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7.4 Trim Drag in the Light of Munk's Stagger Theorem

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Abstract

Munk's stagger theorem holds that the induced drag of a multiplane is independent of the streamwise position (the stagger) of its lifting elements so long as the gap/span ratios and the element/element lift ratios are specified. In particular, a monoplane-tailplane or a monoplane-foreplane (canard) arrangement can be regarded as a biplane of zero gap and the trim drag due to tailplane download or foreplane upload can be readily calculated. The trim drag penalty is the same for both configurations. Relations are given for trim drag estimates for various practical arrangements.

Max Munk was one of the first generation of Goettingen aerodynamicists. Later he worked for the old NACA, and was largely responsible for the concept of, and the first test programs carried out in, the Variable Density wind tunnel. He contributed greatly to our present understanding of aerodynamic drag. While still at Goettingen he discovered some general laws about the induced drag of multiplanes, one of which is set forth in Figure 1.

Prandtl used this law as one of the cornerstones of a monograph on the "Induced Drag of Multiplanes", which appears in German in the Technical Reports (Technische Berichte) Vol. III, No. 7, pp. 309-315 of the aerodynamics research establishment at Goettingen. This report was immediately translated into English and published by the NACA as Technical Note No. 182 in 1924. Its contents also appear in Glauert's "Elements of Aerofoil and Airscrew Theory".

Figure 2 gives Prandtl's formula for the induced drag of a biplane. It is written as the sum of the self-induced drag of the elements of the biplane, plus twice the induced drag of one element due to the flow about the other for the case of an unstaggered array. Munk showed that the cross induced drags of the two elements were equal for an unstaggered biplane, but that also, by virtue of his stagger theorem, the <u>sum</u> of the cross induced drags was <u>unchanged</u> by stagger so long as the lift distribution between the elements is preserved. Thus the cross induced drag of the forward element of a biplane is reduced by the upwash about it due to the aft element; conversely, the cross induced drag of the aft element is increased by the downwash about it due to the forward element.

The magnitude of the cross induced drag is specified by the biplane interference factor, σ, defined by the relation on Figure 2. Its numerical value was calculated by Pohlhausen, who graphically evaluated an integral which gave the cross induced drag of one element of an unstaggered biplane carrying an elliptic span loading in the presence of the other element, also assumed to be elliptically loaded, and creating the downwash field appropriate to an elliptic span loading at the arbitrary location of the first element. The values of σ were evaluated for three discrete element span ratios and several gap to average span ratios. The results are presented in Figure 3. It might be worthwhile to refine Pohlhausen's calculations with a modern calculating machine. For our purposes it will suffice to note that σ approaches the span ratio in the limit as the gap/span ratio approaches zero.

Professor Ober of M.I.T., now emeritus, always taught his students (and I teach mine) that the induced drag of a monoplane-tailplane combination can be closely estimated by treating it as a staggered biplane of zero gap. Figure 4 presents some results of such a calculation. It is seen that the induced drag penalty for carrying a <u>download</u> of 10% of the total lift on the tailplane of a conventional airplane is slightly more than 10% of the minimum induced drag of the wing alone for a tailplane to wing span ratio of 0.3; surprisingly, the trim drag penalty for carrying an <u>upload</u> of 10% of the total lift on a canard foreplane of the same span ratio is identical. The trim penalty disappears for tailplane (or foreplane) span equal to the wing span, as one might expect.

Figure 5 compares the induced trim drag penalties for two representative wing-body-tailplane combinations in which the tail off pitching moment of the two wing body combinations differ only in the magnitude of the pitching moment about the wing aerodynamic center, the first example corresponding to a conventional NACA 4 digit airfoil, and the other corresponding to a heavily cambered airfoil of the Whitcomb supercritical, or general aviation type. It is seen that the drag penalties at the rather high total lift coefficient of 0.6 amount to about 0.2 counts and 2.2 counts, respectively; amounts which would be difficult to establish by wind tunnel testing.

Figure 6 presents an experimental verification of this technique for calculating the additional induced drag due to tail load by comparison with experimental drag data obtained on an 1/8 scale model of the XP-87 airplane during the course of wind tunnel tests conducted to determine the average downwash angle at the tailplane as a function of airplane angle of attack with deflected flaps. The experimental tail loads were large and the additional tail drag could be measured accurately.

