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THEORETICAL PERFORMANCE OF CROSS-WIND AXIS TURBINES WITH RESULTS FOR A CATENARY VERTICAL AXIS CONFIGURATION

By

Ralph J. Muraca, Maria V. Stephens and J. Ray Dagenhart

Langley Research Center
Hampton, Virginia

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SUMMARY

A general analysis capable of predicting performance characteristics of vertical axis windmills (VAW) has been developed. This analysis includes the effects of airfoil geometry, support struts, blade aspect ratio, windmill solidity, blade interference and curved flow. The results from the analysis are compared with available wind tunnel results for a catenary blade shape. A theoretical performance curve for an aerodynamically efficient straight blade configuration is also presented. In addition, a linearized analytical solution applicable for straight blade configurations is developed. This analysis is useful for parameter studies when the aerodynamic characteristics of the airfoil can be assumed to behave linearly with angle of attack. The errors introduced by this assumption are demonstrated by comparing numerical with analytical results. A listing of the computer program developed for numerical solutions of the general performance equations is included in the appendix.

SYMBOLS

AR	aspect ratio
A_s	swept or projected area in vertical plane
a	lift curve slope for two dimensional airfoil
C_D	drag coefficient
C_L	lift coefficient
C_M	moment coefficient
C_{M_a}	coefficient of the average moment produced during one revolution of the windmill
C_P	power coefficient
C_p	pressure coefficient
C_{p_L}	pressure coefficient - lower surface
C_{p_U}	pressure coefficient - upper surface
C_T	thrust coefficient
c	chord length
D	drag
d	chordwise distance from airfoil leading edge to center of pressure
h	height of windmill
K_1	lift curve slope including aspect ratio and downwash corrections
L	lift
l	length
M	moment

SYMBOLS CONTINUED

N	normal force
n	number of blades
P	power
P_t	total wind stream power
q	dynamic pressure
R	tip speed to wind speed ratio
Rey	Reynolds number
r	distance from axis of rotation to half chord point of blade element
r'	distance from axis of rotation to any particular point along blade chord line
s	spanwise distance
T_h	thrust
V	wind speed at windmill
V_r	wind speed relative to blades
V_∞	wind speed in free stream
x	chordwise distance from leading edge
y	vertical distance
α	angle of attack
β	local blade slope angle
ϵ	angle in plane of rotation between r and r' directions
Θ	angle of rotation from wind direction
ξ	chordwise distance from half chord point forward
ρ_∞	air density
ω	rotational speed

SYMBOLS CONTINUED

Subscripts:

a	average
m, max	maximum
r	relative
T	total
0	reference
∞	free stream

Superscripts:

—	nondimensional
•	time derivative

INTRODUCTION

As a consequence of the current shortages of fossil fuels interest is being revived in alternate sources of energy. One means for converting solar energy to a more useful form is through the use of windmills. Windmills may take many forms from drag devices to those which utilize the efficiency of airfoil shapes. An interesting survey of various types of windmills is given in reference 1. The most common windmill using airfoil shapes is the conventional multibladed horizontal axis type. These range from two blades to as high as twenty-seven blades depending on application, and the blades may range from propeller type to flexible sails. Analyses of horizontal axis windmills are well documented.

Another category of windmill is the vertical axis (VAW) or Darrieus windmill. The primary attraction of a VAW is the simplicity of manufacture compared to a HAW, and the manner in which the loads developed during operation are resisted by the structure. These differences indicate that if the efficiency of a VAW is equal to or not much less than a conventional windmill, use of a VAW might result in a more economical energy conversion system.

In recent years a small body of data has been published on the vertical axis windmill. Some of these reports present the results from tests of the vertical axis concept, ref. 2 and 3, while others present various analyses, structural, aerodynamic, and economic, of vertical axis windmills, references 4 through 7.

This report presents a general analysis of vertical axis windmills. Airfoil theory is used to determine the torque generated by the blades about the axis of rotation, and momentum theory is used to determine the effective wind velocity at the blades. These yield a set of integral equations which require an iterative approach for their solution. The nonlinear characteristics of the blade airfoils are accounted for in the general analysis. A more simplified analysis assuming linear aerodynamics and applicable to straight blade configuration is also presented.

The results of this analysis in addition to being in good agreement with available test data on one VAW concept indicate that the VAW can be designed such that their efficiency is comparable to the best horizontal axis designs.

This report presents the details of the general nonlinear analysis and the linearized analytical solutions for the straight blade concept. The result produced from the general analysis are compared with existing test data. The results from a parameter study of VAW's using the general analysis and the linearized solutions are compared and limitation of the linear analysis are identified. One straight blade configuration having significantly improved aerodynamic performance while maintaining the same structural capability of the tested configuration is also analyzed and the results are presented. Also included in the Appendix of this report is a listing, of a computer program for general analyses of vertical axis windmills along with input and results for a sample case.

GENERAL THEORY

In this section, general nondimensional equations for aerodynamic torque and average windmill thrust are developed by use of actuator disk theory and blade element aerodynamics. Real effects such as aspect ratio, downwash and flow curvature are accounted for. An average nondimensional power coefficient is defined and its relation to wind speed and average windmill torque is given.

Defining Equations

The basic elements of the vertical axis windmill are shown on figure 1. It is composed of an arbitrary number of blades (the figures are shown with two) attached to a rotating shaft which is nominally perpendicular to free stream velocity vector. The blades shown in the figure are straight; however, this is not a requirement. The blade cross section must be such that when subjected to an angle of attack a lift force is generated. The performance of the windmill is directly related to the ratio of this lift force to the blade drag. Since the angle of attack which the blades experience is cyclic the blade cross sections are restricted to symmetric shapes.

The coordinate system used in this analysis is shown in figure 2. The origin is located at the midpoint of the rotor with the y axis corresponding to the spin axis. The wind velocity is arbitrarily assumed to be directed along the x-axis. The rotor position as given by the angle Θ is measured from the x-axis. The local radius is measured from the y-axis and the local slope, β , is measured with respect to the radius vector.

The determination of local lift and drag characteristics of a blade element represents an unsteady flow problem. However, it has been assumed that the static aerodynamic data for the subject airfoil section is applicable.

Consider an arbitrary blade section as shown in figure 3. For a blade element the incremental forces acting in radial and tangential directions are derived as follows. The total velocity at any location on a blade is,

$$V_r^2 = (r\omega - V \sin \Theta)^2 + (V \cos \Theta)^2 \tag{1}$$

The angle of attack is given by

$$\tan \alpha = \frac{V \cos \Theta \sin \beta}{r\omega - V \sin \Theta} \tag{2}$$

The incremental forces developed as a consequence of the angle of attack are defined as, dL, in the direction normal to the relative velocity vector, and dD, in the direction of the relative velocity. These forces act to produce a moment about the axis of rotation given by

$$dM = (dL \sin \alpha - dD \cos \alpha)\, r + (dL \cos \alpha + dD \sin \alpha) \sin \beta\left(\frac{c}{2} - d\right) \tag{3}$$

The elemental lift and drag force can be written as

$$dL = C_L\, q\, cds \tag{4a}$$

and

$$dD = C_D\, q\, cds \tag{4b}$$

where the element length, ds, is given by $ds = dy/\sin\beta$.

Substituting eq. (4) into eq. (3) yields

$$dM = (C_L\, q\, c(dy/\sin\beta)\sin\alpha - C_D\, q\, c(dy/\sin\beta)\cos\alpha)r$$

$$+ (C_L\, q\, c(dy/\sin\beta)\cos\alpha + C_D\, q\, c(dy/\sin\beta)\sin\alpha)\sin\beta\left(\frac{c}{2} - d\right), \quad (5)$$

where $q = \frac{1}{2}\rho_\infty V_r^2$.

Integrating over y yields the total moment due to the blade as a function of the angle Θ.

$$M(\Theta) = \int_{-h/2}^{h/2} (C_L \sin\alpha - C_D \cos\alpha)\, \frac{rqc\, dy}{\sin\beta}$$

$$+ \int_{-h/2}^{h/2} (C_L \cos\alpha + C_D \sin\alpha)\, qc\left(\frac{c}{2} - d\right) dy \qquad (6)$$

The following nondimensionalization scheme is used; all velocities are nondimensionalized using the free stream velocity, V_∞, and all lengths are nondimensionalized using the maximum radius of the windmill blades r_{max}. The nondimensional variables are denoted by a bar, i.e. $\bar{V} = V/V_\infty$. With this notation equation (6) becomes,

$$M(\Theta) = \frac{1}{2}\rho_\infty V_\infty^2 (r_{max})^3 \int_{-\bar{h}/2}^{\bar{h}/2} (C_L \sin\alpha - C_D \cos\alpha)\, \frac{\bar{r}\,\bar{c}\,\bar{V}_r^2\, d\bar{y}}{\sin\beta}$$

$$+ \frac{1}{2}\rho_\infty V_\infty^2 (r_{max})^3 \int_{-\bar{h}/2}^{\bar{h}/2} (C_L \cos\alpha + C_D \sin\alpha)\, \bar{V}_r^2\, \bar{c}\left(\frac{\bar{c}}{2} - \bar{d}\right) d\bar{y} \qquad (7)$$

If a moment coefficient, C_M, is defined as

$$C_M \equiv \frac{M}{\frac{1}{2}\rho_\infty V_\infty{}^2 r_{max}{}^3 \bar{A}_s} \tag{8}$$

where $\bar{A}_s$ is the area swept out by the windmill then eq. (7) becomes

$$C_M(\Theta) = \frac{1}{\bar{A}_s}\int_{-\bar{h}/2}^{\bar{h}/2} [(C_L \sin\alpha - C_D \cos\alpha)\frac{\bar{r}}{\sin\beta}$$

$$+ (C_L \cos\alpha + C_D \sin\alpha)(\frac{\bar{c}}{2} - \bar{d})]\, \bar{c}\, \bar{V}_r{}^2\, d\bar{y}$$

This can be rewritten as

$$C_M(\Theta) = \frac{2}{\bar{A}_s}\int_{o}^{\bar{h}/2} \frac{\bar{c}\,\bar{r}\,\bar{V}_r{}^2}{\sin\beta} [(C_L \sin\alpha - C_D \cos\alpha)$$

$$+ \frac{\bar{c}}{\bar{r}}(\frac{1}{2} - \frac{\bar{d}}{\bar{c}}) \sin\beta\, (C_L \cos\alpha + C_D \sin\alpha)]\; d\bar{y} \tag{9}$$

Equation (9) holds for one blade, if there are n blades in the configuration the contribution for each blade can be obtained by replacing Θ in equations (1) and (2) by $\Theta + (i - 1)\,\Delta\Theta$ and re-evaluating equation (9).

Doing this the total moment coefficient can be written

$$C_{M_T}(\Theta) = \sum_{i=1}^{i=n} C_{M_i}(\Theta + (i-1)\,\Delta\Theta) \tag{10}$$

where $\Delta\Theta = \frac{2\pi}{n}$.

The integral of the total moment through one revolution of the windmill is given by

$$C_{M_N} = n \int_0^{2\pi/n} C_{M_T} (\Theta) \; d\Theta \tag{11}$$

The average moment is then given by

$$C_{M_a} = \frac{C_{M_N}}{2\pi} = \frac{n}{2\pi} \int_0^{2\pi/n} C_{M_T} (\Theta) \; d\Theta \tag{12}$$

The average power developed by the windmill is given by

$$P = \frac{\omega n}{2\pi} \int_0^{2\pi/n} M_T (\Theta) \; d\Theta \tag{13}$$

Defining a power coefficient C_p as

$$C_p = \frac{P}{P_t}$$

where

$$P_t = \frac{1}{2} \dot{m} \, V_\infty^{\;2} = \frac{1}{2} \rho_\infty \, A_s \, V_\infty^{\;3}$$

represents the total kinetic energy available per unit time in the stream tube. Using the definition of eq. (9), eq. (13) can be written in terms of the average moment coefficient as

$$P = \frac{1}{2} \rho_\infty (V_\infty^{2}) A_s C_{M_a} (r_m \omega) \tag{14}$$

thereby giving

$$C_p = \frac{(r_{max} \omega)}{V_\infty} C_{M_a}$$

or

$$C_p = R \, C_{M_a} \tag{15}$$

where

$$R \equiv \frac{(r_{max} \omega)}{V_\infty}$$

In nondimensional form equations (1) and (2) become

$$\bar{V}_r^{\,2} = (R \, \bar{r} - \bar{V} \sin \Theta)^2 + \bar{V}^2 \cos^2 \Theta \tag{16}$$

and

$$\tan \alpha = \frac{\bar{V} \cos \Theta \sin \beta}{\bar{r} \, R - \bar{V} \sin \Theta} \tag{17}$$

Momentum Loss and Windmill Thrust

The thrust acting on any device extracting energy from the wind is directly related to the change in momentum which occurs in the wind stream. Using momentum theory, reference 8 gives

$$T_h = 2\ \rho_\infty\ \bar{V}\ (1 - \bar{V})\ A_s\ V_\infty^{\ 2} \tag{18}$$

or

$$C_{T_a} = 4\ \bar{V}\ (1 - \bar{V})$$

where C_{T_a} is an average thrust coefficient and is defined as

$$C_{T_a} \equiv \frac{T_h}{\frac{1}{2}\ \rho_\infty\ V_\infty^{\ 2}\ A_s} \tag{19}$$

In terms of the elemental forces acting on a blade section the average thrust coefficient can be written

$$C_{T_a} = \frac{n}{2\pi} \int_{o}^{2\pi/n} \frac{2\bar{c}}{\bar{A}_s} \int_{-\bar{h}/2}^{\bar{h}/2} \frac{\bar{V}_r^{\ 2}}{\sin\beta}\ [(C_L \cos\alpha + C_D \sin\alpha) \cos\theta \sin\beta$$

$$+ (C_L \sin\alpha - C_D \cos\alpha) \sin\theta\ ] d\bar{y}\ d\theta \tag{20}$$

The determination of C_{T_a} and $\bar{V}$ for specified values of V_∞ requires an iterative procedure.

Real Effects

Although determining the flow field around a rotating windmill airfoil constitutes a 3 dimensional unsteady flow problem, the assumption has been made in this analysis that the flow is steady and that a two dimensional analysis can be used. However, a number of other effects including aspect ratio, downwash and curved flow can influence the rotor performance and some attempt has been made to evaluate these.

Aspect Ratio. - During a rotation through 360 degrees a blade section will experience large angles of attack as the ratio of $r\omega/V_\infty$ becomes small. When this occurs blade stall will result and to account for stall effects, it is necessary to use experimental data on section lift, drag and moment coefficients. These data are generally presented for infinite aspect ratio wings with appropriate aspect ratio correction being required. From reference 9 using Prandtl's lifting line theory, aspect ratio correction can be written as

$$C_L = C_{L_o} \,/\, \left(1 + \frac{a_o}{\pi\, AR}\right) \tag{21a}$$

and

$$C_D = C_{D_o} + \frac{C_L^{\,2}}{\pi\, AR} \tag{21b}$$

where C_{L_o} & C_{D_o} represents the lift and drag coefficient for an infinite aspect ratio wing, and $AR = h/c$.

Downwash. - For a multibladed configuration each blade is in the downwash from the preceding blade thus the angle of attack is reduced by this downwash angle. Again using the approach outlined in reference 9 the effect of downwash can be accounted for by using an aspect ratio correction. This yields

$$C_L = C_{L_o} \quad [\frac{1 - a_o / \pi \; AR}{1 + a_o / \pi \; AR}] . \qquad (22)$$

Flow Curvature. - Since the airfoil is traveling in a circular arc the angle of attack varies along the length of the chord. The effect of flow curvature is a function of the spin rate and the chord to radius ratio. This effect can be evaluated by considering the flow over a flat plate. The force distribution on a plate of unit width in a uniform stream of velocity V_∞, is given by

$$\frac{dN}{dx} = q \; (C_{p_L} - C_{p_U}) \qquad (23)$$

For a flat plate the velocity ratio at angle of attack is given by reference 10 as

$$\frac{V}{V_\infty} = \cos \alpha \;\; \pm \sin \alpha \;\; (\frac{c - x}{x}), \qquad (24)$$

where the (+) and (-) signs refer to the upper and lower surfaces, respectively. Now the relation between V/V_∞ and C_p is

$$C_p = 1 - (V/V_\infty)^2 \qquad (25)$$

Substituting yields

$$\frac{dN}{dx} = 4 \; q \cos \alpha \sin \alpha \;\; (\frac{c - x}{x}) \qquad (26)$$

Since the blade is actually moving along a curved path, the local angle of attack will vary. Referring to figure 4 the local angle of attack can be written as

$$\cot \alpha = \frac{r'\omega \cos \varepsilon - V_\infty \sin \theta}{V_\infty \cos \theta - r'\omega \sin \varepsilon}$$

or

$$\cot \alpha = \frac{r\omega - V_\infty \sin \theta}{V_\infty \cos \theta - \xi \omega}$$

or

$$\cot \alpha = \frac{R - \sin \theta}{\cos \theta - \xi R} \tag{27}$$

where $R = r\omega/V_\infty$.

In terms of the coordinate $\bar{x}$ measured from the leading edge of the plate this becomes

$$\cot \alpha = \frac{R - \sin \theta}{\cos \theta + R(\bar{x} - \bar{c}/2)} . \tag{28}$$

In terms of the lift distribution on the plate eq. (26) becomes

$$\frac{dL}{d\bar{x}} = 4 q \cos^2 \alpha \sin \alpha \left(\frac{\bar{c} - \bar{x}}{\bar{x}}\right). \tag{29}$$

In the limiting case $\bar{c} \to o$ and the expression for angle of attack becomes

$$\cot \alpha_o = \frac{R - \sin \theta}{\cos \theta} .$$

Defining a reference lift distribution as

$$\left(\frac{dL}{d\bar{x}}\right)_o = 4 q \cos^2 \alpha_o \sin \alpha_o \left(\frac{\bar{c} - \bar{x}}{\bar{x}}\right)$$

the effect of curvature can be displayed as a function of the ratio of lift distributions, with and without curvature effects. That is

$$\frac{(dL/d\bar{x})}{(dL/d\bar{x})_o} = \frac{\cos^2 \alpha \sin \alpha}{\cos^2 \alpha_o \sin \alpha_o} \tag{30}$$

The ratio of lift with and without curvature effects can be written

$$\frac{L}{L_o} = \frac{\int_o^{\bar{c}} \cos^2 \alpha \sin \alpha \, d\bar{x}}{\int_o^{\bar{c}} \cos^2 \alpha_o \sin \alpha_o \, d\bar{x}} = \frac{\int_o^{\bar{c}} \cos^2 \alpha \sin \alpha \, d\bar{x}}{\cos^2 \alpha_o \sin \alpha_o \, \bar{c}} \tag{31}$$

The numerator can be integrated giving

$$\frac{L}{L_o} = \frac{\frac{R - \sin \Theta}{R} (\cos \alpha_1 - \cos \alpha_2)}{\cos^2 \alpha_o \sin \alpha_o \, \bar{c}} \tag{32}$$

where

$$\alpha_o = \tan^{-1} \left(\frac{\cos \Theta}{R - \sin \Theta}\right)$$

$$\alpha_1 = \tan^{-1} \left(\frac{\cos \Theta - \frac{R\bar{c}}{2}}{R - \sin \Theta}\right)$$

$$\alpha_2 = \tan^{-1} \left(\frac{\cos \Theta + \frac{R\bar{c}}{2}}{R - \sin \Theta}\right)$$

This correction term has been applied to account for flow curvature effects. For most typical configurations the correction is less than one percent. However, if $\bar{c}$ becomes large this effect would become more important.

VERIFICATION OF GENERAL THEORY

In this section, the difficulties of modeling windmill performance using airfoil data with corrections for three dimensional flow effects are discussed. Theoretical results are compared to test data.

Reynolds Number, Roughness and Crossflow

As indicated in reference 11 the NACA 0012 airfoil falls into the category which experiences either leading edge or trailing edge stall depending on Reynolds Number.

Furthermore, the section characteristics are very dependent on surface roughness. When these effects are combined with the three dimensional nature of the flow over the windmill blades it becomes apparent that using two dimensional data at a fixed Reynolds Number represents a significant approximation.

Fortunately, use of two dimensional data yields a good approximation to the actual performance of a windmill; however, differences between measured and computed performance can easily be explained within the framework of these aerodynamics uncertainties. The data used in this analysis come from reference 12, however, some cases are presented in which these data have been modified to simulate leading edge stall and surface roughness.

Comparison with Test Data

The results predicted by the analysis for a two blade configuration ($n = 2$) were compared with test data from references

3 and 4. The blade shape is a catenary with maximum radius of 2.13 m (7 ft) and a height of 4.27 m (14 ft). The shape of the upper half of a blade is represented by the curve shown on figure 5. The test Reynolds number was about 0.3×10^6 consequently low Reynolds number data were used for the NACA 0012 airfoil. The airfoil data were obtained from reference 12, and a tabulation of lift and drag coefficient and center of pressure versus angle of attack is presented in table 1. These data are for a Reynolds Number of 0.5×10^6.

From the theoretical curve it is seen that the catenary VAW does not begin to produce sufficient power to overcome the drag due to stall until a velocity ratio of about 2.5 is reached. At this point the power coefficient increases with spin rate until a peak value of about .37 is reached. The power coefficient then decreases until at a velocity ratio of $R \simeq 10$ it reaches zero. This determines the no load spin rate at which the windmill would operate.

The test data follows the same trends; however, there are differences between these results. The test data seem to indicate that stall occurred at a higher velocity ratio and more abruptly than predicted, possibly indicating leading edge stall. Also, the no load velocity ratio reached in the tests was slightly greater than 8 indicating the zero lift drag was greater than predicted by smooth airfoil data. To demonstrate the effect of these variations on the predicted performance curve a computation was made using a zero lift drag coefficient representative of a rough surface.

