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THE DETERMINATION OF ORBITS USING PICARD ITERATION

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The topic of this presentation is the determination of orbits using Picard iteration, which is a direct extension of the classical method of Picard that has been used in finding approximate solutions of nonlinear differential equations for a variety of problems. The application of the Picard method of successive approximations to the initial value and the two-point boundary value problems is given below.

The initial value problem,

$$\dot{z} = F(z, t)$$

$$z(t_1) = c, \qquad n = 1, 2, \ldots$$

is solved by means of the iterative scheme,

$$\dot{z}_n = F(z_{n-1}, t)$$

$$z_n(t_1) = c,$$

whereby the *nth* approximation to the state vector z is computed from the (n − 1) approximation (beginning with some initial approximation, $z_0(t)$). The computation involves performing a simple integration or quadrature at each stage of the iterative process. The constant of integration is chosen such that each iterate satisfies the initial condition, $z(t_1) = c$.

Similarly, the two-point boundary value problem,

$$\dot{z} = F(z, t)$$

$$g(z(t_1), z(t_2)) = 0,$$

is solved by means of the iterative scheme,

$$\dot{z}_n = F(z_{n-1}, t)$$

$$g(z_n(t_1), z_n(t_2)) = 0, \qquad n = 1, 2, \ldots$$

where g denotes certain constraints or boundary conditions that must be satisfied by the solution at t_1 and t_2.

This paper presents an investigation of the suitability of this type of iterative scheme for the problem of estimating orbits. In the estimation problem, we again have to solve the differential equation for the state. However, we are now given a set of imprecise observations, $y(t_i)\ _{i=1}^{m}$, which are, in general, nonlinear functions of the state. The problem is to find a z(t) which comes close (in some sense) to satisfying the observations. The estimation problem can thus be written as

$$\dot{z} = F(z, t)$$

subject to the condition that z(t) minimizes some function Q, where

$$Q = Q\,[z(t_1), z(t_2), \ldots, z(t_m), y(t_1), y(t_2), \ldots, y(t_m)].$$

For instance, Q could be the sum of the squares of the residuals (the residual being the difference between the observed and the computed values of the observations). Again, the iteration is set up,

$$\dot{z}_n = F(z_{n-1}, t),$$

now choosing the constant of integration for $z_n(t)$ such that

$$Q\,[z_n(t_1), z_n(t_2), \ldots, z_n(t_m), y(t_1), \ldots, y(t_m)]$$

is minimized. As before, each stage of the iteration requires that a quadrature is performed. In order to perform the integration, we have chosen to approximate the right-hand side, $F(z_{n-1}, t)$, by a series of Chebyshev polynomials, $T_j(\tau)$, and to integrate term-by-term. Thus, the approximation

$$F(z_{n-1}, t) \cong \sum_{j=0}^{N}{}' \ b_j T_j(\tau)$$

is made where $\tau = 1 - 2\,t/(t_m - t_1)$ is normalized time. The coefficients of the series are determined readily using the orthogonality properties from the relation

$$b_j = \sum_{k=0}^{N}{}'' \ F(\tau_k)\, T_j(\tau_k)$$

where $\tau j = \cos(j\pi/N)$. The integration can be performed by manipulating these coefficients, giving

$$z_n \cong \sum_{j=0}^{N}{}' \ a_j T_j(\tau)$$

where

$$a_j = (b_{j-1} - b_{j+1})/2j \qquad \text{for } j = 1, 2, \ldots N$$

and a_0 = the arbitrary constant of integration that is determined from the requirement that Q be minimized.

We will now consider the application of this technique to an orbit estimation problem, namely, the determination of the orbit of an earth satellite from observations of the range and range rate by several tracking stations.