The minimum induced drag of a wing (W) tail (H) configuration may be written as:

$$C_{D_{\substack{\text{induced}\\ \text{minumum}}}} = \frac{C^2_{L_W}}{\pi A\!\!R_W} + \frac{S_H}{S_W} \quad \frac{2\sigma}{\pi} \; \frac{C_{L_W} C_{L_H}}{(b_W b_H/S_W)} \; + \frac{C^2_{L_H}}{\pi A\!\!R_H}$$

where

$$A\!\!R_W = b^2 w/S_W$$

$$A\!\!R_H = b^2 H/S_H$$

In this particular case the drag of the complete airplane was computed from the relation

$$C_D = C_{D_{P_{wBNF}}} + \frac{C^2_{L_W}}{\pi A\!\!R_{We_{wBNF}}} + \frac{S_V}{S_W} c_{d_V} + \frac{S_H}{S_W} c_{d_H}$$

$$+ \frac{S_H}{S_W} \quad \frac{2\sigma}{\pi} \; \frac{C_{L_W} C_{L_H}}{(b_W/b_H/S_W)} + \frac{C^2_{L_H}}{\pi A\!\!R_H e_H}$$

where

$$C_{L_W} = C_L - \frac{S_H}{S_W} C_{L_H}$$

$C_{D_{P_{wBNF}}} = 0.0660$ $\qquad$ $e_{wBNF} = 0.877$

$A\!\!R_W = 6$

$S_V/S_W = 0.146$ $\qquad$ $c_{dV} = 0.01$

$S_H/S_W = 0.219$ $\qquad$ $c_{dH} = 0.01$

$b_H/b_W = 0.373$

$2 \text{ gap}/(b_W + b_H) = 0.0995 \rightarrow \sigma = 0.325$

$A\!\!R_H = 3.81, \; e_H = 0.85, \; b_W b_H/S_W = 2.236$

The experimental tail loads corresponding to the various tailplane incidences are given by

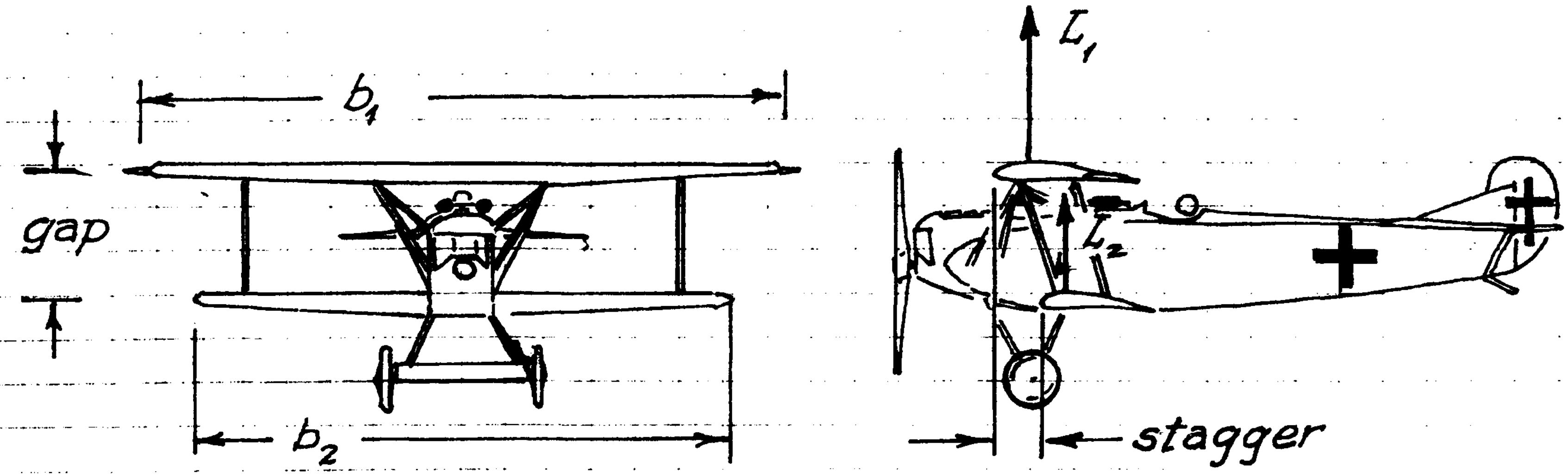
$$C_{L_H} = \frac{c\, S_W}{\ell_H S_H} \left(C_{m_{\text{tail off}}} - C_{m_{\text{tail on}}} \right) \qquad \alpha = \text{constant}$$

