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THE TOTAL ASSESSMENT PROFILE

VOLUME II

by

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APPENDIX A

Parametric Modeling Of Discounted Benefit-Sacrifice Streams

INTRODUCTION

This Appendix provides an introduction to modeling of benefit-sacrifice streams by parametric approach: benefit and sacrifice streams are shaped in terms of mathematical functions with an appropriate number of shaping parameters that are interpreted in some relevant manner. Each parameter can be treated also as a "random variable" whereby the benefit-cost streams can be subjected into effects of a multidimensional uncertainty. Although this Appendix treats primarily internal rate of return models, the extensions can be made to any other discounted streams.

SOME BASIC DEFINITIONS AND CONCEPTS OF BENEFIT-SACRIFICE STREAMS

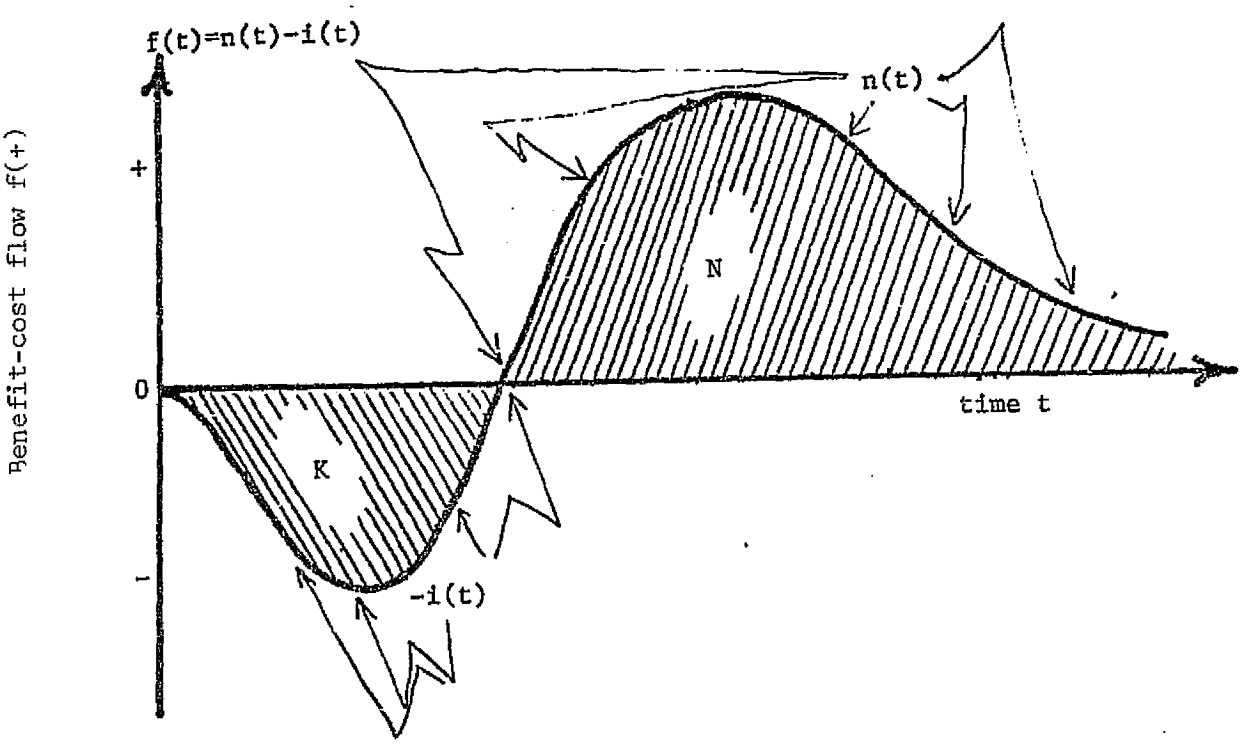


Figure A-1

Figure A-1 illustrates a typical net benefit-cost or benefit-sacrifice stream given by a function f(t). By definition, when the net f(t) is negative, the flow is called net sacrifice flow or "investment flow", -i(t); and when it is nonnegative, it is called net returns benefit flow, +n(t). i(+) is then a positive flow running out from the pocket, while n(+) is a positive flow running into the pocket.

Further,

$$K = \int_{t=0}^{\infty} n(+)dt$$

is the total volume of sacrifice or total invested value in the program while

$$N = \int_{t=0}^{\infty} n(+)dt$$

is the total net returns benefit volume.

The internal rate of return from the program, r, is of the form

$$r = \frac{1}{Tpb} f\left(\frac{N}{K}\right)$$

Where Tpb is the so-called pay-back period possible only if N>K in Figure A-1. $f\left(\frac{N}{K}\right)$ is a monotonic increasing function of the benefit to sacrifice volume ratio N/K. The payback period Tpb for N>K is defined as follows:

$$K = \int_{0}^{Tpb} n(+)dt$$

In Figure A-2, also the break-even point T_{be} is indicated.

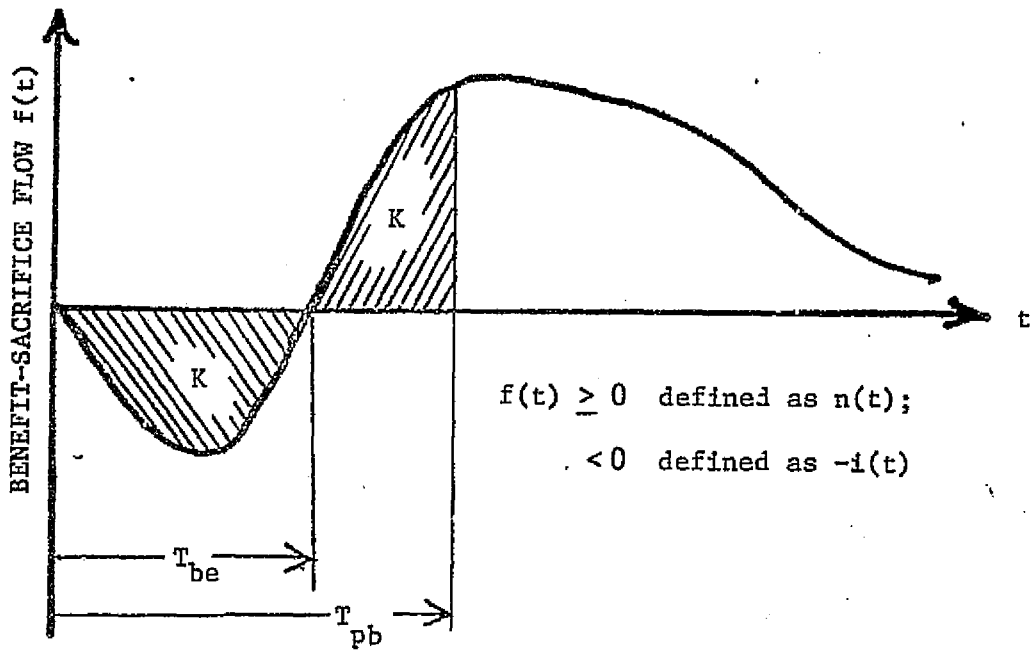


Figure A-2. Break-even and pay-back periods of a Benefit-sacrifice flow.

The discounted net benefit-net sacrifice flow g(t) is related to the net benefit-net sacrifice flow f(t) as follows:

$$g(t) = f(t)\, e^{-rt}$$

r here is the rate of discounting and has dimension 1/year or inverse year. Figure A-3 illustrates a flow f(t) and its discounted flow g(t).

The internal rate of return on investment, r, of the total flow f(t) is that rate of discounting for which

$$\int_0^\infty f(t)\, e^{-rt} dt = \int_0^\infty g(t)\, dt = 0$$

i.e. is that value of r for which the discounted total value of volume invested equals to the discounted total net returns volume. Let K' be this discounted total dollar value invested, and N' be the total value volume net returns. Then the internal rate of return on investment is that value of discount rate r for which

$$K' = \int_0^\infty i(t) e^{-rt} dt = \int_0^\infty n(t)\, e^{-rt} dt = N'$$

One notes then that there is this one particular rate of discounting that satisfies the above equality on the nose.

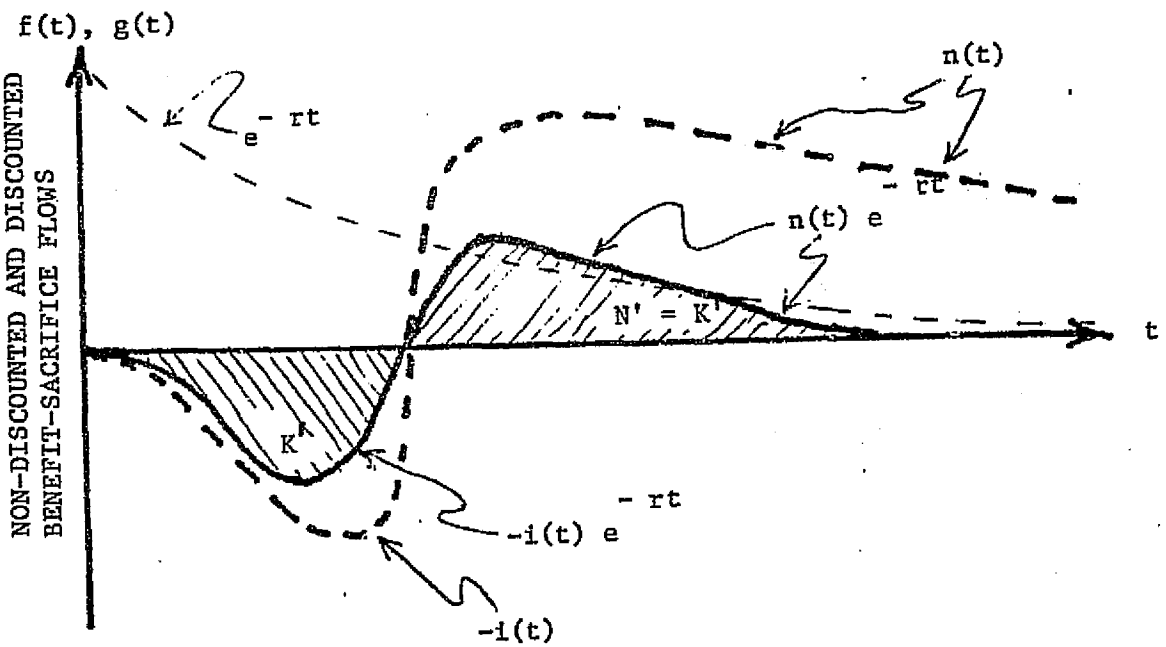


Figure A-3.

For a POSITIVE rate of return on investment, r, one must have initially $N>K$ in order that $N' = K'$. If $N = K$, then $r = 0$, and if $N<K$, then r must be made negative in order to make $N' = K'$. The effect of how n(t) and i(t) are distributed over time will become clarified by numerous subsequent examples.

The method of continuous discounting has been chosen here rather than the conventionally accepted method of discrete discounting. There are some very good reasons for this. First, continuous discounting turns out to be mathematically simpler than the discrete discounting method. Second, it is much easier to find the right and unique value of the rate of return on investment for a particular problem in the case of continuous discounting than in the case of discrete discounting. Third, it is much easier to introduce parametric cash flow models subject to continuous discounting than those subject to discrete discounting in order to find the rates of returns in terms of these parameters in closed forms. Fourth, it is rather simple to discount discrete cash flow models by continuous discounting so that basically nothing is lost even in discrete cases by using continuous discounting techniques. Fifth, appropriate dimensionality of various financial and other business parameters becomes properly accounted for by the use of continuous discounting techniques. The general overall advantages of continuous cash flow and discounting techniques over the conventional discrete methods of discounting becomes obvious as one introduces several examples in the subsequent treatment. It is useful to review a simple case of discrete discounting and then show how one can introduce from it the case of continuous discounting.

A REVIEW OF CONCEPTUAL RELATIONSHIPS AND DIFFERENCES BETWEEN THE DISCRETE PERIODIC DISCOUNTING METHODS AND THE CONTINUOUS DISCOUNTING METHODS

A commonly used conventional method of discounting is the discrete one where the event of discounting occurs at definite intervals of time. Typically the event of discounting occurs once per annum, semi-annually, or quarterly.

Consider a program for which it is desired to invest K dollars in a productive asset at the <u>beginning</u> of the first period ($k = 1$). Let N_k be the lump sum dollar <u>net return</u> for the kth period and let J be the residue value of the asset in dollars at the end of its life at the mth period's end. Then the usual "present value equation" for this case is as follows:

$$K = \sum_{k=1}^{m} \frac{N_k}{(1 + r_o)^k} + \frac{J}{(1 + r_o)^m}$$

It is very important to note that $N_1, N_2, \ldots, N_k, \ldots, N_m$ are all sums of net dollars obtained over the respective periods. That is, the dimensionality of N_k is dollars. Thus the above sequence of dollar sums is not yet properly a "cash flow" although it is so called at times. Further, one should note that r_o here is a dimensionless discounting factor (i.e. not a discounting RATE) that can be eventually related to a discounting RATE. If $K, J, N_1, N_2, \ldots, N_k, \ldots, N_m$ are all given then the problem is to find an appropriately UNIQUE value r_o satisfying the above equation. By a fundamental theorem of algebra one will find m roots for the polynomial in r_o from the above equation. Many of these roots could be negative or complex, and many of them could be positive. A tedious problem facing an analyst is to pick up a unique real valued root that corresponds to the RELEVANT fraction of return on investment r_o for his problem. It will be seen that this tedious aspect of the discounting becomes greatly simplified by shifting to the continuous discounting methods.

The relationship between the fraction or return on investment r_o and the rate of return on investment r (note, r must have a dimension of INVERSE TIME) can be handled by introducing a standard unit period of time (say, one year) θ whereby

$$r = r_o/\theta \quad \text{or} \quad r_o = r\,\theta\ .$$

Thus, since r_o is a dimensionless fraction, r now has the dimensionality of inverse years or 1/years. The percentage rate of return on investment is just 100 r. In terms of the standard period θ and the RATE of return on investment r one can rewrite the present value equation in the following form:

$$K = \sum_{k=1}^{m} N_k/(1 + \theta\, r)^k + J/(1 + \theta\, r)^m$$

Here then the discounting is done once per period θ (say, once a year).

Assume now that one wants to discount p times per period θ where p is a positive real number greater than one. Thus the NEW discounting period is NO MORE θ but is now θ/p. Further, the old index $k = 1,2,\ldots,m-1$, m must be replaced by a new index $k(p)$ which depends on p, $k(p) = 1,2,3,4, \ldots, pm$. Note also that the net return dollar sums over the old periods of length θ, $N_1, N_2, \ldots, N_k, \ldots, N_m$ are now to be replaced by new dollar sums of about only 1/p times the old values over the respective time periods. However, one still has the following equality:

$$N = \sum_{k=1}^{m} N_k = \sum_{k(p)=1}^{mp} N_{k(p)}$$

Note, the old k^{th} interval of time was divided to p parts so that over it the old N_k would be replaced roughly b new values $N_k/p = N_{k(p)}$. Thus one now has a present value equation where the discounting is done not once per period θ but p times over this period, or once per a new period θ/p:

$$K = \sum_{k(p)=1}^{pm} N_{k(p)}/ [1 = (\theta/p)r]^{k(p)} + J/[1 + (\theta/p)r]^{mp}$$

$$= \sum_{k(p)=1}^{pm} [[N_{k(p)}/(\theta/p)](\theta/p)]/[(1+\frac{\theta}{p} r)^{p/\theta r}]^{(\theta r/p)k(p)} + J/[(1 + \frac{\theta}{p} r)^{p/\theta r}]^{(\theta r/p)mp}$$

It is now relatively easy to see what happens if one lets p become large or θ/p, respectively, small. As p approaches ∞, one can then introduce the following notations and limit values:

1. $\theta/p \to dt$, an infinitesimal increment of time;

2. $[\theta/p]\, k(p) \to t$, the continuous time variable;

3. $N_{k(p)}/(\theta/p) \to dN(t)/dt = n(t)$ the true CASH FLOW with the correct dimensionality of DOLLARS/YEAR; n(t) is then the expected net returns cash flow.

4. Further one obtains the following well known limit:

$$\lim_{p \to \infty} [1 + (\theta/p)r]^{p/\theta r} = e$$

Thus, as p goes to infinity, one obtains in the limit the following continuously discounted present value equation:

$$K = \int_{t=0}^{t=T} n(t)\, e^{-rt} dt + J\, e^{-rt}$$

If K, n(t), J and T are given then one is supposed to find the rate of discounting r that satisfies the above equation. This r is then called the rate of return on investment. For this equation recall now

K is the sacrified volume generating productive asset.

T is the planning time horizon or the planned life of the asset;

J is the expected residue value of the asset at the end t=T of its life;

n(t) is the expected net returns benefit flow in dollars per annum from the venture utilizing the productive asset;

r is the rate of return expected from the venture, in 1/years;

t is the continuous time variable in units of years.

One can also define the pay-back period T_{pb} for this case as follows: T_{pb} is that period of time for which

$$K - J = \int_{t=0}^{t=T_{pb}} n(t)\, dt \; .$$

It should be noted that if one cannot find a finite T_{pb} for a given K-J and n(t), then one should not get involved in such a venture. It should be intuitively clear that a venture should have a finite pay-back period in order that it would have a positive rate of return on investment.

Insofar as future events are concerned, one does not deal with certainties but rather with expectations subject to uncertainties. Thus T,K,J,n(t), T and r as well as T_{pb} are expected values in most cases when contemplated for future actions. It should also be clear that all such expected values are subject to the relevancy of technological, financial, production, etc. information necessary for putting together a meaningful benefit-sacrifice flow picture for the contemplated venture. Typically such pieces of information must be integrated together to a total meaningful picture or pattern caricaturing a venture in a reasonable manner. Once a relevant model for the venture is obtained, one can proceed to find whether a finite pay-back period would exist. If it does, then it is appropriate to find the magnitude for the rate of return on sacrifice for this contemplated venture.

REVIEW OF MATHEMATICAL CONCEPTS USEFUL FOR BENEFIT-SACRIFICE FLOW MODELS

The subsequent treatments of several benefit-sacrifice (B-S) flow models can be greatly faciliated and simplified by utilizing concepts of applied mathematics. For the cases of continuously discounted flows techniques of Laplace transforms and related mathematical concepts are useful.

UNIT STEP FUNCTION u(t-T) is defined as follows:

$$u(t-T) = 1 \text{ for all } t \geq T$$
$$= 0 \text{ for all other values of } t.$$

An example of a linear combination of unit step functions is the following "pulse function":

$$u(t) - u(t-T) = 1 \text{ for } 0 \leq t \leq T$$
$$= 0 \text{ for all other values of } t.$$

This kind of a pulse function allows one to cut off an appropriate portion of any time function. For example,

$$\int_0^T n(t)\, e^{-rt} dt = \int_0^\infty [u(t) - u(t-T)]\, n(t)\, e^{-rt} dt$$

DELTA FUNCTION OR IMPULSE FUNCTION $\delta\ (t-T)$ is defined as follows:

$$\delta\ (t-T) = 0 \text{ fol all } t \neq T$$
$$\int_{-\infty}^{\infty} \delta(t-T)\ dt = 1$$

An immediately useful application for this function is the case where K dollars are laid out instantly for the purchase of an asset. In this case the investment outlay cash flow would be as follows:

$$-\ i(t) = -\ K\delta(t)\ .$$

This would be then the investment outlay cash flow in the "present value" case, as will be pointed out later on.

THE LAPLACE TRANSFORM OF A FUNCTION f(t) is defined as follows:

$$L[f(t)] = F(r) = \int_0^{+\infty} f(t)\, e^{-rt} dt$$

The Laplace transforms for many different given time functions f(t) are tabulated in tables.

Some useful examples of Laplace transforms are the following ones:

$L[i_o\ u(t)] = i_o/r$ $\qquad$ $L[K\delta(t)] = K$

$L[n_o u(t-T)] = (n_o/r)\ e^{-rT}$ $\qquad$ $L[N\delta(t-T)] = N\ e^{-rT}$

For any function f(t) that is shifted to the right by the amount T of time one has:

$$L[u(t-T)f(t-T)] = e^{-rT} L[f(t)] = e^{-rT} F(r)\ .$$

Further discussion on how to use Laplace transform techniques will be done in the conjunction of several examples which are introduced subsequently.

PERIODIC FUNCTION: The function $f(t)$ is said to be periodic if for $n = 0,1,2,3,4,\ldots$ it is true that $f(t) = f(t-nT)$. Then T is called the period of this periodic function.

If one has the integral of the form

$$\int_0^T f(t)e^{-rt}dt$$

then it can be rewritten into the following form:

$$\int_0^T f(t)e^{-rt}dt = \int_0^\infty [u(t)-u(t-T)]f(t)e^{-rt}dt = L[(u(t)-u(t-T))f(t)]$$

$$= F(r) - \int_0^\infty u(t-T)f(t)e^{-rt}dt.$$

If $f(t)$ is either constant or periodic with period T, then

$\int_0^\infty u(t-T)f(t)e^{-rt}dt = e^{-rT}F(r)$ and so for this particular case

$$\int_0^T f(t)e^{-rt}dt = F(r)[1 - e^{-rT}] \text{ or } F(r) = \frac{\int_0^T f(t)e^{-rt}dt}{[1 - e^{-rT}]}$$

Further, note if $rT>>1$ then one can use the approximation where

$\int_0^T f(t)\, e^{-rt}dt \approx F(r)$. This can, indeed, simplify treatment of several cash flow discounting problems.

THE INTERNAL RATE OF RETURN ON SACRIFICE

The intrinsic or internal rate of return on sacrifice for a B-S flow associated with the respective venture is a commonly accepted and used measure for expected success. Consider the total B-C flow $f(t)$ whose negative portions represent investment outlay (and thus with negative sign) and whose positive portions represent the expected net returns flow (hence with positive sign). If $i(t)$ is the investment cash flow and $n(t)$ is the net returns cash flow, then

$$f(t) = n(t) - i(t) .$$

In general then the condition for obtaining the internal rate of return on investment is as follows:

$$\int_0^\infty f(t)\, e^{-rt} dt = \int_0^\infty [n(t) - i(t)]\, e^{-rt} dt$$

$$= \int_0^\infty n(t)\, e^{-rt} dt - \int_0^\infty i(t)\, e^{-rt} dt = 0$$

i.e.

$$\int_0^\infty i(t)\, e^{-rt} dt = \int_0^\infty n(t)\, e^{-rt} dt$$

or $L[i(t)] = I(r) = L[n(t)] = N(r)$.

In this case it has been assumed that the residue value of the asset is zero, i.e. $J = 0$. This assumption is also reasonable if $rT \gg 1$ for the case at hand. If the residue value cannot be ignored, then

$$f(t) = n(t) + J\delta(t-T) - i(t)$$

and

$$I(r) = N(r) + e^{-rT} J \ .$$

The "present value" case is one where $i(t) = K\delta(t)$ whereby then $I(r) = K$. In such a case one obtains the condition shown below:

$$K = N(r) + e^{-rT} J \ .$$

Here the outlay occurs at the time $t = 0$ in a one lump sum of a magnitude K so that it is a delta function with a strength K.

In an interim summary, then, the basic equations of interest are as follows:

In a general case

$$I(r) = N(r) + e^{-rT} J$$

where $I(r) = L[i(t)]$ and $N(r) = L[n(t)]$, J is the residue value or benefit of the asset at the end of its life occurring at $t=T$ where T is the life of the asset or planning time horizon, and r is the expected internal rate of return on the investment.

EXAMPLES OF DISTRIBUTED BENEFIT-LUMPSUM SACRIFICE MODELS

The distributed benefit-lump sum sacrifice streams represent cases where the sacrifice lump sum is the present value of the discounted benefit stream, and the rate of discounting is the internal rate of return.

EXAMPLE NO. 1

A program requires an asset of K dollars at the time t=0. Thus the investment cash outflow is just $- i(t) = - K\delta(t)$. The net returns from the venture occurs in a lump sum of N dollars at the time t=T so that the net returns cash inflow is $+ n(t) = + N\delta(t-T)$. The overall cash flow of this venture is then $f(t) = n(t) - i(t) = N\delta(t-T) - K\delta(t)$. For the internal rate of return on investment one has then the following requirement:

$$L[f(t)] = L[n(t)] - L[i(t)]$$

$$= N e^{-rT} - K = 0.$$

Solving for r yields the following expression for the rate of return on investment r:

$$r = (1/T) \ln (N/K) .$$

Figure A-4 illustrates the cash flow plot for this case.

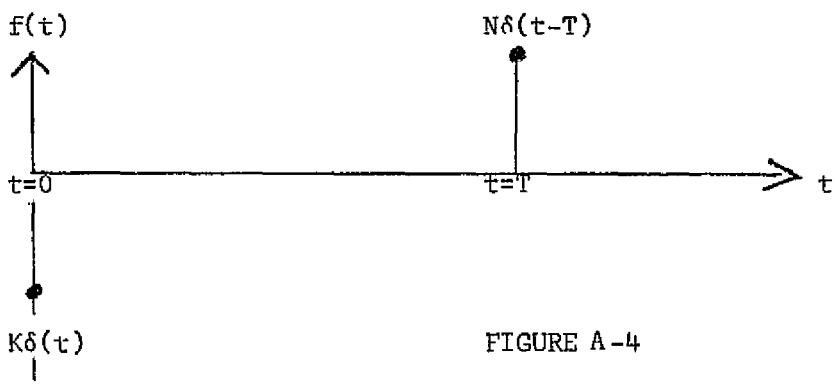


FIGURE A-4

For this model K (capital outlay) and N (net returns volume) are financial parameters while T is a non-financial time parameter. It is noted that the rate of return on investment is inversely proportional to T and directly proportional to the logarithm of the financial ratio N/K. Note then how strongly r varies with T and how mildly it varies with N/K. This example illustrates simply how important the element of time is in ventures. One can readily obtain the total differential for r in order to see how sensitive r is to the changes in T and N/K:

$$\Delta r = -[(\ln(N/K))/T]^2 \Delta T + [1/T(N/K)]\Delta(N/K)$$

EXAMPLE NO. 2

Let $i(t) = K\delta(t)$ as before but assume now $n(t) = n_o$ for $0 \leq t \infty$.

Then $$L[i(t)] = K = L[n(t)] = n_o/r \ .$$

Thus $$r = n_o/K \ .$$

Note K is in units of dollars while n_o is in dollars per year. Thus r has the correct dimension of a rate, i.e. inverse year. If $n)t) = n_o$ for $0 \leq t \leq T$ and zero elsewhere, then

$$K = (n_o/r)[1 - e^{-rT}] \ .$$

If $rT > 3$, then $\exp(-rT)$ is much smaller than unity whereby $r = n_o/K$ is quite a good approximation. Note that in this case $N = n_o T$ is now the net returns dollar volume so that one has the following expression:

$$rT/(1 - e^{-rT}) = N/K$$

or

$$K = [(1 - e^{-rT})/rT] \ N$$

The term in the square brackets is a dimensionless "time price of benefit value or money", i.e. the worth of N when "purchased" by K. Note that for high rate of return r N is cheap and for low rate of return r it becomes dear. For high r little K is needed to purchase a unit of N; for low r much K is needed to purchase a unit of N. Each venture has its N or net returns value volume that must be "purchased" by investing a value volume K. Thus one has a clear relationship between the time price of money and the rate of return on investment. However, other time parameters besides the rate of return on investment must enter into this expression of the time price of value. This will becomes more evident in subsequent discussions. It is for this and other good reasons that parametric flow models are extremely useful in venture analysis. Again one notes N/K enters in as a "financial" parameter or a benefit/sacrifice ratio while T and r are time parameters.

EXAMPLE NO. 3

Consider a venture facing a competition with a constant force of mortality, $h(t) = a$, and an exponertial survival probability, $\exp(-at)$. Assume no renovation force. The expected net returns cash flow for the venture is assumed to be proportional to the probability of its survival. It is appropriate to note the mean life of the venture is just 1/a in this case. Then the following net returns cash flow is assumed:

$$n(t) = n_o \ e^{-at}$$

$$N = n_o/a = n_o \ T_{pl}$$

Here $T_{pl} = 1/a$ is the expected life of the venture in the competitive market. The capital outlay is K for this venture. Then one obtains the following result for the rate of return on investment:

$$r = n_o/K - a = [(N/K)-1]a \ .$$

Thus the internal rate of return on investment is reduced linearly by the force of product mortality. That is, the force of mortality to an added discount rate on the benefit stream. Note further:

$$K = [a/(r+a)]\ N\ .$$

Thus the time price of N is the higher a gets in comparison to r, and approaches unity in the ultimate limit. One notes again that a and r are the two time parameters associated with the expression for the time price of N. But a is now the frequency called the force of venture mortality.

EXAMPLE NO. 4

The previous model can be expanded to a bit more interesting one by introducing some additional relevant parameters. Figure A-5 illustrates the basic setting. In this case the net returns flow need not rise instantly to its maximum value but rather gradually if this be relevant to the venture at hand. Thus the cash flow model has a $t=T_{max}$ point. It also has the force of product mortality, a. The other parameters include a characteristic time T_o and the financial parameters K and N or their equivalents. Thus one has gained some additional degrees of freedom to portray more complex cases than the ones discussed previously. With these added degrees of freedom or descriptive parameters one can shape net return flows with growth, maturation and decline characteristics found so often in everyday ventures. Yet one wishes to obtain closed form expressions that simplify greatly any computational efforts while retaining at the same time an easy pattern recognition for the total behavior of the venture.

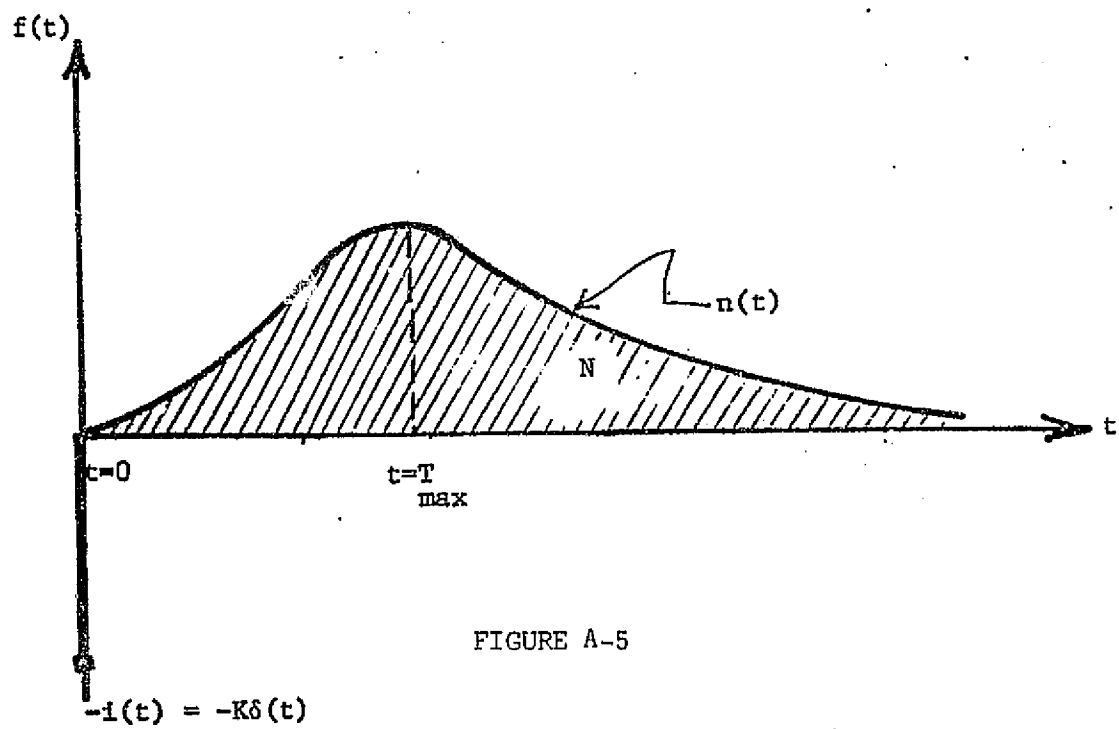


FIGURE A-5

The maximum expected net returns cash flow occurs at $t=T_{max}$. The expected net returns B-S flow model that could portray or caricature such phenomena is as follows:

$$n(t) = n_o(t/T_o)^k e^{-at} = [k!\ n_o/T_o^{\ k}](t^k/k!)\ e^{-at}$$

The constant n_o, in value units per annum, scales the general amplitude of the whole flow cycle. The exponential term with the force of product mortality a will assure eventual decline of the whole net benefit flow. The characteristic time parameter T_o and the dimensionless parameter k can be used to caricature the growth portion while k and a can be used to portray the maximum cash flow time T_{max}. Assuming J=0 dollars for the residual value of the productive assets, one has the following result after appropriate discounting operation:

$$K = [k!\ n_o/T^k][1/(r + a)]^{k+1}$$

$$r = [k!\ n_o/KT_o^k]^{1/(k+1)} - a$$

One can easily verify that the maximum of the expected net returns benefit flow occurs when

$$t = T_{max} = k/a\ .$$

Further, the net returns volume is obtained as follows:

$$N = \int_0^\infty n(t)\ dt = (n_o/T^k) \int_0^\infty t^k e^{-at}\ dt$$

$$= k!\ n_o/T_o^{\ k}\ a^{k+1}\ .$$

One has then, also, the following maximum value for the net returns benefit flow:

$$n_{max} = [k/eaT_o]^k\ n_o = (k/e)^k[aN/k!]\ .$$

This flow model can be then expressed in the following alternative but equivalent forms:

$$n(t) = (a^{k+1}/k!)\ N\ t^k e^{-at} = (ea/k)^k\ n_{max} t^k\ e^{-at}\ .$$

The internal rate of return on investment could be expressed in the form involving the benefit-cost ration N/K, the force of mortality a and the constant k:

$$r = a\ [(N/K)^{1/(k+1)} - 1]\ .$$

As an illustration, consider a case where the expected venture life is five years or, thus, the force of product mortality is a = 0.2 failures per year. Assume the net returns dollar volume N = 1,000,000 dollars. Let the capital outlay at the time t=0 be K= 200,000 dollars. Let then

```
100 DATA 4,0,40,1,0,7.5E4
200 LET A=(1E6)*((0.2↑2)/1)*(X↑1)*EXP((-0.2)*X)
210 LET B=(1E6)*((0.2↑3)/2)*(X↑2)*EXP((-0.2)*X)
220 LET C=(1E6)*((0.2↑4)/6)*(X↑3)*EXP((-0.2)*X)
230 LET D=(1E6)*((0.2↑5)/24)*(X↑4)*EXP((-0.2)*X)
RUNKEY

PLØTTØ     20:53  ACTS  WED. 09/03/69

              MULTIPLE PLØT ØF THE FUNCTIONS A, B, C, D

                     HØRIZ INCREMENT = 1875
                0                                                75000
                :....:....:....:....:....:....:....:....:....:
X-VALUE
 0              A
 1              C B                 A        For A, r = 0.247 or 24.7 %
 2              DC    B                      A
 3              D C          B For B, r=0.142 or 14.2 %   A
 4               D  C            B                        A
 5                D    C             B                    A
 6                 D     C              B                 A
 7                  D       C            B            A
 8                    D         C            B      A
 9                      D         C            B  A
 10                       D         C          A
 11                         D         C    A  B
 12                          D         CA    B
 13                            D     A C   B
 14                              DA     CB
 15                             A D     B
 16                           A    D  B C
 17                          A       DB C  For C, r=0.0997;or 9.97 %
 18                        A         BD  C
 19                       A        B   DC
 20                      A        B    C
 21                     A       B     CD
 22                    A       B     CD
 23                   A     B     C  D
 24                  A     B     C  D
 25                  A    B    C   D
 26                 A    B     C   D   For D, r =0.0759 or 7.59 %
 27                 A   B     C   D
 28                A   B     C   D
 29                A  B     C   D
 30                A  B    C   D
 31               A  B    C   D
 32               A  B   C   D
 33               A B   C   D
 34               A B  C    D
 35               AB   C   D
 36               AB  C   D
 37              A B C   D
 38              A B C  D
 39              AE C   D
 40              AB C  D
                          16
                :....:....:....:....:....:....:....:....:....:
                0                                                75000
```

FIGURE A-6

k vary from 1 to 4. First one wishes to plot the net returns cash flow. The table below facilitates scaling a multiple plot of the four cases for comparison:

k	T_{max}	n_{max}	n(t)
1	5	7.35E+4	A = (1E+6)*((0.2↑2)/1)*(T↑1)*EXP(-0.2*T)
2	10	5.40E+4	B = (1E+6)*((0.2↑3)/2)*(T↑2)*EXP(-0.2*T)
3	15	4.51E+4	C = (1E+6)*((0.2↑4)/6)*(T↑3)*EXP(-0.2*T)
4	20	3.89E+4	D = (1E+6)*((0.2↑5)/24)*(T↑4)*EXP(-0.2*T)

One can now scale and provide appropriate plotting programming at some ease. The time scale should run from t=0 to t=40 years in steps of one year. For the cash flow the range should be from zero to 7.5E+4.

The time price of money N in this case is $[1+(r/a)]^{-(k+1)}$. Figure A-6 illustrates the general shape of the cash flows as k goes from 1 to 4. The respective rates of returns on investment, r, are shown for each case.

EXAMPLE NO. 5

In the discrete periodic discounting process one has as many roots for r as there are discounting periods. This is no more so with the continuous discounting. Rather the number of roots obtained for r now depend on the general shapes of cash flows and is typically much smaller a number than in the discrete case. Further, the choice of an appropriate root for r is greatly simplified by considering at a relative ease a number of relevant limiting conditions that allow one to disregard all r except the relevant one.

Consider a venture with seasonal variations. Management considers to purchase such a venture at the time t=0 for K dollars. The net returns flow is expected to have the following periodic form:

$$n(t) = n_o\ [1 - \cos w\ t]$$

$$w = 2\pi/T$$

w is the angular frequency of the seasonal variation in the flow and T is the respective time period for the repetitive fluctuations in the net returns cash flow. Typically T is one year. Using the previous principles of discounting one obtains the following result:

$$K = L[n(t)] = (n_o w^2)/[r(r^2+w^2)].$$

This is a cubic equation in r. If K, n_o, and w are, as they should be, positive real numbers, then this cubic equation has only <u>one real root</u>. The other two roots form a complex conjugate pair and are not used for any relevant interpretation for the rate of return on investment. The real root turns out to be as follows:

$$r = (n_o w^2/2K)^{1/3} \left\{ [1 + \sqrt{1 + (1/27)(2Kw/n_o)^2}]^{1/3} + [1 - \sqrt{1 + (1/27)(2Kw/n_o)^2}]^{1/3} \right\} .$$

One notes this expression behaves correctly when $w \to \infty$ as in this case one has an average cash flow of a constant amplitude n_o for which in this limit $r \to n_o/K$. Also, as $w \to 0$ then $n(t) \to 0$ for which $r \to 0$.

In order to illustrate how the rate of return on investment, r, now depends on the seasonal periodicity w, consider a particular numerical example. Let K = 400,000 dollars for the purchase of an asset. Let n_o = 100,000 dollars per annum for the amplitude of the oscillation of the net returns flow. Then

$$r = (1/2)\, w^{2/3} \left\{ [1 + \sqrt{1 + (64/27)\, w^2}]^{1/3} + [1 - \sqrt{1 + (64/27)\, w}]^{1/3} \right\} .$$

Using time sharing computer terminal one readily finds the following results:

w	r	w	r
0.00	0.000000	0.55	0.216468
0.05	0.075799	0.60	0.220301
0.10	0.111475	0.65	0.223556
0.15	0.136633	0.70	0.226337
0.20	0.155677	0.75	0.228727
0.25	0.170582	0.80	0.230792
0.30	0.182482	0.85	0.232586
0.35	0.192116	0.90	0.234151
0.40	0.200000	0.95	0.235524
0.45	0.206509	1.00	0.236733
0.50	0.211927	1.05	0.237803

The ultimate limit for r is 0.25 as w increases toward infinity. Thus here the most sensitive range for r is when w goes from zero to unity years $^{-1}$.

Figure A-7 illustrates the behavior of the B-S flow for this kind of a periodic case.

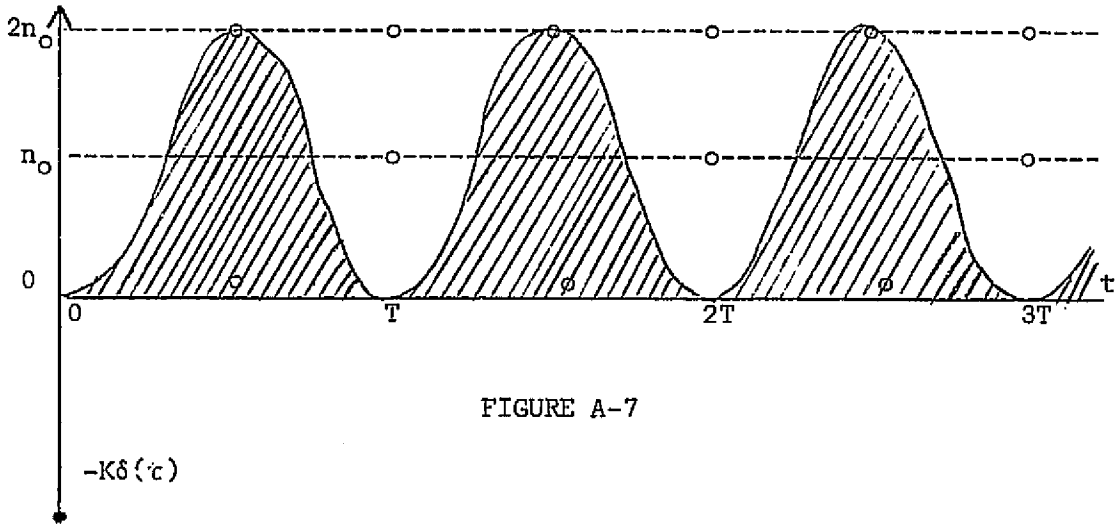


FIGURE A-7

EXAMPLE NO. 6

If one assumes a case where the B-S flow has the form

$$f(t) = n_o \cdot \cos wt - K\delta(t) ,$$

then by the previous discounting procedures one finds for r the following expressions:

$$r = (n_o/2K)[1 \pm \sqrt{1 - (2Kw/n_o)^2}] .$$

which r would be correct? If $w \to 0$ one should get a limiting case where $n(t) = n_o$ for all $t \geq 0$. Then + sign is appropriate since for w=0 one obtains the correct limiting case where $r = n_o/K$. Therefore, one can pick up uniquely an appropriate rate of return on investment. The above case is somewhat academic since there will be no real valued r unless

$$(2Kw/n_o) \leq 1 .$$

Note that in this example the flow has periodic negative portions that could be then interpreted as investments for survival of the venture. This interpretation was discussed previously.

EXAMPLE NO. 7

A potentially usable flow model is as follows:

$$f(t) = n_o e^{-at} \cos wt - K\delta(t) .$$

Here a is again the force of venture mortality. If w = 0 then one should obtain for r the expression

$$r = (n_o/K) - a$$

and if a = 0 then one should obtain the results of the previous example. The condition after discounting is as follows:

$$K = [n_o(r+a)]/[(r+a)^2 + w^2]$$

This is a quadratic equation in r yielding two roots. The correct relevant root satisfying the above mentioned limiting conditions is as follows:

$$r = [(n_o/2K) - a] + \sqrt{(n_o/2K)^2 - w^2}$$

For a real valued r it is necessary that $(n_o/2K)^2 > w^2$. Figure A-8 illustrates the general shape for this kind of a flow with oscillations and negative portions, which, again should be interpreted as sacrifices or investments for survival.

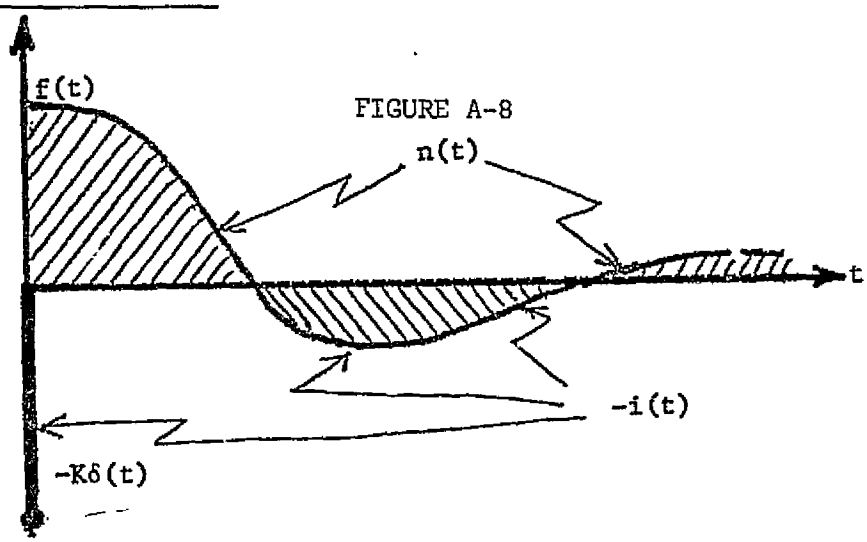


FIGURE A-8

EXAMPLE NO. 8

It is appropriate now to illustrate how to treat the discounting of "sample data" flows or discrete periodic flows by using the principles of continuous discounting. The first such example is illustrated in Figure A-9

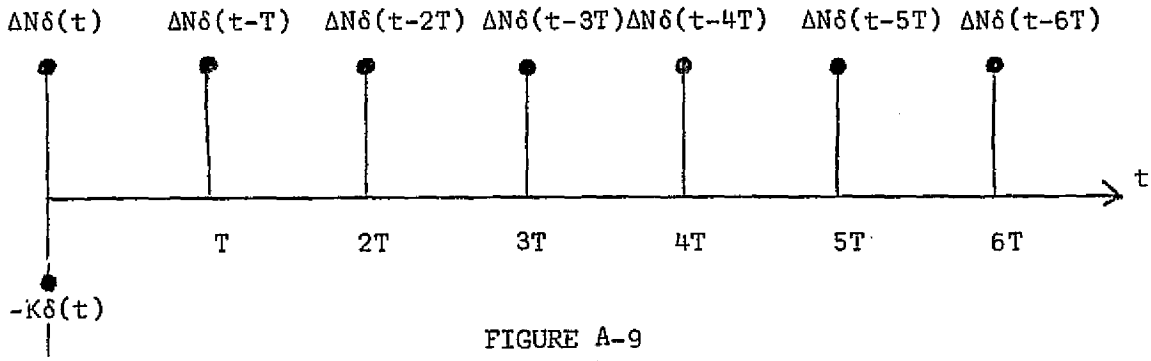


FIGURE A-9

In the above illustration lump sum net returns occur uniformly periodically as shown. They are impulse functions $\Delta N\delta(t - nT)$, $n = 1,2,3, \ldots$. Then, using the previous principles of discounting, one obtains the following condition:

$$K = \sum_{n=0}^{\infty} \Delta N\, e^{-rnT} = \Delta N/(1 - e^{-rT}) \ .$$

Let $\Delta N/T = n_o$. If $0<rT<<1$, and so $e^{-rT} \simeq 1 - rT$, then $r = n_o/K$. For the above case one can solve for r yielding the following result:

$$r = (1/T) \ln [1/(1 - \Delta N/K)] \ .$$

EXAMPLE NO. 9

If the net returns dollar pulses are attenuated by a factor g^n, $o \leq g \leq 1$, then these pulses have the form

$$\begin{cases} \Delta N\, g^n \delta(t - nT) \ , \ n = 0,1,2,3,\ldots\ldots \\ 0<ge^{-rT}<1 \ . \end{cases}$$

Then

$$K = \sum_{n=0}^{\infty} \Delta N\, g^n\, e^{-rnT} = \Delta N/(1-ge^{-rT})$$

and

$$r = (1/T) \ln [g/(1 - \Delta N/K)] \ .$$

EXAMPLE NO. 10

One can introcuce the force of venture mortality, a, and the probability of product survival exp(-at) as follows:

$$\Delta N\, e^{-anT}\delta(t - nT) \ ; \ n = 1,2,3,4,\ldots\ldots$$

For such a train of net returns pulses one obtains the following results of discounting:

$$\begin{cases} K = \sum_{n=0}^{\infty} \Delta N\, e^{-(r+a)nT} = N/(1-e^{-(r+a)T}) \\ r = (1/T) \ln [1/(1 - \Delta N/K)] - a \end{cases}$$

If now $\Delta N<<K$, and if $\Delta N/T = n_o$ then

$$r = (n_o/K) - a$$

approximating thus the case with the continuous flow $n(t) = n_o e^{-at}$. At this point it should be noted that the continuous flows can be approximated by sample data discrete flows or vice versa.

EXAMPLE NO. 11

Consider now the net returns pulses of the following form:

$$\Delta N\ n\ g^n \delta(t - nT);\ n = 1,2,3,4,\ldots\ldots\ldots$$

In this case

$$K = \sum_{n=0}^{\infty} \Delta N\ n\ g^n\ e^{-rnT} = (\Delta N\ g\ e^{-rT})/(1 - g\ e^{-rT})^2$$

$$0 < ge^{-rT} < 1\ .$$

Solving for r yields the following expression:

$$r = (1/T)\ \ln\ [g/(1 - (\Delta N/2K)(\sqrt{1 + 4K/\Delta N} - 1)]\ .$$

If $0<\Delta N<<K$, then for $g=1$ this sample data discrete case approximates a continuous case with the net returns cash flow of the form

$$n(t) = (n_o/T)\ t\ .$$

BENEFIT-SACRIFICE STREAM MODELING: A GENERALIZATION

The previous B-S flow models for ventures did not take into account the fact that in many cases the assets for a venture cannot be obtained at an instant of time but rather the investment activity must occur over extended periods of time. The general B-S flow has the form

$$f(t) = n(t) - i(t) + J\delta(t-T)$$

as pointed out previously. Figure A-2 provides an illustration of a rather typical B-S flow cycle. Its nature has been discussed in some detail. It is the purpose of this section to present several B-S flow models of this more general type.

