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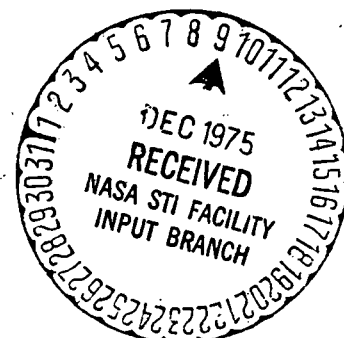
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*Attitude Dynamics Simulation Subroutines for  
Systems of Hinge-Connected Rigid Bodies  
With Nonrigid Appendages*

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## **Preface**

The work described in this report was performed by the Guidance and Control Division of the Jet Propulsion Laboratory.

# Contents

<b>I. Introduction</b> . . . . .	1
<b>II. Unrestricted Systems</b> . . . . .	2 <sup>(1)</sup>
A. Mathematical Model . . . . .	2
B. The Equations . . . . .	6
<b>III. Systems With Rotating Appendages</b> . . . . .	12 <sup>(2)</sup>
A. Equations . . . . .	12
B. Subroutine MBDYFR . . . . .	24
C. A Sample Problem Simulation . . . . .	30
<b>IV. Systems With Nonrotating Appendages</b> . . . . .	44 <sup>(3)</sup>
A. Equations . . . . .	44
B. Subroutines MBDYFN, MBDYFL . . . . .	55
C. Sample Problems . . . . .	57
<b>V. Summary and Conclusions</b> . . . . .	67
<b>References</b> . . . . .	74
<b>Appendixes</b>	
A. Effects of Damping on Rotating Appendage Equations . . . . .	75
B. System Angular Momentum Computation . . . . .	78
C. Subroutine MBDYFR Listing and User Requirements . . . . .	79
D. Subroutine MBDYFN Listing and User Requirements . . . . .	100
E. Subroutine MBDYFL Listing and User Requirements . . . . .	117
 <b>Figures</b>	
1. Definitions for the $k$ th substructure, with $j < k$ . . . . .	3
2. An 8-body, 10-hinge system illustrating the labeling convention . . . . .	26
3. MBDYFR simulation test vehicle . . . . .	31
4. Substructure $s_1$ . . . . .	37
5. Substructure $s_2$ . . . . .	38
6. Commanded slew rates . . . . .	41
7. Simulation program for test vehicle dynamics using MBDYFR . . . . .	42
8. Test vehicle (with spinning rotor) simulation results using MBDYFR . . . . .	45

9. Simulation printout for test vehicle with spinning rotor . . . . .	.49
10. MBDYFN, MBDYFL simulation test vehicle . . . . .	.57
11. Simulation program for test vehicle dynamics using MBDYFN . . . . .	.61
12. Test vehicle simulation results using MBDYFN . . . . .	.63
13. Simulation printout for program using MBDYFN . . . . .	.68
14. Simulation printout for program using MBDYFL . . . . .	.69
15. Simulation program for test vehicle with prescribed platform motion using MBDYFN . . . . .	.70
16. Simulation printout for program using MBDYFN with prescribed platform motion . . . . .	.72

## **Abstract**

This report describes three computer subroutines designed to solve the vector-dyadic differential equations of rotational motion for systems that may be idealized as a collection of hinge-connected rigid bodies assembled in a tree topology, with an optional flexible appendage attached to each body. Deformations of the appendages are mathematically represented by modal coordinates and are assumed small. Within these constraints, the subroutines provide equation solutions for (1) the most general case of unrestricted hinge rotations, with appendage base bodies nominally rotating at a constant speed, (2) the case of unrestricted hinge rotations between rigid bodies, with the restriction that those rigid bodies carrying appendages are nominally nonspinning, and (3) the case of small hinge rotations and nominally nonrotating appendages, i.e., the linearized version of case 2. Sample problems and their solutions are presented to illustrate the utility of the computer programs. Complete listings and user instructions are included for these routines (written in Fortran), which are intended as general-purpose tools in the analysis and simulation of spacecraft and other complex electromechanical systems.

# Attitude Dynamics Simulation Subroutines for Systems of Hinge-Connected Rigid Bodies With Nonrigid Appendages

## I. Introduction

Equations of motion which characterize the small, time-varying deformations of an elastic appendage attached to a rigid body experiencing arbitrary motions have been derived in detail for distributed-mass finite element models in Ref. 1, and for discrete mass models in Ref. 2. With the general structure of the appendage deformation equations established in these references, coordinate transformations are developed in Refs. 1 and 3 in order to allow representation of the elastic appendage in terms of a set of truncated modal coordinates far fewer in number than the original set. In Ref. 4, additional equations of motion are derived to describe the rotations of typical bodies in a *system* of hinge-connected rigid bodies arranged as a topological tree, with optional arbitrary *nonrigid* appendages attached to each rigid body in the system. In this respect, the results of Hooker in Ref. 5 and those of Ref. 4 are parallel.

It is the purpose of this report first to draw together the appendage equations and the equations describing rigid body motions of the tree system, assuming that some or all of the rigid bodies carry nonrigid appendages, and to derive a consistent and detailed set of system dynamical equations suitable for digital computer solution. Secondly, it is the purpose here to present general-purpose computer subroutines capable of solving the resulting system equations of rotational motion, and to demonstrate their utility and applicability to a wide class of spacecraft.

In generating the equations of motion for the hinge-connected tree of rigid bodies with nonrigid appendages, two specific formulations are obtained. The first formally constrains<sup>1</sup> appendage base motion to small deviations from a nominal constant angular velocity in inertial space, thus allowing appendage rotation but with only small deviations from a constant rate of spin. The second formulation formally permits no spin and constrains appendage base motion to small deviations from a nominally zero angular velocity (and acceleration) in the inertial frame. However, both formulations permit otherwise unrestricted motions of the system rigid bodies consistent with the fundamental assumption of small appendage deformations from some nominal state. Computer subroutines (written in Fortran) are described which solve the equations produced by each of these approaches. In addition, a third subroutine is presented which solves the completely linearized equations for the nonrotating case, under the assumption that all rigid body rotations and their derivatives are small.

The computer programs are direct descendants of those described in Refs. 6 and 7, which are applicable to the hinge-connected rigid body tree *without* nonrigid appendages. All of the programs are designed to calculate the angular accelerations for every rigid and nonrigid body in the system but do *not* perform numerical integration. Thus, the routines are intended as general-purpose tools, to be called into action by the user's own particular simulation language, whether this be CSSL, CSMP, MIMIC, or some "homemade" variety. Each of the routines allows the user to prescribe the motion of any *rigid* body in the system rather than allow it to be calculated, a feature often useful for eliminating unwanted dynamics or for "rigidizing" certain joints in sensitivity studies.

## II. Unrestricted Systems

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### A. Mathematical Model

Any problem of dynamic analysis must begin with the adoption of a mathematical model representing the physical system of interest. In what follows, it is assumed that the model consists of  $n + 1$  rigid bodies (labeled  $\mathcal{L}_0, \dots, \mathcal{L}_n$ ) interconnected by  $n$  line hinges (implying no closed loops and, hence, tree topology), with each body containing no more than three orthogonal rigid rotors, each with an axis of symmetry fixed in the housing body, and moreover with the possibility of attaching to each of the  $n + 1$  bodies a nonrigid appendage, with appendage  $\alpha_k$  attached to body  $\mathcal{L}_k$ .

If the actual connection between two massive portions of the physical system admits two (or three) degrees of freedom in rotation, then the analyst simply introduces one (or two) massless and dimensionless imaginary bodies into his model (as though they were massless gimbals). Since the number of equations to be derived here matches the number of degrees of freedom of the system, no price is paid in problem dimension by the introduction of imaginary bodies.

Each combination of a rigid body and its internal rotors and attached flexible appendage comprises a basic building block, referred to here as a *substructure*;

<sup>1</sup> Deviations from nominal appendage base motion are treated as small in the sense that their products with appendage deformations are ignored, but nonlinear terms in these base motion deviations alone are retained. Thus, there is a *formal* limitation to small base motion deviations from nominal, but in practical applications, substantial deviations are accommodated quite satisfactorily.

thus, there are  $n + 1$  substructures in the total system, so labeled that  $\mathcal{A}_k$  encompasses  $\mathcal{B}_k$ ,  $\mathcal{A}_k$ , and any rotors in  $\mathcal{B}_k$ .

## Definitions and Notations

Definitions and notational conventions are as follows (see Fig. 1):

- Def. 1. Let  $n$  be the number of hinges interconnecting a set of  $n + 1$  substructures.
- Def. 2. Define the integer set  $\mathcal{B} \equiv \{0, 1, \dots, n\}$ .
- Def. 3. Define the integer set  $\mathcal{P} \equiv \{1, \dots, n\}$ .
- Def. 4. Let  $\mathcal{B}_0$  be a label assigned to one rigid body chosen arbitrarily as a reference body, and let  $\mathcal{B}_1, \dots, \mathcal{B}_n$  be labels assigned to the rest of the rigid bodies in such a way that if  $\mathcal{B}_j$  is located between  $\mathcal{B}_0$  and  $\mathcal{B}_k$ , then  $0 < j < k$ .
- Def. 5. Define dextral, orthogonal sets of unit vectors  $\mathbf{b}_1^k, \mathbf{b}_2^k, \mathbf{b}_3^k$  so as to be imbedded in  $\mathcal{B}_k$  for  $k \in \mathcal{B}$ , and such that in some arbitrarily selected nominal configuration of the total system,  $\mathbf{b}_\alpha^k = \mathbf{b}_\alpha^j$  for  $\alpha = 1, 2, 3$  and  $k, j \in \mathcal{B}$ .

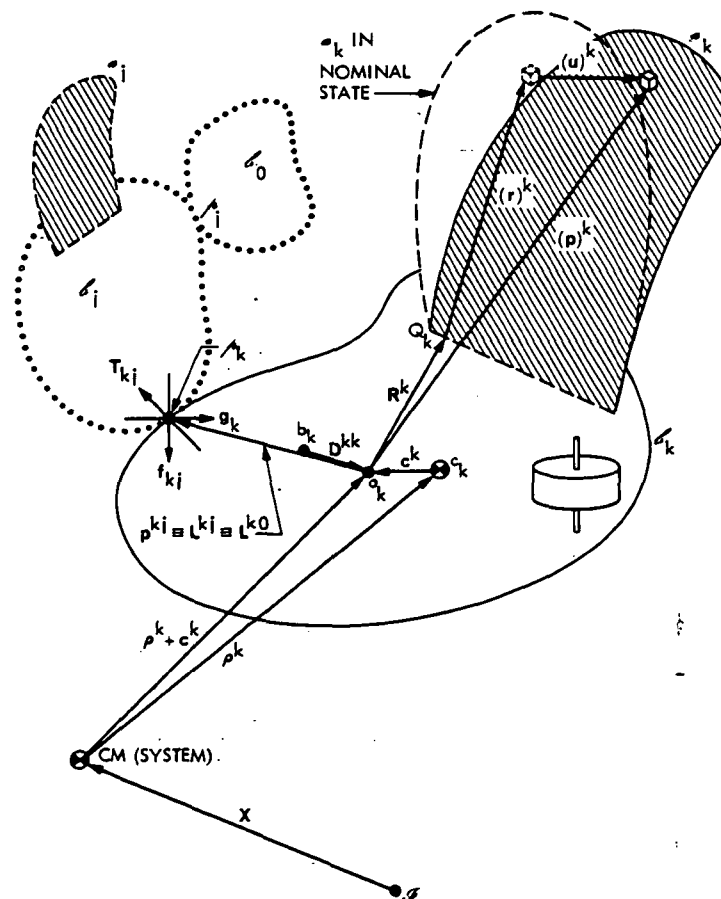


Fig. 1. Definitions for the  $k$ th substructure, with  $j < k$



Def. 6. Define

$$(\mathbf{b}^k) \equiv \begin{pmatrix} \mathbf{b}_1^k \\ \mathbf{b}_2^k \\ \mathbf{b}_3^k \end{pmatrix} \quad k \in \mathfrak{B}$$

Def. 7. Define  $\{\mathbf{i}\}$  as a column array of inertially fixed, dextral, orthogonal unit vectors  $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$ .

Def. 8. Let  $C$  be the direction cosine matrix defined by

$$\{\mathbf{b}^0\} = C\{\mathbf{i}\}$$

Def. 9. Let  $\boldsymbol{\omega}^0 \equiv \{\mathbf{b}^0\}^T \boldsymbol{\omega}^0$  be the inertial angular velocity vector of  $\mathcal{B}_0$ , so that  $\boldsymbol{\omega}^0$  is the corresponding  $3 \times 1$  matrix in basis  $\{\mathbf{b}^0\}$ .

Def. 10. Let  $c_k$  be the mass center of the  $k$ th substructure,  $k \in \mathfrak{B}$ .

Def. 11. Let  $p_k$  be a point on the hinge axis common to  $\mathcal{B}_k$  and  $\mathcal{B}_j$ , for  $j < k$  and  $k \in \mathfrak{P}$ .

Def. 12. Let  $\mathbf{p}^{kj}$  be the position vector of the hinge point connecting  $\mathcal{B}_j$  and  $\mathcal{B}_k$  from the point  $o_k$  occupied by  $c_k$  when the  $k$ th substructure is in its nominal state.

Def. 13. Let  $\mathbf{c}^k$  be the position vector from  $c_k$  to  $o_k$ .

Def. 14. Let  $\mathbf{p}^k$  be the position vector to  $c_k$  from the system mass center CM.

Def. 15. Let  $\mathbf{X}$  be the position vector to CM from an inertially fixed point  $\mathcal{G}$ , and let  $X = \mathbf{X} \cdot \{\mathbf{i}\}$ .

Def. 16. Let  $\mathfrak{M}_k$  be the mass of the  $k$ th substructure, for  $k \in \mathfrak{B}$ .

Def. 17. Let  $(\mathbf{p})^k$  be a generic position vector from  $o_k$  to any point in the  $k$ th substructure.

Def. 18. Let  $Q_k$  be a point common to rigid body  $\mathcal{B}_k$  and flexible appendage  $\alpha_k$ .

Def. 19. Let  $\mathbf{R}^k = \{\mathbf{b}^k\}^T \mathbf{R}^k$  be the position vector fixed in  $\mathcal{B}_k$  locating  $Q_k$  with respect to  $o_k$ .

Def. 20. Let  $(\mathbf{r})^k = \{\mathbf{b}^k\}^T (\mathbf{r})^k$  be a generic symbol such that  $\mathbf{R}^k + (\mathbf{r})^k$  locates a typical field point in  $\alpha_k$  with respect to  $o_k$  when the flexible appendage is in some nominal state (perhaps undeformed). For a discretized appendage  $\alpha_k$ , let  $(\mathbf{r}^s)^k = \{\mathbf{b}^k\}^T (\mathbf{r}^s)^k$  locate the  $s$ th node in the nominal state.

Def. 21. Define the generic deformation vector  $(\mathbf{u})^k$  in such a way that<sup>2</sup>

$$(\mathbf{p})^k \equiv \mathbf{R}^k + (\mathbf{r})^k + (\mathbf{u})^k$$

and

$$(\mathbf{p})^k = \mathbf{R}^k + (\mathbf{r})^k + (\mathbf{u})^k$$

<sup>2</sup> Superscripts on generic symbols such as  $\mathbf{p}$ ,  $\mathbf{r}$ , and  $\mathbf{u}$  will be omitted when obvious, as when the symbol appears within an integrand of a definite integral.

For a discretized appendage  $\alpha_k$ , let  $(u^s)^k = \{b^k\}^T (u^s)^k$  be the deformation vector for node  $s$ .

- Def. 22. Let  $g^k \equiv \{b^k\}^T g^k$  be a unit vector parallel to the hinge axis through  $\mathcal{A}_k$ .
- Def. 23. For  $k \in \mathcal{P}$ , let  $\gamma_k$  be the angle of a  $g^k$  rotation of  $\mathcal{A}_k$  with respect to the body attached at  $\mathcal{A}_k$ . Let  $\gamma_k$  be zero when  $b_\alpha^k = b_\alpha^j$  ( $\alpha = 1, 2, 3; j, k \in \mathcal{B}$ ).
- Def. 24. Let  $J^k \equiv \{b^k\}^T J^k \{b^k\}$  be the inertia dyadic of the  $k$ th substructure for  $o_k$ , so that  $J^k$  is time-variable by virtue of deformations.
- Def. 25. Let  $F^k \equiv \{b^k\}^T F^k$  be the resultant vector of all forces applied to the  $k$ th substructure except for those due to interbody forces transmitted at hinge connections.
- Def. 26. Let  $T^k \equiv \{b^k\}^T T^k$  be the resultant moment vector with respect to  $c_k$  of all forces applied to the  $k$ th substructure except for those due to interbody forces transmitted at hinge connections.
- Def. 27. Let  $\tau_k$  be the scalar magnitude of the torque component applied to  $\mathcal{A}_k$  in the direction of  $g^k$  by the body attached at  $\mathcal{A}_k$ .
- Def. 28. Let  $F \equiv \sum_{k \in \mathcal{B}} F^k = \{b^0\}^T F$  be the external force resultant for the total system.
- Def. 29. Define the scalar  $\epsilon_{sk}$  such that for  $k \in \mathcal{B}$  and  $s \in \mathcal{P}$

$$\epsilon_{sk} \equiv \begin{cases} 1 & \text{if } \mathcal{A}_s \text{ lies between } \mathcal{A}_0 \text{ and } \mathcal{A}_k \\ 0 & \text{otherwise} \end{cases}$$

(The  $n(n+1)$  scalars  $\epsilon_{sk}$  are called *path elements*.)

- Def. 30. Define  $\mathcal{M} \equiv \sum_{k \in \mathcal{B}} \mathcal{M}_k$ , the total system mass.
- Def. 31. Let  $C^{rj}$  be the direction cosine matrix defined by  $\{b^r\} = C^{rj} \{b^j\}$ ,  $r, j \in \mathcal{B}$ . (Note that in the nominal state,  $C^{rj} = U$ , the unit matrix.)
- Def. 32. Let  $N_{kr}$  denote the index of the body attached to  $\mathcal{A}_k$  and on the path leading to  $\mathcal{A}_r$ , and let  $N_{kk} \equiv k$ . (These are the *network elements*.) For notational simplicity, use  $N_k$  for  $N_{k0}$ .
- Def. 33. For<sup>3</sup>  $r \in \mathcal{B} - k$ , let  $L^{kr} \equiv p^{kN_r}$ , and let  $L^{kk} \equiv 0$ .
- Def. 34. Define  $D^{kk} \equiv - \sum_{j \in \mathcal{B}} L^{kj} \mathcal{M}_j / \mathcal{M}$  for  $k \in \mathcal{B}$ .
- Def. 35. Let  $b_k$  be a point fixed in  $\mathcal{A}_k$  such that  $D^{kk}$  is the position vector of  $o_k$  with respect to  $b_k$ . (This point  $b_k$  is called the *barycenter* of the  $k$ th substructure in the nominal state.)
- Def. 36. Define  $\{b^k\}^T D^{kj} \equiv D^{kj} \equiv D^{kk} + L^{kj}$  for  $k, j \in \mathcal{B}$ .
- Def. 37. Define the dyadic

$$K^k \equiv \sum_{r \in \mathcal{B}} \mathcal{M}_r (D^{kr} \cdot D^{kr} U - D^{kr} D^{kr})$$

<sup>3</sup> For notational brevity, the set  $\mathcal{B} - \{k\}$  is designated  $\mathcal{B} - k$ .

where  $\mathbf{U}$  is the unit dyadic, and define the corresponding matrix  $K^k \equiv \{\mathbf{b}^k\} \cdot \mathbf{K}^k \cdot \{\mathbf{b}^k\}^T$ .

Def. 38. Define

$$\Phi^{kk} \equiv \mathbf{K}^k + \mathbf{J}^k \quad \text{and} \quad \Phi^{kk} \equiv \{\mathbf{b}^k\} \cdot \Phi^{kk} \cdot \{\mathbf{b}^k\}^T$$

Def. 39. Define

$$\Phi^{kj} \equiv -\mathfrak{N}(\mathbf{D}^{jk} \cdot \mathbf{D}^{kj} \mathbf{U} - \mathbf{D}^{jk} \mathbf{D}^{kj})$$

with

$$\{\mathbf{b}^j\} \cdot \Phi^{kj} \cdot \{\mathbf{b}^k\}^T = -\mathfrak{N}(C^{jk} \mathbf{D}^{jk} C^{jk} \mathbf{D}^{kj} - \mathbf{D}^{jk} \mathbf{D}^{kj})$$

Def. 40. Let  $\boldsymbol{\omega}^k = \{\mathbf{b}^k\}^T \omega^k$  be the inertial angular velocity of  $\mathcal{S}_k$ .

Def. 41. Let  $\mathbf{h}^k$  be the contribution of rotors in  $\mathcal{S}_k$  to the angular momentum of the  $k$ th substructure relative to  $\mathcal{S}_k$  with respect to  $o_k$ , and let  $h^k \equiv \mathbf{h}^k \cdot \{\mathbf{b}^k\}$ .

Def. 42. Let  $\mathfrak{B}_r$  be the  $r$ th neighbor set for  $r \in \mathfrak{B}$ , such that  $k \in \mathfrak{B}_r$  if  $\mathcal{S}_k$  is attached to  $\mathcal{S}_r$ .

Def. 43. Let  $\mathfrak{B}_{jk}$  be the branch set of integers  $r$  such that  $r \in \mathfrak{B}_{jk}$  if  $k = N_{jr}$ . Thus,  $\mathfrak{B}_{jk}$  consists of the indices of those bodies attached to  $\mathcal{S}_j$  on a branch which begins with  $\mathcal{S}_k$ .

Def. 44. Let the tilde symbol ( $\tilde{\phantom{x}}$ ) signify, in application to a 3 by 1 matrix  $V$  with elements  $V_\theta$  ( $\theta = 1, 2, 3$ ), transformation to a skew-symmetric 3 by 3 matrix  $\tilde{V}$  given by

$$\tilde{V} \equiv \begin{bmatrix} 0 & -V_3 & V_2 \\ V_3 & 0 & -V_1 \\ -V_2 & V_1 & 0 \end{bmatrix}$$

## B. The Equations

The objective of this section is to begin with the general vector-dyadic equations derived in Ref. 4 and to proceed by sacrificing some of their generality in favor of a *particular* appendage model. Explicit results, in the form of both vector and matrix equations suitable for computer programming, will thereby be obtained.

In what follows, attention is confined to a special case of the finite element appendage model of Ref. 1, for which, as in Ref. 2, all mass of appendage  $k$  is concentrated in the  $n_k$  discrete nodal bodies of the appendage (with no distributed mass for the internodal elastic elements). All deformations from a nominal appendage state are assumed arbitrarily small, so that terms above the first degree in these deformations (and corresponding rates) can be neglected. Further, any rigid body  $\mathcal{S}_k$  will be assumed to carry rotors, and they will consist of an orthogonal triad whose axes parallel  $\mathbf{b}_1^k$ ,  $\mathbf{b}_2^k$ , and  $\mathbf{b}_3^k$ .

