GRAVITATIONAL-WAVE BURSTS FROM THE NUCLEI OF DISTANT GALAXIES AND QUASARS: PROPOSAL FOR DETECTION USING DOPPLER TRACKING OF INTERPLANETARY SPACECRAFT[*][†]

KIP S. THORNE

California Institute of Technology, Pasadena, California, U.S.A.

and

VLADIMIR B. BRAGINSKY

Physics Faculty, Moscow State University, Moscow, U.S.S.R.

(NASA-CR-145432) GRAVITATIONAL-WAVE BURSTS
FROM THE NUCLEI OF DISTANT GALAXIES AND
QUASARS: PROPOSAL FOR DETECTION USING
DOPPLER TRACKING OF INTERPLANETARY
SPACECRAFT (California Inst. of Tech.) 17 p G3/93

N76-10977

Unclas
39447

---

[*] Supported in part by the Ministry of Higher Education, U.S.S.R.; and by the National Aeronautics and Space Administration [NGR-05-002-256] and the National Science Foundation [MPS75-01398], U.S.A.

[†] Based in part on lectures by V. B. Braginsky at Caltech, Stanford, L.S.U., M.I.T., and Princeton in October 1974, and by K. S. Thorne at the Symposium on Theoretical Principles in Astrophysics and Relativity, Chicago, May 27, 1975

ABSTRACT

It is likely that supermassive black holes ($M \sim 10^6$ to 10^{10} $M_\odot$) exist in the nuclei of many quasars and galaxies. The collapse which forms these holes and subsequent collisions between them should produce strong, broad-band bursts of gravitational waves: for a source of mass M at the Hubble distance of $\sim 10^{10}$ light years, the dimensionless amplitude would be $h \sim 2 \times 10^{-17} \times (M/10^6\ M_\odot)$, and the duration of the burst would be $\tau \sim$ (90 sec)$\times (M/10^6\ M_\odot)$. Such bursts might arrive at Earth as often as 50 times per year--or as rarely as once each 300 years. The detection of such bursts may be possible within the next few years using dual-frequency doppler tracking of interplanetary spacecraft.

Subject headings: galactic nuclei - gravitation - quasistellar sources or objects - relativity

I. SOURCES OF THE BURSTS

When building models for the cores of quasars, of strong radio sources, and of active galactic nuclei, theorists typically conclude that, whatever may be in the core, it is likely to generate one or more supermassive black holes ($M \sim 10^6$ to 10^{10} $M_\odot$) in a time short compared to the age of the Universe. See, e.g., Lynden-Bell (1969), Wolfe and Burbidge (1970), Spitzer (1971), and Saslaw (1974, 1975).

A typical scenario involves the collapse of supermassive stars, gas clouds, or star clusters to form holes, and perhaps subsequent collisions between holes. Each such collapse or collision involving masses $M \gtrsim 10^6$ $M_\odot$ should produce a burst of gravitational radiation so strong that it might be observable at Earth. However, in the alternative scenario of a single hole which forms small ($M \ll 10^6$ $M_\odot$) and then grows by gradual accretion, no strong burst of gravitational waves can be expected (Davis et al. 1971).

There are so few quasars and galaxies in the Universe ($\sim 10^{10}$), and each one can produce so few strong gravitational-wave bursts (perhaps $\lesssim 10$) that to detect several bursts per year, we must search for them over the entire volume of the observable Universe. Most such bursts will probably come from a redshift $z \simeq 2.5$, when the Universe was about 2 billion years old, since quasar activity seems to have peaked very sharply at that time (Schmidt 1970, 1972).