EXAMPLE NO. 12

Figure A-10 illustrates a case where a constant outflow of investment lasts for a period of time T whereafter a constant net returns flow will take place over another time period T.

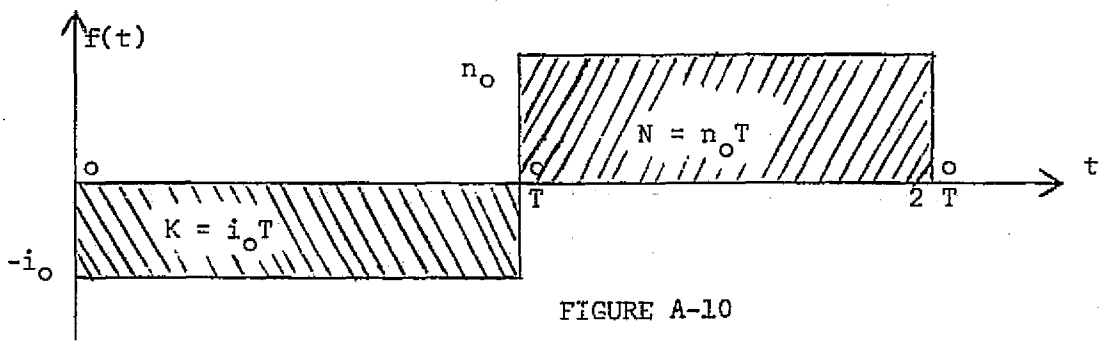


FIGURE A-10

Then the discounting relationship for the total cash flow is as follows:

$$\int_0^{2T} f(t)\, e^{-rt} dt = \int_0^{T} (-i_o) e^{-rt} dt + \int_T^{2T} n_o\, e^{-rt} dt = 0$$

This relationship yields the following equivalent condition:

$$(e^{-rT})^2 - 2[(n_o + i_o)/2n_o]\, e^{-rT} + (i_o/n_o) = 0 \ .$$

Thus

$$e^{rT} = (2n_o)/[(n_o + i) \pm (n_o - i_o)] \ .$$

In a limiting case where $rT \ll 1$ one must have $e^{rT} \approx 1 + rT$. If the - sign is used one gets

$$r \approx (1/T)[(n_o - i_o)/i_o]$$

whereas if + sign is used this would imply that $r=0$ which is not of any relevant interest. Therefore - sign is the appropriate one. Further,

$$r = (1/T) \ln (n_o/i_o) = (1/T) \ln (N/K)$$

It is noted again how the rate of return on investment is inversely proportional to the investment duration or lag time T while it is only logarithmically proportional to the relevant B-S ratio. For a given N/K ratio it is desirable to organize the program in such a fashion that T is minimized. This again is related to basic realization of the program's goals in a minimum time. It is, indeed, possible to illustrate that the maximization of the rate of return on investment may require a well defined restriction for the financial ration N/K, i.e. the real performance for a program is of such an essential nature that the B-S ration must be matched for it in order to maximize the rate of return on investment. This demonstration of the principle can be done as follows: A short duration T or a large inverse duration 1/T cannot be obtained at no cost in a typical modern economy. Let s be some non-negative real number. Assume that 1/T is proportional to K raised to this number s, i.e.

$$1/T \propto K^s \ ; \ s>0 \ .$$

Then

$$r \propto K^s \ln(N/K) \ .$$

If N is estimated or given, then the only manipulated factor is K. Then r can be maximized with respect to K. The necessary condition is as follows:

$$dr/dk \propto sK^{s-1} \ln(N/K) + K^s(-K^{-1}) = 0$$

or

$$N/K = e^{1/s} \ .$$

If s is a positive number much less than unity, the B-S ratio N/K for the maximum return on investment can be high. However, if s becomes larger than unity, then this benefit-sacrifice ratio must be reduced for the realization of a maximum rate of return on investment. Indeed, it is illustrative to plot the function $K^s \ln(N/K)$ for a given N and few

```
PLØTTØ
READY.

100 DATA 3,1E3,1E6,5E4,0,5E5
200 LET A=(4E2)*(X↑(1/2))*LØG(1E6/X)
210 LET B=X*LØG(1E6/X)
220 LET C=(2E-6)*(X↑2)*LØG(1E6/X)
RUN
```

```
PLØTTØ     17:04    THURS. 09/04/69

              MULTIPLE PLØT ØF THE FUNCTIØNS A, B, C

                      HØRIZ INCREMENT = 12500
                 0                                                  500000
                 :.....:.....:.....:.....:.....:.....:.....:.....:
X-VALUE
 1000            CB      A                                  K is X here.
 51000            C            B           A                  s
 101000              C                 B    A                r      K /s
 151000                C                    BA                max
 201000                   C                 A  B
 251000                       C         A      B             Case A: s=1/2
 301000                          C   A         B
 351000                            AC          B             Case B: s= 1
 401000                          A     C       B
 451000                         A          C  B             Case C: s= 2
 501000                        A             B
 551000                      A             B  C
 601000                     A            B    C
 651000                  A            B       C
 701000                 A           B        C
 751000              A         B          C
 801000            A         B          C
 851000           A      B        C
 901000         A    B      C
 951000        A B  C

                 :.....:.....:.....:.....:.....:.....:.....:.....:
                 0                                                  500000
```

FIGURE A-11

different s. Choose N = 1,000,000 dollars. Let s take values 1/2, 1 and 2. Figure A-11 illustrates the results for these cases indicating the limited role of benefit-sacrifice ratios for best results.

One might think then that in this illustration the cost of obtaining the shortened time T for an increased rate of return on investment is overriding the importance of the B-S ratio N/K and that the control of such a cost of reduced time T is not a problem of, say, financial management as much as it is the problem of the management responsible for realization of actual performance for the goals of the program.

EXAMPLE NO. 13

If the problem remains the same as the previous one except in that now the net returns flow is $n(t) = n_o$ for all $t>T$, then one finds that the rate of return on investment is given by the following expression:

$$r = (1/T)\ \ln[1 + (n_o/i_o)] \ .$$

Again the time period T is the duration of the investment activity or the investment lag time. Thus $K = i_oT$. Figure A-12 illustrates this case.

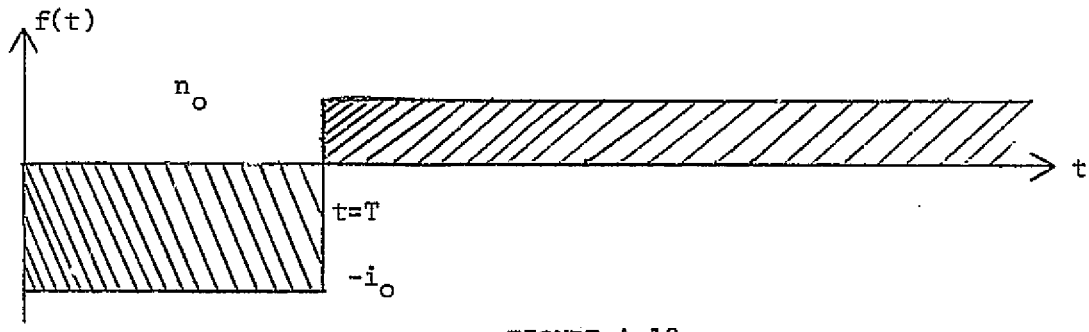


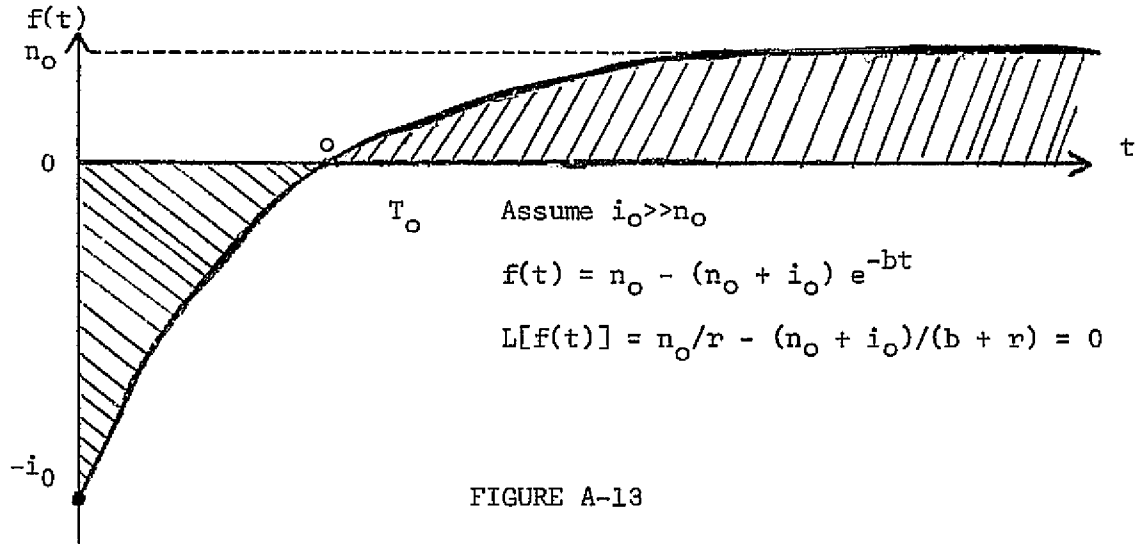
FIGURE A-12

EXAMPLE NO. 14

Considerable mathematical simplifications can be obtained by constructing B-S models using appropriate superposition of relatively simple component functions which, nevertheless, could be characterized by relevant parameters for the discription of a program. Consider, for example, a case where the force of venture mortality is not a factor but where the lag in realizing productive assets by a process of investment is controlled by the force or rate of renovation or innovation. If such a rate is low, one would expect long lag times. What kind of a model could be constructed to portray this situation with a reasonable

simplicity? In the most simple case one might assume the force of innovation or rate of innovation is just constant, say, b. The probability that the job will not be realized within a given time would be then an exponentially decaying one in time, i.e. exp(-bt). This concept will be now used to develop a B-S flow model for an innovation limited venture facing no product mortality.

It is assumed that the venture is characterized by a one way process from a state of no business and building up to the state of steady business. Figure A-13 illustrates the model for the cash flow of this venture.



Assume $i_o >> n_o$

$$f(t) = n_o - (n_o + i_o)\, e^{-bt}$$

$$L[f(t)] = n_o/r - (n_o + i_o)/(b + r) = 0$$

FIGURE A-13

For the given total cash flow the rate of return on investment is simply as follows:

$$r = (n_o/i_o)\, b$$

Note then that this rate of return on investment is directly proportional to the rate of innovation b. If b=0 then r=0. The brake even period T_o (or zero crossing time) occurs when f(t) = 0, i.e. is given by the following expression:

$$T_o = (1/b) \ln [1 + (i_o/n_o)] \; .$$

Thus the time T_o to the brake even condition (f(t)=0) is inversely proportional to the rate of innovation. Since in this case 1/b is the mean time of realizing the productive asset by the investment process, then T_o is directly proportional to this mean realization time of the productive asset for the venture. The total dollar volume of the investment is given as follows:

$$K = \int_0^{T_o} [-f(t)]dt = \int_0^{T_o} [(n_o+i_o)e^{-bt} - n_o]dt = [(n_o+i_o)/b](1-e^{-bT_o}) - n_oT_o.$$

Noting the expression for T_o one has $e^{-bT_o} = n_o/(n_o+i_o)$. Thus

$$K = i_o/b - n_oT_o = (1/b)[i_o - n_o \ln (1 + i_o/n_o)]$$

$$= (i_o/b)[1 - (n_o/i_o) \ln [1 + (i_o/n_o)]]$$

As an illustration, if $i_o/n_o = 10$, then $\ln 11 = 2.4$. Thus $k=0.76(i_o/b)$.

Thus far, an earlier model illustrated a venture with no force of innovation acquiring an asset at the time t=0, worth K dollars. It was subjected to a constant force of mortality, a. Its net B-S flow was exponentially decaying and the consequent rate of return on investment was expressed as follows:

$$r = (n_o/K) - a$$

In the last example there were no such force of mortality present. The only limitation was the rate of innovation or force of innovation. For this case one had the following expression for r:

$$r = (n_o/i_o)\, b\ .$$

It would be, indeed, interesting to develop a model where a productive asset is built up in an innovation limited way while the output of the venture would be subjected to a force of mortality without subsequent renovation processes.

EXAMPLE NO. 15

Consider a venture which starts in a state of being conceived as an opportunity for investment in productive assets. The process of investment would then take this venture from its state of being conceived to the state where it has realized the planned productive assets. Meanwhile there is a force of mortality to which force this venture is subjected.

A relatively simple B-S flow model for this kind of situation can be constructed by a superposition of two exponential functions, one portraying the investment process with a constant force or rate of innovation, and the other portraying the process of product mortality with a constant force of mortality. Figure A-14 illustrates this case. For a meaningful venture "rat race" it is assumed that the rate of innovation b is magnificantly greater than the rate or force of product mortality a.

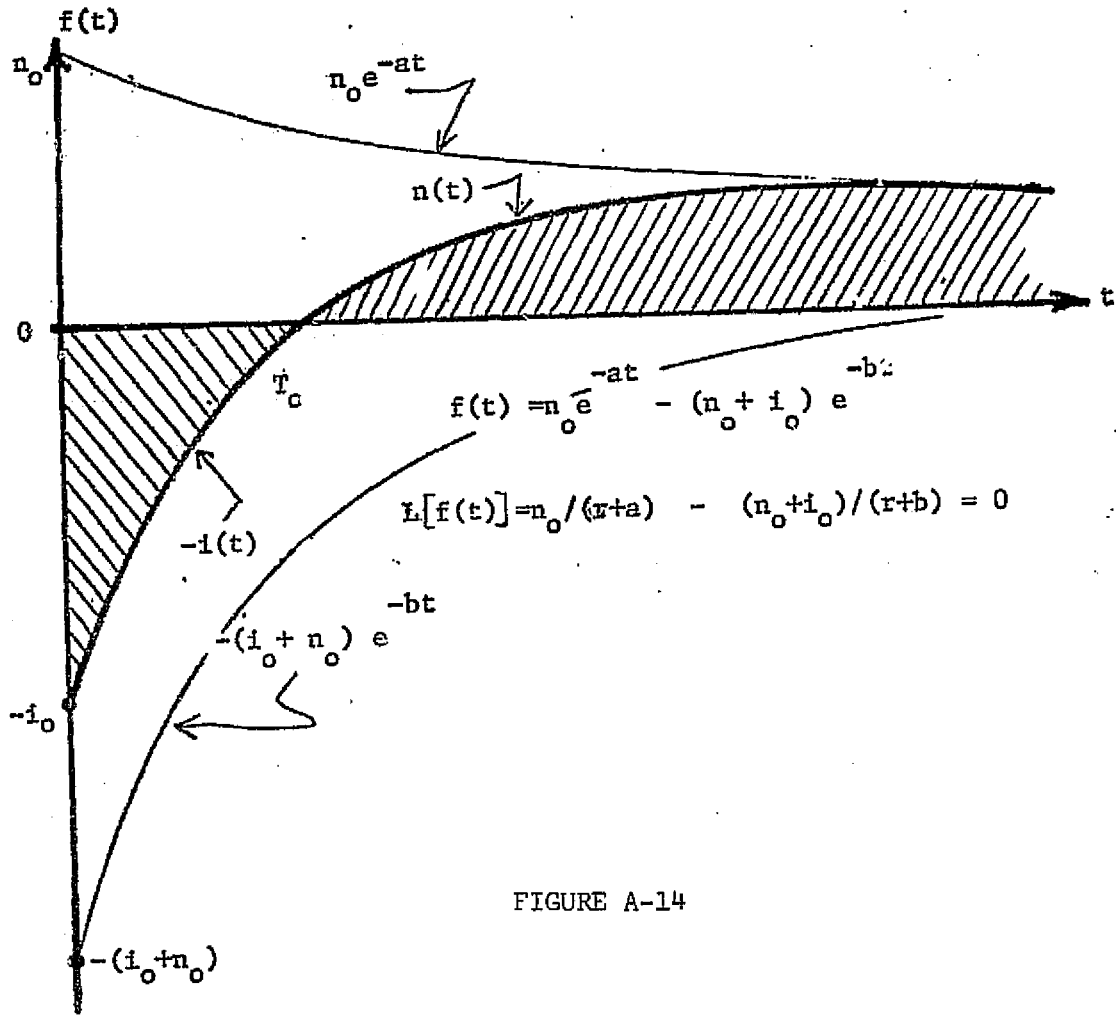


FIGURE A-14

In this case one interprets the investment and net returns flows as follows:

$$i(t) = -f(t) = (n_o+i_o)e^{-bt} - n_o e^{-at} \text{ for } 0 \leq t \leq T_o$$

$$= 0 \text{ elsewhere.}$$

$$n(t) = +f(t) = n_o e^{-bt} - (n_o+i_o)e^{-bt} \text{ for } t \geq T_o$$

$$= 0 \text{ elsewhere.}$$

For this case the zero crossing time or time to the break even condition occurs when f(t) = 0 which condition yields the following expression for T_o:

$$T_o = [1/(b-a)] \ln [1 + (i_o/n_o)]$$

The rate of return on investment in this case is obtained by the method discussed previously, and is as follows:

$$r = (n_o/i_o)[b - a] - a$$

With this result one obtains the following interpretation for (b - a): b - a is the net rate of innovation in an environment of a continuously existing force of mortality a. It is not enough that a venture provides just some innovation rate in realizing its new productive existence. It must provide a rate of innovation in net over the existing rate of mortality of its products and methods of production and marketing, etc.

For this model the total investment dollar volume K is as follows:

$$K = \int_0^{T_o} i(t)\,dt = \int_0^{T_o} [(n_o+i_o)e^{-bt} - n_o e^{-at}]dt = [(n_o+i_o)/b](1-e^{-bT_o}) - (n_o/a)(1-e^{-aT_o}).$$

The total net returns dollar volume N is as follows:

$$N = \int_{T_o}^{\infty} n(t)\,dt = \int_{T_o}^{\infty} [n_o e^{-at} - (n_o+i_o)e^{-bt}]dt = (n_o/a)e^{-aT_o} - [(n_o+i_o)/b]e^{-bT_o}\ .$$

Noting the expression for T_o one finds $\begin{cases} e^{-aT_o} = [1+(i_o/n_o)]^{-a/(b-a)} \\ e^{-bT_o} = [1+(i_o/n_o)]^{-b/(b-a)} \end{cases}$.

Therefore, K and N as well as N/K can be computed readily if i_o, n_o, a and b are specified, a four parameter model. There are two financial parameters and two "performance" parameters.

The descriptive flexibility and power of parametric flow models can be increased by increasing the number of usable relevant parameters. Although the above may be of interest in caricaturing some ventures, it may be far too simple for others. Therefore it may be desirable to introduce more complex models.

One way to increase the descriptive power of simple models is to add into a model a sufficient number of independent parameters or "degrees of freedom" which allow one to match such a model to various empirical or conceptual situations with appropriate levers for a reasonably good fit so that a model, indeed, serves as an adequate MAP for its intended purposes of representation.

EXAMPLE NO. 16

As an example of a multiple parameter flow model, consider a particular example where

$$i(t) = i_o(t/T)^{k-1}e^{-bt}$$

and $$n(t) = \left[M_o\ (t/\theta)^{m-1} e^{-at} - (t/T)^{R-1} b^{-bt}\right]$$

used in a particular case with $b \geq 2a$. If one chooses $\theta = T = 1$ unit of time (years), then

$$K = \int_0^\infty i(t)\ dt = [\Gamma(k)/T^{k-1} b^k]\ i_o$$

$$N = \int_0^\infty n(t)\ dt = [\Gamma(m)/\theta^{m-1} a^m]\ n_o$$

The discounting equation is as follows: $L[i(t)] = L[n(t)]$; thus

$$(r+b)^k/(r+a)^m = [1 + (i_o/n_o)]\ [\Gamma(k)/\Gamma(m)]\ [\theta^{m-1}/T^{k-1}] = [\Gamma(k)/\Gamma(m)]$$
$$[\theta^{m-1}/T^{k-1}] \left\{1 + [(\Gamma(m) T^{k-1} b^k/\Gamma(k) \theta^{m-1} a^m) - 1](K/N)\right\} = A\ .$$

A subclass of such potentially useful flow models is obtained if $m = 2k$. In such a case one has the following relationship:

$$(r+b)^k/(r+a)^{2k} = A \text{ or } (r+b) = A^{1/k}(r+a)^2\ .$$

This quadratic equation has two roots one of which is relevant since it satisfies the necessary condition $r>0$ if $N/K>1$:

$$r = (1/2)\ A^{-1/k}\ [1 + \sqrt{1 + 4\ (b - a)\ A^{1/k}}] - a\ .$$

For this model one still has the following relationships so long $b \geq 2a$:

$$i_{max} = [(k-1)/ebT]^{k-1} i_o \text{ at } t = T_i = (k-1)/b$$

$$n_{max} = [(m-1)/ea\theta]^{m-1} n_o \text{ at } t = T_n = (m-1)/a\ .$$

If one lets $T = \theta = 1$ and $m = 2k$, then the zero crossing or break even time T_o is as follows:

$$[k/(b-a)]\ \ln T_o + T_o = [1/(b-a)]\ \ln [1 + (i_o/n_o)].$$

Consider a particular case for which $T = \theta = 1$; $b = 1$, $a = 1/4$, $k = 3$, $m = 2k = 6$, $K = 1{,}000{,}000$ \$, $N = 4{,}000{,}000$ \$. For this case

$i_o = 500{,}000$ \$/year

$i_{max} = 275{,}000$ \$/year at $T_i = 2$ years

$n_o = 8.15$ \$/year

$n_{max} = 186{,}000$ \$/year at $T_n = 20$ years

The zero crossing time occurs when $4 \ln T_o + T_o = 14.65$ years. This would yield T_o around 7 years. Figure A-15 is a plot for this case. The points C correspond to f(t), A to i(t), and B to n(t). Thus f(t)

MULTIPLE PLOT OF THE FUNCTIONS A, B, C

HORIZ INCREMENT = 12500

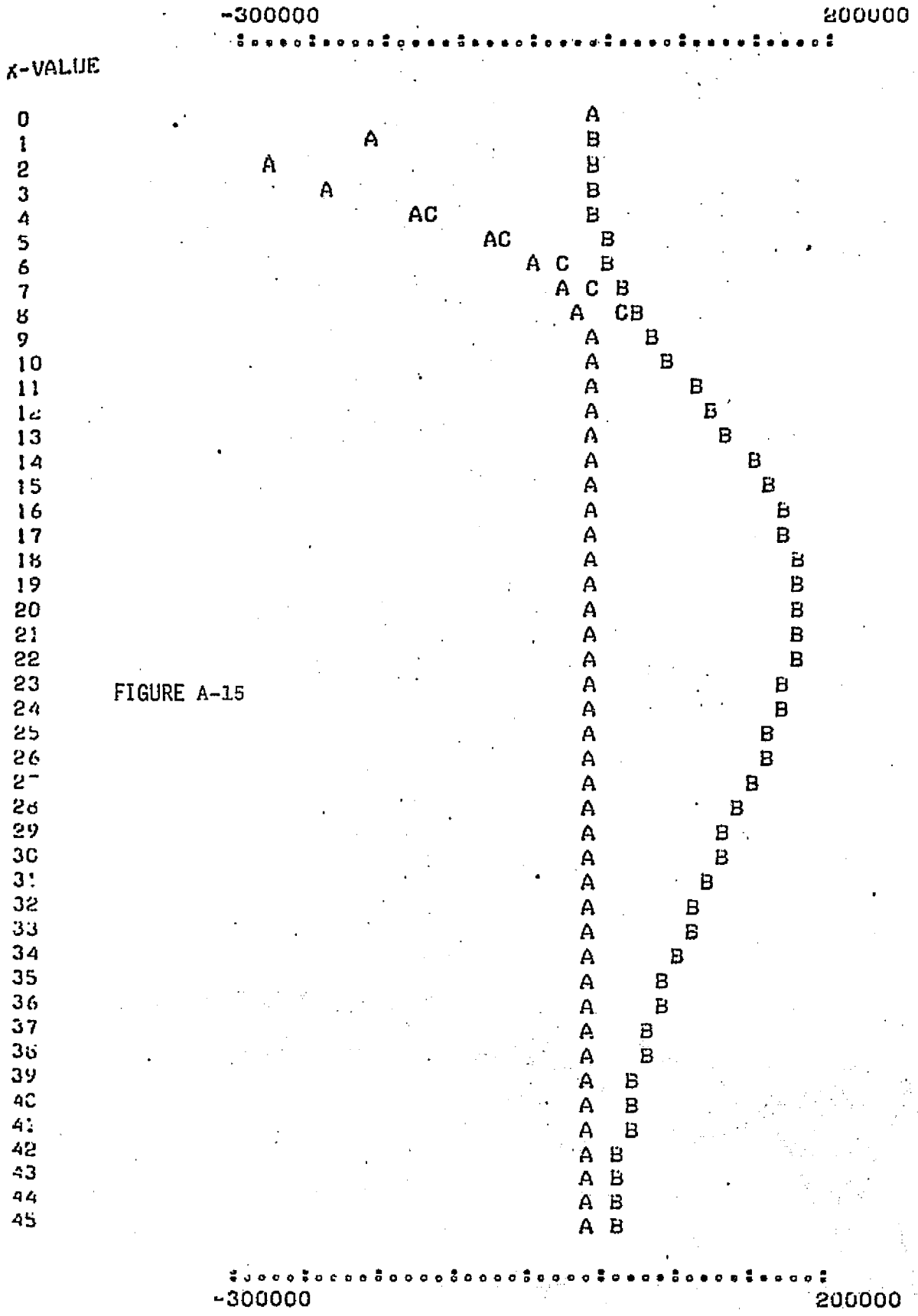


FIGURE A-15

corresponds very closely to i(t) and n(t) except in a relatively small neighborhood of the zero crossing.

Note k need not be an integer. However if it is an integer while $m = 2k$, $N/K = 4$, $a = 1/4$ inverse years and $b = 1$ inverse year, then

$$A = [(k-1)!/(2k-1)!]\left\{1 + [(2k-1)!4^{2k}/(k-1)! - 1](1/4)\right\} = 4^{2k-1} .$$

This approximation is the better the higher k becomes. Then for this example one has the following expression for the rate of return on investment:

$$r = (1/2)(1/4^{(2k-1)/k})[1 + 1 + 3\cdot 4^{(2k-1)/k}] - (1/4)$$

In this case one obtains the following time parameters, and the values for the rate of return on investment:

k	%r	T_i	T_n
1	32.57	0	4
2	12.50	1	12
3	7.69	2	20
4	5.54	3	28
5	4.33	4	36
6	3.56	5	44
7	3.02	6	52
8	2.62	7	60
9	2.31	8	68
10	2.07	9	76

The behavior of this cash flow model, as k increases, is illustrated in Figure A-16.

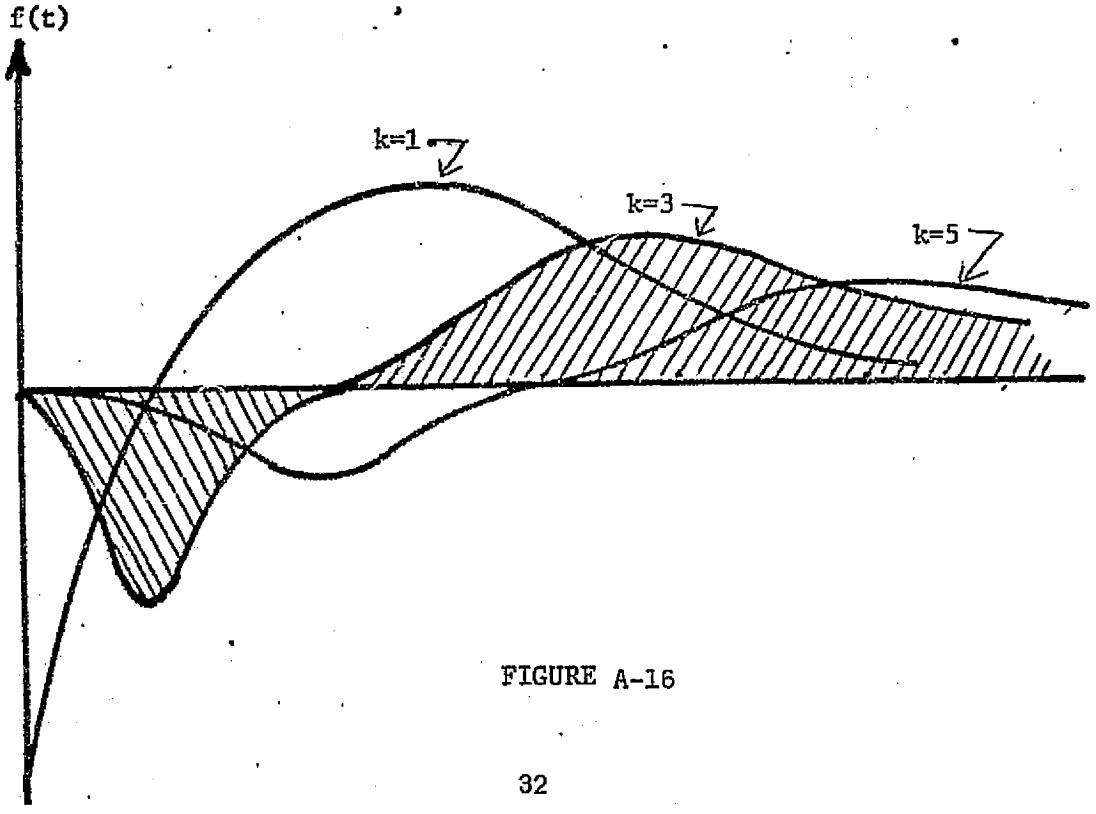


FIGURE A-16

It is seen that the general features of this model can indeed provide a relevant portrayal of "typical" features of cash flow cycles.

AN INTRODUCTION TO RISK ANALYSIS UTILIZING PARAMETRIC BENEFIT-SACRIFICE FLOW MODELS

An extension of the parametric B-S flow models is into the areas of risk analysis. Each parameter in the cash flow model can be treated as a random variable to which one can assign some appropriate probability density function. In particular, one could consider a set of parameters as a set of mutually independent random variables if this seems desirable. The general idea is to find the mean and variance of a general performance measure such as the rate of return on investment or pay-back period in terms of the statistical measures of the relevant parameters of the cash flow. In order to illustrate a variety of cases, four examples will be given.

EXAMPLE NO. 17

In the Example No. 3 one was considering a persent value model where the net returns flow was as follows:

$$n(t) = n_o e^{-a t} .$$

For this case the rate of return on investment with a total dollar volume outlay of K dollars was as follows:

$$r = (n_o/K) - a = [(N/K)-1] a.$$

Let us assume a decision maker has come up with information to his satisfaction allowing time to assign from the past and present acceptable experiences a probability density distribution for the random variables n_o and a. For an illustration, assume K is given with certainty by the situation facing a decision maker. Assume, further, n_o and a are two mutually statistically independent variables. The decision maker assigns the relevant probability density distributions $g(n_o)$ and $s(a)$ to the random variables n_o and a, respectively

$$\begin{cases} g(n_o) \geq 0 \text{ for all } n_o \geq 0 \\ \int_0^\infty g(n_o)dn_o = 1 \end{cases} \qquad \begin{cases} s(a) \geq 0 \text{ for all } a \geq 0 \\ \int_0^\infty s(a)ds = 1 \end{cases}$$

The expected value of r is then as follows:

$$E(r) = \int_0^\infty \int_0^\infty [(n_o/i_o) - a]g(n_o)s(a)dn_o da$$

$$= (1/K)\int_0^\infty n_o g(n_o)dn_o - \int_0^\infty as\ (a)da = (1/K)E(n_o) - E(a)\ .$$

For the above case one has the following relationships for the expected value E(r) and variance Var(r) of r:

$$r = (n_o/K) - a$$

$$E(r) = (1/K)E(n_o) - E(a)$$

$$Var(r) = \sigma_r^2 = (1/K)^2\sigma_{n_o}^2 + \sigma_a^2$$

Let the random variable r lie within the interval $E(r) \pm k_i\ \sigma r$ where $k_i \geq 0$ is a real number chosen so that the probability P of r being within this interval is a desired one. Then one can define the following ranges for r:

The range of pessimistic rates of returns on investment:

$r < r_{pe} = E(r) - k_p\sigma r$, kp chosen for pessimistic limit

The range of neutral rates of return on investment:

$r_{pe} = E(r) - k_p\sigma_r \leq r \leq E(r) + k_o\sigma_r = r_{op}$

The range of optimistic rates of return on investment:

$r > r_{op} = E(r) + k_o\sigma_r$, k_o chosen for optimistic limit

There could be several ways of picking k_p and k_o. The way a decision maker wishes to choose these values depends on the particular criteria he wishes to emphasize. For example, if $k_o + k_p = 1$, both being non-negative, then

$r < r_{pe}$ with the probability $k_p(1-P)$

$r_{pe} \leq r \leq r_{op}$ with the probability P

$r > r_{op}$ with the probability $k_o(1-F)$.

With this interpretation, k_p is the conditional probability that r is in the pessimistic range given it is not within the middle range. Also k_o is the conditional probability r is in the optimistic range given it is not in the middle or normal range.

EXAMPLE NO. 18

In the previous example No. 15 one had a special case for a B-C flow characterized by a constant "force of product mortality" a and a constant

"rate of innovation" b. It was found in this very special case that the rate of return on investment was as follows:

$$r = (n_o/i_o)(b - a) - a \ .$$

Consider now a risk analysis where i_o is a given deterministic parameter while n_o, b, and a are considered as mutually independent random variables. With the calculus of means and variances of a set of mutually independent random variables one can derive the following basic expressions:

$$E(r) = (1/i_o)E(n_o)[E(b)-E(a)] - E(a)$$

$$\sigma_r^2 = E(r^2) - [E(r)]^2$$

$$= (1/i_o)^2 \left\{ E(n_o^2)[E(b^2)+E(a^2)]-E(n_o)^2[E(b)^2+E(a)^2]-2E(b)E(a)\sigma_n^2 + [(2/i_o)E(n_o)+1]\sigma_a^2 \right\} .$$

Again one can choose k_o and k_p which are non-negative and whose sum is unity such that one obtains the desired pessimistic, "normal" and optimistic ranges for r.

EXAMPLE NO. 19

Consider again Example No. 13. In this case the total investment dollar volume was $K = i_oT$. In order to get back this money one needs to collect n_o dollars per year for T_1 years so that $i_oT = n_oT_1$. Thus $T_1 = (i_o/n_o)T$ and the pay-back time would be then as follows:

$$T_{pb} = T + T_1 = [1 + (n_o/i_o)]\ T_1 \ .$$

Assume now n_o and T_1 are random variables while i_o is a given deterministic parameter. Assume also n_o and T_1 are mutually independent. Then for the pay-back period one obtains the following mean and variance:

$$E(T_{pb}) = (1/i_o)E(n_o)E(T_1) + E(T_1)$$

$$\sigma_{T_{pb}}^2 = (1/i_o^2)[\sigma_{n_o}^2\sigma_{T_1}^2 + E(T_1)^2\sigma_{n_o}^2 + E(n_o)^2\sigma_{T_1}^2] + \sigma_{T_1}^2$$

EXAMPLE NO. 20

In Example No. 3 one found the time price relationship between K and N to be as follows:

$$K = [a/(r+a)]\ N.$$

Suppose a decision maker sets r to be some desired value set, for example by capital markets. But he feels uncertain about a and N, and therefore he feels uncertain about the required investment dollar volume which is clearly a function of the two random variables a and N. He would conceive some probability density functions for a and N, respectively. Assuming a and N are mutually independent random variables one obtains the following expressions:

$$E(K) = E[a/(r+a)]\ E(N)$$

$$\sigma_K{}^2 = E(K^2) - [E(K)]^2$$

$$= E[a^2/(r+a)^2]\ E(N^2) - [E(a/(a+r)]^2[E(N)]^2 \ .$$

The ranges of interest may then be as follows:

The optimistic range for the total capital outlay:

$$K < E(K) - k_o\sigma_K$$

The "normal" range for the total capital outlay:

$$E(K) - k_o\sigma_K \leq K \leq E(K) + k_p\sigma_K$$

The pessimistic range for the total capital outlay:

$$K > E(K) + k_p\sigma_K \ .$$

EXAMPLES OF MAXIMIZATION OF A DECISION MAKER'S UTILITY UNDER A PATTERN OF CONJOINED SIMULTANEOUS BENEFIT-SACRIFICE STREAMS

The earlier classical utility maximization problems fell in the category where the one-dimensional scalar utility measure was considered as a response to a pattern of conjoined simultaneous stimuli each of which was necessary among others for the generation of a positive reaction. It is the purpose of this section to introduce few examples of this type.

EXAMPLE NO. 21

Consider a "decision maker" who is completely convinced that he needs to execute simultaneously two distinctly different programs in order to be happy at all. For the program No. 1 he expects a net return of benefits N_1 at a time price p_1. For the program No. 2 he expects a net return of benefits N_2 at a time price p_2. Both N_1 and

N_2 are measured in same units as is the total resources K for the required sacrifice. Let N_{1m} and N_{2m} be the respective saturation values for N_1 and N_2, respectively, to which this "decision maker" will be sensitive at the maximum. Also, let N_{1o} and N_{2o} be the respective norms for N_1 and N_2. Assume that for a "typical case" $N_{1o} << N_{1m}$ and $N_{2o} << N_{2m}$. Then the utility response relationship to these stimuli is assumed to be of the following form:

$$U/U_o = [(N_1/N_{1o})/[1 + (N_1/N_{1m})]]^{h_1} [(N_2/N_{2o})/[1 + N_2/N_{2m})]]^{h_2}$$

Here h_1 and h_2 are two non-negative dimensionless real numbers characterizing the power law coefficients. This relationship satisfies the following additional requirements assuming all N_1 and N_2 are non-negative real measures:

1. The ordering of the utility U is preserved no matter how the positive values U_o, N_{1m}, $N_{1o} << N_{1m}$, N_{2m}, $N_{2o} << N_{2m}$, h_1 and h_2 are chosen. Thus this function has the ordinal property in the Pareto sense and, respectively, and appropriate structure for psycho-physiological scaling for a stimulus-response relationship.

2. U/U_o is a monotonic increasing function of, both, N_1 and N_2. That is, the relative utility will not ever decrease if either N_1 or N_2 increase, given any positive values of U_o, N_{1m}, N_{1o}, N_{2m}, N_{2o}, h_1 and h_2.

3. U/U_o has continuous first and second partial derivatives for all non-negative values of N_1 and/or N_2. The total second differential will exist for all such non-negative values of N_1 and N_2 in whatsoever direction one wishes to obtain it. Thus the necessary properties for the classical marginal utility theory are provided.

4. The saturation property built into this model is equivalent to the principle of an eventually diminishing marginal utility.

The total sacrifice volume K relates to the net benefits N_1 and N_2 received from the two programs as follows:

$$K = p_1 N_1 + p_2 N_2$$

p_1 and p_2 are the dimensionless time prices of N_1 and N_2, respectively. The general objective of the optimization process is to maximize U/U_o under this constraint of available resources.

This simple case depending on the two conjoined benefit variables N_1 and N_2 and on a sacrifice constraint that the total allowable resources in the same units as N_1 and N_2 amounts to K can be now generalized to a finite number of benefit variables N_1, N_2, ... N_n as follows:

Consider a total sacrifice K measured in any appropriate units to be allocated for the realization of a conjoined set of n simultaneous programs each with its respective total net benefit $N_1, N_2, \ldots, N_n$ where all these benefits are measured in the same units as K. Let $p_1, p_2, \ldots, p_n$ be the respective time prices of the above benefits.

Then the problem is to maximize

$$U/U_o = \prod_{i=1}^{n} [(N_1/N_{1o})/[1 + (N_1/N_{1m})]]^{h_i}$$

under the constraint

$$K = \sum_{i=1}^{n} p_i N_i \ .$$

This kind of a problem is best solved by the method of Lagrange's multipliers. Introduce first the modified function

$$V(N_1, N_2, \ldots, N_n) = U/U_o - \lambda[\sum_{i=1}^{n} p_i N_i - K]$$

The necessary first order conditions for maximum value are as follows:

$$\partial V/\partial N_i = \partial(U/U_o)/\partial N_i - \lambda p_i = 0 \ ; \ i = 1, 2, 3, \ldots, n \ ;$$

$$\partial V/\partial \lambda = \sum_{i=1}^{n} p_i N_i - K = 0 \ .$$

Actually the second order conditions are satisfied automtically by the way the function U/U_o was constructed, but they can be assured formally by investigating the sign of the bordered Hessian determinants will alternate properly. It follows then from the first order conditions that

$$(1/p_1)[\partial U/\partial N_1] = (1/p_2)[\partial U/\partial N_2] = \ldots\ldots\ldots = (1/p_n)[\partial U/\partial N_n].$$

One may now consider a particular illustration how some individual might optimize the allocation of sacrifice K between two conjoined simultaneous programs characterized by the following two sacrifice-benefit flows:

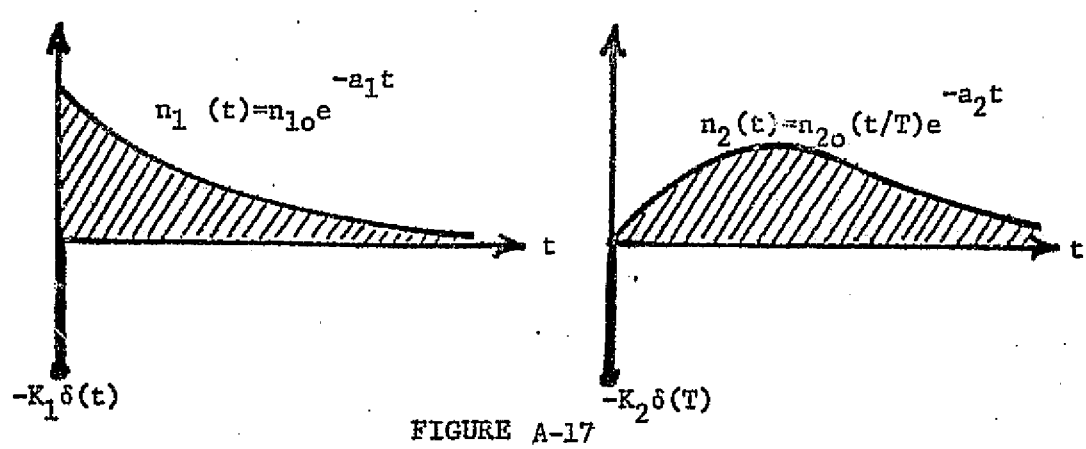


FIGURE A-17

Assume now saturation effects are not important. Thus, consider the following utility function for N_1 and N_2:

$$U/U_o = [N_1/N_{1o}]^h[N_2/N_{2o}]^{1-h} .$$

The rates of returns on sacrifices and the respective time price relationships for the two programs to be executed conjointly are, respectively, as follows:

$$r_1 = a_1[(N_1/K_1)-1] \text{ or } K_1 = [a_1/(r_1+a_1)]\, N_1 .$$

$$p_1 = a_1/(r_1+a_1).$$

$$r_2 = a_2[(N_2/K_2)^{1/2}-1] \text{ or } K_2 = [a_2/(r_2+a_2)]^2 N_2 .$$

$$p_2 = [a_2/(r_2+a_2)]^2 .$$

Then one has further the following constraint:

$$K = K_1 + K_2 = p_1N_1 + p_2N_2 = [a_1/(r_1+a_1)]N_1+[a_2/(r_2+a_2)]^2N_2 .$$

The first order condition yields the following relationship for the maximum relative utility:

$$(1/p_1)[\partial U/\partial N_1] = (1/p_2)[\partial U/\partial N_2] \text{ or}$$

$$[(r_1+a_1)/a_1]\, h\, N_1^{h-1}N_2^{1-h} = [(r_2+a_2)/a_2]^2\, (1-h)\, N_1^h\, N_2^{-h} \text{ or}$$

$$h\, K_2 = (1 - h)\, K_1 \text{ or } \underline{K_1 = h\, K \text{ and } K_2 = (1-h)\, K} .$$

This represents the optimal splitting of resources between the two conjoined programs. Since

$$N_1 = hK/p_1 \quad \text{and} \quad N_2 = (1-h)K/p_2 ,$$

the maximum utility would have the following relative value:

$$U/U_o = [N_{1o}\, N_{2o}^{1-h}]^{-1}[(r_1+a_1)/a_1]^h[(r_2+a_2]^{2(1-h)}K .$$

One notes the higher r_1 and r_2 becomes and the smaller a_1 and a_2 become the higher the relative utility. a_1 and a_2 represent venture failure rates. If $h = 1/2$, then

$$U/U_o = (N_{1o}\, N_{2o})^{-1/2}[(r_1+a_1)/a_1]^{1/2}[(r_2+a_2)/a_2]\, K .$$

CONCLUSIONS

The several examples presented here were done in order to illustrate how parametric modeling of benefit-sacrifice flows works. Each para-

meter introduced into the flow model has a relevant interpretation, and can be treated also as a variable. The number of parameters is also a number of degrees of freedom in shaping benefit and sacrifice streams to fit data or provide an adequately complete portrayal of a situation involving several performance, time, and financial factors. Benefit-Sacrifice streams relate also to product institutional life cycle processes not specifically discussed here. Some references on life-cylce modeling appear in Appendix M.