The starting point for this development is the set of vector-dyadic equations of vehicle translation and substructure rotation as derived in Ref. 4 (Eqs. 9, 31-35):

$$\mathbf{F} = \mathcal{N} \ddot{\mathbf{X}} \quad (1)$$

$$\sum_{k \in \mathcal{B}} \mathbf{W}^k = 0 \quad (2)$$

$$\tau_s + \mathbf{g}' \cdot \sum_{k \in \mathcal{P}} \epsilon_{sk} \mathbf{W}^k = 0 \quad (s \in \mathcal{P}) \quad (3)$$

where

$$\begin{aligned} \mathbf{W}^k \equiv & \mathbf{T}^k + \sum_{r \in \mathcal{B}} \mathbf{D}^{kr} \times \mathbf{F}^r + \mathbf{c}^k \times \left( \frac{\mathcal{N}_k}{\mathcal{N}} \mathbf{F} - \mathbf{F}^k \right) \\ & + \sum_{r \in \mathcal{B}} \mathcal{N}_r \mathbf{D}^{kr} \times [\ddot{\mathbf{c}}^r + 2\boldsymbol{\omega}' \times \dot{\mathbf{c}}^r + \dot{\boldsymbol{\omega}}' \times \mathbf{c}^r + \boldsymbol{\omega}' \times (\boldsymbol{\omega}' \times \mathbf{c}^r)] \\ & + \mathcal{N}_k \mathbf{c}^k \times \sum_{r \in \mathcal{B}} [\dot{\boldsymbol{\omega}}' \times \mathbf{D}^{rk} + \boldsymbol{\omega}' \times (\boldsymbol{\omega}' \times \mathbf{D}^{rk})] \\ & - \Phi^{kk} \cdot \dot{\boldsymbol{\omega}}^k - \sum_{r \in \mathcal{B} - k} \Phi^{kr} \cdot \dot{\boldsymbol{\omega}}^r + \mathcal{N}_k \sum_{r \in \mathcal{B} - k} \mathbf{D}^{kr} \times [\boldsymbol{\omega}' \times (\boldsymbol{\omega}' \times \mathbf{D}^{rk})] \\ & - \boldsymbol{\omega}^k \times \Phi^{kk} \cdot \boldsymbol{\omega}^k - \dot{\mathbf{h}}^k - \boldsymbol{\omega}^k \times \mathbf{h}^k - \dot{\Phi}^{kk} \cdot \boldsymbol{\omega}^k \\ & - \int_{a_k} \mathbf{p} \times \ddot{\mathbf{p}} \, dm - \boldsymbol{\omega}^k \times \int_{a_k} (\mathbf{p} \times \dot{\mathbf{p}}) \, dm \end{aligned} \quad (4)$$

and

$$\boldsymbol{\omega}^k = \boldsymbol{\omega}^0 + \sum_{r \in \mathcal{P}} \epsilon_{rk} \dot{\gamma}_r \mathbf{g}^r \quad (5)$$

$$\dot{\boldsymbol{\omega}}^k = \dot{\boldsymbol{\omega}}^0 + \sum_{r \in \mathcal{P}} \epsilon_{rk} [\dot{\gamma}_r \mathbf{g}^r + \boldsymbol{\omega}' \times \mathbf{g}^r \dot{\gamma}_r] \quad (6)$$

The adoption of a nodal body appendage model leads (as in Ref. 2, Eq. 58) to the following useful relation:

$$\mathbf{c}^k = - \sum_{s=1}^{n_k} \frac{m_s}{\mathcal{N}_k} \mathbf{u}^s \quad (7)$$

where appendage  $a_k$  has been idealized as  $n_k$  nodal bodies interconnected by massless elastic structure, with  $m_s$  the mass of nodal body  $s$ , and  $\mathbf{u}^s$  the displacement of the body  $s$  relative to  $b_k$  from the position occupied in the nominal state.

It will also be necessary to develop an expression for  $\dot{\Phi}^{kk}$  in terms of appendage variables. From Def. 38, we know that

$$\Phi^{kk} = \mathbf{K}^k + \mathbf{J}^k \quad (8)$$

where  $\mathbf{K}^k$ , the "augmented" inertia dyadic, is a constant.  $\mathbf{J}^k$ , the inertia dyadic of the  $k$ th substructure for  $o_k$ , is time-variable due to appendage deformations and may be obtained from

$$\mathbf{J}^k = \int (\mathbf{p} \cdot \mathbf{p} \mathbf{U} - \mathbf{p} \mathbf{p}) dm \quad (9)$$

where  $\mathbf{U}$  is the unit dyadic.

For the small-deformation appendage model adopted here,  $\mathbf{J}^k$  may be evaluated (see Ref. 2, Eq. 126) as

$$\begin{aligned} \mathbf{J}^k = \bar{\mathbf{J}}^k + \{\mathbf{b}^k\}^T \left[ \sum_{s=1}^{n_k} \left\{ m_s [2(R^k + r^s)^T u^s \mathbf{U} - (R^k + r^s) u^s{}^T \right. \right. \\ \left. \left. - u^s (R^k + r^s)^T] + \tilde{\beta}^s I^s - I^s \tilde{\beta}^s \right\} \right] \{\mathbf{b}^k\} \end{aligned} \quad (10)$$

where  $\bar{\mathbf{J}}^k$  is the nominal (constant) value of  $\mathbf{J}^k$ , and  $I^s$  is the constant inertia matrix of the  $s$ th nodal body for its own mass center and in its own body-fixed vector basis  $\{\mathbf{n}^s\}^k$ , where in the nominal state,  $\{\mathbf{n}^s\}^k = \{\mathbf{b}^k\}$ .

Combining (8) and (10), we have

$$\begin{aligned} \dot{\Phi}^{kk} = \{\mathbf{b}^k\}^T \left[ \sum_{s=1}^{n_k} \left\{ m_s [2(R^k + r^s)^T \dot{u}^s \mathbf{U} - (R^k + r^s) \dot{u}^s{}^T \right. \right. \\ \left. \left. - \dot{u}^s (R^k + r^s)^T] + \dot{\tilde{\beta}}^s I^s - I^s \dot{\tilde{\beta}}^s \right\} \right] \{\mathbf{b}^k\} \end{aligned} \quad (11)$$

Finally, Eq. (4) requires more explicit expressions for the integrals over the appendage  $a_k$ . The appropriate expressions in this case may be found in Eq. (114) of Ref. 2, which simplifies to

$$-\frac{d}{dt} \int_{a_k} \mathbf{p} \times \dot{\mathbf{p}} dm = - \int_{a_k} \mathbf{p} \times \ddot{\mathbf{p}} dm - \boldsymbol{\omega}^k \times \int_{a_k} \mathbf{p} \times \dot{\mathbf{p}} dm$$

or

$$\begin{aligned} -\frac{d}{dt} \int_{a_k} \mathbf{p} \times \dot{\mathbf{p}} dm = - \sum_{s=1}^{n_k} (\mathbf{R}^k + \mathbf{r}^s) \times m_s \ddot{\mathbf{u}}^s - \boldsymbol{\omega}^k \times \sum_{s=1}^{n_k} (\mathbf{R}^k + \mathbf{r}^s) \times m_s \dot{\mathbf{u}}^s \\ - \sum_{s=1}^{n_k} (\mathbf{r}^s \cdot \ddot{\boldsymbol{\beta}}^s + \boldsymbol{\omega}^k \times \mathbf{r}^s \cdot \dot{\boldsymbol{\beta}}^s) \end{aligned} \quad (12)$$

Note that in Eqs. (7), (10), (11), and (12), the superscript  $k$  has been dropped from nodal body variables in the  $k$ th appendage (such as  $u^s$ , which replaces  $(u^s)^k$ ).

Turning now to the appendage equations, we will make use of the nodal body finite element model case described by Eq. (95) of Ref. 2 (correcting the last algebraic sign within the braces on the right side of Eq. 95 by changing  $-$  to  $+$ , and subtracting all nominally nonzero terms from the right side so as to make  $q$  a measure of the deviation from a nominal state in which the appendage might be deformed). In matrix form, the equation for the  $k$ th appendage becomes

$$\begin{aligned}
& M^k \left( U - \sum_{U0} \sum_{U0}^T \frac{M^k}{\mathfrak{N}_k} \right) \ddot{q}^k + \left\{ 2M^k \left[ (\sum_{U0} \omega^k)^- - \sum_{U0} \tilde{\omega}^k \sum_{U0}^T \frac{M^k}{\mathfrak{N}_k} \right] \right. \\
& \quad \left. + M^k (\sum_{0U} \omega^k)^- + (\sum_{0U} \omega^k)^- M^k - (M^k \sum_{0U} \omega^k)^- \right\} \dot{q}^k \\
& \quad + \left\{ M^k (\sum_{0U} \dot{\omega}^k)^- - (M^k \sum_{0U} \dot{\omega}^k)^- - (\sum_{0U} \omega^k)^- (M^k \sum_{0U} \dot{\omega}^k)^- \right. \\
& \quad \left. + (\sum_{0U} \omega^k)^- M^k (\sum_{0U} \dot{\omega}^k)^- + M^k \left[ (\sum_{U0} \dot{\omega}^k)^- \right. \right. \\
& \quad \left. \left. - \sum_{U0} (\tilde{\omega}^k + \tilde{\omega}^k \tilde{\omega}^k) \sum_{U0}^T \frac{M^k}{\mathfrak{N}_k} + (\sum_{U0} \omega^k)^- (\sum_{U0} \dot{\omega}^k)^- \right] + K^k \right\} q^k \\
& = -M^k \left\{ \sum_{0U} \dot{\omega}^k + \sum_{U0} [\Theta \ddot{X} - \tilde{R}^k \dot{\omega}^k + \tilde{\omega}^k \tilde{\omega}^k R^k - \tilde{\Omega}^k \tilde{\Omega}^k R^k] \right. \\
& \quad \left. + (\sum_{U0} \omega^k)^- (\sum_{U0} \dot{\omega}^k)^- \tilde{r}_k - (\sum_{U0} \Omega^k)^- (\sum_{U0} \dot{\Omega}^k)^- \tilde{r}_k \right. \\
& \quad \left. - \tilde{r}_k \sum_{U0} \dot{\omega}^k \right\} - (\sum_{0U} \omega^k)^- M^k (\sum_{0U} \dot{\omega}^k) + (\sum_{0U} \Omega^k)^- M^k (\sum_{0U} \dot{\Omega}^k) + \lambda^k \quad (13)
\end{aligned}$$

where the assumption has been made that the appendage structure contains no damping. The symbol  $\lambda^k$  is a column matrix containing any forces or torques applied to the  $n_k$  sub-bodies of the appendage other than the structural interaction forces induced by deformations. For example, gravity forces or attitude control jet thrust would contribute to  $\lambda^k$ . Also,

$$q^k \equiv [u_1^1 u_2^1 u_3^1 \beta_1^1 \beta_2^1 \beta_3^1 u_1^2 \cdots \beta_3^{n_k}]^T$$

a  $6n_k$  by 1 matrix which fully characterizes the appendage deformations relative to some nominal state of deformation induced by the nominal constant value  $\Omega^k$  of  $\omega^k$ .

$$M^k \equiv \begin{bmatrix} m^1 & 0 & 0 & 0 & \dots & 0 \\ 0 & I^1 & 0 & 0 & \dots & 0 \\ 0 & 0 & m^2 & 0 & \dots & 0 \\ 0 & 0 & 0 & I^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & I^{n_k} \end{bmatrix}$$

a constant, symmetric  $6n_k$  by  $6n_k$  matrix defined in terms of the 3 by 3 partitioned matrices  $m^s, I^s$ .

$$m^s \equiv \begin{bmatrix} m_s & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_s \end{bmatrix}, \quad I^s = \begin{bmatrix} I_{11}^s & I_{12}^s & I_{13}^s \\ I_{21}^s & I_{22}^s & I_{23}^s \\ I_{31}^s & I_{32}^s & I_{33}^s \end{bmatrix} \quad (s = 1, \dots, n_k)$$

$$\Sigma_{U0} = [U \quad 0 \quad U \quad 0 \quad \dots \quad U \quad 0]^T$$

$$\Sigma_{0U} = [0 \quad U \quad 0 \quad U \quad \dots \quad 0 \quad U]^T$$

$6n_k$  by 3 Boolean operator matrices, where  $U$  and  $0$  are the 3 by 3 unit and null matrices, respectively.

$$r_k \equiv [r^{1T} \quad 0 \quad r^{2T} \quad 0 \quad \dots \quad r^{n_k T} \quad 0]^T$$

$$\tilde{r}_k \equiv \begin{bmatrix} \tilde{r}^1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \tilde{r}^2 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \tilde{r}^{n_k} & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

a constant  $6n_k$  by  $6n_k$  matrix.

$K^k \equiv$  the stiffness matrix that determines the structural interaction forces and torques induced by deformation of the  $k$ th appendage from its nominal state (a constant, symmetric  $6n_k$  by  $6n_k$  matrix).

It should now be recognized that the term  $\Theta \ddot{X}$  in Eq. (13) must be replaced by the inertial acceleration of the mass center of the corresponding substructure in the local vector basis, which is assumed for each  $k$  to be zero in the "nominal" state. For substructure  $s_k$ , this term is given by (see Eq. 54, Ref. 4)

$$\Theta \ddot{X} = C^{k0} C \ddot{X} - (\ddot{c}^k + \ddot{\omega}^k c^k + 2\dot{\omega}^k \dot{c}^k + \dot{\omega}^k \dot{\omega}^k c^k) + \sum_{r \in \mathfrak{B}} C^{kr} \left[ (\ddot{\omega}^r + \dot{\omega}^r \dot{\omega}^r) \left( D^{rk} + \frac{\mathfrak{M}_r}{\mathfrak{M}} c^r \right) + \frac{\mathfrak{M}_r}{\mathfrak{M}} (\ddot{c}^r + 2\dot{\omega}^r \dot{c}^r) \right] \quad (14)$$

and

$$C \ddot{X} = \frac{F}{\mathfrak{M}} \quad (15)$$

treated as zero in the nominal state.

Equations (1)–(15) provide a rather complete system description (although the contribution of rigid rotors, i.e.,  $h^k$ , will be developed in more detail later). Since the number of nodes  $n_k$  in a single finite-element model of an elastic appendage is typically rather large, it is to be understood that the nodal body vibration equations, (Eqs. 13–15), will provide the basis for a transformation to distributed or modal coordinates for appendage deformations and that most of these will be deleted from consideration by truncating the matrix of deformation variables. Thus, the variables labeled  $u^j$  and  $\beta^j$  above will be replaced by appropriate combinations of new modal deformation variables.

The equations actually to be programmed for digital computer solution will therefore be the transformed and truncated versions of Eqs. (1)–(15). These will be described in the following sections as the system motions are confined to two particular cases of interest: (1) the case in which all appendage base-body angular rates  $\omega^k$  experience only slight deviations from some constant nonzero value (i.e.,  $\omega^k \approx \Omega^k$ ,  $\dot{\omega}^k \approx 0$ ), or (2) the case in which  $\Omega^k \approx 0$  (i.e.,  $\omega^k \approx 0$ ,  $\dot{\omega}^k \approx 0$ ) for all appendage base bodies.

In the first case, i.e., where  $\omega^k \approx \Omega^k$  and  $\dot{\omega}^k \approx 0$ , the approach taken in developing the system equations of motion, including linearization, coordinate transformation, and truncation, may be described as follows:

- (1) For the purposes of constructing a coordinate transformation for the appendages, assume that  $\omega^k$  experiences only small deviations from a constant  $\Omega^k$ , and write the homogeneous form of the appendage equations.
- (2) Construct a coordinate transformation from these linear, constant-coefficient equations, and select the truncation level.
- (3) Return to the unrestricted  $\omega^k$  assumption, and substitute the transformations from (2) into *all* equations of motion.
- (4) In the homogeneous part of the appendage vibration equations *only*, ignore products of deformation variables and deviations of  $\omega^k$  from  $\Omega^k$ . *This step is not formally correct*, since mathematically we cannot justify treating the deviation of  $\omega^k$  from  $\Omega^k$  as small only when it is multiplied by a deformation variable. On the basis of engineering judgment, however, the authors feel that it is probably justifiable and would be a less significant source of error than either modeling or truncation. The resulting equations contain all terms formally required for the analysis of a system with appendage base bodies experiencing small deviations from their nominal motions, but in applying these equations to systems with large deviations of base bodies from their



nominal motion one is suppressing products of these deviations with deformation variables. In fact, a very large change in base-body spin rate would change the effective structural stiffness of the appendage, and invalidate the modal analysis on which the appendage modal coordinate selection is based. In this respect, the equations would be tainted by truncation even if the suppressed terms were retained, and, since these terms would substantially complicate the analysis by coupling all variables into each vibration equation, they have been rejected here.

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### III. Systems With Rotating Appendages

#### A. Equations

Inspection of the appendage equations (Eqs. 13-15) reveals that the coefficients of  $q^k$  and  $\dot{q}^k$  depend upon  $\omega^k$ , which characterizes the rotational motion of the appendage base. In general,  $\omega^k$  is an unknown function of time, to be determined only after the appendage equations are augmented by other equations of dynamics and control for the total vehicle and solved. Only if  $\omega^k$  can be assumed to experience, in a given time interval, small excursions about a constant nominal value  $\Omega^k$  is there any possibility of transforming Eq. (13) to a new set of *uncoupled* appendage coordinates. Any methods involving modal coordinates (see Ref. 1, Sect. I) depend formally upon this assumption.

Assuming then that  $\omega^k \approx \Omega^k$  and  $\dot{\omega}^k \approx 0$ , Eqs. (13)-(15) can be combined to provide the following appendage equation:

$$\begin{aligned}
 & M^k \left( U - \sum_{U0} \Sigma_{U0}^T \frac{M^k}{\mathcal{N}} \right) \dot{q}^k + \left\{ 2M^k \left[ (\Sigma_{U0} \Omega^k)^{-} - \sum_{U0} \tilde{\Omega}^k \Sigma_{U0}^T \frac{M^k}{\mathcal{N}} \right] \right. \\
 & \quad \left. + M^k (\Sigma_{0U} \Omega^k)^{-} + (\Sigma_{0U} \Omega^k)^{-} M^k - (M^k \Sigma_{0U} \Omega^k)^{-} \right\} \dot{q}^k \\
 & \quad + \left\{ -(\Sigma_{0U} \Omega^k)^{-} (M^k \Sigma_{0U} \Omega^k)^{-} + (\Sigma_{0U} \Omega^k)^{-} M^k (\Sigma_{0U} \Omega^k)^{-} \right. \\
 & \quad \left. + M^k \left[ -\sum_{U0} (\tilde{\Omega}^k \tilde{\Omega}^k) \Sigma_{U0}^T \frac{M^k}{\mathcal{N}} + (\Sigma_{U0} \Omega^k)^{-} (\Sigma_{U0} \Omega^k)^{-} \right] + K^k \right\} q^k \\
 & = \left( -M^k \Sigma_{0U} + M^k \sum_{U0} \tilde{R}^k + M^k \tilde{r}_k \Sigma_{U0} \right) \dot{\omega}^k - M^k \sum_{U0} \sum_{r \in \mathfrak{B}} C^{kr} (\tilde{\omega}^r + \tilde{\omega}' \tilde{\omega}^r) D^{rk} \\
 & \quad - M^k \left[ \sum_{U0} C^{k0} \frac{F}{\mathcal{N}} + \sum_{U0} \tilde{\omega}^k \tilde{\omega}^k R^k + (\Sigma_{U0} \omega^k)^{-} (\Sigma_{U0} \omega^k)^{-} r_k \right] \\
 & \quad - (\Sigma_{0U} \omega^k)^{-} M^k (\Sigma_{0U} \omega^k) + \lambda^k + M^k \left[ \sum_{U0} \tilde{\Omega}^k \tilde{\Omega}^k R^k + (\Sigma_{U0} \Omega^k)^{-} (\Sigma_{U0} \Omega^k)^{-} r_k \right] \\
 & \quad + (\Sigma_{0U} \Omega^k)^{-} M^k (\Sigma_{0U} \Omega^k) \\
 & \quad - M^k \sum_{U0} \sum_{r \in \mathfrak{B} - k} C^{kr} \left[ -\sum_{U0}^T \frac{M^r}{\mathcal{N}} \dot{q}^r + 2\tilde{\omega}^r \frac{\mathcal{N}_r}{\mathcal{N}} \dot{c}^r + \tilde{\omega}' \tilde{\omega}^r \frac{\mathcal{N}_r}{\mathcal{N}} c^r \right] \quad (16)
 \end{aligned}$$

Equation (16) consists of  $6n_k$  second-order scalar equations and can be written as a matrix equation with the following structure:

$$M'_k \ddot{q}^k + D'_k \dot{q}^k + G'_k \dot{q}^k + K'_k q^k + A'_k \dot{q}^k = L'_k \quad (17)$$

where

$$M'_k = M^k \left( U - \sum_{U0} \Sigma_{U0}^T \frac{M^k}{\mathfrak{N}} \right)$$

$$D'_k = 0$$

$$G'_k = 2M^k \left[ (\Sigma_{U0} \Omega^k)^{-} - \sum_{U0} \tilde{\Omega}^k \Sigma_{U0}^T \frac{M^k}{\mathfrak{N}} \right] + M^k (\Sigma_{0U} \Omega^k)^{-}$$

$$+ (\Sigma_{0U} \Omega^k)^{-} M^k - (M^k \Sigma_{0U} \Omega^k)^{-}$$

$$A'_k = -(\Sigma_{0U} \Omega^k)^{-} (M \Sigma_{0U} \Omega^k)^{-}$$

$$K'_k = (\Sigma_{0U} \Omega^k)^{-} M^k (\Sigma_{0U} \Omega^k)^{-} + K^k$$

$$+ M^k \left[ -\sum_{U0} (\tilde{\Omega}^k \tilde{\Omega}^k) \Sigma_{U0}^T \frac{M^k}{\mathfrak{N}} + (\Sigma_{U0} \Omega^k)^{-} (\Sigma_{U0} \Omega^k)^{-} \right]$$

and

$$L'_k = -M^k \left[ \Sigma_{0U} - \sum_{U0} (\tilde{R}^k + \tilde{D}^{kk}) - \tilde{r}_k \Sigma_{U0} \right] \dot{\omega}^k - M^k \Sigma_{U0} C^{k0} \frac{F}{\mathfrak{N}} + \lambda^k$$

$$- M^k \Sigma_{U0} \sum_{r \in \mathfrak{B} - k} C^{kr} (\tilde{\omega}^r + \tilde{\omega}^r \tilde{\omega}^r) D^{rk} + N_k^c - N_{k_u}^c$$

$$- M^k \Sigma_{U0} \sum_{r \in \mathfrak{B} - k} C^{kr} \left[ -\Sigma_{U0}^T \frac{M^r}{\mathfrak{N}} \dot{q}^r + 2\tilde{\omega}^r \frac{\mathfrak{N}_r}{\mathfrak{N}} \dot{c}^r + \tilde{\omega}^r \tilde{\omega}^r \frac{\mathfrak{N}_r}{\mathfrak{N}} c^r \right]$$

with

$$N_k^c = -M^k \left[ \sum_{U0} \tilde{\omega}^k \tilde{\omega}^k (R^k + D^{kk}) \right]$$

$$+ (\Sigma_{U0} \omega^k)^{-} (\Sigma_{U0} \omega^k)^{-} r_k - (\Sigma_{0U} \omega^k)^{-} M^k (\Sigma_{0U} \omega^k)$$

and

$$N_{k_u}^c = -M^k \left[ \sum_{U0} \tilde{\Omega}^k \tilde{\Omega}^k (R^k + D^{kk}) + (\Sigma_{U0} \Omega^k)^{-} (\Sigma_{U0} \Omega^k)^{-} r_k \right] - (\Sigma_{0U} \Omega^k)^{-} M^k (\Sigma_{0U} \Omega^k)$$

Matrices  $M'_k$ ,  $D'_k$ , and  $K'_k$  are constant symmetric matrices, while  $G'_k$  is a constant skew-symmetric matrix, and  $A'_k$  has both symmetric and skew-symmetric parts.  $N_k^c$

contains the nonlinear terms in  $\omega^k$  due to centripetal accelerations of the appendage due to  $\omega^k$ , and  $N_{k_u}^c$  represents the nominal steady-state value of  $N_k^c$ .