There are no external forces except the gravity field of the earth. The equations of motion for the satellite are written as a set of six, first-order equations for the six state variables: x, y, z, u, v, and w. For instance, $R_s(t_i)$ represents the range observed at t_i from the fifth tracking station, and range rate is the time derivative of the range denoted by $\dot{R}_s(t_i)$; these are given. For the computer simulation presented here, we have taken the nominal values of the range and the range rate and added random noise. The standard deviations assumed are 30 meters in range and about 50 cm per second in range rate.

For this particular problem, the Q function should be considered. This is the sum of the weighted squares of the residuals:

$$Q = \sum_{s=1}^{s} \sum_{i=1}^{m} [R_s(t_i) - g_s(t_i)]^2 \; W_{si}$$

$$+ \sum_{s=1}^{s_1} \sum_{i=1}^{m_1} [\dot{R}_s(t_i) - \dot{g}_s(t_i)]^2 \; W_{si}$$

where

$$\begin{aligned} g_s(t_i) = & \Big\{ [x(t_i) - x_s(t_i)]^2 \\ & + [y(t_i) - y_s(t_i)]^2 \\ & + [z(t_i) - z_s(t_i)]^2 \Big\}^{1/2} \end{aligned}$$

and

$x_s(t_i)$, $y_s(t_i)$, $z_s(t_i)$ = the fifth station coordinates at t_i,

$R_s(t_i)$ = the range from the particular tracking station,

g_s = the range computed using the present iterate,

$\dot{R}_s(t_i)$ = observed range rate, and

W_{si} = weight for the particular observation.

We can see that g_s is the function of the state at t_i, and x_s represents the coordinates of the station itself. The minimization of this function Q with respect to the integration constant is carried out using Newton's method.

Table 1 provides the data for the three tracking stations we have assumed. The longitude and latitude, number of observations, and the total interval considered—about 1500 seconds—are listed. The elliptic orbit considered has an eccentricity of 0.0557, with a semimajor axis of about 7178 km. This corresponds to a perigee at 400 km altitude, an apogee at 1200 km altitude, and an inclination of 20° to the equatorial plane.

Table 1
Station and Orbit Data

Parameters	Station		
	1	2	3
Latitude (deg)	18.0	12.0	10.0
Longitude (deg)	0.0	28.0	14.0
Number of Observations	10	20	30
Interval Between Observations (s)	168.0	79.0	52.0
Semimajor Axis (km)	a = 7178.145		
Eccentricity	e = 0.0557		
Inclination (deg)	i = 20.0		
Longitude of Ascending Node	Ω = 0.0		
Argument of Perigee	ω = 0.0		

Some of the results obtained are depicted in figure 1, which shows the error as a function of time for different iterations. The initial guess is off by about 70 km from the true solution, and with two iterations the error is brought down to something like 15 km.

In order to see the convergence properly, the log of absolute error is plotted as a function of the iteration number in figure 2. The error is plotted for several points along the trajectory. The first point is T = 0, and the last point is T = 1513.1. It is seen that the convergence is linear as was expected because of its relation to the classical Picard method.

One of the disadvantages of this method is that the convergence is obtained only for arcs of length less than about one-third of a revolution. Another problem is that of developing a procedure for connecting two successive arcs.