APPENDIX B

Definitions And Interest Factor Notation Used In Engineering Economic Calculations

TABLE 1

DEFINITIONS AND SYMBOLS USED FOR PARAMETERS
(Suggested Standards)

No.	Definition of Parameter	Symbol
1.	Effective interest rate per interest period.	i
2.	Nominal interest rate per year.	r
3.	Number of compounding periods.	N
4.	Number of compounding periods per year.	M
5.	Present sum of money. The letter "P" implies present. (or equivalent present value)	P
6.	Future sum of money. The letter "F" implies future. (or equivalent future value)	F
7.	End-of-period cash flows (or equivalent end-of-period values) in a uniform series continuing for a specified number of periods. The letter "A" implies annual or annuity.	A
8.	Uniform period-by-period increase or decrease in cash flows (or equivalent values); the arithmetic gradient.	G
9	Amount of money (or equivalent value) flowing continuously and uniformly during a given period.	$\overline{P}$ or $\overline{F}$
10.	Amount of money (or equivalent value) flowing continuously and uniformly during each and every period continuing for a specific number of periods.	$\overline{A}$

Appendix B was obtained from [130]

TABLE 2

MNEMONIC/FUNCTIONAL FORMS
OF
COMPOUND INTEREST FACTORS
(Suggested Standards)

Ref. No.	Name of Factor	Mnemonic Format	Functional Format
Group I. All cash flows discrete: end-of-period compounding			
1.	Compound Amount Factor (Single Payment)	(CA-i%-N)	(F/P,i%,N)
2.	Present Worth Factor (Single Payment)	(PW-i%-N)	(P/F,i%,N)
3.	Sinking Fund Factor	(SF-i%-N)	(A/F,i%,N)
4.	Capital Recovery Factor	(CR-i%-N)	(A/P,i%,N)
5.	Compound Amount Factor (Uniform Series)	(SCA-i%-N)	(F/A,i%,N)
6.	Present Worth Factor (Uniform Series)	(SPW-i%-N)	(P/A,i%,N)
7.	Arithmetic Gradient Conversion Factor (to Uniform Series)	(GUS-i%-N)	(A/G,i%,N)
8.	Arithmetic Gradient Conversion Factor (to Present Value)	(GPW-i%-N)	(P/G,i%,N)
Group II. All cash flows discrete: continuous compounding			
9.	Continuous Compounding Compound Amount Factor (Single Payment)	(CCA-r%-N)	(F/P,r%,N)
10.	Continuous Compounding Present Worth Factor (Single Payment)	(CPW-r%-N)	(P/F,r%,N)
11.	Continuous Compounding Sinking Fund Factor	(CSF-r%-N)	(A/F,r%,N)
12.	Continuous Compounding Capital Recovery Factor	(CCR-r%-N)	(A/P,r%,N)
13.	Continuous Compounding Compound Amount Factor (Uniform Series)	(CSCA-r%-N)	(F/A,r%,N)
14.	Continuous Compounding Present Worth Factor (Uniform Series)	(CSPW-r%-N)	(P/A,r%,N)

TABLE 2 *Continued*

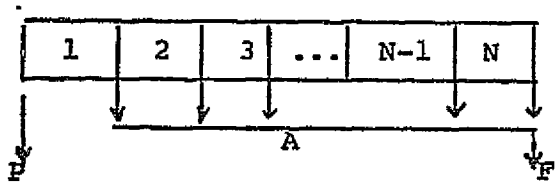
Ref. No.	Name of Factor	Mnemonic Format	Functional Format
Group III.	Continuous, uniform cash flows: continuous compounding (payments during one period only)		
15.	Continuous Compounding Present Worth Factor (single, continuous payment)	($\overline{CPW}$-i%-N)	(P/$\overline{F}$,i%,N)
16.	Continuous Compounding Compound Amount Factor (single, continuous payment)	($\overline{CCA}$-i%-N)	(F/$\overline{P}$,i%,N)
Group IV.	Continuous, uniform cash flows: continuous compounding (payments during a continuous series of periods)		
17.	Continuous Compounding Sinking Fund Factor (continuous, uniform payments)	($\overline{CSF}$-i%-N)	($\overline{A}$/F,i%,N)
18.	Continuous Compounding Capital Recovery Factor (continuous, uniform payments	($\overline{CCR}$-i%-N)	($\overline{A}$/P,i%,N)
19.	Continuous Compounding Present Worth Factor (continuous, uniform payments)	($\overline{CSCA}$-i%-N)	(F/$\overline{A}$,i%,N)
20.	Continuous Compounding Present Worth Factor (continuous, uniform payments)	($\overline{CSPW}$-i%-N)	(P/$\overline{A}$,i%,N)

TABLE 3

DIAGRAMS, ALGEBRAIC FORMS, AND USES FOR COMPOUND INTEREST FACTORS
(Explanatory Supplement to Table 2)

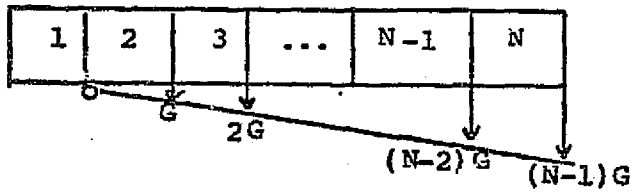
Group I. All cash flows discrete: end-of-period compounding

Cash Flow diagram for factors 1 through 6 (and 9 through 14):



No.	Name of Factor	Algebraic Form	Use when:
1	Compound Amount Factor (Single Payment)	$(1+i)^N$	Given P, to find F
2	Present Worth Factor (Single Payment)	$(1+i)^{-N}$	Given F, to find P
3	Sinking Fund Factor	$\frac{i}{(1+i)^N-1}$	Given F, to find A
4	Capital Recovery Factor	$\frac{i(1+i)^N}{(1+i)^N-1}$	Given P, to find A
5	Compound Amount Factor (Uniform Series)	$\frac{(1+i)^N-1}{i}$	Given A, to find F
6	Present Worth Factor (Uniform Series)	$\frac{(1+i)^N-1}{i(1+i)^N}$	Given A, to find P

Cash flow diagram for factors 7 and 8



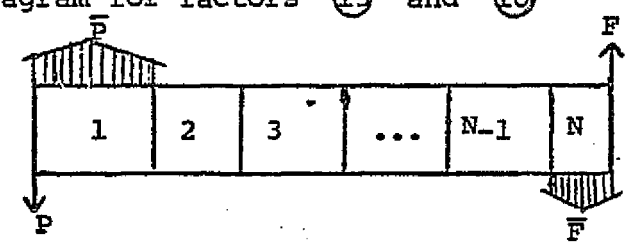
7	Arithmetic Gradient Conversion Factor (to uniform series)	$\left[\frac{1}{i} - \frac{N}{(1+i)^N-1}\right]$	Given G, to find A
8	Arithmetic Gradient Conversion Factor (to present value)	$\left[\frac{1}{i} - \frac{N}{(1+i)^N-1}\right]\left[\frac{(1+i)^N-1}{i(1+i)^N}\right]$	Given G, to find P

Group II. All cash flows discrete: continuous compounding

No.	Name of Factor	Algebraic Form	Use when:
9	Continuous Compounding Compound Amount Factor (Single Payment)	e^{rN}	Given P, to find F
10	Continuous Compounding Present Worth Factor (Single Payment)	e^{-rN}	Given F, to find P
11	Continuous Compounding Sinking Fund Factor	$\frac{e^{r}-1}{e^{rN}-1}$	Given F, to find A
12	Continuous Compounding Capital Recovery Factor	$\frac{e^{rN}(e^{r}-1)}{e^{rN}-1}$	Given P, to find A
13	Continuous Compounding Compound Amount Factor (Uniform Series)	$\frac{e^{rN}-1}{e^{r}-1}$	Given A, to find F
14	Continuous Compounding Present Worth Factor (Uniform Series)	$\frac{e^{rN}-1}{e^{rN}(e^{r}-1)}$	Given A, to find P

Group III. Continuous, uniform cash flows: continuous compounding (Payments during one period only)

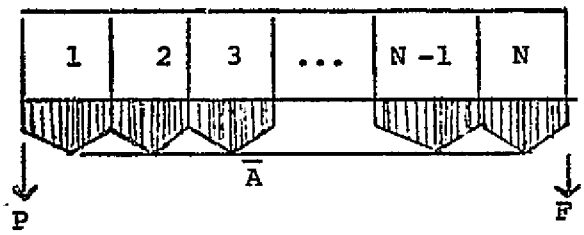
Cash flow diagram for factors 15 and 16



No.	Name of Factor	Algebraic Form	Use when:
15	Continuous Compounding Present Worth Factor (Single Continuous Payment)	$\frac{e^{r}-1}{re^{rN}} = \frac{i(1+i)^{-N}}{\ln(1+i)}$	Given $\bar{F}$, to find P
16	Continuous Compounding Compound Amount Factor (Single Continuous Payment)	$\frac{e^{rN}(e^{r}-1)}{re^{r}} = \frac{i(1+i)^{N-1}}{\ln(1+i)}$	Given $\bar{P}$, to find F

Group IV. Continuous, uniform cash flows
(Payments during a continuous series of periods)

Cash flow diagram for factors 17 through 20



No.	Name of Factor	Algebraic Form	Use when:
17	Continuous Compounding Sinking Fund Factor (Continuous, Uniform Payments)	$\frac{r}{e^{rN}-1} = \frac{\ln(1+i)}{(1+i)^N-1}$	Given F, to find $\bar{A}$
18	Continuous Compounding Capital Recovery Factor (Continuous, Uniform Payments)	$\frac{re^{rN}}{e^{rN}-1} = \frac{(1+i)^N \ln(1+i)}{(1+i)^N-1}$	Given P, to find $\bar{A}$
19	Continuous Compounding Compound Amount Factor (Continuous, Uniform Payments)	$\frac{e^{rN}-1}{r} = \frac{(1+i)^N-1}{\ln(1+i)}$	Given $\bar{A}$, to find F
20	Continuous Compounding Present Worth Factor (Continuous, Uniform Payments)	$\frac{e^{rN}-1}{re^{rN}} = \frac{(1+i)^N-1}{(1+i)^N \ln(1+i)}$	Given $\bar{A}$, to find P

APPENDIX C

Interest Formula Values
Discrete Compounding

Single Payment Compound Amount Factor
Discrete Compounding
(F/P, r%, N)

N	1% To find F Given P F/P	2% To find F Given P F/P	4% To find F Given P F/P	6% To find F Given P F/P	8% To find F Given P N/P	10% To find F Given P F/P	12% To find F Given P F/P	15% To find F Given P F/P	20% To find F Given P F/P	25% To find F Given P F/P	N
1	1.0100	1.0200	1.0400	1.0600	1.0800	1.1000	1.1200	1.1500	1.2000	1.2500	1
2	1.0201	1.0404	1.0816	1.1236	1.1664	1.2100	1.2544	1.3225	1.4400	1.5625	2
3	1.0303	1.0612	1.1249	1.1910	1.2597	1.3310	1.4049	1.5209	1.7280	1.9531	3
4	1.0406	1.0824	1.1699	1.2625	1.3605	1.4641	1.5735	1.7490	2.0736	2.4414	4
5	1.0510	1.1041	1.2167	1.3382	1.4693	1.6105	1.7623	2.0114	2.4883	3.0518	5
6	1.0615	1.1262	1.2653	1.4185	1.5869	1.7716	1.9738	2.3131	2.9860	3.8147	6
7	1.0721	1.1487	1.3159	1.5036	1.7138	1.9487	2.2107	2.6600	3.5832	4.7684	7
8	1.0829	1.1717	1.3686	1.5938	1.8509	2.1436	2.4760	3.0590	4.2998	5.9605	8
9	1.0937	1.1951	1.4233	1.6895	1.9990	2.3579	2.7731	3.5179	5.1598	7.4506	9
10	1.1046	1.2190	1.4802	1.7908	2.1589	2.5937	3.1058	4.0456	6.1917	9.3132	10
11	1.1157	1.2434	1.5395	1.8983	2.3316	2.8531	3.4785	4.6524	7.4301	11.6415	11
12	1.1268	1.2682	1.6010	2.0122	2.5182	3.1384	3.8960	5.3502	8.9161	14.5519	12
13	1.1381	1.2936	1.6651	2.1329	2.7196	3.4523	4.3635	6.1528	10.6993	18.1899	13
14	1.1495	1.3195	1.7317	2.2609	2.9372	3.7975	4.8871	7.0757	12.8392	22.7374	14
15	1.1610	1.3459	1.8009	2.3966	3.1722	4.1772	5.4736	8.1371	15.4070	28.4217	15
16	1.1726	1.3728	1.8730	2.5404	3.4259	4.5950	6.1304	9.3576	18.4884	35.5271	16
17	1.1843	1.4002	1.9479	2.6928	3.7000	5.0545	6.8660	10.7613	22.1861	44.4089	17
18	1.1961	1.4282	2.0258	2.8543	3.9960	5.5599	7.6900	12.3755	26.6233	55.5112	18
19	1.2081	1.4568	2.1068	3.0256	4.3157	6.1159	8.6128	14.2318	31.9480	69.3889	19
20	1.2202	1.4859	2.1911	3.2071	4.6610	6.7275	9.6463	16.3665	38.3376	86.7362	20
21	1.2324	1.5157	2.2788	3.3996	5.0338	7.4002	10.8038	18.8215	46.0051	108.420	21
22	1.2447	1.5460	2.3699	3.6035	5.4365	8.1403	12.1003	21.6447	55.2061	135.525	22
23	1.2572	1.5769	2.4647	3.8197	5.8715	8.9543	13.5523	24.8915	66.2474	169.407	23
24	1.2697	1.6084	2.5633	4.0489	6.3412	9.8497	15.1786	28.6252	79.4968	211.758	24
25	1.2824	1.6406	2.6658	4.2919	6.8485	10.8347	17.0001	32.9189	95.3962	264.698	25
26	1.2953	1.6734	2.7725	4.5494	7.3964	11.9182	19.0401	37.8568	114.475	330.872	26
27	1.3082	1.7069	2.8834	4.8223	7.9881	13.1100	21.3249	43.5353	137.371	413.590	27
28	1.3213	1.7410	2.9987	5.1117	8.6271	14.4210	23.8839	50.0656	164.845	516.988	28
29	1.3345	1.7758	3.1187	5.4184	9.3173	15.8631	26.7499	57.5754	197.814	646.235	29
30	1.3478	1.8114	3.2434	5.7435	10.0627	17.4494	29.9599	66.2118	237.376	807.794	30
35	1.4166	1.9999	3.9461	7.6861	14.7853	28.1024	52.7996	133.176	590.668	2465.19	35
40	1.4889	2.2080	4.8010	10.2857	21.7245	45.2592	93.0509	267.863	1469.77	7523.16	40
45	1.5648	2.4379	5.8412	13.7646	31.9204	72.8904	163.988	538.769	3657.26	22958.9	45
50	1.6446	2.6916	7.1067	18.4201	46.9016	117.391	289.002	1083.66	9100.43	70064.9	50
55	1.7285	2.9717	8.6464	24.6503	68.9138	189.059	509.320	2179.62	22644.8		55
60	1.8167	3.2810	10.5196	32.9876	101.257	304.481	897.596	4384.00	56347.5		60
65	1.9094	3.6225	12.7987	44.1449	148.780	490.370	1581.87	8817.78			65
70	2.0068	3.9996	15.5716	59.0758	218.606	789.746	2787.80	17735.7			70
75	2.1091	4.4158	18.9452	79.0568	321.204	1271.89	4913.05	35672.8			75
80	2.2167	4.8754	23.0498	105.796	471.955	2048.40	8658.47	71750.8			80
85	2.3298	5.3829	28.0436	141.579	693.456	3298.97					85
90	2.4486	5.9431	34.1193	189.464	1018.92	5313.02					90
95	2.5735	6.5617	41.5113	253.546	1497.12	8556.67					95
100	2.7048	7.2446	50.5049	339.301	2199.76	13780.6					100
∞											∞

Single Payment Present Worth Factor
Discrete Compounding
(P/F, r%, N)

N	1% To find P Given F P/F	2% To find P Given F P/F	4% To find P Given F P/F	6% To find P Given F P/F	8% To find P Given F P/F	10% To find P Given F P/F	12% To find P Given F P/F	15% To find P Given F P/F	20% To find P Given F P/F	25% To find P Given F P/F	N
1	0.9901	0.9804	0.9615	0.9434	0.9259	0.9091	0.8929	0.8696	0.8333	0.8000	1
2	0.9803	0.9612	0.9246	0.8900	0.8573	0.8264	0.7972	0.7561	0.6944	0.6400	2
3	0.9706	0.9423	0.8890	0.8396	0.7938	0.7513	0.7118	0.6575	0.5787	0.5120	3
4	0.9610	0.9238	0.8548	0.7921	0.7350	0.6830	0.6355	0.5718	0.4823	0.4096	4
5	0.9515	0.9057	0.8219	0.7473	0.6806	0.6209	0.5674	0.4972	0.4019	0.3277	5
6	0.9420	0.8880	0.7903	0.7050	0.6302	0.5645	0.5066	0.4323	0.3349	0.2621	6
7	0.9327	0.8706	0.7599	0.6651	0.5835	0.5132	0.4523	0.3759	0.2791	0.2097	7
8	0.9235	0.8535	0.7307	0.6274	0.5403	0.4665	0.4039	0.3269	0.2326	0.1678	8
9	0.9143	0.8368	0.7026	0.5919	0.5002	0.4241	0.3606	0.2843	0.1938	0.1342	9
10	0.9053	0.8203	0.6756	0.5584	0.4632	0.3855	0.3220	0.2472	0.1615	0.1074	10
11	0.8963	0.8043	0.6496	0.5268	0.4289	0.3505	0.2875	0.2149	0.1346	0.0859	11
12	0.8874	0.7885	0.6246	0.4970	0.3971	0.3186	0.2567	0.1869	0.1122	0.0687	12
13	0.8787	0.7730	0.6006	0.4688	0.3677	0.2897	0.2292	0.1625	0.0935	0.0550	13
14	0.8700	0.7579	0.5775	0.4423	0.3405	0.2633	0.2046	0.1413	0.0779	0.0440	14
15	0.8613	0.7430	0.5553	0.4173	0.3152	0.2394	0.1827	0.1229	0.0649	0.0352	15
16	0.8528	0.7284	0.5339	0.3936	0.2919	0.2176	0.1631	0.1069	0.0541	0.0281	16
17	0.8444	0.7142	0.5134	0.3714	0.2703	0.1978	0.1456	0.0929	0.0451	0.0225	17
18	0.8360	0.7002	0.4936	0.3503	0.2502	0.1799	0.1300	0.0808	0.0376	0.0180	18
19	0.8277	0.6864	0.4746	0.3305	0.2317	0.1635	0.1161	0.0703	0.0313	0.0144	19
20	0.8195	0.6730	0.4564	0.3118	0.2145	0.1486	0.1037	0.0611	0.0261	0.0115	20
21	0.8114	0.6598	0.4388	0.2942	0.1987	0.1351	0.0926	0.0531	0.0217	0.0092	21
22	0.8034	0.6468	0.4220	0.2775	0.1839	0.1228	0.0826	0.0462	0.0181	0.0074	22
23	0.7954	0.6342	0.4057	0.2618	0.1703	0.1117	0.0738	0.0402	0.0151	0.0059	23
24	0.7876	0.6217	0.3901	0.2470	0.1577	0.1015	0.0659	0.0349	0.0126	0.0047	24
25	0.7798	0.6095	0.3751	0.2330	0.1460	0.0923	0.0588	0.0304	0.0105	0.0038	25
26	0.7720	0.5976	0.3607	0.2198	0.1352	0.0839	0.0525	0.0264	0.0087	0.0030	26
27	0.7644	0.5859	0.3468	0.2074	0.1252	0.0763	0.0469	0.0230	0.0073	0.0024	27
28	0.7568	0.5744	0.3335	0.1956	0.1159	0.0693	0.0419	0.0200	0.0061	0.0019	28
29	0.7493	0.5631	0.3207	0.1846	0.1073	0.0630	0.0374	0.0174	0.0051	0.0015	29
30	0.7419	0.5521	0.3083	0.1741	0.0994	0.0573	0.0334	0.0151	0.0042	0.0012	30
35	0.7059	0.5000	0.2534	0.1301	0.0676	0.0356	0.0189	0.0075	0.0017	0.0004	35
40	0.6717	0.4529	0.2083	0.0972	0.0460	0.0221	0.0107	0.0037	0.0007	0.0001	40
45	0.6391	0.4102	0.1712	0.0727	0.0313	0.0137	0.0061	0.0019	0.0003		45
50	0.6080	0.3715	0.1407	0.0543	0.0213	0.0085	0.0035	0.0009	0.0001		50
55	0.5785	0.3365	0.1157	0.0406	0.0145	0.0053	0.0020	0.0005			55
60	0.5505	0.3048	0.0951	0.0303	0.0099	0.0033	0.0011	0.0002			60
65	0.5237	0.2761	0.0781	0.0227	0.0067	0.0020	0.0006	0.0001			65
70	0.4983	0.2500	0.0642	0.0169	0.0046	0.0013	0.0004				70
75	0.4741	0.2265	0.0528	0.0126	0.0031	0.0008	0.0002				75
80	0.4511	0.2051	0.0434	0.0095	0.0021	0.0005	0.0001				80
85	0.4292	0.1858	0.0357	0.0071	0.0014	0.0003					85
90	0.4084	0.1683	0.0293	0.0053	0.0010	0.0002					90
95	0.3886	0.1524	0.0241	0.0039	0.0007	0.0001					95
100	0.3697	0.1380	0.0198	0.0029	0.0005						100
∞											∞

Uniform Series Compound Amount Factor
Discrete Compounding
(F/A, r%, N)

N	To find F Given A F/A	To find F Given A F/A	To find F Given A F/A	To find F Given A F/A	To find F Given A F/A	To find F Given A F/A	To find F Given A F/A	To find F Given A F/A	To find F Given A F/A	To find F Given A F/A	N
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
2	2.0100	2.0200	2.0400	2.0600	2.0800	2.1000	2.1200	2.1500	2.2000	2.2500	2
3	3.0301	3.0604	3.1216	3.1836	3.2464	3.3100	3.3744	3.4725	3.6400	3.8125	3
4	4.0604	4.1216	4.2465	4.3746	4.5061	4.6410	4.7793	4.9934	5.3680	5.7656	4
5	5.1010	5.2040	5.4163	5.6371	5.8666	6.1051	6.3528	6.7424	7.4416	8.2070	5
6	6.1520	6.3081	6.6330	6.9753	7.3359	7.7156	8.1152	8.7537	9.9299	11.2588	6
7	7.2135	7.4343	7.8983	8.3938	8.9228	9.4872	10.0890	11.0668	12.9159	15.0735	7
8	8.2857	8.5830	9.2142	9.8975	10.6366	11.4359	12.2997	13.7268	16.4991	19.8419	8
9	9.3685	9.7546	10.5828	11.4913	12.4876	13.5795	14.7757	16.7858	20.7989	25.8023	9
10	10.4622	10.9497	12.0061	13.1808	14.4866	15.9374	17.5487	20.3037	25.9587	33.2529	10
11	11.5668	12.1687	13.4863	14.9716	16.6455	18.5312	20.6546	24.3493	32.1504	42.5661	11
12	12.6825	13.4121	15.0258	16.8699	18.9771	21.3843	24.1331	29.0017	39.5805	54.2077	12
13	13.8093	14.6803	16.6268	18.8821	21.4953	24.5227	28.0291	34.3519	48.4966	68.7596	13
14	14.9474	15.9739	18.2919	21.0151	24.2149	27.9750	32.3926	40.5047	59.1959	86.9495	14
15	16.0969	17.2934	20.0236	23.2760	27.1521	31.7725	37.2797	47.5804	72.0351	109.687	15
16	17.2578	18.6393	21.8245	25.6725	30.3243	35.9497	42.7533	55.7175	87.4421	138.109	16
17	18.4304	20.0121	23.6975	28.2129	33.7502	40.5447	48.8837	65.0751	105.931	173.636	17
18	19.6147	21.4123	25.6454	30.9056	37.4502	45.5992	55.7497	75.8363	128.117	218.045	18
19	20.8109	22.8405	27.6712	33.7600	41.4463	51.1591	63.4397	88.2118	154.740	273.556	19
20	22.0190	24.2974	29.7781	36.7856	45.7620	57.2750	72.0524	102.444	186.688	342.945	20
21	23.2391	25.7833	31.9692	39.9927	50.4229	64.0025	81.6987	118.810	225.026	429.681	21
22	24.4715	27.2990	34.2480	43.3923	55.4567	71.4027	92.5026	137.632	271.031	538.101	22
23	25.7162	28.8449	36.6179	46.9958	60.8933	79.5430	104.603	159.276	326.237	673.626	23
24	26.9734	30.4218	39.0826	50.8155	66.7647	88.4973	118.155	184.168	392.484	843.033	24
25	28.2431	32.0303	41.6459	54.8645	73.1059	98.3470	133.334	212.793	471.981	1054.79	25
26	29.5256	33.6709	44.3117	59.1563	79.9544	109.182	150.334	245.712	567.377	1319.49	26
27	30.8208	35.3443	47.0842	63.7057	87.3507	121.100	169.374	283.569	681.853	1650.36	27
28	32.1290	37.0512	49.9676	68.5281	95.3388	134.210	190.699	327.304	819.223	2063.95	28
29	33.4503	38.7922	52.9663	73.6397	103.966	148.631	214.583	377.170	984.068	2580.94	29
30	34.7848	40.5681	56.0849	79.0581	113.283	164.494	241.333	434.745	1181.88	3227.17	30
35	41.6602	49.9944	73.6522	111.435	172.317	271.024	431.663	881.170	2948.34	9856.76	35
40	48.8863	60.4019	95.0255	154.762	259.056	442.592	767.091	1779.09	7343.85	30088.7	40
45	56.4809	71.8927	121.029	212.743	386.506	718.905	1358.23	3585.13	18281.3	91831.5	45
50	64.4630	84.5793	152.667	290.336	573.770	1163.91	2400.02	7217.71	45497.2	280256	50
55	72.8523	98.5864	191.159	394.172	848.923	1880.59	4236.00	14524.1	113219		55
60	81.6695	114.051	237.991	533.128	1253.21	3034.81	7471.63	29220.0	281732		60
65	90.9364	131.126	294.968	719.082	1847.25	4893.71	13173.9	58778.5			65
70	100.676	149.978	364.290	967.931	2720.08	7887.47	23223.3	118231			70
75	110.912	170.792	448.631	1300.95	4002.55	12708.9	40933.8	237812			75
80	121.671	193.772	551.245	1746.60	5886.93	20474.0	72145.6	478332			80
85	132.979	219.144	676.090	2342.98	8655.71	32979.7					85
90	144.86	247.16	827.98	3141.07	12723.9	53120.2					90
95	157.35	278.08	1012.78	4209.10	18071.5	85556.7					95
100	170.48	312.23	1237.62	5638.36	27484.5	137796					100
∞											∞

Uniform Series Sinking Fund Factor
Discrete Compounding
(A/F, r%, N)

N	To find A Given F A/F	To find A Given F A/F	To find A Given F A/F	To find A Given F A/F	To find A Given F A/F	To find A Given F A/F	To find A Given F A/F	To find A Given F A/F	To find A Given F A/F	To find A Given F A/F	N
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
2	0.4975	0.4950	0.4902	0.4854	0.4808	0.4762	0.4717	0.4651	0.4545	0.4444	2
3	0.3300	0.3268	0.3203	0.3141	0.3080	0.3021	0.2963	0.2880	0.2747	0.2623	3
4	0.2463	0.2426	0.2355	0.2286	0.2219	0.2155	0.2092	0.2003	0.1863	0.1734	4
5	0.1960	0.1922	0.1846	0.1774	0.1705	0.1638	0.1574	0.1483	0.1344	0.1218	5
6	0.1625	0.1585	0.1508	0.1434	0.1363	0.1296	0.1232	0.1142	0.1007	0.0888	6
7	0.1386	0.1345	0.1266	0.1191	0.1121	0.1054	0.0991	0.0904	0.0774	0.0663	7
8	0.1207	0.1165	0.1085	0.1010	0.0940	0.0874	0.0813	0.0729	0.0606	0.0504	8
9	0.1067	0.1025	0.0945	0.0870	0.0801	0.0736	0.0677	0.0596	0.0481	0.0388	9
10	0.0956	0.0913	0.0833	0.0759	0.0690	0.0627	0.0570	0.0493	0.0385	0.0301	10
11	0.0865	0.0822	0.0741	0.0668	0.0601	0.0540	0.0484	0.0411	0.0311	0.0235	11
12	0.0788	0.0746	0.0666	0.0593	0.0527	0.0468	0.0414	0.0345	0.0253	0.0184	12
13	0.0724	0.0681	0.0601	0.0530	0.0465	0.0408	0.0357	0.0291	0.0206	0.0145	13
14	0.0669	0.0626	0.0547	0.0476	0.0413	0.0357	0.0309	0.0247	0.0169	0.0115	14
15	0.0621	0.0578	0.0499	0.0430	0.0368	0.0315	0.0268	0.0210	0.0139	0.0091	15
16	0.0579	0.0537	0.0458	0.0390	0.0330	0.0278	0.0234	0.0179	0.0114	0.0072	16
17	0.0543	0.0500	0.0422	0.0354	0.0296	0.0247	0.0205	0.0154	0.0094	0.0058	17
18	0.0510	0.0467	0.0390	0.0324	0.0267	0.0219	0.0179	0.0132	0.0078	0.0046	18
19	0.0481	0.0438	0.0361	0.0296	0.0241	0.0195	0.0158	0.0113	0.0065	0.0037	19
20	0.0454	0.0412	0.0336	0.0272	0.0219	0.0175	0.0139	0.0098	0.0054	0.0029	20
21	0.0430	0.0388	0.0313	0.0250	0.0198	0.0156	0.0122	0.0084	0.0044	0.0023	21
22	0.0409	0.0366	0.0292	0.0230	0.0180	0.0140	0.0108	0.0073	0.0037	0.0019	22
23	0.0389	0.0347	0.0273	0.0213	0.0164	0.0126	0.0096	0.0063	0.0031	0.0015	23
24	0.0371	0.0329	0.0256	0.0197	0.0150	0.0113	0.0085	0.0054	0.0025	0.0012	24
25	0.0354	0.0312	0.0240	0.0182	0.0137	0.0102	0.0075	0.0047	0.0021	0.0009	25
26	0.0339	0.0297	0.0226	0.0169	0.0125	0.0092	0.0067	0.0041	0.0018	0.0008	26
27	0.0324	0.0283	0.0212	0.0157	0.0114	0.0083	0.0059	0.0035	0.0015	0.0006	27
28	0.0311	0.0270	0.0200	0.0146	0.0105	0.0075	0.0052	0.0031	0.0012	0.0005	28
29	0.0299	0.0258	0.0189	0.0136	0.0096	0.0067	0.0047	0.0027	0.0010	0.0004	29
30	0.0287	0.0246	0.0178	0.0126	0.0088	0.0061	0.0041	0.0023	0.0008	0.0003	30
35	0.0240	0.0200	0.0136	0.0090	0.0058	0.0037	0.0023	0.0011	0.0003	0.0001	35
40	0.0205	0.0166	0.0105	0.0065	0.0039	0.0023	0.0013	0.0006	0.0001		40
45	0.0177	0.0139	0.0083	0.0047	0.0026	0.0014	0.0007	0.0003			45
50	0.0155	0.0118	0.0066	0.0034	0.0017	0.0009	0.0004	0.0001			50
55	0.0137	0.0101	0.0052	0.0025	0.0012	0.0005	0.0002				55
60	0.0122	0.0088	0.0042	0.0019	0.0008	0.0003	0.0001				60
65	0.0110	0.0076	0.0034	0.0014	0.0005	0.0002					65
70	0.0099	0.0067	0.0027	0.0010	0.0004	0.0001					70
75	0.0090	0.0059	0.0022	0.0008	0.0002						75
80	0.0082	0.0052	0.0018	0.0006	0.0002						80
85	0.0075	0.0046	0.0015	0.0004	0.0001						85
90	0.0069	0.0040	0.0012	0.0003							90
95	0.0064	0.0036	0.0010	0.0002							95
100	0.0059	0.0032	0.0008	0.0002							100

Uniform Series Present Worth Factor
Discrete Compounding
(P/A, r%, N)

N	To find P Given A P/A	To find P Given A P/A	To find P Given A P/A	To find P Given A P/A	To find P Given A P/A	To find P Given A P/A	To find P Given A P/A	To find P Given A P/A	To find P Given A P/A	To find P Given A P/A	N
1	0.9901	0.9804	0.9615	0.9434	0.9259	0.9091	0.8929	0.8696	0.8333	0.8000	1
2	1.9704	1.9416	1.8861	1.8334	1.7833	1.7355	1.6901	1.6257	1.5278	1.4400	2
3	2.9410	2.8839	2.7751	2.6730	2.5771	2.4869	2.4018	2.2832	2.1065	1.9520	3
4	3.9020	3.8077	3.6299	3.4651	3.3121	3.1699	3.0373	2.8550	2.5887	2.3616	4
5	4.8534	4.7135	4.4518	4.2124	3.9927	3.7908	3.6048	3.3522	2.9906	2.6893	5
6	5.7955	5.6014	5.2421	4.9173	4.6229	4.3553	4.1114	3.7845	3.3255	2.9514	6
7	6.7282	6.4720	6.0021	5.5824	5.2064	4.8684	4.5638	4.1604	3.6046	3.1611	7
8	7.6517	7.3255	6.7327	6.2098	5.7466	5.3349	4.9676	4.4873	3.8372	3.3289	8
9	8.5660	8.1622	7.4353	6.8017	6.2469	5.7590	5.3282	4.7716	4.0310	3.4631	9
10	9.4713	8.9826	8.1109	7.3601	6.7101	6.1446	5.6502	5.0188	4.1925	3.5705	10
11	10.3676	9.7868	8.7605	7.8869	7.1390	6.4951	5.9377	5.2337	4.3271	3.6564	11
12	11.2551	10.5753	9.3851	8.3838	7.5361	6.8137	6.1944	5.4206	4.4392	3.7251	12
13	12.1337	11.3484	9.9856	8.8527	7.9038	7.1034	6.4235	5.5831	4.5327	3.7801	13
14	13.0037	12.1062	10.5631	9.2950	8.2442	7.3667	6.6282	5.7245	4.6106	3.8241	14
15	13.8650	12.8493	11.1184	9.7122	8.5595	7.6061	6.8109	5.8474	4.6755	3.8593	15
16	14.7178	13.5777	11.6523	10.1059	8.8514	7.8237	6.9740	5.9542	4.7296	3.8874	16
17	15.5622	14.2919	12.1657	10.4773	9.1216	8.0216	7.1196	6.0472	4.7746	3.9099	17
18	16.3982	14.9920	12.6593	10.8276	9.3719	8.2014	7.2497	6.1280	4.8122	3.9279	18
19	17.2260	15.6785	13.1339	11.1581	9.6036	8.3649	7.3658	6.1982	4.8435	3.9424	19
20	18.0455	16.3514	13.5903	11.4699	9.8181	8.5136	7.4694	6.2593	4.8696	3.9539	20
21	18.8570	17.0112	14.0292	11.7641	10.0168	8.6487	7.5620	6.3125	4.8913	3.9631	21
22	19.6603	17.6580	14.4511	12.0416	10.2007	8.7715	7.6446	6.3587	4.9094	3.9705	22
23	20.4558	18.2922	14.8568	12.3034	10.3711	8.8832	7.7184	6.3988	4.9245	3.9764	23
24	21.2434	18.9139	15.2470	12.5504	10.5288	8.9847	7.7843	6.4338	4.9371	3.9811	24
25	22.0231	19.5234	15.6221	12.7834	10.6748	9.0770	7.8431	6.4641	4.9476	3.9849	25
26	22.7952	20.1210	15.9828	13.0032	10.8100	9.1609	7.8957	6.4906	4.9563	3.9879	26
27	23.5596	20.7069	16.3296	13.2105	10.9352	9.2372	7.9426	6.5135	4.9636	3.9903	27
28	24.3164	21.2813	16.6631	13.4062	11.0511	9.3066	7.9844	6.5335	4.9697	3.9923	28
29	25.0657	21.8444	16.9837	13.5907	11.1584	9.3696	8.0218	6.5509	4.9747	3.9938	29
30	25.8077	22.3964	17.2920	13.7648	11.2578	9.4269	8.0552	6.5660	4.9789	3.9950	30
35	29.4085	24.9986	18.6646	14.4982	11.6546	9.6442	8.1755	6.6166	4.9915	3.9984	35
40	32.8346	27.3555	19.7928	15.0463	11.9246	9.7791	8.2438	6.6418	4.9966	3.9995	40
45	36.0945	29.4902	20.7200	15.4558	12.1084	9.8628	8.2825	6.6543	4.9986	3.9998	45
50	39.1961	31.4236	21.4822	15.7619	12.2335	9.9148	8.3045	6.6605	4.9995	3.9999	50
55	42.1471	33.1748	22.1086	15.9905	12.3186	9.9471	8.3170	6.6636	4.9998		55
60	44.9550	34.7609	22.6235	16.1614	12.3766	9.9672	8.3240	6.6651	4.9999		60
65	47.6265	36.1975	23.0467	16.2891	12.4160	9.9796	8.3281	6.6659			65
70	50.1684	37.4986	23.3945	16.3845	12.4428	9.9873	8.3303	6.6663			70
75	52.5870	38.6771	23.6804	16.4558	12.4611	9.9921	8.3316	6.6665			75
80	54.8881	39.7445	23.9154	16.5091	12.4735	9.9951	8.3324	6.6666			80
85	57.0776	40.7113	24.1085	16.5489	12.4820	9.9970					85
90	59.161	41.5869	24.2673	16.5787	12.4877	9.9981					90
95	61.143	42.3800	24.3978	16.6009	12.4917	9.9988					95
100	63.029	43.0983	24.5050	16.6175	12.4943	9.9993					100
∞	100.000	50.0000	25.0000	18.182	12.5000	10.0000					∞

Uniform Series Capital Recovery Factor
Discrete Compounding
(A/P, r%, N)

N	1% To find A Given P A/P	2% To find A Given P A/P	4% To find A Given P A/P	6% To find A Given P A/P	8% To find A Given P A/P	10% To find A Given P A/P	12% To find A Given P A/P	15% To find A Given P A/P	20% To find A Given P A/P	25% To find A Given P A/P	N
1	1.0100	1.0200	1.0400	1.0600	1.0800	1.1000	1.1200	1.1500	1.2000	1.2500	1
2	0.5075	0.5150	0.5302	0.5454	0.5608	0.5762	0.5917	0.6151	0.6545	0.6944	2
3	0.3400	0.3468	0.3603	0.3741	0.3880	0.4021	0.4163	0.4380	0.4747	0.5123	3
4	0.2563	0.2626	0.2755	0.2886	0.3019	0.3155	0.3292	0.3503	0.3863	0.4234	4
5	0.2060	0.2122	0.2246	0.2374	0.2505	0.2638	0.2774	0.2983	0.3344	0.3718	5
6	0.1725	0.1785	0.1908	0.2034	0.2163	0.2296	0.2432	0.2642	0.3007	0.3388	6
7	0.1486	0.1545	0.1666	0.1791	0.1921	0.2054	0.2191	0.2404	0.2774	0.3163	7
8	0.1307	0.1365	0.1485	0.1610	0.1740	0.1874	0.2013	0.2229	0.2606	0.3004	8
9	0.1167	0.1225	0.1345	0.1470	0.1601	0.1736	0.1877	0.2096	0.2481	0.2888	9
10	0.1056	0.1113	0.1233	0.1359	0.1490	0.1627	0.1770	0.1993	0.2385	0.2801	10
11	0.0965	0.1022	0.1141	0.1268	0.1401	0.1540	0.1684	0.1911	0.2311	0.2735	11
12	0.0888	0.0946	0.1066	0.1193	0.1327	0.1468	0.1614	0.1845	0.2253	0.2684	12
13	0.0824	0.0881	0.1001	0.1130	0.1265	0.1408	0.1557	0.1791	0.2206	0.2645	13
14	0.0769	0.0826	0.0947	0.1076	0.1213	0.1357	0.1509	0.1747	0.2169	0.2615	14
15	0.0721	0.0778	0.0899	0.1030	0.1168	0.1315	0.1468	0.1710	0.2139	0.2591	15
16	0.0679	0.0737	0.0858	0.0990	0.1130	0.1278	0.1434	0.1679	0.2114	0.2572	16
17	0.0643	0.0700	0.0822	0.0954	0.1096	0.1247	0.1405	0.1654	0.2094	0.2558	17
18	0.0610	0.0667	0.0790	0.0924	0.1067	0.1219	0.1379	0.1632	0.2078	0.2546	18
19	0.0581	0.0638	0.0761	0.0896	0.1041	0.1195	0.1358	0.1613	0.2065	0.2537	19
20	0.0554	0.0612	0.0736	0.0872	0.1019	0.1175	0.1339	0.1598	0.2054	0.2529	20
21	0.0530	0.0588	0.0713	0.0850	0.0998	0.1156	0.1322	0.1584	0.2044	0.2523	21
22	0.0509	0.0566	0.0692	0.0830	0.0980	0.1140	0.1308	0.1573	0.2037	0.2519	22
23	0.0489	0.0547	0.0673	0.0813	0.0964	0.1126	0.1296	0.1563	0.2031	0.2515	23
24	0.0471	0.0529	0.0656	0.0797	0.0950	0.1113	0.1285	0.1554	0.2025	0.2512	24
25	0.0454	0.0512	0.0640	0.0782	0.0937	0.1102	0.1275	0.1547	0.2021	0.2509	25
26	0.0439	0.0497	0.0626	0.0769	0.0925	0.1092	0.1267	0.1541	0.2017	0.2508	26
27	0.0424	0.0483	0.0612	0.0757	0.0914	0.1083	01.259	0.1535	0.2015	0.2506	27
28	0.0411	0.0470	0.0600	0.0746	0.0905	0.1075	0.1252	0.1531	0.2012	0.2505	28
29	0.0399	0.0458	0.0589	0.0736	0.0896	0.1067	0.1247	0.1527	0.2010	0.2504	29
30	0.0387	0.0446	0.0578	0.0726	0.0888	0.1061	0.1241	0.1523	0.2008	0.2503	30
35	0.0340	0.0400	0.0536	0.0690	0.0858	0.1037	0.1223	0.1511	0.2003	0.2501	35
40	0.0305	0.0366	0.0505	0.0665	0.0839	0.1023	0.1213	0.1506	0.2001	0.2500	40
45	0.0277	0.0339	0.0483	0.0647	0.0826	0.1014	0.1207	0.1503	0.2001	0.2500	45
50	0.0255	0.0318	0.0466	0.0634	0.0817	0.1009	0.1204	0.1501	0.2000	0.2500	50
55	0.0237	0.0301	0.0452	0.0625	0.0812	0.1005	0.1202	0.1501	0.2000	0.2500	55
60	0.0222	0.0288	0.0442	0.0619	0.0808	0.1003	0.1201	0.1500	0.2000		60
65	0.0210	0.0276	0.0434	0.0614	0.0805	0.1002	0.1201	0.1500	0.2000		65
70	0.0199	0.0267	0.0427	0.0610	0.0804	0.1001	0.1200	0.1500			70
75	0.0190	0.0259	0.0422	0.0608	0.0802	0.1001	0.1200	0.1500			75
80	0.0182	0.0252	0.0418	0.0606	0.0802	0.1000	0.1200	0.1500			80
85	0.0175	0.0246	0.0415	0.0604	0.0801	0.1000	0.1200	0.1500			85
90	0.0169	0.0240	0.0412	0.0603	0.0801	0.1000					90
95	0.0164	0.0236	0.0410	0.0602	0.0801	0.1000					95
100	0.0159	0.0232	0.0408	0.0602	0.0800	0.1000					100
∞	0.0100	0.0200	0.0400	0.0600	0.0800	0.1000					

APPENDIX D

Gradient Series Formula Values
Discrete Compounding

Gradient Series To Present Worth
Discrete Compounding
(P/G, r%, N)

n	1%	2%	4%	6%	8%	10%	12%	15%	20%	25%	n
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
2	0.98	0.96	0.92	0.89	0.86	0.83	0.80	0.76	0.69	0.64	2
3	2.92	2.85	2.70	2.57	2.45	2.33	2.22	2.07	1.85	1.66	3
4	5.80	5.62	5.27	4.95	4.65	4.38	4.13	3.79	3.30	2.89	4
5	9.61	9.24	8.55	7.93	7.37	6.86	6.40	5.78	4.91	4.20	5
6	14.32	13.68	12.50	11.46	10.52	9.68	8.93	7.94	6.58	5.51	6
7	19.92	18.90	17.07	15.45	14.02	12.76	11.64	10.19	8.26	6.77	7
8	26.38	24.88	22.18	19.84	17.81	16.03	14.47	12.48	9.88	7.95	8
9	33.69	31.57	27.80	24.58	21.81	19.42	17.36	14.75	11.43	9.02	9
10	41.84	38.95	33.88	29.60	25.98	22.89	20.25	16.98	12.89	9.99	10
11	50.80	47.00	40.38	34.87	30.27	26.40	23.13	19.13	14.23	10.85	11
12	60.57	55.67	47.25	40.34	34.63	29.90	25.95	21.18	15.47	11.60	12
15	94.48	85.20	69.74	57.55	47.89	40.15	33.92	26.69	18.51	13.33	15
20	165.46	144.60	111.56	87.23	69.09	55.41	44.97	33.58	21.74	14.89	20
25	252.89	214.26	156.10	115.97	87.80	67.70	53.10	38.03	23.43	15.56	25
30	355.00	291.72	201.06	142.36	103.46	77.08	58.78	40.75	24.26	15.83	30
35	470.15	374.88	244.88	165.74	116.09	83.99	62.61	42.36	24.66	15.94	35
40	596.85	461.99	286.53	185.96	126.04	88.95	65.12	43.28	24.85	15.98	40
45	733.70	551.56	325.40	203.11	133.73	92.45	66.73	43.81	24.93	15.99	45
50	879.41	642.36	361.16	217.46	139.59	94.89	67.76	44.10	24.97	16.00	50
60	1192.80	823.70	423.00	239.04	147.30	97.70	68.81	44.34	24.99	—	60
70	1528.64	999.83	472.48	253.33	151.53	98.99	69.21	44.42	—	—	70
80	1879.87	1166.79	511.12	262.55	153.80	99.56	69.36	44.47	—	—	80
90	2240.55	1322.17	540.77	268.39	154.99	99.81	—	—	—	—	90
100	2605.76	1464.75	563.12	272.05	155.61	99.92	—	—	—	—	100

Gradient Series To Uniform Series
Discrete Compounding
(A/G, r%, N)

n	1%	2%	4%	6%	8%	10%	12%	15%	20%	25%	n
1	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1
2	0.4974	0.4950	0.4902	0.4854	0.4808	0.4762	0.4717	0.4651	0.4545	0.4444	2
3	0.9932	0.9868	0.9739	0.9612	0.9487	0.9366	0.9246	0.9071	0.8791	0.8525	3
4	1.4874	1.4752	1.4510	1.4272	1.4040	1.3812	1.3589	1.3263	1.2742	1.2249	4
5	1.9799	1.9604	1.9216	1.8836	1.8465	1.8101	1.7746	1.7228	1.6405	1.5631	5
6	2.4708	2.4422	2.3857	2.3304	2.2763	2.2236	2.1720	2.0972	1.9788	1.8683	6
7	2.9600	2.9208	2.8433	2.7676	2.6937	2.6216	2.5515	2.4498	2.2902	2.1424	7
8	3.4476	3.3961	3.2944	3.1952	3.0985	3.0045	2.9131	2.7813	2.5756	2.3872	8
9	3.9335	3.8680	3.7391	3.6133	3.4910	3.3724	3.2574	3.0922	2.8364	2.6048	9
10	4.4177	4.3367	4.1773	4.0220	3.8713	3.7255	3.5847	3.3832	3.0739	2.7971	10
11	4.9003	4.8021	4.6090	4.4213	4.2395	4.0641	3.8953	3.6549	3.2893	2.9663	11
12	5.3813	5.2642	5.0343	4.8113	4.5957	5.3884	4.1897	3.9082	3.4841	3.1145	12
15	6.8141	6.6309	6.2721	5.9260	5.5945	5.2789	4.9803	4.5650	3.9588	3.4530	15
20	9.1692	8.8433	8.2091	7.6051	7.0369	6.5081	6.0202	5.3651	4.4643	3.7667	20
25	11.4829	10.9744	9.9925	9.0722	8.2254	7.4580	6.7708	5.8834	4.7352	3.9052	25
30	13.7555	13.0251	11.6274	10.3422	9.1897	8.1762	7.2974	6.2066	4.8731	3.9628	30
35	15.9869	14.9961	13.1198	11.4319	9.9611	8.7086	7.6577	6.4019	4.9406	3.9858	35
40	18.1774	16.8885	14.4765	12.3590	10.5699	9.0962	7.8988	6.5168	4.9728	3.9947	40
45	20.3271	18.7033	15.7047	13.1413	11.0447	9.3740	8.0572	6.5830	4.9877	3.9980	45
50	22.4362	20.4420	16.8122	13.7964	11.4107	9.5704	8.1597	6.6205	4.9945	3.9993	50
60	26.5331	23.6961	18.6972	14.7909	11.9015	9.8023	8.2664	6.6530	4.9989	—	60
70	30.4701	26.6632	20.1961	15.4613	12.1783	9.9113	8.3082	6.6627	—	—	70
80	34.2490	29.3572	21.3718	15.9033	12.3301	9.9609	8.3241	6.6656	—	—	80
90	37.8723	31.7929	22.2826	16.1891	12.4116	9.9831	—	—	—	—	90
100	41.3424	33.9863	22.9800	16.3711	12.4545	9.9927	—	—	—	—	100

APPENDIX E

Interest Formula Values
Continuous Compounding - Discrete Flows

Single Payment Compound Amount Factor
Continuous Compounding - Discrete Flow
(F/P, r%, N)

N	1% To find F Given P F/P	2% To find F Given P F/P	4% To find F Given P F/P	6% To find F Given P F/P	8% To find F Given F F/P	10% To find F Given P F/P	12% To find F Given P F/P	15% To find F Given P F/P	20% To find F Given P F/P	25% To find F Given P F/P	N
1	1.0101	1.0202	1.0408	1.0618	1.0833	1.1052	1.1275	1.1618	1.2214	1.2840	1
2	1.0202	1.0408	1.0833	1.1275	1.1735	1.2214	1.2712	1.3499	1.4918	1.6487	2
3	1.0305	1.0618	1.1275	1.1972	1.2712	1.3499	1.4333	1.5683	1.8221	2.1170	3
4	1.0408	1.0833	1.1735	1.2712	1.3771	1.4918	1.6161	1.8221	2.2255	2.7183	4
5	1.0513	1.1052	1.2214	1.3499	1.4918	1.6487	1.8221	2.1170	2.7183	3.4903	5
6	1.0618	1.1275	1.2712	1.4333	1.6161	1.8221	2.0544	2.4596	3.3201	4.4817	6
7	1.0725	1.1503	1.3231	1.5220	1.7507	2.0138	2.3164	2.8577	4.0552	5.7546	7
8	1.0833	1.1735	1.3771	1.6161	1.8965	2.2255	2.6117	3.3201	4.9530	7.3891	8
9	1.0942	1.1972	1.4333	1.7160	2.0544	2.4596	2.9447	3.8574	6.0496	9.4877	9
10	1.1052	1.2214	1.4918	1.8221	2.2255	2.7183	3.3201	4.4817	7.3891	12.1825	10
11	1.1163	1.2461	1.5527	1.9348	2.4109	3.0042	3.7434	5.2070	9.0250	15.6426	11
12	1.1275	1.2712	1.6161	2.0544	2.6117	3.3201	4.2207	6.0496	11.0232	20.0855	12
13	1.1388	1.2969	1.6820	2.1815	2.8292	3.6693	4.7588	7.0287	13.4637	25.7903	13
14	1.1503	1.3231	1.7507	2.3164	3.0649	4.0552	5.3656	8.1662	16.4446	33.1155	14
15	1.1618	1.3499	1.8221	2.4596	3.3201	4.4817	6.0496	9.4877	20.0855	42.5211	15
16	1.1735	1.3771	1.8965	2.6117	3.5966	4.9530	6.8210	11.0232	24.5325	54.5982	16
17	1.1853	1.4049	1.9739	2.7732	3.8962	5.4739	7.6906	12.8071	29.9641	70.1054	17
18	1.1972	1.4333	2.0544	2.9447	4.2207	6.0496	8.6711	14.8797	36.5982	90.0171	18
19	1.2092	1.4623	2.1383	3.1268	4.5722	6.6859	9.7767	17.2878	44.7012	115.584	19
20	1.2214	1.4918	2.2255	3.3201	4.9530	7.3891	11.0232	20.0855	54.5981	148.413	20
21	1.2337	1.5220	2.3164	3.5254	5.3656	8.1662	12.4286	23.3361	66.6863	190.566	21
22	1.2461	1.5527	2.4109	3.7434	5.8124	9.0250	14.0132	27.1126	81.4509	244.692	22
23	1.2586	1.5841	2.5093	3.9749	6.2965	9.9742	15.7998	31.5004	99.4843	314.191	23
24	1.2712	1.6161	2.6117	4.2207	6.8120	11.0232	17.8143	36.5982	121.510	403.429	24
25	1.2840	1.6487	2.7183	4.4817	7.3891	12.1825	20.0855	42.5211	148.413	518.013	25
26	1.2969	1.6820	2.8292	4.7588	8.0045	13.4637	22.6464	49.4024	181.272	665.142	26
27	1.3100	1.7160	2.9447	5.0531	8.6711	14.8797	25.5337	57.3975	221.406	854.059	27
28	1.3231	1.7507	3.0649	5.3656	9.3933	16.4446	28.7892	66.6863	270.426	1096.63	28
29	1.3364	1.7860	3.1899	5.6973	10.1757	18.1741	32.4597	77.4785	330.299	1408.10	29
30	1.3499	1.8221	3.3201	6.0496	11.0232	20.0855	36.5982	90.0171	403.429	1808.04	30
35	1.4191	2.0138	4.0552	8.1662	16.4446	33.1155	66.6863	190.566	1096.63	6310.69	35
40	1.4918	2.2255	4.9530	11.0232	24.5325	54.5981	121.510	403.429	2980.96	22026.5	40
45	1.5683	2.4596	6.0496	14.8797	36.5982	90.0171	221.406	854.059	8103.08	76879.9	45
50	1.6487	2.7183	7.3891	20.0855	54.5982	148.413	403.429	1808.04	22026.5	268337	50
55	1.7333	3.0042	9.0250	27.1126	81.4509	244.692	735.095	3827.63	59874.1		55
60	1.8221	3.3201	11.0232	36.5982	121.510	403.429	1339.43	8103.08	162755		60
65	1.9155	3.6693	13.4637	49.4024	181.272	665.142	2440.60	17154.2			65
70	2.0138	4.0552	16.4446	66.6863	270.426	1096.63	4447.07	36315.5			70
75	2.1170	4.4817	20.0855	90.0171	403.429	1808.04	8103.08	76879.9			75
80	2.2255	4.9530	24.5325	121.510	601.845	2980.96	14764.8	162755			80
85	2.3396	5.4739	29.9641	164.022	897.847	4914.77					85
90	2.4596	6.0496	36.5982	221.406	1339.43	8103.08					90
95	2.5857	6.6859	44.7012	298.867	1998.20	13359.7					95
100	2.7183	7.3891	54.5982	403.429	2980.96	22026.5					100