II. MEAN TIME BETWEEN BURSTS

The mean time Δt between such gravitational-wave bursts at Earth can be expressed in terms of: (i) the Hubble expansion rate H_o and deceleration parameter q_o of the Universe today (which we assume globally Friedmann with zero cosmological constant); (ii) the redshift $z \simeq 2.5$ at which most of the bursts were generated; (iii) the number density n_o today of "centers" where bursts originated (each center was probably a galactic nucleus); (iv) the mean number of bursts N generated in each center during its active life; and (v) the speed of light c:

$$\Delta t = \frac{1}{4\pi R^2 n_o N c} \quad , \quad R \approx \frac{[\, 1 - q_o + q_o z - (1-q_o)(1 + 2q_o z)^{1/2}]}{(H_o/c) q_o^{\,2} (1 + z)} \; ; \qquad (1)$$

cf. equation (29.33) of Misner, Thorne, and Wheeler (1973)--cited henceforth as MTW. Because number densities of cosmological objects are usually computed using $H_o = 100$ km sec^{-1} Mpc^{-1} and $q_o = 1$, we shall assume these parameters throughout. The more modern values $H_o = 55$ and $q_o << 0.5$ (Sandage 1972; Gunn *et al.* 1974) would give values of Δt smaller by a factor of ~ 3. With $H_o = 100$, $q_o = 1$, $z = 2.5$ the time between bursts is

$$\Delta t = \text{(0.06 years)} \, (n_o/\text{Mpc}^{-3})^{-1} \, N^{-1} \; . \qquad (2)$$

Various assumptions--each quite reasonable--lead to widely differing estimates of n_o, the density of burst centers. If nearly every quasar (radio-quiet or radio-loud) produced at least one black hole and accompanying burst, then n_o is the density of dead quasars in the Universe:

$$n_o = \int_0^{t_o} \frac{n_{live}(t)}{t_{life}} \, dt \simeq \left(\frac{t_o}{t_{life}}\right) \left(\frac{1 \times 10^{-6}}{\text{Mpc}^3}\right) \int_0^1 10^{5\tau} d\tau = \left(\frac{0.8 \times 10^{-4}}{\text{Mpc}^3}\right) \left(\frac{t_o}{t_{life}}\right) .$$

Here $n_{live}(t)$ is the comoving density of live quasars at time t with $M_V < -22$ as estimated by Schmidt (1970, 1972); t_{life} is the mean lifetime of quasar activity in each center; and t_o is the present age of the Universe. Schmidt's evolution law, $n_{live} \propto 10^{5t/t_o} = e^{11.5t/t_o}$ requires an upper limit $t_{life} \leq t_o/11.5$; and a lower limit might be $t_{life} \geq$ (typical size, 100 kpc, of extended radio-emitting regions)/(typical velocity, 0.1 c, expected for expansion of extended regions in models of radio sources) $\simeq 3 \times 10^6$ yrs (cf. Longair, Ryle, and Scheuer 1973; Blandford and Rees 1974). Thus

$$1 \times 10^{-3} \text{ Mpc}^{-3} \leq n_o \leq 0.2 \text{ Mpc}^{-3} \qquad \text{if all quasars were burst centers.} \qquad (3)$$

Suppose, alternatively, that all galaxies with violently active nuclei produce supermassive holes and wave bursts during their violence. Presumably the violence seen in strong radio galaxies is sufficient. This leads to an n_o roughly the same as for quasars (eq. [3]) (Schmidt 1966, 1972).