Increasing C_{D_o} reduces the peak efficiency somewhat and causes the no load tip speed ratio to drop to a value of about 8.5. To demonstrate the effect of leading edge stall at low Reynolds number the lift and drag coefficient data were modified to reflect a sudden stall at an angle of attack of 10 degrees. These changes cause the peak value of power coefficient to be reduced slightly, and to occur at a higher tip speed ratio. Also, the no load point drops to about 8.6. For the catenary configuration the angle of attack variation along the blade is shown on figure 7 for $R = 4$ and $R = 5$, and at one angular location $\theta = 0$. As can be seen the angle of attack is almost constant over the majority of the blade and doesn't increase appreciable until the last 15 percent of the blade is reached. However, the contribution of this part of the blade to the total power production is small. Shown on figure 8 is the variation of peak angle of attack with R at the maximum radius. From this it can be seen that at values of $R \leq 4.5$ the peak angle of attack exceeds 10^o consequently stall begins to set in. Since the angle of attack is essentially constant over the major part of the blade the whole blade stalls at about the same time thus causing the rapid reduction in the power coefficient curve as shown in figure 6.

If a system could be built with no losses other than aerodynamic its starting characteristics would consist of an initial rotation caused by the differential drag on the two blades. However, the angles of attack experienced would be higher than the stall angle and limiting spin rate would be reached where the moment generated by the lift force would just balance that generated by the drag force. To get the windmill over this point some energy must be added to the system. Once the spin rate attains a value high enough such that the net torque due to lift is greater than that due to drag the system will spin up to the velocity ratio corresponding to no load condition.

The computed results for the case n = 2 are also shown plotted on figure 9 versus the induced velocity at the blades $\tilde{V}$. Included on this figure is a plot of the theoretical maximum curve obtained by setting the drag coefficient of the airfoil to zero.

As indicated previously no power is available until R reaches a value of about 2.5. As the spin rate increases the power curve follows the trend of the theoretical maximum curve although it is displaced somewhat. As the spin rate increases to values greater than about 6 the drag losses become more important since the angles of attack are decreasing and the contribution of the lift forces to the turning moment become of the same order as the retarding effects of the drag forces. Consequently the actual curve departs from the theoretical maximum by a large amount.

LINEAR THEORY FOR STRAIGHT BLADE CONFIGURATIONS

By restricting the possible windmill configurations to those with straight blades and by assuming that the blade aerodynamic coefficients vary linearly with angle of attack, closed form analytical expressions for windmill performance parameters are obtained. These expressions are used to study the effects on windmill performance of variations in aspect ratio and solidity.

Reduction to Analytical Form

Consider the special case of a straight blade configuration where $\bar{r} = 1.0$ and $\beta = 90^{\circ}$, both constant with respect to $\bar{y}$. From figure 3 we have

$$\bar{V}_r \sin\alpha = \bar{V}\cos\Theta \tag{33a}$$

and

$$\bar{V}_r \cos\alpha = R - \bar{V}\sin\Theta \tag{33b}$$

Also, if only small angles of attack are considered

$$C_L = K_1 \sin\alpha \tag{34a}$$

$$C_D \simeq C_{D_o} \tag{34b}$$

With these approximations the trust coefficient for one blade becomes

$$C_T(\Theta) = \frac{\bar{c}}{2}\{[K_1 \sin\alpha\cos\alpha\cos\Theta + K_1 \sin^2\alpha\sin\Theta] + C_{D_o}\ [\sin\alpha\cos\Theta - \cos\alpha\sin\Theta]\}\ \bar{V}_r^{\,2} \tag{35}$$

Substituting for $\bar{V}_r$ yields

$$C_T(\Theta) = \frac{\bar{c}}{2} \quad \{ K_1 \quad [\bar{V} (R - \bar{V} \sin \Theta) \cos^2 \Theta + \bar{V}^2 \cos^2 \Theta \sin \Theta]$$

$$+ \bar{V}_r \, C_{D_o} \quad [\bar{V} - R \sin \Theta]\} \tag{36}$$

and integrating over Θ an average thrust coefficient C_{T_a} for n blades is obtained,

$$C_{T_a} = \frac{n\bar{c}}{4} \; K_1 \; \bar{V}R + \frac{n\bar{c}}{4\pi} \int_\Theta \; [(R - \bar{V} \sin \Theta)^2$$

$$+ (\bar{V} \cos \Theta)^2]^{\frac{1}{2}} \; (\bar{V} - R \sin \Theta) \; C_{D_o} \; d \, \Theta \tag{37}$$

Simplifying by assuming $(R - \bar{V} \sin \Theta) \gg \bar{V} \cos \Theta$ this becomes

$$C_{T_a} \simeq \frac{n\bar{c} \; K_1 \; \bar{V}R}{4} + \frac{n\bar{c} \; C_{D_o}}{4\pi} \int_\Theta \; (R - \bar{V} \sin \Theta) \; (\bar{V} - R \sin \Theta) \; d \, \Theta$$

or

$$C_{T_a} = \frac{n\bar{c}}{4} \, R \, K_1 \, \bar{V} + 3 \, \frac{n\bar{c}}{4} \; \bar{V} \, R \, C_{D_o} \tag{38}$$

From momentum theory we have

$$C_{T_a} = 4 \, \bar{V} \; (1 - \bar{V})$$

therefore, equating these yields an expression for $\bar{V}$, the nondimensional velocity at the blade

$$\bar{V} = 1 - \frac{n\bar{c} \, R}{16} \quad (K_1 + 3 \, C_{D_o}) \tag{39}$$

Using eq. (9) with $\bar{r} = 1$ and $\beta = 90^{o}$, the moment coefficient $C_M(\Theta)$ can be written as

$$C_M(\Theta) = \frac{2\,\bar{Y}_m}{\bar{A}_s}\,\bar{c}\,\bar{V}_r^{\,2} \quad [(C_L \sin\alpha - C_D \cos\alpha)$$

$$+ \bar{c}(.5 - \bar{d}/\bar{c})\ (C_L \cos\alpha + C_D \sin\alpha)] \tag{40}$$

but for a straight blade configuration $\bar{A}_s = 2\,\bar{h}$

Using the relationships between $\bar{V}_r$ & $\bar{V}$ in equation (33)

yields

$$C_M(\Theta) = \frac{\bar{c}\,K_1}{2}\,[\,\bar{V}^2 \cos^2\Theta + \bar{c}\,(.5 - \bar{d}/\bar{c})\,\bar{V}\cos\Theta\,(R - \bar{V}\sin\Theta)]$$

$$+ \frac{\bar{c}}{2} \quad (R - \bar{V}\sin\Theta)^2 + \bar{V}^2\cos^2\Theta\,[\bar{c}\,(.5 - \bar{d}/\bar{c})\ C_{D_o}\ \bar{V}\cos\Theta$$

$$- C_{D_o} \quad (R - \bar{V}\sin\Theta)] \tag{41}$$

Restricting this analysis to cases in which $R - \bar{V}\sin\Theta >> \bar{V}\cos\Theta$ equation 41 can be integrated over Θ to yield the average moment C_{M_a} as

$$C_{M_a} = \frac{n\bar{c}\,K_1}{4}\ \bar{V}^2 - \frac{n\bar{c}\ C_{D_o}}{4}\ (2R^2 + \bar{V}^2) \tag{42}$$

The power coefficient C_P is given by $C_P = R\,C_{M_a}$, substituting yields

$$C_p = R \frac{n\bar{c}\,K_1}{4} \left(1 - \frac{n\bar{c}\,R}{16} \left(K_1 + 3\,C_{D_o}\right)\right)^2$$
$$- \frac{n\bar{c}\,C_{D_o}}{4} \left[2R^2 + \left(1 - \frac{n\bar{c}\,R}{16} \left(K_1 + 3\,C_{D_o}\right)\right)^2\right] \qquad (43)$$

where

$$K_1 = a_o \left[\frac{1 - a_o/\pi\,AR}{1 + a_o/\pi\,AR}\right]$$

Now for a straight blade

$$AR = \frac{h}{c} = \bar{h}/\bar{c} \qquad (44)$$

therefore,

$$K_1 = a_o \left(\frac{1 - a_o\,\bar{c}/\pi\,\bar{h}}{1 - a_o\,\bar{c}/\pi\,\bar{h}}\right) \qquad (45)$$

Equation (43) can be used to evaluate the effects of $n\bar{c}$, $\bar{h}$, and C_{D_o} on performance of windmills. For the case where $C_{D_o} = 0$ the windmills performance reaches the theoretical maximum. This case along with a case using a representative value of C_{D_o} are shown in figure 10. Both C_p and R are shown as a function of velocity ratio $\bar{V}$. Also shown on this figure are the results of the numerical analysis using nonlinear aerodynamics. As would be expected the linear and nonlinear results are in fairly good agreement at the higher spin rates; however, comparing these with the limiting value given by the $C_{D_o} = 0$ curve indicates that significant improvements in performance could be realized if the airfoil drag could be reduced. For low spin rates the analytical and numerical results are in poor agreement due to the stall effects. The use of linear aerodynamics in the analytical solution causes the drag term to appear erroneously small so that the analytical results approach the theoretical maximum for low spin rates. This is due to the fact that the airfoil experiences high angles of attack and the contribution of the lift force is orders of magnitude greater than the retarding effect of the zero lift drag force.

Parameter Study

The analytical solutions can be used to evaluate the effects of solidity, and windmill aspect ratio on power coefficient. For small values of solidity, which is proportional to $n\bar{c}$, and for high tip speed ratios these results are adequate to identify trends; however, at low tip speed ratios the effects of nonlinear airfoil characteristics become dominant and the analytical results are inaccurate. Figure 11 shows the analytical results as a function of solidity at a fixed aspect ratio. Power coefficient is plotted versus tip speed ratio and as indicated on the figure peak efficiency increases as solidity increases; however, the peak value occurs at a lower tip speed ratio and the range of tip speed ratios at which power can be supplied by the windmill varies inversely with solidity. Figure 12 shows the variation of power coefficient with $\bar{h}$ as tip speed ratio varies. As can be seen $\bar{h}$ does not have a major effect on windmill performance for the range of values cited. In general as $\bar{h}$ increases the aspect ratio of each blade increases consequently its performance improves. However, due to structural considerations it is desirable to design a windmill with as small a value of $\bar{h}$ as possible without drastically and impairing performance.

When a numerical analysis based on the general theory is used to determine the effect of solidity on performance a major difference occurs for small values of tip speed ratio. These results are shown on figure 13. Since in this analysis the occurrence of airfoil stall is not dependent on solidity the minimum speed ratio at which power is produced is essentially the same for all configurations. Using the aerodynamic data for the NACA 0012 airfoil this value of tip speed ratio is about 2.5. The efficiency increases rapidly as tip speed ratio increases from this minimum value. However, for some values of solidity the peak efficiency would occur at tip speed ratios less than this minimum value were it not for stall effects. Consequently, with stall effects included, these peak values are never achieved. If a windmill could be designed using a blade which did not stall or stalled at a higher angle of attack it might be possible to achieved the higher performance indicated by the analytical solution. However, it remains questionable if one would want a design which responded so radically to changes in wind velocity.

CONFIGURATION EFFECTS ON AERODYNAMIC EFFICIENCY

Considering only aerodynamic efficiency the optimum VAW design as depicted in figure 14a would consist of straight blades attached to the center shaft by low drag struts. The windmill aspect ratio must be large enough to minimize losses due to blade aspect ratio. Unfortunately such a design leads to severe structural limitations due to the high bending loads which develop in the blade. It is for this reason that curved blades have been selected for designs currently in operation. With a curved blade the bending loads are greatly reduced and the centrifugal loads are reacted as almost pure tension loads in the blades. However, it is possible to design a straight bladed VAW which has the same structural capability as a curved blade VAW. This can be accomplished by supporting the blade at appropriate intervals as shown in figure 14c. Secondly, by placing the support struts at an angle other than 90 degrees from the axis of rotation they will produce enough lift force such that their lift induced torque is greater than the drag induced torque and they in effect contribute to the performance of the windmill.

The results of such a design are shown in figure 15 along with a comparison of the catenary VAW performance. For this configuration the support struts and the primary blade are an NACA 0012 airfoil. The strut to blade chord ratio is 1/2. As the data indicates the peak power coefficient for the chevron design is about .43 compared to the value of .37 for the catenary. However, the catenary can produce power over a wider range of spin ratios than the chevron.

No attempt has been made to optimize the chevron design and one would expect that for higher values of h/r some improvement in performance could be expected. Also, the higher the angle of the support struts the greater will be their contribution. It is also possible to select a support strut angle such that the angle of attack at some point on the strut is the same as the angle of attack at the blade.

CONCLUSIONS

The results from the analysis presented herein indicate that a numerical solution using non-linear aerodynamic data and assuming a uniform induced velocity over the complete volume swept by the windmill can be used to predict performance of vertical axis windmills. The major parameters affecting windmill performance are the blade shape, blade aspect ratio and downwash effects. The effects of curved flow are insignificant providing the ratio of blade chord to diameter is small. The use of linear aerodynamics allows the defining torque equation to be integrated; however, these analytic solutions are incorrect at lower values of spin parameter where the aerodynamics are nonlinear. They are useful for assessing trends at higher spin parameter values. The performance of a VAW can be improved by using straight rather than curved blades provided the blade is supported in the proper manner. However, optimization of any VAW design will ultimately be based on the costs per installed unit of power consequently not only aerodynamic but also structural considerations must be taken into account.

REFERENCES

1. Wilson, Robert E.; and Lissaman, Peter B. S.: Applied Aerodynamics of Wind Power Machines. National Science Foundation, Oregon State University, May 1974.

2. South, P.; and Rangi, R. S.: Preliminary Tests of a High Speed Vertical Axis Windmill Model, National Research Council Canada, LTR-LA-74. March 1971.

3. South, P.; and Rangi, R. S.: A Wind Tunnel Investigation of 14 Ft. Diameter Vertical Axis Windmill, Canadian National Research Council, LTR-LA-105. September 1972.

4. Templin, R. J.: Aerodynamic Performance Theory for the NRC Vertical Axis Wind Turbine, National Research Council of Canada, LTR-LA-160. June 1974.

5. Feltz, L. V.; and Blackwell, B. F.: An Investigation of Rotation Induced Stresses of Straight and of Curved Vertical Axis Wind Turbine Blades. Sandia Laboratories, March 1975.

6. Weingarten, L. I.; and Nickell, R. E.: Nonlinear Stress Analysis of Vertical Axis Wind Turbine Blades. Sandia Laboratories, April 1975.

7. Vadot, L.: A Synoptic Study of Different Types of Windmills, La Houille Blanche, No. 2, March/April 1957.

8. Dommasch, Daniel O.; Sherby, Sydney S.; and Cannolly, Thomas F.: Airplane Aerodynamics. Pitman Publishing Corporation, 1967.

9. Millikan, Clark B.: Aerodynamics of the Airplane. John Wiley and Sons, Inc., 1941.

10. Riegels, Friedrich W.: Aerofoil Sections, Butterworth and Co., LTD., 1961.

11. Gault, Donald E.: A Correlation of Low-Speed, Airfoil Section Stalling Characteristics with Reynolds Number and Airfoil Geometry, NACA TN-3963, March 1957.

12. Critzos, Chris C.; Heyson, Harry H.; and Boswinkle, Robert W., Jr.: NACA 0012 Report - Aerodynamic Characteristics of the NACA 0012 Airfoil Section at Angles of Attack from 0° to 180°, NACA Technical Note 3361.

APPENDIX

WINDMILL PERFORMANCE PREDICTION COMPUTER PROGRAM

Order of Input

1. Header Card, Columns 1 through 80.

2. Namelist $NAM1 including:

Fortran Symbol	Engineering Symbol	Definition
AR	AR	Aspect ratio of blade
ARS	AR_s	Aspect ratio of lifting struts
AZ	a_z	Section lift curve slope
BETA(I)	β	Array of local slope of blade for each YBAR(I)
BETS	β_s	Angle of lifting struts
CBAR	$\bar{c}$	Nondimensional blade chord
CBARS	$\bar{c}_s$	Chord of lifting and nonlifting struts
CD(I)	C_D	Array of Section drag coefficient of blade and struts for each GAM(I)
CD1	C_{D_1}	Zero lift drag coefficient of nonlifting struts
CK1	CK1	Number of nonlifting struts
CL(I)	C_L	Array of Section lift coefficient of blade and struts for each GAM(I)
CMC4(I)	$C_{M_{c/4}}$	Quarter chord pitching moment coefficient
DY	DY	Increment used to calculate y coordinate array for blade, from YMIN to YMAX.
DYS	DY_s	Increment used to calculate Y coordinate array for struts from YSMIN to YSMAX.
GAM(I)	γ	Array of angle of attack, where I = 1, NGAM.

Fortran Symbol	Engineering Symbol	Definition
ITHPRNT		Total number of THPRNT values, maximum is 10.
N	n	Number of blades
NGAM		Total number of GAM values, maximum is 100.
NS	n_s	Number of lifting struts, if none set = 0.
NTHETA		Total number in THETA array, maximum is 200.
NVR		Total number in VRRY array, maximum is 10.
NY		Total number in YBAR array, maximum is 50.
RBAR(I)	$\bar{r}$	Array of windmill blade radius for each YBAR(I).
TC	τ	Blade airfoil thickness ratio.
THETA(I)	θ	Array of windmill angular coordinate, degrees, where I = 1, NTHETA.
THPRNT(I)		Array of THETA values at which the lift, drag and moment values can be printed, where I = 1, ITHPRNT.
VRRY(I)	R	Array of specified velocity ratios at which the windmill is to be analyzed, where I = 1, NVR.
YBAR(I)	$\bar{y}$	Array of coordinates along the axis of rotation of the windmill, where I = 1, NY.
YMAX	$\bar{Y}_{max}$	Maximum Y coordinate of blade.
YMIN	$\bar{Y}_{min}$	Minimum Y coordinate of blade.
YSMAX	$\bar{Y}_{smax}$	Maximum y coordinate of lifting struts
YSMIN	$\bar{Y}_{smin}$	Minimum y coordinate of lifting struts

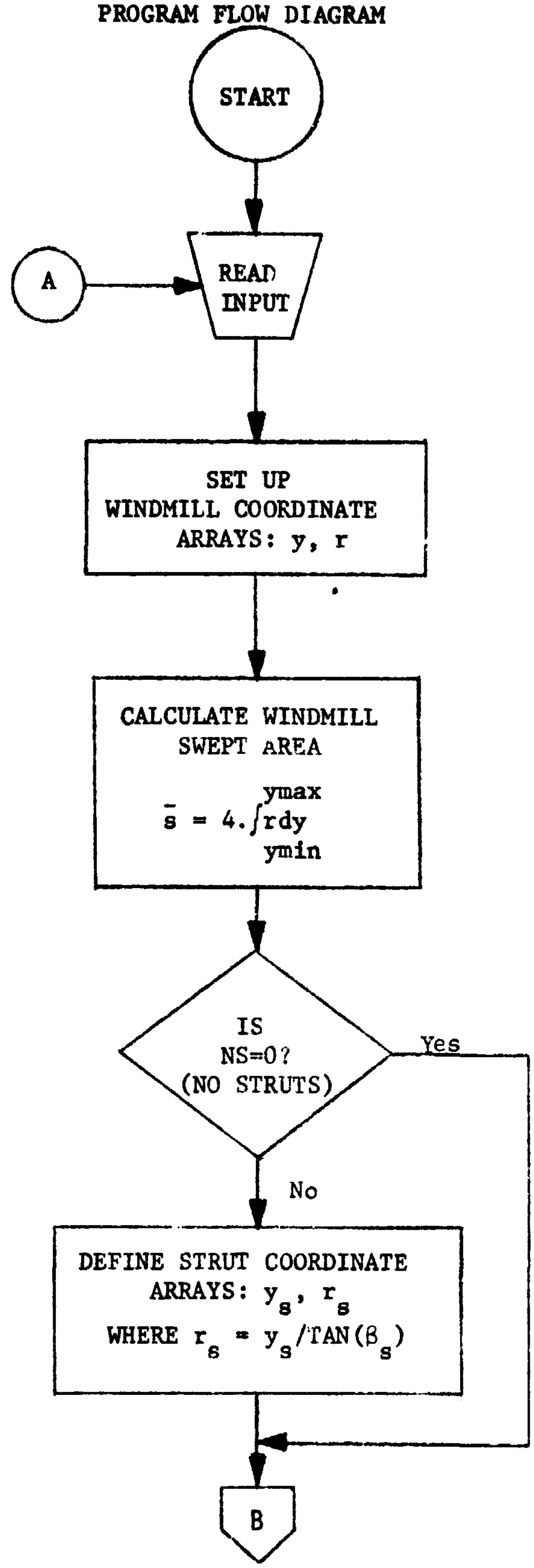
Output Parameters

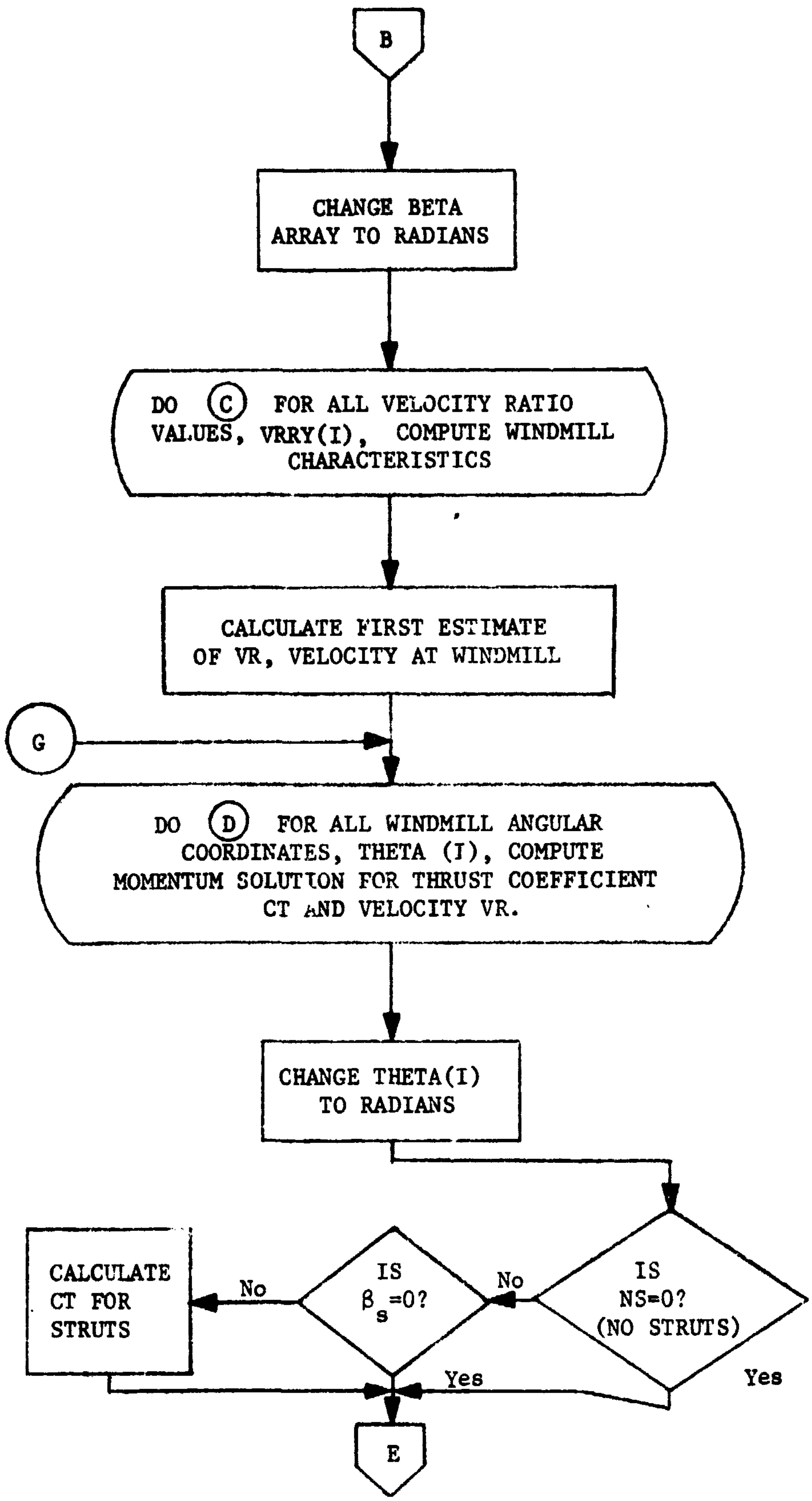
Output Symbol	Engineering Symbol	Definition
BARCT	C_{T_a}	Windmill thrust coefficient averaged over a revolution
CD	C_D	Airfoil section drag coefficient
CL	C_L	Airfoil section lift coefficient
CM(THETA)	$C_M(\Theta)$	Blade moment coefficient for angular location Θ
CMA	C_{M_a}	Average windmill moment coefficient
CMAS	$C_{M_{as}}$	Average moment coefficient due to struts
CMC4	$C_{M_{c/4}}$	Airfoil section quarter chord moment coefficient
CMS(THETA)	$C_{M_s}(\Theta)$	Strut moment coefficient for angular location Θ
CMT	$C_{M_T}(\Theta)$	Total moment coefficient for n blades at angular location Θ
CMTS	$C_{M_{T_S}}(\Theta)$	Total moment coefficient for all struts at angular location Θ
CPA	C_{P_a}	Power coefficient due to lifting blades and struts
CPL	C_{P_1}	Power coefficient loss due to nonlifting struts
CPN	C_{P_n}	Net power coefficient
D/C	d/c	Location of center of pressure measured chordwise from the airfoil leading edge in fractions of the chord length