Single Payment Present Worth Factor
Continuous Compounding - Discrete Flow
(F/F, r%, N)

N	1% To find P Given F P/F	2% To find P Given F P/F	4% To find P Given F P/F	6% To find P Given F P/F	8% To find P Given P P/F	10% To find P Given F P/F	12% To find P Given F P/F	15% To find P Given F P/F	20% To find P Given F P/F	25% To find P Given F P/F	N
1	0.9900	0.9802	0.9608	0.9418	0.9231	0.9048	0.8869	0.8607	0.8187	0.7788	1
2	0.9802	0.9608	0.9231	0.8869	0.8521	0.8187	0.7866	0.7408	0.6703	0.6065	2
3	0.9704	0.9418	0.8869	0.8353	0.7866	0.7408	0.6977	0.6376	0.5488	0.4724	3
4	0.9608	0.9231	0.8521	0.7866	0.7261	0.6703	0.6188	0.5488	0.4493	0.3679	4
5	0.9512	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4724	0.3679	0.2865	5
6	0.9418	0.8869	0.7866	0.6977	0.6188	0.5488	0.4868	0.4066	0.3012	0.2231	6
7	0.9324	0.8694	0.7558	0.6570	0.5712	0.4966	0.4317	0.3499	0.2466	0.1738	7
8	0.9231	0.8521	0.7261	0.6188	0.5273	0.4493	0.3829	0.3012	0.2019	0.1353	8
9	0.9139	0.8353	0.6977	0.5827	0.4868	0.4066	0.3396	0.2592	0.1653	0.1054	9
10	0.9048	0.8187	0.6703	0.5488	0.4493	0.3679	0.3012	0.2231	0.1353	0.0821	10
11	0.8958	0.8025	0.6440	0.5169	0.4148	0.3329	0.2671	0.1920	0.1108	0.0639	11
12	0.8869	0.7866	0.6188	0.4868	0.3829	0.3012	0.2369	0.1653	0.0907	0.0498	12
13	0.8781	0.7711	0.5945	0.4584	0.3535	0.2725	0.2101	0.1423	0.0743	0.0388	13
14	0.8694	0.7558	0.5712	0.4317	0.3263	0.2466	0.1864	0.1225	0.0608	0.0302	14
15	0.8607	0.7408	0.5488	0.4066	0.3012	0.2231	0.1653	0.1054	0.0498	0.0235	15
16	0.8521	0.7261	0.5273	0.3829	0.2780	0.2019	0.1466	0.0907	0.0408	0.0183	16
17	0.8437	0.7118	0.5066	0.3606	0.2567	0.1827	0.1300	0.0781	0.0334	0.0143	17
18	0.8353	0.6977	0.4868	0.3396	0.2369	0.1653	0.1153	0.0672	0.0273	0.0111	18
19	0.8270	0.6839	0.4677	0.3198	0.2187	0.1496	0.1023	0.0578	0.0224	0.0087	19
20	0.8187	0.6703	0.4493	0.3012	0.2019	0.1353	0.0907	0.0498	0.0183	0.0067	20
21	0.8106	0.6570	0.4317	0.2837	0.1864	0.1225	0.0805	0.0429	0.0150	0.0052	21
22	0.8025	0.6440	0.4148	0.2671	0.1720	0.1108	0.0714	0.0369	0.0123	0.0041	22
23	0.7945	0.6313	0.3985	0.2516	0.1588	0.1003	0.0633	0.0317	0.0101	0.0032	23
24	0.7866	0.6188	0.3829	0.2369	0.1466	0.0907	0.0561	0.0273	0.0082	0.0025	24
25	0.7788	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0235	0.0067	0.0019	25
26	0.7711	0.5945	0.3535	0.2101	0.1249	0.0743	0.0442	0.0202	0.0055	0.0015	26
27	0.7634	0.5827	0.3396	0.1979	0.1153	0.0672	0.0392	0.0174	0.0045	0.0012	27
28	0.7558	0.5712	0.3263	0.1864	0.1065	0.0608	0.0347	0.0150	0.0037	0.0009	28
29	0.7483	0.5599	0.3135	0.1755	0.0983	0.0550	0.0308	0.0129	0.0030	0.0007	29
30	0.7408	0.5488	0.3012	0.1653	0.0907	0.0498	0.0273	0.0111	0.0025	0.0006	30
35	0.7047	0.4966	0.2466	0.1225	0.0608	0.0302	0.0150	0.0052	0.0009	0.0002	35
40	0.6703	0.4493	0.2019	0.0907	0.0408	0.0183	0.0082	0.0025	0.0003		40
45	0.6376	0.4066	0.1653	0.0672	0.0273	0.0111	0.0045	0.0012	0.0001		45
50	0.6065	0.3679	0.1353	0.0498	0.0183	0.0067	0.0025	0.0006			50
55	0.5769	0.3329	0.1108	0.0369	0.0123	0.0041	0.0014	0.0003			55
60	0.5488	0.3012	0.0907	0.0273	0.0082	0.0025	0.0007	0.0001			60
65	0.5220	0.2725	0.0743	0.0202	0.0055	0.0015	0.0004				65
70	0.4966	0.2466	0.0608	0.0150	0.0037	0.0009	0.0002				70
75	0.4724	0.2231	0.0498	0.0111	0.0025	0.0006	0.0001				75
80	0.4493	0.2019	0.0408	0.0082	0.0017	0.0003					80
85	0.4274	0.1827	0.0334	0.0061	0.0011	0.0002					85
90	0.4066	0.1653	0.0273	0.0045	0.0007	0.0001					90
95	0.3867	0.1496	0.0224	0.0033	0.0005						95
100	0.3679	0.1353	0.0183	0.0025	0.0003						100

Uniform Series Compound Amount Factor
Continuous Compounding - Discrete Flow
(F/A, r%, N)

N	1% To find F Given A F/A	2% To find F Given A F/A	4% To find F Given A F/A	6% To find F Given A F/A	8% To find F Given A F/A	10% To find F Given A F/A	12% To find F Given A F/A	15% To find F Given A F/A	20% To find F Given A F/A	25% To find F Given A F/A	N
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1
2	2.0101	2.0202	2.0408	2.0618	2.0833	2.1052	2.1275	2.1618	2.2214	2.2840	2
3	3.0303	3.0610	3.1241	3.1893	3.2568	3.3266	3.3987	3.5117	3.7132	3.9327	3
4	4.0607	4.1228	4.2516	4.3866	4.5280	4.6764	4.8321	5.0800	5.5353	6.0497	4
5	5.1015	5.2061	5.4251	5.6578	5.9052	6.1683	6.4481	6.9021	7.7609	8.7680	5
6	6.1528	6.3113	6.6465	7.0077	7.3970	7.8170	8.2703	9.0191	10.4792	12.2584	6
7	7.2146	7.4388	7.9178	8.4410	9.0131	9.6391	10.3247	11.4787	13.7993	16.7401	7
8	8.2871	8.5891	9.2409	9.9630	10.7637	11.6528	12.6411	14.3364	17.8545	22.4947	8
9	9.3704	9.7626	10.6180	11.5790	12.6602	13.8784	15.2528	17.6565	22.8075	29.8837	9
10	10.4646	10.9598	12.0513	13.2950	14.7147	16.3380	18.1974	21.5139	28.8572	39.3715	10
11	11.5698	12.1812	13.5432	15.1172	16.9402	19.0563	21.5176	25.9956	36.2462	51.5539	11
12	12.6860	13.4273	15.0959	17.0519	19.3511	22.0604	25.2610	31.2026	45.2712	67.1966	12
13	13.8135	14.6985	16.7119	19.1064	21.9628	25.3806	29.4817	37.2522	56.2944	87.2821	13
14	14.9524	15.9955	18.3940	21.2878	24.7920	29.0499	34.2405	44.2809	69.7581	113.073	14
15	16.1026	17.3186	20.1346	23.6042	27.8569	33.1051	39.6061	52.4471	86.2028	146.188	15
16	17.2645	18.6685	21.9668	26.0638	31.1770	37.5867	45.6557	61.9348	106.288	188.709	16
17	18.4380	20.0456	23.8632	28.6755	34.7736	42.5398	52.4767	72.9580	130.821	243.397	17
18	19.6233	21.4505	25.8371	31.4487	38.6698	48.0137	60.1673	85.7651	160.785	313.413	18
19	20.8205	22.8839	27.8916	34.3934	42.8905	54.0634	68.8384	100.645	197.383	403.430	19
20	22.0298	24.3461	30.0298	37.5202	47.4627	60.7493	78.6151	117.933	242.084	519.014	20
21	23.2512	25.8380	32.2554	40.8403	52.4158	68.1383	89.6383	138.018	296.682	667.427	21
22	24.4849	27.3599	34.5717	44.3657	57.7813	76.3045	102.067	161.354	363.369	857.993	22
23	25.7309	28.9126	36.9826	48.1091	63.5938	85.3295	116.080	188.467	444.820	1102.69	23
24	26.9895	30.4967	39.4919	52.0840	69.8903	95.3037	131.880	219.967	544.304	1416.88	24
25	28.2608	32.1128	42.1036	56.3047	76.7113	106.327	149.694	256.565	665.814	1820.30	25
26	29.5448	33.7615	44.8219	60.7864	84.1003	118.509	169.780	299.087	814.227	2338.31	26
27	30.8417	35.4435	47.6511	65.5452	92.1048	131.973	192.426	348.489	995.500	3003.46	27
28	32.1517	37.1595	50.5958	70.5983	100.776	146.853	217.960	405.886	1216.91	3857.52	28
29	33.4748	38.9102	53.6607	75.9639	110.169	163.298	246.749	472.573	1487.33	4954.15	29
30	34.8113	40.6962	56.8506	81.6612	120.345	181.472	279.209	550.051	1817.63	6362.26	30
35	41.6976	50.1824	74.8626	115.889	185.439	305.364	515.200	1171.36	4948.60	22215.2	35
40	48.9370	60.6663	96.8625	162.092	282.547	509.629	945.203	2486.67	13459.4	77547.5	50
45	56.5476	72.2528	123.733	224.458	427.416	846.404	1728.72	5271.19	36594.3	270676	45
50	64.5483	85.0578	156.553	308.645	643.535	1401.65	3156.38	11166.0	99481.4	944762	50
55	72.9593	99.2096	196.640	422.285	965.947	2317.10	5757.75	23645.3	270426		55
60	81.8015	114.850	245.601	575.683	1446.93	3826.43	10497.8	50064.1	735103		60
65	91.0971	132.135	305.403	782.748	2164.47	6314.88	19134.6	105993			65
70	100.869	151.238	378.445	1062.26	3234.91	10417.6	34872.0	224393			70
75	111.143	172.349	467.659	1439.56	4831.83	17182.0	63547.3	475047			75
80	121.942	195.682	576.625	1948.85	7214.15	28334.4	115797	1005680			80
85	133.296	221.468	709.717	2636.34	10768.1	46721.7					85
90	145.232	249.966	872.275	3564.34	16070.1	77037.3					90
95	157.780	281.461	1070.82	4817.01	23979.7	127019					95
100	170.971	316.269	1313.33	6507.94	35779.3	209425					100

Uniform Series Present Worth Factor
Continuous Compounding - Discrete Flow
(P/A, r%, N)

N	1% To find P Given A P/A	2% To find P Given A P/A	4% To find P Given A P/A	6% To find P Given A P/A	8% To find P Given A P/A	10% To find P Given A P/A	12% To find P Given A P/A	15% To find P Given A P/A	20% To find P Given A P/A	25% To find P Given A P/A	N
1	0.9900	0.9802	0.9608	0.9418	0.9231	0.9048	0.8869	0.8607	0.8187	0.7788	1
2	1.9703	1.9410	1.8839	1.8287	1.7753	1.7236	1.6735	1.6015	1.4891	1.3853	2
3	2.9407	2.8828	2.7708	2.6640	2.5619	2.4644	2.3712	2.2392	2.0379	1.8577	3
4	3.9015	3.8059	3.6230	3.4506	3.2880	3.1347	2.9900	2.7880	2.4872	2.2256	4
5	4.8527	4.7107	4.4417	4.1914	3.9584	3.7412	3.5388	3.2603	2.8551	2.5121	5
6	5.7945	5.5976	5.2283	4.8891	4.5771	4.2900	4.0256	3.6669	3.1563	2.7352	6
7	6.7269	6.4670	5.9841	5.5461	5.1483	4.7866	4.4573	4.0168	3.4029	2.9090	7
8	7.6500	7.3191	6.7103	6.1649	5.6756	5.2360	4.8402	4.3180	3.6048	3.0443	8
9	8.5639	8.1544	7.4079	6.7477	6.1624	5.6425	5.1798	4.5773	3.7701	3.1497	9
10	9.4688	8.9731	8.0783	7.2965	6.6117	6.0104	5.4810	4.8004	3.9054	3.2318	10
11	10.3646	9.7756	8.7223	7.8133	7.0265	6.3433	5.7481	4.9925	4.0162	3.2957	11
12	11.2515	10.5623	9.3411	8.3001	7.4094	6.6445	5.9850	5.1578	4.1069	3.3455	12
13	12.1296	11.3333	9.9356	8.7585	7.7629	6.9170	6.1952	5.3000	4.1812	3.3843	13
14	12.9990	12.0891	10.5068	9.1902	8.0891	7.1636	6.3815	5.4225	4.2420	3.4145	14
15	13.8597	12.8299	11.0556	9.5968	8.3903	7.3867	6.5468	5.5279	4.2918	3.4380	15
16	14.7118	13.5561	11.5829	9.9797	8.6684	7.5886	6.6934	5.6186	4.3325	3.4563	16
17	15.5555	14.2678	12.0895	10.3402	8.9250	7.7713	6.8235	5.6967	4.3659	3.4706	17
18	16.3908	14.9655	12.5763	10.6798	9.1620	7.9366	6.9388	5.7639	4.3932	3.4817	18
19	17.2177	15.6494	13.0439	10.9997	9.3807	8.0862	7.0411	5.8217	4.4156	3.4904	19
20	18.0365	16.3197	13.4933	11.3009	9.5826	8.2215	7.1318	5.8715	4.4339	3.4971	20
21	18.8470	16.9768	13.9250	11.5845	9.7689	8.3440	7.2123	5.9144	4.4489	3.5023	21
22	19.6496	17.6208	14.3398	11.8516	9.9410	8.4548	7.2836	5.9513	4.4612	3.5064	22
23	20.4441	18.2521	14.7383	12.1032	10.0998	8.5550	7.3469	5.9830	4.4713	3.5096	23
24	21.2307	18.8709	15.1212	12.3402	10.2464	8.6458	7.4030	6.0103	4.4795	3.5121	24
25	22.0095	19.4774	15.4891	12.5633	10.3817	8.7278	7.4528	6.0338	4.4862	3.5140	25
26	22.7806	20.0719	15.8425	12.7734	10.5067	8.8021	7.4970	6.0541	4.4917	3.5155	26
27	23.5439	20.6547	16.1821	12.9713	10.6220	8.8693	7.5362	6.0715	4.4963	3.5167	27
28	24.2997	21.2259	16.5084	13.1577	10.7285	8.9301	7.5709	6.0865	4.5000	3.5176	28
29	25.0480	21.7858	16.8219	13.3332	10.8267	8.9852	7.6017	6.0994	4.5030	3.5183	29
30	25.7888	22.3346	17.1231	13.4985	10.9174	9.0349	7.6290	6.1105	4.5055	3.5189	30
35	29.3838	24.9199	18.4609	14.1913	11.2765	9.2212	7.7257	6.1467	4.5125	3.5203	35
40	32.8034	27.2591	19.5562	14.7046	11.5172	9.3342	7.7788	6.1638	4.5151	3.5207	40
45	36.0563	29.3758	20.4530	15.0848	11.6786	9.4027	7.8079	6.1719	4.5161	3.5208	45
50	39.1505	31.2910	21.1872	15.3665	11.7868	9.4443	7.8239	6.1757	4.5165	3.5208	50
55	42.0939	33.0240	21.7883	15.5752	11.8593	9.4695	7.8327	6.1775	4.5166	--	55
60	44.8936	34.5921	22.2804	15.7298	11.9079	9.4848	7.8375	6.1784	4.5166		60
65	47.5569	36.0109	22.6834	15.8443	11.9404	9.4940	7.8401	6.1788			65
70	50.0902	37.2947	23.0133	15.9292	11.9623	9.4997	7.8416	6.1790			70
75	52.5000	38.4564	23.2834	15.9920	11.9769	9.5031	7.8424	6.1791			75
80	54.7923	39.5075	23.5045	16.0386	11.9867	9.5051	7.8428	6.1791			80
85	56.9727	40.4585	23.6856	16.0731	11.9933	9.5064					85
90	59.0468	41.3191	23.8338	16.0986	11.9977	9.5072					90
95	61.0198	42.0978	23.9552	16.1176	12.0007	9.5076					95
100	62.8965	42.8023	24.0545	16.1316	12.0026	9.5079					100

APPENDIX F

Interest Factor Values
Continuous Compounding - Continuous Uniform Flow

Uniform Series Compound Amount Factor
Continuous Compounding - Continuous Uniform Flow
($F/\bar{A}$, r%, N)

N	1% To find F Given $\bar{A}$ $F/\bar{A}$	2% To find F Given $\bar{A}$ $F/\bar{A}$	4% To find F Given $\bar{A}$ $F/\bar{A}$	6% To find F Given $\bar{A}$ $F/\bar{A}$	8% To find F Given $\bar{A}$ $F/\bar{A}$	10% To find F Given $\bar{A}$ $F/\bar{A}$	12% To find F Given $\bar{A}$ $F/\bar{A}$	15% To find F Given $\bar{A}$ $F/\bar{A}$	20% To find F Given $\bar{A}$ $F/\bar{A}$	25% To find F Given $\bar{A}$ $F/\bar{A}$	N
1	1.0050	1.0101	1.0203	1.0306	1.0411	1.0517	1.0625	1.0789	1.1070	1.1361	1
2	2.0201	2.0405	2.0822	2.1249	2.1689	2.2140	2.2604	2.3324	2.4591	2.5949	2
3	3.0455	3.0918	3.1874	3.2870	3.3906	3.4986	3.6111	3.7887	4.1106	4.4680	3
4	4.0811	4.1644	4.3378	4.5208	4.7141	4.9182	5.1340	5.4808	6.1277	6.8731	4
5	5.1271	5.2585	5.5351	5.8310	6.1478	6.4872	6.8510	7.4467	8.5914	9.9614	5
6	6.1837	6.3748	6.7812	7.2222	7.7009	8.2212	8.7869	9.7307	11.6006	13.9268	6
7	7.2508	7.5137	8.0782	8.6994	9.3834	10.1375	10.9697	12.3843	15.2760	19.0184	7
8	8.3287	8.6755	9.4282	10.2679	11.2060	12.2554	13.4308	15.4674	19.7652	25.5562	8
9	9.4174	9.8609	10.8332	11.9334	13.1804	14.5960	16.2057	19.0495	25.2482	33.9509	9
10	10.5171	11.0701	12.2956	13.7020	15.3193	17.1828	19.3343	23.2113	31.9453	44.7300	10
11	11.6278	12.3038	13.8177	15.5799	17.6362	20.0417	22.8618	28.0465	40.1251	58.5705	11
12	12.7497	13.5625	15.4019	17.5739	20.1462	23.2012	26.8391	33.6643	50.1159	76.3421	12
13	13.8828	14.8465	17.0507	19.6912	22.8652	26.6930	31.3235	40.1913	62.3187	99.1614	13
14	15.0274	16.1565	18.7668	21.9394	25.8107	30.5520	36.3796	47.7745	77.2232	128.462	14
15	16.1834	17.4929	20.5530	24.3267	29.0015	34.8169	42.0804	56.5849	95.4277	166.084	15
16	17.3511	18.8564	22.4120	26.8616	32.4580	39.5303	48.5080	66.8212	117.663	214.393	16
17	18.5305	20.2474	24.3469	29.5532	36.2024	44.7395	55.7551	78.7140	144.820	276.422	17
18	19.7217	21.6665	26.3608	32.4113	40.2587	50.4965	63.9261	92.5315	177.991	356.068	18
19	20.9250	23.1142	28.4569	35.4461	44.6528	56.8589	73.1390	108.585	218.506	458.337	19
20	22.1403	24.5912	30.6385	38.6686	49.4129	63.8906	83.5265	127.237	267.991	589.653	20
21	23.3678	26.0981	32.9092	42.0904	54.5694	71.6617	95.2383	148.907	328.432	758.265	21
22	24.6077	27.6354	35.2725	45.7237	60.1555	80.2501	108.443	174.084	402.254	974.768	22
23	25.8600	29.2037	37.7323	49.5817	66.2067	89.7418	123.332	203.336	492.422	1252.76	23
24	27.1249	30.8037	40.2924	53.6783	72.7620	100.232	140.119	237.322	602.552	1609.72	24
25	28.4025	32.4361	42.9570	58.0282	79.8632	111.825	159.046	276.807	737.066	2068.05	25
26	29.6930	34.1014	45.7304	62.6470	87.5559	124.637	180.386	322.683	901.361	2656.57	26
27	30.9964	35.8003	48.6170	67.5515	95.8892	138.797	204.448	375.983	1102.03	3412.23	27
28	32.3130	37.5336	51.6214	72.7593	104.917	154.446	231.577	437.909	1347.13	4382.53	28
29	33.6427	39.3019	54.7483	78.2891	114.696	171.741	262.164	509.856	1646.50	5628.42	29
30	34.9859	41.1059	58.0029	84.1608	125.290	190.855	296.652	593.448	2012.14	7228.17	30
35	41.9068	50.6876	76.3800	119.436	193.058	321.154	547.386	1263.78	5478.17	25238.8	35
40	49.1825	61.2770	98.8258	167.053	294.157	535.982	1004.25	2682.86	14899.8	88101.9	40
45	56.8312	72.9802	126.241	231.329	444.978	890.171	1836.72	5687.06	40510.4	307516	45
50	64.8721	85.9141	159.726	318.092	669.977	1474.13	3353.57	12046.9	110127	1073350	50
55	73.3253	100.208	200.625	435.211	1005.64	2436.92	6117.46	25510.8	299366		55
60	82.2119	116.006	250.579	593.304	1506.38	4024.29	11153.6	54013.9	813769		60
65	91.5541	133.465	311.593	806.708	2253.40	6641.42	20330.0	114355			65
70	101.375	152.760	386.116	1094.772	3367.83	10956.3	37050.6	242097			70
75	111.700	174.084	477.138	1483.619	5030.36	18070.7	67517.4	512526			75
80	122.554	197.652	588.313	2008.507	7510.56	29799.6	123032	1085030			80
85	133.965	223.697	724.102	2717.032	11210.6	49137.7					85
90	145.960	252.482	889.956	3673.440	16730.4	81020.8					90
95	158.571	284.295	1092.530	4964.457	24964.9	133587					95
100	171.828	319.453	1339.954	6707.146	37249.5	220255					100

Uniform Series Present Worth Factor
Continuous Compounding - Continuous Uniform Flow
(P/A, r%, N)

N	1% To find P Given $\bar{A}$ $P/\bar{A}$	2% To find P Given $\bar{A}$ $P/\bar{A}$	4% To find P Given $\bar{A}$ $P/\bar{A}$	6% To find P Given $\bar{A}$ $P/\bar{A}$	8% To find P Given $\bar{A}$ $P/\bar{A}$	10% To find P Given $\bar{A}$ $P/\bar{A}$	12% To find P Given $\bar{A}$ $P/\bar{A}$	15% To find P Given $\bar{A}$ $P/\bar{A}$	20% To find P Given $\bar{A}$ $P/\bar{A}$	25% To find P Given $\bar{A}$ $P/\bar{A}$	N
1	0.9950	0.9901	0.9803	0.9706	0.9610	0.9516	0.9423	0.9286	0.9063	0.8848	1
2	1.9801	1.9605	1.9221	1.8847	1.8482	1.8127	1.7781	1.7279	1.6484	1.5739	2
3	2.9554	2.9118	2.8270	2.7455	2.6672	2.5918	2.5194	2.4158	2.2559	2.1105	3
4	3.9211	3.8442	3.6964	3.5562	3.4231	3.2968	3.1768	3.0079	2.7534	2.5285	4
5	4.8771	4.7581	4.5317	4.3197	4.1210	3.9347	3.7599	3.5176	3.1606	2.8540	5
6	5.8235	5.6540	5.3343	5.0387	4.7652	4.5119	4.2771	3.9562	3.4940	3.1075	6
7	6.7606	6.5321	6.1054	5.7159	5.3599	5.0341	4.7357	4.3337	3.7670	3.3049	7
8	7.6884	7.3928	6.8463	6.3536	5.9088	5.5067	5.1426	4.6587	3.9905	3.4587	8
9	8.6069	8.2365	7.5581	6.9542	6.4156	5.9343	5.5034	4.9384	4.1735	3.5784	9
10	9.5163	9.0635	8.2420	7.5198	6.8834	6.3212	5.8234	5.1791	4.3233	3.6717	10
11	10.4166	9.8741	8.8991	8.0525	7.3152	6.6713	6.1072	5.3863	4.4460	3.7443	11
12	11.3080	10.6686	9.5304	8.5541	7.7138	6.9881	6.3589	5.5647	4.5464	3.8009	12
13	12.1905	11.4474	10.1370	9.0266	8.0818	7.2747	6.5822	5.7182	4.6286	3.8449	13
14	13.0642	12.2108	10.7198	9.4715	8.4215	7.5340	6.7802	5.8503	4.6959	3.8792	14
15	13.9292	12.9591	11.2797	9.8905	8.7351	7.7687	6.9558	5.9640	4.7511	3.9059	15
16	14.7856	13.6925	11.8177	10.2851	9.0245	7.9810	7.1116	6.0619	4.7962	3.9267	16
17	15.6335	14.4115	12.3346	10.6568	9.2917	8.1732	7.2498	6.1461	4.8331	3.9429	17
18	16.4730	15.1162	12.8312	11.0067	9.5384	8.3470	7.3723	6.2186	4.8634	3.9556	18
19	17.3041	15.8069	13.3083	11.3363	9.7661	8.5043	7.4810	6.2810	4.8881	3.9654	19
20	18.1269	16.4840	13.7668	11.6468	9.9763	8.6466	7.5774	6.3348	4.9084	3.9730	20
21	18.9416	17.1477	14.2072	11.9391	10.1703	8.7754	7.6628	6.3810	4.9250	3.9790	21
22	19.7481	17.7982	14.6304	12.2144	10.3494	8.8920	7.7387	6.4208	4.9386	3.9837	22
23	20.5466	18.4358	15.0370	12.4737	10.5148	8.9974	7.8059	6.4550	4.9497	3.9873	23
24	21.3372	19.0608	15.4277	12.7179	10.6674	9.0928	7.8655	6.4845	4.9589	3.9901	24
25	22.1199	19.6735	15.8030	12.9478	10.8083	9.1791	7.9184	6.5099	4.9663	3.9923	25
26	22.8948	20.2740	16.1636	13.1644	10.9384	9.2573	7.9654	6.5317	4.9724	3.9940	26
27	23.6621	20.8626	16.5101	13.3684	11.0584	9.3279	8.0070	6.5505	4.9774	3.9953	27
28	24.4216	21.4395	16.8430	13.5604	11.1693	9.3919	8.0439	6.5667	4.9815	3.9964	28
29	25.1736	22.0051	17.1628	13.7413	11.2716	9.4498	8.0766	6.5806	4.9849	3.9972	29
30	25.9182	22.5594	17.4701	13.9117	11.3660	9.5021	8.1056	6.5926	4.9876	3.9978	30
35	29.5312	25.1707	18.8351	14.6257	11.7399	9.6980	8.2084	6.6317	4.9954	3.9994	35
40	32.9680	27.5336	19.9526	15.1547	11.9905	9.8168	8.2648	6.6501	4.9983	3.9998	40
45	36.2372	29.6715	20.8675	15.5466	12.1585	9.8889	8.2957	6.6589	4.9994	3.9999	45
50	39.3469	31.6060	21.6166	15.8369	12.2711	9.9326	8.3127	6.6630	4.9998	4.0000	50
55	42.3050	33.3564	22.2299	16.0519	12.3465	9.9591	8.3220	6.6649	4.9999		55
60	45.1188	34.9403	22.7321	16.2113	12.3971	9.9752	8.3271	6.6658	5.0000		60
65	47.7954	36.3734	23.1432	16.3293	12.4310	9.9850	8.3299	6.6663			65
70	50.3415	37.6702	23.4797	16.4167	12.4538	9.9909	8.3315	6.6665			70
75	52.7633	38.8435	23.7553	16.4815	12.4690	9.9945	8.3323	6.6666			75
80	55.0671	39.9052	23.9809	16.5295	12.4792	9.9966	8.3328	6.6666			80
85	57.2585	40.8658	24.1657	16.5651	12.4861	9.9980					85
90	59.3430	41.7351	24.3169	16.5914	12.4907	9.9988					90
95	61.3259	42.5216	24.4407	16.6109	12.4937	9.9993					95
100	63.2121	43.2332	24.5421	16.6254	12.4958	9.9995					100

APPENDIX G

Shaping Mustang I Life Cycle By An Interplay Of Product Innovation Rates Versus Forces Of Product Mortality

SUMMARY

The Mustang I life cycle is portrayed as a dynamic resultant of an interplay of two countervailing forces: the force of an innovating performance to keep the product alive against a force of mortality generated by the introduction of internal and external competing effects to replace the product by other similar or equivalent products. A generalized two state Markov process is used to caricature this dynamic interplay of innovative forces against forces of mortality. The actual Mustang I product life cycle is first smoothed out to dampen random, seasonal and cyclic effects. Thereafter, a "best fit" is obtained to match the Markov model results against the smoothed out empirical results. This matching process synthesizes a force of innovation and a force of mortality, which are then interpreted in terms of pertinent observable phenomena. This approach can provide a method of accounting for performance of an enterprise in a specific environment and in relation to a specific product beyond conventional measures of business success. A discussion is provided on the applicability of the concept for marketing and product planning needs of an enterprise.

INTRODUCTION

Life cycles of products, enterprises, societies and cultures are considered as a more or less established empirical fact. The use of the life cycle concept in new product and marketing planning is a far less obvious fact. We may divorce ourselves from the heroic attempts to interpret the rise and fall of Roman Empire; nevertheless, we remain immensely curious about the life cycles of very familiar entities over our rather short life spans of experience. This is particularly true in the case of practical business planning of new products in a setting of competition and uncertainty. The rather well recognized life cycle concept appearing through historical, political, social and business literature closely parallels that of the biological life cycle processes of living organisms: there is a birth, growth, maturation, decline and eventual termination of the process. Actuarial practices utilize this concept in insurance business. In the practice of new products planning and marketing the practical and operational usefulness of the life cycle concept is not so obvious before the fact. Usually, the life cycle concept appears as a post mortem judgment after the fact. An important aspect of planning is prediction and forecasting of the faith of a product in the future market settings. A life cycle of a product should be understood in such relevant terms that are subject to some meaningful forecasting processes. The main purpose of this article is to investigate this possibility in the context of a specific example.

The literature on product life cycle concepts is considerable. A small sample of references are provided here [360-372]. A general impression obtained from this literature is that the product life cycle

concept is not particularly useful in planning a new product and predicting its faith in a marketplace. The purpose of this article is not to review in detail this literature, but merely point out that there is an agreement that a process of innovation is needed to support a product against a variety of forces tending to kill this product in the marketplace.

It is possible to account for innovative and renovative performance of an enterprise in terms of the life cycles of its products, one by one or in an appropriate composition. Such an accounting could be done in financial terms, in terms of real output, or in terms of some relevant measure of the "worth" of the activity generated by this enterprise. With a dimensional consistency of a measure of performance, the same could be done with the effects that promote the mortality of this enterprise within and without in a setting of a particular environment. At least in some relative terms one can introduce a measure for a force of renovation and innovation on one hand, and a force of mortality on the other hand. Then is is possible to develop a simple model in which these two opposing forces produce in a dynamic interplay a bounded product life cycle [370]. With a meaningful interpretation such a life cycle model can be applied in post mortem to interpret such phenomena as the railroad passenger service life cycle in the United States [371].

A TWO-STATE LIFE CYCLE MARKOV MODEL

References [370,371] provide a derivation of a simple generalized two-state Markov process life cycle model repeated here. The two states for the model are as follows:

State #0: A product is produced by the enterprise and is desired by the society or the marketplace.

State #1: The product cannot be produced by the enterprise or it is not desired by the society or the marketplace.

Let $A(t)$ be the probability that the product is "alive", i.e. in the state #0, at the time t. Let $D(t)$ be the probability that the product is dead, i.e. in the state #1, at the time t. Further, let $h(t)$ be the hazard or force of product mortality (events of death per unit time) pushing the life of the product from the state #0 toward the state #1. Let $v(t)$ be the innovative recovery rate or force of innovation or force of renovation forcing the product from the state #1 toward the state #0. Then

$$(1) \qquad \begin{aligned} &dA(t)/dt = -h(t)A(t) + v(t)D(t) \\ &A(t) + D(t) = 1 \\ &A(0) = A_o \; ; \; 0 \leq A_o \leq 1 \end{aligned}$$

As shown in references [370,371], the appropriate solution to this differential equation with the indicated constraints is as follows:

$$(2) \qquad A(t) = S(t)U(t)[\int_0^t [v(x)/S(x)U(x)]dx + A_o]$$

where

$$(3) \qquad S(t) = \exp[-\int_0^t h(x)\, dx]$$

and

$$(4) \qquad U(t) = \exp[-\int_0^t v(x)\, dx]$$

S(t) is the survival probability of the product without any supportive innovation, and U(t) is the probability that the innovation will not occur within time t; S(0) = U(0) = 1. Therefore, knowing h(t) and v(t) will suffice to shape the probability A(t) that the product is alive. Let K be an appropriate dimensional scaling constant. Then K A(t) can be treated as the measure of the output for a particular product by the enterprise. In the spirit of references [370,371] Figure G-1 illustrates the flow-diagram of the two-state Markov process life cycle generator. h(t) and v(t) represent a pair of relative forces which with A_o and K suffice to provide a match to a bounded life cycle of a "reasonably well behaved type". The idea is now to match an empirically observed life cycle in an appropriate manner and then extract out h(t) and v(t) for further interpretation. If h(t) and v(t) can be explained in some relevant manner subject to possible forecasting, then it would be possible to predict also the shape of an expected product life cycle. Such a possibility would be operationally helpful to new products and marketing planning processes. A particular product life cycle history could be helpful in testing out such a possibility. In a previous study the product life cycle of the US passenger railroad service was investigated [371]. The results had some encouraging correlation with other empirical investigations [372]. This preliminary study motivated the investigation described subsequently.

THE MUSTANG I PRODUCT LIFE CYCLE

Specific lines of automobile products provide good examples of product life cycles. The Mustang I product life cycle is a particular example. It could have been just as well a life cycle of some other specific brand and line of automobiles. Figure G-1 illustrates the flow diagram of the two state Markov process discussed previously [370,371]. The quarterly data for Mustang I actual production and the respective smoothed data by a four quarter moving average process from the year 1964 to the year 1972 is given in Table 1. Figures G-2 and G-3 illustrate this data. The purpose of this investigation was to match the life cycle generated by the Markov Life Cycle Model to the smoothed data for Mustang I, and thereby extract out the respective force of innovation v(t) and the force of mortality h(t) that generated this seemingly "best fit". This is a trial and error process indicated by the flow diagram

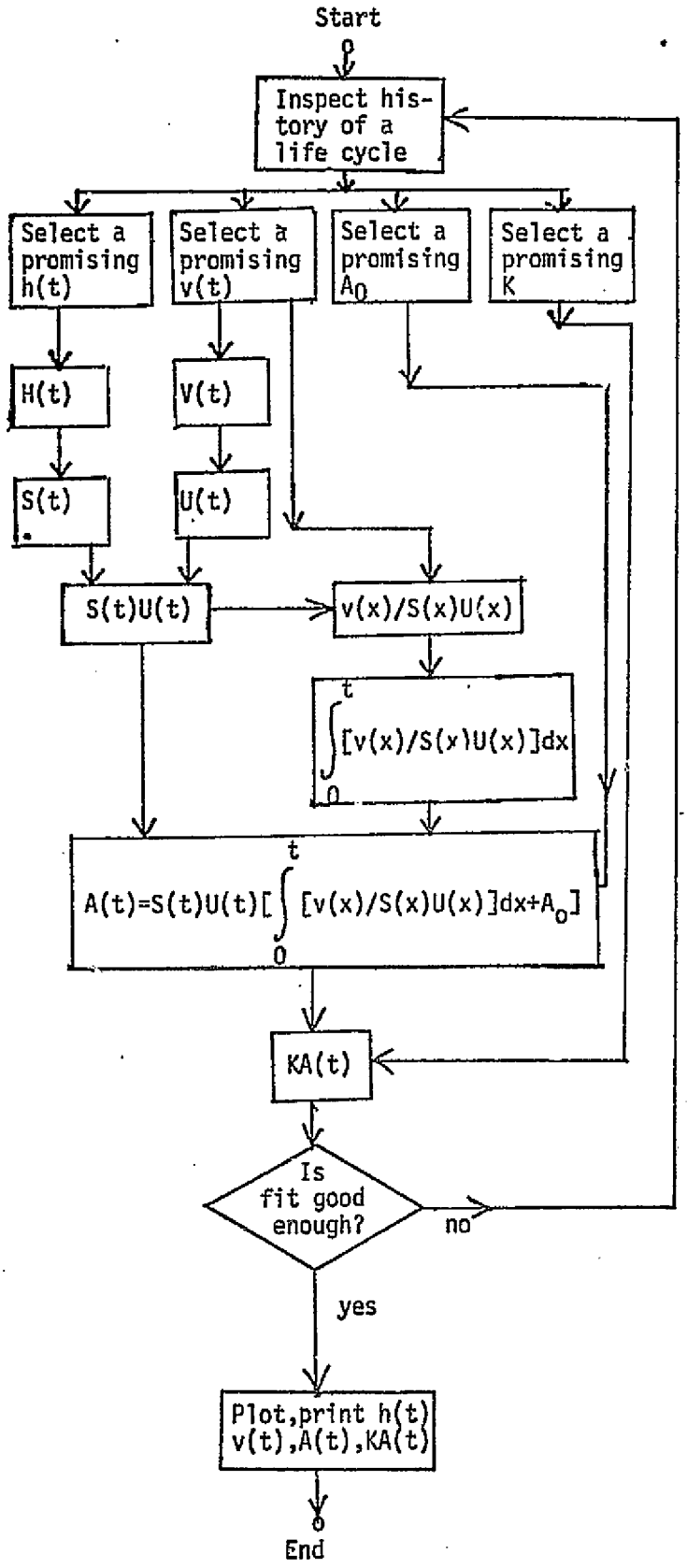

Start
Inspect history of a life cycle
Select a promising h(t)
Select a promising v(t)
Select a promising A_0
Select a promising K
H(t)
V(t)
S(t)
U(t)
S(t)U(t)
v(x)/S(x)U(x)
$\int_0^t [v(x)/S(x)U(x)]dx$
$A(t)=S(t)U(t)[\int_0^t [v(x)/S(x)U(x)]dx+A_0]$
KA(t)
Is fit good enough?
no
yes
Plot,print h(t) v(t),A(t),KA(t)
End

FIGURE G-1

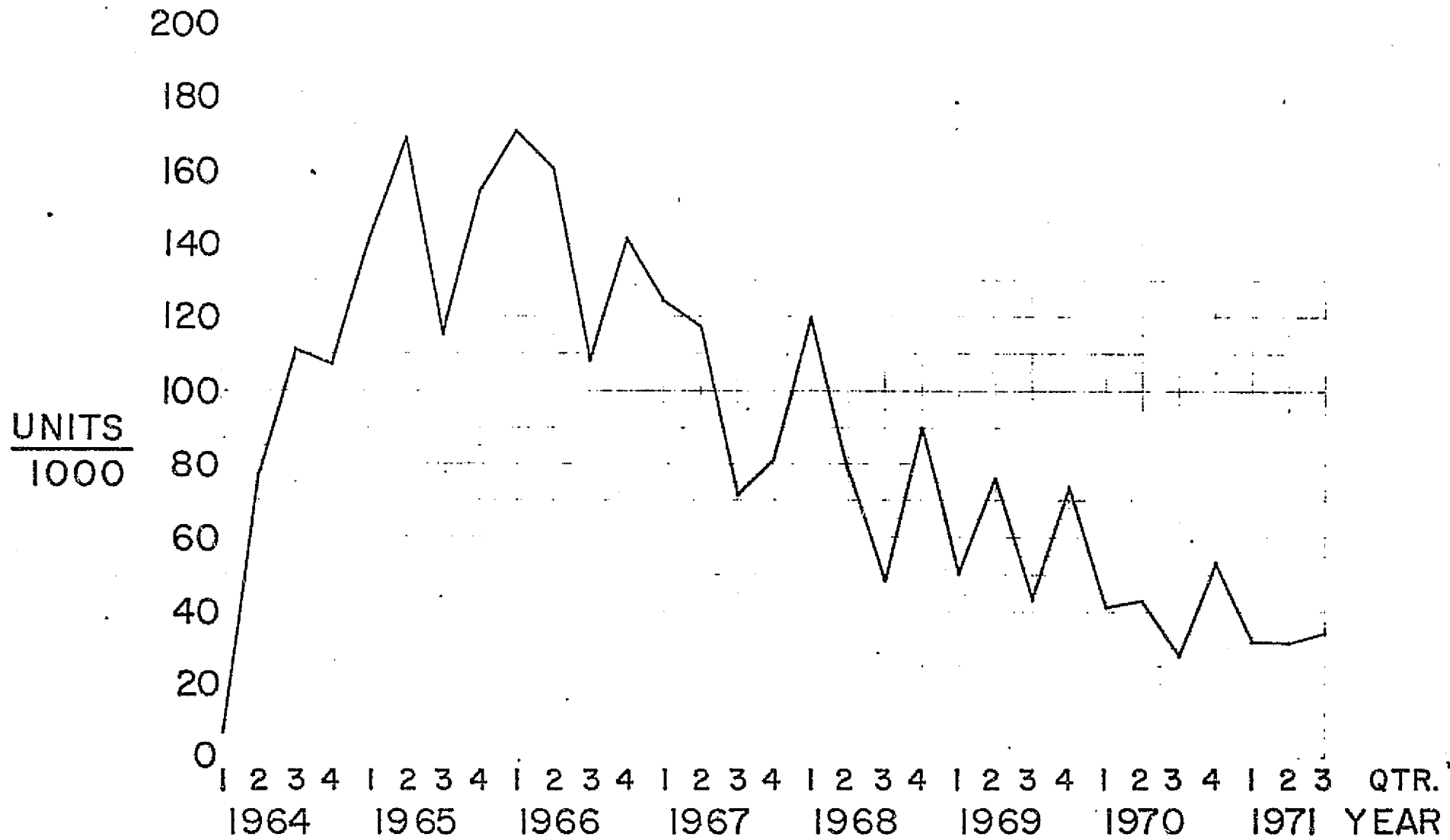


FIGURE G-2 ACTUAL QUARTERLY MUSTANG PRODUCTION

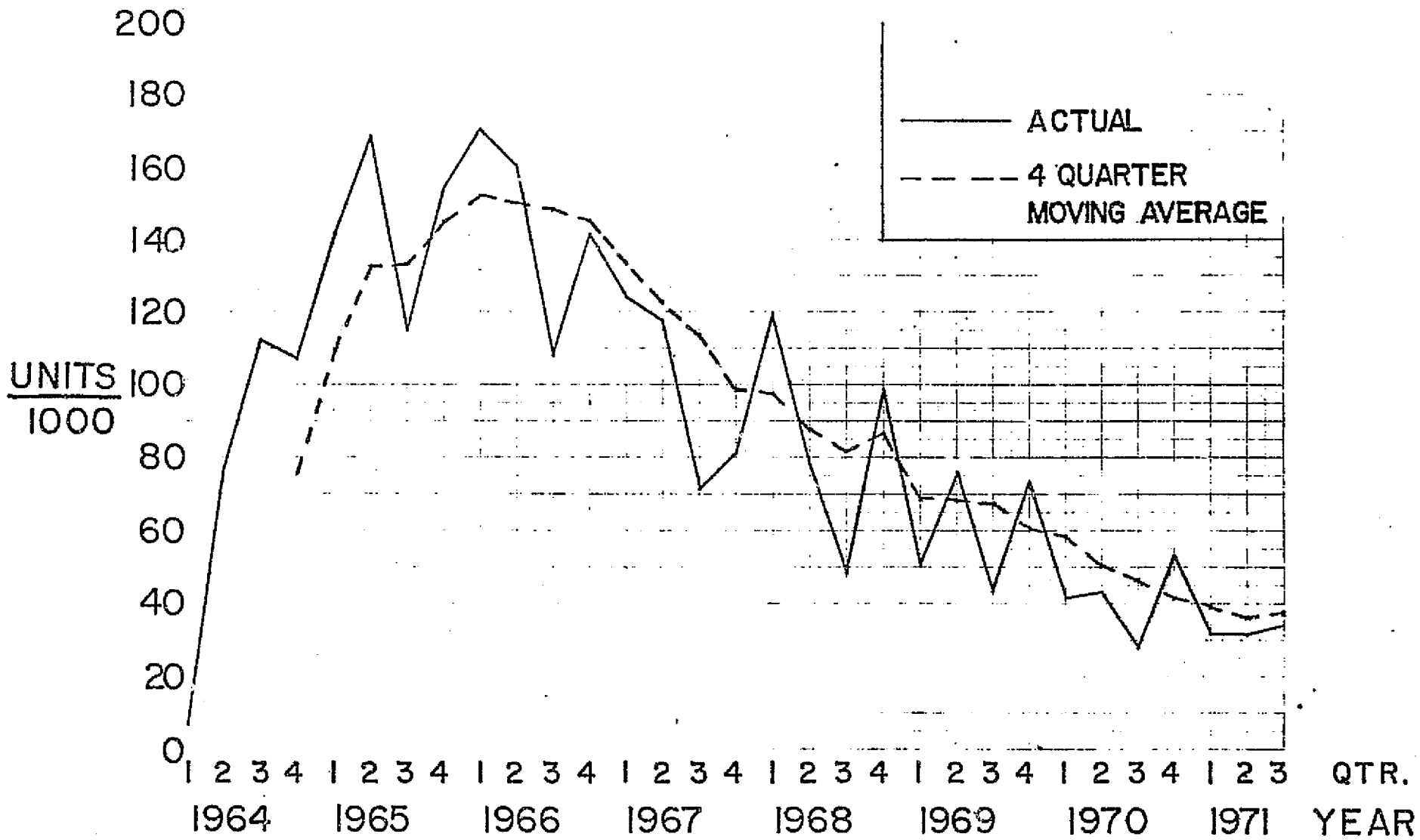


FIGURE G-3 MUSTANG PRODUCTION – ACTUAL QUARTERLY AND "4 QUARTER MOVING AVERAGE"

TABLE 1

Calendar Year	Quarter	Actual Production	4 Quarter Moving Average	Calendar Year	Quarter	Actual Production	4 Quarter Moving Average
1964	I	6,954		1968	I	119,671	97,536
	II	77,315			II.	78,084	87,719
	III	112,120			III	48,556	81,918
	IV	107,019	75,852		IV	98,906	86,554
1965	I	141,702	109,539	1969	I	50,323	68,968
	II	168,796	132,409		II	76,064	68,462
	III	115,034	133,138		III	43,420	67,178
	IV	154,655	145,047		IV	73,384	60,798
1966	I	170,592	152,269	1970	I	41,256	58,531
	II	160,638	150,255		II	43,135	50,299
	III	108,114	148,499		III	27,750	46,631
	IV	141,423	145,192		IV	53,184	41,331
1967	I	124,009	133,546	1971	I	31,983	39,013
	II	117,354	122,722		II	31,577	36,123
	III	71,758	113,636		III	34,028	37,793
	IV	81,361	98,620		IV	33,104	32,673
				1972	I	27,968	31,674

Sources: 1. Automotive Industries, March 15, 1965-1970 Issues.