Presumably the violence of a Seyfert galaxy also qualifies. The number density of Seyferts active today is ~ 2×10^{-4} Mpc^{-3} (Huchra and Sargent 1973 adjusted from $H_o = 75$ to $H_o = 100$). However, it is likely that Seyfert activity is transient, and that in the past a large fraction of all spiral galaxies with $M_V < -19$ experienced nuclear violence comparable to that in Seyferts ($n_o \sim 0.01$ Mpc^{-3}; Shapiro 1971). Arguments for such widespread violence in spirals include these: (i) Of all non-Virgo-cluster and non-local-group spiral galaxies with corrected recession velocities $v < 1000$ km/sec, ~ 10 per cent have small radio sources in their nuclei, and ~ 5 per cent are "optically active" in ways that could be remnants of earlier, more violent behavior (Ulrich 1974). (ii) The only big ($M_V < -19$) spirals close enough to be studied from the ground with optical resolution < 10 pc are our galaxy and M31 (Andromeda); and they both exhibit peculiar nuclear activity: (a) both show radial gas outflows suggestive of violent nuclear explosions in the past, perhaps 10^8 years ago (Münch 1960, Rubin and Ford 1969, Sanders and Prendergast 1974, and references therein); (b) M31, as viewed by Stratoscope II (Light, Danielsen, and Schwarzschild 1974), has a nucleus so compact ($M \sim 5 \times 10^7$ $M_\odot$; scale height 0.5 pc) that all of its stars more massive than ~ 2 $M_\odot$ may by now have sunk to the center, producing violence (cf. Spitzer and Schull 1975); (c) the central 3 pc of our own galactic nucleus contains a radio source with diameter ≤ 200 AU and luminosity $\simeq 10^{33}$ erg/sec at $\nu \sim 10$ GHz (Lo et al. 1975), which coincides with a discrete near-infrared source (Becklin and Neugebauer 1975 source 16) that does not show CO absorption (Becklin, private communication) and that therefore could be a cluster of red dwarfs emitting $L_{\text{total}} \simeq 10^5$ $L_\odot$ and congregating around a supermassive black hole (Sanders and Lowinger 1972); (d) one cannot rule out our nucleus containing a black hole of mass as large as $1 \times 10^8\ M_\odot$ (Sanders and Wrixon 1973).

If, as suggested by these observations, most spiral galaxies in the Universe have experienced enough nuclear violence in the past to produce supermassive holes and strong gravitational-wave bursts, n_o may be as large as $0.01\ \mathrm{Mpc}^{-3}$. If dwarf spirals ($M_V > -19$) also contain holes, but show little nuclear activity now because of a paucity of gas to accrete into the holes, then n_o could be $0.3\ \mathrm{Mpc}^{-3}$. On the other hand, if the only burst centers were the currently active Seyferts, then $n_o \sim 2 \times 10^{-4}$. Thus,

$$2 \times 10^{-4}\ \mathrm{Mpc}^{-3} \lesssim n_o \lesssim 0.3\ \mathrm{Mpc}^{-3} \quad \text{if galaxy nuclei are burst centers.} \tag{4}$$

Note the striking resemblance of equation (4) to equation (3)--a resemblance which suggests to Lynden-Bell (1969) that most galaxies were quasars at one time and now contain supermassive holes.

The average number of strong gravitational-wave bursts from each burst center is not likely to be large, perhaps

$$1 \leq N \lesssim 10, \tag{5}$$

because the gravitational and collisional effects of each massive object are likely to inhibit the formation and evolution-to-collapse of other massive objects. However, our ignorance here, like our ignorance of n_o, is enormous.

By combining equations (2) - (4) we arrive at a range of "reasonable values" for the time between gravitational-wave bursts at Earth

$$1\ \text{week} \lesssim \Delta t \lesssim 300\ \text{years}. \tag{6}$$

However, we must admit that hardly any strong bursts at all ($\Delta t >> 300$ years) is also reasonable; holes might usually form small and grow by gradual accretion.

III. EXPECTED CHARACTERISTICS OF THE BURSTS

Consider the collapse of a massive star or star cluster to form a black hole, or the collision between two black holes with impact parameter of the order of their size. Let the total mass involved be M and let the total energy radiated as gravitational waves be ϵMc^2. (A reasonable efficiency is $\epsilon \sim 0.1$). Most of the gravitational waves will come off in a burst with timescale $3\sqrt{3}\ GM/c$ at the source (see, e.g., Press 1971), and redshifted timescale at Earth

$$\tau \simeq (3\sqrt{3}\ GM/c^3)\ (1+z) = (90\ \mathrm{sec})\ (M/10^6\ M_\odot)\quad \text{if } z = 2.5. \tag{7}$$

