution \(\left\langle g_{t}^{2}\right\rangle\) has increased in this new data set. This is reasonable; since the unobserved areas had an expected value of zero, the fewer observed areas there are, the lower the variance is. However, the information in the satellite solution \(\left\langle\mathrm{g}_{\mathrm{s}}^{2}\right\rangle\) has decreased, a fact that is confirmed by a decrease in \(D\). Therefore, the information in SE III was too high. The residual \(<\left(g_{t}-g_{S}\right)^{2}>\) has remained roughly the same, while the information in the higher harmonics is estimated to be large:. live estimate of \(E\left(\varepsilon_{S}^{2}\right)\) cannot be good, as these sets of data, \(g_{S}\) and \(g_{t}\), are not independent.

The spherical-harmonic coefficients are listed in Tables 9 and 10. Figure 4 is a plot of the mean potential coefficient as a function of degree, and Figures 5 and 6 show the geoid height and gravity anomalies for this solution.

FUTURE WORK

The obvious next step is to complete another iteration, taking the solution to perhaps degree and order 24. To this can be added the normal equations for zonal harmonics and sets of resonant harmonics. When the orbital accuracy approaches a few meters, then we must reduce the error in the station coordinates by solving for them. Finally, of course, the aim is to add altimetry data to this system of normal equations.

In summary, we find that the improved accuracy that has been apparent from laser data is becoming realized.

\section*{ACKNOWLEDGMENT}

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Table 1. Surface-gravity data available.
\begin{tabular}{|c|c|c|c|}
\hline \multirow[t]{2}{*}{Source} & \multirow[t]{2}{*}{\[
1^{\circ} \times 1^{\text {No. of }} \text { means }
\]} & \multicolumn{2}{|l|}{No, of \(550 \times 550 \mathrm{~km}\) गlocks} \\
\hline & & n 2 & \(n=25\) \\
\hline \begin{tabular}{l}
Gaposchkin (1974) \\
Williamson and Gaposchkin
\end{tabular} & 19328 & 1183 & 145 \\
\hline Maximum number (1975) & \[
\begin{aligned}
& 31636 \\
& 64800
\end{aligned}
\] & 1452
1690 & 485
1690 \\
\hline
\end{tabular}

Table 2. Comparison of \(1^{\circ} \times 1^{\circ}\) mean gravity anomalies with DMAAC (1973).
\begin{tabular}{lccc}
\hline Source & \begin{tabular}{c} 
No. of points \\
compared
\end{tabular} & \begin{tabular}{c} 
Mean difference \\
(mgal)
\end{tabular} & \begin{tabular}{c} 
rms \\
(mgal)
\end{tabular} \\
\hline \begin{tabular}{l} 
Australia (Mather, 1970) \\
North America and North Atlantic \\
(Talwani et al., 1972)
\end{tabular} & 1364 & 1.64 & 24.16 \\
\begin{tabular}{l} 
Indian Ocean \\
(Kahle and Talwani, 1973) \\
Worldwide (f.CIC, 1971)
\end{tabular} & 3613 & -0.18 & 15.29 \\
\hline
\end{tabular}

Table 3. Lasers stations used in this analysis.
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{3}{|c|}{Station} & ISAGEX & EPSOC & Geos 3 \\
\hline Number & Location & Agency & & & \\
\hline 7902 & 01 fantsfontein, S. Africa & SAO & x & x & \(\times\) \\
\hline 7907 & Arequipa, Peru & SAO & x & x & X \\
\hline 7921 & Mt. Hopkins, Arizona & SAO & x & x & x \\
\hline 7928 & Natal, Brazil & SAO & x & x & x \\
\hline 7930 & Athens, Greece & SAO & x & & x \\
\hline 7050 & GSFC & NASA & x & & x \\
\hline 7060 & Guam & NASA & x & & \\
\hline 7061 & San Diego, Calif. & NASA & & x & \\
\hline 7080 & Quincy, Calif. & NASA & & x & \\
\hline 7068 & Grand Turk Island & NASA & & & x \\
\hline 7804 & San Fernando. Spain & CNES & \({ }^{x}\) & & \\
\hline 7809 & Haute Prov il France & CNES & x & & \\
\hline
\end{tabular}