A comparison of the computed tail on drag for the complete airplane with the experimental drag shows that the cross induced drag term

$$\frac{S_H}{S} \frac{2\sigma}{\pi} \frac{C_{L_H} C_{L_W}}{(b_H b_W / S)}$$

is very important when the tail is carrying a download; its calculated value overcomes the skin friction drag of the tail assembly and the self induced drag of the horizontal tail itself at $C_{L_H} = -0.4$. The experimental data do not quite confirm this result: The experimental skin friction of the tail assembly and the self induced drag of the horizontal tail were underestimated; but note that the general shape of the complete airplane drag with tail plane lift curve is correctly predicted.

It is concluded that biplane theory presents a simple method for calculating tail drag, and that the trim drag penalties are generally small, for foreplanes or tailplanes of reasonable span and loading.



The induced drag of a multiplane is independent of the streamwise location of its lifting elements (the stagger) so long as its front view is unchanged (the gap/span ratios), and so long as the lift distribution between the elements (L_2/L_1) is preserved. (1918)

Figure 1. Munk's Stagger Theorem

$$D_{induced} = \frac{1}{\pi q}\left[\frac{L_1^2}{b_1^2} + 2\sigma\frac{L_1 L_2}{b_1 b_2} + \frac{L_2^2}{b_2^2}\right] \quad \left(\begin{matrix} \text{L. Prandtl} \\ 1923 \end{matrix}\right)$$

self-induced drag of wing 1

induced drag of wing 1 due to wing 2 plus induced drag of wing 2 due to wing 1.