Notice that the form of Eq. (17) is identical to that of Eq. (140) in Ref. 2 (or Eq. 64, Ref. 1), with the exception of the additional right-hand-side terms

$$-M^k \sum_{r \in \mathfrak{B} - k} \sum_{\nu} C^{kr} \left[ -\sum_{\nu}^T \frac{M^r}{\mathfrak{M}} \ddot{q}^r + 2\tilde{\omega}^r \frac{\mathfrak{M}_r}{\mathfrak{M}} \dot{c}^r + \tilde{\omega}^r \tilde{\omega}^r \frac{\mathfrak{M}_r}{\mathfrak{M}} c^r + (\tilde{\omega}^r + \tilde{\omega}^r \tilde{\omega}^r) D^{rk} \right]$$

which describe the coupling of appendage  $a_k$  to other rigid bodies and appendages of the system. Also, in comparing Eq. (17) to Eq. (140) of Ref. 2, note that  $\mathbf{R}$  has been replaced by  $(\mathbf{R}^k + \mathbf{D}^{kk})$ , a vector from the mass center (barycenter) of the undeformed *augmented* substructure to the point  $Q_k$  (see Fig. 1 and Def. 35).

At this point in the development of the appendage equations, it is appropriate to elaborate upon what is meant by "nominal appendage state," and what relationship this idea has to Eq. (17). We have already indicated that the approach to be taken is that of Ref. 1 (see pp. 1-3), namely that appendages are ideally considered as linearly elastic and that  $\mathbf{u}$  and  $\beta$  are "small," oscillatory appendage deformations, i.e., variational deformations. It is quite possible that these small oscillatory deformations will be superimposed on relatively large steady-state deformations, due to spin, for example.

In order to derive a suitable appendage equation, applicable for a "variational deformation"  $\mathbf{q}$ , the substitution of an expansion for the total deformation  $\mathbf{q}'$  such as

$$\mathbf{q}' = \mathbf{q} + \mathbf{q}_{ss}$$

has been made in Eq. (17), where  $\mathbf{q}_{ss}$  (= constant) is understood to be the steady-state appendage deformation due to spin. The steady-state deformation is given by

$$(K_k' + A_k') \mathbf{q}_{ss} = N_{k_u}^c$$

where

$$N_{k_u}^c = -M^k \left[ \sum_{\nu} \tilde{\Omega}^k \tilde{\Omega}^k (R^k + D^{kk}) + (\sum_{\nu} \tilde{\Omega}^k)^{-1} (\sum_{\nu} \tilde{\Omega}^k)^{-1} r_k \right] \\ - (\sum_{\nu} \tilde{\Omega}^k)^{-1} M^k (\sum_{\nu} \tilde{\Omega}^k)$$

In effect then, in Eq. (17), we have linearized about the steady-state deformation induced by centrifugal forces due to spin of the  $k$ th substructure, with the mass center of this substructure inertially fixed. It should also be remembered that the original definitions of  $o_k$ ,  $c_k$ , and the vectors  $(\mathbf{r})^k$ ,  $\mathbf{R}^k$ , etc., remain intact but that the term "nominal state" is more clearly specified as the "steady state" of deformation due to the nominal (constant) spin of the  $k$ th substructure, with the mass center of that substructure inertially fixed. Also, the value of  $K^k$  should include whatever increment to the elastic stiffness of the appendage is attributable to structural preload due to this spin; that is,  $K^k$  includes the so-called "geometric stiffness matrix" of the structure.

The matrix  $D'$ , which in the general case would accommodate any viscous damping that may be introduced to represent energy dissipation due to structural vibrations, is zero here since such terms have been omitted. But they can still be inserted if one accepts the practice common among structural dynamicists of incorporating the equivalent of a term  $D'_k \dot{q}^k$  into equations of vibration only *after* derivation of equations of motion and transformation of coordinates.

The nature of terms contributing to  $G'_k$ ,  $K'_k$ , and  $A'_k$  is discussed in some detail in Ref. 1. In particular, the matrix  $A'_k$  is shown in Ref. 1 to disappear for the case of small base excursions about a nonzero constant spin only if the nodal bodies are particles or spheres, or if in the steady state of deformation, all nodal bodies have principal axes of inertia aligned with the nominal value of the angular velocity  $\omega^k$  (i.e.,  $\omega^k \approx \{b^k\} \Omega^k$ ). The *latter* restriction will henceforth be adopted in this report since it greatly reduces the computational task in transforming the homogeneous form of Eq. (17) to a set of completely uncoupled differential equations.

In order to transform Eq. (17) to a set of uncoupled equations, it is first necessary to rewrite it in first-order form, such as

$$\mathcal{U}_k \dot{Q}^k + \mathcal{V}_k Q^k = \mathcal{E}_k \quad (18)$$

where

$$Q^k \equiv \begin{bmatrix} q^k \\ \dot{q}^k \end{bmatrix} \quad \mathcal{E}_k \equiv \begin{bmatrix} 0 \\ L'_k \end{bmatrix}$$

$$\mathcal{U}_k \equiv \begin{bmatrix} K'_k & | & 0 \\ \hline 0 & | & M'_k \end{bmatrix} \quad \mathcal{V}_k \equiv \begin{bmatrix} 0 & | & -K'_k \\ \hline K'_k & | & G'_k \end{bmatrix}$$

Now let  $\Phi$  be a  $(12n_k \times 12n_k)$  matrix of (complex) eigenvectors of the differential operator in Eq. (18), and let  $\Phi'$  be a  $(12n_k \times 12n_k)$  matrix of (complex) eigenvectors of the homogeneous adjoint equation

$$\mathcal{U}_k^T \dot{Q}'^k + \mathcal{V}_k^T Q'^k = 0 \quad (19)$$

so that  $\Phi_k$  and  $\Phi'_k$  are related by

$$\Phi_k^{-1} = l \Phi_k'^T$$

with  $l$  a  $(12n_k \times 12n_k)$  diagonal matrix which depends upon the normalization of  $\Phi_k$  and  $\Phi'_k$ . Substitution into Eq. (18) of the transformation

$$Q^k = \Phi_k Y^k$$

and premultiplication by  $\Phi_k'^T$  furnishes

$$(\Phi_k'^T \mathcal{U}_k \Phi_k) \dot{Y}^k + (\Phi_k'^T \mathcal{V}_k \Phi_k) Y^k = \Phi_k'^T \mathcal{E}_k$$

The two coefficient matrices enclosed in parentheses are diagonal. If  $\Lambda_k$  is the  $(12n_k \times 12n_k)$  matrix of the (complex) eigenvalues of the differential operator in Eq. (18), then upon premultiplication by  $(\Phi_k'^T \mathcal{U}_k \Phi_k)^{-1}$ , one obtains

$$\dot{Y}^k = \Lambda_k Y^k + (\Phi_k'^T \mathcal{U}_k \Phi_k)^{-1} \Phi_k'^T \mathcal{P}_k \quad (20)$$

Note that the matrix inversion in Eq. (20) consists simply of calculating the reciprocals of the diagonal elements of  $\Phi_k'^T \mathcal{U}_k \Phi_k$ .

In practice, one may expect that physical interpretation of the new (complex) state variables  $Y_1^k, \dots, Y_{12n_k}^k$  will permit truncation to a reduced set of variables

$$\bar{Y}^k \equiv [Y_1^k \dots Y_{N_k}^k Y_1^{k*} \dots Y_{N_k}^{k*}]^T \quad (21)$$

where  $N_k$  is the number of modes to be preserved in the simulation. The transformation matrix  $\Phi_k$  is accordingly truncated to the  $(12n_k \times 2N_k)$  matrix  $\bar{\Phi}_k$ , where

$$\bar{\Phi}_k \equiv [\Phi_k' \dots \Phi_k^{N_k} \Phi_k'^* \dots \Phi_k^{N_k*}]$$

The equation of motion of the appendage now becomes

$$\dot{\bar{Y}}^k = \begin{bmatrix} \lambda_1 & & & & & & & & & & & 0 \\ & \ddots & & & & & & & & & & \\ & & \lambda_{N_k} & & & & & & & & & \\ & & & \lambda_1^* & & & & & & & & \\ 0 & & & & \ddots & & & & & & & \\ & & & & & \lambda_{N_k}^* & & & & & & \end{bmatrix} \bar{Y}^k + (\bar{\Phi}_k'^T \mathcal{U}_k \bar{\Phi}_k)^{-1} \bar{\Phi}_k'^T \mathcal{P}_k \quad (22)$$

Since, in the particular case studied here, the matrices  $\mathcal{U}_k$  and  $\mathcal{V}_k$  in Eq. (18) are, respectively, symmetric and skew-symmetric, so that Eq. (19) becomes

$$\mathcal{U}_k \dot{Q}^{*k} - \mathcal{V}_k Q^{*k} = 0 \quad (23)$$

the adjoint eigenvector matrix is available as the complex conjugate

$$\Phi_k' = \Phi_k^* \quad (24)$$

The final equations, after truncation of Eq. (24) and substitution into (22), are therefore obtained without the necessity of actually computing the eigenvectors constituting  $\Phi^k$ . Thus, Eq. (22) becomes

$$\dot{\bar{Y}}^k = \bar{\Lambda}_k \bar{Y}^k + (\bar{\Phi}_k^{*T} \mathcal{U}_k \bar{\Phi}_k)^{-1} \bar{\Phi}_k^{*T} \mathcal{P}_k \quad (25)$$

Since the appendage modeling process thus far has assumed that the structure contains no damping, the diagonal matrix,  $\Lambda_k$ , will contain only eigenvalues that

are purely imaginary, e.g.,  $\lambda_m = \pm i\sigma_m$ . Conventional practice in structural dynamics, if some energy dissipation in the model is desired, is to rather arbitrarily add what amounts to a viscous damping term  $D'_k \dot{q}^k$  to the appendage equation *after* completing the modal analysis, assuming that the structure of  $D'_k$  is such that eigenvectors  $\Phi'_k, \dots, \Phi_k^{12n_k}$  are undisturbed by this addition. Specifically, one substitutes  $\lambda_m = -\xi_m \sigma_m \pm i\sigma_m$  into Eq. (22) or (25), where  $\xi_m$  is the "percent of critical damping" and is chosen based on experience (including tests) with similar structures. (See Appendix A for a discussion of some ramifications of adding damping *after* transforming the appendage equations to modal coordinates.)

An apparent disadvantage of Eq. (25) is the fact that the quantities  $\bar{Y}^k, \bar{\Lambda}_k,$  and  $\bar{\Phi}_k$  are complex. However, Eq. (25) can be written in terms of its real and imaginary parts and the resulting equations greatly simplified by the use of certain orthogonality relationships. The detailed development of the equations is shown in Ref. 3, and only the results are presented here.

Realizing that  $\Phi_k^j$  must have the form

$$\Phi_k^j = \begin{bmatrix} -\phi^j \\ \phi^j \lambda_j \end{bmatrix}, \quad (\Phi_k^j = j\text{th column of } \Phi_k, j = 1, \dots, 12n_k)$$

where  $\phi^j = \psi^j + i\Gamma^j$ , ( $6n_k \times 1$ ), and letting  $Y_\alpha^k = \delta_\alpha^k + i\eta_\alpha^k$ ,  $Y_\alpha^{k*} = \delta_\alpha^k - i\eta_\alpha^k$ , ( $\alpha = 1, \dots, 6n_k$ ), one can see from Ref. 3 that the real  $N_k \times 1$  (truncated) matrices,  $\bar{\delta}^k$  and  $\bar{\eta}^k$ , are the solutions to the equations

$$\bar{\delta}^k = -\bar{\sigma}^k \bar{\eta}^k - \bar{\sigma}^k \bar{\Gamma}_k^T L_k' - \bar{\xi}^k \bar{\sigma}^k \bar{\delta}^k \quad (26a)$$

and

$$\bar{\eta}^k = \bar{\sigma}^k \bar{\delta}^k - \bar{\sigma}^k \bar{\Psi}_k^T L_k' - \bar{\xi}^k \bar{\sigma}^k \bar{\eta}^k \quad (26b)$$

As a result, the relationships between the *real* quantities  $q^k, \dot{q}^k, \delta^k,$  and  $\eta^k$ , in matrix terms, are as follows:

$$q^k = 2(\bar{\Psi}_k \bar{\delta}^k - \bar{\Gamma}_k \bar{\eta}^k) \quad (27a)$$

and

$$\dot{q}^k = -2(\bar{\Gamma}_k \bar{\sigma}^k \bar{\delta}^k + \bar{\Psi}_k \bar{\sigma}^k \bar{\eta}^k) \quad (27b)$$

so that

$$\ddot{q}^k = -2(\bar{\Gamma}_k \bar{\sigma}^k \dot{\bar{\delta}}^k + \bar{\Psi}_k \bar{\sigma}^k \dot{\bar{\eta}}^k) \quad (27c)$$

In order to complete the set of model equations, particularly in the form suitable for computer solution, it is necessary to return to the vehicle equations, substituting the relations developed in Eqs. (7), (11), (12), etc., into Eq. (4), to obtain

$$\begin{aligned}
W^k &= T^k + \sum_{r \in \mathfrak{B}} D^{kr} \times F^r + c^k \times \left( \frac{\mathfrak{N}_k}{\mathfrak{N}} F - F^k \right) \\
&+ \sum_{r \in \mathfrak{B}} \mathfrak{N}_r D^{kr} \times \left[ - \sum_{s=1}^{n_r} \frac{m_r}{\mathfrak{N}_r} \ddot{u}^s + 2\omega^r \times \dot{c}^r + \dot{\omega}^r \times c^r + \omega^r \times (\omega^r \times c^r) \right] \\
&+ \mathfrak{N}_k c^k \times \sum_{r \in \mathfrak{B}} [\dot{\omega}^r \times D^{rk} + \omega^r \times (\omega^r \times D^{rk})] \\
&- \sum_{r \in \mathfrak{B}} \Phi^{kr} \cdot \dot{\omega}^r + \mathfrak{N}_k \sum_{r \in \mathfrak{B}-k} D^{kr} \times [\omega^r \times (\omega^r \times D^{rk})] \\
&- \omega^k \times \Phi^{kk} \cdot \omega^k - \dot{h}^k - \omega^k \times h^k \\
&- \{b^k\}^T \sum_{s=1}^{n_k} \left[ m_s \{2(R^k + r^s)^T \dot{u}^s U - (R^k + r^s) \dot{u}^{sT} - \dot{u}^s (R^k + r^s)^T\} \right. \\
&\left. + \tilde{\beta}^s I^s - I^s \tilde{\beta}^s \right] \{b^k\} \cdot \omega^k \\
&- \sum_{s=1}^{n_k} (R^k + r^s) \times m_s \ddot{u}^s - \omega^k \times \sum_{s=1}^{n_k} (R^k + r^s) \times m_s \dot{u}^s \\
&- \sum_{s=1}^{n_k} (I^s \cdot \ddot{\beta}^s + \omega^k \times I^s \cdot \dot{\beta}^s) \tag{28}
\end{aligned}$$

Eliminating the use of  $R^k$  for simplicity (noting that this is an arbitrary vector fixed in  $\mathcal{L}_k$  and it can always be chosen as zero) and substituting  $q^k$  and  $q^r$  where appropriate, the matrix form of Eq. (28) becomes

$$\begin{aligned}
W^k &= T^k + \sum_{r \in \mathfrak{B}} \tilde{D}^{kr} C^{kr} F^r + \left[ \tilde{F}^k - \left( C^{k0} \frac{\mathfrak{N}_k}{\mathfrak{N}} F \right)^- \right] c^k \\
&+ \sum_{r \in \mathfrak{B}} \mathfrak{N}_r \tilde{D}^{kr} C^{kr} \left[ - \sum_{U0}^T \frac{M^r}{\mathfrak{N}_r} \ddot{q}^r + 2\tilde{\omega}^r \dot{c}^r - \dot{c}^r \tilde{\omega}^r + \tilde{\omega}^r \tilde{\omega}^r c^r \right] \\
&+ \mathfrak{N}_k \tilde{c}^k \sum_{r \in \mathfrak{B}} C^{kr} [- \tilde{D}^{rk} \dot{\omega}^r + \tilde{\omega}^r \tilde{\omega}^r D^{rk}] \\
&- \sum_{r \in \mathfrak{B}} \Phi^{kr} C^{kr} \dot{\omega}^r + \mathfrak{N}_k \sum_{r \in \mathfrak{B}-k} \tilde{D}^{kr} C^{kr} \tilde{\omega}^r \tilde{\omega}^r D^{rk} - \tilde{\omega}^k \Phi^{kk} \omega^k \\
&- \dot{h}^k - \tilde{\omega}^k h^k - \left[ 2(M^k r_k)^T \dot{q}^k U - r_k^T (M^k \dot{q}^k)^{TT} - (M^k \dot{q}^k)^T r_k^{TT} \right. \\
&\left. + \sum_{U0}^T (\tilde{q}^k M^k - M^k \tilde{q}^k) \sum_{U0} \right] \omega^k - \sum_{U0}^T \tilde{r}_k M^k \tilde{q}^k - \tilde{\omega}^k \sum_{U0}^T \tilde{r}_k M^k \dot{q}^k \\
&- \sum_{U0}^T M^k \ddot{q}^k - \tilde{\omega}^k \sum_{U0}^T M^k \dot{q}^k \tag{29}
\end{aligned}$$

where the operator † reassembles the 3 by 1 submatrices of a column matrix into a three-row matrix, as illustrated by

$$r_k^\dagger \equiv [r^1 \quad 0 \quad r^2 \quad 0 \quad \dots \quad r^{n_k} \quad 0]$$

Using the identity

$$(M^k \dot{q}^k)^T r_k \equiv (M^k r_k)^T \dot{q}^k$$

and regrouping some of the terms in (29), we have

$$\begin{aligned} W^k = & - \sum_{r \in \mathfrak{B}} [\Phi^{kr} C^{kr} + \mathfrak{N}_k \bar{c}^k C^{kr} \bar{D}^{rk} + \mathfrak{N}_r \bar{D}^{kr} C^{kr} \bar{c}^r] \dot{\omega}^r \\ & - [\Sigma_{0U}^T + \Sigma_{U0}^T \bar{r}_k] M^k \dot{q}^k - \sum_{r \in \mathfrak{B}} \bar{D}^{kr} C^{kr} \Sigma_{U0}^T M^r \dot{q}^r - \dot{h}^k \\ & + T^k + \sum_{r \in \mathfrak{B}} \bar{D}^{kr} C^{kr} F^r + \left[ \bar{F}^k - \left( C^{k0} \frac{\mathfrak{N}_k}{\mathfrak{N}} F \right)^- \right] c^k \\ & + \sum_{r \in \mathfrak{B}} \mathfrak{N}_r \bar{D}^{kr} C^{kr} (2\bar{\omega}^r \dot{c}^r + \bar{\omega}^r \dot{\omega}^r c^r) + \mathfrak{N}_k \bar{c}^k \sum_{r \in \mathfrak{B}} C^{kr} \bar{\omega}^r \dot{\omega}^r D^{rk} \\ & + \mathfrak{N}_k \sum_{r \in \mathfrak{B} - k} \bar{D}^{kr} C^{kr} \bar{\omega}^r \dot{\omega}^r D^{rk} - \bar{\omega}^k \Phi^{kk} \omega^k - \bar{\omega}^k h^k \\ & - [2(M^k \dot{q}^k)^T r_k U - r_k^\dagger (M^k \dot{q}^k)^{\dagger T} - (M^k \dot{q}^k)^\dagger r_k^{\dagger T} \\ & + \Sigma_{0U}^T (\bar{q}^k M^k - M^k \bar{q}^k) \Sigma_{0U}] \omega^k - \bar{\omega}^k (\Sigma_{0U}^T + \Sigma_{U0}^T \bar{r}_k) M^k \dot{q}^k \end{aligned} \quad (30)$$