Table 4. Summary of dynamical data.
\begin{tabular}{lllllll}
\hline \multicolumn{2}{c}{ Satellite } & Inclination & Eccentricity & \begin{tabular}{c} 
Perigee \\
height \\
\((\mathrm{km})\)
\end{tabular} & \begin{tabular}{c} 
a \\
\((\mathrm{km})\)
\end{tabular} & \begin{tabular}{c} 
Number \\
of \\
arcs
\end{tabular} \\
\hline 7010901 & Peole & \(15^{\circ}\) & 0.017 & 635 & 7070 & 5 \\
6701401 & D1D & 39 & 0.053 & 569 & 7337 & 3 \\
6701101 & DIC & 40 & 0.052 & 579 & 7336 & 2 \\
6503201 & BE-C & 41 & 0.026 & 941 & 7311 & 9 \\
7501001 & Starlette & 50 & 0.0207 & 805 & 7335 & 5 \\
6508901 & Geos 1 & 59 & 0.073 & 1121 & 8074 & 14 \\
7502701 & Geos 2 & -65 & 0.0005 & 840 & 7222 & 4 \\
6800201 & Geos 3 & -75 & 0.031 & 1101 & 7709 & 8 \\
6406401 & BE-B & 80 & 0.012 & 912 & 7362 & 2 \\
& & & & & & - \\
& & & & & Total & 52 \\
\hline
\end{tabular}

Table 5. Resonant periods.
\begin{tabular}{ccc}
\hline Satellite & \begin{tabular}{c} 
Resonant with \\
order m
\end{tabular} & \begin{tabular}{c} 
Period \\
(days)
\end{tabular} \\
\hline 7010901 & none & \\
6701401 & 13 & 9.4 \\
6701101 & 14 & 2.6 \\
6503201 & 13 & 5.6 \\
7501001 & 14 & 3.2 \\
6508901 & 12 & 7.2 \\
7502701 & 14 & 3.9 \\
6800201 & 13 & 6.3 \\
6406401 & 14 & 2.9 \\
\hline
\end{tabular}

Table 6. Constants used in orbit computation.