self-induced drag of wing 2

$q = \frac{1}{2}\rho V^2$, flight dynamic pressure

σ = biplane interference factor

Figure 2. Induced Drag of Biplanes

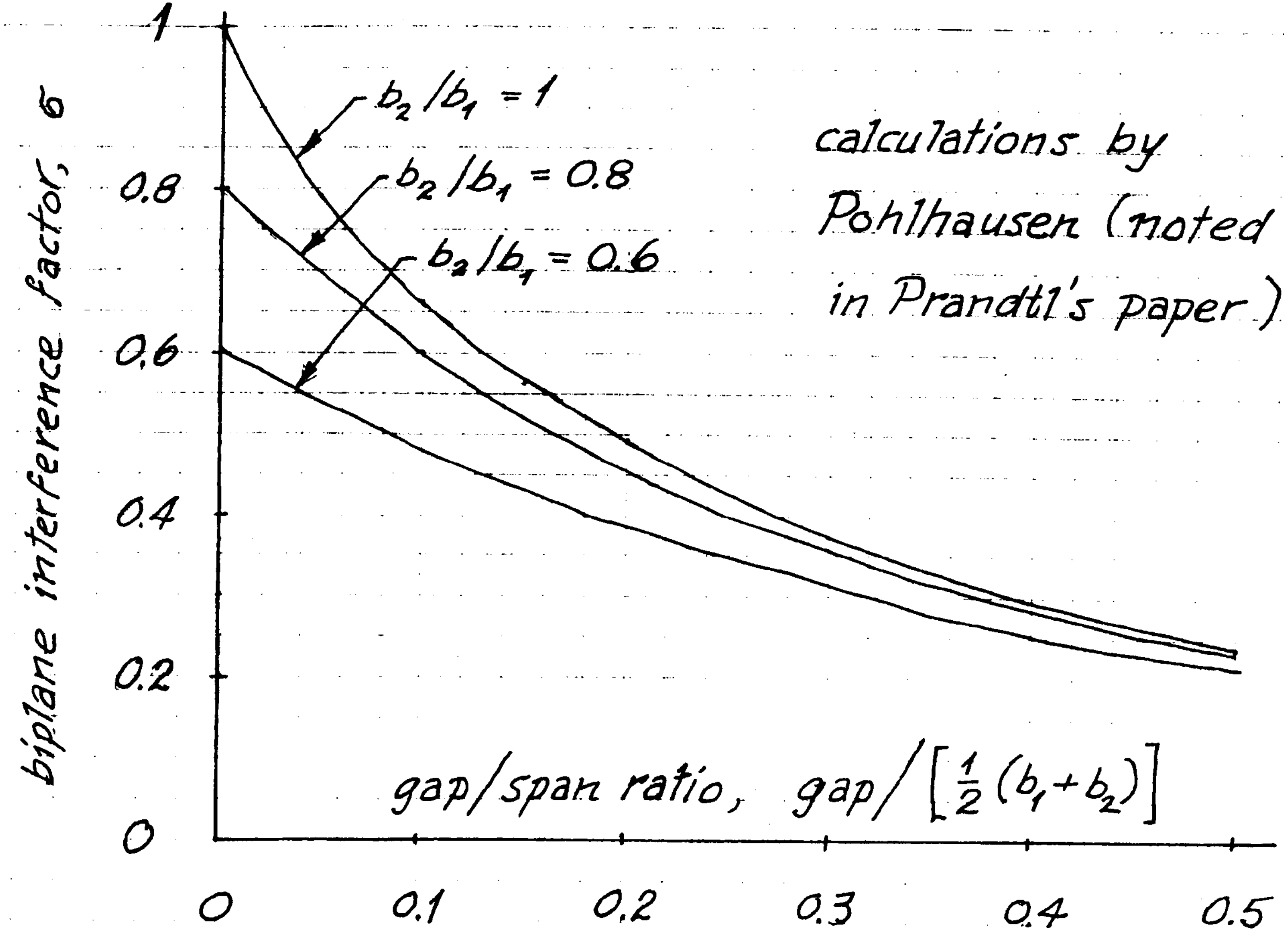
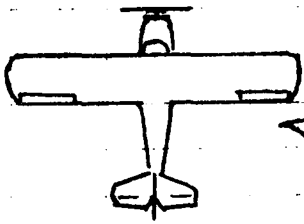


Figure 3. Biplane Interference Factor

DOWNLOAD FOR TRIM



staggered "biplanes" of zero gap

$b_1 = b \qquad b_2 = 0.3b$

$L_1 = 1.1L \qquad L_2 = -0.1L$

$\sigma \cong b_2/b_1 = 0.3$

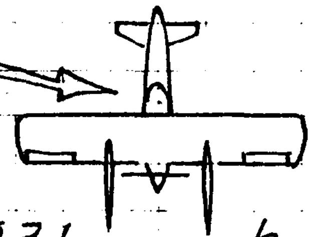
$$D_{ind} = \frac{L^2}{\pi q b^2}\left[1.21 - 0.22 + 0.1111\cdots\right]$$

$$= 1.1011\cdots\left(\frac{L^2}{\pi q b^2}\right)$$

$$= \frac{L^2}{\pi q b^2}\left[1 + \left(\frac{\delta}{r}\right)^2 - \delta^2\right]$$

δ = download fraction, r = span ratio

UPLOAD FOR TRIM



$b_1 = 0.3b \qquad b_2 = b$

$L_1 = +0.1L \qquad L_2 = 0.9L$

$\sigma \cong b_1/b_2 = 0.3$

$$D_{ind} = \frac{L^2}{\pi q b^2}\left[0.1111\cdots + 0.18 + 0.81\right]$$

$$= 1.1011\left(\frac{L^2}{\pi q b^2}\right)$$

$$= \frac{L^2}{\pi q b^2}\left[1 + \left(\frac{\epsilon}{r}\right)^2 - \epsilon^2\right]$$