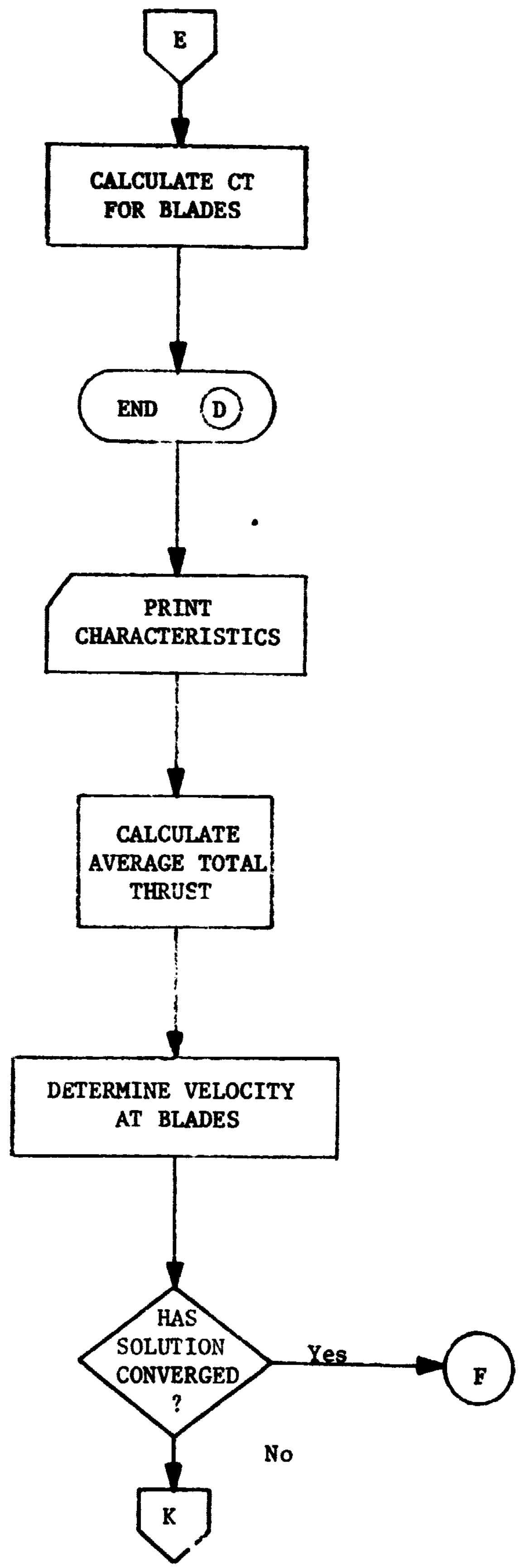
Output Symbol	Engineering Symbol	Definition
DEL CM	ΔC_M	Incremental moment coefficient due to a blade element
GAMMA	γ	Local blade angle of attack
OMEGA	ω	Windmill rotational speed
RMAX	r_{max}	Maximum windmill radius
SBAR	$\bar{S}$	Nondimensional windmill projected area in a vertical plane
THETA	Θ	Angle of windmill rotation measured from the free wind direction
VINF	V_∞	Wind stream speed at a large distance from the windmill
VR	V	Wind stream speed at the windmill

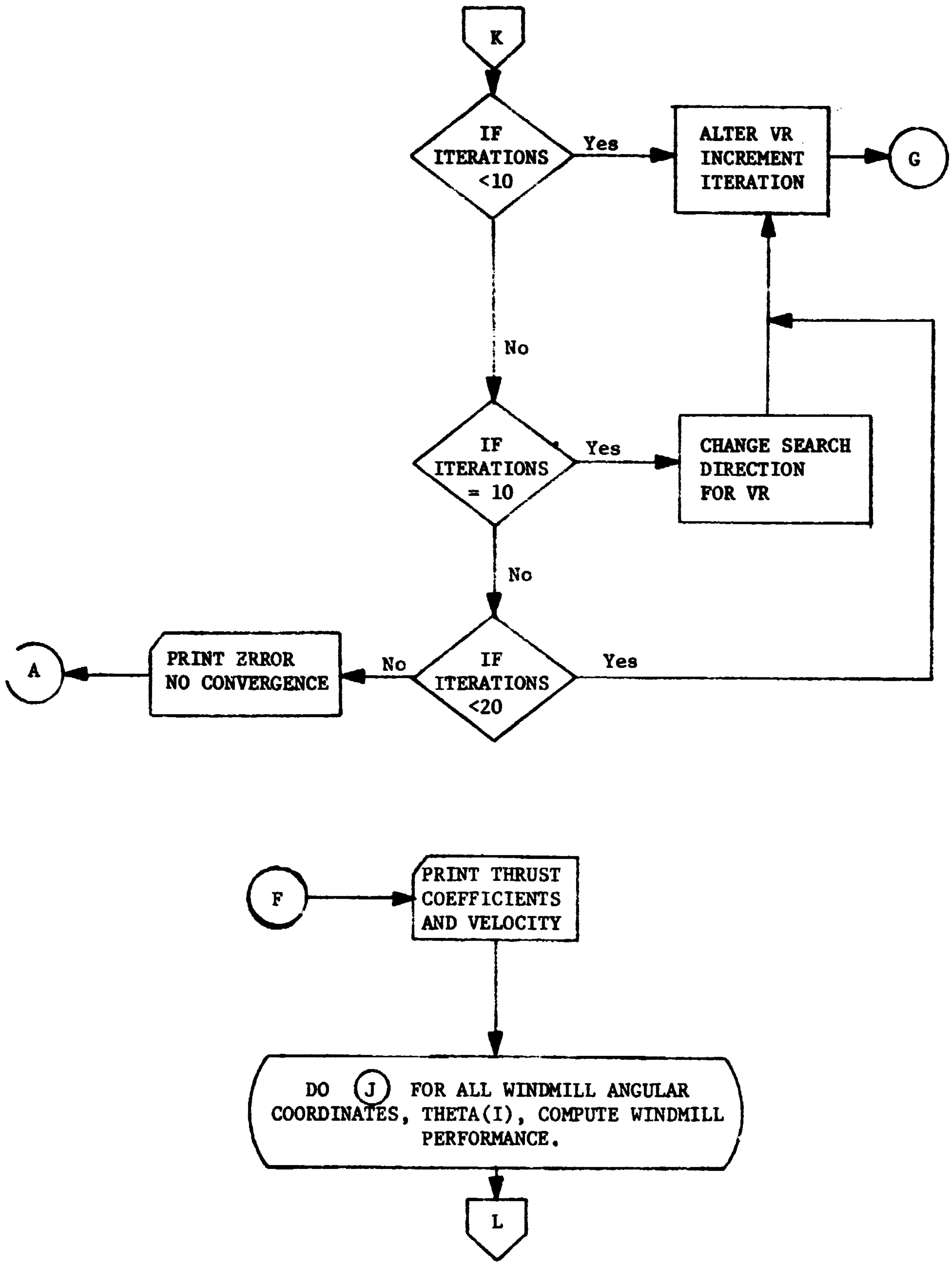
APPENDIX - CONTINUED

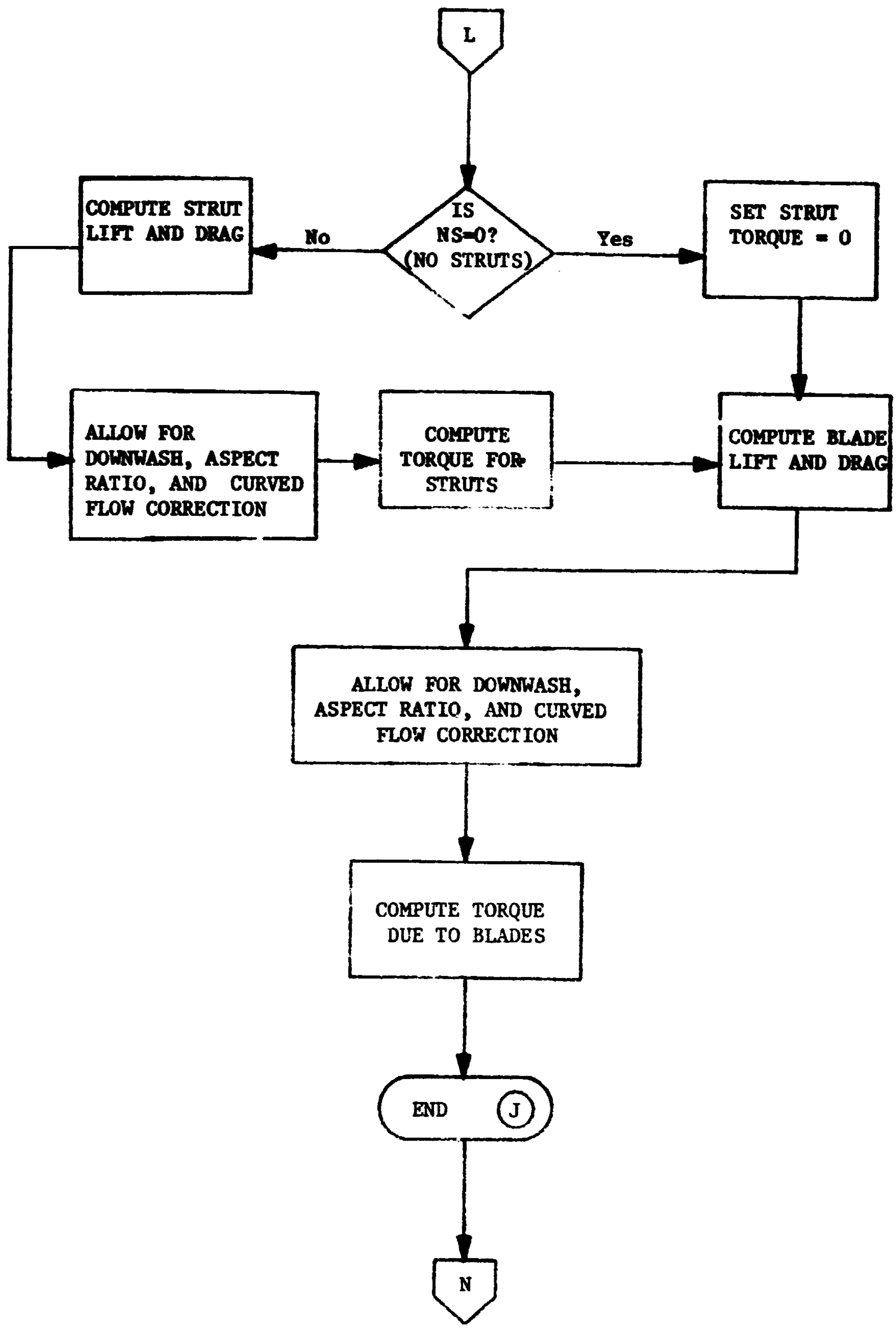
PROGRAM FLOW DIAGRAM

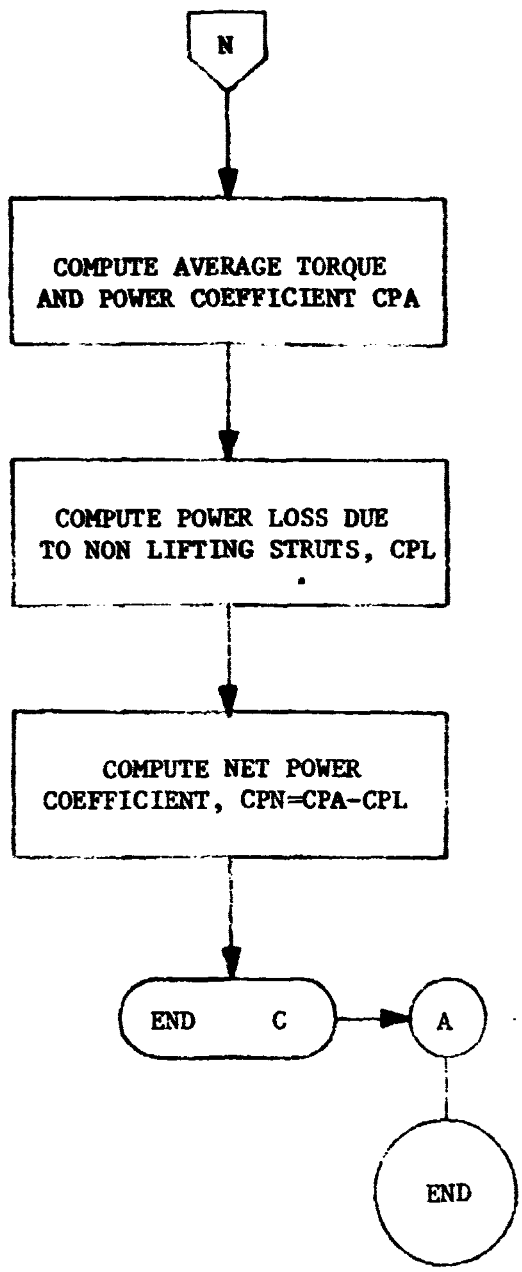



B
CHANGE BETA ARRAY TO RADIANS
DO C FOR ALL VELOCITY RATIO VALUES, VRRY(I), COMPUTE WINDMILL CHARACTERISTICS
CALCULATE FIRST ESTIMATE OF VR, VELOCITY AT WINDMILL
G
DO D FOR ALL WINDMILL ANGULAR COORDINATES, THETA (I), COMPUTE MOMENTUM SOLUTION FOR THRUST COEFFICIENT CT AND VELOCITY VR.
CHANGE THETA(I) TO RADIANS
IS NS=0? (NO STRUTS)
No
IS β_s=0?
No
CALCULATE CT FOR STRUTS
Yes
Yes
E


E
CALCULATE CT
FOR BLADES
END
D
PRINT
CHARACTERISTICS
CALCULATE
AVERAGE TOTAL
THRUST
DETERMINE VELOCITY
AT BLADES
HAS
SOLUTION
CONVERGED
?
Yes
F
No
K


K
IF ITERATIONS <10
Yes
ALTER VR INCREMENT ITERATION
G
No
IF ITERATIONS = 10
Yes
CHANGE SEARCH DIRECTION FOR VR
No
IF ITERATIONS <20
Yes
No
PRINT ZRROR NO CONVERGENCE
A
F
PRINT THRUST COEFFICIENTS AND VELOCITY
DO J FOR ALL WINDMILL ANGULAR COORDINATES, THETA(I), COMPUTE WINDMILL PERFORMANCE.
L


L
IS
NS=0?
(NO STRUTS)
No
Yes
COMPUTE STRUT LIFT AND DRAG
SET STRUT TORQUE = 0
ALLOW FOR DOWNWASH, ASPECT RATIO, AND CURVED FLOW CORRECTION
COMPUTE TORQUE FOR STRUTS
COMPUTE BLADE LIFT AND DRAG
ALLOW FOR DOWNWASH, ASPECT RATIO, AND CURVED FLOW CORRECTION
COMPUTE TORQUE DUE TO BLADES
END
J
N


N
COMPUTE AVERAGE TORQUE AND POWER COEFFICIENT CPA
COMPUTE POWER LOSS DUE TO NON LIFTING STRUTS, CPL
COMPUTE NET POWER COEFFICIENT, CPN=CPA-CPL
END C
A
END

APPENDIX - CONTINUED

COMPUTER PROGRAM LISTING

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```
      PROGRAM A4425(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C   TITLE - WINDMILL EFFICIENCY ANALYSIS
C
C   ENGINEERS- RALPH MURACA AND RAY DAGENHART
C   PROGRAMMER - MARIA STEPHENS
C
      EXTERNAL CLCL0
      DIMENSION BETA(50), CD(100), CL(100), CM(100), CMC4(100), CT(200),
     1CTS(200), GAM(100), HEADER(8), MC(100), PSI(50), RBAR(50),
     2RBETA(50), RR(200), RS(100), SUM(200), SUMCM(200), SUMCMS(200),
     3SUMCTS(200), SUMS(200), THETA(200), THPRNT(10), VRBAR(50),
     4VRRM(20), VRRY(20), YBAR(50), YR(200), YS(100)
      COMMON/GAMSUB/CAPR
      NAMELIST/NAM1/AR,ARS,AZ,BETA,BETS,CBAR,CBARS,CD,CD1,CK1,CL,CMC4,
     1DY,DYS,GAM,  ITHPRNT,N,NGAM,NS,NTHETA,NVR,NY,RBAR,TC,THETA,
     2THPRNT,VRRY,YBAR,YMAX,YMIN,YSMAX,YSMIN
C
C   ORDER OF I N P U T
C
C   1. HEADER CARD
C
C   2. NAMELIST NAM1 INCLUDING-
C
C   AR       - ASPECT RATIO OF BLADE
C   ARS      - ASPECT RATIO OF LIFTING STRUTS
C   AZ       - SECTION LIFT CURVE SLOPE
C   BETA     - LOCAL SLOPE OF BLADE FOR EACH YBAR
C   BETS     - ANGLE OF LIFTING STRUTS
C   CBAR     - NONDIMENSIONAL BLADE CHORD
C   CBARS    - CHORD OF LIFTING AND NONLIFTING STRUTS
C   CD       - SECTION DRAG COEFFICIENT OF BLADE   AND STRUTS FOR EACH GAM
C   CD1      - ZERO LIFT DRAG COEFFICIENT OF NONLIFTING STRUTS
C   CK1      - NUMBER OF NONLIFTING STRUTS
C   CL       - SECTION LIFT COEFFICIENT OF BLADE AND STRUTS FOR EACH GAM
C   CMC4     - QUARTER CHORD MOMENT COEFFICIENT
C   DY       - INCREMENT USED TO CALCULATE Y COORDINATE ARRAY FOR BLADE
C              FROM YMIN TO YMAX
C   DYS      - INCREMENT USED TO CALCULATE Y COORDINATE ARRAY FOR STRUTS
C              FROM YSMIN TO YSMAX
C   GAM      - ANGLE OF ATTACK ARRAY
C   ITHPRNT  - TOTAL NUMBER OF THPRNT VALUES. MAXIMUM IS 10.
```

```
      C   N        - NUMBER OF BLADES
      C   NGAM     - TOTAL NUMBER OF GAM VALUES, MAXIMUM IS 100
      C   NS       - NUMBER OF LIFTING STRUTS, IF NONE SET =0
      C   NTHETA   - TOTAL NUMBER IN THETA ARRAY, MAXIMUM IS 200
      C   NVR      - TOTAL NUMBER IN VRRY ARRAY, MAXIMUM IS 10
      C   NY       - TOTAL NUMBER IN YBAR ARRAY, MAXIMUM IS 50
      C   RBAR     - WINDMILL BLADE RADIUS FOR EACH YBAR
      C   TC       - BLADE AIRFOIL THICKNESS RATIO
      C   THETA    - ARRAY OF WINDMILL ANGULAR COORDINATES,DEGREES,
      C   THPRNT   - ARRAY OF THETA VALUES AT WHICH THE LIFT, DRAG, AND MCMENT
      C              VALUES CAN BE PRINTED
      C   VRRY     - ARRAY OF SPECIFIED VELOCITY RATIOS AT WHICH THE WINDMILL IS TO BE
      C              ANALYSED
      C   YBAR     - ARRAY OF COORDINATES ALONG THE AXIS OF ROTATION OF THE WINDMILL
      C   YMAX     - MAXIMUM Y COORDINATE OF BLADE
      C   YMIN     - MINIMUM Y COORDINATE OF BLADE
      C   YSMAX    - MAXIMUM Y COORDINATE OF LIFTING STRUTS
      C   YSMIN    - MINIMUM Y COORDINATE OF LIFTING STRUTS
      C
      C
000003       CBARS=0.
000004       RADIANS=.0174532925
      C
      C   READ HEADER CARD AND INPUT DATA
      C
000006     1 READ(5,104)HEADER
000014       IF(EOF,5)2,3
000017     2 STOP
000021     3 READ(5,NAM1)
000024       DO 5 I=1,NGAM
000026       G = GAM(I)*RADIANS
000030       CN = CL(I)*COS(G)+CD(I)*SIN(G)
000040       DELTA = 0.01*ABS(CN)
000042       IF (-DELTA.LE.CN.AND.CN.LE.DELTA) GO TO 7
000054       GO TO 6
000054     7 HC(I) = 0.25
000056       GO TO 5
000057     6 HC(I) = 0.25 - CMC4(I)/CN
000063     5 CONTINUE
000066       WRITE(6,105)HEADER
000073       WRITE(6,NAM1)
000076       WRITE(6,115)(GAM(I),CL(I),CD(I),CMC4(I),HC(I),I=1,NGAM)
      C
```