Table 7. Comparison of solutions.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{Solution} & \multirow[t]{2}{*}{No. of arcs} & \multicolumn{2}{|l|}{\[
\begin{gathered}
\text { Surface gravity } \\
<\left(\Delta g_{t}-\Delta g_{\mathrm{S}}\right)^{2}>, \ell \leq 18 \\
\left(m g a l^{2}\right)
\end{gathered}
\]} & \multicolumn{2}{|l|}{Geos 1} & \multicolumn{2}{|l|}{BE-C} & \multicolumn{2}{|l|}{Geos 2} & \multicolumn{2}{|l|}{Starlette} & \multicolumn{2}{|l|}{Peole} & \multicolumn{2}{|l|}{Geos 3} \\
\hline & & \(n \geq 1\) & \(n \geq 20\) & \(\sigma_{0}\) & n & \(\sigma_{0}\) & a & \(\sigma_{0}\) & n & \(\sigma^{\circ}\) & n & \% & \(n\) & \({ }^{\circ}\) & n \\
\hline SE III & (203) & 193 & 135 & 4.53 & 3021 & 5.81 & 1699 & 5.58 & 1112 & 16.61 & 2443 & 15.41 & 805 & 13.16 & 1076 \\
\hline Geos 1 & (5) & 123 & 86 & 3.15 & 3033 & 38.87 & 1688 & 32.60 & 1126 & 24.14 & 2336 & 16.16 & 797 & & \\
\hline \begin{tabular}{l}
and \\
BE-C
\end{tabular} & (5) & 130 & 89 & 3.12 & 3004 & 4.22 & 1701 & 133 & 1140 & 770 & 2344 & 12.35 & 796 & & \\
\hline and Geos 2 & (5) & 132 & 91 & 3.38 & 3012 & 4.20 & 1689 & 3.23 & 1125 & 18.11 & 2316 & 12.59 & 794 & & \\
\hline \begin{tabular}{l}
Geos 1 \\
BE-C \\
Geos 2 \\
BE-B \\
DID \\
Peole \\
Starlette
\end{tabular} & \[
\begin{array}{r}
(14) \\
(9) \\
(8) \\
(2) \\
(3) \\
(2) \\
(5) \\
(5) \\
(5)
\end{array}
\] & 139 & 99 & 3.58 & 3011 & 4.50 & 1696 & 3.32 & 1130 & 4.96 & 2259 & 12.40 & 807 & & \\
\hline \begin{tabular}{l}
and \\
Geos 3
\end{tabular} & (4) & 142 & 100 & 3.72 & 3022 & 4.52 & 1695 & 3.13 & 1122 & 5.12 & 2258 & 11.38 & 794 & 11.37 & 10i5 \\
\hline & &  & & & & & 16 & & & & & & & & \\
\hline
\end{tabular}
Table 8. Comparison of surface gravity with solution \(\left(\mathrm{mgal}{ }^{2}\right)\); all solutions are to the 18 th degree and order
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline Solution & \(\left.\left\langle g_{t}-g_{s}\right)^{2}\right\rangle\) & \(\left\langle g_{t} g_{s}\right\rangle^{\prime}\) & \(\left.{ }^{\langle g}{ }_{s}^{2}\right\rangle\) & D & \(\left\langle g_{t}^{2}\right\rangle\) & \(E\left(\varepsilon_{s}^{2}\right)\) & \(E\left(\varepsilon_{t}^{2}\right)\) & \(E\left(\delta_{\text {g }}{ }^{2}\right)\) & n* \\
\hline SE \(\mathrm{HII}^{+}\) & 156 & 202 & 258 & 237 & 302 & 56 & 24 & 75 & \(n \geq 1\) (1183) \\
\hline SE III \({ }^{+}\) & 117 & 255 & 281 & 237 & 345 & 26 & 19 & 63 & \(n \geq 10 \quad\) (659) \\
\hline SE IIIt & 105 & 221 & 236 & 237 & 311 & 15 & 13 & 77 & \(n \geq 20\) (301) \\
\hline SE III & 135 & 237 & 254 & 235 & 356 & 16 & 15 & 104 & \(n \geq 20 \quad\) (678) \\
\hline SE IV. 1 & 142 & 192 & 217 & 227 & 309 & 25 & 19 & 98 & \(n \geq 1\) (1452) \\
\hline SE IV. 1 & 103 & 232 & 235 & 227 & 333 & 2 & 16 & 85 & \(n \geq 10 \quad(1056)\) \\
\hline SE IV. 1 & 100 & 250 & 244 & 227 & 356 & -6 & 15 & 92 & \(n \geq 20\) (678) \\
\hline
\end{tabular}


\section*{FIGURE CAPTIONS}

Figure 1. Distribution of \(1^{\circ} \times 1^{\circ}\) mean surface-gravity data.
Figure 2. Locations of the observing stations included in SE IV.I.
Figure 3. Distribution of perigee heights and inclinations of the satellites used in SE IV.I.

Figure 4. Mean posential coefficient by degree.
Figure 5. SE IV.l geoid height in meters calculated with respect to the bestfitting ellipsoid, \(f=1 / 298.256\).

Figure 6. SE IV.l gravity anomalies in milligals calculated with respect to the best-fitting ellipsoid, \(f=1 / 298.256\).

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ORIGINAL PaGE IS
OF POOR QUALITY

Figure 2. Locations of the observing stations included in SE IV.1.

Figure 3. Distribution of perigee heights and inclinations of the satellites used in SE IV.1.


Figure 5. SE IV. 1 geoid height in meters calculated with respect to the

Figure 6. SE IV. 1 gravity anomalies in milligals calculated with respect```