2. Economics, Research and Statistics Dept., Automobile Mfrs. Asso., Inc., Detroit, Michigan 48202.

of Figure G-1. Figures G-4 and G-5 illustrate the result of such a trial and error search process. With some insight to this matching process the number of trials is not very large. Corresponding to this seemingly "best fit" life cycle modeling, the following forces were found:

The force of innovation $v(t) = 1.1$ innovation events/year
The force of mortality $h(t) = 0.25\ t^{1.5}$ failure events/year

It is readily noted that the force of innovation or renovation is essentially constant over time while the force of product mortality increases proportional to the 3/2 power of time from the starting moment of the product life cycle at the time $t = 0$ (at about third quarters of the year 1964 for the smoothed data).

The interpretation of the force of innovation or renovation seems not so hard: the automobile business by tradition introduces new models on an annual basis on the top of which there are some technological improvements (such as learning curve effects). Therefore, each year the potential automobile buyer is exposed to a "new" model. In addition to the new model, there are also new optional features including such items as window defoggers, sun roofs, "Fire-Frost" paints, ride and handling packages, and so on. It is reasonable to assume that the consumers' perception of a "new model" represents a nearly constant innovation event per year. Such an innovative event is generated by total business planning. This innovation per year can be related to new options, minor technical improvements, new consumer product perceptions created by advertisement, improvements in sales promotion, and so on. It should be noted that the concept and perception of a "New Model" is an aspect of American consumer expectations.

The force of mortality extracted out by fitting the model into the smoothed data, $h(t)$, is not so obvious to interpret at the outset. There can be a multitude of factors affecting product survival in the marketplace, such as:

Decline in the number of buyers in a specific age group, e.g. 18-35 year old males, or females, etc.

Decline of number of buyers in a specific income bracket, e.g. \$10,000 - \$15,000 per year, etc.

Introduction of new "product substitutions" by competitors or by the firm producing the product subject to forces of mortality.

Increase in vehicle sales prices.

Increase and change in vehicle size, weight, characteristics, etc.

Decrease in vehicle operating cost efficiencies.

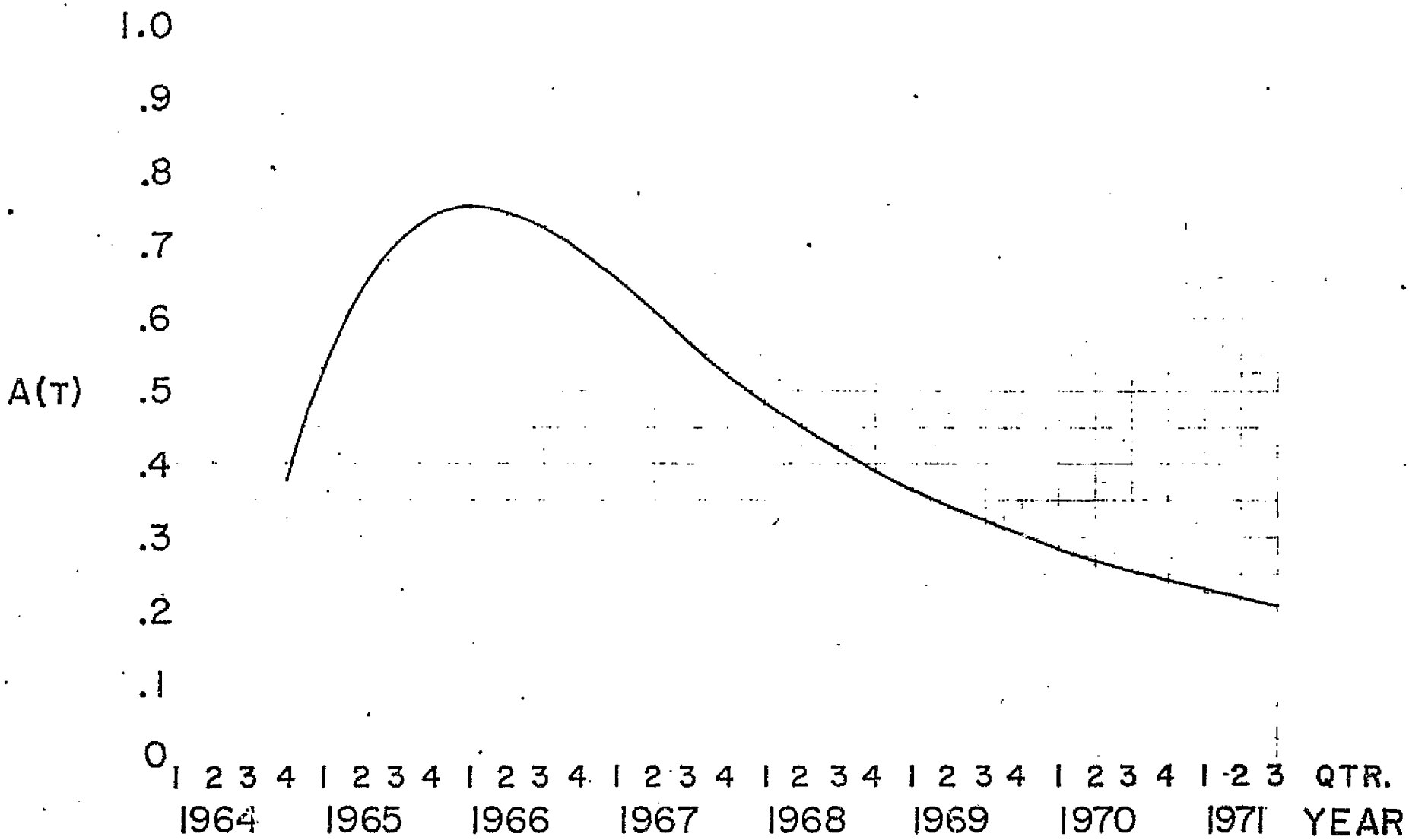


FIGURE 4 MARKOV PROCESS SIMULATION RESULTS

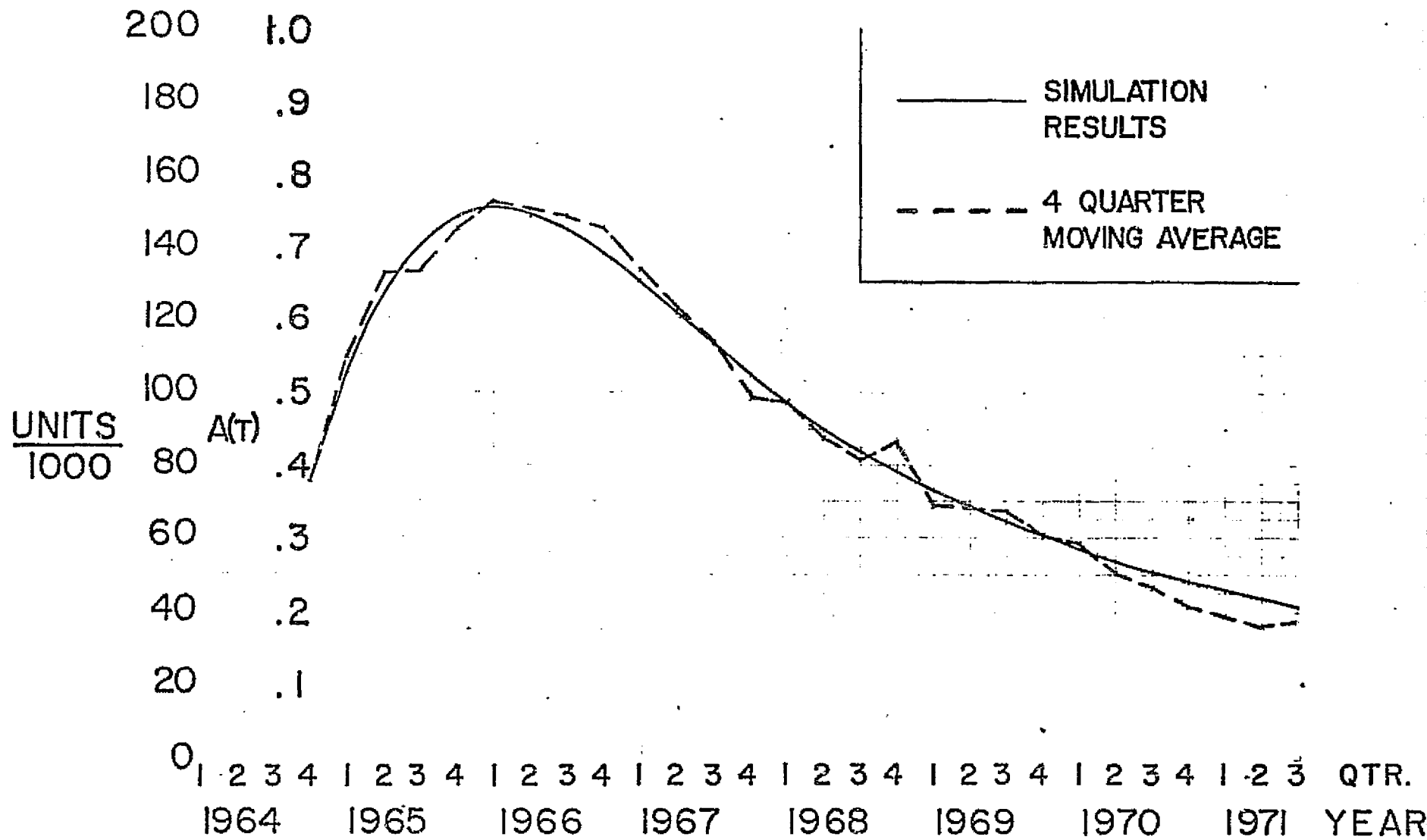


FIGURE G-5 MARKOV PROCESS SIMULATION RESULTS AND "4 QUARTER MOVING AVERAGE"

Many of these specific forces affecting the product life cycle might be aggregated into some relevant single measure such as the total competitors' market share of effective "product substitutes" that could affect a specific particular product perception in the minds of customers associated with, for example, Mustang I.

In order to compare Mustang I to "Mustang-like" automobiles certain assumptions must be made. First, the number of people interested in this type of automobile is assumed to be slowly growing and with some relatively fixed characteristics such as a certain age bracket (say, 18-35 years) and income. With a slowly growing total market for the "Mustang-like" cars the Mustang I sales deteriorated while that for all the substitute vehicles increased. This would suggest that the force of mortality for Mustang I is somehow related to the total market share of the competition. Table 2 lists the automobiles that the authors of this article considered reasonably "Mustang-like". The table gives the four quarter moving average of the actual production divided by 1,000 for these various "substitute-products" of Mustang I. Since there was some question whether VW and Toyota would really be sufficiently "Mustang-like", totals were obtained with and without these two types of cars. The final results did not seem to be affected strongly either way.

Figure G-6 illustrates the total market share of competition including VW and Toyota. This markest share is called the actual mortality market share, and it exhibits a cyclical trend with a period of some three years (corresponding, perhaps, to a three year automobile replacement period of customers?). A Theoretical Mortality Market Share (TMMS) was developed to conform to the force of mortality, h(t), obtained from the "best fit" life cycle results discussed previously. The assumed form of TMMS was as follows:

(5) $$TMMS = A + B\ t^{1.5}$$

Figure G-6 illustrates the "best fit" match of this theoretical mortality market share to the data. The relative cyclical trend, AMMS/TMMS, is given in Table 2 and illustrated in Figure G-6.

The numerical expression for the "best fit" TMMS is as follows:

(6) $$TMMS = 150{,}000 + 65{,}530\ (.25\ t^{1.5})$$

The initial condition of the Mustang I life cycle occurs at t = 0 years (1964, I). Therefore, noting the initial value of TMMS, the difference impact of the competing market on Mustang is from Equation 5,

(7) $$TMMS - A = B\ t^{1.5}$$

This represents the competing market share over and above the initial competing market share that Mustang I faced at the beginning of the life-cycle. Therefore, the force of mortality, h(t) is calculated from the equation

TABLE 2

4 Quarter Moving Average of
Actual Production : 1000

Year	Quarter	American	Barracuda	Corvair	Cougar	Camaro	Firebird	AMX & Javelin	Challenger	Gremlin	Pinto	Vega	VW[1]	Toyota[1]	Actual Total	Theoretical Mortality Market Share	Cyclical[3] Trend	Actual* Total
1964	1	11.9	4.3	13.3									76.8		106.3			29.5
	2	22.6	8.7	26.8									76.8		134.9			58.1
	3	28.9	12.3	34.1									76.8		152.1			75.3
	4	37.8	16.7	49.0									76.8		180.3	150,000	1.202	103.5
1965	1	32.3	17.1	54.1									96.0		199.5	152,047	1.312	103.5
	2	29.5	15.5	57.3									96.0		194.3	155,792	1.273	102.3
	3	25.7	14.6	56.2									96.0		192.5	160,640	1.198	96.5
	4	25.0	13.7	51.0									96.0		185.7	166,383	1.116	84.7
1966	1	24.3	11.0	41.9									105.0	3.9	186.1	172,895	1.076	77.2
	2	33.7	10.1	30.4									105.0	3.9	183.1	180,096	1.017	64.2
	3	24.3	7.8	25.2	1.4	3.9							105.0	3.9	171.5	187,926	.913	62.6
	4	21.3	10.5	18.3	12.0	23.6							105.0	3.9	194.6	196,337	.990	85.7
1967	1	18.0	14.5	10.6	20.0	38.1	8.3						113.2	8.3	231.0	205,291	1.125	109.5
	2	16.7	15.8	6.7	32.6	52.5	18.7						113.2	8.3	264.5	214,757	1.232	143
	3	16.7	18.3	6.4	37.2	57.3	23.1						113.2	8.3	280.5	224,710	1.248	159
	4	15.8	15.5	4.7	32.9	54.1	30.6						113.2	8.3	275.1	235,126	1.170	153.6
1968	1	18.9	12.0	4.2	36.7	52.8	28.4	4.1					140.9	17.2	315.2	245,986	1.281	157.1
	2	20.5	11.0	3.8	31.1	54.8	26.5	8.7					140.9	17.2	314.5	257,271	1.222	156.4
	3	20.6	10.8	3.7	29.9	56.5	25.8	11.0					140.9	17.2	316.4	268,967	1.176	158.3
	4	22.4	9.6	2.9	32.4	57.3	26.4	14.5					140.9	17.2	323.6	281,060	1.151	165.5
1969	1	16.7	9.1	1.8	26.6	61.1	26.5	14.0					134.5	29.3	319.6	293,537	1.089	155.8
	2	9.5	8.3	1.1	26.0	55.5	20.3	12.9					134.5	29.3	297.4	306,387	.971	133.6
	3	6.2	9.5	.5	25.6	52.3	18.8	12.9	4.3				134.5	29.3	293.9	319,598	.920	130.1
	4	0	12.4	0	22.6	39.8	12.4	11.3	13.3				134.5	29.3	275.6	333,156	.827	111.8
1970	1		12.8		20.0	31.1	9.9	10.3	16.8	1.5			142.3	46.2	290.9	347,070	.838	102.4
	2		13.6		19.1	37.2	14.4	8.1	19.9	5.2			142.3	46.2	306.0	361,312	.847	117.5
	3		11.4		17.9	40.0	16.3	7.[illegible]	17.9	8.8	4.7	5.5	142.3	46.2	318.7	375,882	.848	130.2
	4		7.6		17.9	35.9	15.2	9.4	10.7	12.0	22.0	11.2	142.3	46.2	329.4	390,772	.843	140.9
1971	1		6.3		16.8	40.9	17.8	7.3	9.1	13.9	40.7	33.5	127.3	67.6	381.2	405,977	.939	186.3
	2		5.2		14.9	34.3	15.4	7.6	8.2	13.7	59.9	60.6	127.3	67.6	414.7	421,488	.984	219.8
	3		4.6		15.5	28.5	12.9	6.9	7.4	13.5	74.3	78.8	127.3	67.6	437.3	437,300	1.000	242.4
	4		4.3		13.4	37.1	16.5	6.2	7.3	13.6	74.9	98.5	127.3	67.6	466.7	453,408	1.029	271.8
1972	1		4.1		14.3	27.4	11.7	6.5	6.6	13.6	75.4	91.6	111.8	64.4	427.4	469,807	.910	251.2

[1] Figures reflect "U.S. New Imported Car Registrations" (Production data not available). Since only annual figures were available, 4 quarter moving average was not calculated.

[2] Theoretical Mortality Market Share (TMMS) calculated from: $TMMS = 150{,}000 + 65{,}530(25t^{1.5})$

[3] Cyclical Trend = Actual Total/TMMS

* Without VW and Toyota

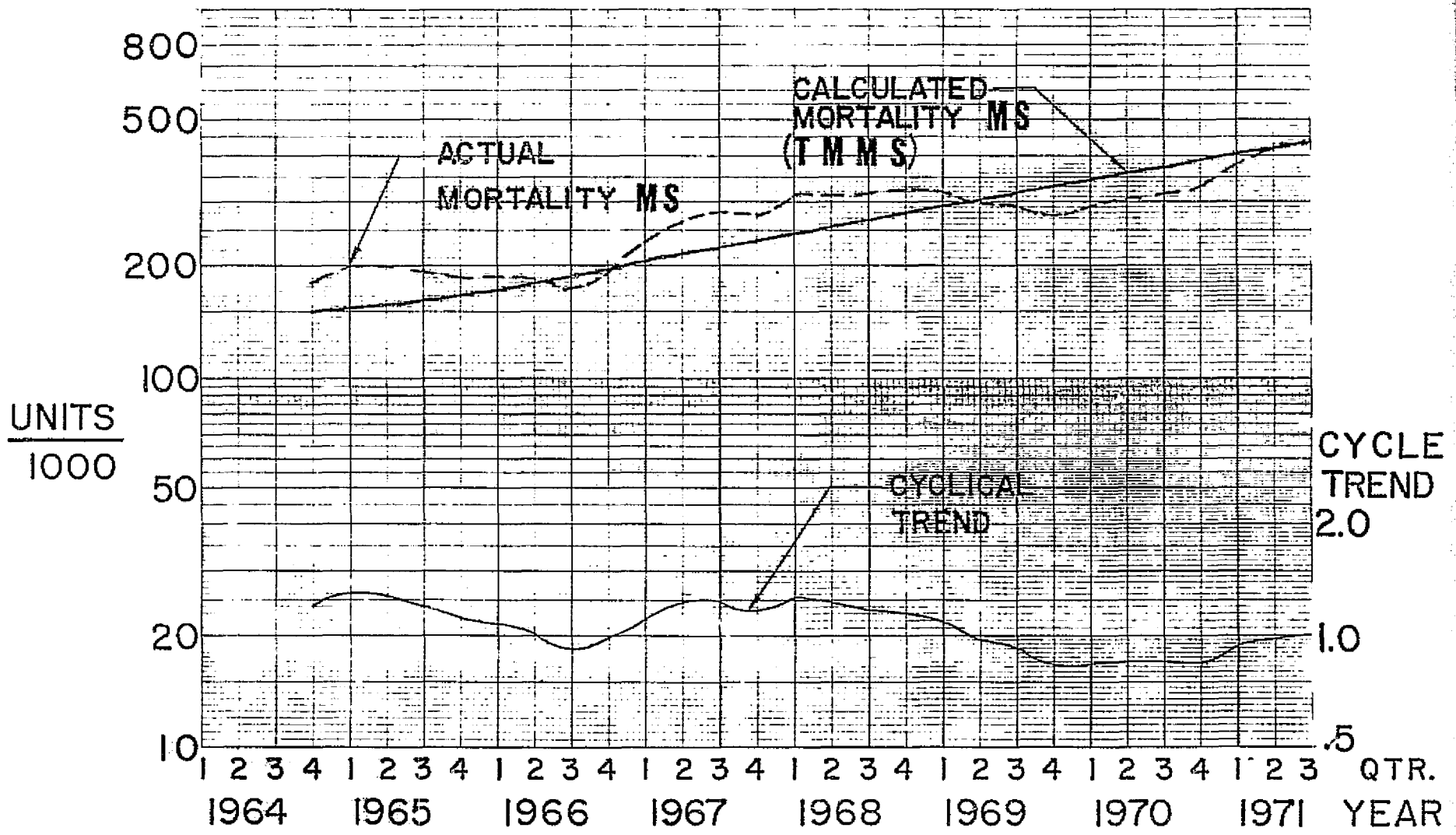


FIGURE 6 MORTALITY FORCE ANALYSIS

$$(8) \qquad h(t) = 0.25\ t^{1.5} = (0.25/B)[TMMS(t) - A]$$
$$= (0.25/65530)[TMMS(t) - 150000]$$

While the cyclic trend is present, its effects tend to cancel and smooth out in the integration process indicated in Equations 2 and 3.

The heuristic method used here in connection with Mustang I life cycle yields a result that can be interpreted: For Mustang I the innovation rate remains essentially constant over its life cycle, while the force of mortality is proportional to the competitors total market share minus their initial market share at the start of the Mustang I life cycle.

CONCLUSIONS

The heuristic shaping of product life cycles presented here generate a relative pair of countervailing forces: the force of product innovation and renovation, v(t), and the force of product mortality, h(t). These are actuarial concepts that can be related to the actual performance of a firm in a specified competitive setting. They are sort of generalized performance measures potentially useful for new products and strategic planning of a corporation. The illustration here was kept intentionally at a simplified level sufficient, however, for meaningful interpretations. The method can be useful for planning purposes provided that the analyst has a proper insight into the actual processes of a firm and understands the basic nature of its competitive environment.

APPENDIX H

Methodologies For Regionalization Of Nodal, Network, And Spatially Continuous Phenomena

A major concern in the implementation of the total assessment profile relates to the identification of the regional scale at which a study would be carried out and the identification of key impact regions such as the total impact area, fringe impact areas and filter areas. It will be the purpose of this Appendix to review the basic methodology geographers utilize in the delineation and analysis of spatial/temporal phenomena. The Appendix will represent a compendium of pertinent geographic methodologies and their applications to the operationalization of the Total Assessment Profile. Through the implementation of these methodologies the researcher will be able to introduce and evaluate the spatial dimensions and impacts of new technological innovation.

THE ANALYSIS OF NODAL DISTRIBUTIONS

A major methodological thrust in geographic methodology is associated with the analysis of phenomena which exhibits a point or nodal character over space. Much of the characteristics of human settlement and activity patterns will reveal either an abstracted point configuration or an orientation to specific points in the space economy. An example of this, would be the location of human settlements, employment opportunities, and movement pattern orientations. Areas that have a concentration of such phenomena would be those likely to be hardest hit with the unslaught of a natural disaster. This necessitates the development of methodology that can evaluate the probable impact area and the extent of damage to the region.

The geographic analysis of nodal patterns in space is normally structured on a two-dimensional coordinate system. Within the coordinates each node is located in respect to an x and y coordinate position. The preceding provides the basic information required to locate the node, define its relationship or spatial situation to other nodes in the system, and to study the aggregate relationships between subsets of nodes. Geographers have noted that every spatial arrangement of nodes possesses the following three characteristics:

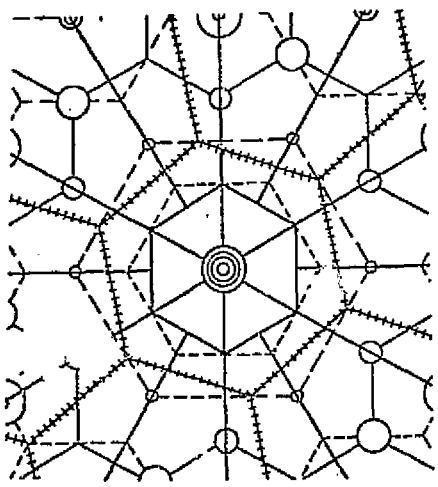
1. Shape - This quality was defined by Bunge [373] and is described as a closed curve which eliminates the collection of objects and provides an areal measure of the distribution in two-dimensional space.

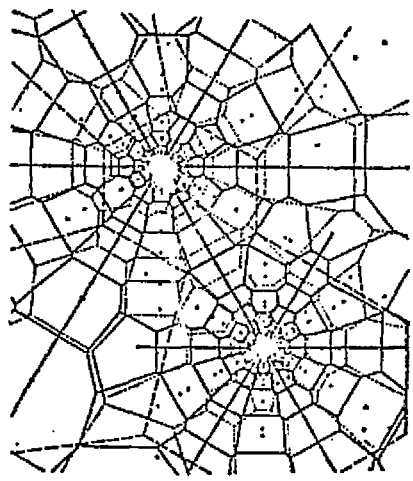
2. Pattern - This is a zero-dimensional characteristic of a spatial arrangement which describes the spacing of a set of objects with respect to one another [374].

3. Dispersion - This quality may be viewed as a one-dimensional characteristic of a spatial arrangement which measures the spacing of a set of objects in relation to one particular shape of a given area [375].

THE ANALYSIS OF SHAPE IN A GEOGRAPHIC DISTRIBUTION

The study of shape in geography has currently only limited methodology associated with it. The implications of shape, however, may be of substantial interest in technology assessment. Elongated political and cultural regions for example, provide significant difficulties in the provision and accessibility to essential services [376]. Service bottlenecks and administrative complications may arise in such a regional configuration. Normative location theories for the allocation of industrial and service center locations over space have provided a comprehensive and convincing literature on the optimality of circular service regions. Hierarchical models and the restrictions associated with spatial packing of service areas on triangular network lattices has forced a general compression of these circular shaped service regions into a generally hexagonal network system. The economic optimality of such arrangements in conjunction with the minimization of movement costs has been verified by an extensive theoretical and empirically based literature [377]. The following figures show respectively a theoretical hexagonal configuration for settlements and an adjusted empirically based service distribution system:





An emergency logistics system structured on fast and error free services may be seriously hampered by an administrative regionalization that is inefficient in geographic structure. The full implications of shape, in particular the shape of administrative and functional regions is of concern to the geographic researcher. The measurement of shape, however, presents many difficulties.

Most measurements of shape employ various indices that are ratios involving parameters of a distribution. The range for most indices fall between the value of zero and one. Departures from a line or circle tend to be the normative shapes utilized in these ratios:

$$s_1 = (A/0.282)P$$
$$s_2 = (A/0.866L)$$
$$s_3 = R_1 / R_2$$
$$s_4 = (A/\pi(0.5L)^2$$
$$s_5 = (1.27A/L^2)$$

Where:

A = Area of shape being measured
L = Length of Longest Axis
R_1 = Radius of largest inscribing circle
R_2 = Radius of smallest circumscribing circle
p = Length of perimeter

In all of the above ratios a value approaching zero would suggest a linear elongated shape, while a value approaching one would imply a circular shape to the region in question [378].

Two other approaches have been commonly used in geography to define the quality of shape. The Boyce-Clark [379] method is based on a formula that calculates an index which varies between 0 and 175 regardless of size of the area concerned. The index is represented below:

$$SBC = \Sigma \left|(r_1/\Sigma r_i)100 - 100/n\right|$$

Where:

SBC = Boyce Clark Shape Index
r_i = the length of the ith radial
n = number of radials

A major problem with the above shape index is the determination of the point within the shape from which the radials are to be drawn. Most researchers use the center of gravity as this orientation point. Interpretation of the ratio does provide some problems with the identification of ground truth. An approximate shape correlation with the Boyce-Clark scale is provided in the following:

0	12	18	25	28	175
	Square	Cruciform	Star	Rectangle (2x+y)	Rectangles (x>y)

Bunge provides an alternative measure of shape that is much more complex than the previous ratios and indices [373]. The method is based on two theorems:

1. Any simply connected shape can be matched by a polygon of any number of sides in which the lengths of each side are equal, but the lengths can of course vary from shape to shape.

2. If the distance between all vertices of the polygon lag 1 are summed and then squared and summed, lag 2 are summed and then squared and summed, and so on, there will exist a unique set of sums that define the shape of the polygon.

Thus, the first step, in the Bunge method is to define for any given shape an abstract polygon which adequately represents the shape to be defined:

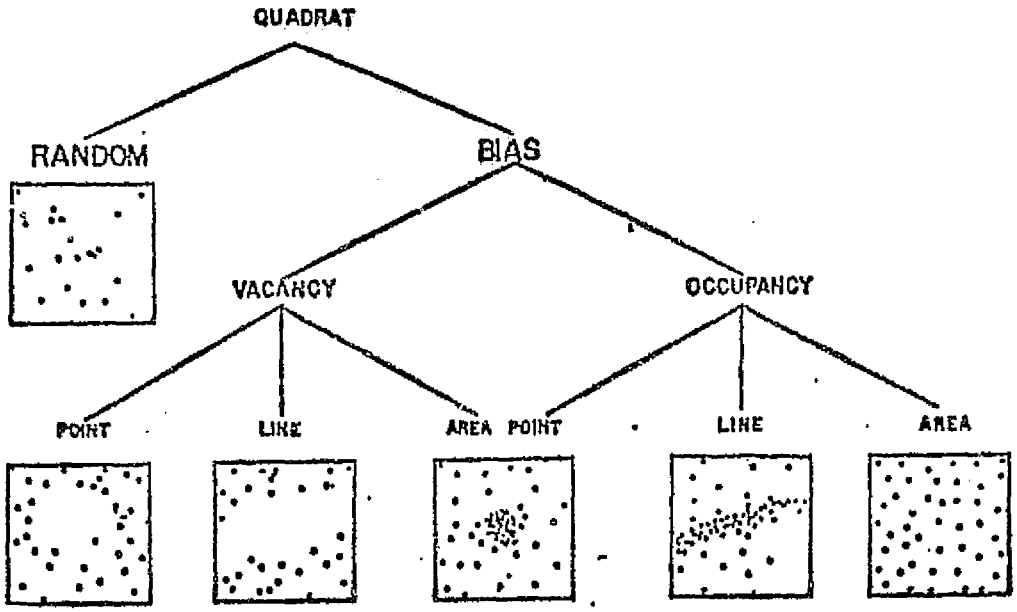
Actual Shape

Abstract Polygon

The unique set of sums which describe that polygon can then be calculated with respect to its vertices. All of the above methods have computational problems associated with them as well as interpretation problems with defining the ground truths of the index values. In a cost benefit model the shape indices would be employed with "hard data" to define the degree of relationship and its respective implications.

THE ANALYSIS OF SPATIAL PATTERNS

The analysis of spatial patterns, is of perhaps, greater significance in a cost benefit application than the function of shape. It is through the use of patterns that spatial relationships may be defined. The diffusion network, its areal extent, and its configuration would be defined by the concept of pattern. Point pattern analysis has been of major methodological interest to geographers. The general orientation of these methods is to establish an observed set of points and compare these points to a theoretical set of points that would be generated by a number of probability processes. A basic division of pattern classification may be structured from the utilization of probability distributions. Point patterns will either reflect a random spatial arrangement or will exhibit a bias spatial pattern that would reveal a vacancy or occupancy bias:

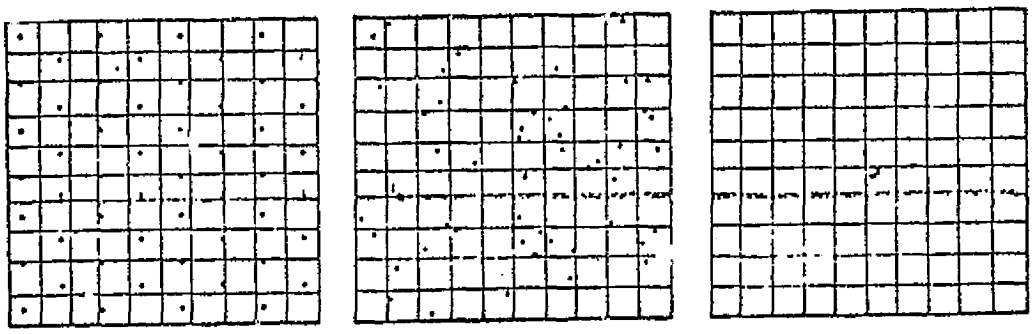


This figure demonstrates a classification structure for a point distribution analysis. A major division in the taxonomic chart is reflected in the determination of a pattern reflecting a random arrangement opposed to the bias arrangements. To define the implications of randomness in space the following conditions are assumed:

1. Any point on the surface has equal probability of occurring at any position on the surface.

2. The position of a point at the above place is independent of any other point.

A major issue in the analysis of point distributions is based on identifying whether the point pattern reflects a regular, random, or clustered spatial distribution. Two basic methodologies may be employed to accomplish the above identification: quadrat analysis, and point to point analysis. In quadrat analysis a grid is laid over the study region with a frequency count of points contained in each cell being calculated:



a. Perfectly regular b. Random c. Perfectly clustered

The observed frequency distribution is then compared to a theoretical one. For example, a Poisson distribution may be used in the case of randomness, or a negative binomial function for a clustered distribution.

Point to Point distributions, on the other hand, have been developed from nearest neighbor statistics. This method involves computation of the mean distance and associated variances for each order of nearest neighbor points. The preceding are then used in a comparison to define the parameters that would characterize the expected distances:

$$R = (\Sigma r/n)/(1/2(\sqrt{n/A})$$

Where:
R = Nearest Neighbor Statistic
r = Observed Nearest Neighbor Distance
n = Total Number of Nodes
A = Area of the Study Region

Thus, under the assumption that the first order distances are drawn from a normal population, a density dependent expected mean would be defined.

Randomness can then be tested using the standard normal curve. Alternatively a ratio of observed and expected mean distances can be computed (R, the nearest neighbor statistic). Employing the nearest neighbor statistic the following range in values may be derived [380,381]:

0----------1.0----------2.14

Absolute Clustering	Random	Absolute Regularity

The relationship of the spatial pattern to a cost benefit analysis of an early warning disaster system rests with the extent, orientation, and duration of the impacted region. If the distribution of settlements, for example, indicate a clustered pattern the expected impact would be far greater than may be anticipated from a region in which the settlements reflect a regular or random spatial arrangement. It is also through the measurement of pattern that disaster agents and their expected impacts may be isolated. Disaster events that reflect a random spatial occurrence would prove to be more difficult to develop an adequate warning network compared to those that reveal a definite and predictable spatial pattern.

THE ANALYSIS OF SPATIAL DISPERSION

Closely related to the element of pattern, is the concept of spatial dispersion. Dispersion, is perhaps, the most important spatial concept to be considered in introducing the areal dimension to a cost benefit analysis. It is by means of this dimension that the areal dispersion, and to a great extent, the necessary intensity of impact over the earth's surface may be isolated. The relationship of dispersion to the preceding concepts may be defined as an attribute of a pattern that is located within a particular shape at a given distance.

The analysis of spatial dispersion draws heavily on the utilization of centrographic statistical procedures. The centroid or arithmetic mean center of an areal distribution (bivariate distribution) is analogous to the concept of the arithmetic mean of a linear distribution. Neft [382] indicates that virtually all arithmetic mean centers have been calculated as the point representing the arithmetic mean of the X values and the arithmetic mean of the Y values where X and Y are a pair of orthogonal axes:

$$\overline{X} = \frac{\Sigma X_i}{N} \qquad \overline{Y} = \frac{\Sigma Y_i}{N}$$

The standard deviation of a linear distribution has for a counterpart in bivariate statistics the standard distance deviation. Hence, this statistic represents a dispersion measure over space. Lee [383] notes that the standard deviation is minimized about lines X_0 and Y_0 drawn through the mean centers of the X and Y reference axes:

$$\sigma_x = \sqrt{\frac{\Sigma(X_i - X_o)}{N}} \quad = \text{ Standard Distance Deviation X}$$

$$\sigma_y = \sqrt{\frac{\Sigma(Y_i - Y_o)}{N}} \quad = \text{ Standard Distance Deviation Y}$$

Normally the standard distance deviation about each of the two orthogonal axes will not be of the same length except in the case where a distribution is circular in shape.

A measure of spatial dispersion computed along the axis of the distribution is referred to as the standard distance or the standard radius of the distribution. The standard radius is defined as the hypotenuse of a right triangle formed from the sides of the distance variance of the X and Y axes:

$$\text{Standard Radius (SR)} = \sqrt{\sigma_x^2 + \sigma_y^2}$$

The combination of the arithmetic mean center with the standard distance deviation, distance variance, and the standard radius provide a quantitative expression of an areal distribution.

The standard distance deviations will vary in their orientation to a set of perpendicular central axes which are parallel to the X and Y reference axes. The central reference axes do not necessarily maximize or minimize the standard distance deviation. Lee indicates that a set of central axes called the principal axes can be found by rotation which will maximize the standard distance deviations. The principal axis about which the standard distance deviation is minimized will be referred to as the major principal axis. Consequently the central axis perpendicular to the major principal axis that maximizes the standard distance deviation is called the minor principal axis. The above minimum and maximum standard distance deviations about the central reference axis can be used to provide an index of circularity. The coefficient of circularity is simply a ratio of the standard distance deviations about the major principal axis to the standard distance deviation about the minor principal axis:

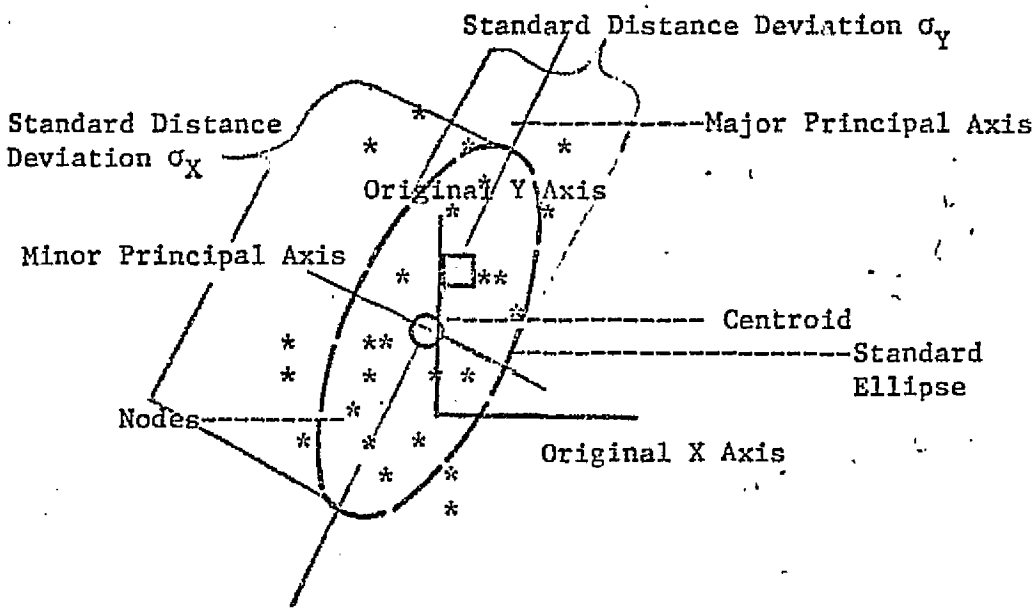
$$\text{Coefficient of Circularity (CC)} = \frac{\sigma_{yp}}{\sigma_{xp}}$$

Where: σ_{yp} = Standard distance deviations about the major principal axis

σ_{xp} = Standard distance deviations about the minor principal axis

If the preceding ratio is equivalent to one this would imply both the maximum and minimum standard distance deviations about the principal axis

are of the same length, thus the distribtuion would be circular in shape. As the index approaches zero the standard distance deviation about the minor principal axis would become very small suggesting a linear pattern in the distribution. In most cases the distributions of spatial phenomena are not evenly dispersed; thus an elliptical shape will characterize the majority. The preceding is provided by a graphic device called the Standard Ellipse. The larger the ellipse the more dispersed will be the phenomena being studied. On the other hand, a small circular ellipse would indicate a concentration of the phenomena in space. The following illustrates the ellipse and the general structure of a centrographic analysis of a point distribution:



THE ANALYSIS OF SPATIAL NETWORKS

The preceding discussion has provided a means of identifying the nodal intensity and distributional characteristics for a regional system. Included within the above system, but not yet introduced would be the connecting networks between points and phenomena demonstrating a nodal character. The demand and level of interaction between nodes would exhibit over space a meshed fabric of intersecting lines and routes. The intensity and connectivity of these networks would indicate the level of potential interaction and the degree of redundancy in potential circulation circuits. Both of the above would be critical considerations within a cost/benefit analysis of technological innovation. Specifically, in the case of a disaster warning system, the nature of the existing network may be a critical factor in defining the temporal lag between the

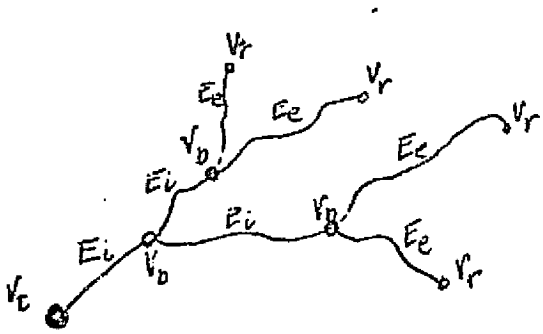
warning sequence and the probable response time for evacuation and emergency aid.

The geographic analysis of the spatial structure of networks has primarily utilized finite graph theory as the key methodology. The geometry of a graph network system is reduced to an abstracted finite set of nodes (V) and to a possible empty set of edges (E). Hence, graphs are defined as sets of points or nodes which may or may not be connected by edges or links to one another. It is through the use of graph theory that the basic topological structure of a network may be defined [384,385,386]. Three major types of networks appear to be relevant in a cost benefit analysis centered on a disaster warning system:

1. Branching Networks

The study of branching networks has its applications focused primarily in the area of the earth sciences. In finite graph theory a branching network represents a minimally connected graph in which all nodes are connected; however, no circuits or loops exist in the system. The major emphasis on the study of branching networks, or what are sometimes referred to as tree graphs, are in applications associated with the configuration and operation of river drainage systems.

A stream network, for example, represents one of the simplest configurations from a topological perspective. All bifurcating networks are, in an abstract sense, topologically identical regardless of their unique attributes. An example of a simple branching graph is provided below:

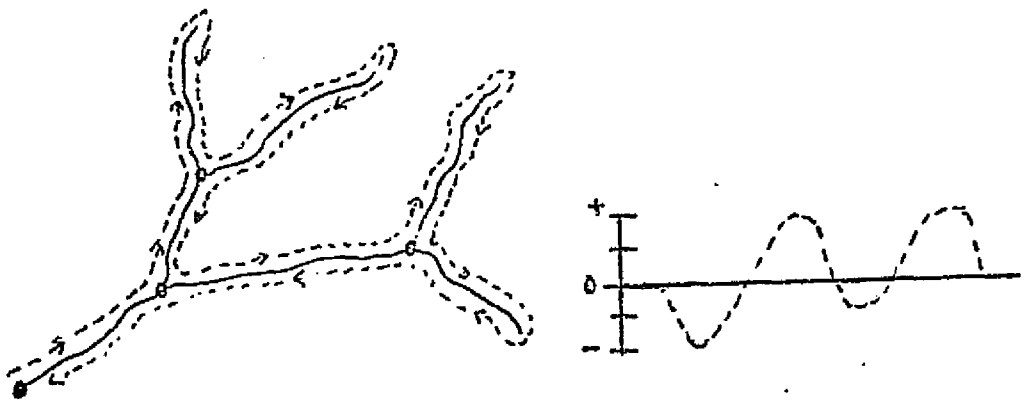


Where: E_i - Internal Edge
E_e - External Edge
V_t - Terminal Node
V_b - Branch Node
V_r - Source Node

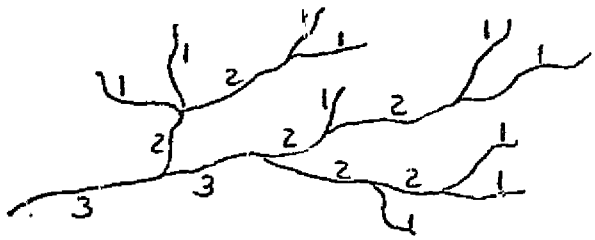
The symbolization associated with the graph is derived from Melton's [387] research on drainage system configurations are components. The geometry of a bifurcation graph permits the identification of the following relationships between nodes and edges [387,388]:

$$E = (V_t + V_b + V_s) - 1$$
$$E = 2V_t - 1$$
$$V_b + V_r = V_t$$

Based on the preceding Shreve [389] has developed a procedure to describe the topological structure of the tree network. The method simply involves tracing a systematic route along a graph while recording all interior and exterior links traversed. Each time an interior link is traversed a +1 is recorded and each exterior link encountered is recorded as a -1:



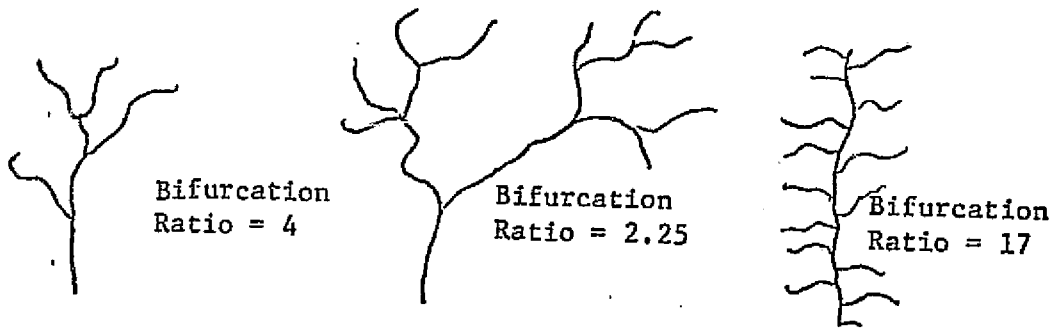
An important consideration in the research associated with branching networks has been oriented to the differentiation in edges based on a hierarchical order. This concept is of particular importance in the study of a drainage system, since each segment may exhibit different morphometric and hydrologic features. Most of the hierarchical models assign a level of relative importance to each segment in a branching network. The approach is based on a sequential arrangement of tributary and major stream segments. An example of a hierarchical ordering system is illustrated in the work of Horton [390] and in the modifications provided by Woldenberg [388]. Horton's approach defines the first order stream as one receiving no tributaries with the second order stream as one receiving two first order streams. Subsequent orders to stream segments are then based on the junctions of the previous order segments:



This diagram is a modification of the Horton system by Woldenberg [388]. Several other systems of hierarchical ordering methods are available and it is suggested that the interested reader review the work of Strahler [391], Melton [387], Warntz [413], and Woldenberg [388].

A review of this literature will define two major research implications: (1) the significance of the relationship between different orders

of stream segments, and (2) the relationship of the various orders of stream edges to the entire drainage network. Haggett [392] defined the relationship between the different orders of the network edges as the "Law of Path Numbers". This concept suggests that the number of edge links for different orders will approximate an inverse geometric series in which the first term equals unity, while the ratio between each order in the sequence would represent the bifurcation ratio. Individual orders of bifurcation ratios may then be used to define a mean bifurcation ratio (the aggregate ratio may be defined by a simple arithmetic mean, a weighted mean, or a geometric mean) for the entire network:



The study of bifurcation ratios has susggested a relatively high stability between regional drainage systems. The minimum bifurcation ratio between 3.00 and 5.00 is seldom found in nature for it requires a basin topography of relative homogeneity. It is through the utilization of the bifurcation ratio and its relationship to the order of the network that descriptive relationships may be defined.

The application of branching networks to a disaster warning system analysis are primarily related to hydrologic effects during the impact phase. The simulation of water flow through a hierarchically ordered drainage system determines warning time and necessary lag times for an effective response. The applications of the study of branching networks has several implications to the human interaction system. Evacuation and recovery efforts may reflect over space a tree like collection or dispersal system. The identification of minimal travel cost spanning paths would be critical in defining major emergency escape routes. It would also be essential to define high risk links within the graph that could function as potential bottlenecks or breaks which would inhibit or prevent movement over the system. Most human interaction systems, however, are more complex and the branching network would merely represent a subgraph or partial graph of the more complex system.

2. Circuit Networks

A circuit network would represent a more complete graph that contains closed loops and circuits. Attempts at measuring the structural character of circuit networks have resulted in a basic set of metrics. The cyclomatic index, for example, defines the number of circuits within

the graph. The alpha index is a ratio measure of the number of actual circuits to the maximum number possible in a given network. The third measure is the Gamma index which is also a ratio which compares the number of existing linkages to the maximum possible. Thus, the cyclomatic number, the alpha index, and the gamma index provide a set of measures of the degree of connectivity and circuitry which characterizes a particular network:

Cyclomatic Number:

$$C = E - V + G$$

Alpha Index:

$$\alpha = ((E - V + G)/(2V - 5)\ 100 \quad \text{For Planar Graphs}$$

$$\alpha = ((E - V + G)/(V(V - 1)/2 - (V - 1))\ 100 \quad \text{For Non-Planar Graphs}$$

Gamma Index:

$$\gamma = (E/(V(V - 1)/2)\ 100 \quad \text{For Planar Graphs}$$

$$\gamma = (E/(3V - 2))\ 100 \quad \text{For Non-Planar Graphs}$$

Where:

C = Cyclomatic Number
E = Number of Edges
V = Number of Nodes
G = Number of Subgraphs

The combinations of the Gamma and Alpha indices may be used to define the degree of complexity characterizing the network. Taaffe and Gauthier [393] indicate three major stages in network structure: spinal networks, grid networks, and delta networks. The spinal network represents the previously discussed minimally connected graph or tree, while the grid and delta networks characterize intermediate and maximally connected graphs respectively. According to Taaffe and Gauthier [393] the following breaking points may be defined for the relationship of the Gamma and Alpha indices to the level of the network:

	Gamma Index		Alpha Index	
Spinal Network	$.33\ \gamma\ .50$	Where $v \geq 4$	$\alpha = 0$	Where $v = e + 1$
Grid Network	$.50\ \gamma\ .66$	Where $v \geq 4$	$0\ \alpha\ .50$	Where $v \geq 3$
Delta Network	$.67\ \gamma\ 1.00$	Where $v \geq 3$	$.51\ \alpha\ 1.00$	Where $v \geq 3$

Hence, by using the above measures in association with one another the degree of connectivity and circuit redundancy may be defined for a network.