```
              C    SET UP WINDMILL COORDINATES RR VS. YR
              C
000121                I=0
000122                YR(1)=YMIN
000124             10 I=I+1
000126                CALL FTLUP(YR(I),RR(I),1,NY,YBAR,RBAR)
000132                YR(I+1)=YR(I)+DY
000135                IF(YR(I+1).LT.YMAX)GO TO 10
000140                I=I+1
000141                YR(I)=YMAX
000142                RR(I)=RBAR(NY)
000145                IT=I
              C    CALCULATE SBAR - WINDMILL SWEPT AREA
              C
000146                CALL TRAP(RR,YR,IT,SUM,0.)
000151                SBAR=4.*SUM(IT)
000154                WRITE(6,106)SBAR
000161                BARCTS=0.
              C
              C    SET UP STRUT COORDINATES RS VS. YS
              C
              C    NS=0, NO STRUTS
000162                IF(NS.EQ.0)GO TO 19
000163                BETSR=BETS*RADIANS
000165                TB=TAN(BETSR)
000167                YS(1)=YSMIN
000171                K=1
000172             15 RS(K)=YS(K)/TB
000175                K=K+1
000176                YS(K)=YS(K-1)+DYS
000200                IF(YS(K).LT.YSMAX)GO TO 15
000203                YS(K)=YSMAX
000204                RS(K)=YS(K)/TB
000206                KS=K
000207             19 CONTINUE
000207                DO 20 I=1,NY
000211                RBETA(I)=RADIANS*BETA(I)
000213             20 CONTINUE
              C
              C    BEGIN COMPUTATION OF WINDMILL CHARACTERISTICS FOR EACH VELOCITY RATIO-VRRY
              C
000215                DO 70 IVR=1,NVR
000217                CAPR=VRRY(IVR)
```

```
000221          WRITE(6,109)CAPR
000226          VVR=1./CAPR
000230          ITER=1
          C
          C   FIRST ESTIMATE OF VR, VELOCITY AT THE WINDMILL
          C
000231          VR = 1. - N*CBAR*3.14159*CAPR/8.
000237       25 CONTINUE
          C
          C   CALCULATE MOMENTUM SOLUTION FOR THRUST COEFFICIENT CT AND VELOCITY VR
          C
000237          DO 40  I=1,NTHETA
000241          TH=THETA(I)*RADIANS
000243          IF(NS.EQ.0)GO TC 35
000244          IF(BETS.EQ.0.)GO TO 35
          C
          C   CALCULATE CT FOR STRUTS
          C
000245          DO 30 K=1,KS
000247          CALL GAMMA(AGAMS,SGAMS,CGAMS,VBAR2S,TH,BETSR,VR,RS(K))
000261          GAMM=ABS(AGAMS)/RADIANS
000264          CALL FTLUP(GAMM,TCL,1,NGAM,GAM,CL)
000270          CALL FTLUP(GAMM,TCD,1,NGAM,GAM,CD)
000274          CALL FTLUP(GAMM,THC,1,NGAM,GAM,HC)
000300          IF(AGAMS.LT.0.)TCL=-TCL
000303          TCL=TCL/(1.+AZ/(3.14159*ARS))*(1.-AZ/(3.14159*ARS))
          C   CURVED FLOW CORRECTICN
000313          TCL=TCL*CLCLO(CAPR,CBAR,TH)
000317          TCD=TCD+TCL**2/(3.14159*ARS)
000323          CT(K)=VBAR2S*((TCL*SGAMS-TCD*CGAMS)*SIN(TH)+(TCL*CGAMS+TCD*SGAMS)*
               1SIN(BETSR)*COS(TH))/SIN(BETSR)
000351       30 CONTINUE
000353          CALL TRAP(CT,YS,KS,SUM,0.)
000357          SUMCTS(I)=SUM(KS)*2.*NS*CBARS/SEAR
000365       35 CONTINUE
          C
          C   CALCULATE CT FOR BLADES
          C
000365          DO 39 J=1,IT
000367          CALL FTLUP(YR(J),BET,1,NY,YBAR,RBETA)
000373          CALL GAMMA(AGAM,SGAM,CGAM,VBAR2,  TH,BET,VR,RR(J))
000405          GAMM =ABS(AGAM)/RADIANS
000410          CALL FTLUP(GAMM , TCL,1,NGAM,GAM,CL)
```

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```
000414          CALL FTLUP(GAMM , TCD,1,NGAM,GAM,CD)
000420          CALL FTLUP(GAMM , THC,1,NGAM,GAM,HC)
000424          IF(AGAM.LT.0.)TCL=-TCL
000427          TCL=TCL/(1.+AZ/(3.14159*AR))*(1.-AZ/(3.14159*AR))
            C   CURVED FLOW CORRECTION
000437          TCL=TCL*CLCLO(CAPR,CBAR,TH)
000443          TCD=TCD+TCL**2/(3.14159*AR)
000447          CT(J)=VBAR2*((TCL*SGAM-TCD*CGAM)*SIN(TH)+(TCL*CGAM+TCD*SGAM)*
               1SIN(BET)*COS(TH))/SIN(BET)
000475       39 CONTINUE
000477          CALL TRAP(CT,YR,IT,SUM,0.)
000503          SUMCM(I)=SUM(IT)*2.*N*CBAR/SBAR
000511       40 CONTINUE
            C
            C   CALCULATE AVERAGE TOTAL THRUST
            C
000514          CALL TRAP(SUMCM,THETA,NTHETA,SUM,0.)
000517          BARCT=1./360.*SUM(NTHETA)
000522          IF(NS.EQ.0)GO TO 44
000523          IF(BETS.EQ.0.)GO TO 42
000524          CALL TRAP(SUMCTS,THETA,NTHETA,SUM,0.)
000530          CTSSUM=SUM(NTHETA)
000532          BARCTS=1./360.*CTSSUM
000535          WRITE(6,114)SUMCTS(NTHETA)
000542          GO TO 44
000543       42 BARCTS=NS*CD(1)*CBARS*VR/(2.*SBAR)
000551       44 BARTT=BARCT+BARCTS
            C
            C   DETERM VELOCITY AT BLADES
            C
000553          FV=VR**2-    VR+BARTT/4.
            C
            C   CHECK ON CONVERGENCE OF SOLUTION
            C
000557          IF(ITER.NE.1)GO TO 46
000561          C=.1
000562          SAVVR=VR
000564          SAVFV=FV
000566          GO TO 48
000566       46 IF(STF*FV.LT.0.)GO TO 49
000571          IF(ITER.LT.10)GO TO 48
000573          IF(ITER.EQ.10)GO TO 150
000574          IF(ITER.GT.10.AND.ITER.LT.20)48,70
```

```
000604        150 VR=SAVVR
000605            FV=SAVFV
000607            C=-.1
000611         48 STF=FV
000612            STV=VR
000614            VR=VR+C*VR
000616            ITER=ITER+1
000620            GO TO 25
              C
              C   SOLUTION HAS CONVERGED,CT AND VR
              C
000620         49 XV=-STF*(VR-STV)/(FV-STF)+STV
000626            VR=XV
000630            WRITE(6,113)BARCT,VR,ITER
000642            IF=1
000643            IF(ITHPRNT.EQ.0)IF=0
              C
              C   CALCULATE THE PERFORMANCE FOR A GIVEN VRRY, FOR ALL THETAS
              C
000645            DO 90 I=1,NTHETA
000647            NPRNT=0
000650            IF(IF.EQ.0)GO TO 54
000651            IF(THETA(I).EQ.THPRNT(IF))53,54
000656         53 WRITE(6,100)THETA(I)
000664            NPRNT=1
000665            IF=IF+1
000667            IF(IF.GT.ITHPRNT)IF=0
000672         54 CONTINUE
000672            TH=THETA(I)*RADIANS
000674            IF(NS.EQ.0)GO TO 51
000676            IF(BETS.NE.0.)GO TO 52
000677         51 SUMCMS(I)=0.
000701            GO TO 58
              C
              C   COMPUTE STRUT LIFT AND DRAG
              C
              C
000701         52 CONTINUE
000701            IF(NPRNT.EQ.1)WRITE(6,116)
000707            DO 56 K=1,KS
000711            CALL GAMMA(AGAMS,EQ2,EQ1,EQ,TH,BETSR,VR,RS(K))
000723            GAMDEG=AGAMS/RADIANS
000725            GAMM=ABS(AGAMS)/RADIANS
```

```
000727          CALL FTLUP(GAMM , TCL,1,NGAM,GAM,CL)
000733          CALL FTLUP(GAMM , TCD,1,NGAM,GAM,CD)
000737          CALL FTLUP(GAMM , THC,1,NGAM,GAM,HC)
000743          IF(AGAMS.LT.0.)TCL=-TCL
          C
          C   DOWNWASH AND ASPECT RATIO CORRECTION
          C
000746          TCL=TCL/(1.+AZ/(3.14159*ARS))*(1.-AZ/(3.14159*ARS))
          C   CURVED FLOW CORRECTION
000757          TCL=TCL*CLCLO(CAPR,CBAR,TH)
000763          TCD=TCD+TCL**2/(3.14159*ARS)
000767          IF(RS(K).EQ.0.)GO TO 55
          C
          C   TORQUE DUE TO STRUTS
          C
000771          CM(K)=2.*CBARS*RS(K)/SBAR*EQ* (CBARS/RS(K)*(.5-THC)*(EQ1*TCL+EQ2*
               1TCD)+1./SIN(BETSR)*(EQ2*TCL-EQ1*TCD))
001020          GO TO 57
001020       55 CM(K)=0.
001022       57 IF(NPRNT.EQ.1)WRITE(6,101)YS(K),GAMDEG,TCL,TCD,THC,CM(K)
001044       56 CONTINUE
001047          CALL TRAP(CM,YS,KS,SUM,0.)
001052          SUMCMS(I)=SUM(KS)
001055       58 CONTINUE
001055          IF(NPRNT.EQ.1)WRITE(6,117)
001063          DO 50 J=1,IT
          C
          C   COMPUTE BLADE LIFT AND DRAG
          C
001065          CALL FTLUP(YR(J),BET,1,NY,YBAR,RBETA)
001071          CALL GAMMA(AGAM,EQ2,EQ1,EQ,TH,BET,VR,RR(J))
001103          GAMDEG=AGAM/RADIANS
001105          GAMM =ABS(AGAM)/RADIANS
001107          CALL FTLUP(GAMM , TCL,1,NGAM,GAM,CL)
001113          CALL FTLUP(GAMM , TCD,1,NGAM,GAM,CD)
001117          CALL FTLUP(GAMM , THC,1,NGAM,GAM,HC)
001123          IF(AGAM.LT.0.)TCL=-TCL
          C
          C   DOWNWASH AND ASPECT RATIO CORRECTION
          C
001126          TCL=TCL/(1.+AZ/(3.14159*AR))*(1.-AZ/(3.14159*AR))
          C   CURVED FLOW CORRECTION
001137          TCL=TCL*CLCLO(CAPR,CBAR,TH)
```

```
001143         TCD=TCD+TCL**2/(3.14159*AR)
          C
          C   TORQUE DUE TO BLADE
          C
001147         CM(J)=2.*CBAR*RR(J)/SBAR*EQ* (CBAR/RR(J)*(.5-THC)*(EQ1
              1*TCL+EQ2*TCD)+1./SIN(BET)*(EQ2*TCL-EQ1*TCD))
001176         IF(NPRNT.EQ.1)WRITE(6,101)YR(J),GAMDEG,TCL,TCD,THC,CM(J)
001217      50 CONTINUE
001222         CALL TRAP(CM,YR,IT,SUM,0.)
001225         SUMCM(I)=SUM(IT)
001230         IF(NPRNT.EQ.1)WRITE(6,102)THETA(I),SUMCM(I)
001241      90 CONTINUE
001244         WRITE(6,103)(THETA(I),SUMCM(I),SUMCMS(I),I=1,NTHETA)
          C
          C   CCMPUTE TOTAL AVERAGE TORQUE AND POWER COEFFICIENT CPA
          C
001262         NN=NTHETA/N
001265         DO 80 I=1,NN
001267         CTS(I)=0.
001270      80 CT(I)=0.
001274         DO 60 J=1,N
001275         NJ=(J-1)*NN
001300         DO 60 I=1,NN
001302         CT(I)=CT(I)+SUMCM(I+NJ)
001306         CTS(I)=CTS(I)+NS*SUMCMS(I+NJ)
001312      60 CONTINUE
001317         CALL TRAP(CT,THETA,NN,SUM,0.)
001323         CALL TRAP(CTS,THETA,NN,SUMS,0.)
001327         DO 65 I=1,NN
001331         SUM(I)=N*SUM(I)/360.
001334         SUMS(I)=N*SUMS(I)/360.
001337      65 CONTINUE
001341         WRITE(6,108)N,(THETA(I),CT(I),CTS(I),I=1,NN)
001361         WRITE(6,111)SUM(NN),SUMS(NN)
001371         CPA=(SUM(NN)+SUMS(NN))/VVR
          C
          C   COMPUTE POWER LOSS DUE TO NCNLIFTING STRUTS,CPL
          C
001374         CPL=(CK1*CBARS*CD1)*(1.+(VR/CAPR)**2)/(4.*SBAR*VVR**3)
001405         CPN=CPA-CPL
001407         WRITE(6,110)CPA,CPL,CPN
001421      70 CONTINUE
001424         GO TO 1
```

```
              C
              C   F O R M A T S
              C
001424          100 FORMAT(//5X,*THETA =*F12.2)
001424          101 FORMAT(E15.7,F10.2,5E15.7)
001424          102 FORMAT(/1X*CM(*F5.1*)=*E15.7)
001424          103 FORMAT(1H1,10X*MOMENT COEFFICIENT FOR ONE BLADE AND SUPPORTING STR
                   1UTS*/5X*THETA*5X*CM(THETA)*6X*CMS(THETA)*/(F10.2,2E16.7))
001424          104 FORMAT(8A10)
001424          105 FORMAT(1H1,10X*PROGRAM A4426 - WINDMILL EFFICIENCY ANALYSIS*/11X
                   18A10)
001424          106 FORMAT(1H1,20X*OUTPUT*//* NONDIMENSIONAL WINDMILL SWEPT AREA SBAR=
                   1*E15.7)
001424          107 FORMAT(//* CP =*E15.7)
001424          108 FORMAT(1H1,5X*NUMBER OF BLADES =*I3/5X*THETA*8X*CMT*12X*CMTS*
                   1/(F10.2,2E15.7))
001424          109 FORMAT(1H1,33H VELOCITY RATIO RMAX*OMEGA/VINF =,F12.4//)
001424          110 FORMAT(//* POWER COEFFICIENT WITHOUT NONLIFTING STRUT LOSSES,CPA=*
                   1E15.7/* POWER LOST DUE TO NONLIFTING STRUTS, CPL=*E15.7/
                   2* NET POWER COEFFICIENT, CPN=*E15.7)
001424          111 FORMAT(//* AVERAGE BLADE MOMENT COEFFICIENT, CMA =*E15.7/* AVERAGE
                   1 STRUT MOMENT COEFFICIENT, CMAS =*E15.7)
001424          113 FORMAT(//* WINDMILL THRUST COEFFICIENT BARCT =*E15.7/* VR =*E15.7/
                   1* ITERATIONS =*I5)
001424          114 FORMAT(* CTS =*E15.7)
001424          115 FORMAT(1H1,20X*INPUT TABLES*//6X*GAMMA*7X*CL*14X*CD*13X*CMC4*13X*D
                   1/C*/(F12.4,4E15.7))
001424          116 FORMAT(//5X*WINDMILL STRUT LIFT,DRAG AND MOMENT*/6X*YSBAR*,8X
                   1*GAMMA*10X*CL*13X*CD*12X*D/C*10X*DEL CMS*)
001424          117 FORMAT(//,5X*WINDMILL BLADE LIFT,DRAG AND MOMENT*/7X*YBAR*8X*GAMMA
                   1*10X*CL*13X*CD*12X*D/C*10X,*DEL CM*)
001424              END
```

```
              SUBROUTINETRAP(Y,X,NL,SUM,SUM1)
000010        DIMENSION Y(NL),X(NL),SUM(NL)
000010        SUM(1)=SUM1
000011        NLM=NL-1
000012        DO1N=1,NLM
000014        NP=N+1
000015      1 SUM(NP)=SUM(N)+(Y(NP)+Y(N))*(X(NP)-X(N))/2.
000034        RETURN
000035        END
```

```
                 SUBROUTINE GAMMA(GAM,SCAM,CGAM,VBAR2,TH,BET,VR,RR)
            C
            C   CALCULATE THE LOCAL ANGLE OF ATTACK GAMMA(I.E. ALPHA), AND THE
            C   LOCAL VELOCITY VBAR
            C
000013           CCMMON/GAMSUB/CAPR
            C
000013           PART1=RR*CAPR-VR*SIN(TH)
000024           PART2=VR*COS(TH)*SIN(BET)
000042           GAM=ATAN2(PART2,PART1)
000051           DENC=SQRT(PART1**2+PART2**2)
000055           SGAM=PART2/DENO
000062           CGAM=PART1/DENO
000064           VBAR2=PART1**2+(VR*COS(TH))**2
000074           RETURN
000075           END
```

```
                FUNCTION CLCLO(CAPR,CBAR,TH)
          C     FUNCTION CLCLO CORRECTS LIFT COEFFICIENT FOR FLOW CURVATURE EFFECTS
000006          REAL NUM
000006          COSTH=COS(TH)
000011          SINTH=SIN(TH)
000015          A=CAPR*CBAR/2.
000017          ALPHA0=ATAN2(COSTH,CAPR-SINTH)
000026          ALPHA1=ATAN2(COSTH-A,CAPR-SINTH)
000037          ALPHA2=ATAN2(COSTH+A,CAPR-SINTH)
000050          NUM=(CAPR-SINTH)*(COS(ALPHA1)-COS(ALPHA2))/CAPR
000063          DENOM=(COS(ALPHA0))**2*SIN(ALPHA0)*CBAR
000075          CLCLO=NUM/DENOM
000077          RETURN
000077          END
```

```
                SUBROUTINE  FTLUP(X,Y,M,N,VARI,VARD)                          FTLU0010
000011          DIMENSION VARI(1),VARD(1),V(3),YY(2)                          FTLU0020
000011          IF (M.EQ.0.AND.N.EQ.0) GO TO 1                                FTLU0030
000016          IF(M.EQ.0.AND.N.NE.0)GO TO 97                                 FTLU0040
000024          IF(N.LE.IABS(M)) GO TO 97                                     FTLU0050
000026          IF(M.GT.0)GO TO 31                                            FTLU0060
          C              M.LT.0                                               FTLU0070
000031          DO 44 IYY=1,N                                                 FTLU0080
000032          I=IYY                                                         FTLU0090
000033          IF(VARI(I)-X)800,119,44                                       FTLU0100
000036       44 CONTINUE                                                      FTLU0110
000041          I=N+M                                                         FTLU0120
          C          IF X.LT.X(N),EXTRAPOLATE                                 FTLU0130
000042          IF(M.EQ.-1)GO TO 801                                          FTLU0140
000044          GO TO 802                                                     FTLU0150
          C          IF X.GT.X(1),EXTRAPOLATE                                 FTLU0160
000044      800 IF(I.EQ.1.AND.M.EQ.-1)GO TO 801                               FTLU0170
000054          IF(I.EQ.1.AND.M.EQ.-2)GO TO 802                               FTLU0180
000063          IF(M.NE.-1)GO TO 622                                          FTLU0190
          C          M=-1                                                     FTLU0200
000065          I=I-1                                                         FTLU0210
000066      801 IF(VARI(I).LE.VARI(I+1))GO TO 97                              FTLU0220
000072          GO TO 1701                                                    FTLU0230
          C          M=-2                                                     FTLU0240
000072      622 IF(I.NE.N)GO TO 1622
000074          I=N-2
000075          GO TO 802
          C          COMPARE WITH NEXT                                        FTLU0250
000076     1622 IF(VARI(I+1)-X)803,97,97
000101      803 I=I-1                                                         6TLU0270
000103          IF(I.EQ.1)GO TO 802                                           FTLU0280
          C          SEE WHICH THREE                                          FTLU0290
000104          IF((VARI(I-1)-X).LT.(X-VARI(I+2)))I=I-1                       FTLU0300
000112      802 IF(VARI(I).LE.VARI(I+1).OR.VARI(I+1).LE.VARI(I+2))GO TO 97    FTLU0310
000125          GO TO 1702                                                    FTLU0320
          C              M.GT.0                                               FTLU0330
000125       31 DO 4 IYY=1,N                                                  FTLU0340
000127          I=IYY                                                         FTLU0350
000130          IF(X-VARI(I))700,119,4                                        FTLU0360
000133        4 CONTINUE                                                      FTLU0370
000136          I=N-M                                                         FTLU0380
          C          IF X.GT.X(N),EXTRAPOLATE                                 FTLU0390
```

```
000137          IF(M.EQ.1)GO TO 701                                                      FTLU0400
000141          GO TO 702                                                                FTLU0410
           C          IF X.LT.X(1), EXTRAPOLATE                                          FTLU0420
000141      700 IF(I.EQ.1.AND.M.EQ.1)GO TO 701                                           FTLU0430
000150          IF(I.EQ.1.AND.M.EQ.2)GO TO 702                                           FTLU0440
000155          IF(M.NE.1)GO TO 222                                                      FTLU0450
           C          M=1                                                                FTLU0460
000157          I=I-1                                                                    FTLU0470
000160      701 IF(VARI(I+1).LE.VARI(I))GO TO 97                                         FTLU0480
           C          LINEAR                                                             FTLU0490
000164     1701 Y=(VARD(I)*(VARI(I+1)-X)-VARD(I+1)*(VARI(I)-X))/(VARI(I+1)-VARI(I)FTLU0500
               C)                                                                        FTLU0510
000177          RETURN                                                                   FTLU0520
           C          M=2                                                                FTLU0530
000200      222 IF(I.NE.N)GO TO 1222
000202          I=N-2
000203          GO TO 702
           C          COMPARE WITH NEXT                                                  FTLU0540
000204     1222 IF(X-VARI(I+1))703,97,97
000207      703 I=I-1                                                                    FTLU0560
000211          IF(I.EQ.1)GO TO 702                                                      FTLU0570
           C          SEE WHICH THREE                                                    FTLU0580
000212          IF((X-VARI(I-1)).LT.(VARI(I+2)-X))I=I-1                                  FTLU0590
000220      702 IF(VARI(I+1).LE.VARI(I).OR.VARI(I+2).LE.VARI(I+1))GO TO 97               FTLU0600
           C          SECOND ORDER                                                       FTLU0610
000233     1702 V(1)=VARI(I)-X                                                           FTLU0620
000235          V(2)=VARI(I+1)-X                                                         FTLU0630
000237          V(3)=VARI(I+2)-X                                                         FTLU0640
000241          K=I                                                                      FTLU0650
000242          DO 704 J=1,2                                                             FTLU0660
000243          YY(J)=(VARD(K)*V(J+1)-VARD(K+1)*V(J))/(VARI(K+1)-VARI(K))                FTLU0670
000257      704 K=K+1                                                                    FTLU0680
000263          Y=(YY(1)*V(3)-YY(2)*V(1))/(VARI(I+2)-VARI(I))                            FTLU0690
000273          RETURN                                                                   FTLU0700
           C             ZERO ORDER(Y=Y(1))                                              FTLU0710
000274        1 Y =VARD(1)                                                               FTLU0720
000275          RETURN                                                                   FTLU0730
           C             Y=Y(I)                                                          FTLU0740
000276      119 Y=VARD(I)                                                                FTLU0750
000300          RETURN                                                                   FTLU0760
           C                                                                             FTLU0770
           C          ERROR PRINT                                                        FTLU0780
000301       97 PRINT 103                                                                FTLU0790
```

```
000305       103 FORMAT( /31H ERROR WAS ENCOUNTERED IN FTLUP)                FTLU0800
000305           PRINT 1103,M,N,X                                            FTLU0810
000323      1103 FORMAT(  1X,2HM=I5,5X,2HN=I5,5X,2HX=E20.8)                  FTLU0820
000323           IF(N.EQ.0)STOP                                              FTLU0830
000332           IF(M.EQ.0)STOP                                              FTLU0840
000335           PRINT 1104                                                  FTLU0850
000341      1104 FORMAT( 19H TABLE OUT OF ORDER)                             FTLU0860
000341           STOP                                                        FTLU0870
000343           END                                                         FTLU0880
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PROGRAM A4426 - WINDMILL EFFICIENCY ANALYSIS
CATENARY (N=2) - NEW AERODYNAMICS