The truncated modal coordinates,  $\bar{\delta}^k$  and  $\bar{\eta}^k$ , may now be introduced into the  $k$ th substructure equation by way of Eq. (27), as follows:

$$\begin{aligned} W^k = & - \sum_{r \in \mathfrak{B}} [\Phi^{kr} C^{kr} + \mathfrak{N}_k \bar{c}^k C^{kr} \bar{D}^{rk} + \mathfrak{N}_r \bar{D}^{kr} C^{kr} \bar{c}^r] \dot{\omega}^r \\ & - \bar{\Delta}_R^{kT} \bar{\delta}^k - \bar{\Delta}_I^{kT} \bar{\eta}^k + \sum_{r \in \mathfrak{B}} \bar{D}^{kr} C^{kr} [G_r \bar{\sigma}^r \bar{\delta}^r + P_r \bar{\sigma}^r \bar{\eta}^r] \\ & - \dot{h}^k + T^k + \sum_{r \in \mathfrak{B}} \bar{D}^{kr} C^{kr} F^r + \left[ \bar{F}^k - \left( C^{k0} \frac{\mathfrak{N}_k}{\mathfrak{N}} F \right)^- \right] c^k \\ & + \sum_{r \in \mathfrak{B}} [\mathfrak{N}_r \bar{D}^{kr} C^{kr} (2\bar{\omega}^r \dot{c}^r + \bar{\omega}^r \dot{\omega}^r c^r) + \mathfrak{N}_k \bar{c}^k C^{kr} \bar{\omega}^r \dot{\omega}^r D^{rk}] \\ & + \mathfrak{N}_k \sum_{r \in \mathfrak{B} - k} \bar{D}^{kr} C^{kr} \bar{\omega}^r \dot{\omega}^r D^{rk} - \bar{\omega}^k \Phi^{kk} \omega^k - \bar{\omega}^k h^k \\ & - j^k \omega^k - \bar{\omega}^k (\bar{\Delta}_R^{kT} \bar{\delta}^k + \bar{\Delta}_I^{kT} \bar{\eta}^k) \end{aligned} \quad (31)$$



where

$$\bar{\Delta}_R^k = -2\bar{\sigma}^k \bar{\Gamma}_k^T M^k [\Sigma_{0U} - \bar{r}_k \Sigma_{U0}]$$

$$\Delta_I^k = -2\bar{\sigma}^k \bar{\Psi}_k^T M^k [\Sigma_{0U} - \bar{r}_k \Sigma_{U0}]$$

$$\bar{P}_k = 2\Sigma_{U0}^T M^k \bar{\Psi}_k$$

$$\bar{G}_k = 2\Sigma_{U0}^T M^k \bar{\Gamma}_k$$

$$j^k = 2(M^k \dot{q}^k)^T r_k - r_k^T (M^k \dot{q}^k)^{\dagger T} - (M^k \dot{q}^k)^{\dagger} r_k^{\dagger T} + \Sigma_{0U}^T (\bar{q}^k M^k - M^k \bar{q}^k) \Sigma_{0U}$$

$$\Phi^{kk} = K^k + J^k$$

$$\dot{\Phi}^{kk} = j^k$$

Using the relation in Eq. (6), the vehicle equations, (2) and (3), become (in matrix form)

$$A^{00} \dot{\omega}^0 + \sum_{j \in \mathcal{P}} A^{0j} \dot{\gamma}_j + \sum_{m \in \mathcal{F}} A_R^{0m} \dot{\delta}^m + \sum_{m \in \mathcal{F}} A_I^{0m} \dot{\eta}^m = \sum_{k \in \mathcal{B}} C^{0k} E^k \quad (32)$$

and for  $s \in \mathcal{P}$ ,

$$A^{s0} \dot{\omega}^0 + \sum_{j \in \mathcal{P}} A^{sj} \dot{\gamma}_j + \sum_{m \in \mathcal{F}} A_R^{sm} \dot{\delta}^m + \sum_{m \in \mathcal{F}} A_I^{sm} \dot{\eta}^m = g^{sT} \sum_{k \in \mathcal{P}} \epsilon_{sk} C^{sk} E^k + \tau_s \quad (33)$$

where

$$A^{00} = \sum_{k \in \mathcal{B}} \sum_{r \in \mathcal{B}} C^{0k} (\Phi^{kr} C^{kr} + \mathcal{N}_k \bar{c}^k C^{kr} \bar{D}^{rk} + \mathcal{N}_r \bar{D}^{kr} C^{kr} \bar{c}^r) C^{r0}, \quad 3 \text{ by } 3 \quad (34)$$

$$A^{0j} = \sum_{k \in \mathcal{B}} \sum_{r \in \mathcal{P}} C^{0k} (\Phi^{kr} C^{kr} + \mathcal{N}_k \bar{c}^k C^{kr} \bar{D}^{rk} + \mathcal{N}_r \bar{D}^{kr} C^{kr} \bar{c}^r) \epsilon_{jr} C^{rj} g^j, \quad 3 \text{ by } 1 \quad (35)$$

$$A^{s0} = g^{sT} \sum_{k \in \mathcal{P}} \sum_{r \in \mathcal{B}} \epsilon_{sk} C^{sk} (\Phi^{kr} C^{kr} + \mathcal{N}_k \bar{c}^k C^{kr} \bar{D}^{rk} + \mathcal{N}_r \bar{D}^{kr} C^{kr} \bar{c}^r) C^{r0}, \quad 1 \text{ by } 3 \quad (36)$$

$$A^{sj} = g^{sT} \sum_{k \in \mathcal{P}} \sum_{r \in \mathcal{B}} \epsilon_{sk} \epsilon_{jr} C^{sk} (\Phi^{kr} C^{kr} + \mathcal{N}_k \bar{c}^k C^{kr} \bar{D}^{rk} + \mathcal{N}_r \bar{D}^{kr} C^{kr} \bar{c}^r) C^{rj} g^j, \quad 1 \text{ by } 1 \quad (37)$$

$$A_R^{0m} = C^{0m} \bar{\Delta}_R^{mT} - \sum_{r \in \mathcal{B}} C^{0r} \bar{D}^{rm} C^{rm} \bar{G}_m \bar{\sigma}^m, \quad 3 \text{ by } N_m \quad (38)$$

$$A_I^{0m} = C^{0m} \bar{\Delta}_I^{mT} - \sum_{r \in \mathcal{B}} C^{0r} \bar{D}^{rm} C^{rm} \bar{P}_m \bar{\sigma}^m, \quad 3 \text{ by } N_m \quad (39)$$

$$A_R^{sm} = g^{sT} \left( \epsilon_{sm} C^{sm} \bar{\Delta}_R^{mT} - \sum_{r \in \mathfrak{B}} \epsilon_{sr} C^{sr} \bar{D}^{rm} C^{rm} \bar{G}_m \bar{\sigma}^m \right), \quad 1 \text{ by } N_m \quad (40)$$

$$A_I^{sm} = g^{sT} \left( \epsilon_{sm} C^{sm} \bar{\Delta}_I^{mT} - \sum_{r \in \mathfrak{B}} \epsilon_{sr} C^{sr} \bar{D}^{rm} C^{rm} \bar{P}_m \bar{\sigma}^m \right), \quad 1 \text{ by } N_m \quad (41)$$

$\mathfrak{F} \equiv$  the integer set containing the labels of only those rigid bodies of the system that possess a nonrigid appendage.

$$\begin{aligned} E^k &= T^k - \tau_R^k - \bar{\omega}^k \mathcal{G}^k (\bar{\omega}^k + \dot{\psi}_R^k) + \sum_{r \in \mathfrak{B}} \bar{D}^{kr} C^{kr} F^r \\ &+ \left[ \bar{F}^k - \left( C^{k0} \frac{\mathfrak{N}_k}{\mathfrak{N}} F \right)^- \right] c^k + \mathfrak{N}_k \sum_{r \in \mathfrak{B} - k} \bar{D}^{kr} C^{kr} \bar{\omega}^r \bar{\omega}^r D^{rk} \\ &- \bar{\omega}^k \Phi^{kk} \omega^k - j^k \omega^k - \bar{\omega}^k (\bar{\Delta}_R^{kT} \bar{\delta}^k + \bar{\Delta}_I^{kT} \bar{\eta}^k) \\ &- \sum_{r \in \mathfrak{B}} \left[ (\Phi^{kr} C^{kr} + \mathfrak{N}_k \bar{c}^k C^{kr} \bar{D}^{rk} + \mathfrak{N}_r \bar{D}^{kr} C^{kr} \bar{c}^r) \cdot \sum_{j \in \mathfrak{F}} \epsilon_{jr} C^{rj} \bar{\omega}^j g^j \dot{\gamma}_j \right. \\ &\left. - \mathfrak{N}_r \bar{D}^{kr} C^{kr} (2\bar{\omega}^r \dot{c}^r + \bar{\omega}^r \bar{\omega}^r \dot{c}^r) - \mathfrak{N}_k \bar{c}^k C^{kr} \bar{\omega}^r \bar{\omega}^r D^{rk} \right], \quad 3 \text{ by } 1 \quad (42) \end{aligned}$$

and substitutions have been made for  $\dot{h}^k$  and  $h^k$  based on restriction to three orthogonal axisymmetric rotors in  $\mathcal{A}_k$ , with spin axes aligned to the unit vectors  $\{\mathbf{b}^k\}$ , and the following equations:

$$h^k = \mathcal{G}^k \dot{\psi}_R^k \quad (43)$$

$$\tau_R^k = \mathcal{G}^k (\dot{\psi}_R^k + \bar{\omega}^k) \quad (44)$$

$$\therefore \dot{h}^k = \tau_R^k - \mathcal{G}^k \bar{\omega}^k \quad (45)$$

where

$\dot{\psi}_R^k \equiv \dot{\psi}_R^k \cdot \{\mathbf{b}^k\} = 3$  by 1 matrix of components of spin rate relative to  $\mathcal{A}_k$  for three orthogonal axisymmetric rotors in  $\mathcal{A}_k$ .

$\mathcal{G}^k \equiv$  spin-axis inertia matrix (diagonal) for the three axisymmetric rotors in  $\mathcal{A}_k$ .

$\tau_R^k \equiv \tau_R^k \cdot \{\mathbf{b}^k\} = 3$  by 1 matrix of applied torque on the three axisymmetric rotors in  $\mathcal{A}_k$ .

It is to be understood that when symmetric rotors are present in the  $k$ th substructure, the rotors' mass and moments of inertia are to be included in  $\bar{\mathbf{J}}^k$ , the undeformed substructure's inertia dyadic for  $o_k$ . Of course, the mass of the rotors is also to be included in the substructure mass and c.m.-location calculations.

Equation (44) then provides up to three scalar differential equations which are uncoupled in acceleration from the system's vehicle/appendage equations. They

may be integrated and, with  $\omega^k$  and  $\tau_R^k$  known, can be solved for  $\dot{\psi}_R^k$ , which is then supplied to Eq. (42).

If one now operates on the appendage equations, Eqs. (26), in a similar way, they may be expressed as

$m \in \mathcal{F}$ :

$$\mathcal{Q}^{m0}\dot{\omega}^0 + \sum_{j \in \mathcal{F}} \mathcal{Q}^{mj}\ddot{y}_j + \sum_{n \in \mathcal{F}} \mathcal{Q}_R^{mn}\dot{\delta}^n + \sum_{n \in \mathcal{F}} \mathcal{Q}_I^{mn}\dot{\eta}^n = \mathcal{Q}_R^m \quad (46)$$

$$\mathcal{V}^{m0}\dot{\omega}^0 + \sum_{j \in \mathcal{F}} \mathcal{V}^{mj}\ddot{y}_j + \sum_{n \in \mathcal{F}} \mathcal{V}_R^{mn}\dot{\delta}^n + \sum_{n \in \mathcal{F}} \mathcal{V}_I^{mn}\dot{\eta}^n = \mathcal{Q}_I^m \quad (47)$$

where

$$\mathcal{Q}^{m0} = \frac{1}{2} \left[ \bar{\Delta}_R^m C^{m0} + \bar{\sigma}^m \bar{G}_m^T \sum_{r \in \mathcal{B}} C^{mr} \bar{D}^{rm} C^{r0} \right], \quad N_m \text{ by } 3 \quad (48)$$

$$\mathcal{Q}^{mj} = \frac{1}{2} \left[ \bar{\Delta}_R^m \epsilon_{jm} C^{mj} + \bar{\sigma}^m \bar{G}_m^T \sum_{r \in \mathcal{B}} C^{mr} \bar{D}^{rm} \epsilon_{jr} C^{rj} \right] g^j, \quad N_m \text{ by } 1 \quad (49)$$

$$\mathcal{Q}_R^{mn} = -\frac{1}{2} \bar{\sigma}^m \bar{G}_m^T C^{mn} \bar{G}_n \frac{\bar{\sigma}^n}{\mathcal{J}\mathcal{L}}, \quad (m \neq n); \quad N_m \text{ by } N_n \quad (50)$$

$$\mathcal{Q}_R^{mn} = U, \quad (m = n); \quad N_m \text{ by } N_m$$

$$\mathcal{Q}_I^{mn} = -\frac{1}{2} \bar{\sigma}^m \bar{G}_m^T C^{mn} \bar{P}_n \frac{\bar{\sigma}^n}{\mathcal{J}\mathcal{L}}, \quad (m \neq n); \quad N_m \text{ by } N_n \quad (51)$$

$$\mathcal{Q}_I^{mn} = 0, \quad (m = n); \quad N_m \text{ by } N_m$$

$$\mathcal{V}^{m0} = \frac{1}{2} \left[ \bar{\Delta}_I^m C^{m0} + \bar{\sigma}^m \bar{P}_m^T \sum_{r \in \mathcal{B}} C^{mr} \bar{D}^{rm} C^{r0} \right], \quad N_m \text{ by } 3 \quad (52)$$

$$\mathcal{V}^{mj} = \frac{1}{2} \left[ \bar{\Delta}_I^m \epsilon_{jm} C^{mj} + \bar{\sigma}^m \bar{P}_m^T \sum_{r \in \mathcal{B}} C^{mr} \bar{D}^{rm} \epsilon_{jr} C^{rj} \right] g^j, \quad N_m \text{ by } 1 \quad (53)$$

$$\mathcal{V}_R^{mn} = -\frac{1}{2} \bar{\sigma}^m \bar{P}_m^T C^{mn} \bar{G}_n \frac{\bar{\sigma}^n}{\mathcal{J}\mathcal{L}}, \quad (m \neq n); \quad N_m \text{ by } N_n \quad (54)$$

$$\mathcal{V}_R^{mn} = 0, \quad (m = n); \quad N_m \text{ by } N_m \quad (55)$$

$$\mathcal{V}_I^{mn} = -\frac{1}{2} \bar{\sigma}^m \bar{P}_m^T C^{mn} \bar{P}_n \frac{\bar{\sigma}^n}{\mathcal{J}\mathcal{L}}, \quad (m \neq n); \quad N_m \text{ by } N_n \quad (56)$$

$$\mathcal{V}_I^{mn} = U, \quad (m = n); \quad N_m \text{ by } N_m \quad (57)$$

$$Q_R^m = +\bar{\sigma}^m \left[ -\bar{\eta}^m - \bar{\xi}^m \bar{\delta}^m + \frac{1}{2} \bar{G}_m^T V_m - \bar{\Gamma}_m^T X_m \right] - Z_R^m, \quad N_m \text{ by } 1 \quad (58)$$

$$Q_I^m = +\bar{\sigma}^m \left[ \bar{\delta}^m - \bar{\xi}^m \bar{\eta}^m + \frac{1}{2} \bar{P}_m^T V_m - \bar{\Psi}_m^T X_m \right] - Z_I^m, \quad N_m \text{ by } 1 \quad (59)$$

$$V_m = C^{m0} \frac{F}{\mathfrak{N}} + \sum_{r \in \mathfrak{B}} C^{mr} \bar{\omega}' \bar{\omega}'^T D^{rm} \\ + \sum_{r \in \mathfrak{B} - m} C^{mr} \frac{\mathfrak{N}_r}{\mathfrak{N}} (2\bar{\omega}' \bar{c}' + \bar{\omega}' \bar{\omega}'^T c') - \bar{\Omega}^m \bar{\Omega}^m D^{mm}, \quad 3 \text{ by } 1 \quad (60a)$$

$$X_m = \lambda^m - M^m (\Sigma_{U0} \omega^m)^{-1} (\Sigma_{U0} \omega^m)^{-1} r_m - (\Sigma_{0U} \omega^m)^{-1} M^m (\Sigma_{0U} \omega^m)^{-1} \\ + M^m (\Sigma_{U0} \Omega^m)^{-1} (\Sigma_{U0} \Omega^m)^{-1} r_m + (\Sigma_{0U} \Omega^m)^{-1} M^m (\Sigma_{0U} \Omega^m)^{-1}, \quad n_m \text{ by } 1 \quad (60b)$$

$$Z_R^m = \frac{1}{2} \sum_{j \in \mathfrak{P}} \left( \bar{\Delta}_R^m \epsilon_{jm} C^{mj} + \bar{\sigma}^m \bar{G}_m^T \sum_{r \in \mathfrak{B}} C^{mr} \bar{D}^{rm} \epsilon_{jr} C^{rj} \right) \bar{\omega}^j g^j \dot{\gamma}_j, \quad N_m \text{ by } 1 \quad (61a)$$

$$Z_I^m = \frac{1}{2} \sum_{j \in \mathfrak{P}} \left( \bar{\Delta}_I^m \epsilon_{jm} C^{mj} + \bar{\sigma}^m \bar{P}_m^T \sum_{r \in \mathfrak{B}} C^{mr} \bar{D}^{rm} \epsilon_{jr} C^{rj} \right) \bar{\omega}^j g^j \dot{\gamma}_j, \quad N_m \text{ by } 1 \quad (61b)$$

Recapping, the system equations (minus the rotor equations) are as follows:

$$A^{00} \dot{\omega}^0 + \sum_{j \in \mathfrak{P}} A^{0j} \dot{\gamma}_j + \sum_{m \in \mathfrak{F}} A_R^{0m} \dot{\delta}^m + \sum_{m \in \mathfrak{F}} A_I^{0m} \dot{\eta}^m = \sum_{k \in \mathfrak{B}} C^{0k} E^k \quad (62a)$$

$s \in \mathfrak{P}$ :

$$A^{s0} \dot{\omega}^0 + \sum_{j \in \mathfrak{P}} A^{sj} \dot{\gamma}_j + \sum_{m \in \mathfrak{F}} A_R^{sm} \dot{\delta}^m + \sum_{m \in \mathfrak{F}} A_I^{sm} \dot{\eta}^m = g^{sT} \sum_{k \in \mathfrak{P}} \epsilon_{sk} C^{sk} E^k + \tau_s, \quad (62b)$$

$m \in \mathfrak{F}$ :

$$Q^{m0} \dot{\omega}^0 + \sum_{j \in \mathfrak{P}} Q^{mj} \dot{\gamma}_j + \sum_{n \in \mathfrak{F}} Q_R^{mn} \dot{\delta}^n + \sum_{n \in \mathfrak{F}} Q_I^{mn} \dot{\eta}^n = Q_R^m \quad (62c)$$

$m \in \mathfrak{F}$ :

$$\mathfrak{Q}^{m0} \dot{\omega}^0 + \sum_{j \in \mathfrak{P}} \mathfrak{Q}^{mj} \dot{\gamma}_j + \sum_{n \in \mathfrak{F}} \mathfrak{Q}_R^{mn} \dot{\delta}^n + \sum_{n \in \mathfrak{F}} \mathfrak{Q}_I^{mn} \dot{\eta}^n = Q_I^m \quad (62d)$$

and these may be combined into the single matrix equation of the form  $A\dot{x} = B$ , as shown in Eq. (63).

$$\begin{bmatrix}
 A^{00} & A^{0j} & \sum_{m \in \mathcal{F}} A_R^{0m} & \sum_{m \in \mathcal{F}} A_I^{0m} \\
 (3 \times 3) & (3 \times n_n) & (3 \times N_m) & (3 \times N_m) \\
 \hline
 A^{s0} & A^{sj} & \sum_m A_R^{sm} & \sum_m A_I^{sm} \\
 (n_n \times 3) & (n_n \times n_n) & (n_n \times N_m) & (n_n \times N_m) \\
 \hline
 \mathcal{Q}^{m0} & \mathcal{Q}^{mj} & \sum_n \mathcal{Q}_R^{mn} & \sum_n \mathcal{Q}_I^{mn} \\
 (N_m \times 3) & (N_m \times n_n) & (N_m \times N_n) & (N_m \times N_n) \\
 \hline
 \mathcal{D}^{m0} & \mathcal{D}^{mj} & \sum_n \mathcal{D}_R^{mn} & \sum_n \mathcal{D}_I^{mn} \\
 (N_m \times 3) & (N_m \times n_n) & (N_m \times N_n) & (N_m \times N_n)
 \end{bmatrix}
 \begin{bmatrix}
 \dot{\omega}^0 \\
 (3 \times 1) \\
 \ddot{\gamma}_j \\
 (n_n \times 1) \\
 \ddot{\delta}^m \\
 (N_m \times 1) \\
 \ddot{\eta}^m \\
 (N_m \times 1)
 \end{bmatrix}$$

$$= \begin{bmatrix}
 \sum_{k \in \mathcal{B}} C^{0k} E^k \\
 (3 \times 1) \\
 \hline
 g^{sT} \sum_{k \in \mathcal{P}} \epsilon_{sk} C^{sk} E^k + \tau_s \\
 (n_n \times 1) \\
 \hline
 Q_R^m \\
 (N_m \times 1) \\
 \hline
 Q_I^m \\
 (N_m \times 1)
 \end{bmatrix} \tag{63}$$

Except for  $\mathcal{Q}_R^{mn}$ ,  $\mathcal{Q}_I^{mn}$ ,  $\mathcal{D}_R^{mn}$ , and  $\mathcal{D}_I^{mn}$  when  $m = n$ , the elements of system matrix  $A$  are, in general, time-variable. Note also that, if the appendage equations are multiplied through by the factor 2, matrix  $A$  becomes symmetric.

### B. Subroutine MBDYFR

Equation (63) provides a complete set of rotational dynamics equations which lend themselves to solution by means of a generic computer program or subroutine for the rotating appendage case. When augmented by the rotor equations, control equations, and kinematical equations, they are fully descriptive of the system behavior.

The kinematical variables adopted in the preceding sections are as follows:  $\gamma_k$  for  $k \in \mathcal{P}$  (Def. 23);  $C^j$  for  $r, j \in \mathcal{B}$  (Def. 31); and  $\omega^0 \equiv \{b^0\} \cdot \omega^0$  (Def. 9). Although the equations of motion have been expressed in terms of these quantities, the latter are not all independent. Relationships among kinematical variables developed in this section must therefore either be considered in conjunction with the dynamical equations or be substituted into them to remove redundant variables whenever a solution is sought.

The direction cosine matrix  $C^j$  (Def. 31) relates sets of orthogonal unit vectors fixed in  $\mathcal{A}_r$  and  $\mathcal{A}_j$ , and hence depends upon those angles  $\gamma_\alpha$  for which  $\mu_\alpha$  lies between  $\mathcal{A}_r$  and  $\mathcal{A}_j$ , and also upon the corresponding unit vectors  $\bar{g}^\alpha$  defining the intervening hinge axes. For the special case in which  $\mathcal{A}_r$  and  $\mathcal{A}_j$  are *contiguous* and  $j < r$ , it is always possible to express  $C^j$  (and  $C^j$ ) in terms of the single angle  $\gamma_r$  and the single matrix  $g^r$ , as follows:

$$C^j = U \cos \gamma_r - \bar{g}^r \sin \gamma_r + \bar{g}^r g^{rT} (1 - \cos \gamma_r)$$

and

$$C^{jr} = U \cos \gamma_r + \bar{g}^r \sin \gamma_r + \bar{g}^r g^{rT} (1 - \cos \gamma_r) = (C^j)^T$$

It is only required that  $C^j$  be determined where  $\mathcal{A}_r$  and  $\mathcal{A}_j$  are contiguous and, since  $C^{0r} = C^{0j} C^{jr}$ , to then derive matrices  $C^{0r}$  for  $r \in \mathcal{Q}$ . An algorithm for accomplishing this task is described in Ref. 6, Appendix A.