The polarization-averaged dimensionless amplitude $\langle h\rangle \equiv (h_{ij}^{TT}\, h_{ij}^{TT}/4)^{1/2}$ of the waves at Earth can be computed from

$$(\epsilon\ Mc^2) = (\text{total energy radiated}) \simeq \left(\frac{c^3\langle h\rangle^2}{8\pi G\tau^2}\right)\ (4\pi R^2)\,\tau\quad (1+z) \tag{8}$$

$$= \begin{pmatrix}\text{energy flux}\\ \text{at Earth}\end{pmatrix} \times \begin{pmatrix}\text{surface area}\\ \text{around source}\end{pmatrix} \times \begin{pmatrix}\text{duration}\\ \text{of burst}\end{pmatrix} \times \begin{pmatrix}\text{blue-}\\ \text{shift}\end{pmatrix};$$

see equation (35.23) of MTW. Here R is as in equation (1). For $H_o = 100$, $q_o = 1$, $z = 2.5$ this gives

$$\langle h\rangle \simeq (6\sqrt{3}\ \epsilon)^{1/2}\ GM/Rc^2 = 2 \times 10^{-17}\ (\epsilon/0.1)^{1/2}\ (M/10^6\ M_\odot). \tag{9}$$

Table 1 shows the expected $\langle h\rangle$ and τ for sources of different mass M.

By observational studies of wave forms h(t) one may deduce much about the details of sources. For example: (i) If h(t) returns to its initial value after the burst, $h_{\text{final}} = h_{\text{initial}}$, then the source involved only one object (e.g., a single star or star cluster which collapsed to form a hole); but if $h_{\text{final}} \neq h_{\text{initial}}$, then the source involved two or more discrete objects (e.g., two black holes that collided and coalesced). (ii) If part or all of the burst is a rather monochromatic, sinusoidal signal, then the source may have been a rapidly rotating, relativistic star which went unstable by the "Chandrasekhar-Dedekind bar-mode process" (Chandrasekhar 1970). (iii) If part or all of the burst involves a rather sinusoidal signal with a variety of harmonics present, then the source may have been a rapidly rotating, highly relativistic star which went unstable by the "Friedman (1975) ergotoroid process". (iv) If the burst is entirely broad-band, except for a rapidly damped ringing at the end, the source may have been a star that encountered a "dynamical instability" (see, e.g., Friedman and Schutz 1975), and collapsed to form a black hole.

IV. DETECTION OF THE BURSTS BY DOPPLER TRACKING OF SPACECRAFT

In the next few years the best detector for the gravitational-wave bursts of Table 1 will probably be doppler tracking of one or more distant interplanetary spacecraft: A tracking station on Earth transmits monochromatic electromagnetic waves with frequency locked onto a highly stable clock (master oscillator). The spacecraft receives the waves, amplifies them, and transmits them back to Earth (i.e., it "transponds" them). The tracking antenna at Earth receives the transponded waves, compares them with the transmitted waves by counting the number of cycles and fractions thereof during an integration time t_{int}, and derives a frequency shift from the comparison. (For further detail see, e.g., Anderson 1973). The measured frequency shift is due primarily to the velocity of the spacecraft relative to Earth (doppler shift), but it also contains contributions ("noise") from frequency fluctuations and drift of the master oscillator, from time-varying dispersion of the electromagnetic waves in the Earth's ionosphere and troposphere and in the interplanetary medium, and from passing gravitational waves.