The implications of the level of network development as measured by the above provides a framework to define the configuration and complexity of an existing transport or communications network. Individual roads and their relative locations to the graph are measured through the utilization of incidence and connectivity matrices. The simplest connectivity matrices are binary in structure and symmetrical. The elements of the matrix defines the existence or lack of existence of a linkage (unity is used to indicate a link while zero indicates the lack of a linkage). By powering the connectivity matrix both the direct and indirect connections between nodes may be defined.

An analysis of the incidence matrix for the networks will provide an initial measure of accessibility. The incidence matrix is an n by m rectangular array with the columns of the matrix representing the edges and the rows associated with the network nodes:

$$\begin{array}{c|cccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & \cdots\cdots & e_n \\ \hline v_1 & & & & & & & & \\ v_2 & & & & & & & & \\ v_3 & & & & & & & & \\ \vdots & & & & & & & & \end{array} = A$$

The elements of the preceding matrix are equivalent to one if an edge is incident to the node (a_{ij} = 1 when v_i is linked to e) or zero if a lack of incidence exists. Multiplication of the incidence matrix (A) by its transposed matrix (A^T) will give a connectivity matrix (C) which is symmetrical:

$$_nA_m \quad \cdot \quad _mA^T_n \quad = \quad _nC_n$$

Garrison [415] suggests a removal of the positive diagonal elements of the connection matrix replacing them with 0's based on the rationale that a reflexive incidence structure at a node is meaningless.

Powering the connection matrix to the diameter of the network and then summing the rows of the powered matrices will provide a measure of node accessibility for both direct and indirect linkages:

$$nCn + nCn^2 + nCn^3 + \cdots\cdots\cdots + nCn^{\xi} = nTn$$

Where ξ = The Diameter of the Network (ξ = max. $d(v_i v_j)$

The above nodal accessibility metric has significant shortcomings because it includes redundant paths generated by powering the connection matrix. Removal of the redundant paths in matrix T is mathematically prohibitive, thus creating an additional complication to the problem. A potential solution to the above is provided by a technique developed by Shimbel [394]. Shimbel's approach consists of recording the power, in a distance matrix (D), at which a one enters the cells of the powered connectivity matrices. Thus, if a one enters element c_{44} on the third powering of the connection matrix a three is recorded for cell c_{44} of the distance matrix:

$$
\begin{array}{c|cccccc|}
 & v_1 & v_2 & v_3 & v_4 & \cdots\cdots\cdots & v_n \\
\hline
v_1 & 0 & p & p & p & & p \\
v_2 & p & 0 & p & p & & p \\
v_3 & p & p & 0 & p & & p \\
\vdots & & & & & & \\
v_n & & & & & & p \\
\hline
\end{array}
= {}_nD_n
$$

Where p = the power at which a one enters the connectivity matrix

It may be noted that the diagonal remains zero with the off diagonal elements indicating minimal path structures in the network.

Summing the rows of the Shimbel distance matrix provides a measure of node accessibility. A high row sum would indicate that linkages to other nodes in the system are indirect and do not occur until the higher powers of the connection matrices are reached, thus the node may be considered inaccessible to the system.

The preceding techniques define nodal accessibility on the basis of equal length linkages between nodes. All linkages in the preceding were assumed to be of equal distance or travel time. The preceding employed a shortest path matrix procedure in which the distance metric was purely topological in structure. Distance was defined in the above simply as the number of linkages between nodes. Hence, all linkages were assumed to be of the same value. If additional information regarding linkage structure is available the network may be studied as a valued graph. In the latter, linkage distances may be weighted by actual mileage or travel time. A similar procedure to define nodal accessibility may be used in a weighted or valued graph approach [393]. The initial step is based on the construction of a connection matrix in which the cell entries are not binary data but rather the weighted values of the linkages. Where linkages are absent infinity is used to define the value of the linkage, while direct links are recorded as their actual distance value. Diagonal cells remain as zero cells in the connection matrix:

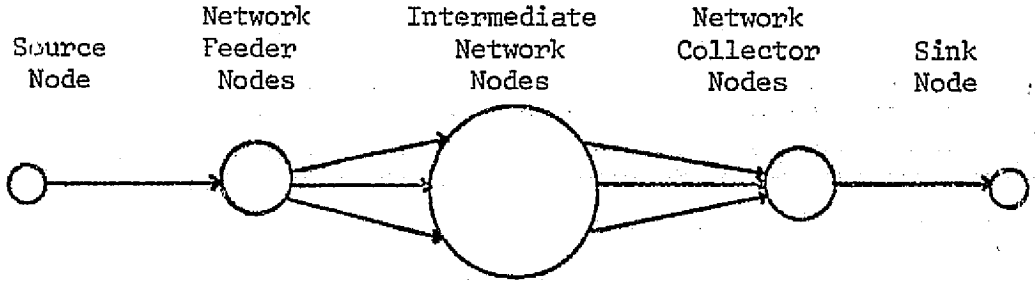
	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	10	∞	∞	∞	∞
v_2	10	0	20	∞	∞	∞
v_3	∞	20	0	10	30	∞
v_4	∞	∞	10	0	∞	∞
v_5	∞	∞	30	∞	0	5
v_6	∞	∞	∞	∞	5	0

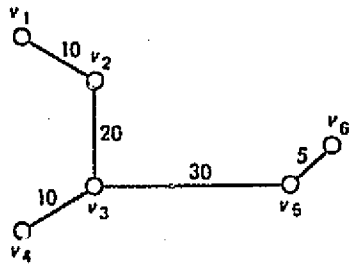
= nCn

This matrix provides the direct linkages between nodes in the network. Indirect linkages are determined by matrix powering procedures similar to the one used to define accessibility in a binary connection matrix. Unlike the previous multiplication procedures Boolean mathematical rules are employed. Instead of element by element multiplication of the rows times the columns of the connection matrix we utilize an element by element addition. Rather than summing the results, we use the minimum value and insert this minimum value in the cells of a new matrix [393]. Hence, the cell ij value in the new matrix is not the sum of the products of all links but is rather the minimum values of the sums of links from the origin to the destination nodes. An example of this is provided in the study by Taaffe and Gauthier [393] on the following page. The matrix L represents a valued connection matrix with each power of the matrix defining the two, three, and four step indirect linkages respectively. Summing the rows of the fourth power linkage matrix (L^4) provides a measure of nodal accessibility of individual nodes to the entire network.

This study represents a static treatment of network structure. An important consideration in the spatial organization of a network may be related to the dynamics of flows through the system. These flows may represent population migration and interaction, the communication of ideas or messages, or the transmission of goods and services over space. The basic typological elements discussed earlier are important considerations in defining a networks flow capabilities. The static typological treatment of graphs must be modified so that dynamic concepts such as direction and velocity may be included in addition to the analysis of connections between nodes. A digraph is constructed where the linkages are represented as having a direction orientation. In the binary case, a +1 in the incidence matrix would represent a destination node with a -1 indicating an origin node. Due to the limitations in length of this paper an extensive treatment of flow problems will not be undertaken. The general structure of these problems, however, will be briefly discussed.

An operational format for a typical network flow problem is presented in the following diagram:





Nodes

Nodes	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	10	∞	∞	∞	∞
v_2	10	0	20	∞	∞	∞
v_3	∞	20	0	10	30	∞
v_4	∞	∞	10	0	∞	∞
v_5	∞	∞	30	∞	0	5
v_6	∞	∞	∞	∞	5	0

L

Nodes

Nodes	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	10	30	∞	∞	∞
v_2	10	0	20	30	50	∞
v_3	30	20	0	10	30	35
v_4	∞	30	10	0	40	∞
v_5	∞	50	30	40	0	5
v_6	∞	∞	35	∞	5	0

L^2

Nodes

Nodes	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	10	30	40	60	∞
v_2	10	0	20	30	50	55
v_3	30	20	0	10	30	35
v_4	40	30	10	0	40	45
v_5	60	50	30	40	0	5
v_6	∞	55	35	45	5	0

L^3

Nodes

Nodes	v_1	v_2	v_3	v_4	v_5	v_6	
v_1	0	10	30	40	60	65	= 205
v_2	10	0	20	30	50	55	= 165
v_3	30	20	0	10	30	35	= 125
v_4	40	30	10	0	40	45	= 165
v_5	60	50	30	40	0	5	= 185
v_6	65	55	35	45	5	0	= 205

L^4

In the diagram the source and sink nodes are used to interject a flow into the feeder nodes and to recover it from the collector nodes of the network. The digraph may or may not be symmetrical with feeder, collector, and intermediate nodes having one-way or two-way linkages.

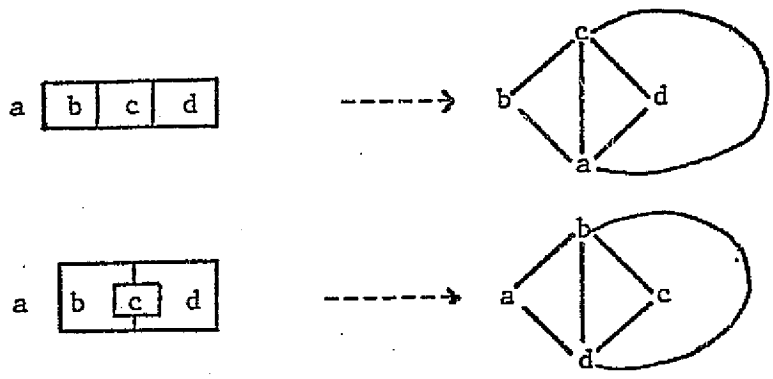
A study of the circulation through a network may have several objectives. A general application of a flow problem would be one oriented to defining a minimal travel spanning tree through the network. Several algorithms are available which determine the minimal paths within networks.

A second application is associated with problems of maximal flow in a network system. In this case, constraints of capacity are interjected on network linkages and sometimes on the nodes themselves. A third major application is associated with combinations of the above. A minimal cost maximal flow problem, for example, would be illustrative of this type. Several labeling algorithms are available for the solution of the minimal cost problem having maximal flow constraints [395].

The study of circuit networks and the treatment of flows over these networks are critical elements in the evaluation of costs and benefits over-space. The potential capability of an emergency delivery system would be constrained by the network's configuration and its linkage and nodal capacities. Changes in the emergency warning technology would require an evaluation of its impact on the communications and transportation networks.

3. Barrier Networks

Barrier networks reveal a substantial difference in function from the previous two types. Branching and circuit networks consist of channels that conduct communications, goods, and people over space while branching networks consist of links which either block or resist flows. Chorley and Hagget [386] observe that such barrier systems demonstrate aspects of closure by consisting of closed loops which are isolated or contiguous. We may regard barrier networks as the "duals of flow networks in both a functional sense and in a topological sense". To illustrate the above point the following diagrams were taken from a study by Warntz [396]:

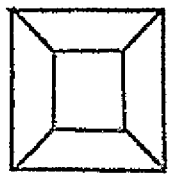


The cellular nets are a form of graph theory that is associated with planar graphs (two-dimensional graphs). They are what Ore [397] calls polygonal graphs in which the edges form a set of adjoining polygons on the plane. An example of such a polygonal plane would be a map of U.S. counties. Euler's formulation of the relationship between edges and vertices in a simple polyhedron establishes the foundation for the geometric structure of polygonal graphs:

$$V - E + F = 2$$

Where: V = Number of Nodes
E = Number of Edges
F = Number of Faces

A polyhedron is defined as a solid whose surface consists of a number of polygonal faces. A simple polyhedron has no "holes" in its surface, so that its surface is continuous. An example is provided below:



$$V - E + F = 2$$
$$8 - 12 + 6 = 2$$

Euler's formula permits one to establish the conditions for the distribution of regular polygons over space. This condition is defined when the following relationship is fulfilled:

$$1 + \frac{P}{P^*} = \frac{P}{2}$$

Where: P = Number of edges at each node
P* = Number of edges bounding each cell

Chorley and Haggett observe that this relationship will only work when both the number of edges at each vertex and the number of edges bounding each cell are equal. These conditions would be demonstrated in regular polygons such as triangles, quadrangles, and hexagons. In a regular hexagonal pattern, for example, the contact number would equal six since each cell would be contiguous with six neighboring cells.

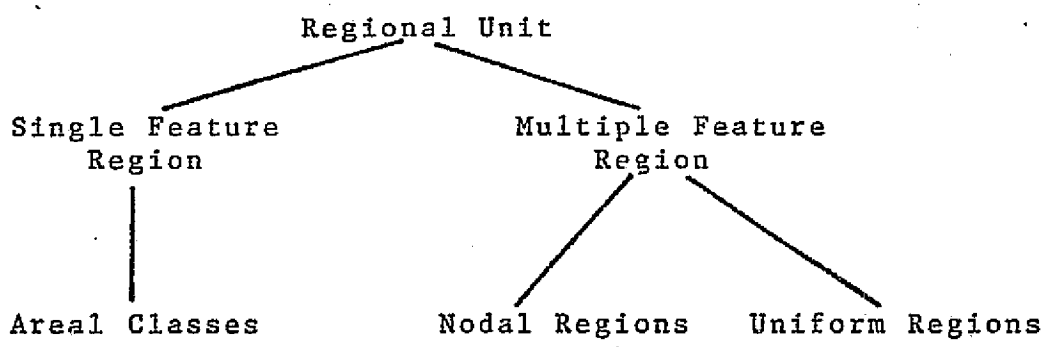
The implications of the barrier networks or polygonal graphs rests with problems associated with the partitioning of space. Efficient service and administrative districting plans would be dependent on an optimal spatial allocation system. The preceding permits the researcher to evaluate an existing or projected areal arrangement of service and administrative districts according to basic geometric concepts of efficient space packing and partitioning. Inefficiencies in the configurations of service hinterlands would reflect a cost to the delivery system. An emergency warning technology that would be dependent upon reaching a maximum number of people in the shortest period of time requires the consideration of geometric and areal dimensions.

The preceding provides the basis for the analysis of the spatial infrastructure. The methodologies noted earlier define the regional geometry and its applications to the distribution and interaction of locations in space. Critical to the application in this Appendix is the identification of aggregate patterns in the construction of regions. It is through the concept of a region that the key impacts and interactions of an emergency disaster warning system may be evaluated. The region provides a spatial benchmark through which pertinent information may be categorized and later evaluated. In addition, it gives the researcher

an areal unit in which accounts of benefits and costs for the new system may be measured. It is critical that the identification of meaningful planning regions be a central issue of the temporal/spatial dimensions of a cost benefit analysis.

REGIONAL IDENTIFICATION

Regions appear to be one of the most logical ways of recording and evaluating geographic information. Geographers have defined several major categories of regions. One of the most popular typologies was developed by Whettlesey [398] and is presented in the following diagram:



The first level of regional classification is based on the number of features or phenomena considered in the definition of the region. A region is defined as a human construct in which an area of space is found to be uniform or homogeneous in relation to a feature or group of features. The notion of a human construct is important for regions are mental images and are real only to the extent of their applications. In a cost benefit application we are dealing with a potential impact area in which a multiplicity of phenomena would be of interest. This application requires that a multiple feature impact zone be defined in the initial stages of the cost benefit analysis. The degree of homogeneity and its distributional characteristics are basic considerations at the second level of the regional typology. Uniform regions are those in which the phenomena is spread over the area at approximately the same intensity. This differs from a nodal region where homogeneity is not based on intensity, but rather on orientation to a node or point.

Most human interaction systems reflect a nodal regional character in which activities are focused on a core or central area. In the nodal region intensity of activity or orientation appears to weaken with increasing distance from the core. The majority of applications in the cost benefit analysis will deal with the interface of both nodal and uniform multiple feature regions.

Uniform regions reflect the degree of overall impact and would be utilized in defining a threshold level. All areas that fall above the

threshold would represent the impact region to be considered in the analysis. Critical to the utilization of the uniform regional concept is the definition of the intensity level or threshold. The definition of risk versus non-risk areas, impact versus non-impact, fringe impact versus non-fringe impact, and filter areas versus non-filter zones are all problems of regionalization which require the identification of all contiguous locations experiencing a specific intensity of a phenomena.

Nodal regional systems relate to the orientation of activities and are probably more relevant when viewing the delivery and interaction aspects of technology. Both perceived and objectively determined regional orientations would be significant considerations in the evaluation of an early warning emergency disaster network. It may be noted that a stochastic element may exist within the regional identification problem. This is demonstrated in the probabilities and uncertainities associated with the identification of risk regions and with the orientation of interactions within nodal regions.

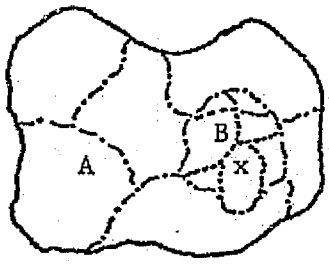
APPROACHES TO REGIONALIZATION

I. REGIONAL IDLNTIFICATION BY BOUNDARY DEFINITION:

The procedures employed in regional boundary definitions is a problem in the delineation of the limits of a phenomena and its spatial configuration. Fletcher [399] suggested that set theory could be employed in the problem of regional delineation. An example of Fletcher's approach is illustrated below:

$A \neq B$

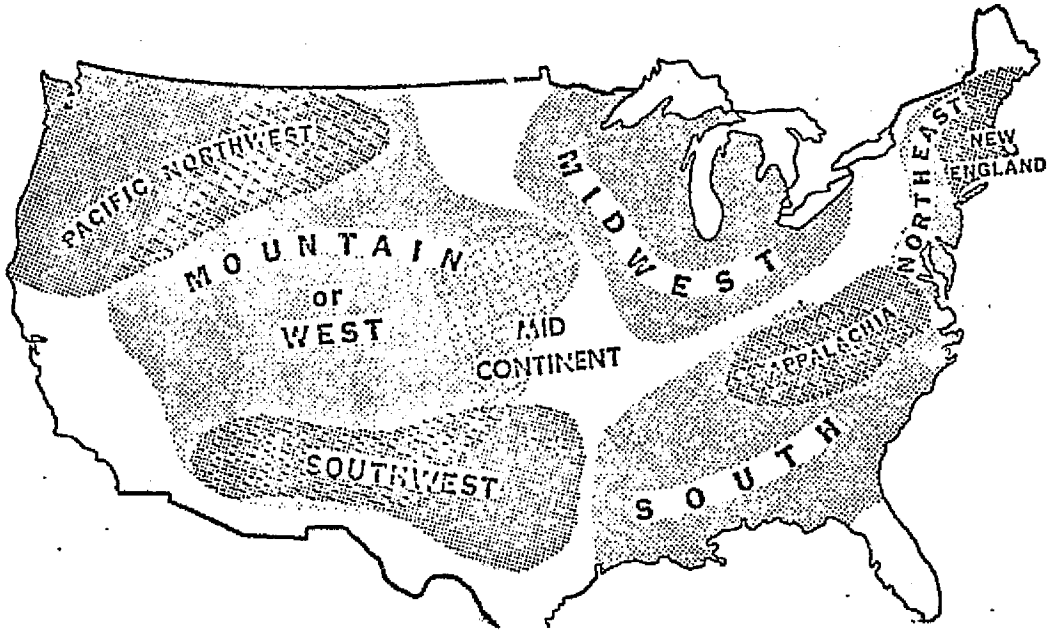
The Venn diagrams A and B reflects a collection of counties associated with two states: A and B. In the above regional format a simple locational discrimination problem between two mutually exclusive groups is defined. In the following example a set of hierarchical relationships are demonstrated:



$x \subset B; \quad B \subset A \quad \therefore \quad x \subset A$

In the diagram x represents a set of counties which are included in state B which is part of a nation A. Hagget [392] notes that this concept of set theory demonstrates that maps are a special type of Venn diagrams.

Problems of regional delineation may be solved by two major approaches: qualitative and quantitative. Qualitative approaches to regional delineation are based on intuitive decision processes. The Midwest, for example, is a regional construct that intuitively may demonstrate substatial variability in its perceived location. Several problems in the perception literature relate to qualitative delineations of regions. The map below represents a study by Cox [400] in which students were asked to define the location of specific cultural regions in the U.S.



The most serious problem of qualitative approaches to regionalization occurs in the identification of boundaries between regions. The overlaps in regions shows confusion among the respondents and indicates their failure to generate mutually exclusive regional concepts. By the superimposition of boundaries, core areas of regions are usually definable, however, a serious element of uncertainty remains in the location of boundaries.

Quantitative analysis for regional delineation appears to be more promising than qualitative attempts. Several approaches to regionalization may be found in the current geographic literature:

1. Distance - Minimization Functions

This approach utilizes methodology from the operations research literature associated with the solution of the transportation problem. Demands and supplies are allocated in a manner which minimizes the interaction costs of the following objective function:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} x_{ij} = \text{minimum}$$

Where:
d_{ij} = distance
x_{ij} = units being allocated

This type of analysis is useful in defining optimal spatial allocations of nodal regional systems. King, Muraco, and Vezner [401] provide an example of the distance minimization model in a problem associated with the regional allocation for mental health care service centers. The diagram on page 103 shows a minimal distance solution to the regionalization problem. On page 104 a modified version of the model is demonstrated with weighted linkages based on a predicted case load over space. The regional differences in the proposed catchment areas (service regions) comparing the weighted versus the distance minimization model is provided on page .

2. Discriminant Analysis

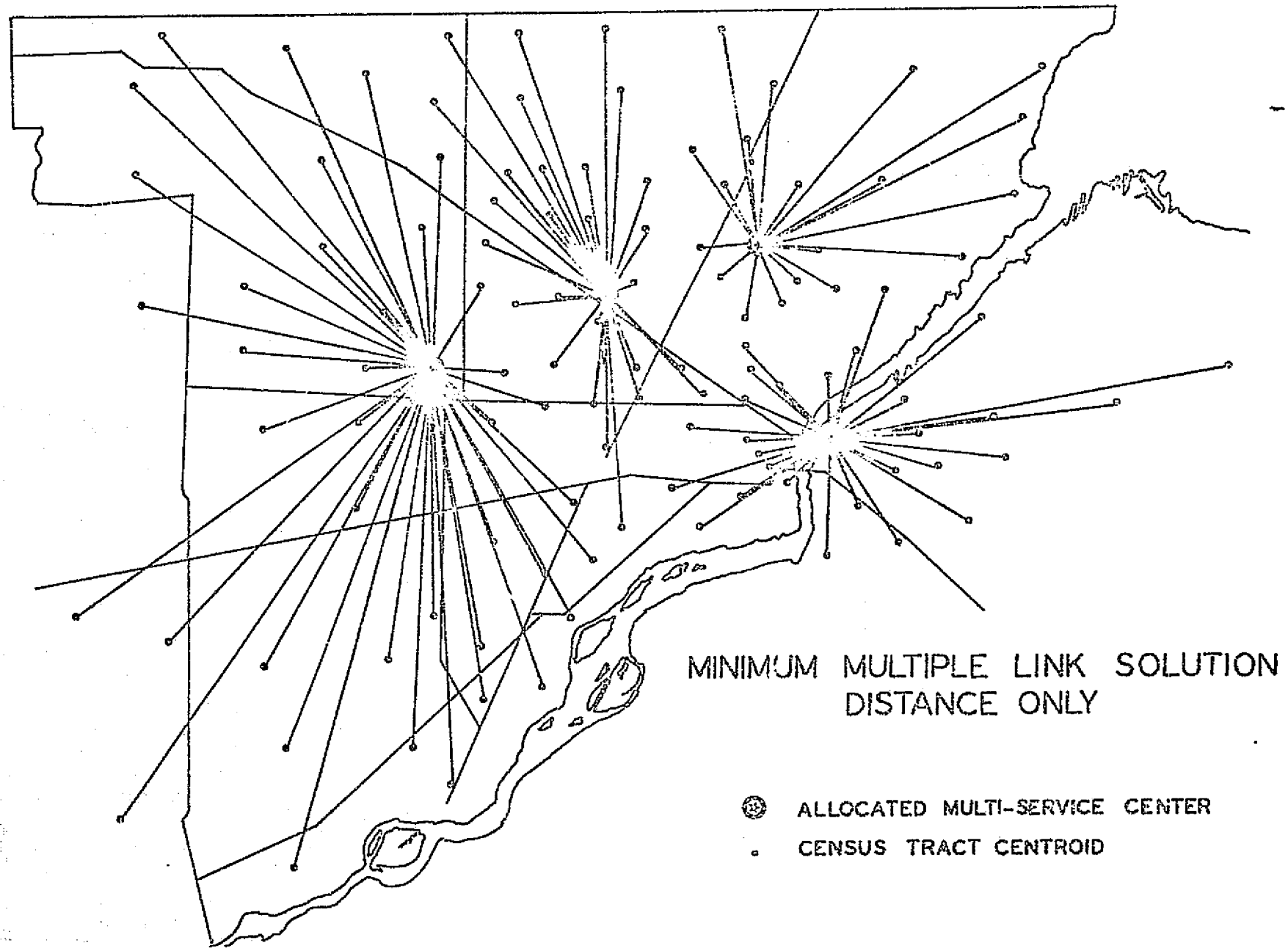
Discriminant analysis may also be used in the regional identification problem. Discriminant analysis is used to classify a set of observations which have already been classified according to some categories. The techniques associated with this form of analysis are drawn from the work of statisticians concerned with biological and anthropological data. The analysis attempts to define or classify observations which could conceivably be associated with several classes.

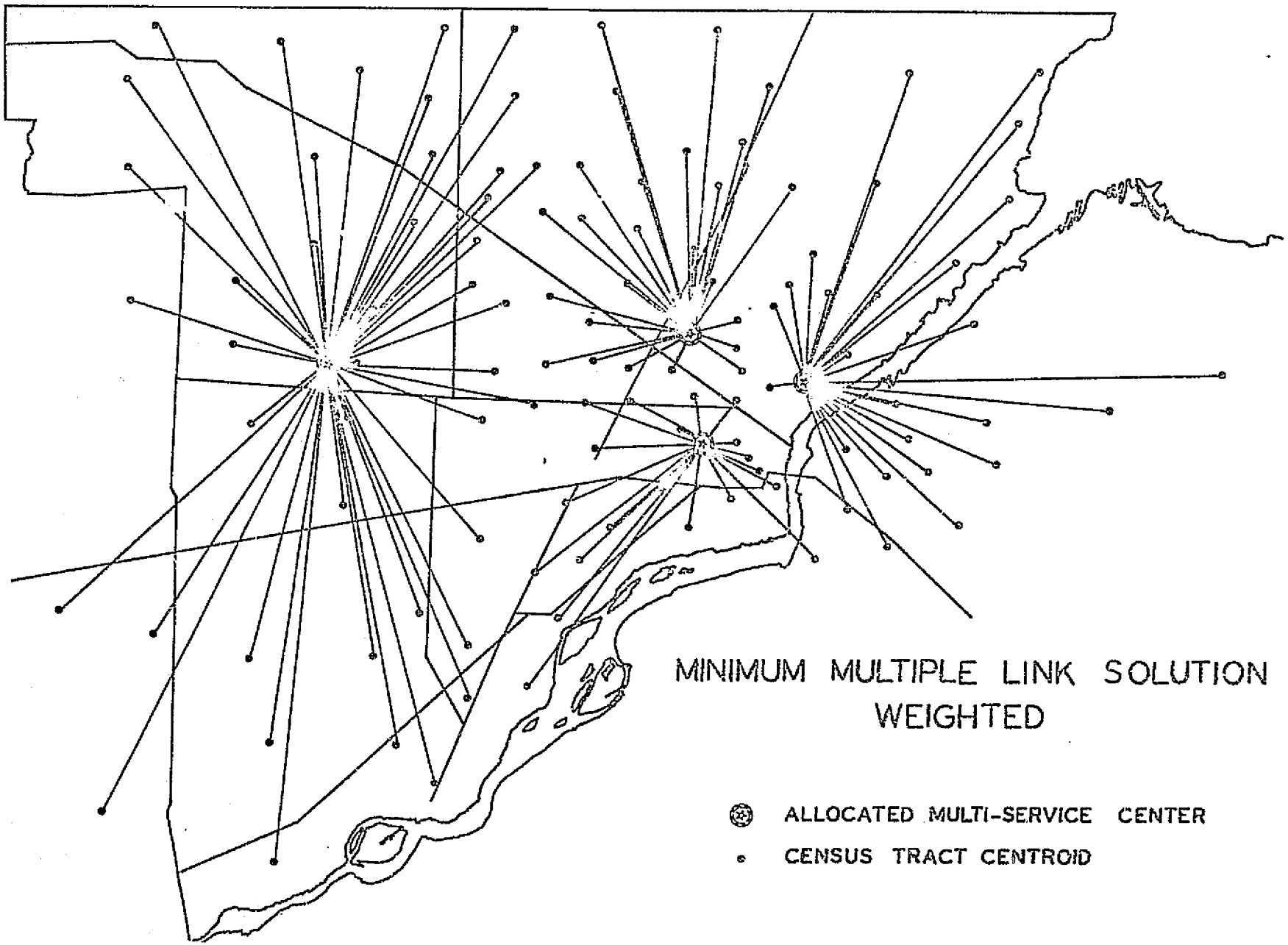
The approach rests on taking observations which may have arisen from several populations and differentiating them in relation to one another. The geographic analogy to this approach would be the identification of two core regions and an intermediate area which may be associated with one or the other of the two core areas.

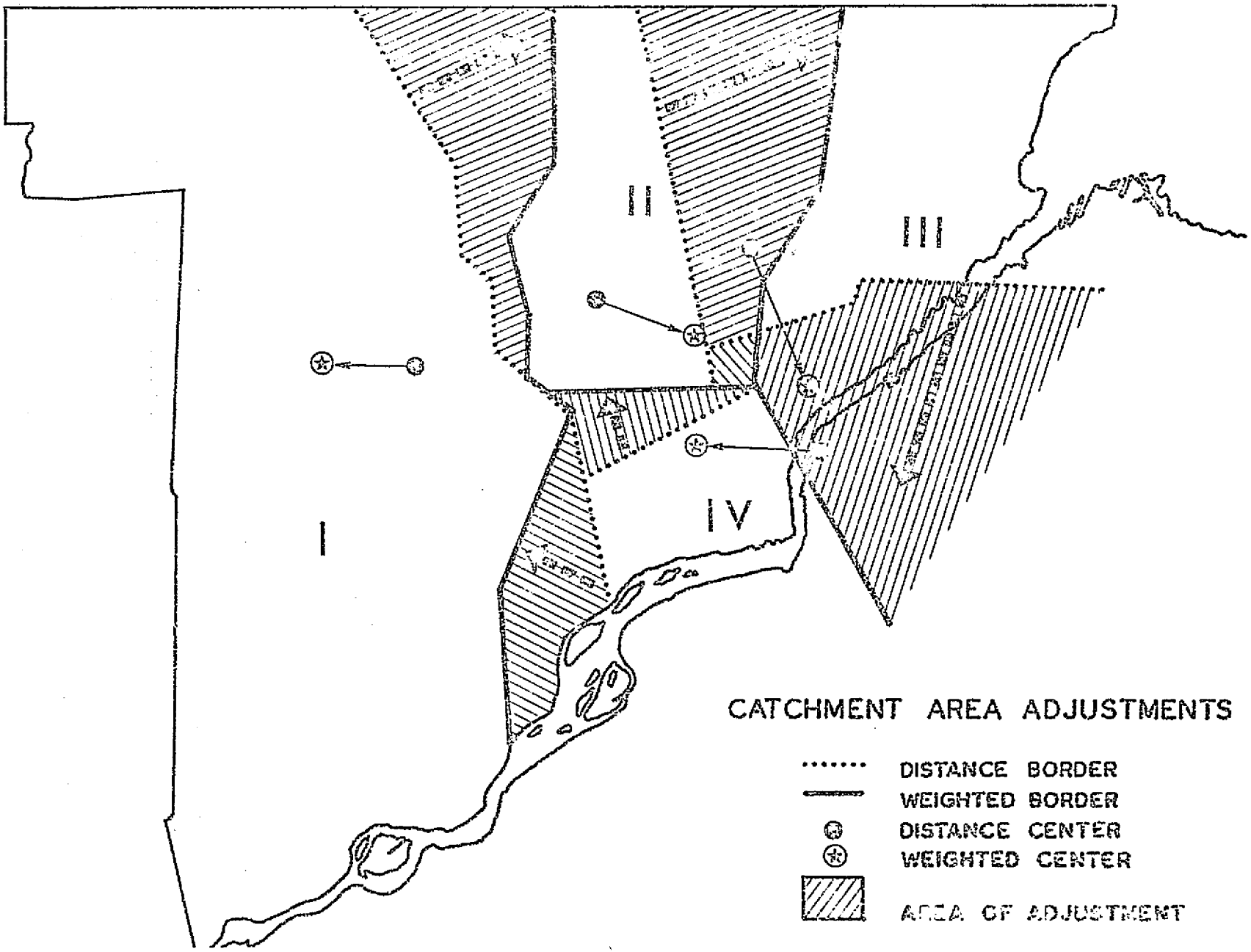
A solution to this problem is provided by the generation of a single linear discriminate function for the p variables so as to ensure maximum discrimination between the two populations while minimizing the probability of error in the assignment of new cases to one of the two populations (core areas). The more general problem may involve more than two regions and require the derivation of a multiple linear discriminant function.

King [402] notes that two major considerations underlie the utilization of discriminant analysis:

- a pre assumption that some form of classes are given.


MINIMUM MULTIPLE LINK SOLUTION
DISTANCE ONLY
ALLOCATED MULTI-SERVICE CENTER
CENSUS TRACT CENTROID


MINIMUM MULTIPLE LINK SOLUTION
WEIGHTED
ALLOCATED MULTI-SERVICE CENTER
CENSUS TRACT CENTROID


I
II
III
IV
CATCHMENT AREA ADJUSTMENTS
DISTANCE BORDER
WEIGHTED BORDER
DISTANCE CENTER
WEIGHTED CENTER
AREA OF ADJUSTMENT

- the assumption that individuals or objects to be classified belong to one or the other of the classification categories.

Fisher [414] states the problem as one of derivation of linear functions for a set of observed variables: $(X_1, X_2, X_3, \ldots\ldots X_p)$ where $Y = \lambda_1 X_1, \lambda_2 X_2, \ldots\ldots \lambda_p X_p$. The objective of the analysis is to maximize the difference of the ratio between the sample means to the standard error within the two samples.

King [402] provides an operational solution to the above problem where N observations are divided into two groups or samples of m_α and n_β members respectively. Each observation in sample α is given a Y score of (n_α/n), whereas the numbers in sample β are given the value $(-n_\alpha/N)$. According to King, these scores are arbitrary and assure that the mean of the Y scores are zero.

The discriminant function: $Y = b_1X_1 + b_2X_2 + \ldots\ldots + b_pX_p$ represents the multiple regression equation with the constant omitted. The b values in the above equation are obtained by the use of the least squares technique. The mean predicted Y score may be obtained by the utilization of $(\overline{X}_1, \overline{X}_2, \ldots\ldots \overline{X}_p)$ for each group. New individuals are then classified on the basis of their Y scores with reference to the mean values.

The frequency misclassification is defined by the use of a t-statistic:

$$t = \frac{\sqrt{Y_\alpha - Y_\beta}}{\left[\dfrac{n_\alpha n_\beta \; \Sigma b_j d_j \; (1 - \Sigma b_j d_j)}{N(N - p - 1)}\right]^{\frac{1}{2}}}$$

King suggests that the significance of the discrimination may be tested using an analysis of variance model:

$$F = (SS_T/p)/(SS_W/N-p-1)$$

Where:

SS_W = Within Sum of Square
SS_T = Total Sum of Square
p = Number of Variables
N = Number of Obs

Discriminant analysis is similar to principal components analysis in that it allows the researcher the capability of identifying the basis for the groupings. Casetti's [403] work in discriminant analysis has provided much of the basic programming for regional classification applications. His contributions are associated with multiple discriminant iterations and the means by which they are used to force classification of an optimal solution. The normative state is defined when within group variance is minimized and between group variance is maximized.

3. The Utilization of Graph Theory

The growing importance of graph theory in the study of geographic structure has resulted in the emergence of several graph oriented methods of regionalization. The majority of the graph approaches to regionalization are based on hierarchical models of interaction networks. Nystuen and Dacey's [404] study of trade flows between cities in Washington reflects the graph approach to regionalization. The following table indicates a hypothetical flow arrangement:

To centre:	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*Class*
From centre:													
a	00	75	15	20	28	02	03	02	01	20	01	00	Satellite
b	69	00	45	50	58	12	20	03	06	35	04	02	Dominant
c	05	51	00	12	40	00	06	01	03	15	00	01	Satellite
d	19	57	14	00	30	07	06	02	11	18	05	01	Satellite
e	07	40	48	26	00	07	10	02	37	39	12	06	Dominant
f	01	06	01	01	10	00	27	01	03	04	02	00	Satellite
g	02	16	03	03	13	31	00	04	18	08	03	01	Dominant
h	00	04	00	01	03	03	06	00	12	38	04	00	Satellite
i	02	28	03	06	43	04	16	12	00	98	13	01	Satellite
j	07	40	10	08	40	05	17	34	98	00	35	12	Dominant
k	01	08	02	01	18	00	06	05	12	30	00	15	Satellite
l	00	02	00	00	07	00	01	00	01	06	12	00	Satellite
Total:	113	337	141	128	290	071	118	065	202	311	091	039	
Rank order:	8	1	5	6	3	10	7	11	4	2	9	12	

Within the table four dominant trade centers are identified. Associated with each of these four are a set of satellite communities. These observations are based on the size and direction of the flows within the transaction matrix. A recent extension of the preceding approach may be found in the research of William Black [405]. Black utilizes a dyadic factor analysis to extract the basic dimensions of a set of interregional commodity flows.

The preceding approaches define regions by taking defined locations and clustering them so as to maximize or minimize basic distributional components. Distance minimization, for example, may be used to define the boundary separating two distinctive nodal regions. Through the use of discriminant analysis, components associated with the variance of clusters of observation may be utilized to obtain an optimal structure. More recently, the graph theoritic approach has provided a means of regionalization based on functional interactions between locations and the assumption that a hierarchical structure exists. The traditional non-quantitative approaches also suggest several new directions to the problem of regionalization. As the paradigm of the behavioral environment becomes a dominant research theme the measurement of cognitive regional structure becomes a central issue. Social and cultural affinities demonstrate a spatial component [406] that can only be isolated through measurement of cognitive spatial dimensions.

The problem of regional identification as treated in the preceding methodologies assumed that the researcher had complete control over the

basic areal units from which the regions would be derived. Often this is not the case and the geographer must utilize data that is already aggregated. When the data is already defined by areal units the problem of isolating regions becomes an assignment task. The researcher is forced to cluster these pre-defined areal units so as to create meaingful classifications that would represent true regions. The following section examines the approaches that may be utilized in assigning areal units to regions.

II. THE REGIONAL ASSIGNMENT PROBLEM:

In the assignment problem, regionalization becomes primarily a process of classification. Berry [407] defines four basic principles which underlie the regional assignment problem:

- every location must ultimately be placed in a region
- only under special conditions will the assignment problem not be mutually exclusive.
- regions may be defined by successive splitting of bigger regions into the smaller regions.
- larger regions will exhibit greater internal variability than the smaller regions.

Berry observes that the methodology of regionalization is based on defining the degree of similarity for each pair of places. These places are then grouped with other pairs with the objective of maximizing the similarity. The above is subjected to the constraint that the groupings of places should be contiguous.

Several methodological approaches may be utilized in the assignment problem. The major regionalization procedure utilize the following combinations of techniques:

- Factor Analysis - A multivariate approach which permits the researcher to define major components of the correlation matrix and to establish the areal structure of these components. It is from the areal distribution of the factors or dimensions that regions may be defined.
- Dimensional Analysis - This approach utilizes the preceding factor output or may work with empirically derived data to define the degree of similarity of each location by its attributes. The multidimensional scaling approaches apply this basic concept and extend it into complex and sophisticated research designs [408].

- Grouping or Cluster Analysis - It is through this technique that the observations are clustered into regions of contiguous spatial sets.

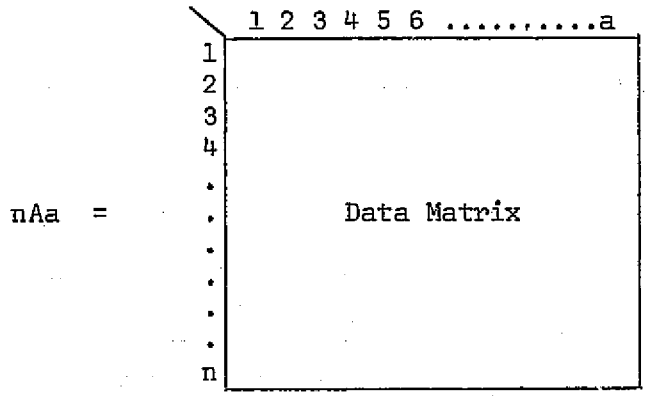
The following section of the paper will briefly examine each of the above methodologies and indicate how they may be used in a regionalization problem.

1. Factor Analysis

A variety of research designs may be defined under the general title of factor analysis. All of these models utilize the relationships that exist between sets of variates to define their underlying roots. The models take a large number of variables and attempt to define a composite set of variables that are fewer in number while minimizing the variability of the original relationships.

THE BASIC MODEL

The mathematical structure of the factor analysis problem applied to a spatial case may be demonstrated by the following data matrix:



In the above, matrix A represents an n by a matrix in which a set of attributes (variables) a and a set of observations (areal units) n and their associated elements (magnitude of a at n) are presented. The at-atribute set, a, may represent physical, environmental, or cultural qualities which are the characteristics to be regionalized. The observation set n, on the other hand, are the areal locations which may be represented as townships, counties, states, census tracts, nations, etc.

A principal axis factor model would then take the above matrix A and transform it to a matrix Z in which the original variables would be normalized:

$$n\,A\,a \longrightarrow n\,Z\,a$$

The above normalized matrix (standardized data matrix) would then be subjected to a correlation analysis to generate matrix R:

$$n\ Z\ a \dashrightarrow \text{Correlation Analysis} \dashrightarrow a\ R\ a$$

The symmetrical matrix a R a would represent the zero-order correlations among the standardized variable set. The correlation matrix would then be subjected to a principal axis factor analysis of $(R - U^2)$ in which U^2 would represent a diagonal matrix which would contain the unique variance of each of the original a variables. The result of the above produces a matrix F of order a by s in which $(R - U^2) = FF^T$ and $F^TF = \Omega$. Matrix F is a matrix of factor loadings and may be interpreted as the loading or relation of an attribute a on a generated set of factor components or dimensions s. Multiplying the transformed factor loadings matrix by F would generate Ω which is the diagonal matrix containing the eigenvalues associated with each factor. The above expresses that portion of the total common variance accounted for by each of the underlying dimensions of variation. The extracted dimensions in the orthogonal factor model are not correlated, so each dimension expresses an additive share of the original variable set. In addition, to computation of the factor matrix, a set of factor scores may be generated. The factor scores matrix n S s is computed by the following:

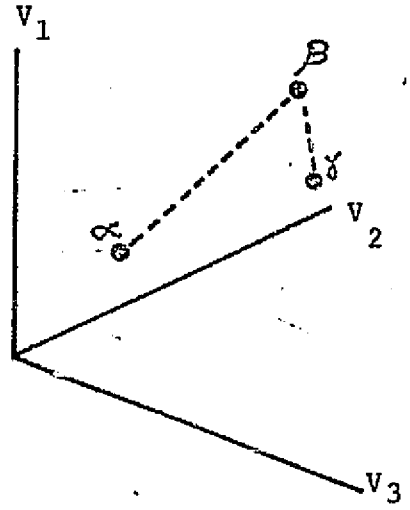
$$S = Z\ F\ \Omega^{-1}$$

The factor scores provide the weighting of the observations on the factor dimensions. The spatial distribution and areal variations of these scores may then be mapped or used to generate multivariate regions.

The preceding approach reflects an R mode factor analytic model. It is possible, however, to carry out the analysis for the rows of matrix A in which case a Q-mode analysis applies. The Q-mode analysis defines an immediate regionalization scheme in which the generated dimensions are composites of areal units. The majority of regionalizations employing factor analysis utilize the R-mode analysis and then attempt to cluster the generated factor scores. This approach permits a stronger definition of the variable structure and the rationale that would provide the base for a regionalization effort.

2. Dimensional Analysis

Dimensional analysis identifies the degree of similarity for each pair of observations by utilizing the concept of a taxonomic distance between observations. The concept of an n-dimensional space is illustrated in the following diagram:



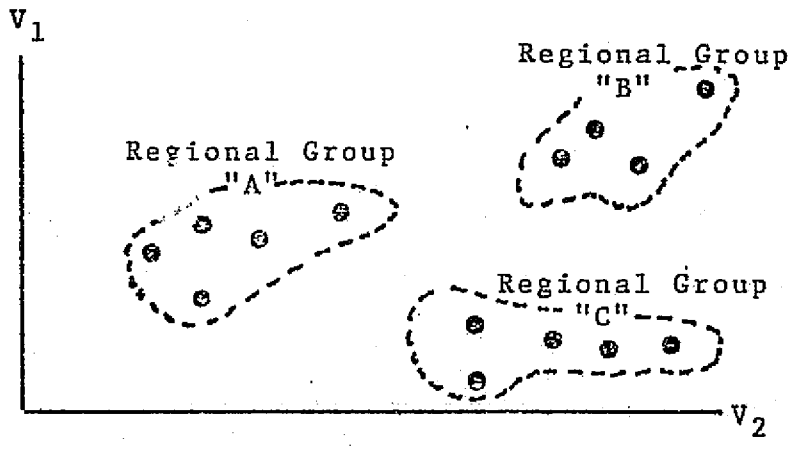
The diagram illustrates the taxonomic space for a three-dimensional area. The observations α, β, and γ which are located in that space relative to their positions on the orthogonal vectors V_1, V_2, and V_3. Distance between points in n-dimensional space follows from the utilization of the pythagorean theorm:

$$D = \sqrt{\Sigma(x_i - y_i)^2}$$

Where:

D = is the distance (similarity) between points
i = 1,2,3,.....n Dimensions
x_i & y_i = are the values of characteristics for observation i

The object of classification is simply to place in one group observations that are closer together in n dimensional space and to separate groups which are farther apart. The following two-dimensional diagram represents the above separation concept:

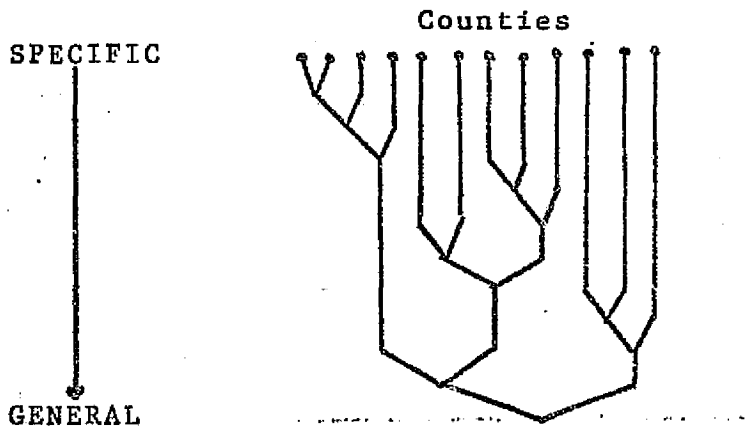


Berry [407] suggests that if the straight line distances are generated between observations it would be possible to compute a similarity matrix. This similarity matrix would then be used as an input to a cluster analysis.

3. Grouping or Cluster Analysis

Grouping or cluster analysis would take the preceding pairwise similarity matrix and through a stepwise procedure provide an optimal classification of observations. There are several grouping algorithms which may be employed to obtain an optimal classification:

- Centroid Groupings - This procedure groups with the objective of minimizing squared distances between groups. The centroid grouping algorithm is the simplest and proceeds by identifying that pair of observations in which d^2_{ij} is at a minimum. It then takes the distance matrix (D) and combines the row and column vectors of D into a single row and column vector representing the new group. The elements of these new vectors are the squared distances from the group centroid to all other points. The preceding is an iterative process with each successive stepwise procedure reducing the elements of the distance matrix until all linkages are defined. The following linkage tree demonstrates a centroid grouping technique for counties:



It may be observed that at each step those minimal distances representing the greatest taxonomic similarity are linked together. Thus, the linkage tree represents an orientation from the specific to the generalized. Through the breakdown process we gain generality at the cost of individual detail.

In the above example, the determination of the breaking points would represent levels of a regional hierarchy. Most researchers define the optimal breaks in the branching tree by noting where substantial jumps occur in the ratio of within to between group distances.

- Total Increment Grouping - This approach minimizes the increment to within group distances. At every step in the linkage analysis a check is made of total within-group distances so that each step minimizes the increment to within group distance. This approach is generally superior to the centroid algorithm for it provides a better control for the size of the final groups. It does require, however, a greater input of computer time.