ϵ = upload fraction r = span ratio

Figure 4. Trim Drag of Conventional vs Canard Monoplane

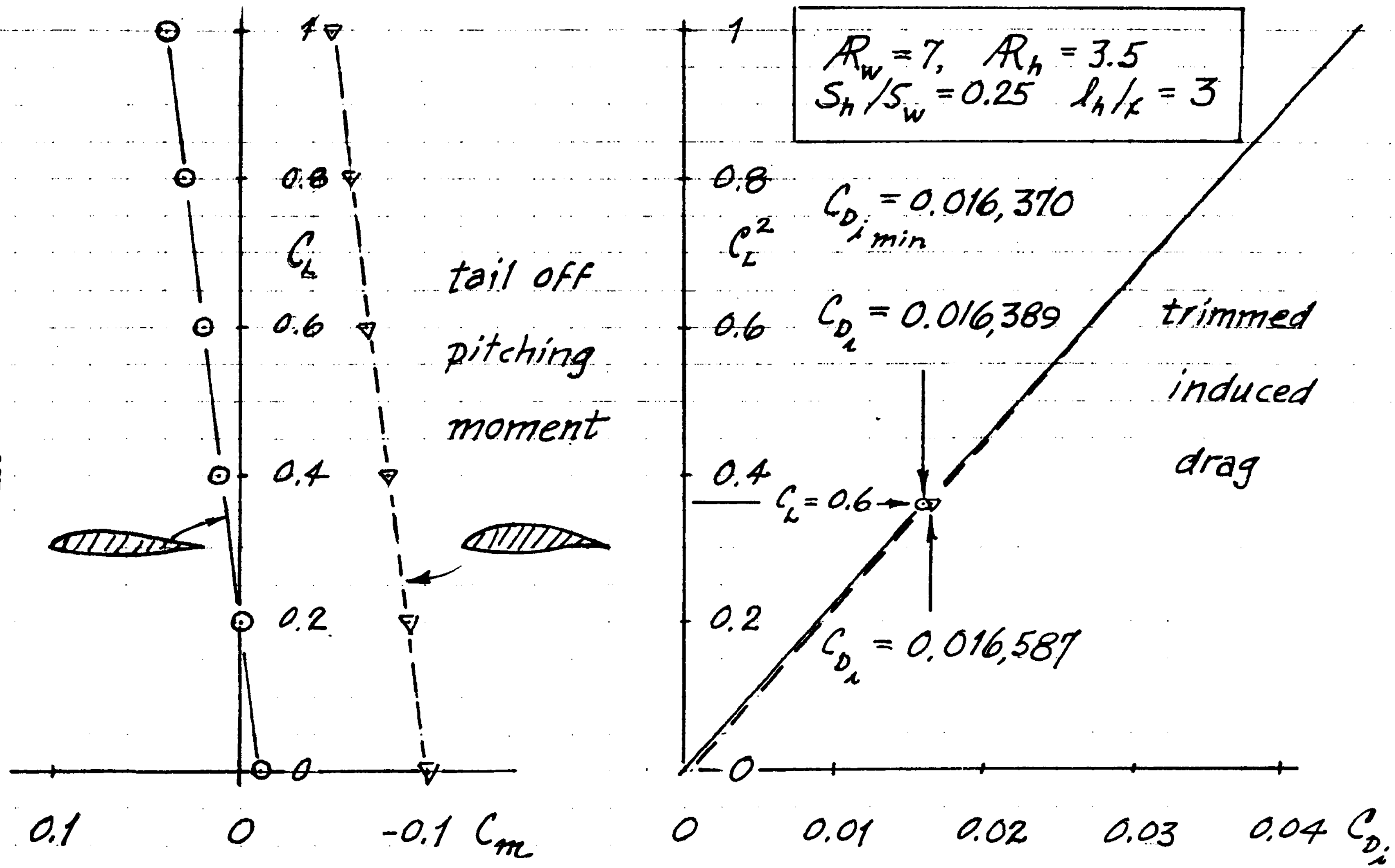


Figure 5. Trim Drag Penalty for Large Wing $C_{m_{ac}}$

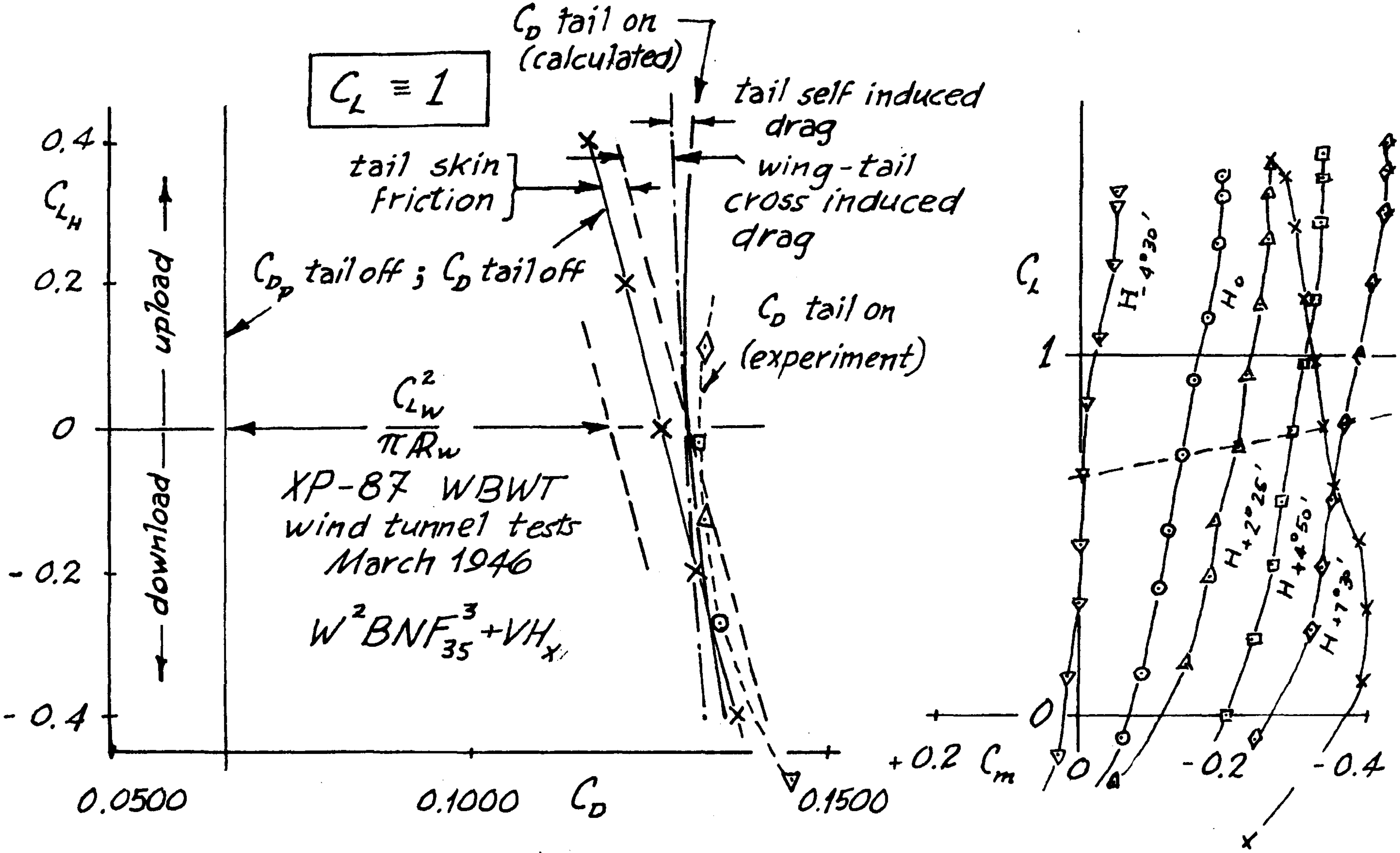


Figure 6. Calculated vs. Experimental Tail Drag

8. PAPERS OF SESSION VI - DRAG OF THE COMPLETE CONFIGURATION

8.1 Overview of Configuration Drag
Part I - Cost Considerations for Aircraft Configuration Changes
R. Tumlinson, Beech Aircraft Corporation
Part II - Aerodynamic Considerations
J. Roskam, University of Kansas

8.2 Learjet Model 25 Drag Analysis
R. Ross and R. D. Neal, Gates Learjet Corporation

8.3 Problems in Propulsion System Integration
W. Henderson and J. Runckel, NASA Langley Research Center

8.4 Determination of the Level Flight Performance of Propeller-Driven Aircraft
E. J. Cross, Jr., Mississippi State University