The Fortran V subroutine, called MBDYFR, which provides the solution to Eq. (63), has been designed in much the same form as those subroutines described in Refs. 6 and 7. The routine may be exercised by means of either of two call statements. An initializing call statement supplies the routine with data that will remain constant throughout the dynamic simulation run.

The description which follows of the subroutine initialization statement includes some new variables which will now be defined. The use of these new variables is necessitated by the desire to make the subroutine MBDYFR more efficient. Therefore, the convention (described in Defs. 1-4) of labeling the system's rigid bodies from 0 to  $n$ , where each connection between bodies is a line hinge, will be modified. Rather than introduce imaginary massless bodies at connections with 2 or 3 degrees of rotational freedom, these types of connections will be handled directly by the routine and no new bodies will be introduced.

*Def. 45.* Let  $n_c$  be the number of *connections* joining a set of  $n_c + 1$  substructures. A connection is a 1-, 2-, or 3-degree-of-freedom joint at which all the rotational axes share a common point. The axes need not be mutually orthogonal.

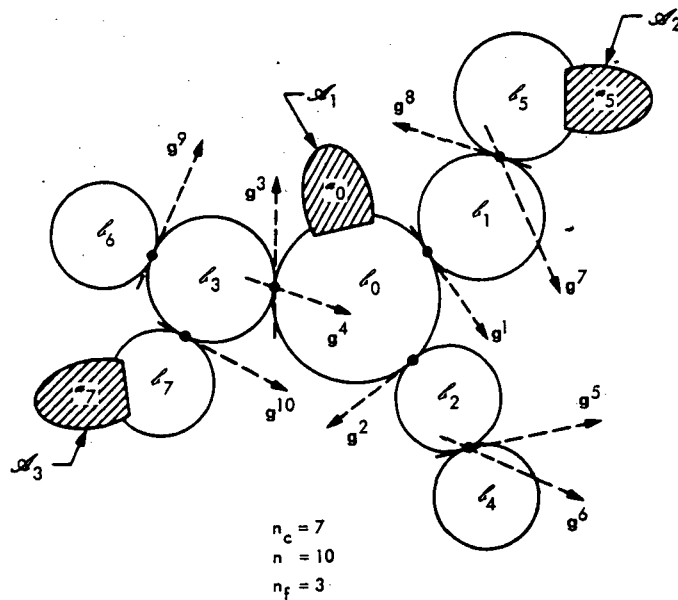
*Def. 46.* Define the integer set  $\mathcal{B}^r \equiv \{0, 1, \dots, n_c\}$ .

*Def. 47.* Define the integer set  $\mathcal{Q}^r \equiv \{1, 2, \dots, n_c\}$ .

*Def. 48.* Let  $\mathcal{B}_j^r$  be the  $j$ th neighbor set for  $j \in \mathcal{B}^r$ , such that  $k \in \mathcal{B}_j^r$  if  $\mathcal{A}_k$  is attached to  $\mathcal{A}_j$ .

The rigid body labeling process is to be carried out precisely as prescribed in Def. 4, except that the last label will be  $\mathcal{A}_n$  rather than  $\mathcal{A}_n$ . Note, however, that the connecting joint degrees of freedom are still labeled from 1 to  $n$ , so that one still has  $\gamma_1, \gamma_2, \dots, \gamma_n$  and  $g^1, g^2, \dots, g^n$  (The joints *must* be in the sequence corresponding to the body label sequence, as shown in Fig. 2). All references to the " $k$ th substructure," when applying the MBDYFR subroutine, imply that  $k \in \mathcal{B}^r$ .

*Def. 49.* For  $k \in \mathcal{Q}^r$ , let  $\mathcal{A}_k$  denote the index label of the body attached to  $\mathcal{A}_k$  and on the path leading to  $\mathcal{A}_0$ . The scalars  $\mathcal{A}_k$  will be termed "connection elements." Thus, it is always true that  $\mathcal{A}_1 = 0$ .



**Fig. 2. An 8-body, 10-hinge system illustrating the labeling convention**

*Def. 50.* Let  $d_k$ ,  $k \in \mathcal{P}^r$ , denote the number of degrees of freedom at the  $k$ th connection.

It is also necessary, when applying the subroutine, to relabel each of the nonrigid appendages  $\alpha_k$  in the same sequence from 1 to  $n_f$  (see Fig. 2) so that the labels become  $\mathcal{Q}_1, \dots, \mathcal{Q}_{n_f}$ .

*Def. 51.* Let  $n_f$  be the number of nonrigid appendages in the system (no more than one per substructure).

The first column of the input array,  $F$ , contains the index labels of those rigid bodies to which nonrigid appendages  $\mathcal{Q}_i$  ( $i = 1, \dots, n_f$ ) are attached.

*Initializing Call Statement*

CALL MBDYFR(NC, H, MB, MS, PB, PS, G, PI,  
NF, F, ER, EI, SR, MF, RF, WF, ZF)

where

NC = the integer  $n_c$  = number of system connections (see Def. 45).

$H(k, m)$  = array containing the connection elements  $h_k$ ,  $k \in \mathcal{P}^r$ , and the number of degrees of freedom,  $d_k$ , at the connection;  $m = 1, 2$ .  
 $(H(1, 1) = h_1, H(2, 1) = h_2, \dots, H(n_c, 1) = h_{n_c}, H(1, 2) = d_1,$   
 $H(2, 2) = d_2, \dots, H(n_c, 2) = d_{n_c})$

$MB(j)$  = array of undeformed reference substructure ( $b_0$ ) inertial constants  $j = 1, \dots, 7$ . (Specifically:  $MB(1) = \bar{J}_{11}^0$ ,  $MB(2) = \bar{J}_{22}^0$ ,  
 $MB(3) = \bar{J}_{33}^0$ ,  $MB(4) = -\bar{J}_{12}^0$ ,  $MB(5) = -\bar{J}_{13}^0$ ,  $MB(6) = -\bar{J}_{23}^0$ ,  
 $MB(7) = \mathcal{M}_{b_0}$ )

$MS(i, j)$  = array of remaining substructure body (undeformed) inertial constants;  $i \in \mathcal{P}^r$ ;  $j = 1, \dots, 7$ . (Thus:  $MS(i, 1) = \bar{J}_{11}^i$ ,  $MS(i, 2) = \bar{J}_{22}^i, \dots$ ,  $MS(i, 7) = \mathcal{N}_i$ .)

$PB(i, j)$  = array containing elements of  $p^{0i}$ ;  $i \in \mathcal{B}_0^r$ ,  $j \in 1, 2, 3$ .

$PS(i, j, k)$  = array containing elements of  $p^{ij}$ ;  $i \in \mathcal{P}^r$ ,  $j \in \mathcal{B}_i^r$ ,  $k = 1, 2, 3$ . (Exception!! If  $j < i$ , set  $PS(i, i, k) = p^{ij}$ . Example:  $PS(3, 3, 1) = p_1^{32}$ . All  $PS(i, j, k)$ , where  $j < i$ , will be ignored.)

$G(i, j)$  = array containing elements of  $g^i$ ;  $i \in \mathcal{P}$ ,  $j = 1, 2, 3$ .

$PI(i)$  = array of indicators;  $i = 1, 2, \dots, n + 1$ . (If  $\gamma_i$  is a prescribed variable,  $PI(i) = 1$ . Otherwise,  $PI(i) = 0$ . Also, if  $PI(n + 1) = 1$ , system angular momentum HM will be calculated; otherwise, HM is set to zero.)

NF = the integer  $n_f$  = number of substructures with nonrigid appendages = number of nonrigid appendages.

$F(n, m)$  = array containing the index labels of those rigid bodies with nonrigid appendages, the number of nodal bodies in each appendage's finite element model, and the number of modes to be used in each appendage's modal model;  $n = 1, 2, \dots, n_f$ ,  $m = 1, 2, 3$ . (Thus:

$F(1, 1)$  = index label of rigid body carrying appendage  $\mathcal{Q}_1$

$F(1, 2)$  = number of nodal bodies in appendage  $\mathcal{Q}_1$

$F(1, 3)$  = number of modes representing appendage  $\mathcal{Q}_1$

$F(2, 1)$  = index label of rigid body carrying appendage  $\mathcal{Q}_2$

⋮

etc.

⋮

$F(n_f, 3)$  = number of modes representing appendage  $\mathcal{Q}_{n_f}$ .)

$ER(n, i, j)$  = array of elements of  $\bar{\psi}_k^i$ ;  $n = 1, 2, \dots, n_f$ ;  $i = 1, 2, \dots, 6n_k$ ;  $k = F(n, 1)$ ;  $j = 1, 2, \dots, N_k$ .

$EI(n, i, j)$  = array of elements of  $\bar{\Gamma}_k^i$ ;  $n = 1, 2, \dots, n_f$ ;  $i = 1, 2, \dots, 6n_k$ ;  $k = F(n, 1)$ ;  $j = 1, 2, \dots, N_k$ .

$SR(n, j)$  = array of substructure nominal spin rates,  $\Omega^k$ ,  $k = F(n, 1)$ ;  $n = 1, 2, \dots, n_f$ ;  $j = 1, 2, 3$ .

$MF(n, i, j)$  = array of nodal body inertial properties,  $M^k$ , for each nonrigid appendage;  $n = 1, 2, \dots, n_f$ ;  $i = 1, 2, \dots, n_k$ ;  $k = F(n, 1)$ ;  $j = 1, 2, \dots, 7$ . (Example:  $MF(2, 3, 1) = I_{11}^3$ ,  $MF(2, 3, 2) = I_{22}^3$ ,  $MF(2, 3, 3) = I_{33}^3$ ,  $MF(2, 3, 4) = -I_{12}^3, \dots$ ,  $MF(2, 3, 7) = m_3$ , all for nonrigid appendage  $\mathcal{Q}_2$ , third nodal body.)



RF( $n, i, j$ ) = array of elements of  $r_k$ ,  $k = F(n, 1)$ , for each nonrigid appendage;  $n = 1, 2, \dots, n_f$ ,  $i = 1, 2, \dots, n_k$ ,  $j = 1, 2, 3$ . (Example: RF(1, 5, 1) =  $r_1^5$ , RF(1, 5, 2) =  $r_2^5$ , RF(1, 5, 3) =  $r_3^5$ , all for appendage  $\mathcal{Q}_1$ .)

WF( $n, j$ ) = array of modal frequencies,  $\bar{\omega}^k$ ,  $k = F(n, 1)$ , for each nonrigid appendage;  $n = 1, 2, \dots, n_f$ ,  $j = 1, 2, \dots, N_k$ .

ZF( $n, j$ ) = array of modal damping factors,  $\bar{\xi}^k$ ,  $k = F(n, 1)$ , for each nonrigid appendage;  $n = 1, 2, \dots, n_f$ ,  $j = 1, 2, \dots, N_k$ .

The statement CALL MBDYFR (NC, H, . . .) need only be executed *once* prior to a simulation run. However, as the simulation proceeds, the routine must be entered at every numerical integration step to compute the angular accelerations  $\dot{\omega}^0, \ddot{\gamma}_1, \dots, \ddot{\gamma}_n$  and the modal coordinate acceleration vectors  $\delta^k$  and  $\dot{\eta}^k$  ( $k \in \mathcal{F}$ ). This is accomplished by executing the "dynamic" call statement.

#### Dynamic Call Statement

CALL MRATE(NC, TH, TB, TS, FB, FS, TF, FF, GM, GMD, GMDD,  
DT, ET, WO, WDOT, DTD, ETD, HM)

where

NC = the integer  $n_c$  = number of system connections.

TH( $i$ ) = array containing the hinge torques,  $\tau_i$ ;  $i \in \mathcal{P}$ .

TB( $j$ ) = array containing the elements of  $T^0$ ;  $j = 1, 2, 3$ .

TS( $i, j$ ) = array containing the elements of  $T^i$ ;  $i \in \mathcal{P}^r$ ,  $j = 1, 2, 3$ .

FB( $j$ ) = array containing the elements of  $F^0$ ;  $j = 1, 2, 3$ .

FS( $i, j$ ) = array containing the elements of  $F^i$ ;  $i \in \mathcal{P}^r$ ,  $j = 1, 2, 3$ .

TF( $n, i, j$ ) = array containing the torque elements of  $\lambda^k$ ;  $n = 1, \dots, n_f$ ,  
 $k = F(n, 1)$ ,  $i = 1, \dots, n_k$ ,  $j = 1, 2, 3$ .

FF( $n, i, j$ ) = array containing the force elements of  $\lambda^k$ ;  $n = 1, \dots, n_f$ ,  
 $k = F(n, 1)$ ,  $i = 1, \dots, n_k$ ,  $j = 1, 2, 3$ .

GM( $i$ ) = array of angles,  $\gamma_i$ ;  $i \in \mathcal{P}$ .

GMD( $i$ ) = array of the angular velocities,  $\dot{\gamma}_i$ ;  $i \in \mathcal{P}$ .

GMDD( $i$ ) = array of the prescribed angular accelerations,  $\ddot{\gamma}_i$ ;  $i \in \mathcal{P}$ .

DT( $n, i$ ) = array of appendage modal coordinates,  $\delta^k$ ;  $n = 1, \dots, n_f$ ,  
 $k = F(n, 1)$ ,  $i = 1, \dots, N_k$ .

ET( $n, i$ ) = array of appendage modal coordinates,  $\dot{\eta}^k$ ;  $n = 1, \dots, n_f$ ,  
 $k = F(n, 1)$ ,  $i = 1, \dots, N_k$ .

WO( $j$ ) = array containing the components of  $\omega^0$ ;  $j = 1, 2, 3$ .

WDOT( $j$ ) = solution vector containing the elements of  $\dot{\omega}^0, \ddot{\gamma}_1, \dots, \ddot{\gamma}_n$ ;  
 $j = 1, \dots, n + 3$ . (WDOT(1) =  $\dot{\omega}_1^0$ , WDOT(2) =  $\dot{\omega}_2^0$ , WDOT(3) =  $\dot{\omega}_3^0$ , WDOT(4) =  $\ddot{\gamma}_1$ , . . . , WDOT( $n + 3$ ) =  $\ddot{\gamma}_n$ .)

DTD( $n, i$ ) = solution matrix for  $\delta^k$ ;  $n = 1, \dots, n_f$ ,  $k = F(n, 1)$ ,  $i = 1, \dots, N_k$ .

ETD( $n, i$ ) = solution matrix for  $\ddot{\eta}^k$ ;  $n = 1, \dots, n_f$ ,  $k = F(n, 1)$ ,  $i = 1, \dots, N_k$ .

HM = magnitude of the system angular momentum vector (see Appendix B for the momentum equations).

In summary, the call to MRATE supplies the subroutine with current instantaneous values for hinge torques and externally applied torques and forces on both rigid bodies and nonrigid appendages. Explicit expressions for computing these forcing functions, which may depend on  $\gamma_i$ ,  $\dot{\gamma}_i$ , and other system or control variables, are located in the main calling program (see sample problem that follows). Current values of  $\omega^0$ ,  $\gamma_i$ ,  $\dot{\gamma}_i$ ,  $\delta^k$ , and  $\ddot{\eta}^k$  are continuously produced by the main program's numerical integration operators and are therefore always available for input to MBDYFR.

It should be noted here that MBDYFR *does not* incorporate the terms in Eq. (42) that describe symmetric rotor torques on body  $\mathcal{L}_k$ . As a result, the *user* is required, if rotors are present, to supply these terms as part of a "new"  $T^k$ , i.e.,

$$T'^k = T^k - \tau_R^k - \bar{\omega}^k g^k (\bar{\omega}^k + \dot{\psi}_R^k)$$

Thus, these terms must be formed in the main program along with Eq. (44), and  $T'^k$  is supplied to the subroutine as TB (if  $k = 0$ ) or TS in the MRATE call statement.

Note also that, if any of the  $\gamma_i$  are to be prescribed, the appropriate values of  $\gamma_i$ ,  $\dot{\gamma}_i$ , and  $\ddot{\gamma}_i$  must be supplied to the subroutine by way of GM, GMD, GMDD, respectively, in the MRATE call statement. An example of this is shown in Section IVC.

When the MBDYFR subroutine is used, the main calling program must contain Fortran V (or IV) statements which specify "type" and allocate storage for the variables and arrays being used. The mandatory specification statements are listed here.

#### Required Specification Statements

```

INTEGER  NC, NF, H( $n_c, 2$ ), F( $n_f, 3$ ), PI( $n + 1$ )
REAL     MB(7), MS( $n_c, 7$ ), PB( $n_c, 3$ ), PS( $n_c, n_c, 3$ ), G( $n, 3$ ),
         TH( $n$ ), TB(3), TS( $n_c, 3$ ), FB(3), FS( $n_c, 3$ ), GM( $n$ ),
         GMD( $n$ ), GMDD( $n$ ), ER( $n_f, 6n_k, N_k$ ), EI( $n_f, 6n_k, N_k$ ),
         MF( $n_f, n_k, 7$ ), RF( $n_f, n_k, 3$ ), WF( $n_f, N_k$ ), ZF( $n_f, N_k$ ),
         TF( $n_f, n_k, 3$ ), FF( $n_f, n_k, 3$ ), DT( $n_f, N_k$ ), ET( $n_f, N_k$ ),
         WO(3), SR( $n_f, 3$ )

DOUBLE PRECISION  WDOT( $n + 3$ ), DTD( $n_f, N_k$ ), ETL( $n_f, N_k$ )

```

Also, in order that storage allocation for arrays internal to MBDYFR be minimized, the following statement must appear *in the subroutine*:

```
PARAMETER QH =  $n$ , QC =  $n_c$ , QF =  $n_f$ , NK =  $n_k$ , NKT =  $N_k$ 
```

The proper placement of this statement in MBDYFR is shown in the listing (Appendix C).

### C. A Sample Problem Simulation

To illustrate the use of subroutine MBDYFR, the dynamical system shown in Fig. 3 will be simulated. It consists of a rigid central body,  $\mathcal{B}_0$ , to which is connected a rigid platform,  $\mathcal{B}_2$ , with 2 degrees of rotational freedom relative to  $\mathcal{B}_0$ . A spinning rotor,  $\mathcal{B}_1$ , is also connected to  $\mathcal{B}_0$ . The platform and the rotor each carry an elastic appendage, which will be modeled as a simple point mass supported by a massless elastic member.

For this test vehicle, the platform will be nominally nonrotating, while the rotor will have a nominal spin rate of  $\omega_s$  about the spin axis fixed in  $\mathcal{B}_0$ . The appendage modal models must now be derived from the appropriate discrete coordinate equations.

#### Rotor Appendage Equations

The general appendage equation is Eq. (17), where the matrices  $M'_k$ ,  $G'_k$ , and  $K'_k$  for the rotor substructure are as follows:

$$M^1 = \begin{bmatrix} m_1 & 0 & 0 & | & \\ 0 & m_1 & 0 & | & 0 \\ 0 & 0 & m_1 & | & \\ \hline & 0 & & | & 0 \end{bmatrix} \quad (6 \times 6)$$

$$\therefore M'_1 = M^1 \left( U - \sum_{u_0} \sum_{v_0}^T \frac{M^1}{\mathcal{N}} \right) = \begin{bmatrix} \mu_1 & 0 & 0 & | & \\ 0 & \mu_1 & 0 & | & 0 \\ 0 & 0 & \mu_1 & | & \\ \hline & 0 & & | & 0 \end{bmatrix} \quad (6 \times 6)$$

where

$$\mu_1 = m_1 - \frac{m_1^2}{\mathcal{N}}$$

$$\mathcal{N} = \mathcal{N}_0 + \mathcal{N}_1 + \mathcal{N}_2$$

The rotor spin rate =  $\Omega^r = [0 \quad 0 \quad \omega_s]^T$ .

$$\therefore G'_1 = 2 \begin{bmatrix} 0 & -\omega_s \mu_1 & 0 & | & \\ \omega_s \mu_1 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & \\ \hline & 0 & & | & 0 \end{bmatrix} \quad (6 \times 6)$$

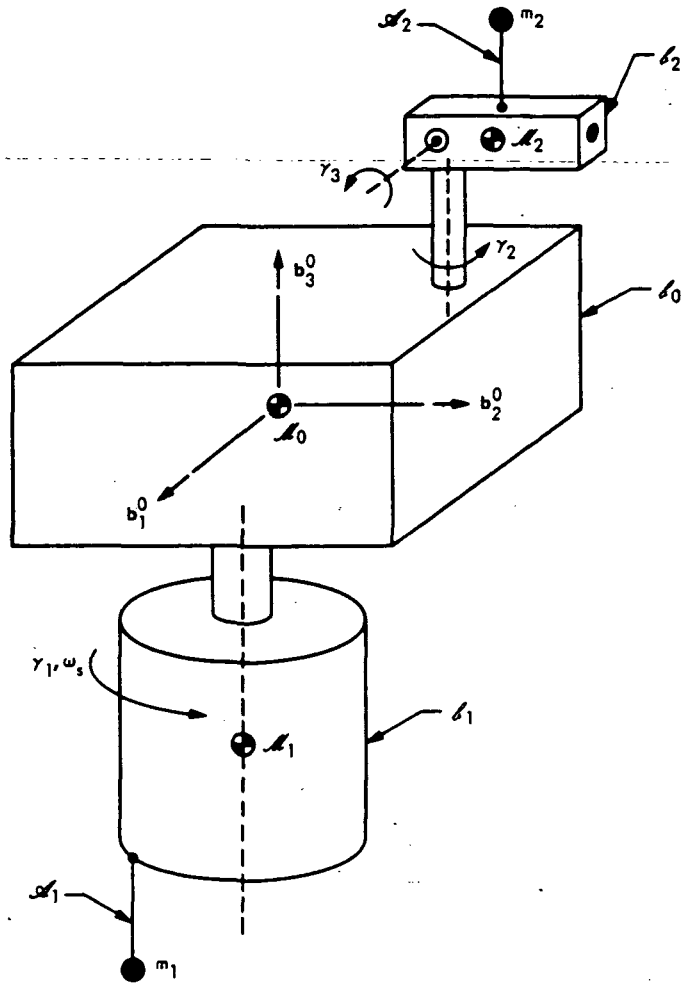


Fig. 3. MBDYFR simulation test vehicle

We will assume a symmetric stiffness matrix,  $K^1$ , of the form

$$K^1 = \begin{bmatrix} k_1 & 0 & 0 & | & 0 \\ 0 & k_2 & 0 & | & 0 \\ 0 & 0 & k_3 & | & 0 \\ \hline & 0 & & | & 0 \end{bmatrix} \quad (6 \times 6)$$

where  $k_1$ ,  $k_2$ , and  $k_3$  are the respective stiffness coefficients which restrain linear motion in the  $b_1^1$ ,  $b_2^1$ , and  $b_3^1$  directions. Thus,

$$K_1^1 = \begin{bmatrix} k_1 - \omega_s^2 \mu_1 & 0 & 0 & | & 0 \\ 0 & k_2 - \omega_s^2 \mu_1 & 0 & | & 0 \\ 0 & 0 & k_3 & | & 0 \\ \hline & 0 & & | & 0 \end{bmatrix}$$

The homogeneous rotor appendage equation may therefore be written as

$$\begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_1 \end{bmatrix} \ddot{q}^1 + 2 \begin{bmatrix} 0 & -\omega_s \mu_1 & 0 \\ \omega_s \mu_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{q}^1 + \begin{bmatrix} k_1 - \mu_1 \omega_s^2 & 0 & 0 \\ 0 & k_2 - \mu_1 \omega_s^2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} q^1 = 0$$

where  $q^1 = [u_1^1 \ u_2^1 \ u_3^1]^T$  (realizing that  $\beta_1^1 = \beta_2^1 = \beta_3^1 = 0$ , since  $m_1$  is a point mass).