```
$NAM1

AR       = 0.28E+C2,

ARS      = I,

AZ       = 0.573E+01,

BETA     = 0.9E+02,  0.888E+02,  0.863E+02,  0.828E+02,  0.7925E+02,  0.76E+02,
            0.725E+02,  0.696E+02,  0.664E+C2,  0.634E+02,  0.606E+02,
            0.582E+02,  0.554E+02,  0.526E+02,  0.5E+02,  0.475E+02,  0.449E+02,
            0.425E+C2,  0.4E+02,  0.377E+02,  0.355E+02,  0.335E+02,  0.315E+02,
            0.295E+02,  0.279E+02,  0.264E+02,  0.25E+02,  0.24E+02, I, I, I, I,
           I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,

BETS     = I,

CBAR     = 0.7142E-01,

CBARS    = 0.0,

CD       = 0.65E-02,  0.8E-02,  0.1E-01,  0.12E-01,  0.14E-01,  0.45E-01,
            0.1E+00,  0.175E+00,  0.21E+00,  0.24E+00,  0.245E+00,  0.59E+00,
            0.97E+00,  0.101E+01,  0.13E+01,  0.159E+01,  0.18E+01,  0.192E+01,
            0.194E+01,  0.189E+01,  0.178E+01,  0.157E+01,  0.132E+01,
            0.99E+00,  0.6E+00,  0.42E+00,  0.28E+00,  0.23E+00,  0.19E+00,
            0.12E+00,  0.8E-01,  0.2E-01, I, I, I, I, I, I, I, I, I, I, I, I, I,
           I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
           I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
           I, I, I, I, I, I, I, I, I,

CD1      = 0.0,

CK1      = 0.0,

CL       = 0.0,  0.21E+00,  0.695E+00,  0.855E+00,  0.96E+00,  0.1015E+01,
            0.8E+00,  0.67E+00,  0.63E+00,  0.63E+00,  0.655E+00,  0.975E+00,
            0.113E+01,  0.1139E+01,  0.109E+01,  0.959E+00,  0.7E+00,  0.41E+00,
            0.8E-01, -0.22E+00, -0.533E+00, -0.79E+00, -0.96E+00, -0.1075E+01,
           -0.9E+00, -0.722E+00, -0.705E+00, -0.755E+00, -0.761E+00, -0.765E+00,
           -0.78E+00, -0.1E-01, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
           I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
           I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
           I, I, I, I, I, I,

CMC4     = 0.0,  0.2E-02,  0.6E-02,  0.1E-01,  0.2E-01, -0.35E-01, -0.65E-01,
           -0.7E-01, -0.72E-01, -0.76E-01, -0.8E-01, -0.165E+00, -0.26E+00,
           -0.27E+00, -0.33E+00, -0.39E+00, -0.43E+00, -0.48E+00, -0.52E+00,
           -0.57E+00, -0.59E+00, -0.595E+00, -0.58E+00, -0.57E+00, -0.47E+00,
           -0.4E+00, -0.34E+00, -0.32E+00, -0.35E+00, -0.39E+00, -0.425E+00,
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        -0.75E-01, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
        I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
        I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
        I, I, I,

DY       = 0.1E-01,

DYS      = I,

GAM      = 0.0,  0.2E+01,  0.6E+01,  0.8E+01,  0.1E+02,  0.12E+02,  0.127E+02,
           0.14E+02,  0.16E+02,  0.18E+02,  0.2E+02,  0.3E+02,  0.4E+02,
           0.42E+02,  0.5E+02,  0.6E+02,  0.7E+02,  0.8E+02,  0.9E+02,
           0.1E+03,  0.11E+03,  0.12E+03,  0.13E+03,  0.14E+03,  0.15E+03,
           0.155E+03,  0.16E+03,  0.164E+03,  0.168E+03,  0.17E+03,  0.172E+03,
           0.18E+03, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
          I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
          I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
          I, I, I,

ITHPRNT  = 1,

N        = 2,

NGAM     = 32,

NS       = 0,

NTHETA   = 73,

NVR      = I,

NY       = 28,

RBAR     = 0.1E+01,  0.9956E+00,  0.9933E+00,  0.9889E+00,  0.9822E+00,
           0.9732E+00,  0.9598E+00,  0.9464E+00,  0.9308E+00,  0.913E+00,
           0.8929E+00,  0.8706E+00,  0.8482E+00,  0.817E+00,  0.788E+00,
           0.7545E+00,  0.7188E+00,  0.6786E+00,  0.6362E+00,  0.5871E+00,
           0.5402E+00,  0.4822E+00,  0.4263E+00,  0.3661E+00,  0.317E+00,
           0.25E+00,  0.1875E+00,  0.1071E+00, I, I, I, I, I, I, I, I, I, I, I,
          I, I, I, I, I, I, I, I, I, I, I,

TC       = I,

THETA    = 0.0,  0.5E+01,  0.1E+02,  0.15E+02,  0.2E+02,  0.25E+02,  0.3E+02,
           0.35E+02,  0.4E+02,  0.45E+02,  0.5E+02,  0.55E+02,  0.6E+02,
           0.65E+02,  0.7E+02,  0.75E+02,  0.8E+02,  0.85E+02,  0.9E+02,
           0.95E+02,  0.1E+03,  0.105E+03,  0.11E+03,  0.115E+03,  0.12E+03,
           0.125E+03,  0.13E+03,  0.135E+03,  0.14E+03,  0.145E+03,  0.15E+03,
           0.155E+03,  0.16E+03,  0.165E+03,  0.17E+03,  0.175E+03,  0.18E+03,
           0.185E+03,  0.19E+03,  0.195E+03,  0.2E+03,  0.205E+03,  0.21E+03,
           0.215E+03,  0.22E+03,  0.225E+03,  0.23E+03,  0.235E+03,  0.24E+03,
```

```
        0.245E+03,  0.25E+03,  0.255E+03,  0.26E+03,  0.265E+03,  0.27E+03,
        0.275E+03,  0.28E+03,  0.285E+03,  0.29E+03,  0.295E+03,  0.3E+03,
        0.305E+03,  0.31E+03,  0.315E+03,  0.32E+03,  0.325E+03,  0.33E+03,
        0.335E+03,  0.34E+03,  0.345E+03,  0.35E+03,  0.355E+03,  0.36E+03,
       I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
       I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
       I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
       I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
       I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
       I, I, I, I, I, I, I, I, I, I, I, I,

THPRNT  = 0.0, I, I, I, I, I, I, I, I, I,

VRRY    = 0.6E+01,  0.3E+01,  0.35E+01,  0.4E+01,  0.45E+01,  0.5E+01,
          0.6E+01,  0.7E+01,  0.8E+01,  0.9E+01,  0.1E+02,  0.11E+02, I, I, I,
       I, I, I, I, I,

YBAR    = 0.0,  0.357E-01,  0.714E-01,  0.1071E+00,  0.1428E+00,  0.1785E+00,
          0.2142E+00,  0.2499E+00,  0.2856E+00,  0.3213E+00,  0.357E+00,
          0.3927E+00,  0.4284E+00,  0.4641E+00,  0.4998E+00,  0.5355E+00,
          0.5712E+00,  0.6069E+00,  0.6426E+00,  0.6783E+00,  0.714E+00,
          0.7497E+00,  0.7854E+00,  0.8211E+00,  0.8568E+00,  0.8925E+00,
          0.9285E+00,  0.9639E+00, I, I, I, I, I, I, I, I, I, I, I, I, I, I,
       I, I, I, I, I, I, I, I,