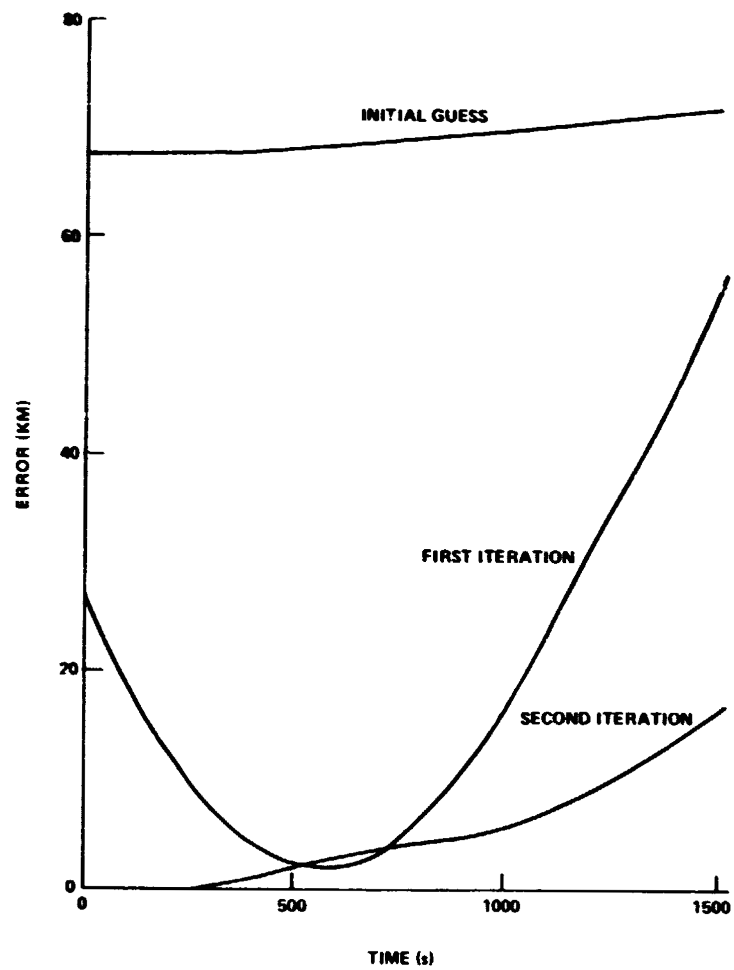

Figure 1. Error as a function of time for different iterations (elliptic orbit).

On the other hand, there are many advantages: For example, the method is quite simple and does not require the linear perturbation equations. The solution is in the form of a polynomial and is convenient to store. There is no interpolation required, should the solution be needed at some intermediate point. The error in representing this by a polynomial can, of course, be estimated by observing the last few terms of the polynomial. Also, the method is not sensitive to a poor initial guess. As we have seen here, the initial guess was off by about 70 km and even then the process will converge without difficulty.

The question is, is this method really more efficient than a more traditional approach to solving a problem? Actually, we have not compared it with the existing methods. However, we have some estimates on the time taken for this particular problem—it takes about 30 seconds on the IBM 360/65.