If the equation is rewritten in first-order form, as in Eq. (18), it becomes

$$\mathcal{Q}_1 \dot{Q}^1 + \mathcal{V}_1 Q^1 = 0$$

where

$$\mathcal{Q}_1 = \begin{bmatrix} k_1 - \omega_s^2 \mu_1 & 0 & 0 & | & 0 \\ 0 & k_2 - \omega_s^2 \mu_1 & 0 & | & 0 \\ 0 & 0 & k_3 & | & 0 \\ \hline & & & | & \mu_1 & 0 & 0 \\ & 0 & & | & 0 & \mu_1 & 0 \\ & & & | & 0 & 0 & \mu_1 \end{bmatrix}$$

$$\mathcal{V}_1 = \begin{bmatrix} & & & | & -k_1 + \omega_s^2 \mu_1 & 0 & 0 \\ & 0 & & | & 0 & -k_2 + \omega_s^2 \mu_1 & 0 \\ & & & | & 0 & 0 & -k_3 \\ \hline k_1 - \omega_s^2 \mu_1 & 0 & 0 & | & 0 & -2\omega_s \mu_1 & 0 \\ 0 & k_2 - \omega_s^2 \mu_1 & 0 & | & 2\omega_s \mu_1 & 0 & 0 \\ 0 & 0 & k_3 & | & 0 & 0 & 0 \end{bmatrix}$$

and

$$Q^1 = [q^1 \ \dot{q}^1]^T$$

The rotor appendage equation eigenvalues,  $\lambda_j$ , and corresponding eigenvectors,  $\Phi_j^1$ , may then be found from

$$[\mathcal{Q}_1 \lambda_j + \mathcal{V}_1] \Phi_j^1 = 0$$

From the characteristic equation, one finds that

$$\lambda_j = \pm i \left[ \frac{k}{\mu_1} + \omega_s^2 \mp 2\omega_s \sqrt{\frac{k}{\mu_1}} \right]^{\frac{1}{2}}$$

and

$$\lambda_j = \pm i \left[ \frac{k_3}{\mu_1} \right]^{\frac{1}{2}}$$

where  $k = k_1 = k_2$ .

If we now arbitrarily let  $\sqrt{k/\mu_1} = 2\omega_s$  and  $\sqrt{k_3/\mu_1} = 5\omega_s$ , the eigenvalues become

$$\lambda_1 = i\omega_s$$

$$\lambda_2 = i3\omega_s$$

$$\lambda_3 = i5\omega_s$$

$$\lambda_4 = -i\omega_s$$

$$\lambda_5 = -i3\omega_s$$

$$\lambda_6 = -i5\omega_s$$

Note that the eigenvalues are imaginary as predicted and that they have been deliberately ordered to correspond to the form of Eq. (22), with conjugates in the lower half of  $\Lambda_1$ .

The eigenvectors corresponding to these eigenvalues may then be determined as

$$\Phi_1 = \begin{bmatrix} i & -i & 0 & -i & i & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ -\omega_s & 3\omega_s & 0 & -\omega_s & 3\omega_s & 0 \\ i\omega_s & i3\omega_s & 0 & -i\omega_s & -i3\omega_s & 0 \\ 0 & 0 & i5\omega_s & 0 & 0 & -i5\omega_s \end{bmatrix} = \begin{bmatrix} \phi_1' \\ \phi_1' \lambda_j \end{bmatrix}$$

Also,

$$\Phi_1^{*T} \mathcal{U}_1 \Phi_1 = \begin{bmatrix} 8\mu_1\omega_s^2 & & & & & \\ & 24\mu_1\omega_s^2 & & & & \\ & & 50\mu_1\omega_s^2 & & & \\ & & & 8\mu_1\omega_s^2 & & \\ & 0 & & & 24\mu_1\omega_s^2 & \\ & & & & & 50\mu_1\omega_s^2 \end{bmatrix}$$

The final form of the appendage modal coordinate equations, shown in Eq. (26), can be obtained only if the eigenvectors are normalized so that  $\Phi_1^{*T} \mathcal{U}_1 \Phi_1 = U$ , the diagonal unit matrix (see Ref. 3). Thus, succeeding columns in  $\Phi_1$  should be multiplied by  $(8\mu_1\omega_s^2)^{-\frac{1}{2}}$ ,  $(24\mu_1\omega_s^2)^{-\frac{1}{2}}$ ,  $(50\mu_1\omega_s^2)^{-\frac{1}{2}}$ , etc., for proper normalization in this case.

If we also arbitrarily truncate this modal transformation to just the first two modes, the resulting real and imaginary parts of  $\bar{\phi}_1$  become

$$\bar{\psi}_1 = \begin{bmatrix} 0 & 0 \\ \frac{1}{2\omega_s\sqrt{2\mu_1}} & \frac{1}{2\omega_s\sqrt{6\mu_1}} \end{bmatrix}, \quad \bar{\Gamma}_1 = \begin{bmatrix} \frac{1}{2\omega_s\sqrt{2\mu_1}} & -\frac{1}{2\omega_s\sqrt{6\mu_1}} \\ 0 & 0 \end{bmatrix}$$

Likewise,

$$\bar{\sigma}^1 = \begin{bmatrix} \omega_s & 0 \\ 0 & 3\omega_s \end{bmatrix}, \quad \bar{\xi}^1 = \begin{bmatrix} \xi_1^1 & 0 \\ 0 & \xi_2^1 \end{bmatrix}$$

#### Platform Appendage Equations

If the same process is applied to the nominally nonspinning platform appendage, its homogeneous equation of motion becomes

$$\begin{bmatrix} \mu_2 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_2 \end{bmatrix} \ddot{q}^2 + \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} q^2 = 0$$

Using the first-order equations again,

$$\mathcal{U}_2 \dot{Q}^2 + \mathcal{V}_2 Q^2 = 0$$

where

$$\mathcal{U}_2 = \begin{bmatrix} k_1 & 0 & 0 & & & \\ 0 & k_2 & 0 & & & 0 \\ 0 & 0 & k_3 & & & \\ \hline & & & \mu_2 & 0 & 0 \\ & 0 & & 0 & \mu_2 & 0 \\ & & & 0 & 0 & \mu_2 \end{bmatrix}$$

$$\mathcal{V}_2 = \left[ \begin{array}{ccc|ccc} & & & -k_1 & 0 & 0 \\ & 0 & & 0 & -k_2 & 0 \\ & & & 0 & 0 & -k_3 \\ \hline k_1 & 0 & 0 & & & \\ 0 & k_2 & 0 & & 0 & \\ 0 & 0 & k_3 & & & \end{array} \right]$$

and

$$\mathcal{Q}^2 = [q^2 \mid \dot{q}^2]^T, \quad \mu_2 = m_2 - \frac{m_2^2}{9\pi}$$

one can easily determine that the eigenvalues are

$$\lambda_j = \pm i \sqrt{\frac{k_1}{\mu_2}}, \pm i \sqrt{\frac{k_2}{\mu_2}}, \pm i \sqrt{\frac{k_3}{\mu_2}}$$

If we let  $k = k_1 = k_2 = k_3$ , and  $\sqrt{k/\mu_2} = \sigma_2$ , then

$$\Lambda_2 = \left[ \begin{array}{cccccc} \sigma_2 i & & & & & \\ & \sigma_2 i & & & 0 & \\ & & \sigma_2 i & & & \\ & & & -\sigma_2 i & & \\ & 0 & & & -\sigma_2 i & \\ & & & & & -\sigma_2 i \end{array} \right]$$

and

$$\Phi_2 = \left[ \begin{array}{cccccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ \sigma_2 i & 0 & 0 & -\sigma_2 i & 0 & 0 \\ 0 & \sigma_2 i & 0 & 0 & -\sigma_2 i & 0 \\ 0 & 0 & \sigma_2 i & 0 & 0 & -\sigma_2 i \end{array} \right] = \left[ \begin{array}{c} \phi_2 \\ -\phi_2 \\ \phi_2 \lambda_j \end{array} \right]$$



The appropriate normalization factor for each  $\phi_j'$  is  $(2\mu_2\sigma_2^2)^{-\frac{1}{2}}$ . Thus, if the platform appendage modal model is truncated to the first two (transverse bending) modes, the needed quantities are

$$\bar{\psi}_2 = \begin{bmatrix} \frac{1}{\sigma_2\sqrt{2\mu_2}} & 0 \\ 0 & \frac{1}{\sigma_2\sqrt{2\mu_2}} \end{bmatrix}, \quad \bar{\Gamma}_2 = 0$$

$$\bar{\sigma}^2 = \begin{bmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{bmatrix}, \quad \bar{\xi}^2 = \begin{bmatrix} \xi_1^2 & 0 \\ 0 & \xi_2^2 \end{bmatrix}$$

### Test Vehicle Constants

To complete the specification of the test configuration shown in Fig. 3, numerical values can now be assigned to its various mass properties and other physical constants. First, let

$$\mathcal{M}_0 = 399.9 \text{ kg}$$

$$\mathcal{M}_1 = 50.1 \text{ kg}$$

$$\mathcal{M}_2 = 50.0 \text{ kg}$$

$$m_1 = 1.0 \text{ kg}$$

$$m_2 = 5.0 \text{ kg}$$

$$\bar{J}^0 = \begin{bmatrix} 250. & 0. & 0. \\ 0. & 275. & 0. \\ 0. & 0. & 350. \end{bmatrix}, \text{ kg-m}^2$$

$$\bar{J}^1 = \begin{bmatrix} 10. & 0. & 0. \\ 0. & 10. & 0. \\ 0. & 0. & 20. \end{bmatrix}, \text{ kg-m}^2$$

$$\therefore \mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1 + \mathcal{M}_2 = 500.0 \text{ kg}$$

$$\therefore \mu_1 = .998 \text{ kg}$$

$$\therefore \mu_2 = 4.95 \text{ kg}$$

$$\bar{J}^2 = \begin{bmatrix} 6. & 0 & 0 \\ 0 & 3. & 0 \\ 0 & 0 & 8. \end{bmatrix}, \text{ kg} \cdot \text{m}^2$$

Also, let

$$\omega_s = 10. \text{ rad/s}, \quad \xi_1^1 = \xi_2^1 = .01$$

$$\sigma_2 = 9. \text{ rad/s}, \quad \xi_1^2 = \xi_2^2 = .01$$

$$\therefore \bar{\psi}_1 = \begin{bmatrix} 0 & 0 \\ .035391 & .020433 \end{bmatrix}, \quad \bar{\Gamma}_1 = \begin{bmatrix} .035391 & -.020433 \\ 0 & 0 \end{bmatrix}$$

$$\bar{\psi}_2 = \begin{bmatrix} .035313 & 0 \\ 0 & .035313 \end{bmatrix}, \quad \bar{\Gamma}_2 = 0$$

The locations of the two point masses (see Figs. 4 and 5) relative to their substructure's mass center when they are in the nominal deformed state will be assumed as

$$r_1 = [.33 \quad 0 \quad -.493]^T \text{ meters}$$

$$r_2 = [0 \quad 0 \quad .56]^T \text{ meters}$$

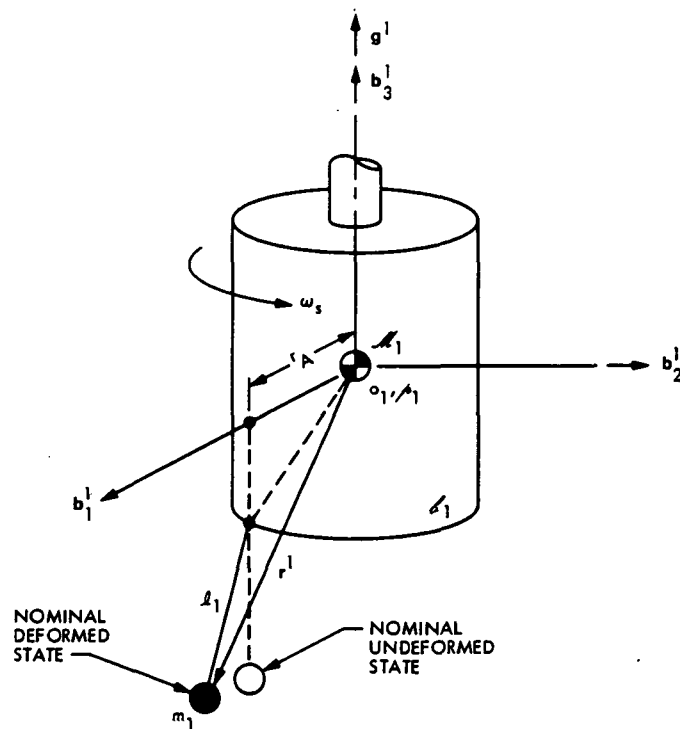


Fig. 4. Substructure  $d_1$

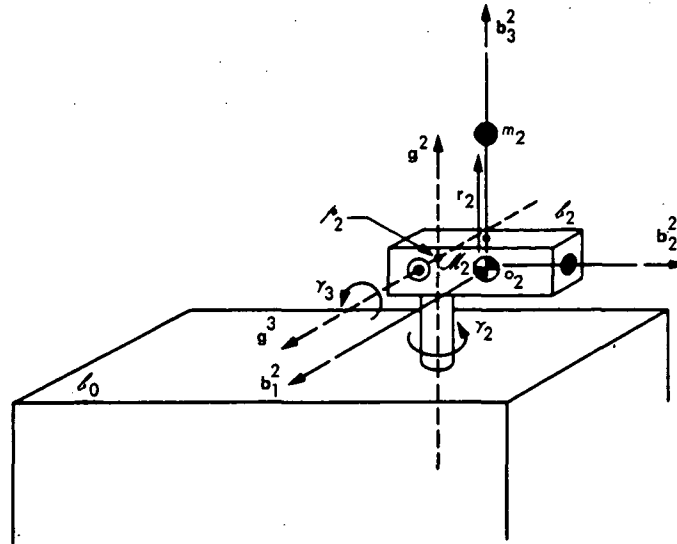


Fig. 5. Substructure  $4_2$

Locations for the interbody connections, relative to substructure mass centers, are

$$p^{01} = [0. \quad 0. \quad -2.]^T \text{ meters}$$

$$p^{02} = [0. \quad 1. \quad 1.]^T \text{ meters}$$

$$p^{10} = [0. \quad 0. \quad 0.] \text{ meters}$$

$$p^{20} = [0. \quad -.3 \quad 0.] \text{ meters}$$

The three hinge directions are given by the direction cosines

$$\text{rotor: } g^1 = [0. \quad 0. \quad 1.]^T$$

$$\text{platform: } g^2 = [0. \quad 0. \quad 1.]^T$$

$$\text{platform: } g^3 = [1. \quad 0. \quad 0.]^T$$

Also,

$$n_c = 2, \quad n_f = 2, \quad n_1 = 1, \quad n_2 = 1, \quad N_1 = 2, \quad N_2 = 2$$

$$h_1 = 0, \quad h_2 = 0, \quad d_1 = 1, \quad d_2 = 2, \quad n = 3$$

As a result of these choices, the initializing call statement arguments become

$$NC = 2$$

$$H = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$$

$$MB = [250. \quad 275. \quad 350. \quad 0. \quad 0. \quad 0. \quad 399.9]$$

$$MS = \begin{bmatrix} 10. & 10. & 20. & 0. & 0. & 0. & 50.1 \\ 6. & 3. & 8. & 0. & 0. & 0. & 50.0 \end{bmatrix}$$

$$PB = \begin{bmatrix} 0. & 0. & -2. \\ 0. & 1. & 1. \end{bmatrix}$$

$$PS(2, 2, j) = [0. \quad -0.3 \quad 0.] \quad (\text{all other PS elements are zero})$$

$$G = \begin{bmatrix} 0. & 0. & 1. \\ 0. & 0. & 1. \\ 1. & 0. & 0. \end{bmatrix}$$

$$PI = [0 \quad 0 \quad 0 \quad 1] \quad (\text{assuming no prescribed hinge motions})$$

$$NF = 2$$

$$F = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$ER(1, i, j) = \begin{bmatrix} 0. & 0. \\ .035391 & .020433 \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \end{bmatrix}$$

$$EI(1, i, j) = \begin{bmatrix} .035391 & -.020433 \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \end{bmatrix}$$

$$ER(2, i, j) = \begin{bmatrix} .035313 & 0. \\ 0. & .035313 \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \end{bmatrix}$$

$$EI(2, i, j) = 0.$$

$$SR = \begin{bmatrix} 0. & 0. & 10. \\ 0. & 0. & 0. \end{bmatrix}$$

$$MF(1, 1, j) = [0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 1.0]$$

$$MF(2, 1, j) = [0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 0. \quad 5.0]$$

$$RF(1, 1, j) = [.3333 \quad 0. \quad -.4930]$$

$$RF(2, 1, j) = [0. \quad 0. \quad .56]$$

$$WF = \begin{bmatrix} 10. & 30. \\ 9. & 9. \end{bmatrix}$$

$$ZF = \begin{bmatrix} .01 & .01 \\ .01 & .01 \end{bmatrix}$$

### *Test Vehicle Dynamics*

Before simulating a specific dynamic case for the test vehicle of Fig. 3, the characteristics of the interbody connections must be defined. The connection between  $\phi_0$  and rotor  $\phi_1$  will be assumed a frictionless bearing so that

$$\tau_1 = 0$$

The platform hinge connections will be assumed to be of the linear spring-damper type, i.e.,

$$\tau_2 = -K_2(\gamma_2 - \gamma_{2c}) - B_2\dot{\gamma}_2$$

$$\tau_3 = -K_3(\gamma_3 - \gamma_{3c}) - B_3\dot{\gamma}_3,$$

where  $\gamma_{2c}$  and  $\gamma_{3c}$  are platform angular position commands. The values of the constants  $K_2, K_3, B_2, B_3$  are arbitrarily chosen as

$$K_2 = 250. \text{ n-m/rad}, \quad B_2 = 50. \text{ n-m-s/rad}$$

$$K_3 = 300. \text{ n-m/rad}, \quad B_3 = 50. \text{ n-m-s/rad}$$

The dynamic response to be simulated here will be that due to a high-rate platform slew sequence. Slew commands  $\gamma_{2c}$  and  $\gamma_{3c}$  will be generated by integrating the time functions shown in Fig. 6. This will result in a 10-deg rotation about  $g^2$  and a 10-deg rotation about  $g^3$ .

Initially, the rotor is spinning at 10 rad/s relative to  $\mathcal{A}_0$ , and the rotor appendage is at rest relative to the rotor but deflected radially outward in its steady-state deformed position. (One can show from Eq. (17), with the assumption  $k/\mu_1 = 4\omega_s^2$ , that the radial deformation (in the  $b_1^1$  direction) due to spin is  $r_A/3$ , where  $r_A$  is the distance from the rotor spin axis to the appendage attachment point.) The platform,  $\mathcal{A}_1$ , as well as the base body,  $\mathcal{A}_0$ , are initially at rest. At  $t = 1$  s, the command is issued to rotate the platform about  $g^2$  at a rate of 10 deg/s until  $t = 2$  s; again at  $t = 3$  s, a command to rotate about the  $g^3$  axis at 10 deg/s appears and ends at  $t = 4$  s. The computer simulation program, employing MBDYFR, for this dynamic maneuver is shown in Fig. 7.

Notice that the necessary dimension specifications for each variable are stated in the JPL CSSL III simulation language as: ARRAY MB(7), MS(2, 7), . . . , etc.

An auxiliary routine, called HCK, is used in the simulation to keep track of the rotations of the reference body ( $\mathcal{A}_0$ ) relative to an inertially fixed frame. HCK uses Euler parameters to do this, and it is initialized using Euler angles. The variable, THET, is calculated in the program by means of HCK and represents the angular deviation of the  $b_3^0$  axis from its initial, inertially fixed position, i.e., the reference body "nutation" angle.

The CSSL III function, "STEP," provides the unit step function when the independent variable, TIME, is greater than the specified constant. "INTEG( $a_1, a_2$ )" signifies the integration of  $a_1$  with respect to TIME, where  $a_2$  is the initial condition.