YMAX    = 0.9639E+00,

YMIN    = 0.0,

YSMAX   = I,

YSMIN   = I,

$END
```

ORIGINAL PAGE IS
OF POOR QUALITY

INPUT TABLES

GAMMA	CL	CD	CMC4	D/C
0.0000	0.	6.5000000E-03	0.	2.5000000E-01
2.0000	2.1000000E-01	8.0000000E-03	2.0000000E-03	2.4048305E-01
6.0000	6.9500000E-01	1.0000000E-02	6.0000000E-03	2.4133246E-01
8.0000	8.5500000E-01	1.2000000E-02	1.0000000E-02	2.3821240E-01
10.0000	9.6000000E-01	1.4000000E-02	2.0000000E-02	2.2889954E-01
12.0000	1.0150000E+00	4.5000000E-02	-3.5000000E-02	2.8492401E-01
12.7000	8.0000000E-01	1.0000000E-01	-6.5000000E-02	3.3100574E-01
14.0000	6.7000000E-01	1.7500000E-01	-7.0000000E-02	3.5109260E-01
16.0000	6.3000000E-01	2.1000000E-01	-7.2000000E-02	3.5851893E-01
18.0000	6.3000000E-01	2.4000000E-01	-7.6000000E-02	3.6287190E-01
20.0000	6.5500000E-01	2.4500000E-01	-8.0000000E-02	3.6440116E-01
30.0000	9.7500000E-01	5.9000000E-01	-1.6500000E-01	3.9481627E-01
40.0000	1.1300000E+00	9.7000000E-01	-2.6000000E-01	4.2459810E-01
42.0000	1.1390000E+00	1.0100000E+00	-2.7000000E-01	4.2736741E-01
50.0000	1.0900000E+00	1.3000000E+00	-3.3000000E-01	4.4451855E-01
60.0000	9.5900000E-01	1.5900000E+00	-3.9000000E-01	4.6007494E-01
70.0000	7.0000000E-01	1.8000000E+00	-4.3000000E-01	4.7269860E-01
80.0000	4.1000000E-01	1.9200000E+00	-4.8000000E-01	4.9464500E-01
90.0000	8.0000000E-02	1.9400000E+00	-5.2000000E-01	5.1804124E-01
100.0000	-2.2000000E-01	1.8900000E+00	-5.7000000E-01	5.5008067E-01
110.0000	-5.3300000E-01	1.7800000E+00	-5.9000000E-01	5.6806794E-01
120.0000	-7.9000000E-01	1.5700000E+00	-5.9500000E-01	5.8909706E-01
130.0000	-9.6000000E-01	1.3200000E+00	-5.8000000E-01	6.0620961E-01
140.0000	-1.0750000E+00	9.9000000E-01	-5.7000000E-01	6.4044907E-01
150.0000	-9.0000000E-01	6.0000000E-01	-4.7000000E-01	6.8541787E-01
155.0000	-7.2200000E-01	4.2000000E-01	-4.0000000E-01	7.3085367E-01
160.0000	-7.0500000E-01	2.8000000E-01	-3.4000000E-01	6.9840155E-01
164.0000	-7.5500000E-01	2.3000000E-01	-3.2000000E-01	6.5550001E-01
168.0000	-7.6100000E-01	1.9000000E-01	-3.5000000E-01	6.9650059E-01
170.0000	-7.6500000E-01	1.2000000E-01	-3.9000000E-01	7.5373558E-01
172.0000	-7.8000000E-01	8.0000000E-02	-4.2500000E-01	7.9240805E-01
180.0000	-1.0000000E-02	2.0000000E-02	-7.5000000E-02	7.7499999E+00

OUTPUT

NONDIMENSIONAL WINDMILL SWEPT AREA SBAR= 2.7700361E+00

VELOCITY RATIO RMAX*OMEGA/VINF = 6.0000

WINDMILL THRUST COEFFICIENT BARCT = 7.9553433E-01
VR = 7.4803774E-01
ITERATIONS = 3

THETA = 0.00

WINDMILL BLADE LIFT,DRAG AND MOMENT

YBAR	GAMMA	CL	CD	D/C	DEL CM
0.	7.11	6.8646994E-01	1.6463737E-02	2.3960619E-01	1.5326166E-01
1.0000000E-02	7.12	6.8706981E-01	1.6481662E-02	2.3959283E-01	1.5305275E-01
2.0000000E-02	7.12	6.8765414E-01	1.6499131E-02	2.3957983E-01	1.5283925E-01
3.0000000E-02	7.13	6.8822286E-01	1.6516141E-02	2.3956717E-01	1.5262121E-01
4.0000000E-02	7.14	6.8861646E-01	1.6527917E-02	2.3955841E-01	1.5243195E-01
5.0000000E-02	7.14	6.8874146E-01	1.6531657E-02	2.3955563E-01	1.5227137E-01
6.0000000E-02	7.14	6.8879260E-01	1.6533188E-02	2.3955449E-01	1.5209097E-01
7.0000000E-02	7.14	6.8876977E-01	1.6532505E-02	2.3955500E-01	1.5189081E-01
8.0000000E-02	7.14	6.8876426E-01	1.6532340E-02	2.3955512E-01	1.5154327E-01
9.0000000E-02	7.14	6.8863344E-01	1.6528425E-02	2.3955803E-01	1.5113793E-01
1.0000000E-01	7.13	6.8835715E-01	1.6520158E-02	2.3956418E-01	1.5069444E-01
1.1000000E-01	7.13	6.8802397E-01	1.6510191E-02	2.3957160E-01	1.5018074E-01
1.2000000E-01	7.12	6.8775992E-01	1.6502294E-02	2.3957747E-01	1.4954992E-01
1.3000000E-01	7.12	6.8734569E-01	1.6489909E-02	2.3958669E-01	1.4888106E-01
1.4000000E-01	7.11	6.8678084E-01	1.6473026E-02	2.3959927E-01	1.4817446E-01
1.5000000E-01	7.11	6.8640328E-01	1.6461745E-02	2.3960767E-01	1.4738181E-01
1.6000000E-01	7.10	6.8602769E-01	1.6450527E-02	2.3961603E-01	1.4653904E-01
1.7000000E-01	7.09	6.8552547E-01	1.6435531E-02	2.3962721E-01	1.4566634E-01
1.8000000E-01	7.09	6.8496678E-01	1.6418855E-02	2.3963964E-01	1.4472817E-01
1.9000000E-01	7.08	6.8467069E-01	1.6410021E-02	2.3964623E-01	1.4355554E-01
2.0000000E-01	7.07	6.8422790E-01	1.6396812E-02	2.3965609E-01	1.4235104E-01
2.1000000E-01	7.07	6.8363736E-01	1.6379204E-02	2.3966923E-01	1.4111523E-01
2.2000000E-01	7.06	6.8316907E-01	1.6365247E-02	2.3967965E-01	1.3991650E-01
2.3000000E-01	7.05	6.8278577E-01	1.6353826E-02	2.3968819E-01	1.3874489E-01
2.4000000E-01	7.05	6.8230065E-01	1.6339376E-02	2.3969898E-01	1.3755388E-01
2.5000000E-01	7.04	6.8171351E-01	1.6321895E-02	2.3971205E-01	1.3634223E-01
2.6000000E-01	7.03	6.8106309E-01	1.6302539E-02	2.3972653E-01	1.3494982E-01
2.7000000E-01	7.02	6.8028664E-01	1.6279444E-02	2.3974381E-01	1.3353569E-01
2.8000000E-01	7.01	6.7938312E-01	1.6252588E-02	2.3976392E-01	1.3210036E-01
2.9000000E-01	6.99	6.7858604E-01	1.6228911E-02	2.3978166E-01	1.3062481E-01
3.0000000E-01	6.99	6.7797180E-01	1.6210674E-02	2.3979533E-01	1.2910724E-01
3.1000000E-01	6.97	6.7724602E-01	1.6189138E-02	2.3981149E-01	1.2757431E-01
3.2000000E-01	6.96	6.7640759E-01	1.6164273E-02	2.3983015E-01	1.2602654E-01
3.3000000E-01	6.96	6.7596509E-01	1.6151157E-02	2.3984000E-01	1.2442896E-01
3.4000000E-01	6.95	6.7550186E-01	1.6137431E-02	2.3985031E-01	1.2281588E-01

APPENDIX - CONTINUED

3.5000000E-01	6.94	6.7494185E-01	1.6120844E-02	[illegible]	[illegible]
3.6000000E-01	6.94	6.7454427E-01	1.6109072E-02	2.3987162E-01	1.1956992E-01
3.7000000E-01	6.94	6.7467802E-01	1.6113032E-02	2.3986864E-01	1.1796013E-01
3.8000000E-01	6.94	6.7474691E-01	1.6115071E-02	2.3986711E-01	1.1634733E-01
3.9000000E-01	6.94	6.7475000E-01	1.6115163E-02	2.3986704E-01	1.1473191E-01
4.0000000E-01	6.93	6.7426276E-01	1.6100739E-02	2.3987789E-01	1.1302178E-01
4.1000000E-01	6.92	6.7352303E-01	1.6078851E-02	2.3989435E-01	1.1127288E-01
4.2000000E-01	6.91	6.7268169E-01	1.6053972E-02	2.3991308E-01	1.0951950E-01