- Gravity Grouping Algorithm - This approach starts in the opposite direction from the preceding approaches. It begins at the highest general level and proceeds to the specific. The method permits larger groups to annex smaller groups over longer distances. The algorithm employs the basic gravity formulation:

$$P_i \, P_j \, / \, d_{ij}^2$$

Where:

P_i = Measure of Mass for i
P_j = Measure of Mass for j
d_{ij}^2 = Distance between i and j

All of the above methods are subjected to the same problems of selection of significant cut-points to define the regional hierarchical levels.

It may be noted that a minor modification should be introduced into these algorithms. The methods just discussed, did not incorporate a linkage structure having a contiguity constraint. These approaches simply linked areal units by similarity, the end result is the derivation of clusters of areal units of high similarity. A basic construct of a region, however, is that the areal units assigned to it must be spatially contiguous. The linkage contiguity constraint may easily be introduced through the use of a binary connection matrix. The elements of the matrix that have a zero would indicate non-contiguous areas, while a one would imply that the places are joined over space. With the contiguity constraint a linkage is made only with clusters that are linked in space. When clustering proceeds under the contiguity restraint, each clustering step reduces the number of remaining parcels by one.

The preceding contiguity concept introduces an important geographic component to the problem of regionalization. Haggett [392] notes that a large share of the regionalization literature is not truly geographic in that it merely classifies geographical data not locations. The concept of contiguity forces the researcher to carry on a "progressive comparison" of each unit with its neighbors. Haggett defines two major approaches that have been reasonably successful in accomplishing the above:

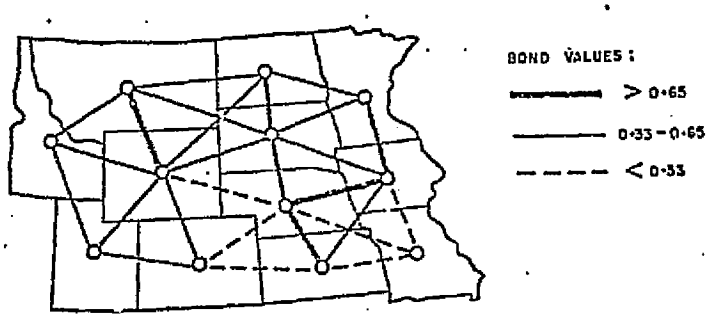
Variance Analysis - Drawing from the work of Zobler [409] regions may be constructed from smaller units by defining their components of variance. Zobler suggests that there is variation among areal units within a region (within-region variation) and that there is variation among the regions (between-region variation). Zobler uses variance analysis to assign West Virginia to three major regions with the following results:

Variance Ratio

	Between Regional Variance (V_b)	Within Regional Variation (V_w)	Variance Ratio V_b/V_w
Alternative Assignments:			
To Mid Atlantic	46.09	8.91	5.17
To South Atlantic	71.55	4.66	15.35
To East-South Central	72.13	4.57	15.78

In the table the between-regional variance demonstrates the variation of regions around the grand mean for all regions. It may be noted that substantial differences are observed between the Mid-Atlantic Group and the two southern regions. The within-regional variance, on the other hand, shows the variations of the states around their respective regional means. The F-ratio generated from the preceding reflects the interregional differential, or in other words, how successful the grouping procedure has functioned. In this case, the optimum allocation for West Virgina would be in the East-South Central region. The high variance ratio indicates maximal between group variation compared to within regional variability.

Correlation Analysis - A second approach is represented by a correlation analysis which denotes a measure of spatial contiguity by defining the strength of functional ties between areal units. Hagood [410,411] utilizes this approach in dividing the United States into contiguous groups of states with the objective that states in each group show a high degree of homogeneity in relation to agricultural and population attributes. Based on this data agricultural and population profiles were generated and then standardized. Correlations were then computed between the profiles of adjacent states. The correlations exhibited the nature of "regional bonds" and were portrayed by lines of varying width on a map:



Hagood later used the correlation linkages in a factor analytic model to generate a single regional index. The preceding map indicates those states that are strongly linked as opposed to those that exhibit relatively weak bonds.

In summary, the problem of regionalization is a basic issue in a cost-benefit analysis. Several methods of approaching the problem from both an identification and an assignment orientation have been illustrated. The choice of methods is dependent on the purpose, type of application, size and configuration of the areal base units, and the attributes or variables to be employed in the regionalization. When the above are utilized in conjunction with the earlier discussed methodologies the structure of the spatial dimensions of a cost-benefit analysis may be derived.

CONCLUSIONS

The function of this paper has been to present a compendium of methodologies that measure the spatial component of events. In general, the paper has been devoid of direct applications. In this final section, the interactions of the previously mentioned methodologies will be placed in the context of a cost-benefit application for an early warning satellite system. It will not be the purpose of the appendix to actually measure the spatial components of the above technology. Its function is directed toward demonstrating which methodologies should be utilized and at what stage they should be introduced into the analysis.

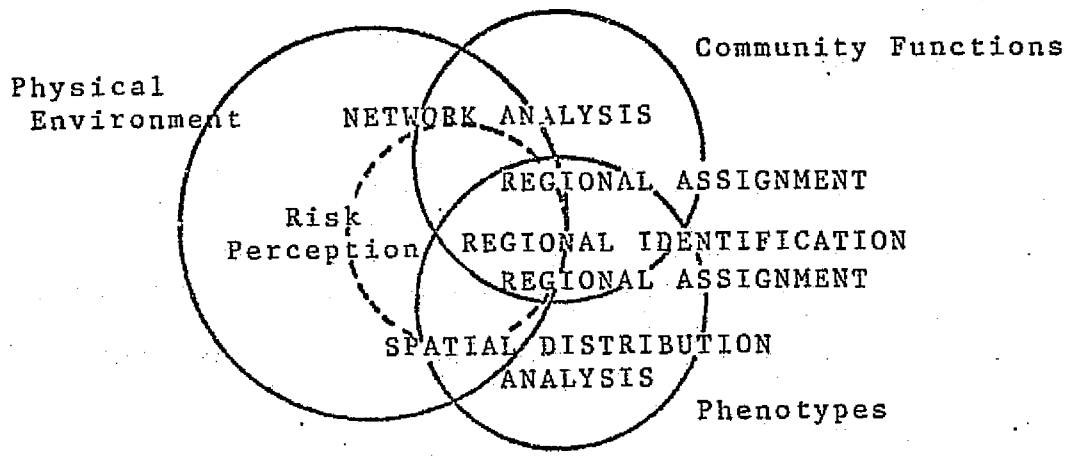
The utilization of a spatial/temporal orientation to events will provide the infrastructure for the methodological review. In an earlier paper Muraco [412] has shown the events leading to and following a natural disaster may be placed in the context of a temporal frame over space. Multiple events characterized each stage of the pre and post disaster experience continuum. At each stage in the sequence various spatial components appeared. The accurate measurement of these components is critical both in the evaluation of the disaster itself and in the assessment of technology that may alter the event sequences.

Pre-Event Phase - In the pre-event stages the major emphasis was placed on the assessment of cultural and physical adoptions, primary community

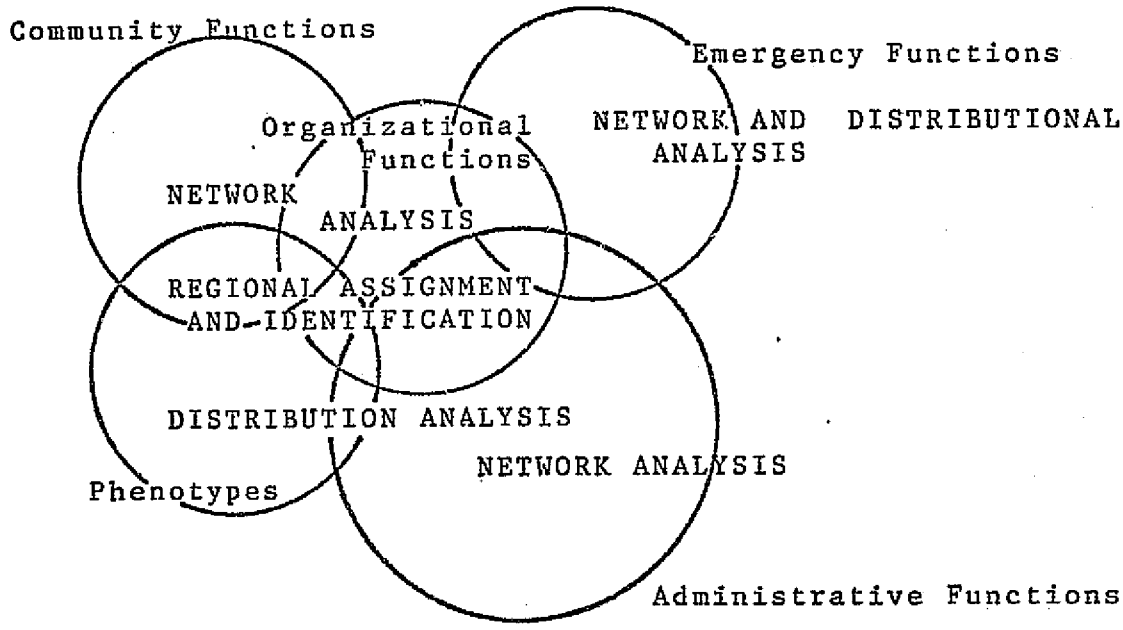
functions, risk and hazard perception and behavior, organizational structure, and the general characteristics of the existing communications system. The interrelationship of these various functional elements as noted by Muraco [412] results in a utility level that is specific in space and time. A change in the utility level, introduced by the adoption of new technology would imply the generation of new costs and benefits to the system. It is essential that the spatial components of these costs and benefits be defined.

In the pre-event stages the cultural and physical setting must be isolated. In a spatial context this would require a definition of the settlement pattern and the nature of functional relationships between settlements. The methodologies that would be employed in this phase of the analysis would be associated with nodal dispersion and network analysis. The intensity of the settlement pattern and its degree of connectivity are important components of the regional infrastructure. The integration of the preceding with the functional linkages would establish the nature of the settlement hierarchy and the general geographic orientation of the area. This establishes the physical-functional setting for the population.

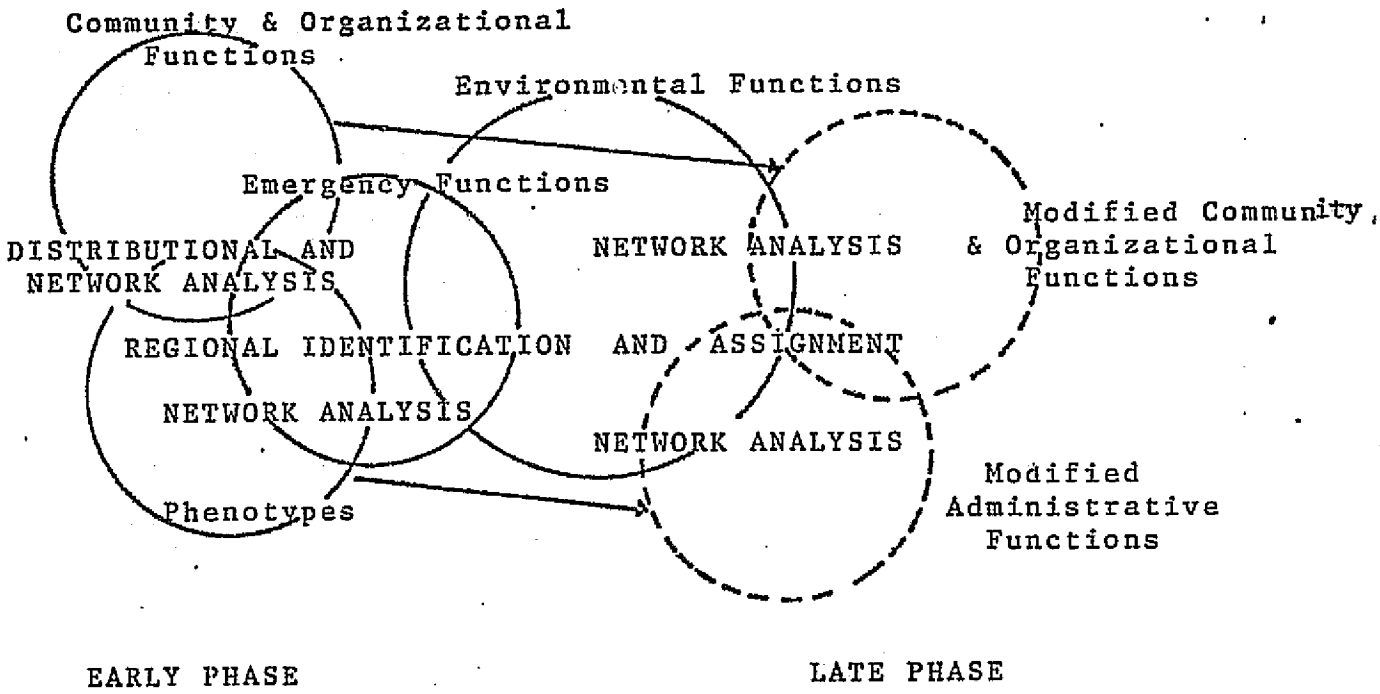
An essential aspect of the pre-event sequence phase is associated with the identification of risks and the dissemination of emergency warnings. This phase of the analysis would utilize the previous methodologies coupled with attempts at multidimensional regionalization. The identification and classification of cultural areas would employ a regional assignment methodology. It is through this approach that the phenotypic composition and distributional character for the area may be differentiated. Based on the spatial distributions of the phenotypes, new behavioral regions may be defined by using a regional identification methodology. These regions would then be employed in isolating perceived risk and non-risk areas, as well as, predicting the phenotypic implications of the perceptions. The following diagrammatically illustrates the preceding:



The Event Phase - The event stage would also utilize a composite of the previously discussed methodologies. During the event, problems of circulation and the identification of major impact regions would be central spatial considerations. The identification of potential bottlenecks and barriers would necessitate a network and flow analysis. Problems of regionalization would also be central issues during this stage. The identification of the location and extent of the event over space are essential elements in measuring the primary costs and the anticipated secondary costs. Much of the information generated during the event phase must be coupled with the earlier distributional analysis of the regional infrastructure. If the event, has a serious impact on that infrastructure, its effect on total utility will have substantial consequences in both time and space:



Post-Event Phase - The post-event stage would also require the inclusion of spatial components. During the early portions of this phase regional assessments of damage and emergency evacuation would be major activities. The derivation of damage regions to concentrate efforts for recovery would be illustrative of a multivariate regional identification problem. The interface of the damage regions against the cultural regions defined in the pre-event stages may represent a means of establishing the distributional aspects of the disaster on various phenotypic groups. In the latter phases of this stage, adjustments of a normative nature may take place in coping with the reoccurrence of the event. The development of optimal interaction networks and administrative regions would be critical considerations in assessing the existing technologies efficiency:



The evaluation of the effectiveness of emergency and administrative functions requires the identification of optimal hinterlands that would minimize barrier effects. The organizational delivery system would be assessed, in part, by its propensity to interact with the impact regions. To identify these interactions requires a comprehensive analysis of the functional networks over space.

The preceding temporal sequences illustrated the methodologies reviewed in this paper within the context of evaluating an event such as a natural disaster. In a technology assessment problem the changes that the technology introduces to the sequences over space would become the central issue. In the event and post-event phases, for example, substantial changes in geographic orientation and interaction may be introduced by an early warning communications technology. Knowledge of the extent, and intensity of these changes over space would be required in assessing the true benefits and costs derived from the innovation.

SUMMARY

In the following table the general application of the methodologies discussed in this paper are displayed:

SPATIAL METHODOLOGIES	APPLICATIONS				
	Diffusion of an Event	Intensity of an Event	Interactions of an Event	Definition of Events Impact	Optimal Response Planning
NODAL ANALYSIS	O		O		O
Shape	O		O		O
Pattern	O	O			O
Dispersion	O	O			O
NETWORK ANALYSIS	O		O		O
Branching	O		O		O
Circuits	O		O		O
Barriers	O		O		O
REGIONALIZATION	O	O		O	O
Boundary Identification		O		O	O
Areal Assignments		O		O	

The table provides a comparison of the different methodologies and their potential applications. The circles in the body of the table suggests under which applications a methodology is suitable. In all of the sample applications multiple methodologies would be required to insure adequate spatial coverage.

In summary, it is the purpose of this ppendix to provide a compendium of geographic methodologies that would have applicability in a cost-benefit problem. The ppendix is largely educational in structure and designed to provide the reader with a background in the techniques geographers employ in the analysis of events over space. The operationalization of these methodologies in a cost-benefit problem is demonstrated in an earlier section of this report.

APPENDIX I

Multidimensional Scaling

This Appendix deals with multidimensional scaling. Multidimensional scaling (MDS) is a generic term that identifies several techniques for obtaining metric, cardinal information from a matrix of ordinal-level rankings. It has been described as a response to "the problem of representing n objects geometrically by n points, so that the interpoint distances correspond in some sense to experimental dissimilarities between objects" [345]. Thus, MDS is concerned with obtaining a satisfactory spatial representation of relationships among stimuli or objects. Usually no tests of significance are computed, although such statistics as STRESS or SQUARIANCE are used to uncover the appropriate final configuration. The points are clustered in multidimensional space so that axes may be inserted. Measurements made along them can describe the relationship of specific points to the several latent dimensions [346].

The purpose of this Appendix is to present the basics of the MDS methodology. Consideration is given to the basic principles of MDS, to the common properties of MDS computer algorithms, and to the methodological limitations of the methods.

PRINCIPLES OF MULTIDIMENSIONAL SCALING

One reason that has been advanced for the lack of empirical theory in the behavioral scienes is that they lack the capability for precise measurement. The importance of this assertion has diminished with the development of many non-parametric tests and measures during the past twenty years.

Simultaneously with these advances in ordinal non-parametric statistics, the behavioral sciences have benefited from developments in the field of cumulative scaling. Table 1 divides attitudinal, behavioral and cognitive scales into direct and indirect types. In direct scaling, respondents are asked to "rank themselves" (ordinal scales) or to locate themselves along a continuum (some interval scale procedures). Experiments also have been made to establish self-rating at the ratio level of measurement. For a variety of reasons, direct ratings are problematic. Preferred alternative scaling approaches involve some third person to do the rating. What this means is that it is deceptively easy to claim that a reliable rank-ordering (or a reliable cardinal measurement) has been reached. Actually, even ordinal measurements are more difficult to attain than it might appear.

Cumulative scales can check upon the truth of ordinality, but they also can be used to generate ordinality from simple nominal information. A cumulative scale assigns magnitudes to entities in relation to the perceived amount of a common attribute that each possesses. That is, it assesses behavior by combining several similar variables into a single composite indicator. It assigns scale scores along a single continuum or "dimension", and these scores constitute an ordinal or higher level of measurement. A wide variety of scales have been constructed, and measurements have been attempted of preferences, of hostility, job prestige,

TABLE 1

APPROACHES TO SCALING

	DIRECT SELF-RATINGS	INDIRECT THIRD-PARTY RATINGS
ORDINAL	CONCORDANCE SCALES	GUTTMAN-TYPE SCALES
INTERVAL	CATEGORY SCALES	THURSTONE SCALES LIKERT SCALES SEMANTIC DIFFERENTIAL
RATIO	CROSS-MODALITY SCALES	EXPERIMENTAL PSYCHOPHYSICS

psychomotor dexterity, friendship, and of untold other concepts. At the ordinal level, the work of Louis Guttman has been very important; Likert and Thurstone scales are representative of work toward interval levels of measurement. Psychometricians have been laboring during the past few years at attitude measurement at even the ratio level--at developing measures with a true zero point.

Table 2 depicts a typical Guttman-type scalogram situation. Simple "yes" and "no" responses (nominal dichotomies) have been entered in accordance with the types of storm damage suffered by the residents of forty-four counties. The categories of loss have been ordered according to their degree of "hardness" as represented by the four marginal totals. The number of counties experiencing each level of loss is specified under N, and the scale score is indicated at the right. Of course, as with all ordinal measures, the interval between scale scores 3 and 4 may bear no relationship to the one between 2 and 3. Only an interval (or ratio) level of measurement is concerned with inter-class magnitudes.

Multidimensional scaling can be understood best in relation to unidimensional scale analysis. In the preceding example, the use of model data for illustrative purposes allows statistical tests of unidimensionality. The units that are scaled may be almost anything--survey respondents, governmental units, verbal concepts--and when found to be unidimensional they can be ranked in relation to the degree that they are perceived to possess an attribute, X.

TABLE 2

MODEL GUTTMAN-TYPE SCALOGRAM

Counties Reporting (N = 44)	Types of Damage Reported				Scale Scores
	Rain Damage	Housing Losses	Disease Outbreaks	Deaths	
15	No	No	No	No	1
12	Yes	No	No	No	2
10	Yes	Yes	No	No	3
5	Yes	Yes	Yes	No	4
2	Yes	Yes	Yes	Yes	5

MULTIDIMENSIONAL SCALING

Multidimensional scaling assumes that objects to be scaled may possess more than a single common attribute, or that Attribute A can be broken down into several sub-attributes. Multidimensional analysis transforms the threat of non-unidimensionality that plagues Guttman-type scales from a debt into an asset. Using multidimensional scaling, for example, it can be demonstrated that color is a composite of three attributes: hue, brightness and saturation. Multidimensional scaling is concerned, then, with organizing the O_i objects so that they can be arranged along two dimensions when a pair of scales exist among data, along three dimensions when three underlying scales are present, or up to ($\underline{n}$ - 1) dimensions when $\underline{n}$ objects are to be scaled multidimensionally. Figure I-1 assumes the existence of three dimensions among a set of hypothetical data bearing upon the "Agnes" hurricane disaster of 1972. It should be pointed out that not only may solutions be sought in even higher dimensionalities, but that they may be found in non-Euclidean space as well.

Figure I-1 shows that objects can be arranged in three-dimensional space. The "objects" indicated here might be units (such as persons who have responded to a series of survey questions) or they could be stimuli (the survey questions themselves). Again, they might be units or stimuli acquired from one of the many non-survey data gathering methods. The model depicted here identifies the first dimension as financial loss incurred from the Hurricane Agnes disaster. Those who suffered more heavily in a purely financial way would be located very close to this axis, and would be scaled high on the concept. Those who actually benefited finan-

cially from "Agnes"--from selling supplies or from profitting on new construction to replace damaged structures--also would be on this dimension but would approaching the value of -1. If all of the n objects were scaled unidimensionally, such tests as coefficient of reproducability, coefficient of scalability and minimum marginal reproducability all would be low because it is apparent from the configuration in Figure I-1 that more than a single dimension is present among the points. Dimension II, the vertical axis, might represent mental stress. The third axis could record the deliterous impact of the hurricane upon personal health. Because a number of points remain at the centroid of the configuration, however, it may be that additional dimensions are required to extract additional explanations from the data. The expectation of higher dimensionality also would come from a more general scattering of points within the cubic space or from any peculiar clusters that are extant within it. Unacceptably high levels of multidimensional scalability (e.g., STRESS, SQUARIANCE) also call for a search at higher dimensionality or in some other Minkowski-p geometric space.

Although the roots of MDS extend into factor analytic concepts and include work in matrix analysis and psychophysics [355,356] its independent development appears to owe more to the innovations of Torgerson and of Coombs during the 1950s. The classic formulation was detailed in two 1962 papers by Shepard, however, which showed that tightly constrained geometric plots of points can be constructed from only rank-ordered information about relationships among a set of n objects. Shepard stressed the need for a monotone relationship between experimental dissimilarities (represented conventionally by the lower-case letter delta--δ) and the distances among points (or d) in the configuration scattergram shown in Figure I-4 (c). Shepard demonstrated that a matrix expressing ordinal relationships among objects could be used to group the objects themselves, and that interval-scale axes could be inserted among the points so as to yield metric outcomes from only ordered inputs.

The first few years after Shepard's paper saw a number of advances in multidimensional scaling. In 1964, Kruskal [345,347] improved upon Shepard's approach by using a measure of "badness of fit" to assist in coverging the distances among the points with the given matrix of dissimilarity orders. In Kruskal's approach, a monotonic regression of distance is preformed on dissimilarity, and the normalized residual variance --STRESS-- is used as a measure of how poorly the point configurations reflect the rankings among the data. Other approaches to MDS followed in short order: Guttman-Lingoes' [348] smallest space analysis (SSA); Torgerson and Young's TORSCA series of computer algorithms, Carroll and Chang's INDSCAL [357] and PROFIT [358] routines, and a number of others. Although these conceptualizations vary in approach, in numerical operations, in size capabilities and even in their ultimate goal, a common model and explanation of the operation of most MDS computing procedures can be offered.

Although the objective of all MDS routines is the spatial representation in ratio-scale of simple ordered relationships among n objects, or the description of ordinal data at a cardinal level, quite a large

number of computing algorithms have been developed to accomplish this task. They vary with regard to (1) the types of associational matrices that may be input, (2) methods of identifying an initial configuration from which the ordinal scaling may begin, (3) types of statistical guides or tests that are used in reaching a solution, (4) methods of "jiggling" the points during the scaling process, (5) types of Minkowski-p spaces in which findings can be reported, and (6) computing efficiency.

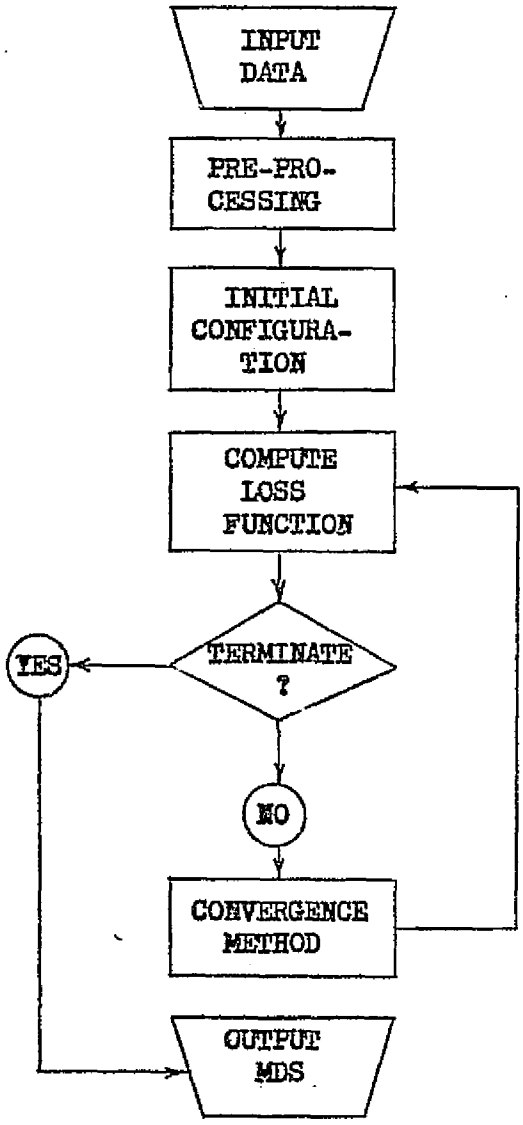
All MDS programs include the steps identified in Figure I-2. They begin from a set of input data that represent similarities or differences in attitudes, behaviors or cognitions. Sometimes extant as an integral part of the algorithm, sometimes supplied externally for it, often there is a routine for pre-processing the data that allows the special treatment of tied or missing cases or for reducing the data from a cardinal to a continuous-ordinal level. Third, some initial configuration of data points is proposed through either randomization or a purposeful assignment process so that the actual clustering might begin from this point. Fourth, the difference between the rank-order of interpoint distances and the extant point configuration is ascertained through computing a "loss function". Whenever the configuration is found to reproduce acceptably the ordinal structure of the data, it could be decided to terminate further iterative "jigglings" of points and to output the present result. Failure to have passed the termination test or to have exhausted the number of iterations called for in the solution (e.g., 25 iterations) again activates the convergence method for carefully moving the configuration of points so that they match with the given input matrix of rankings as nearly as possible. Consider each of these steps more closely.

1. Input Data.

Any type of raw data that satisfy the requirements of a data matrix concept and that can be measured at the ordinal level or higher can be used for input to any MDS algorithm. Many of the early applications of scaling were drawn from the fields of psychology and marketing. Perhaps because the extensive use of "pair comparison" techniques similar to those discussed in operationalizing T.A.P. as a basis of constructing rank-orderings has been common to these areas, many publications involving MDS use this approach. In addition, the focus on identifying dissimilarities among brands of automobiles, foods or colors might lead one to believe that the potential uses of MDS are limited. As Kruskal [345] pointed out a decade ago, however, the techniques can work with measures of similarity or dissimilarity, with correlation/association coefficients, and with information of a "most diverse kind". These data might be gathered in any of a number of ways. They might be "discrete" or "aggregate" sources; they may be exante or post-hoc indicators of attitude, behavior or cognition. Although there are conditions under which the data themselves can be analyzed directly, it is customary to correlate the columns of a raw data matrix (i.e., the stimuli or variables) with one another to obtain a matrix of stimuli (the R-technique of factor analysis). Where interest centers upon the actions of the persons or other units of analysis in the data matrix, however, a resulting matrix

FIGURE I-2

TYPICAL MULTIDIMENSIONAL SCALING FLOWCHART



of units can be constructed by associating the rows with one another (the Q-technique of factor analysis). Six types of resulting or secondary matrices ordinarily are encountered at inputs to MDS algorithms. They are shown in Figure I-3.

The most common input matrix is the symmetric triangular one shown in Figure I-3 (a). In this case, raw data have been associated into n(n-1)/2 unique combinations. Because the main diagonal records the association of variables with themselves and the reflexive coefficient always is unity (1.0), it is usually omitted from further analysis. Symmetric triangular matrices result from correlating all possible pairs of stimuli with one another, or from associating all pairs of units with some sort of coefficient of agreement.

Figure I-3 (b) is a non-symmetric matrix with the main diagonal again being missing. In this case, one or more of the entries in the lower left-hand segment of the matrix will be dissimilar from its analog in the upper right corner. If the coefficients entered in matrices such as these are indicated as $\underline{r}_{ij}$, then $\underline{r}_{ij} \neq \underline{r}_{ji}$. This is a very realistic type of data representation, for it represents the myriad situations in which interactions are not perfectly reciprocal. One individual may think more highly of a second person in a social setting than the latter does of the former. The availability of a scarce raw material will have a stronger impact upon the behavior or a manufacturer in a purely competitive market than the activity of that manufacturer will have upon the raw material supply. The input-output models of Leonteif exemplify this sort of asymmetry, and the development of parametric and non-parametric coefficients of similarity or dissimilarity (e.g., asymmetric uncertain coefficients from information theory; $\underline{\text{lambda}}_a$; Somer's$_{dx}$ and Somer's$_{xd}$) enable these relationships to be summarized neatly.

Figure I-3 (c) depicts an asymmetric matrix in which the main diagonal does not contain unities. Such a square matrix can result when the context in which the stimulus is encountered varies, or because of some inability of respondents to identify a concept as being identical with itself. Again, the substitution of communality estimates in the main diagonal for principal components factoring exemplifies the use of a square matrix.

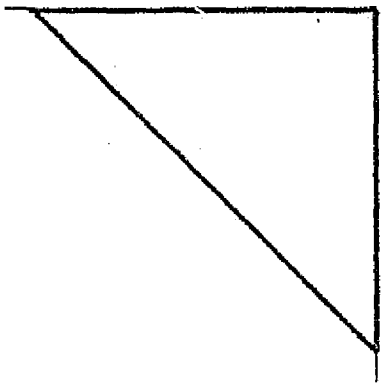
Other types of input matrices are possible as well. Conditional matrices can be described by comparing the row or columnar entries (Figures I-3 (d) and I-3 (e)--something that yields a rectangular effect. Again, Green and Carmone [349] have described partitioned matrices in which unconditional, symmetric relationships are obtained for a core set of units or stimuli, and the other interrelationships are obtained on the basis of their interaction with the respective core set entries.

2. Preprocessing.

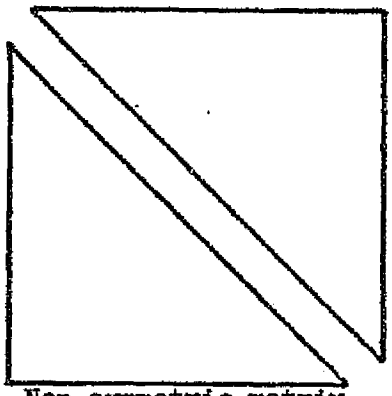
When data about to be scaled multidimensionally are expressed in interval or some other form than that of a strict rank-ordering, they

FIGURE I-3

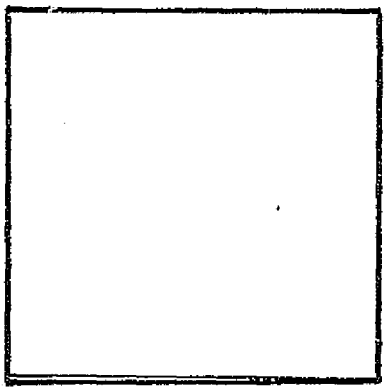
MODEL INPUT MATRICES



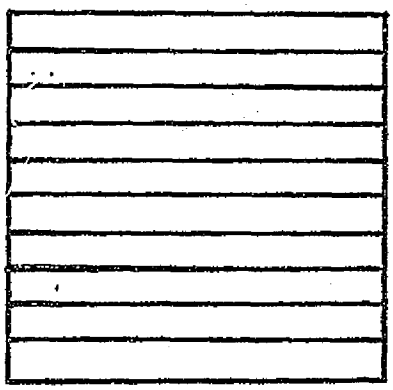
(a) Symmetric triangular matrix



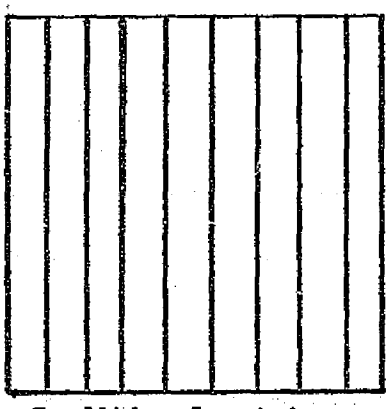
(b) Non-symmetric matrix with missing main diagonal



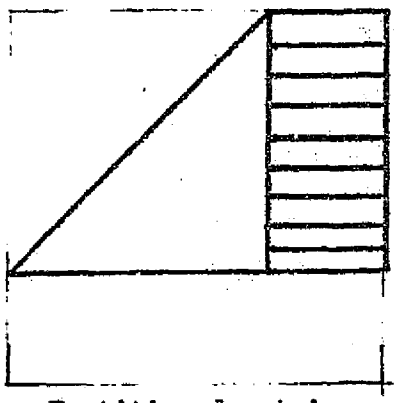
(c) Square matrix with diagonal present



(d) Conditional matrix--intra-row comparability



(e) Conditional matrix--intra-column comparability



(f) Partitioned matrix

must be converted into an ordered matrix before the actual scaling can begin. During the early development of MDS this sometimes was done by an external preprocessing routine. One of these, TRICON [350], constructed an ordinal similarity matrix of up to only fifteen stimuli. Another, WAGS [351], was designed to accomplish this with data encoded into zero-one dichotomies. The use of such small quantities of objects for scaling would impose serious limitations on research. Fortunately, such MDS packages as M-D-SCAL, TORSCA and SSA allow the input of as many as eighty stimuli and do the necessary rank-ordering automatically.

The procedures for preparing non-ordered data into strictly ordered matrices have a common goal, although they do vary with regard to their treatment of intransitives, of missing or tied data, and of asymmetric information. Table 3 provides an example of how this ranking process is done. A matrix of indexes of association is supplied in this case. These happen to be phi coefficients, or coefficients that approximate Pearson's r when x and y are dichotomies. Matrix (b) shows that the information has been reduced to a ranking. Of course, there might be little or great similarity of appearance between the coefficients with which we begin and the ordered matrix with which we conclude. The degree of similarity will depend upon both the data with which we work and the coefficients used to summarize them. In this example, the median phi value among these 45 coefficients is .09. Were these initial coefficients expressed as Yule's Q's, however, the twenty-third (or median) value would have been .25, given the same data configuration.

TABLE 3

CONVERSION TO RANK-ORDERINGS

(a) Phi Coefficients

	299	021	058	278	242	065	270	157	359
		211	171	094	122	121	134	053	166
			062	017	090	078	237	085	068
				106	024	312	012	103	053
					035	077	298	126	161
						024	010	058	154
							033	013	022
								057	260
									034

(b) Rank-Ordering

	3	41	30 $\frac{1}{2}$	5	8	28	6	14	1
		10	11	22	18	19	16	33 $\frac{1}{2}$	12
			29	42	23	25	9	24	27
				20	38 $\frac{1}{2}$	2	44	21	33 $\frac{1}{2}$
					35	26	4	17	13
						38 $\frac{1}{2}$	45	30 $\frac{1}{2}$	15
							37	43	40
								32	7
									36

3. Initial Configuration.

Spurious outcomes can result from MDS techniques under certain conditions. One problem is that of the degenerate solution--one in which the goodness of fit between the scaled points is unacceptable but where STRESS approaches zero. It can occur when the object points cumulate in clusters so that within-cluster dissimilarities are much less than those between clusters, or when all individuals polled have chosen one stimulus in preference to all other possibilities. Therefore, quite different final solutions are obtained when the same data are re-analyzed. (This is not the same as the local minima difficulty. There, the step value that regulates the distance that points are to be moved on the next iteration is too small. This means that points can be trapped in locations that are not optimal solutions. The appropriate remedy in this case is to increase the step values).

It has been found that the choice of an initial configuration of points that is as near to the likely final MDS solution as possible minimizes the possibility of degeneracy. If, when the "jiggling" of points begins, a correct solution is close at hand, the likelihood of a degenerate result is greatly reduced. MDS algorithms exhibit a variety of techniques for selecting initial configurations. Although points might be input randomly, most users today prefer some type of non-random initial configuration. M-D-SCAL allows an input configuration of one's choosing, such as a square, circle, or other shape. Metric inputs also are used, as with the TORSCA series. Here, a semi-metric configuration is prepared by converting the data to scalar products that are factor analyzed to yield a preselected set of r dimensions. In TORSCA-9, five cycles of this semi-metric iterative process normally are repeated prior to the actual non-metric analysis. Often the initial configuration is very close to the final outcome.

4. Converging the Data and the Object Points.

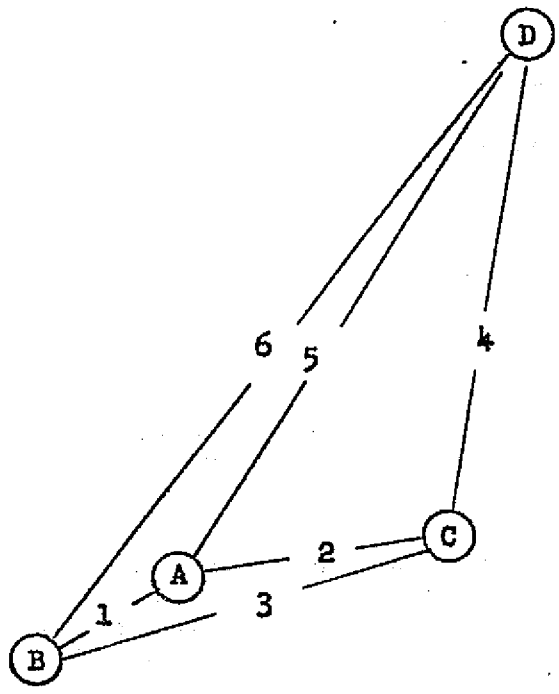
Given an initial configuration of points, the fundamental problem of MDS is to move them in such a way that the distances between them best reproduces the order of entries in the strictly-ordered matrix. Early in the development of multidimensional scaling, solutions were sought by placing pins in a board, measuring the distances among them, comparing the rankings of these distances with the ordered matrix, moving the pins some more, and repeating these steps until the fit between pins and data were compared with the ranks of the inter-pin distances, and not with the actual lengths that separated them.

Figure I-4 shows the interrelationship among ranked data, a configuration of points, and a scattergram that is used for calculating a measure of STRESS useable in re-configuring the scaled objects. Let the interpoint distances between objects i and j be d_{ij}, and the corresponding ordering in the matrix be δ_{ij}.

FIGURE I-4

DATA, POINTS AND STRESS DIAGRAM

(A) DIAGRAM, D



(B) MATRICES, DELTA

CASE I. PERFECT MATCH OF MATRIX AND DIAGRAM

	B	C	D
A	1	2	5
B		3	6
C			4

(STRESS = 0)

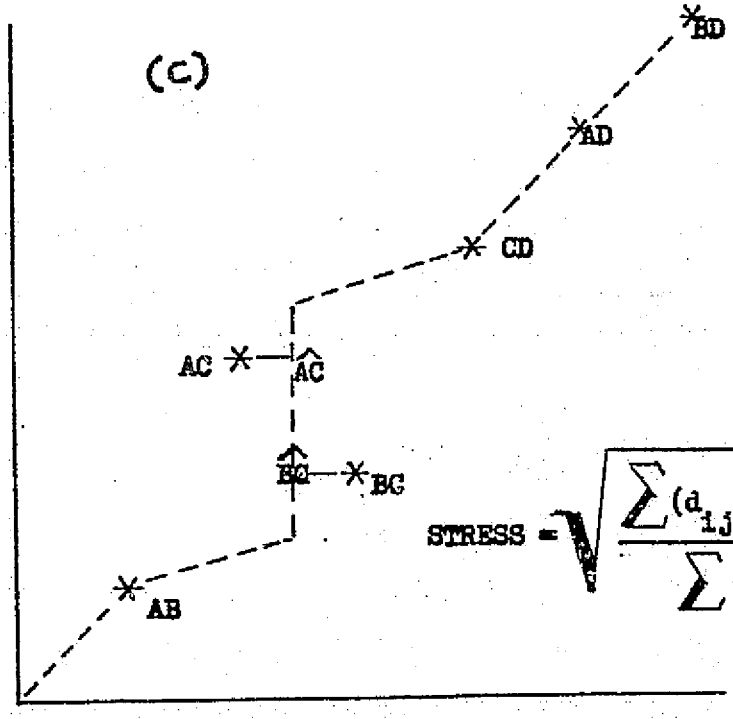
CASE II. IMPERFECT MATCH

	B	C	D
A	1	3	5
B		2	6
C			4

(STRESS > 0)

(C)

MATRIX ORDERED RANKS BETWEEN OBJECTS (DELTAs)



$$\text{STRESS} = \sqrt{\frac{\sum (d_{ij} - \hat{d}_{ij})^2}{\sum d_{ij}^2}}$$

DIAGRAM DISTANCES BETWEEN OBJECTS (D's)

Kruskal's description of a procedure for mapping points is typical in many ways of convergence methods. Given an initial input matrix of ordered relationships, Figure I-4 (b), an initial clustering of objects can be done (Figure I-4 (a). Next, a scattergram can be constructed, I-4 (c) so that the data rankings are represented on the ordinate axis and the "measured" distances on the abscissa. What amounts to a best-fitting line then is fitted among the d_{ij} and the distance between the actual objects d_{ij} and their expected location (denoted as "hat" or $\hat{d}_{ij}$) is found. This is the distance between where the points presently are located and where they would be expected to be if there were a perfect monotonic relationship between points d_{ij} and ranking δ_{ij}. This difference is squared and summed over all cases. Obviously, when the scaled object is located where it "should" be--where a perfect monotone relationship exists between data and point, the difference $\hat{d}_{ij} - d_{ij} = 0$. An equation

$$\sum(d_{ij} - d_{ij})^2$$

can be interpreted as the "residual sum of squares" for the real and expected interpoint distances.

Kruskal's important contribution to MDS is his development of a "badness of fit" statistic that can be used both to judge the adequacy of an intermediate or final solution and to make the minute adjustments or "jiggles" of the points in the iterative process that ensues. The statistic is a normalization of the residual sum of squares, and is called STRESS:

$$\text{STRESS} = \sqrt{\frac{\sum(d_{ij} - \hat{d}_{ij})^2}{\sum d_{ij}^2}}$$

When all of the clustered points fall directly along the monotonic best-fitting line, STRESS will be zero and a perfect fit will exist between points and data. While STRESS always will equal zero when the number of dimensions in which a solution is sought is one less than the number of points according to Bennett and Hays [352], it also has been argued by Guttman [348] that perfect configurations will obtain in n-2 dimensions. Less satisfactory values usually are encountered at lower dimensionalities. Kruskal offered this evaluation of STRESS and of the corresponding goodness of fit:

TABLE 4

STRESS AND GOODNESS FIT

STRESS	Goodness of Fit
20%	Poor
10%	Fair
5%	Good
$2\frac{1}{2}$%	Excellent
0%	Perfect

Values of STRESS and similar indices according to Klahr [359] are larger with greater numbers of scaled objects, with greater error in the data, and with lower-dimensional solutions.

The values of d_{ij} and $\hat{d}_{ij}$ that are found in computing STRESS also are used to guide the direction and magnitude of point jigglings in reducing the discontinuities between points and input data. Kruskal notes [345] the process of accomplishing this can be understood intuitively as a process of successive approximation, theoretically as in iterative technique of numerical analysis called the method of steepest descent, or operationally as any of the applications designed to accomplish the jiggling process. Alternative computational approaches also exist. One of them is Gleanson's alternating algorithm. This process switches between satisfying metric distance requirements first, then by satisfying the ordering requirement. A solution is sought whereby both interval and ordinal needs are fulfilled.

It should be noticed in Figure I-4 (c) that the calculations in the scattergram are executed on the distances (d_{ij}) arranged on the abscissa, rather than on the δ_{ij} shown on the ordinate. To base the calculation in reverse would imply that the distances between the ordinal data were of equal intervals--that the data were interval in level--precisely the assumption about cardinality that MDS was devised to circumvent. As Kruskal [345] points out deviations between the curve and the plotted points are measured along the distance axis because

> ". . . if we measure them along the dissimilarity axis, we shall find ourselves doing arithmetic with dissimilarities. This we must not do, because we are committed to using only the rank ordering of the dissimilarities . . . . We wish to measure good-

ness of fit in such a way that monotone distortion of the dissimilarity axis will not have any effect. This clearly prevents us from measuring deviations along the dissimilarity axis" [345].

5. Dimensionality.

In most scaling situations involving more than a few scalable objects, acceptable convergence of points and data can be attained only multidimensionally. Examples used above suggest correctly that acceptable two-dimensional solutions--and even outcomes in a single dimension--are possible. The development of computerized MDS techniques since 1964, however, have made it possible to attain not only quick and accurate solutions, but also to reach configurations of points in more than two dimensions. Between the extremes of one or two-dimensional solutions on the one hand and n-1 or n-2 solutions on the other, the question of identifying an appropriate dimensionality for obtaining solutions is important.

Identification of an optimal number of dimensions to be used in a solution can be made in at least two ways. First, the values of the "badness of fit" statistic (STRESS in M-D-SCAL, SQUARIANCE in TORSCA-9, COEFFICIENT OF ALIENATION in SSA, etc.) can be plotted for each dimensionality of a given body of data. Figure I-5 shows that a distinct bend appears at the fourth dimension for the Euclidean solution (rho = 2.0), from which it can be inferred that this would be an appropriate number of dimensions. In the case of rho = 1.0, the bend is found at three dimensions. Sometimes there may be no distinct "elbow" and the investigator is provided with no firm clue as to the appropriate dimensionality. A second answer to the question of how many dimensions should be extracted is to obtain only as many as can be interpreted reasonably in a substantive sense.

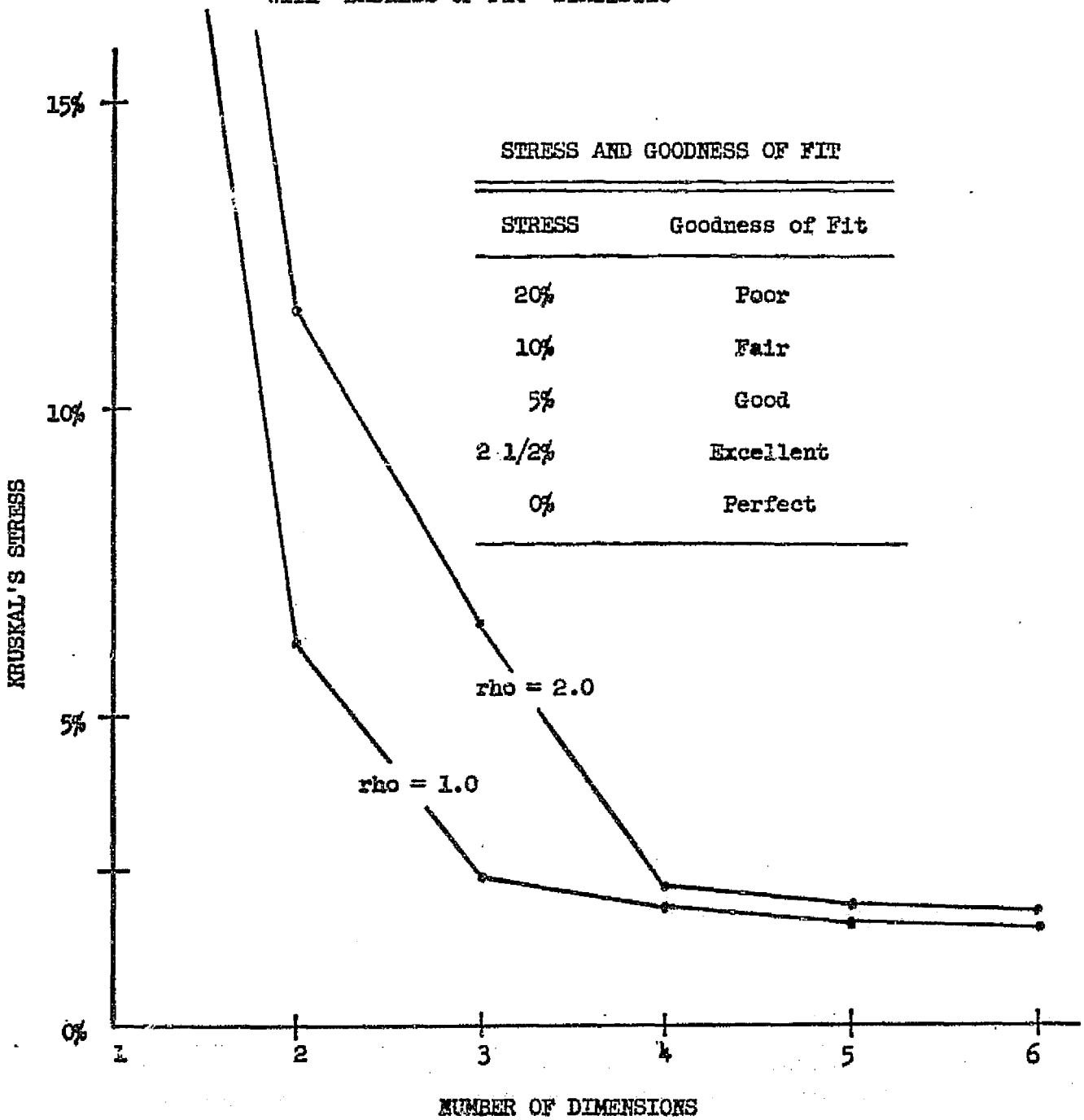
6. Solution Spaces.