The gravitational-wave contribution as computed by Estabrook and Wahlquist (1975) can be described as follows. Orient the spatial axes of a Euclidean coordinate system so the gravitational waves travel in the z direction, and the Earth-spacecraft line of sight has length ℓ and lies in the x-z plane at an angle

θ relative to the z axis. Then the component of the gravitational waves which influences the doppler signal is $h_{xx}^{TT} \equiv h(t - z)$. [For the meaning of h_{xx}^{TT} see chapter 35 of MTW.] The influence of h on the doppler signal is given by

$$\frac{\Delta\nu}{\nu} = -\left(\frac{1 - \cos\theta}{2}\right) h_R - \cos\theta \, h_T + \left(\frac{1 + \cos\theta}{2}\right) h_E \tag{10}$$

(Estabrook and Wahlquist 1975). Here $\Delta\nu/\nu$ is the frequency shift of a specific short piece of the electromagnetic wave train--a piece received at Earth at time t_R; and h_E, h_T, h_R are the values of h encountered by that same piece of wave train at its moments of emission, transponding, and reception:

$$h_E \equiv h(t_R - 2\ell) \ , \quad h_T \equiv h\,(t_R - \ell[1 + \cos\theta]), \quad h_R \equiv h(t_R). \tag{11}$$

Two features of equation (10) are important: First, the magnitude of the gravitational-wave-induced frequency shift is $\Delta\nu/\nu \approx h$; second, (as Estabrook and Wahlquist emphasize) the gravitational wave-form is repeated three times in the Doppler signal, with relative amplitudes and time separations that depend on only one unknown parameter: the angle θ. This second feature may allow one to extract gravitational-wave effects in the presence of much larger doppler noise, and may also allow one to determine the angle θ. By simultaneous tracking of several spacecraft one could dig even deeper into the noise, and/or one could determine the 2-dimensional direction of the source.

When using single-frequency radio waves ("S-band", $\lambda \approx$ 14 cm), the NASA deep-space network can track spacecraft under favorable conditions (no planet or sun or high-density plasma cloud in the radio beam) with rms doppler residuals of[1]

---

[1] Our remarks here and below about NASA doppler capabilities and possible improvements are based largely on discussions with JPL personnel, including John D. Anderson, Richard Davies, Frank B. Estabrook, Richard Sydnor, Oldwig H. Vonroos, and Briant Winn.

$$\delta(\Delta\nu/\nu) \approx (3 \times 10^{-13}) \quad \text{for } t_{int} \gtrsim 7 \text{ min}, \tag{12}$$

where t_{int} is the integration time. This precision is far short of that required for detection of the wave bursts shown in Table 1 [$\delta(\Delta\nu/\nu) \approx h \sim 10^{-14}$ to 10^{-17}]. However--and this is the key point--there has not been much motivation for improvements in doppler tracking in recent years. Given the motivation, one could improve on the present precision by several orders of magnitude.

The following improvements, largely motivated by non-doppler considerations, are underway or could be implemented with small effort and cost: (i) conversion from single-frequency tracking to dual-frequency ("S-band" from Earth to spacecraft ["uplink"]; "S-band" and "X-band" [$\lambda \approx$ 3.6 cm] from spacecraft to Earth ["downlink"]; conversion recently completed); (ii) replacement of rubidium clocks by hydrogen-maser clocks as the master oscillators (replacement to be complete by summer 1976); and (iii) replacement of the present "doppler extractor," which is accurate to 10^{-2} cycles, by a new extractor accurate to 10^{-3} cycles (replacement not now planned, but easily doable and definitely needed for the doppler system to take full advantage of the new master oscillators). With these improvements the system residuals could drop as low as $\delta(\Delta\nu/\nu) \sim 2 \times 10^{-15}$ for $t_{int} \gtrsim 15$ min--though dispersion of the signal, even with dual-frequency monitoring, may make the overall residuals somewhat worse than this. By using the triplet structure of the gravitational-wave signal to dig into the noise, and by transponding off spacecraft more distant than about 10 A.U., one might be able to detect waves with $h \sim 1 \times 10^{-15}$ and $\tau \sim 70$ min (case II of Table 1). NASA could try this in late 1976 with Pioneer 10 and 11 (the two spacecraft which flew by Jupiter in 1973-74); though such an experiment will be degraded by the lack of dual-frequency capabilities in the Pioneer spacecraft. NASA could also try with the Mariner Jupiter-Saturn mission (1977 launch; Jupiter flyby in 1979).