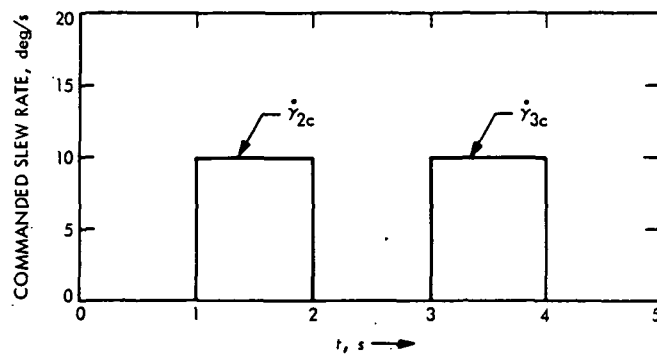


Fig. 6. Commanded slew rates

CSSL III JET PROPULSION LABORATORY 040374-ADD2H 021775-021636

\*\*\* START T(RUN)= 28.032 T(TASK)= .003 CTP = .760  
DT(TASK)= .003 DCTP = .760

PROGRAM 3-BODY VEHICLE WITH SPINNING ROTOR AND 2 FLEXIBLE APPENDAGES  
\*SC4020 BLDG/198,BOX/601,CAMERA/9IN,FRAMES/50

ARRAY MB(7),MS(2,7),PB(2,3),PS(2,2,3),G(3,3),TH(3),TB(3),TS(2,3)  
ARRAY FB(3),FS(2,3),GM(3),GMD(3),GMD0(3),ER(2,6,2),EI(2,6,2)  
ARRAY MF(2,1,7),RF(2,1,3),WF(2,2),ZF(2,2),TF(2,1,3),FF(2,1,3)  
ARRAY SR(2,3),DT(2,2),ET(2,2),WO(3),U(2,1,3),UD(2,1,3)  
DOUBLE PRECISION WDOT(6),DTD(2,2),ETD(2,2),EC(14)  
INTEGER NC,NF,H(2,2),F(2,3),PI(4)  
DATA H/O,0,1,2/PI/O,0,0,1/  
DATA MB/250.,275.,350.,0.,0.,0.,399.9/  
DATA MS(1,1)/10./MS(1,2)/10./MS(1,3)/20./MS(1,7)/50.1/  
DATA MS(2,1)/6./MS(2,2)/3./MS(2,3)/8./MS(2,7)/50./  
DATA PB(1,1)/0./PB(1,2)/0./PB(1,3)/2./  
DATA PB(2,1)/0./PB(2,2)/1./PB(2,3)/1./  
DATA PS(2,2,1)/0./PS(2,2,2)/.3/PS(2,2,3)/0./  
DATA G(1,3)/1./G(2,3)/1./G(3,1)/1./  
DATA F/1,2,1,1,2,2/  
DATA ER(1,2,1)/.03539075/ER(1,2,2)/.02043286/  
DATA EI(1,1,1)/.03539075/EI(1,1,2)/.02043286/  
DATA ER(2,1,1)/.03531343/ER(2,2,2)/.03531343/  
DATA RF(1,1,1)/.3333/RF(1,1,3)/.4930/RF(2,1,3)/.56/  
DATA MF(1,1,7)/1.0/MF(2,1,7)/5.0/  
DATA ZF(1,1)/.01/ZF(1,2)/.01/ZF(2,1)/.01/ZF(2,2)/.01/  
DATA WF(1,1)/10./WF(1,2)/30./WF(2,1)/9.0/WF(2,2)/9.0/  
DATA SR(1,1)/0./SR(1,2)/0./SR(1,3)/10./  
CONSTANT K2=250.,B2=50.,K3=300.,B3=50.  
CONSTANT TFINAL=10.,CLKTIM=900.,PIE=3.14159265  
CONSTANT PHIZ=0.,THETZ=0.,PSIZ=0.  
CONSTANT GMIDI=10.,W11=0.,W21=0.,W31=0.

INITIAL

NC=2 S NF=2  
CALL MBDYFR(NC,H,MB,MS,PB,PS,G,PI,NF,F,ER,EI,SR,MF,RF,WF,ZF)  
PZI,PAI,PBI,PCI=HCK(INITL,PHIZ,THETZ,PSIZ)

END

DYNAMIC

IF(TIME.GE.TFINAL) GO TO S1

DERIVATIVE BODY3F

VARIABLE TIME=0. S CINTERVAL CI=.01  
XERROR W1=1.E-9,W2=1.E-9,W3=1.E-9,GM2D=1.E-9,GM3D=1.E-9,...  
GM2=1.E-9,GM3=1.E-9,PAO=1.E-9,PBO=1.E-9,PCO=1.E-9,GM1=1.E-5  
MERROR W1=1.E-9,W2=1.E-9,W3=1.E-9,GM2D=1.E-9,GM3D=1.E-9,...  
GM2=1.E-9,GM3=1.E-9,PAO=1.E-9,PBO=1.E-9,PCO=1.E-9,GM1=1.E-5  
STPCLK CLKTIM  
OUTPUT IO,W1D,W2D,W3D,W1,W2,W3,GM1D,GM2D,GM3D,GM1,GM2,GM3,THET,...  
DT1A,DT1B,DT2A,DT2B,ET1A,ET1B,ET2A,ET2B,PZO,PAO,PBO,...  
PCO,U1X,U1Y,U1Z,U2X,U2Y,UZZ,U1XD,U1YD,U2XD,U2YD,GM2C,...  
GM3C,GM2CD,GM3CD,ANGH  
PREPAR THET,W1,W2,W3,GM1D,GM2D,GM3D,GM2,GM3,U1X,U1Y,U2X,U2Y,...  
GM1,U1XD,U1YD,U2XD,U2YD,GM2C,GM3C,ANGH

NOSORT

WO(1)=W1 S WO(2)=W2 S WO(3)=W3  
GM(1)=GM1 S GM(2)=GM2 S GM(3)=GM3

Fig. 7. Simulation program for test vehicle dynamics using MBDYFR

```

GM1D=GM1D S GM2D=GM2D S GM3D=GM3D
DT(1,1)=DT1A S DT(1,2)=DT1B S ET(1,1)=ET1A S ET(1,2)=ET1B
DT(2,1)=DT2A S DT(2,2)=DT2B S ET(2,1)=ET2A S ET(2,2)=ET2B
COMMENT...
                                PLATFORM POSITION COMMANDS
COMMENT
GM2CD=(STEP(1.0,TIME)-STEP(2.0,TIME))*PIE/180.
GM3CD=(STEP(3.0,TIME)-STEP(4.0,TIME))*PIE/180.
GM2C=INTEG(GM2CD,0.) S GM3C=INTEG(GM3CD,0.)
COMMENT...
                                REFERENCE BODY MUTATION ANGLE
COMMENT
D1,D2=HCK(MATRIX,PZO,PAO,PBO,PCO)
DC1,DC2,DC3=HCK(BTO1,0.,0.,1.,D1,D2)
DCM=SQRT(DC1**2 + DC2**2)
THET=ASIN(DCM)*180./PIE
COMMENT...
                                HINGE TORQUES
COMMENT
TH(2)=-K2*(GM2-GM2C) - B2*GM2D
TH(3)=-K3*(GM3-GM3C) - B3*GM3D
COMMENT...
                                SYSTEM ANGULAR ACCELERATIONS
COMMENT
CALL HRATE(NC,TH,TB,TS,FB,FS,TF,FF,GH,GMD,GHDD,DT,ET,W0,W0DT,...
           DTD,ETD,HH,U,UD)
U1XD=UD(1,1,1) S U1YD=UD(1,1,2) S U2XD=UD(2,1,1) S U2YD=UD(2,1,2)
U1X=U(1,1,1) S U1Y=U(1,1,2) S U1Z=U(1,1,3)
U2X=U(2,1,1) S U2Y=U(2,1,2) S U2Z=U(2,1,3)
W1D=W0DT(1) S W2D=W0DT(2) S W3D=W0DT(3) S ANGH=HH
COMMENT...
                                SYSTEM ANGULAR RATES AND POSITIONS
COMMENT
W1=INTEG(WDOT(1),W1)
W2=INTEG(WDOT(2),W2)
W3=INTEG(WDOT(3),W3)
GM1D=INTEG(WDOT(4),GM1D)
GM2D=INTEG(WDOT(5),0.)
GM3D=INTEG(WDOT(6),0.)
GM1=INTEG(GM1D,0.) S GM2=INTEG(GM2D,0.) S GM3=INTEG(GM3D,0.)
DT1A=INTEG(DTD(1,1),0.) S DT1B=INTEG(DTD(1,2),0.)
DT2A=INTEG(DTD(2,1),0.) S DT2B=INTEG(DTD(2,2),0.)
ET1A=INTEG(ETD(1,1),0.) S ET1B=INTEG(ETD(1,2),0.)
ET2A=INTEG(ETD(2,1),0.) S ET2B=INTEG(ETD(2,2),0.)
COMMENT...
                                HCK PARAMETER RATES AND POSITIONS
COMMENT
PZOD,PAOD,PBOD,PCOD=HCK(HCK,PZO,PAO,PBO,PCO,W1,W2,W3)
PZO=INTEG(PZOD,PZI) S PAO=INTEG(PAOD,PAI)
PBO=INTEG(PBOD,PBI) S PCO=INTEG(PCOD,PCI)
END
END
END
TERMINAL
SI:: CONTINUE
END
END

```

Fig. 7 (contd)

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All arithmetic statements are in Fortran, although CSSL III allows several statements in a single line if separated by a "\$". Variables to be plotted at every communication interval, CI, are listed in the PREPAR statement. Printed variables are listed in the OUTPUT statement.

The statement "CALL MBDYFR(NC, H, ...)" is located in the INITIAL section and is therefore executed only once, i.e., prior to the dynamic calculations. However, "CALL MRATE(NC, ...)" is in the DERIVATIVE section and is thus executed at every integration step. Note that two *additional* output variables have been added to the MRATE call statement argument list. They are U and UD, containing the appendage deformations  $u_1^1, u_2^1, u_3^1, u_1^2, \dots$  etc. and the deformation rates  $\dot{u}_1^1, \dot{u}_2^1, \dots$ , respectively. These variables are always available internal to MBDYFR using the relations of Eq. (27) and are outputted here only to more clearly illustrate the dynamic response of the system. ( $\beta_1^1, \beta_2^1, \dots, \dot{\beta}_1^1, \dot{\beta}_2^1, \dots$  etc. could also be obtained from the subroutine in those cases where the appendage nodal bodies have inertia.)

Results of the dynamic simulation are shown in the computer plots of Fig. 8, and the sample printout is presented in Fig. 9.

The solutions show, as expected, that all three components of the reference body angular velocity,  $\omega^0$ , are strongly perturbed by the platform as it accelerates or decelerates. Further, induced vibrations of the platform appendage are also in evidence on the reference body rates. Rotor spin rate,  $\dot{\gamma}_1$ , relative to  $\mathcal{L}_0$  remains very close to its initial and nominal value of 10 rad/s, although the effect of slewing the platform about an axis parallel to rotor spin is quite evident as are the subsequent vibrations due to platform appendage motion. Platform hinge rates,  $\dot{\gamma}_2$  and  $\dot{\gamma}_3$ , also show some appendage vibration, although it is very small compared to the slewing rate transients.

The components of rotor appendage deformation  $u_1^1, u_2^1$  exhibit both modal frequencies,  $\omega$ , and  $3\omega$ , but are relatively small in amplitude compared to the platform appendage deflections  $u_1^2, u_2^2$ . An "X-Y" plot of the platform appendage's deflections relative to its locally fixed coordinate frame is also shown.

System angular momentum magnitude in this test simulation should remain constant since no external forces or torques are being applied. The plot of HM shows this to be true very closely. Small deviations from a perfectly constant angular momentum in the simulations are to be expected due to the presence of modal damping (see Appendix A), numerical integration error, and round-off error.

## N76 12094

### IV. Systems With Nonrotating Appendages

#### A. Equations

In Part III, dynamical equations were developed for the substructure tree on the basis of (1) arbitrarily small flexible appendage deformations (and rates) from some nominal state and (2) arbitrarily small deviations of the angular rate of any rigid appendage base from a constant nonzero spin rate,  $\Omega^k$ . In this section, the assumption will be made that  $\Omega^k = 0$  ( $k \in \mathcal{F}$ ), i.e., that the appendage bases are nonrotating.

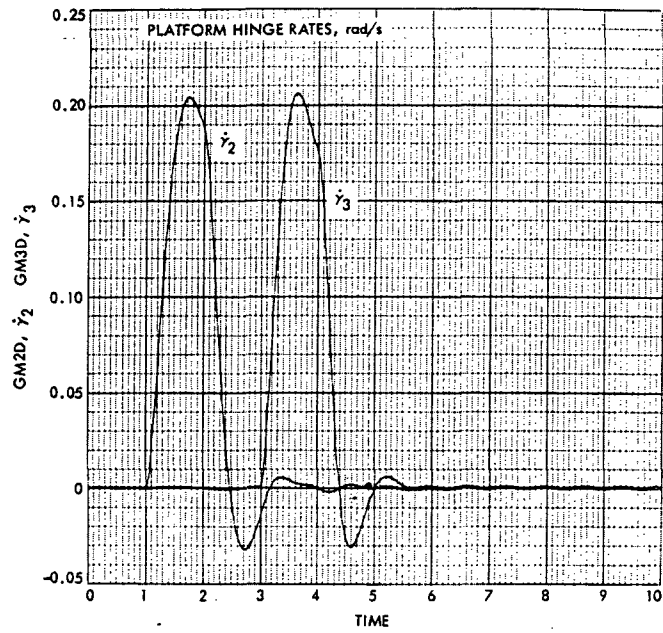
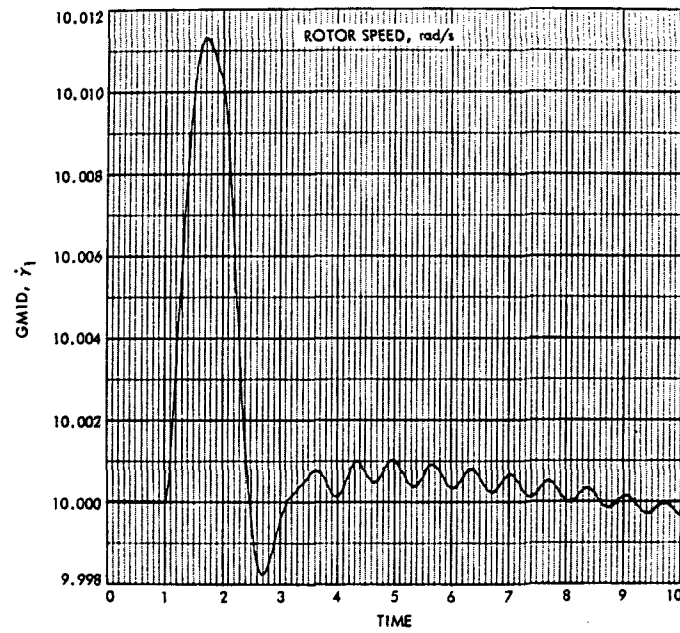
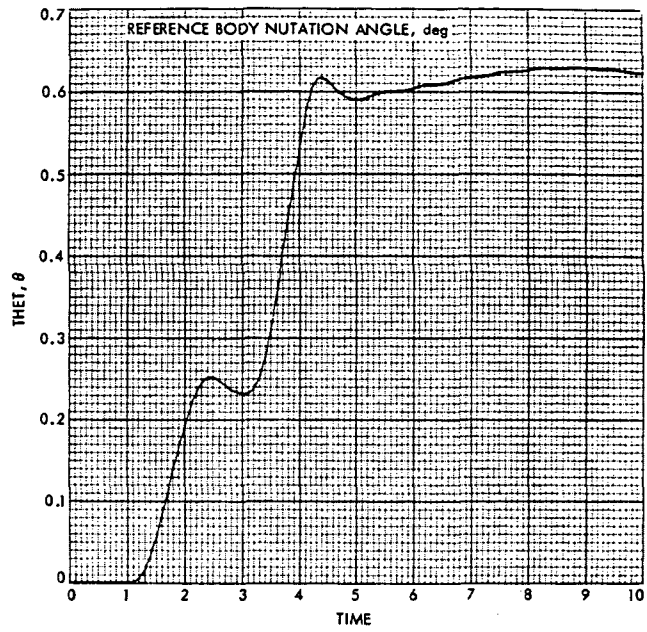
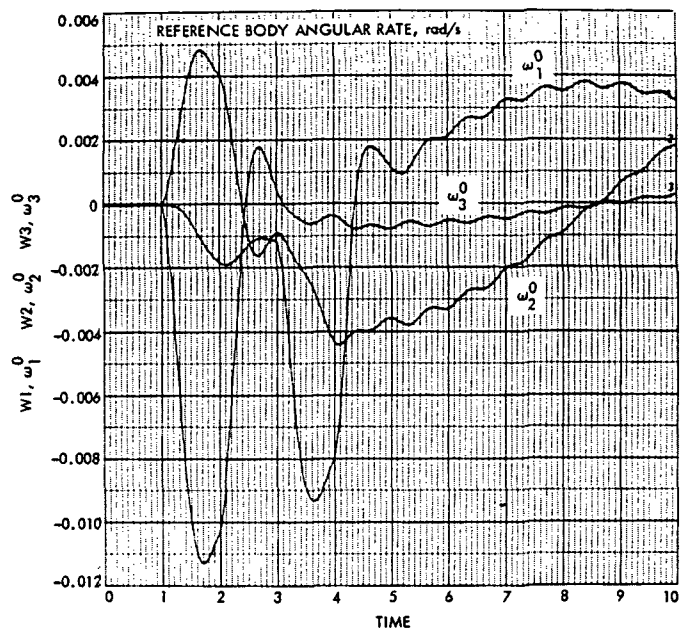


Fig. 8. Test vehicle (with spinning rotor) simulation results using MBDYFR

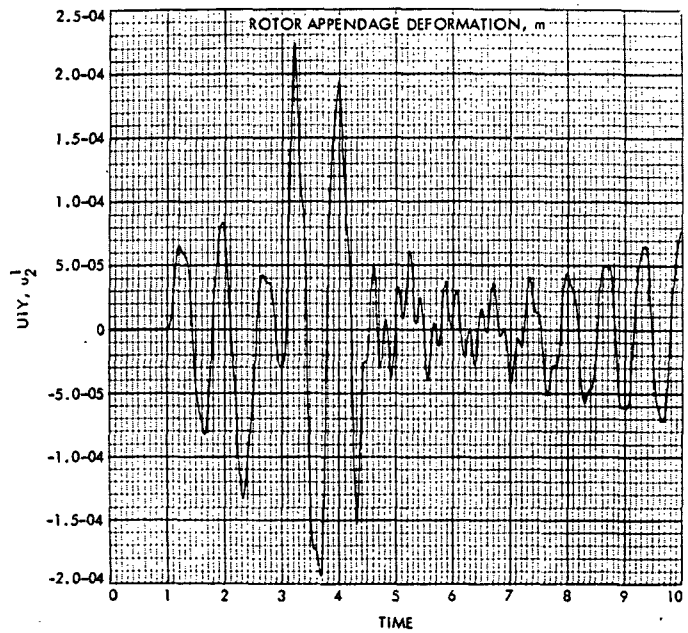
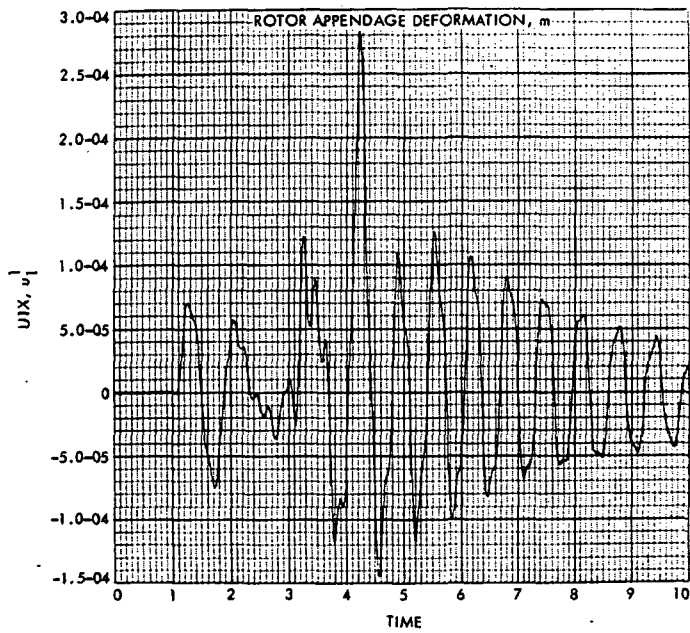
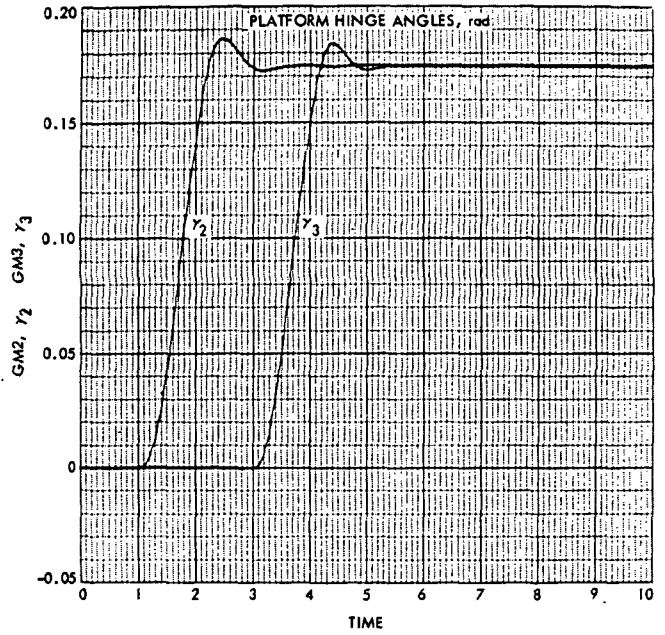
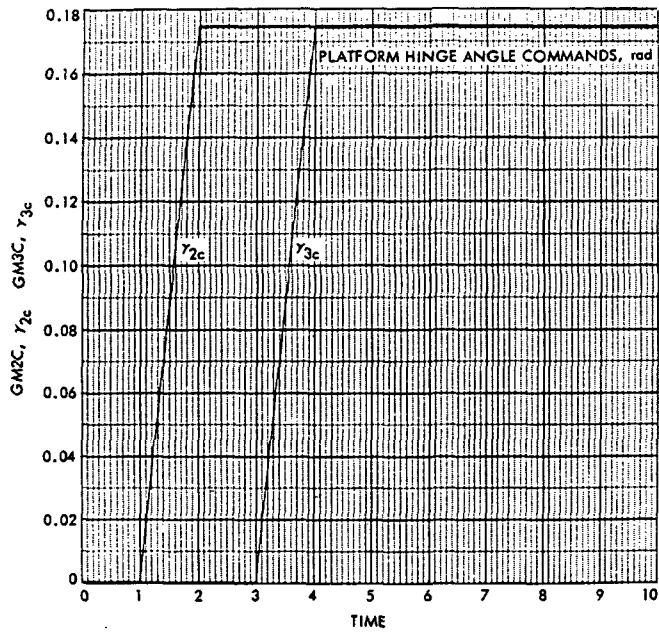


Fig. 8 (contd)