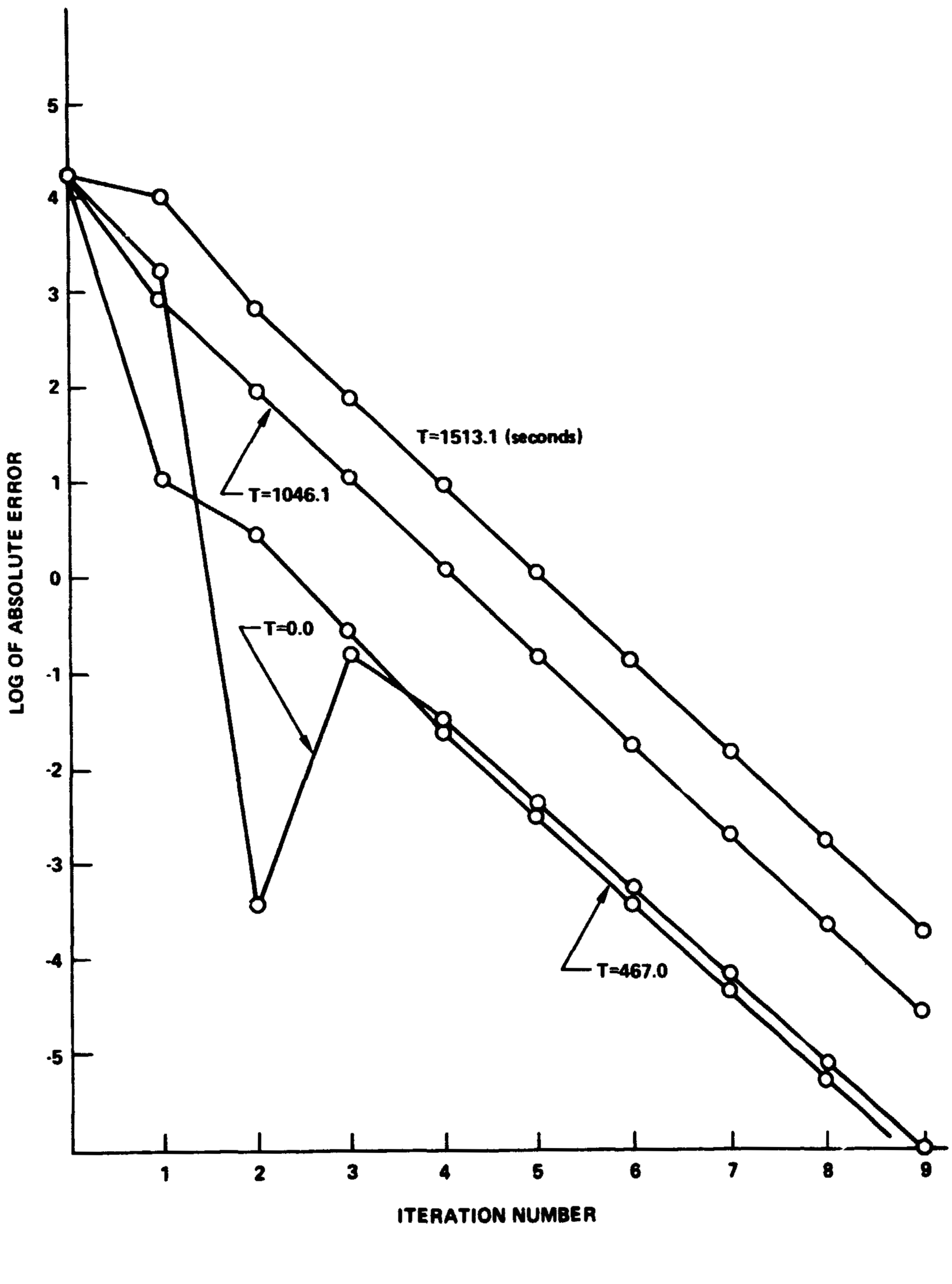

Figure 2. Convergence of the method for an elliptic orbit.

A FINITE ELEMENT GRAVITY FIELD?

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The paper details an approach for constructing a globally valid, piecewise continuous family of locally valid potential functions. The thesis put forth here is that the higher frequency terms of the geopotential can be more efficiently computed from such locally convergent approximations (typically three variable power series of order less than 5) than from any globally convergent gravity representation. This approach appears to be a step in the direction of voiding the recent trend that the better model we have of the geopotential, the more expensive it is to integrate orbits with it! Numerical experiments conducted thus far confirm the validity of the approach and that acceleration errors of $0\,(10^{-10}\ \mathrm{km/s^2})$ are achievable. The trade-offs involved in selecting the finite element shape and size, and the order of the local approximations versus resulting accuracy, computational speed, and storage requirements are currently under study.

MEAN RATES OF THE ORBITAL ELEMENTS OF A SATELLITE PERTURBED BY A LENS SHAPED MASCON

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A set of mean orbital rates are computed for a satellite perturbed by a lens shaped mascon. A disturbing potential in terms of the orbital elements of the satellite and the mascon parameters is developed. The partial derivatives of the potential with respect to the orbital elements are formed. These partials are averaged over the period of the satellite orbit to eliminate the short periodic terms. The averaged partials are substituted into the variation of parameters equations to give the mean orbital rates. In the limiting case, when the radius of the lens shaped mascon reaches zero the mean orbital rate due to a point mass is obtained. The orbital rates developed by the method described here are compared against the rates obtained by numerical differencing. The method developed here is used to reduce the Apollo-15 and -16 subsatellite data for lunar farside gravity determination.