Further doppler improvements might produce a capability for detecting case-III waves ($h \sim 1 \times 10^{-16}$, $\tau \sim 7$ min) and conceivably case-IV by late in this decade or early in the next. (For such waves one can use any spacecraft more distant than 1 A.U.--i.e., almost any interplanetary spacecraft at all). Among the possible improvements are: (i) X-band tracking on uplink as well as downlink (work on this begins in 1978); (ii) replacement of hydrogen-maser master oscillators by superconducting-cavity stabilized oscillators, which in 1974 had a stability $\delta(\Delta\nu/\nu) = 6 \times 10^{-16}$ for $t_{int} > 10$ sec (Stein and Turneaure 1975), which will probably reach $\delta(\Delta\nu/\nu) \approx 1 \times 10^{-16}$ in 1976 (Turneaure, private communication), and which--using niobium-coated sapphire cavities--may ultimately achieve stabilities much higher than 1×10^{-16} (Braginsky and Manukin 1974); (iii) the design and construction of new tracking-station electronics (doppler extractors, receivers, etc.) to go along with the improved master oscillators.

Buffeting of the spacecraft by fluctuating solar radiation pressure and solar wind will hide the gravitational-wave signals at some level (perhaps below case III?) despite their unique triplet structure. If necessary, one can reduce buffeting by using a drag-free ("conscience-guided") spacecraft (Kundt 1974) or a twin-probe spacecraft (Bertotti and Columbo 1972). The TRIAD drag-free spacecraft, which has already been developed, constructed, and flown once (Johns Hopkins and Stanford 1974) is sufficiently drag free ($a < 5 \times 10^{-9}$ cm/sec^2) to reduce time-averaged buffeting noise to $\delta(\Delta\nu/\nu) \approx 1 \times 10^{-17}$ (t_{int}/1 min)--a noise level comparable to the gravitational-wave signals of Table 1. However, the TRIAD "dead band" (maximum $\frac{1}{2}at^2$ displacement between firings of correction jets) was 1 millimeter--too large for gravitational-wave detection. A dead band of 0.1 μ would be needed to <u>completely</u> prevent buffeting at the level of case-IV wave bursts. A twin-probe spacecraft is not subject to such dead-band limitations.

Time-varying dispersion cannot be completely compensated for by dual-frequency ranging. One might ultimately remove dispersion in the Earth's atmosphere and near the Earth, as well as all noise sources at the tracking station (e.g., fluctuations in antenna dimensions), by tracking two spacecraft at different distances (e.g., 1 A.U. and 10 A.U.) but in the same line of sight, and by differencing their doppler signals (Davies 1974). And someday one might remove dispersion altogether by tracking at optical frequencies.

One might be able to search for case-II and case-III bursts without the enormous expense of special-purpose spacecraft (drag-free, twin-probe, or two-in-line-of-sight). Detailed studies are needed to determine this.

Simultaneous with any search for such gravitational-wave bursts one should also watch for associated electromagnetic events (radio, infrared, optical, or X-ray) and for possible redshift changes due to mass outflow in gravitational waves. The electromagnetic flux at Earth and apparent bolometric magnitude would be

$$F = \left(0.3\ \frac{\mathrm{erg}}{\mathrm{cm}^2\,\mathrm{sec}}\right)\left(\frac{E_{em}/E_{gw}}{\tau_{em}/\tau_{gw}}\right), \quad m_{BOL} = -10.2 - 2.5\,\log_{10}\left(\frac{E_{em}/E_{gw}}{\tau_{em}/\tau_{gw}}\right). \tag{13}$$

The duration of the electromagnetic event, τ_{em}, should be much longer than that of the gravitational-wave burst, τ_{gw}, because of absorption and reemission by gas surrounding the hole. The electromagnetic energy, E_{em}, might be far less than the gravitational energy, E_{gw}--or they might be comparable.