Although the implication to this point often has been that objects are being scaled in Euclidean multidimensional space, this need not be the case. STRESS, SQUARIANCE, and other indicators of badness of fit can operate in a wide variety of outcome spaces. Euclidean space is but a special case of L_p-Norms, or of Minkowski rho-metrics. Under more generalized rules of spatial geometry, a Minkowski-p = 2.0 yields the Euclidean solution. When rho = 1.0, however, a different spatial outcome ensues. This is the so-called City Bloc or Manhattan metric space. City bloc solutions are useful in mapping the real distances that persons must travel under certain conditions or the psychological space perceptions that they have internalized. Consider as an example that the distance between two urban locations may not be thought of "as the crow flies", but rather is conceived of as "up to the corner, then turn to the right". Minkowski metrics range from zero through infinity. Theoretically, any Minkowski-p spaced can be treated multidimensionally. Translation of precisely what is meant by a rho-space of 0.8 or of 3.5 can be troublesome, however. Figure I-5 also suggests that measures of stress can be plotted across varying Minkowski spaces to aid in the search for an appropriate dimensionality.

FIGURE I-5

ESTABLISHING APPROPRIATE DIMENSIONALITY

WITH "BADNESS OF FIT" STATISTIC



STRESS AND GOODNESS OF FIT

STRESS	Goodness of Fit
20%	Poor
10%	Fair
5%	Good
2 1/2%	Excellent
0%	Perfect

7. Outputs.

The results obtained from any MDS algorithm vary somewhat, but typically they include a listing of the matrix of similarities or dissimilarities, a table of the cardinal coordinates or configurations of points with respect to each of the dimensions called for in the solution, and a series of scatter diagrams or plots for the visual display of the arrangements of points in the appropriate dimensionalities. Also included may be a scattergram of distances by original rank order, as shown in Figure I-4 (c).

METHODOLOGICAL ISSUES

Before addressing the topic of the use of MDS in total assessment profiling, two methodological issues must be addressed: (1) the reasons that have been advanced for using non-metric MDS, and (2) the limits or caveats that must be understood by those who would use it.

Consider the uses of MDS. First, its use is justifiable when data are ordinal and it is inappropriate to assume that the intervals between the ordered units are equal. Oddly enough, even since the development of MDS the literature of many fields continues to apply metric techniques to ordinal information. Second, it has been pointed out by Lingoes and Guttman [348] that a smaller space is required to reflect ordered relationships alone than is needed to represent both order and intervening distance intervals. Third, reducing stronger data to an ordered level can make clearer the dimensions that can be identified through MDS than would have been the case with factor analysis. This is because ordinal coefficients set aside the requirement of linearity that constrains techniques based upon least-squares measures. However, recent developments have been encouraging in the area of polynominal factor analysis. Finally, as Lingoes and Guttman, also have observed, "when some lawful structure or pattern is present in the data, e.g., a simplex, a circumplex, or a radex, a nonmetric analysis will reveal the configuration whereas a metric approach will obscure the lawfulness" [353].

What can be said about the limitations of this family of approaches? It has been twelve years since the publication of Shepard's major article. The first six years after 1962 were characterized by innovative developments of several sorts in the field of MDS; since 1968, however, more attention has been devoted to consolidating those gains by testing the performance of the several approaches. The application of multidimensional scaling in Total Assessment Profiling must be approached with these recent evaluations in mind.

The best general summary of methodological limitations in MDS probably is that given by Green and Carmone [349] although a growing body of studies consider the relative merits of two or three algorithms at a time. These authors have grouped their methodological caveats and suggestions

for further research into three areas: computational problems, empirical considerations, and conceptual difficulties.

Computational problems are best approached by comparing the performance of several algorithms under similar conditions. Involved here are questions of uniqueness and determinateness of solutions within a single approach (e.g., does M-D-SCAL successfully replicate the first solution on successive runs?) the invariance of solutions across algorithms, and the impact of error upon algorithm efficiency. It may not be at all surprising to learn that comparative studies of performance reveal few differences among the leading three approaches to MDS. One investigation reported by Green and Carmone [349] attempted to recover a synthetic configuration of points (the capital letter "R" represented by 27 points in two dimensions) and found that TORSCA-8, M-D-SCAL IV and SSA-1 performed rather similarly" under both error free and "noisy" data conditions. In a comparative study of the Shepard-Kruskal and the Torgerson approaches to multidimensional scaling, it was found that, in thirty-six simulations, both "models produced highly accurate solutions. . . ." [354]. Again, in a comparison of M-D-SCAL, SSA-1 and TORSCA-9, it was found by Spence [346] that "differences between the solutions obtained by the algorithms were typically so small as to be of little practical importance".

Since the computational considerations raised by Green and Carmone seem difficult to resolve--fortunately, because the leading MDS procedures all perform quite well--what is the best course to follow in any one case? A conventional approach when selecting the best alternative from among a small number of choices probably is to do the analysis each way. Sometimes one solution will be quite superior to the others, and a knowledge of the contrasting techniques allows the investigator to tell why these contradictions have been found.

Methodological matters relevant to the use of MDS in total assessment profiling include, as well as computational considerations, certain empirical and conceptual considerations. Empirically, one must consider the impact of data gathering techniques upon scalability. Although the questions raised by Green and Carmone are important ones, they are common to many critiques of multivariate statistical methods and need not be considered at this point. Conceptually, there is considerable food for thought. Care will have to be given to the ecological fallacy, or to the problem of "analysis at the wrong level". When TAP findings are reported on the basis of survey data, for example, it must be understood that what is true for the individual is not necessarily valid for the group. If the recent concern with ecological problems can be used as one example, we can see that the benefits and costs that accrue at the level of the firm are not identical with those that obtain at a higher level of aggregation. Externalities seldom are foreseen by isolated respondents. Other conceptual issues are significant. Just how well do the coefficients of similarity or preference reflect social realities? How quickly, and why, do the preferences mapped at any given time alter dischronicly? How do they change with distance? These and other important theoretical questions are addressed in the larger context of the Total Assessment Profile.

APPENDIX J

A Tentative Register Of American Values

Source: Baier, Kurt and N. Rescher (eds.), Values and The Future, The Free Press, New York, 1969

I. SELF-ORIENTED VALUES

1. Personal "material" welfare (the right to life and the pursuit of happiness)

   a. health (physical and mental well-being)
   b. economic security and well-being ("materialism" and the American way of life)
   c. personal security (stability of the conditions of life)

2. Self-respect (the right to be treated as a person and as a member in good standing of the community; honor, honorableness)

3. Self-reliance (self-sufficiency, rugged individualism and the pioneer tradition)

4. Personal liberty (the right to endeavor to "shape one's own life", to work out major facets of one's own destiny and to go one's own way)

   a. freedom (from interference)
   b. privacy
   c. property rights

5. Self-Advancement ("success", ambition, diligence)

6. Self-Fulfillment (and "the pursuit of happiness")

7. Skill and Prowess

   a. the intellectual virtues (intelligence, education, know-how, realism, practicality, versatility, etc.)
   b. the physical virtues (strength, dexterity, endurance, good appearance, cleanliness, etc.)
   c. the virtues of the will (strengths of character)
      1. readiness for hard work (industriousness)
      2. toughness (fortitude, endurance, bravery, courage)
      3. initiative and activism (the "go getter" approach)
      4. self-control (temperateness, sobriety)
      5. perseverance and stedfastness
   d. competence (pride of workmanship)
   e. inventiveness and innovativeness
   f. initiative (the "self-starter")
   g. well-informedness (access to information, being "in the know")
   h. faith ("believing in something" including "having a sense of values")
   i. appreciation and appreciativeness (of "the good things of life")

II. GROUP-ORIENTED VALUES

1. Respectability (group acceptance, avoidance of reproach, good repute, conformity, the "done thing" and the "herd instinct")

2. Rectitude and personal morality (honesty, fairness, probity, reliability, truthfulness, trustworthiness--the "man of honor")

3. Reasonableness and rationality (objectivity)

4. The domestic virtues (love, pride in family role, providence, simplicity, thrift, prudence, etc.)

5. The civic virtues (involvement, good citizenship, law-abidance, civic pride--the "greatest little town" syndome)

6. Conscientiousness

   a. devotion to family, duty
   b. personal responsibility and accountability
   c. devotion to principle (especially of one's religion--"the godfearing man")

7. Friendship and friendliness

   a. friendship proper
   b. loyalty (to friends, associates)
   c. friendliness, kindliness, helpfulness, cooperativeness, and courteousness (the good scout; "getting along with people")
   d. fellow-feeling (compassion, sympathy, and "love of one's fellows")
   e. gregariousness
   f. receptivity (openness, patience, "the good listener")
   g. personal tolerance ("live and let live", "getting along with people")
   h. patience

8. Service (devotion to the well-being of others)

9. Generosity (charity, openhandedness)

10. Idealism (hopefulness in human solutions to human problems)

11. Recognition (getting due public credit for the good points scored in the game of life; success and status)

12. Forthrightness (frankness, openness, sincerity, genuineness; keeping things "above board", the fair deal)

13. Fair play (the "good sport")

III. Society-Oriented Values

1. Social welfare (indeed "social consciousness" as such)

2. Equality

   a. tolerance
   b. "fair play". fairness
   c. civil rights

3. Justice (including legality, proper procedure, recourse)

4. Liberty (the "open society"; the various "freedoms")

5. Order (public order, "law and order")

6. Opportunity ("land of opportunity" concept; the square deal for all)

7. Charity (help for the "underdog")

8. Progressivism optimism (faith in the society's ability to solve its problems)

9. Pride in "our culture" and "our way of life"

IV. NATION-ORIENTED VALUES

1. The patriotic virtues (love of country, devotion to country, national pride)

   a. national freedom and independence
   b. national prosperity and national achievement generally
   c. patriotism and national pride
   d. concern for the national welfare
   e. loyalty (to country)
   f. chauvinism (nationalism, pride in national power and preeminence)

2. Democracy and "the American way"

3. "Public service" in the sense of service of country (the nation)

V. MANKIND-ORIENTED VALUES

1. The "welfare of mankind"

a. peace
b. material achievement and progress
c. cultural and intellectual achievement and progress

2. Humanitarianism and the "brotherhood of man"

3. Internationalism

4. Pride in the achievements of "the human community"

5. Reverence for life

6. Human dignity and the "worth of the individual"

VI. ENVIRONMENT-ORIENTED VALUES

1. Aesthetic values (environmental beauty)

2. Novelty

APPENDIX K

Factors In Regionalization

This Appendix contains the results of Zelinsky's findings. The tables are obtained from the article. Careful examination of the results indicates substantial regional variations.

Q-1: THE URBAN-MIGRANT FACTOR

National Geographic	0.710	Saturday Review	0.410
Playboy	0.136	Stag	-0.405
Cosmopolitan	1.159	Real Story	-0.418
Skiing	1.140	Lady's Circle	-0.513
True	1.033	Flower & Garden	-0.554
Glamour	1.003	Outdoor Life	-0.560
Ski	0.977	Field & Stream	-0.705
Esquire	0.606	Sports Afield	-1.054
National Rifle Association	0.057	American Wildlife Federation	-1.602
Argosy	0.032	Secret Romances	-1.717
Psychology Today	0.500	True Story	-2.087
Gourmet	0.589	American Bowling Congress	-2.112
Holiday	0.481		

Q-2: THE MIDDLE WEST FACTOR

American Bowling Congress	10.004	Yachting	-0.401
Outdoor Life	3.083	Common Cause	-0.436
National Geographic	3.064	Penthouse	-0.451
True	2.815	Mademoiselle	-0.463
Popular Mechanics	2.303	Glamour	-0.704
Sports Afield	1.861	Gourmet	-0.732
Field & Stream	1.694	Cosmopolitan	-0.748
True Story	1.427	National Wildlife Federation	-0.845
Argosy	1.226	Holiday	-0.889
Lady' Circle	0.809	Esquire	-1.022
Western Horseman	0.750	Secret Romances	-1.081
Prevention	0.737	Playboy	-1.089
Workbench	0.479	National Council of Garden Clubs	-1.161
Flower & Garden	0.420		

Q-3: THE SOUTHERN FACTOR

True Story	7.580	True Romances	0.457
Field & Stream	3.925	Izaak Walton League	-0.400
Secret Romances	3.443	Saddle & Bridle	-0.402
Sports Afield	3.090	American Artist	-0.403
National Council of State Garden Clubs	2.399	American Humane Association	-0.403
Playboy	2.191	Hit Parader	-0.404
Outdoor Life	2.083	Bloodhorse	-0.407
Modern Romances	1.593	Garden Club of America	-0.407
Flower & Garden	1.582	American Numismatic Society	-0.408
Ingenue	1.456	Water Skiier	-0.409
National Wildlife Federation	1.345	National Field Archery Association	-0.410
Glamour	1.319	Chronicle of the Horse	-0.411
Popular Mechanics	1.135	Thorobred Record	-0.413
Holiday	1.007	Flying Models	-0.414
Modern Screen	0.999	Road & Track	-0.414
True	0.997	American Rabbit Breeder's Association	-0.414
National Geographic	0.965	American Iris Society	-0.414
Hot Rod	0.910	Trains	-0.417
True Confessions	0.890	Popular Dogs	-0.419
Home & Garden	0.886	Gourmet	-0.419
National Rifle Association	0.744	International Brotherhood of Magicians	-0.423
True Love	0.739	American Guild of Handbell Ringers	-0.427
Esquire	0.693	National Horseshoe Pitchers Association	-0.427
Organic Gardening	0.624	Metropolitan Opera Guild	-0.428
Registered Catteries	-0.430	Antique Motor Car Clubs	-0.449
International Federation of Homing Pigeon Fanciers	-0.430	International Arabian Horse Federation	-0.445
Pure-Bred Dogs	-0.430	Camera	-0.460
Specialty Dog Clubs	-0.431	Skin Diver	-0.460
American Orchid Society	-0.431	Model Railroader	-0.462
Railroad Model Craftsman	-0.431	Yachting	-0.465
American Rhododendron Society	-0.433	League of Women Voters	-0.472
American Philatelic Society	-0.434	Amateur Trapshooting Association	-0.477
American Badminton Association	-0.434	Art in America	-0.493
Amateur Chamber Music Players Society	-0.434	Trailer Life	-0.501
Society for the Preservation & Encouragement of Barber Shop Quartet Singing in America	-0.434	Camping Guide	-0.508
		Natural History	-0.524
		Sky & Telescope	-0.539
National Model Railroad Association	-0.437	New Republic	-0.541
Art News	-0.439	Psychology Today	-0.558
Soaring Society of America	-0.440	Common Cause	-0.641
Dune Buggies	-0.440	Western Horseman	-0.663
War Resisters League	-0.442	Skiing	-1.089
National Mustang Society	-0.443	Ski	-1.141
Salt Water Sports	-0.448	American Bowling Congress	-3.511

R-1: URBAN SOPHISTICATION FACTOR

Metropolitan Opera Guild	.883
Esquire	.870
Art News	.861
Mademoiselle	.830
Steren Review	.831
Saturday Review	.815
Common Cause	.814
Gourmet	.792
Parenthood	.780
Holiday	.778
High Fidelity	.768
New Republic	.763
Glamour	.758
National Review	.730
Modern Photography	.729
Hit Parader	.727
Popular Photography	.695
American Philatelic Society	.663
Audio	.661
Amateur Chamber Music Players Society	.652
War Resisters League	.650
Car & Driver	.641
Popular Dogs	.632
Antiques	.606
Motor Boating & Sailing	.605
Trains	.578
Sports Car	.568
Natural History	.543
Chronicle of the Horse	.520
Yachting	.514
Penthouse	.509
Garden Club of America	.501
Model Railroader	.495
Camera	.492
Railroad Model Craftsman	.471
International Fed. of Amer. Homing Pigeon Fanciers	.407
Playboy	.400
Super Stock	.453
Horticulture	.452
Cars	.441
19 Specialty Dog Clubs	.438
Salt Water Sportsmen	.419
Pure-Bred Dogs	.410
Sports Afield	-.408
Shooting Times	-.421
Modern Screen	-.442
Guns & Ammo	-.445
Personal Romances	-.455
Motion Picture	-.467
Trap Shooting	-.491
Horseman	-.498
International Arabian Horse Federation	-.500
National Horseshoe Pitchers Association	-.512
Western Horsemen	-.522
True Confessions	-.555
True Romances	-.555
Master Detective	-.567
True Experiences	-.571
Field & Stream	-.586
Saga	-.615
Official Detective	-.650
Outdoor Life	-.668
True Detective	-.712
Argosy	-.728
True	-.744
Man's Magazine	-.744

R-2: THE INNOVATIVE WEST VS. TRADITIONAL SOUTH FACTOR

Camper Coachman	.555
International Arabian Horse Federation	.547
American Bowling Congress	.524
National Geographic	.520
Trailer Life	.513
Camera	.497
2 Antique Motor Car Clubs	.481
Ski	.447
Amateur Trapshooting Association	.439
Cosmopolitan	.424
Natural History	.420
National Mustang Society	.418
Soaring Society of America	.403
Cycle World	.398
Skiing	.396
Dune Buggies	.395
Psychology Today	.390
Coin World	-.394
Sports Afield	-.408
Personal Romances	-.455
Hunter's Horn	-.503
Secret Romances	-.519
National Council of Garden Clubs	-.520
Field & Stream	-.468
True Experiences	-.549
American Guild of Handbell Ringers	-.596
Ingenue	-.601
National Horsemen	-.604
True Confessions	-.637
True Romances	-.638
Intimate Story	-.650
Saddle & Bridle	-.652
Shooting Times	-.652
American Camellia Society	-.655
American Rose Society	-.688
Flower & Garden	-.696
Real Story	-.702
Modern Romances	-.734
Hounds & Hunting	-.764
True Story	-.775
Real Confessions	-.780
Real Romances	-.788
True Love	-.795
Home & Garden	-.830
Modern Loves	-.832
American Cooper	-.837

R-3: MIGRANT FACTOR

Citizens Band Magazine	.712
Camping Journal	.687
Trailer Life	.650
Dog World	.601
Cycle	.593
Pure-Bred Dogs	.569
Camper Coachman	.537
Cycle World	.525
Radio Relay League	.516
Woodall's Trailer Travel	.494
American Numismatic Society	.469
Coin World	.443
Sports Car	.441
Cycle Guide	.405
Movie Mirror	.399
Soaring Society of America	.395
National Model Railroad Association	.392

R-4: SEX AND ROMANCE FACTOR (or Anti-Sex-and-Romance Factor)

National Geographic	.582	Penthouse	-.515
Radio Relay League	.524	Intimate Story	-.518
Pure-Bred Dogs	.451	Downbeat	-.521
League of Women Voters	.404	Cars	-.601
International Federation of Homing Pigeon Fanciers	.403	Men	-.616
		Lady's Circle	-.652
Secret Romances	-.426	Personal Romances	-.666
Hot Rod	-.448	Man's Magazine	-.713
Real Story	-.469	For Men Only	-.717
Super Stock	-.484	Stag	-.748

R-5: LATITUDINAL FACTOR

Popular Hot Rod	.823	Skin Diver	.470
Golf	.776	Super Stock	.399
Popular Cycling	.761	Sportfishing	.398
Car Craft	.755	National Audubon Society	-.404
Golf Digest	.747	National Rifle Association	-.430
American Orchid Society	.610	National Horseshoe Pitchers Association	-.453
Screen Stars	.604	Outdoor Life	-.466
Modern Movies	.597	Amateur Chamber Music Players Society	-.472
Photo Screen	.565	Creative Crafts	-.535
Motor Trend	.564	Ski	-.620
Hot Rod	.498	Skiing	-.627

R-6: AQUATIC FACTOR

Rudder	.784	19 Specialty Dog Clubs	.453
Yachting	.717	League of Women Voters	.430
Salt Water Sports	.679	Camera	.425
Boating	.660	Registered Catteries	.405
Sport Fishing	.617	International Arabian Horse Association	-.399
Motor Boating & Sailing	.615	Amateur Trap Shooting Association	-.408
Skin Diver	.571	Guns & Ammo	-.411
Road & Track	.557	Field & Stream	-.422
American Badminton Society	.554	Sports Afield	-.433
Popular Dogs	.551	Western Horseman	-.546
Horticulture	.548	Prevention	-.571
Garden Club of America	.542	American Iris Society	-.622
Antiques	.518	Horseman	-.682
Flying Models	.489	Quarter Horses	-.701
Amer. Philatelic Society	.481		

R-7: MIDLAND AND MIDWEST VS. SOUTHWEST FACTOR

Workbench	.841	Ingenue	.420
Better Camping	.805	International Brotherhood of Magicians	.408
Popular Mechanics	.800	National Model Railroad Association	.399
Society for the Preservation & Encouragement of Barber Shop Quartet Singing	.681	Flower & Garden	.399
		Motion Picture	-.390
Camping Guide	.622	True Detective	-.400
Motor Trend	.601	Playboy	-.406
Woodall's Trailer Travel	.549	Saga	-.474
Camping Journal	.506	Cycle World	-.486
Trains	.499	Dune Buggies	-.500
Izaak Walton League	.484	Movieland & TV Time	-.520
Railroad Model Craftsman	.475	Cosmopolitan	-.569
Sports Afield	.467	Amer. Contract Bridge League	-.621
Model Railroader	.459		

TABLE 6

CORRELATION COEFFICIENTS: FACTOR LOADINGS AND SCORES FOR STATES EXTRACTED FROM ROTATED MATRIX OF 163 SPECIAL-INTEREST MAGAZINES AND VOLUNTARY ASSOCIATIONS CORRELATED WITH SELECTED SOCIAL, ECONOMIC, AND DEMOGRAPHIC VARIABLES

	Q-Mode Loadings		
	Q-1: Urban Migrant Factor	*Q-2: Middle West Factor*	*Q-3: Southern Factor*
General demographic			
1. Population change, 1960–1970	.620	–.383	–.286
2. Persons 65 years & older as % of total population	–.317		
3. Median age of population			
4. Negroes as % of total population		–.385	
5. Foreign-born as % of total population	.502		–.609
Place of residence or work			
6. Urban population as % of total population	.510		–.665
7. Population in urbanized areas as % of total population	.445		–.510
8. Population in urban places outside urbanized areas as % of total population		.329	
9. Change in urban population, 1960–1970	.433	–.278	
10. Rural-farm population as % of total population	–.708	.624	
11. % of total employed working outside county of residence		–.341	
Migration			
12. Migrants, 5 yrs. and older as % of total population	.319		
13. % of native population residing in state of birth	–.497		.491
14. % of native population born in different state	.475		–.402
15. % of native population living in same house as 1965	–.363		.267
16. % of native population living in same county as in 1965 but different house			
17. % of native population living in different county but same state as 1965		.361	
18. % of native population living in different state in 1965	.406		–.292
19. Estimated net migration, 1960–1970	.601	–.365	
20. Persons who have always lived in same house as % of total population	–.412		.422
Occupation			
21. Employees in manufacturing industries as % of total employed		.253	.371
22. Employees in white-collar occupations as % of total employed	.511		–.711
23. Government workers as % of total employed			–.330
24. Professional, technical & kindred workers as % of total males employed 14 years and older	.352		–.270
25. Workers in recreational and entertainment services as % of persons employed 14 years and older			
Economic Status			
26. % of labor force unemployed			
27. Median family income, 1969	.423		–.777
28. % of families under poverty level, 1969	–.345		.793
29. % of families with income of $15,000 or more, 1969	.475		–.462
30. Mean family income, 1969	.524		–.743
31. Index of income concentration, 1969			.414
32. % of families receiving public assistance income			.454
33. Change in median family income, 1960–1970		–.255	.436

TABLE 3 (Continued)

	Q-Mode Loadings		
	Q-1: Urban Migrant Factor	Q-2: Middle West Factor	Q-3: Southern Factor
Education			
34. % of persons age 14–17 yrs., attending school		.405	–.754
35. Median school years completed by persons 25 yrs. & older	.654	.278	–.822
36. College students as % of total attending school			–.547
37. Persons 25 yrs. & older who have completed 4 yrs. of college or more as % of total population	.538		–.709

R-Mode Factor Scores

	R-1: Urban Sophistication	R-2: Innovative West vs. Traditional South	R-3: Migrant	R-4: Sex & Romance	R-5: Latitudinal	R-6: Aquatic	R-7: Midland & Midland vs. Southwest
1.			.496			.388	–.255
2.							.436
3.	.344						.296
4.	.505	–.390		–.433			–.621
5.	.409	.498				.470	
6.	.576	.475			.617		
7.	.679	.302			.659		
8.	–.650				–.300		
9.	–.306		.356				–.369
10.	–.483		–.367			–.466	.365
11.	.734						
12.	–.485	.292	.277				–.369
13.		–.432	–.431				.440
14.		.424	.442				–.403
15.	.290		–.384				.548
16.				–.312	.550		
17.	–.301					–.499	
18.	–.342	.306					–.458
19.			.554			.430	
20.		–.343	–.497				
21.	.280	–.405				.383	.460
22.	.593	.511					–.305
23.				–.307			–.580
24.	.407				–.580		
25.			.312				–.604
26.	–.502		.294		–.296		–.346
27.	.454	.580				.355	
28.	–.271	–.661					–.309
29.	.501	.522				.333	
30.	.430	.574				.379	
31.		–.439				.310	–.481
32.		–.396					–.487
33.	.283	–.523	–.315	–.586			
34.		.737					.309
35.		.776					
36.	.414	.432					
37.	.383	.591					–.377

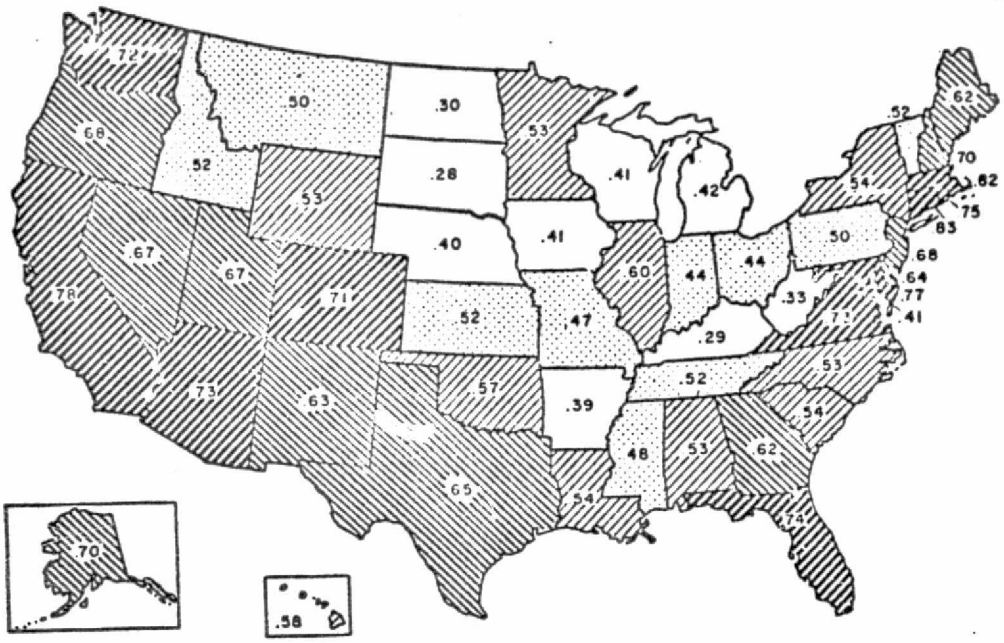


Fig. 1. Q-1: Urban-Migrant Factor (factor loadings in quintiles).

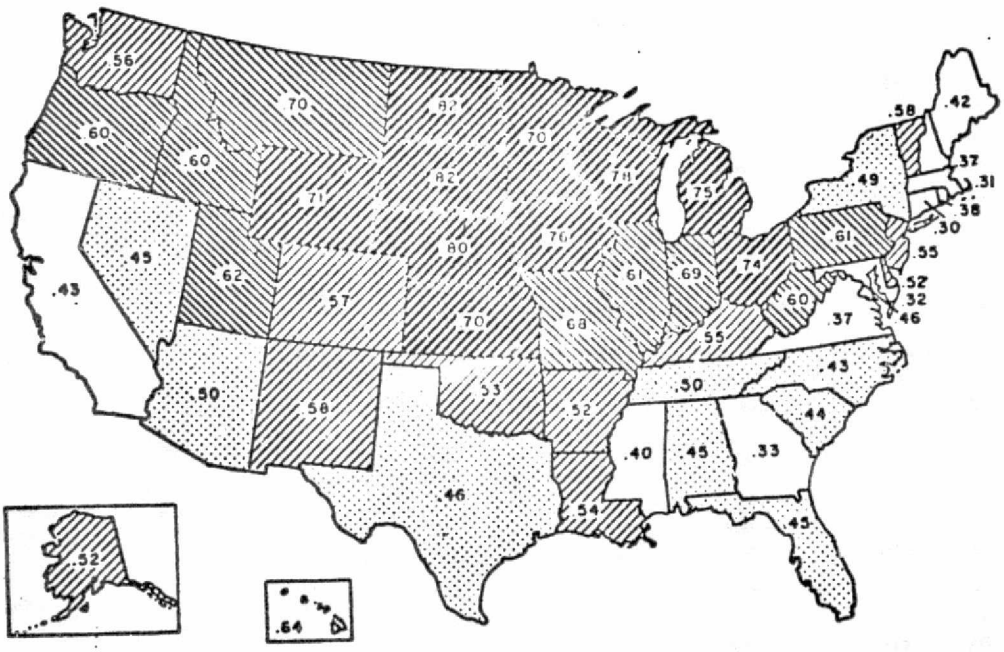


Fig. 2. Q-2: Middle West Factor (factor loadings in quintiles).

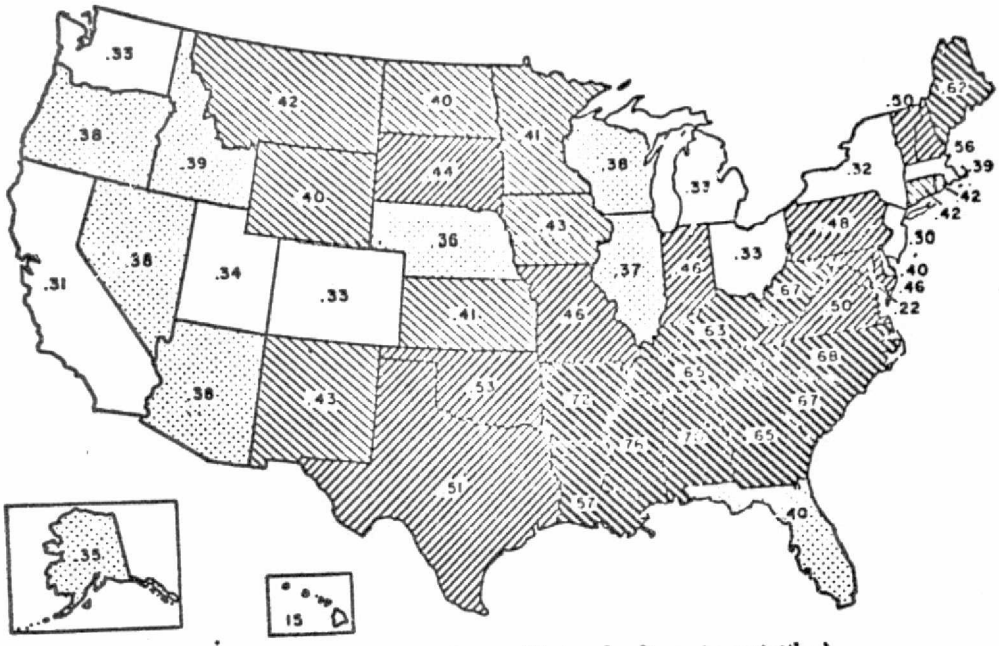


Fig. 3. Q-3: Southern Factor (factor loadings in quintiles).

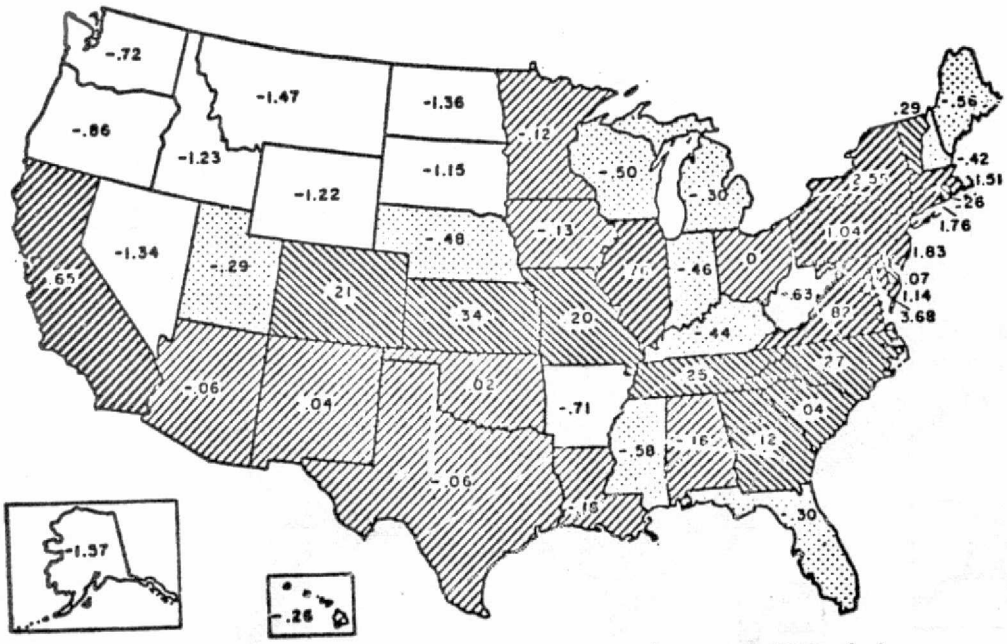


Fig. 4. R-1: Urban Sophistication Factor (factor scores in quintiles).

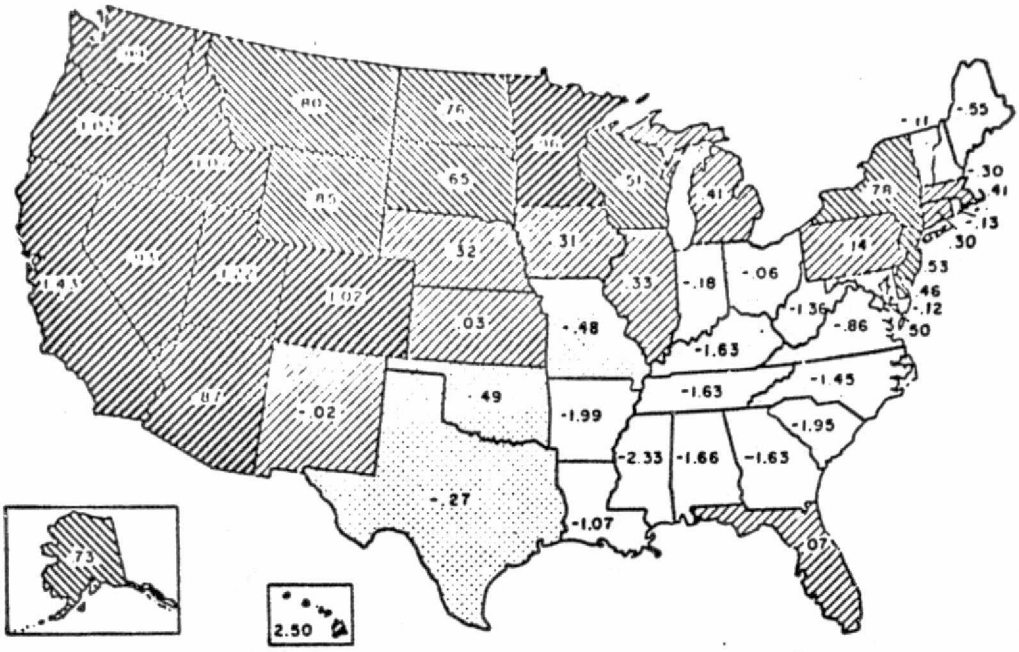


Fig. 5. R-2: Innovative West vs. Traditional South Factor (factor scores in quintiles).

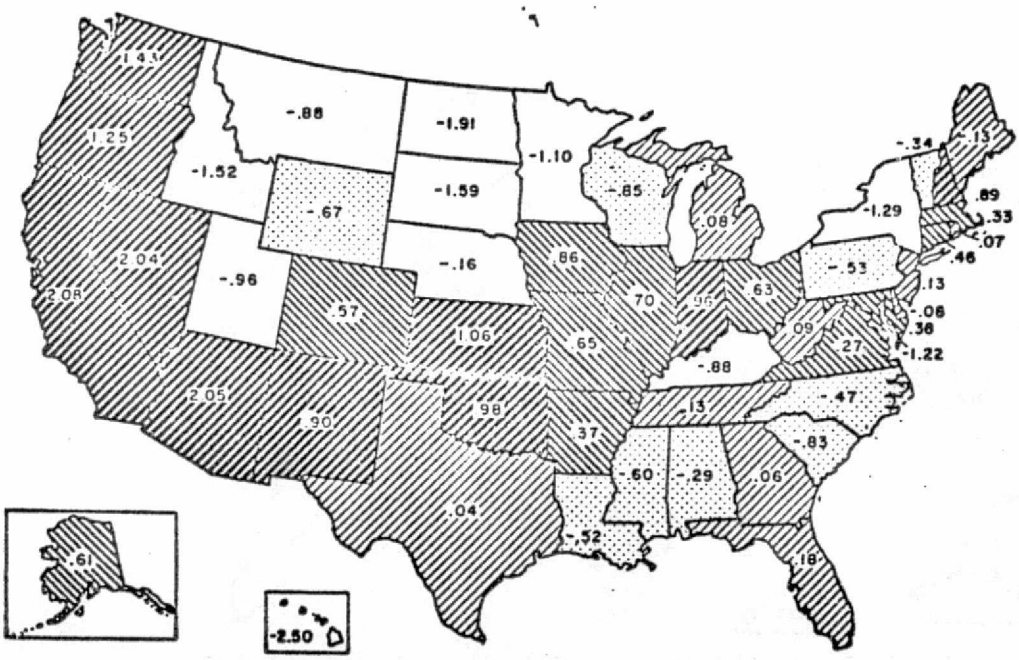


Fig. 6. R-3: Migrant Factor (factor scores in quintiles).

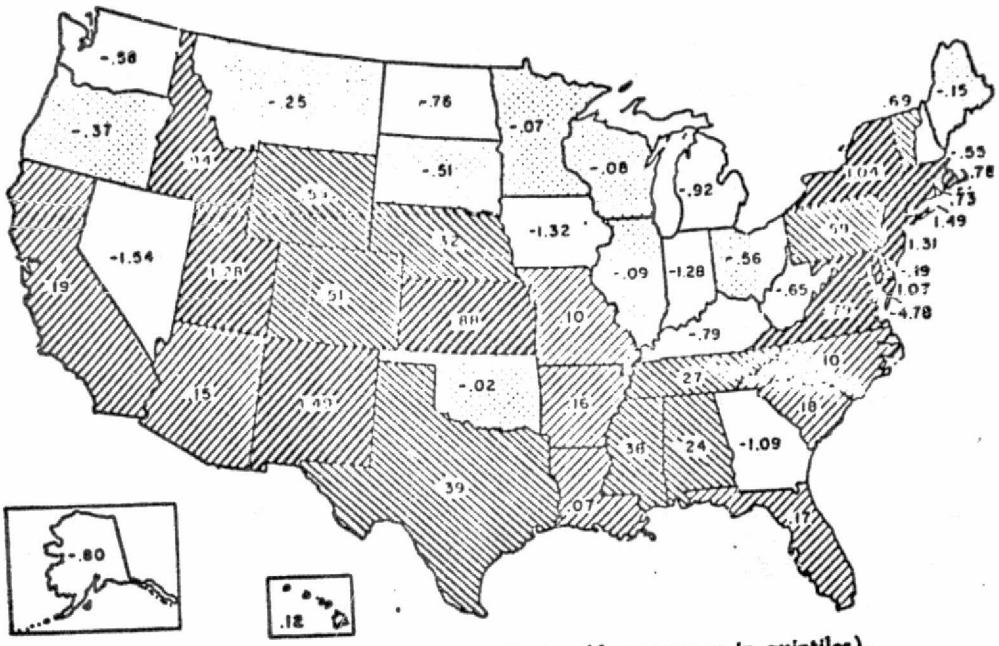


Fig. 7. R-4: Sex and Romance Factor (factor scores in quintiles).

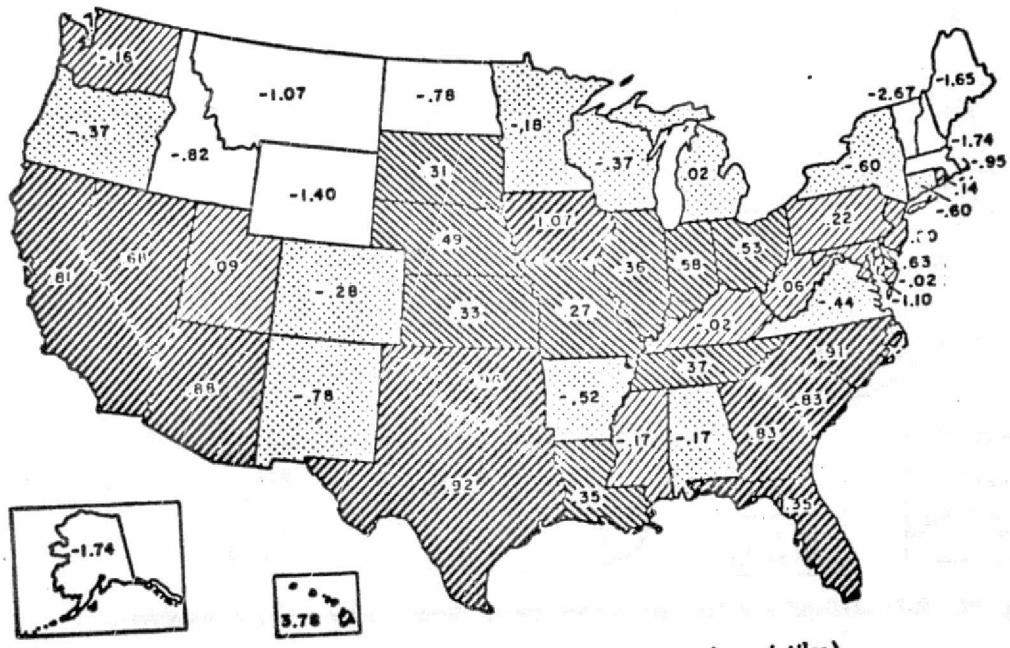


Fig. 8. R-5: Latitudinal Factor (factor scores in quintiles).

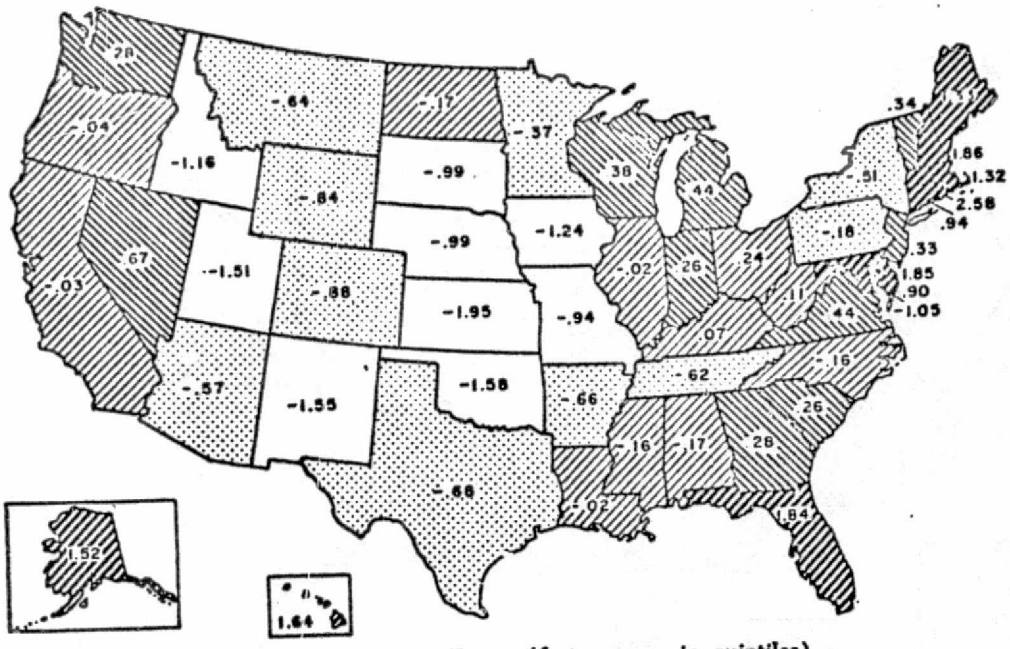


Fig. 9. R-6: Aquatic Factor (factor scores in quintiles).

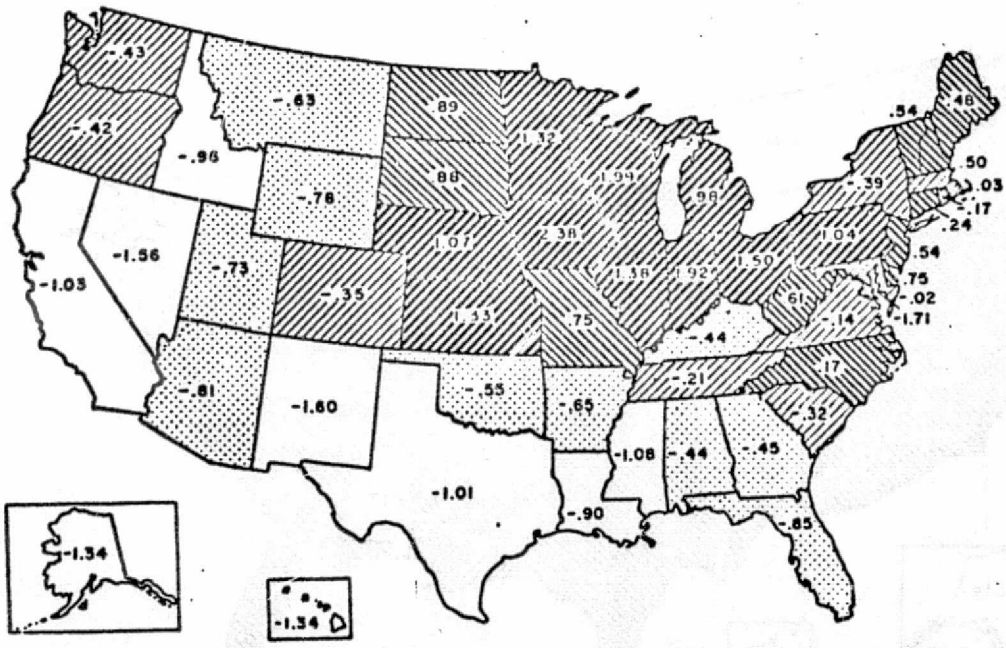


Fig. 10. R-7: Midland and Midwest vs. Southwest Factor (factor scores in quintiles).

APPENDIX L

Pareto's Derivations

DERIVATIONS

CLASS I: ASSERTION

I-a. Assertions of facts, experimental or imaginary
I-b. Assertions of sentiments
I-c. Mixtures of fact and sentiment

CLASS II: AUTHORITY

II-a. Of one individual or a number of individuals
II-b. Of tradition, usages, and customs
II-c. Of divine beings, or personifications

CLASS III: ACCORDS WITH SENTIMENTS OR PRINCIPLES

III-a. Accord with sentiments
III-b. Accord with individual interest
III-c. Accord with collective interest
III-d. Accord with juridical entities
III-e. Accord with metaphysical entities
III-f. Accord with supernatural entities

CLASS IV: VERBAL PROOFS

IV-a. Indefinite terms designating real things; indefinite things corresponding to terms
IV-b. Terms designating things and arousing incidental sentiments, or incidental sentiments determining choice of terms.
IV-c. Terms with numbers of meanings, and different things designated by single terms
IV-d. Metaphors, allegories, analogies
IV-e. Vague, indefinite terms corresponding to nothing concrete

APPENDIX M

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APPENDIX N

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