4.3000000E-01	6.90	6.7196026E-01	1.6032651E-02	2.3992913E-01	1.0770796E-01
4.4000000E-01	6.90	6.7231815E-01	1.6043226E-02	2.3992117E-01	1.0561232E-01
4.5000000E-01	6.91	6.7259283E-01	1.6051345E-02	2.3991505E-01	1.0352056E-01
4.6000000E-01	6.91	6.7278254E-01	1.6056953E-02	2.3991083E-01	1.0143338E-01
4.7000000E-01	6.91	6.7288569E-01	1.6060003E-02	2.3990854E-01	9.9438463E-02
4.8000000E-01	6.91	6.7291024E-01	1.6060729E-02	2.3990799E-01	9.7509286E-02
4.9000000E-01	6.91	6.7285697E-01	1.6059154E-02	2.3990918E-01	9.5585775E-02
5.0000000E-01	6.91	6.7274357E-01	1.6055801E-02	2.3991170E-01	9.3665923E-02
5.1000000E-01	6.92	6.7350865E-01	1.6078426E-02	2.3989467E-01	9.1630049E-02
5.2000000E-01	6.93	6.7421973E-01	1.6099466E-02	2.3987884E-01	8.9605271E-02
5.3000000E-01	6.94	6.7487559E-01	1.6118882E-02	2.3986425E-01	8.7592139E-02
5.4000000E-01	6.95	6.7555494E-01	1.6139004E-02	2.3984913E-01	8.5542070E-02
5.5000000E-01	6.96	6.7627408E-01	1.6160316E-02	2.3983312E-01	8.3445990E-02
5.6000000E-01	6.97	6.7693377E-01	1.6179876E-02	2.3981844E-01	8.1365030E-02
5.7000000E-01	6.98	6.7753251E-01	1.6197637E-02	2.3980511E-01	7.9299796E-02
5.9000000E-01	7.00	6.7925600E-01	1.6248811E-02	2.3976675E-01	7.7182164E-02
5.9000000E-01	7.03	6.8113062E-01	1.6304548E-02	2.3972503E-01	7.5075422E-02
6.0000000E-01	7.06	6.8299826E-01	1.6360157E-02	2.3968346E-01	7.2989712E-02
6.1000000E-01	7.08	6.8491666E-01	1.6417360E-02	2.3964076E-01	7.0894001E-02
6.2000000E-01	7.11	6.8695924E-01	1.6478357E-02	2.3959530E-01	6.8751681E-02
6.3000000E-01	7.14	6.8900100E-01	1.6539425E-02	2.3954985E-01	6.6634477E-02
6.4000000E-01	7.17	6.9104297E-01	1.6600594E-02	2.3950440E-01	6.4542997E-02
6.5000000E-01	7.22	6.9463361E-01	1.6708386E-02	2.3942448E-01	6.2380629E-02
6.6000000E-01	7.28	6.9887251E-01	1.6836014E-02	2.3933014E-01	6.0215318E-02
6.7000000E-01	7.35	7.0323969E-01	1.6967933E-02	2.3923294E-01	5.8082460E-02
6.8000000E-01	7.41	7.0770854E-01	1.7103372E-02	2.3913347E-01	5.5998356E-02
6.9000000E-01	7.47	7.1213975E-01	1.7238118E-02	2.3903484E-01	5.4022851E-02
7.0000000E-01	7.54	7.1671672E-01	1.7377765E-02	2.3893297E-01	5.2078568E-02
7.1000000E-01	7.61	7.2145082E-01	1.7522707E-02	2.3882760E-01	5.0166014E-02
7.2000000E-01	7.71	7.2866856E-01	1.7744672E-02	2.3866695E-01	4.8147363E-02
7.3000000E-01	7.84	7.3786223E-01	1.8029116E-02	2.3846233E-01	4.6075795E-02
7.4000000E-01	7.98	7.4759590E-01	1.8332362E-02	2.3824568E-01	4.4045816E-02
7.5000000E-01	8.13	7.5487440E-01	1.8603746E-02	2.3762694E-01	4.1892227E-02
7.6000000E-01	8.27	7.6160872E-01	1.8866252E-02	2.3694520E-01	3.9824768E-02
7.7000000E-01	8.43	7.6876864E-01	1.9146479E-02	2.3622037E-01	3.7810262E-02
7.8000000E-01	8.59	7.7640009E-01	1.9446445E-02	2.3544781E-01	3.5849235E-02
7.9000000E-01	8.78	7.8508152E-01	1.9789291E-02	2.3456896E-01	3.3887832E-02
8.0000000E-01	9.00	7.9507145E-01	2.0185933E-02	2.3355764E-01	3.1922301E-02
8.1000000E-01	9.23	8.0588271E-01	2.0617741E-02	2.3246317E-01	3.0019143E-02
8.2000000E-01	9.49	8.1762677E-01	2.1089817E-02	2.3127428E-01	2.8179172E-02
8.3000000E-01	9.72	8.2836171E-01	2.1524074E-02	2.3018754E-01	2.6694403E-02
8.4000000E-01	9.97	8.3966576E-01	2.1984184E-02	2.2904319E-01	2.5289692E-02
[illegible]	[illegible]	[illegible]	2.5786602E-02	2.3547068E-01	2.3153613E-02

8.6000000E-01	10.58	8.5501959E-01	3.1275466E-02	2.4510079E-01	2.0770636E-02
8.7000000E-01	11.09	8.6729783E-01	3.9414774E-02	2.5937601E-01	1.8037579E-02
8.8000000E-01	11.66	8.8119455E-01	4.8631107E-02	2.7553294E-01	1.5564121E-02
8.9000000E-01	12.32	8.0237090E-01	7.7692354E-02	3.0618319E-01	9.7762153E-03
9.0000000E-01	13.04	6.7152648E-01	1.2447126E-01	3.3618675E-01	3.2272296E-03
9.1000000E-01	13.84	6.0107442E-01	1.6984401E-01	3.4861168E-01	-4.6723255E-04
9.2000000E-01	14.77	5.7344616E-01	1.9228175E-01	3.5396625E-01	-1.3067786E-03
9.3000000E-01	15.95	5.5288122E-01	2.1255684E-01	3.5832411E-01	-1.5528604E-03
9.4000000E-01	17.88	5.5196186E-01	2.4159121E-01	3.6260024E-01	-1.3516103E-03
9.5000000E-01	20.37	5.8424162E-01	2.6164926E-01	3.6552686E-01	-5.1262216E-04
9.6000000E-01	23.71	6.7790231E-01	3.7824765E-01	3.7568767E-01	-7.5695022E-04
9.6390000E-01	25.34	7.2347547E-01	4.3505390E-01	3.8063168E-01	-7.4488488E-04

CM(0.0)= 8.6889397E-02

MOMENT COEFFICIENT FOR ONE BLADE AND SUPPORTING STRUTS

THETA	CM(THETA)	CMS(THETA)
0.00	8.6899397E-02	0.
5.00	8.5881643E-02	0.
10.00	8.3759314E-02	0.
15.00	8.0609620E-02	0.
20.00	7.6495737E-02	0.
25.00	7.1544750E-02	0.
30.00	6.5879350E-02	0.
35.00	5.9643087E-02	0.
40.00	5.2685125E-02	0.
45.00	4.4548149E-02	0.
50.00	3.6284738E-02	0.
55.00	2.8204365E-02	0.
60.00	2.0529240E-02	0.
65.00	1.3496762E-02	0.
70.00	7.3233915E-03	0.
75.00	2.1874444E-03	0.
80.00	-1.4334443E-03	0.
85.00	-3.7836292E-03	0.
90.00	-4.9034437E-03	0.
95.00	-5.6308808E-03	0.
100.00	-5.1767618E-03	0.
105.00	-3.5307559E-03	0.
110.00	-5.2426713E-04	0.
115.00	3.5560217E-03	0.
120.00	8.5424365E-03	0.
125.00	1.4231414E-02	0.
130.00	2.0399[illegible]E-02	0.
135.00	2.6834633E-02	0.
140.00	3.325532[illegible]E-02	0.
145.00	3.8852253E-02	0.
150.00	4.3839098E-02	0.
155.00	4.8356988E-02	0.
160.00	5.2254892E-02	0.
165.00	5.5421762E-02	0.
170.00	5.7745092E-02	0.
175.00	5.9170773E-02	0.
180.00	5.9627429E-02	0.
185.00	5.9100834E-02	0.
190.00	5.7590322E-02	0.
195.00	5.5121818E-02	0.
200.00	5.1754278E-02	0.
205.00	4.7463005E-02	0.
210.00	4.1696070E-02	0.
215.00	3.5137390E-02	0.
220.00	2.8232691E-02	0.
225.00	2.1236161E-02	0.
230.00	1.4414484E-02	0.
[illegible]	[illegible]	[illegible]

240.00	2.2619981E-03	0.
245.00	-2.6298138E-03	0.
250.00	-6.4786958E-03	0.
255.00	-8.9284351E-03	0.
260.00	-1.0147028E-02	0.
265.00	-1.0171479E-02	0.
270.00	-9.9681143E-03	0.
275.00	-7.6369365E-03	0.
280.00	-5.0513613E-03	0.
285.00	-1.2599894E-03	0.