Even if the gravitational waves of Table 1 do not exist, doppler tracking should ultimately reveal waves at $h \sim 10^{-19}$ due to (i) close binary systems in our galaxy; (ii) the collapse of normal stars ($M < 100\ M_\odot$) to form black holes in nearby galaxies (distance < 10 Mpc); (iii) the capture of globular clusters, containing black holes of $10^3 - 10^4\ M_\odot$, by galactic nuclei at a distance ~ 1000 Mpc, followed by collisions and coalescences of the holes in the nuclei (cf. Bahcall and Ostriker 1975; Silk and Arons 1975; Tremaine, Ostriker, and Spitzer

1975); and/or near encounters between compact stars in a possible dense star cluster in the nucleus of our own galaxy (Zel'dovich and Polnarev 1974). In the case of close binary systems the periodicity of the signal can be used to enhance the accuracy of the experiment to an effective level of $h \sim 10^{-18}$ (Davies 1974, Estabrook and Wahlquist 1975).

V. SUGGESTIONS FOR FURTHER STUDY

In view of the above discussion, we suggest the following: (1) That space-research organizations (NASA, USSR Academy of Sciences, ESRO) initiate detailed studies of the expected limits which various doppler-tracking configurations can place on gravitational waves of the type shown in Table 1. (2) That, if those studies are as optimistic as our cursory analysis, high priority be given to doppler-tracking searches for gravitational-wave bursts. (3) That astronomers and astrophysicists try to firm up our estimates of the density n_o of centers where strong wave bursts were generated, and of the number N of bursts per center. (4) That relativity theorists develop a catalog of gravitational wave forms to be expected from the various objects and events that might occur in the nuclei of galaxies and quasars. (5) That experimenters try to develop alternative detection schemes for the waves shown in Table 1.

For helpful discussions or correspondence we thank the JPL personnel listed in footnote 1; also Eric Becklin, Peter Bender, Bruno Bertotti, Daniel de Bra, Peter Goldreich, Wolfgang Kundt, Charles Misner, Gerry Neugebauer, Jerry Ostriker, Wallace Sargent, Bill Saslaw, Maarten Schmidt, Lyman Spitzer, John Turneaure, and Tony Tyson. One of us (KST) thanks the Aspen Center for Physics for hospitality during part of this research.

TABLE 1

CHARACTERISTICS OF GRAVITATIONAL-WAVE BURSTS FROM BLACK-HOLE EVENTS INVOLVING A MASS M AT A REDSHIFT $z = 2.5$

	Case I	Case II	Case III	Case IV
Mass, M	$5 \times 10^{8}\ M_{\odot}$	$5 \times 10^{7}\ M_{\odot}$	$5 \times 10^{6}\ M_{\odot}$	$5 \times 10^{5}\ M_{\odot}$
Wave amplitude, h	1×10^{-14}	1×10^{-15}	1×10^{-16}	1×10^{-17}
Burst duration, τ	700 min.	70 min.	7 min.	40 sec.

REFERENCES

Anderson, J. D. 1973, in B. Bertotti, ed., *Experimental Gravitation*, (New York: Academic Press), p. 163.

Bahcall, J. N. and Ostriker, J. P. 1975, *Nature*, **256**, 23.

Becklin, E. E. and Neugebauer, G. 1975, *Ap. J. (Letters)*, **200**, L71.

Bertotti, B. and Columbo, G. 1972, *Ap. and Space Sci.*, **17**, 223.

Blandford, R. D. and Rees, M. J. 1974, *M.N.R.A.S.*, **169**, 395.

Braginsky, V. B. and Manukin, A. B. *Measurement of Small Forces in Physics Experiments* (Moscow: Nauka), p. 127; English translation by U. Chicago Press, in preparation.

Chandrasekhar, S. 1970, *Phys. Rev. Letters*, **24**, 611.

Davies, R. W. 1974, in *Ondes et Radiations Gravitionelles* (Paris: Editions de C.N.R.S.), p. 33.

Davis, M., Ruffini, R., Press, W. H., and Price, R. H. 1971, *Phys. Rev. Letters*, **27**, 1466.

Estabrook, F. B. and Wahlquist, H. D. 1975, *Gen. Rel. Grav.*, in press.

Friedman, J. L. 1975, paper in preparation.

Friedman, J. L. and Schutz, B. F. 1975, *Ap. J.*, in press.

Gunn, J. E., Gott, J. R., Schramm, D. N., and Tinsley, B. M. 1974, *Ap. J.*, **194**, 543.

Huchra, J. and Sargent, W. L. W. 1973, *Ap. J.*, **186**, 433.

Johns Hopkins University, Staff of the Space Science Department of the Applied Physics Laboratory, and Stanford University, Staff of the Guidance and Control Laboratory 1974, *J. Spacecraft*, **11**, 637.

Kundt, W. 1974, *Ap. & Space Sci.*, **30**, 455.

Light, E. S., Danielsen, R. E., and Schwarzschild, M. 1974, *Ap. J.*, **194**, 257.

Lo, K. Y., Schilizzi, R. T., Cohen, M. H., and Ross, H. N. 1975, *Ap. J. (Letters)*, in press.

Longair, M. S., Ryle, M., and Scheuer, P. A. G. 1973, *M.N.R.A.S.*, **164**, 243.

Lynden-Bell, D. 1969, *Nature*,**223**, 690

Misner, C. W., Thorne, K. S., and Wheeler, J. A. 1973, *Gravitation* (San Francisco: W. H. Freeman and Co.); cited in text as MTW.

Münch, G. 1960, *Ap. J.*, **131**, 250.

Press, W. H. 1971, *Ap. J. (Letters)*, **170**, L105

Rubin, V. C. and Ford, W. K. 1969, *Ap. J.*, **159**, 379.

Sandage, A. 1972, *Quart. J.R.A.S.*, **13**, 282.

Sanders, R. H. and Lowinger, T. 1972, *A. J.*, **77**, 292.

Sanders, R. H. and Prendergast, K. H. 1974, *Ap. J.*, **188**, 489.

Sanders, R. H. and Wrixon, G. T. 1973, *Astron. and Ap.*, **26**, 365.

Saslaw, W. C. 1974, in J. R. Shakeshaft, ed., *The Formation and Dynamics of Galaxies*, Proceedings of I.A.U. Symposium No. 58 (Dortrecht-Holland: Reidel), p. 305.

——————— 1975, in A. Hayli, ed., *Dynamics of Stellar Systems*, Proceedings of I.A.U. Symposium No. 59 (Dortrecht-Holland: Reidel), p. 379.

Schmidt, M. 1966, *Ap. J.*, **146**, 7.

——————— 1970, *Ap. J.*, **162**, 371.

——————— 1972, *Ap. J.*, **176**, 303.

Shapiro, S. L. 1971, *A. J.*, **76**, 291.

Silk, J. and Arons, J. 1975, *Ap. J. (Letters)*, **200**, L131.

Spitzer, L., Jr. 1971, in *Semain d'etude sur les noyeaux des galaxies*, Pontificae Academiae Scientarum, Script Varia no. 35, p. 443.

Spitzer, L., Jr., and Schull, J. M. 1975, Ap. J., in press.

Stein, S. R. and Turneaure, J. P. 1975, IEEE Proceedings, in press.

Tremaine, S. D., Ostriker, J. P., and Spitzer, L., Jr. 1975, Ap. J., **196**, 407.

Ulrich, M.-H. 1974, in J. R. Shakeshaft, ed., The Formation and Dynamics of Galaxies, Proceedings of I.A.U. Symposium No. 58 (Dortrecht-Holland: Reidel), p. 279.

Wolfe, A. M. and Burbidge, G. R. 1970, Ap. J., **161**, 419.

Zel'dovich, Ya. B. and Polnarev, A. G. 1974, Soviet Astron. - AJ, **18**, 17.