290.00	3.7920792E-03	0.
295.00	1.0353970E-02	0.
300.00	1.7814015E-02	0.
305.00	2.5962460E-02	0.
310.00	3.4565951E-02	0.
315.00	4.3373375E-02	0.
320.00	5.2117018E-02	0.
325.00	6.0507098E-02	0.
330.00	6.8282991E-02	0.
335.00	7.4915154E-02	0.
340.00	7.9567621E-02	0.
345.00	8.3063335E-02	0.
350.00	8.5477176E-02	0.
355.00	8.6758499E-02	0.
360.00	8.6839397E-02	0.

NUMBER OF BLADES = 2

THETA	CMT	CMTS
0.00	1.4651683E-01	0.
5.00	1.4498248E-01	0.
10.00	1.4134564E-01	0.
15.00	1.3573144E-01	0.
20.00	1.2825001E-01	0.
25.00	1.1900776E-01	0.
30.00	1.0757542E-01	0.
35.00	9.4780477E-02	0.
40.00	8.0917816E-02	0.
45.00	6.5784310E-02	0.
50.00	5.0699422E-02	0.
55.00	3.6217730E-02	0.
60.00	2.2791238E-02	0.
65.00	1.0866548E-02	0.
70.00	8.4469570E-04	0.
75.00	-6.7409907E-03	0.
80.00	-1.1580473E-02	0.
85.00	-1.3955109E-02	0.
90.00	-1.3871558E-02	0.
95.00	-1.3267817E-02	0.
100.00	-1.0228123E-02	0.
105.00	-4.8007454E-03	0.
110.00	3.2678120E-03	0.
115.00	1.3909992E-02	0.
120.00	2.6355452E-02	0.
125.00	4.0193874E-02	0.
130.00	5.4965070E-02	0.
135.00	7.0208009E-02	0.
140.00	8.5372340E-02	0.
145.00	9.9339351E-02	0.
150.00	1.1212209E-01	0.
155.00	1.2327214E-01	0.
160.00	1.3182251E-01	0.
165.00	1.3848510E-01	0.
170.00	1.4322226E-01	0.
175.00	1.4592927E-01	0.

AVERAGE BLADE MOMENT COEFFICIENT, CMA = 6.2614295E-02
AVERAGE STRUT MOMENT COEFFICIENT, CMAS = 0.

POWER COEFFICIENT WITHOUT NONLIFTING STRUT LOSSES,CPA= 3.7568577E-01
POWER LOST DUE TO NONLIFTING STRUTS, CPL= 0.
NET POWER COEFFICIENT, CPN= 3.7568577E-01

Table I. Aerodynamic Characteristic of the NACA 0012 Airfoil at Reynolds Numbers of .5 x 10^6 and 1.8 x 10^6

α, degrees	C_L	C_D	$C_{M_{c/4}}$	d/c	Rey
0.0000	0.	6.5000000E-03	0.	2.5000000E-01	
2.0000	2.1000000E-01	8.0000000E-03	2.0000000E-03	2.4048305E-01	
6.0000	6.9300000E-01	1.0000000E-02	6.0000000E-03	2.4133246E-01	
8.0000	8.5500000E-01	1.2000000E-02	1.0000000E-02	2.3821240E-01	
10.0000	9.6000000E-01	1.4000000E-02	2.0000000E-02	2.2889954E-01	.5 x 10^6
12.0000	1.0150000E+00	4.5000000E-02	-3.5000000E-02	2.8492401E-01	
12.7000	8.0000000E-01	1.0000000E-01	-6.5000000E-02	3.3100574E-01	
14.0000	6.7000000E-01	1.7500000E-01	-7.0000000E-02	3.5109260E-01	
16.0000	6.3000000E-01	2.1000000E-01	-7.2000000E-02	3.5851893E-01	
18.0000	6.3000000E-01	2.4000000E-01	-7.6000000E-02	3.6287190E-01	
20.0000	6.5500000E-01	2.4500000E-01	-8.0000000E-02	3.6440116E-01	
30.0000	9.7500000E-01	5.9000000E-01	-1.6500000E-01	3.9481627E-01	
40.0000	1.1300000E+00	9.7000000E-01	-2.6000000E-01	4.2459810E-01	
42.0000	1.1390000E+00	1.0100000E+00	-2.7000000E-01	4.2736741E-01	
50.0000	1.0900000E+00	1.3000000E+00	-3.3000000E-01	4.4451855E-01	1.8 x 10^6
60.0000	9.5900000E-01	1.5900000E+00	-3.9000000E-01	4.6007494E-01	
70.0000	7.0000000E-01	1.8000000E+00	-4.3000000E-01	4.7269860E-01	
80.0000	4.1000000E-01	1.9200000E+00	-4.8000000E-01	4.9464500E-01	
90.0000	8.0000000E-02	1.9400000E+00	-5.2000000E-01	5.1804124E-01	
100.0000	-2.2000000E-01	1.8900000E+00	-5.7000000E-01	5.5008067E-01	
110.0000	-5.3300000E-01	1.7800000E+00	-5.9000000E-01	5.6806794E-01	
120.0000	-7.9000000E-01	1.5700000E+00	-5.9500000E-01	5.8909706E-01	
130.0000	-9.6000000E-01	1.3200000E+00	-5.8000000E-01	6.0620961E-01	
140.0000	-1.0750000E+00	9.9000000E-01	-5.7000000E-01	6.4044907E-01	
150.0000	-9.0000000E-01	6.0000000E-01	-4.7000000E-01	6.8541787E-01	
155.0000	-7.2200000E-01	4.2000000E-01	-4.0000000E-01	7.3085367E-01	
160.0000	-7.0500000E-01	2.8000000E-01	-3.4000000E-01	6.9840155E-01	
164.0000	-7.5500000E-01	2.3000000E-01	-3.2000000E-01	6.5550001E-01	
168.0000	-7.6100000E-01	1.9000000E-01	-3.5000000E-01	6.9650059E-01	
170.0000	-7.6300000E-01	1.2000000E-01	-3.9000000E-01	7.5373558E-01	
172.0000	-7.3000000E-01	8.0000000E-02	-4.2500000E-01	7.9240805E-01	
180.0000	-1.0000000E-02	2.0000000E-02	-7.5000000E-02	7.7499999E+00	

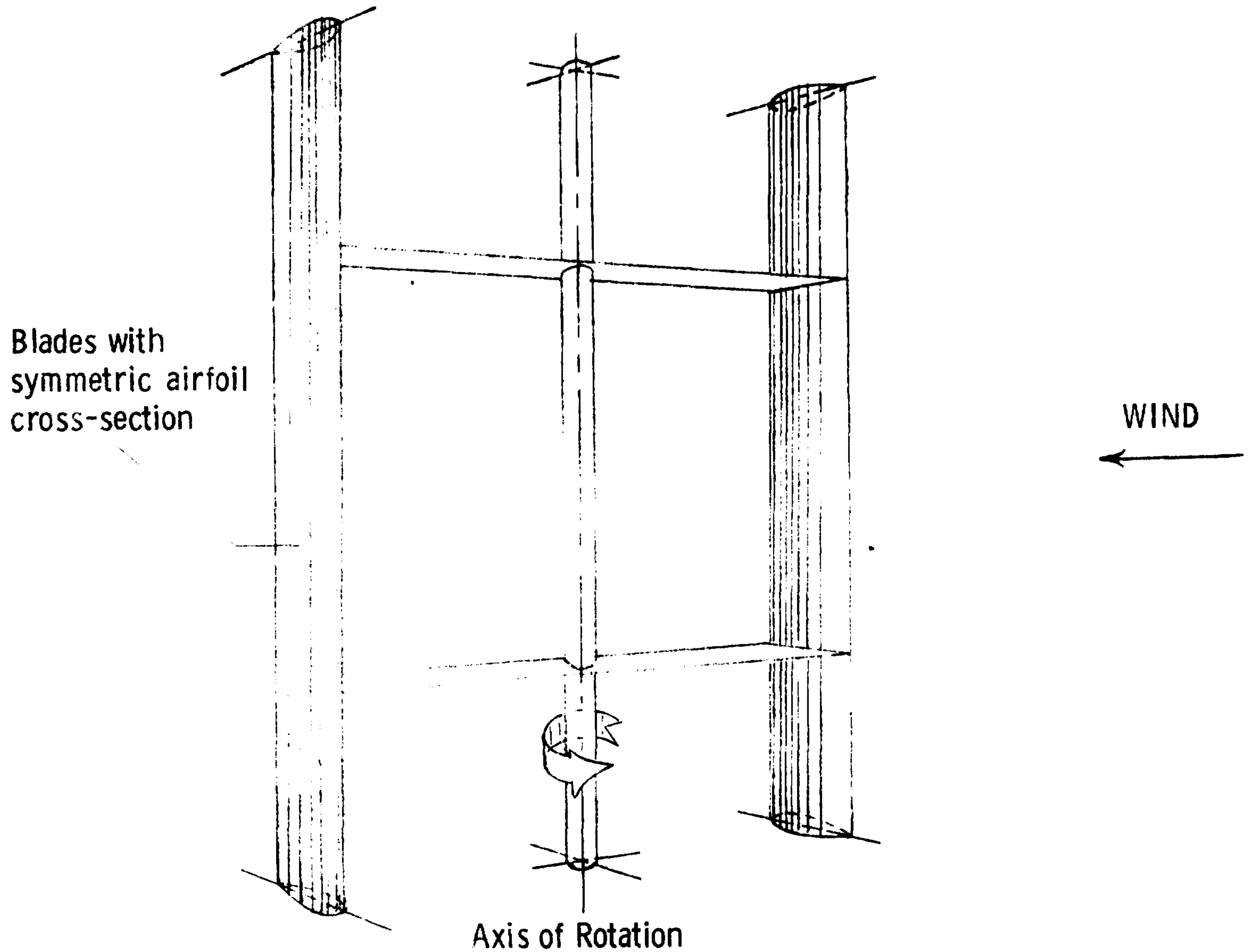


Figure 1. The Vertical Axis Windmill.

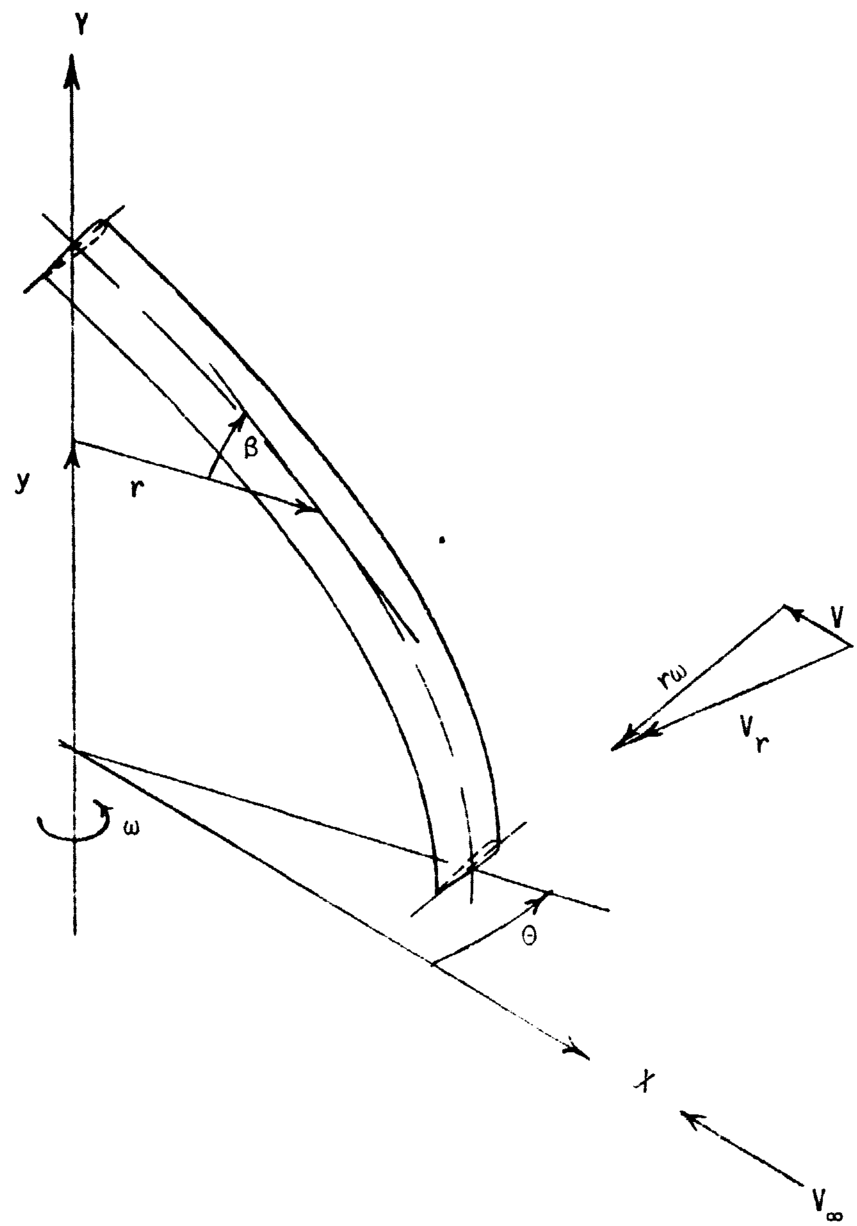


FIGURE 2 - Coordinate System.

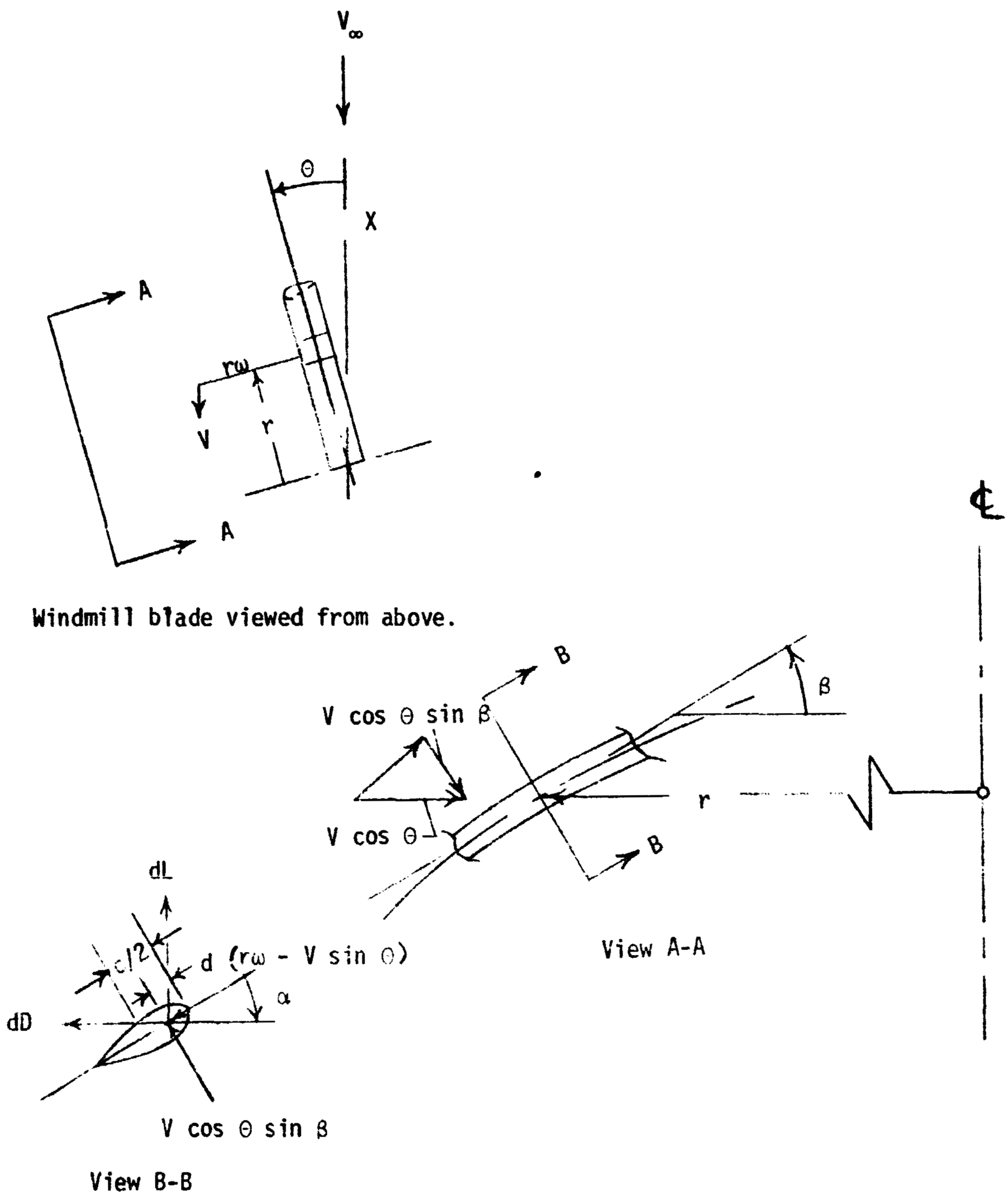


Figure 3. - Velocity and force vector components.

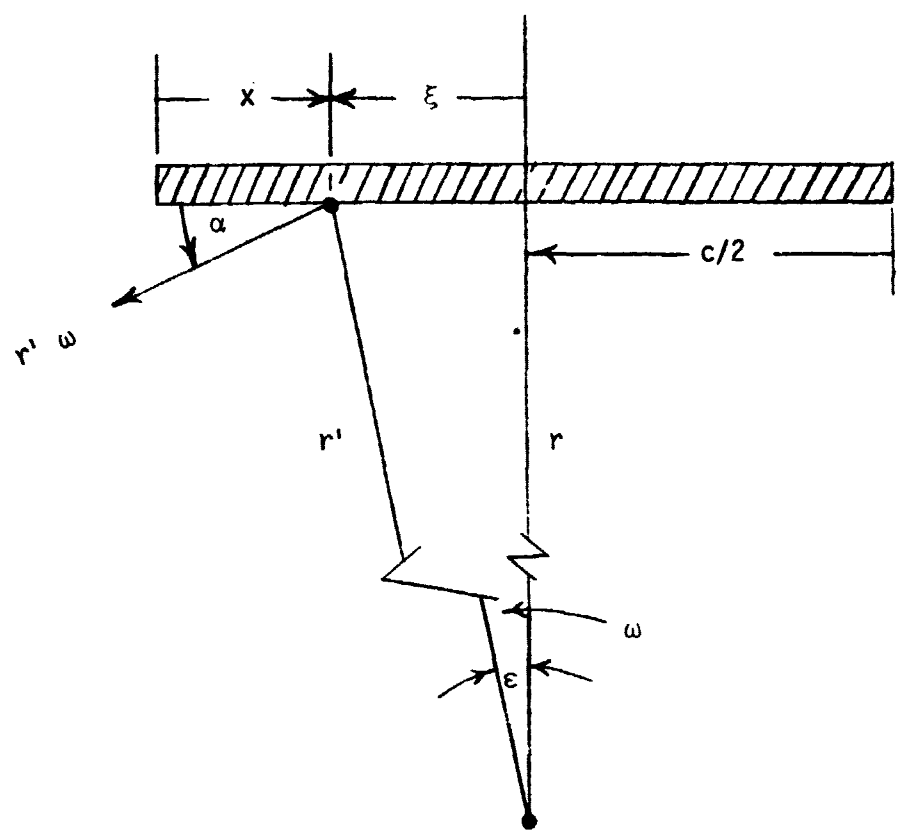


FIGURE 4. - Local Coordinates for Flow Curvature Analysis.

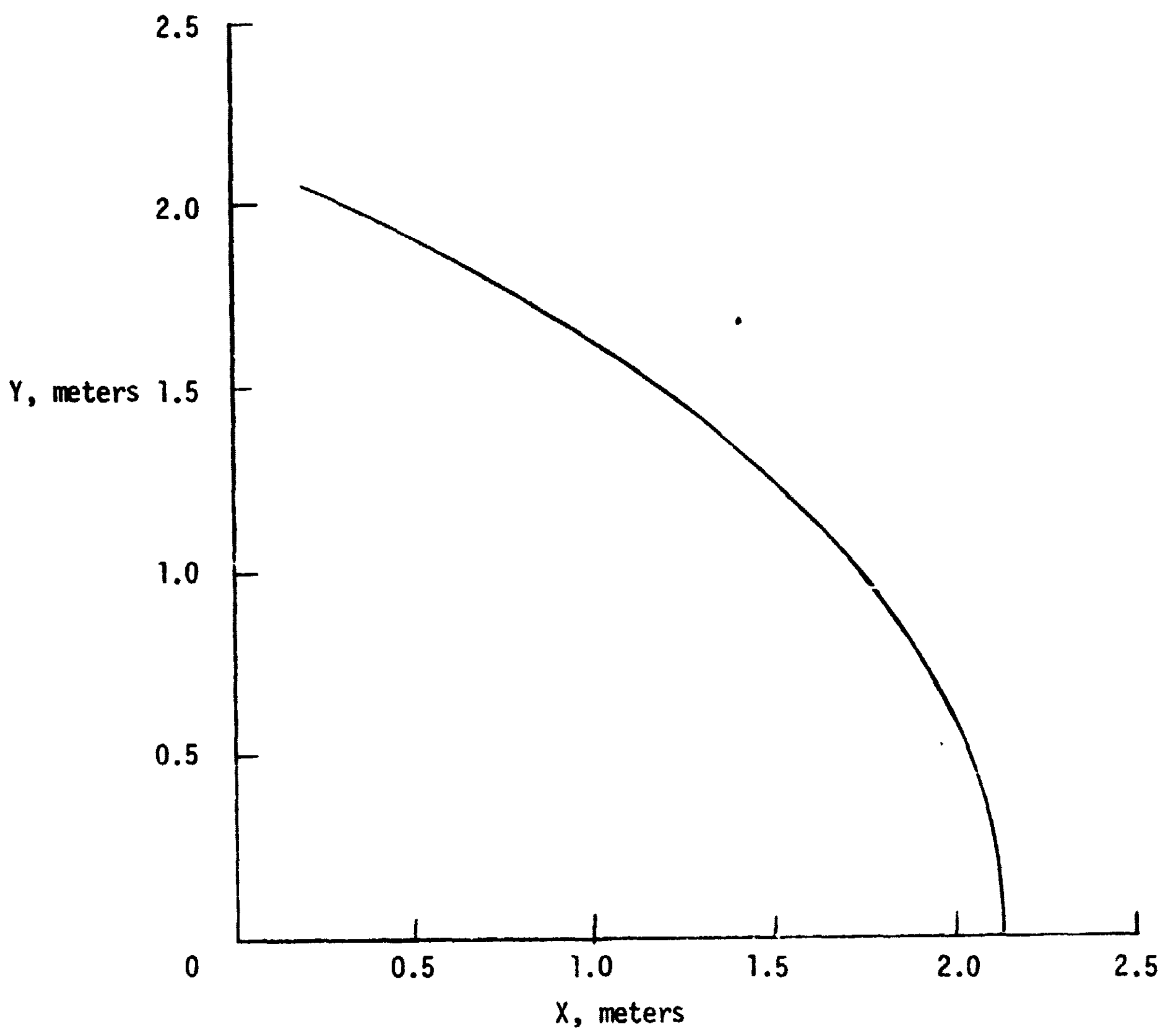


Figure 5. - Side View of the Upper Half of a Blade.

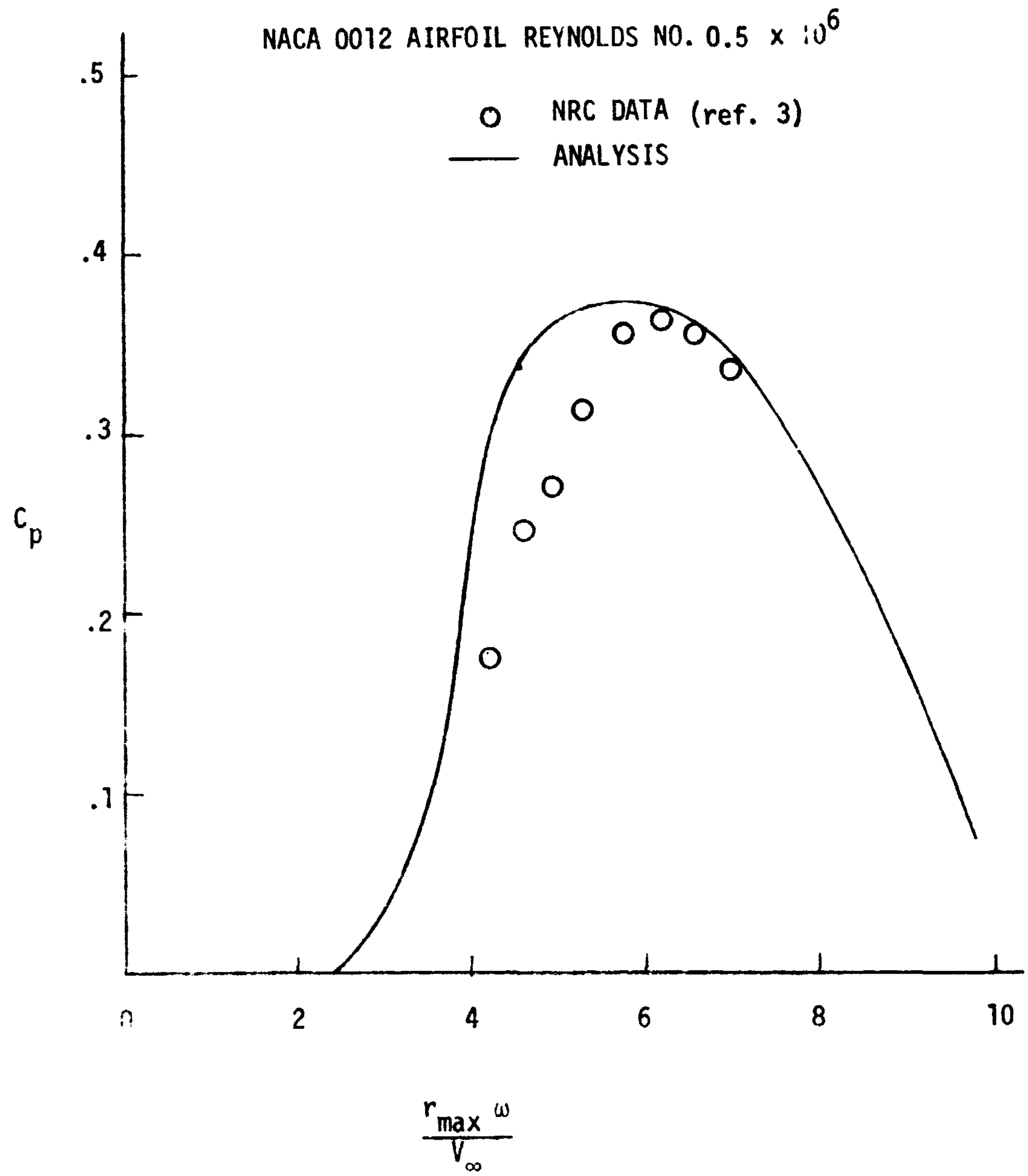


FIGURE 6. - Comparison of Analysis with Test Data for a Two Blade Catenary VAW Using the NACA 0012 Airfoil.

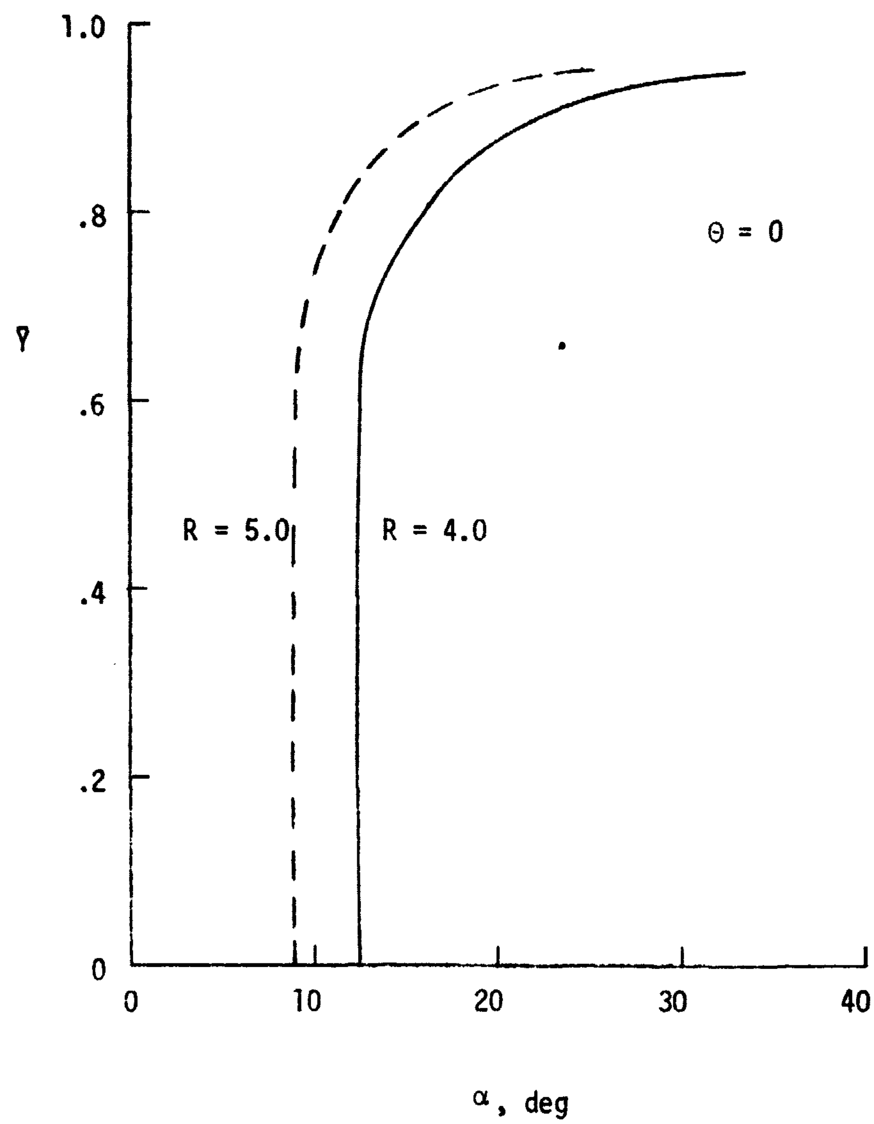


FIGURE 7 - Variation of Angle of Attack Along the Blade - Catenary VAW.

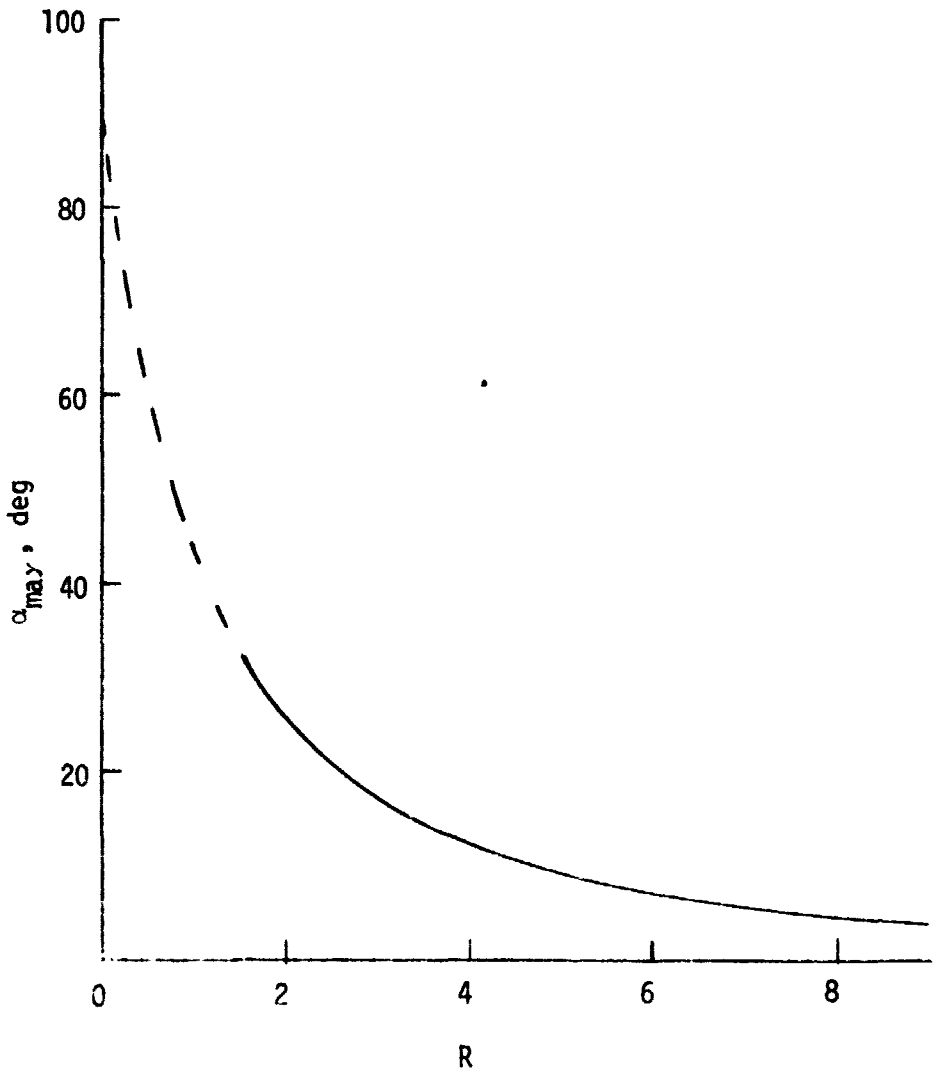


FIGURE 8 - Maximum Angle of Attack Envelope for the Catenary VAW as a Function of R.

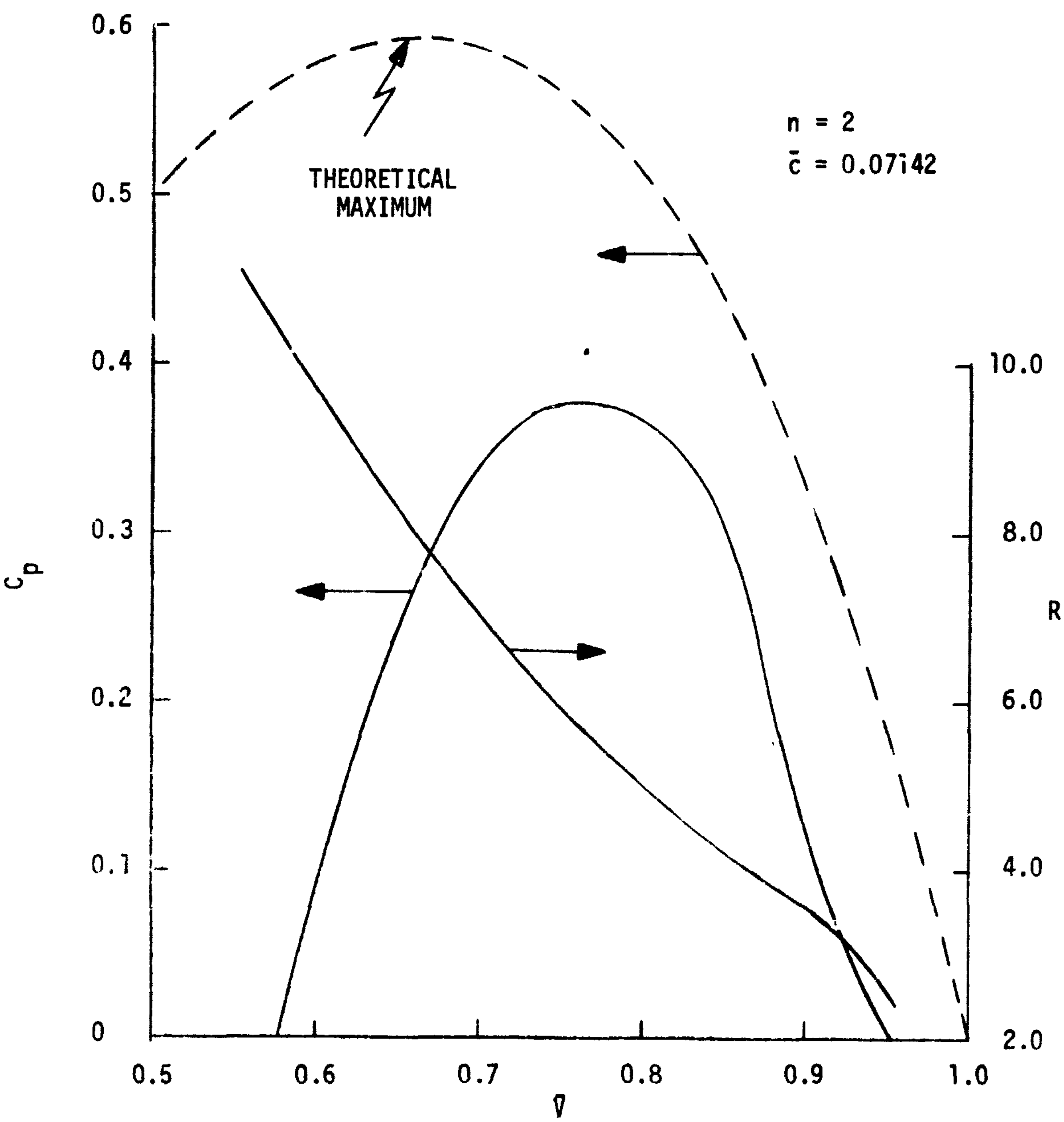


FIGURE 9 - Performance Characteristics of the Catenary Windmill.

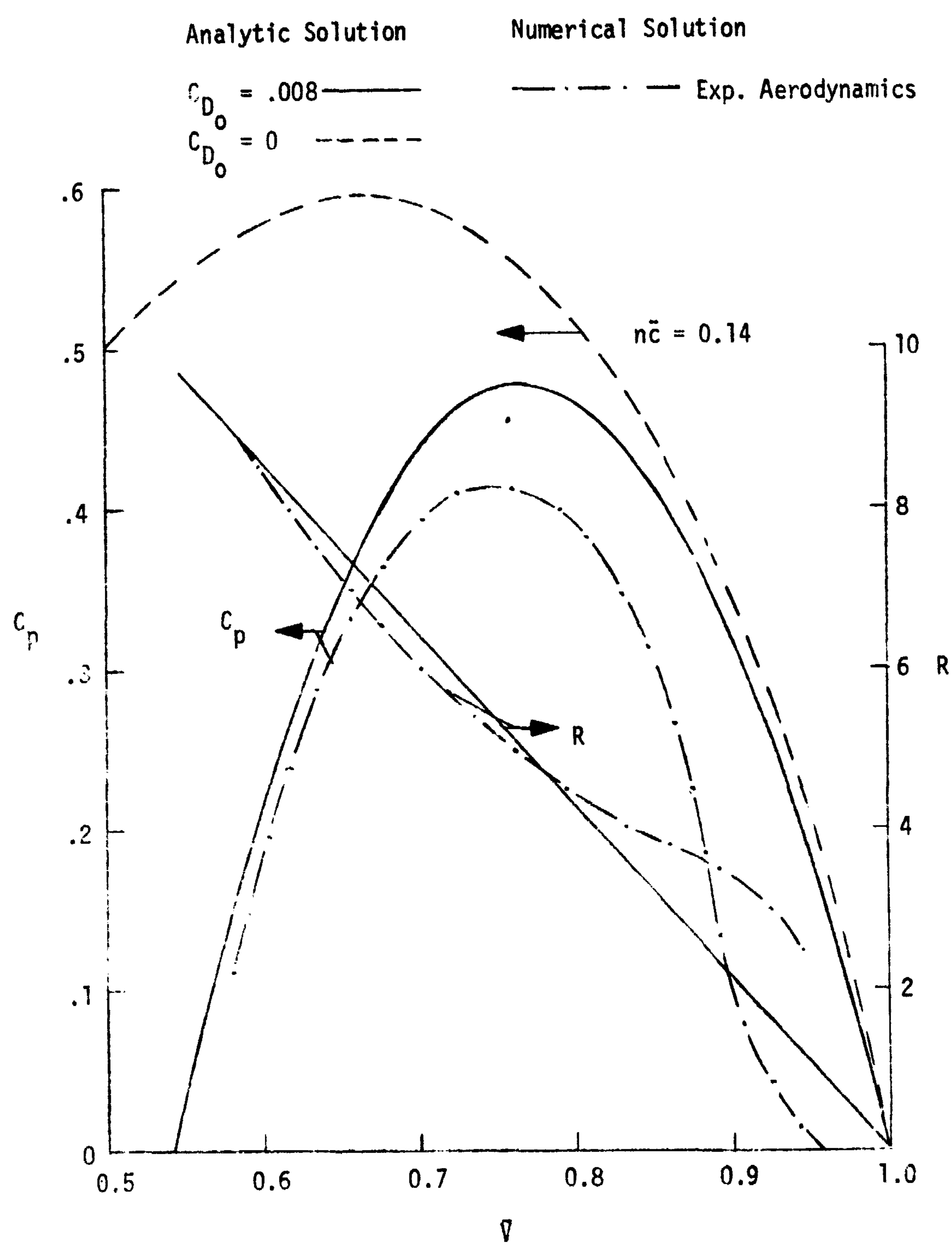


FIGURE 10 - Performance Characteristics of a Straight Blade VAW.

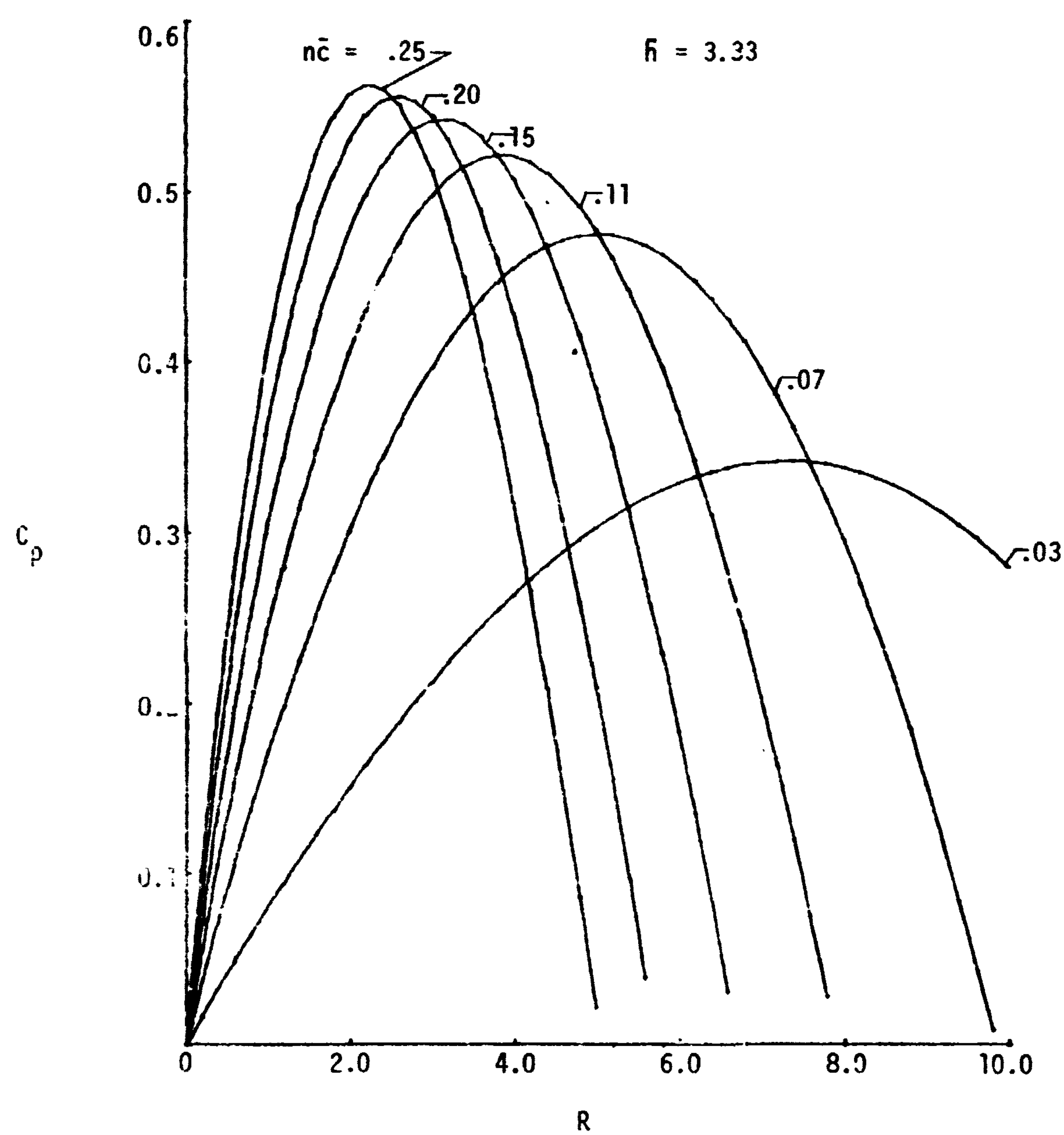


FIGURE 11 - Effect of Solidity on the Performance of a Straight Blade VAW - Analytical Solution.

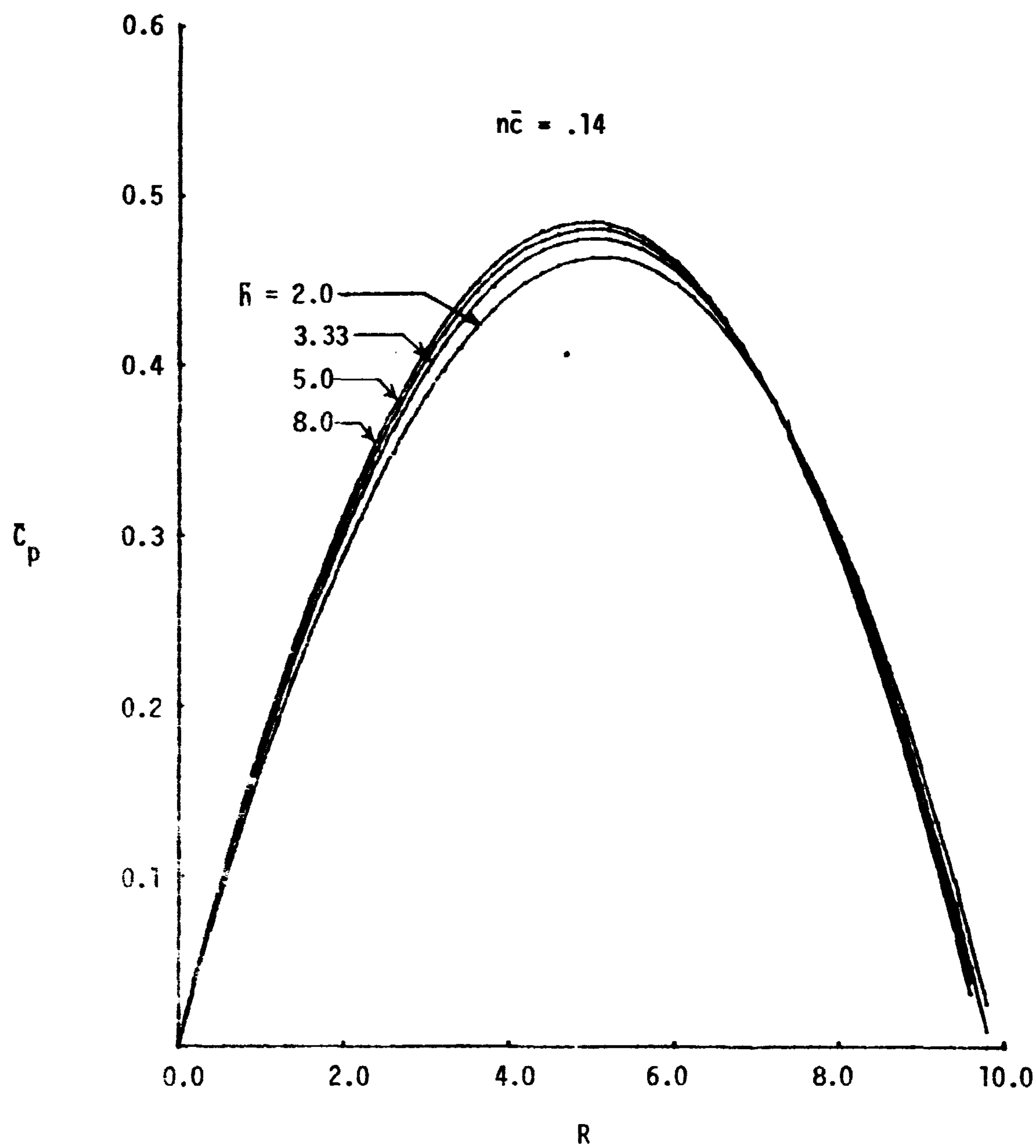


FIGURE 12 - ˙ ˙ of Windmill Aspect Ratio h̄ on ˙rformance for a Straight Blade Design - Analytical Results.

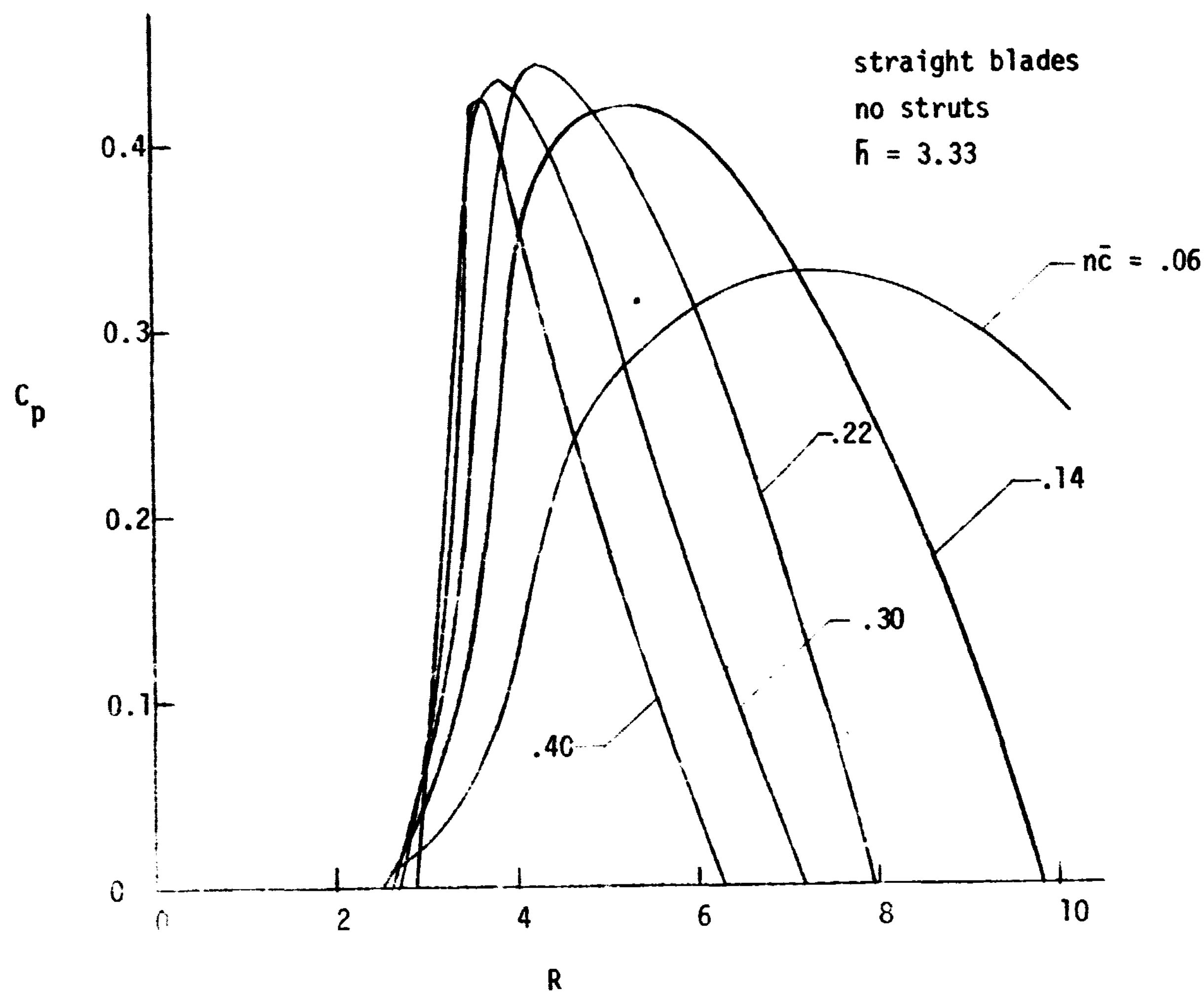
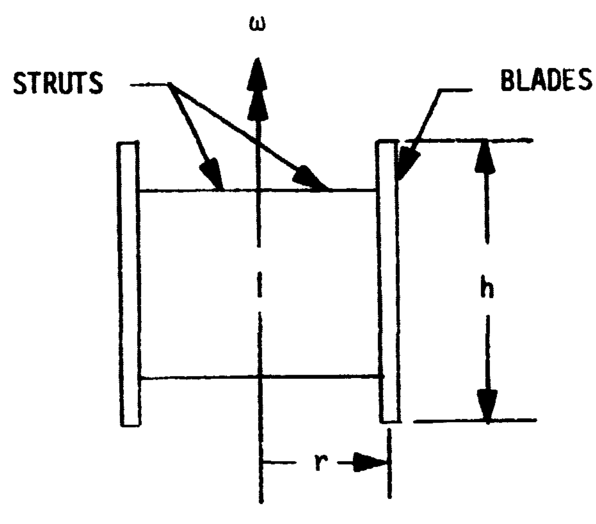
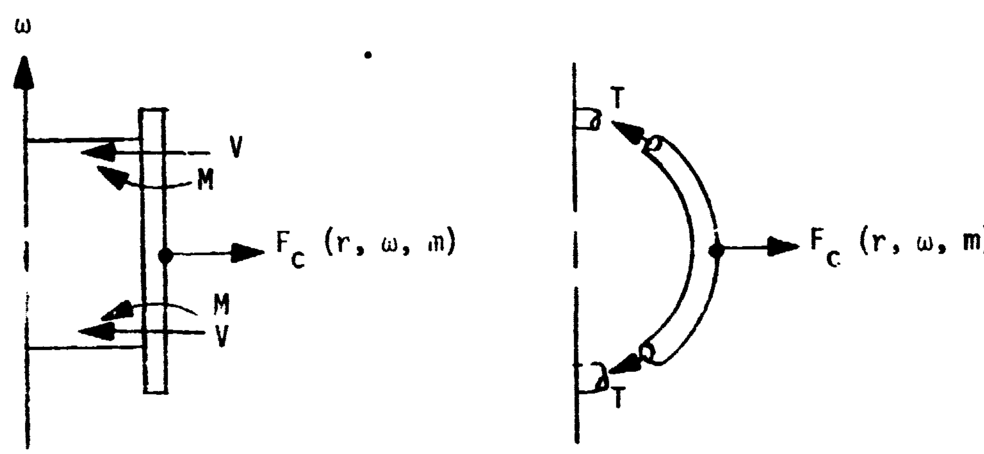


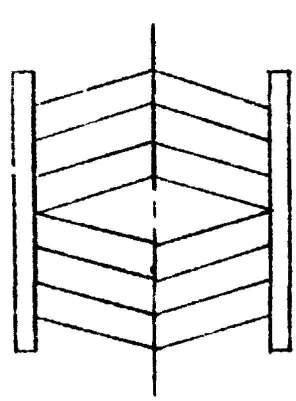
FIGURE 13 - Effect of Solidity on Performance for a Straight Blade Configuration - Numerical Results.



a) STRAIGHT BLADE CONFIGURATION.



b) TYPICAL LOADING FOR STRAIGHT AND CURVED BLADE VAW.



c) CHEVRON STRAIGHT BLADE VAW CONCEPT.

FIGURE 14 - Straight and Curved Blade VAW Designs.

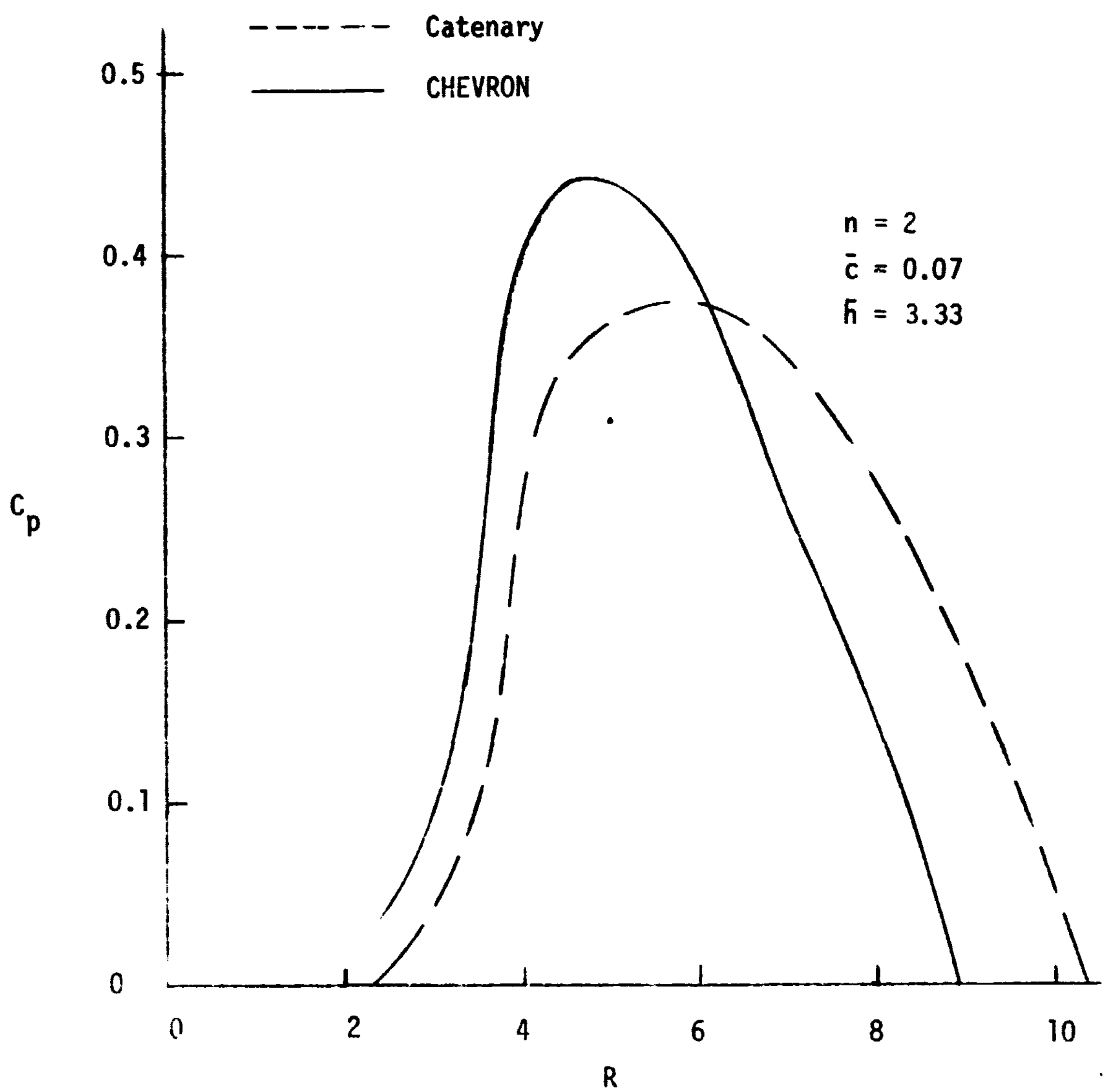


FIGURE 15 - Performance Characteristics of a Chevron Type Straight Blade VAW.