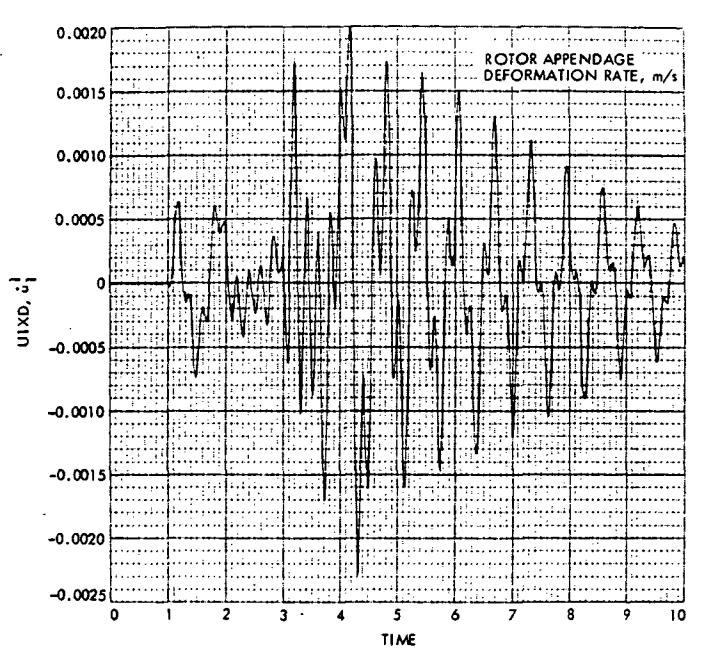
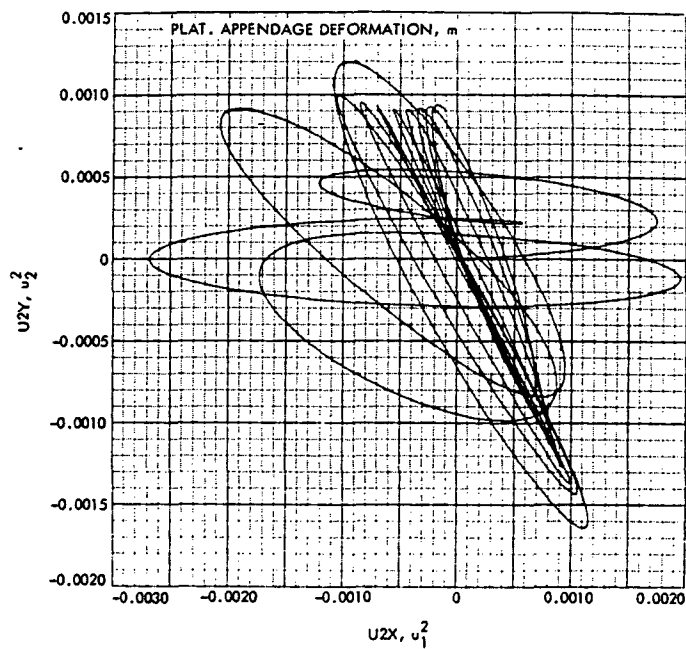
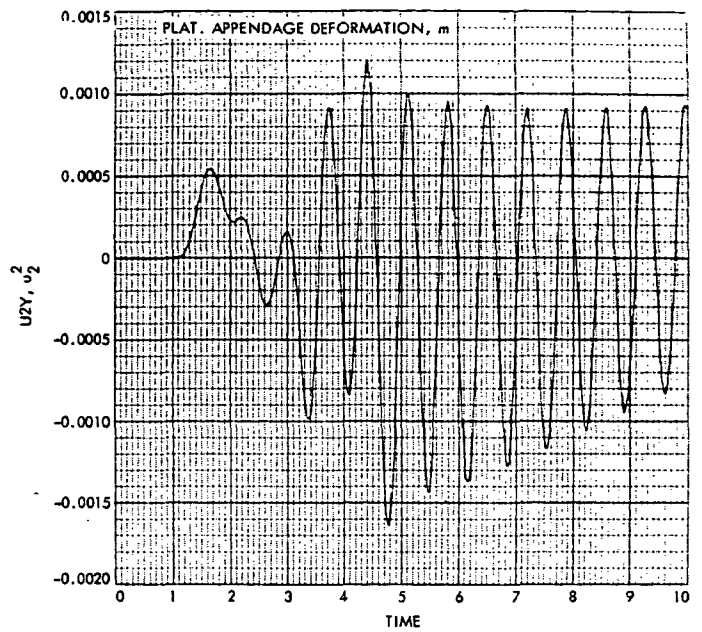
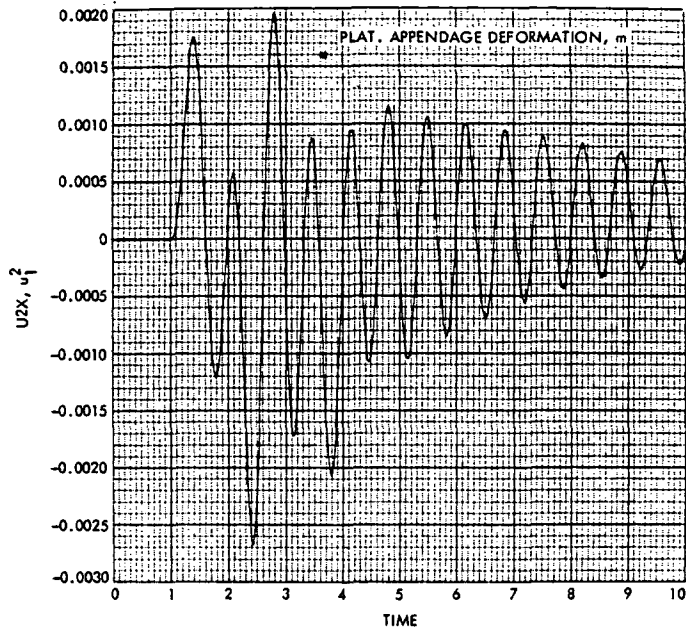


Fig. 8 (contd)

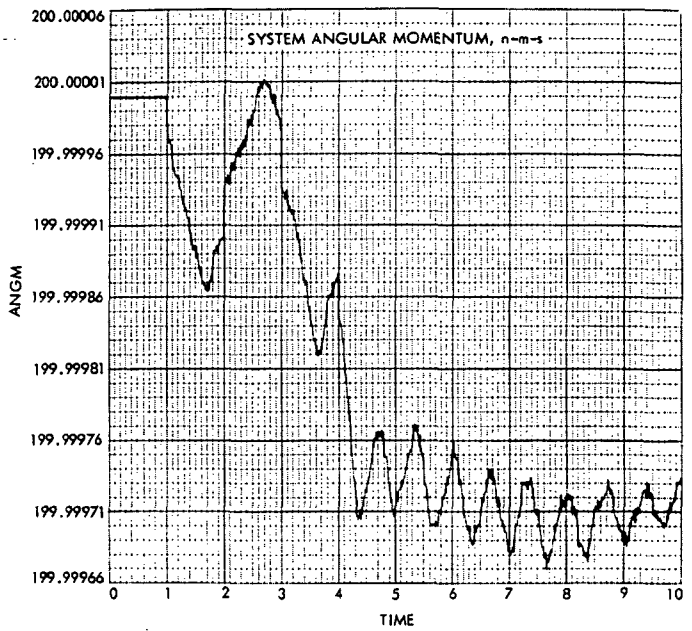
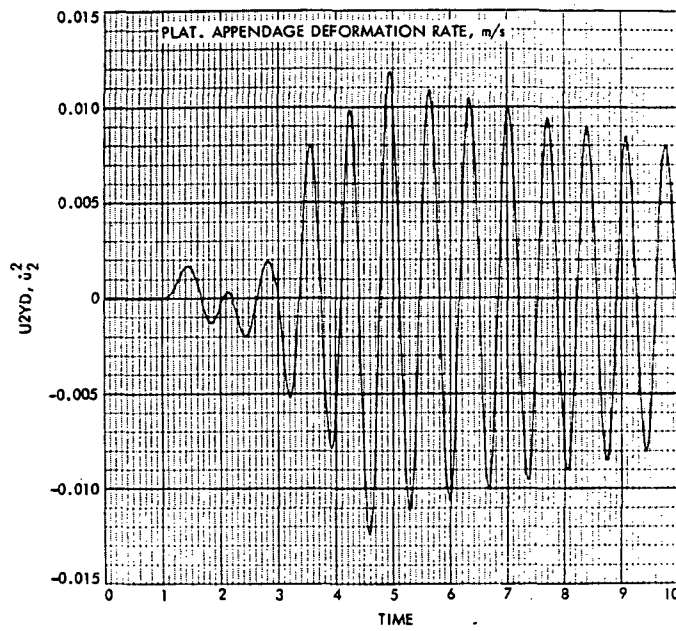
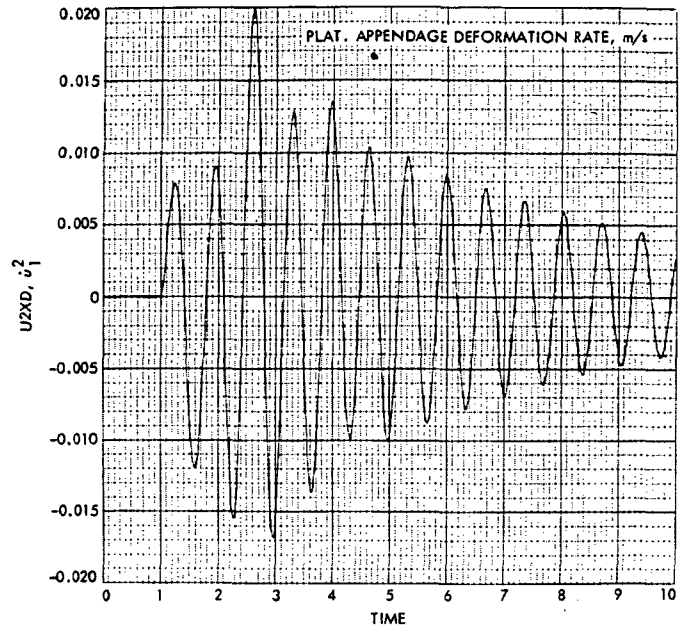
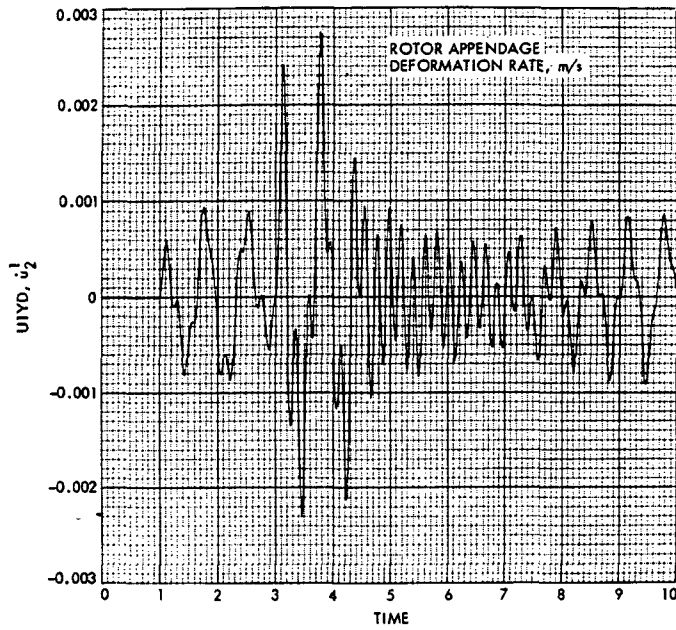


Fig. 8 (contd)

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TIME	9.30000	WID	-1.236511-03	W20	6.946611-04	W30	6.680829-04
		W1	3.556527-03	W2	9.172441-04	W3	1.222779-04
		GM1D	9.99981	GM2D	3.833038-04	GM3D	3.453655-04
		GM1	93.0119	GM2	.174599	GM3	.174527
		THET	.628172	DT1A	5.599171-04	DT1B	4.708252-04
		DT2A	-2.450952-03	DT2B	1.272927-02	ET1A	-2.991362-04
		ET1B	3.456706-05	ET2A	-4.240413-03	ET2B	2.997854-03
		PZO	.999969	PAO	1.258746-03	PBO	-5.335321-03
		PCO	-6.043045-03	UIX	2.258592-05	UIY	5.887238-05
		UIZ	0.000000	UZX	-1.731031-04	UZY	8.990284-04
		UZZ	0.000000	UIXD	1.809007-04	UIYD	1.693549-04
		UZXD	2.727165-03	UZYD	-1.905561-03	GM2C	.174533
		GM3C	.174533	GM2CD	0.000000	GM3CD	0.000000
		ANGM	200.000				
TIME	9.40000	WID	-5.872507-04	W20	1.192746-03	W30	4.019024-04
		W1	3.461807-03	W2	1.007584-03	W3	1.778381-04
		GM1D	9.99971	GM2D	2.937455-04	GM3D	3.525509-04
		GM1	94.0119	GM2	.174635	GM3	.174564
		THET	.627909	DT1A	5.533947-04	DT1B	4.892285-04
		DT2A	3.079498-03	DT2B	5.541556-03	ET1A	-2.872588-04
		ET1B	4.543387-04	ET2A	-7.094131-03	ET2B	1.162703-02
		PZO	.999969	PAO	1.434171-03	PBO	-5.288482-03
		PCO	-6.034436-03	UIX	3.889949-05	UIY	5.916278-05
		UIZ	0.000000	UZX	2.174952-04	UZY	3.913827-04
		UZZ	0.000000	UIXD	2.080791-04	UIYD	-3.536803-04
		UZXD	4.509326-03	UZYD	-7.390628-03	GM2C	.174533
		GM3C	.174533	GM2CD	0.000000	GM3CD	0.000000
		ANGM	200.000				
TIME	9.50000	WID	8.668784-05	W20	1.815489-03	W30	4.299192-07
		W1	3.439150-03	W2	1.160164-03	W3	1.977724-04
		GM1D	9.99973	GM2D	-2.058826-05	GM3D	1.211780-04
		GM1	95.0119	GM2	.174650	GM3	.174589
		THET	.627575	DT1A	2.189948-06	DT1B	-4.258137-04
		DT2A	8.515822-03	DT2B	-5.457229-03	ET1A	1.589932-05
		ET1B	6.865656-04	ET2A	-4.239752-03	ET2B	1.108587-02
		PZO	.999969	PAO	1.606681-03	PBO	-5.235608-03
		PCO	-6.023887-03	UIX	2.693162-05	UIY	-1.724618-05
		UIZ	0.000000	UZX	6.014458-04	UZY	-3.854269-04
		UZZ	0.000000	UIXD	-5.235856-04	UIYD	-8.529636-04
		UZXD	2.694963-03	UZYD	-7.046642-03	GM2C	.174533
		GM3C	.174533	GM2CD	0.000000	GM3CD	0.000000
		ANGM	200.000				

Fig. 9. Simulation printout for test vehicle with spinning rotor

Equation (29) may now be simplified by the assumptions (for  $k \in \mathfrak{F}$ )  $\omega^k \approx 0$ ,  $\dot{\omega}^k \approx 0$ ,  $q^k \approx 0$ ,  $\dot{q}^k \approx 0$ ,  $\ddot{q}^k \approx 0$ ,  $c^k \approx 0$ ,  $\dot{c}^k \approx 0$ , to obtain

$$\begin{aligned}
 (k \in \mathfrak{B}) \quad W^k &= T^k + \sum_{r \in \mathfrak{B}} \bar{D}^{kr} C^{kr} F^r + \left[ \bar{F}^k - \left( C^{k0} \frac{\mathfrak{N}_k}{\mathfrak{N}} F \right)^- \right] c^k \\
 &\quad - \sum_{r \in \mathfrak{F}} \bar{D}^{kr} C^{kr} \Sigma_{U0}^T M^r \ddot{q}^r - \sum_{r \in \mathfrak{B}} \Phi^{kr} C^{kr} \dot{\omega}^r - \dot{h}^k \\
 &\quad - \bar{\omega}^k h^k - \Sigma_{U0}^T \bar{r}_k M^k \ddot{q}^k - \Sigma_{0U}^T M^k \ddot{q}^k \\
 &\quad + \mathfrak{N}_k \bar{c}^k \sum_{r \in \mathfrak{B} - \mathfrak{F}} C^{kr} \bar{\omega}^r \bar{\omega}^r D^{rk} \\
 &\quad + \mathfrak{N} \sum_{r \in \mathfrak{B} - \mathfrak{F}} \bar{D}^{kr} C^{kr} \bar{\omega}^r \bar{\omega}^r D^{rk} - \bar{\omega}^k \Phi^{kk} \omega^k \\
 &\quad - \sum_{r \in \mathfrak{B}} \mathfrak{N}_r \bar{D}^{kr} C^{kr} \bar{c}^r \dot{\omega}^r - \mathfrak{N}_k \bar{c}^k \sum_{r \in \mathfrak{B}} C^{kr} \bar{D}^{rk} \dot{\omega}^r \quad (64)
 \end{aligned}$$

The appendage equation (Eq. 16) may be simplified as well (letting  $R^k = 0$ ) to obtain

$$\begin{aligned}
 (k \in \mathfrak{B}) \quad M^k \left( U - \Sigma_{U0} \Sigma_{U0}^T \frac{M^k}{\mathfrak{N}} \right) \ddot{q}^k + K^k q^k \\
 = -M^k (\Sigma_{0U} - \bar{r}_k \Sigma_{U0}) \dot{\omega}^k - M^k \Sigma_{U0} \sum_{r \in \mathfrak{B}} C^{kr} \bar{\omega}^r D^{rk} \\
 - M^k \Sigma_{U0} C^{k0} \frac{F}{\mathfrak{N}} + \lambda^k + M^k \Sigma_{U0} \sum_{r \in \mathfrak{B} - k} C^{kr} \Sigma_{U0}^T \frac{M^r}{\mathfrak{N}} \ddot{q}^r \\
 - M^k \Sigma_{U0} \sum_{r \in \mathfrak{B} - \mathfrak{F}} C^{kr} \bar{\omega}^r \bar{\omega}^r D^{rk} \quad (65)
 \end{aligned}$$

This appendage equation is analogous to that in Eq. (207) of Ref. 2, whose homogeneous solution has the form

$$q^k = \sum_{j=1}^{6n_k} a_j e^{\lambda_j t} \phi_k^j$$

where  $\lambda_j$  and  $\phi_k^j$  are, respectively, eigenvalues and eigenvectors available from

$$(M' \lambda_j^2 + K') \phi_k^j = 0$$

and

$$M' = M^k \left( U - \Sigma_{U0} \Sigma_{U0}^T \frac{M^k}{\mathfrak{N}} \right)$$

$$K' = K^k$$

If  $\phi_k$  is the  $6n_k$  by  $6n_k$  matrix

$$\phi_k \equiv [\phi_k^1 \phi_k^2 \cdots \phi_k^{6n_k}]$$

the transformation

$$q^k = \phi_k \eta^k \quad (66)$$

may be used to transform Eq. (65) into

$$\ddot{\eta}^k + \sigma_k^2 \eta^k = \phi_k^T L'_k \quad (67)$$

where

$$\begin{aligned} L'_k = & -M^k (\Sigma_{0U} - \bar{r}_k \Sigma_{U0}) \dot{\omega}^k - M^k \Sigma_{U0} \sum_{r \in \mathfrak{B}} C^{kr} \bar{\omega}^r D^{rk} \\ & - M^k \Sigma_{U0} C^{k0} \frac{F}{\mathfrak{M}} + \lambda^k + M^k \Sigma_{U0} \sum_{r \in \mathfrak{B} - k} C^{kr} \Sigma_{U0}^T \frac{M^r}{\mathfrak{M}} \phi_r \bar{\eta}^r \\ & - M^k \Sigma_{U0} \sum_{r \in \mathfrak{B} - \mathfrak{F}} C^{kr} \bar{\omega}^r \bar{\omega}^r D^{rk} \end{aligned}$$

If the modal coordinates  $\eta_1^k, \eta_2^k, \dots, \eta_{6n_k}^k$  are now truncated to the set  $\eta_1^k, \dots, \eta_{N_k}^k$  (as symbolized by the overbar) and modal damping is also incorporated, Eq. (67) becomes

$$\ddot{\bar{\eta}}^k + 2\bar{\xi}_k \bar{\sigma}_k \dot{\bar{\eta}}^k + \bar{\sigma}_k^2 \bar{\eta}^k = \bar{\phi}_k^T L'_k \quad (68)$$

Returning to the vehicle substructure equation, Eq. (64), the truncated modal transformation,  $q^k \approx \bar{\phi}_k \bar{\eta}^k$ , may be substituted and the result combined with Eqs. (2), (3), (5), and (6) to give

$$A^{00} \dot{\omega}^0 + \sum_{j \in \mathfrak{P}} A^{0j} \ddot{\gamma}_j + \sum_{k \in \mathfrak{F}} A^{0k} \ddot{\eta}^k = \sum_{k \in \mathfrak{B}} C^{0k} E^k \quad (69a)$$

$$(i \in \mathfrak{P}) \quad A^{i0} \dot{\omega}^0 + \sum_{j \in \mathfrak{P}} A^{ij} \ddot{\gamma}_j + \sum_{k \in \mathfrak{F}} A^{ik} \ddot{\eta}^k = g^{iT} \sum_{k \in \mathfrak{P}} \epsilon_{ik} C^{ik} E^k + \tau_i \quad (69b)$$

where

$$A^{00} = \sum_{k \in \mathfrak{B}} \sum_{r \in \mathfrak{B}} C^{0k} \Phi^{kr*} C^{r0}, \quad 3 \text{ by } 3$$

$$A^{0j} = \sum_{k \in \mathfrak{B}} \sum_{r \in \mathfrak{P}} C^{0k} \Phi^{kr*} C^{rj} \epsilon_{jr} g^j, \quad 3 \text{ by } 1$$

$$A^{0k} = C^{0k} \left( \bar{\Delta}^{kT} + \sum_{r \in \mathfrak{B}} C^{kr} \bar{D}^{rk} C^{rk} \bar{p}^k \right), \quad 3 \text{ by } N_k$$

$$A^{i0} = g^{iT} \sum_{k \in \mathfrak{P}} \sum_{r \in \mathfrak{B}} C^{ik} \epsilon_{ik} \Phi^{kr*} C^{r0}, \quad 1 \text{ by } 3$$

$$A^{ij} = g^{iT} \sum_{k \in \mathfrak{P}} \sum_{r \in \mathfrak{P}} C^{ik} \epsilon_{ik} \epsilon_{jr} \Phi^{kr*} C^{rj} g^j, \quad 1 \text{ by } 1$$



$$A^{ik} = g^{iT} \left( \epsilon_{ik} C^{ik} \bar{\Delta}^{kT} + \sum_{r \in \mathfrak{B}} \epsilon_{ir} C^{kr} \bar{D}^{rk} C^{rk} \bar{P}^k \right), \quad 1 \text{ by } N_k$$

$$\begin{aligned} E^k &= T^k - \tau_R^k - \bar{\omega}^k g^k (\bar{\omega}^k + \psi_R^k) + \sum_{r \in \mathfrak{B}} \bar{D}^{kr} C^{kr} F^r \\ &+ \left[ \bar{F}^k - \left( C^{k0} \frac{\mathfrak{N}_k}{\mathfrak{N}} F \right) \right] c^k + \mathfrak{N} \sum_{r \in \mathfrak{B} - \mathfrak{F}} \bar{D}^{kr} C^{kr} \bar{\omega}^r \bar{\omega}^r D^{rk} \\ &- \bar{\omega}^k \Phi^{kk} \omega^k - \sum_{r \in \mathfrak{B}} \Phi^{kr*} \sum_{j \in \mathfrak{F}} C^{rj} \epsilon_{jr} \bar{\omega}^j g^j \dot{y}_j \\ &+ \mathfrak{N}_k \bar{c}^k \sum_{r \in \mathfrak{B} - \mathfrak{F}} C^{kr} \bar{\omega}^r \bar{\omega}^r D^{rk}, \quad 3 \text{ by } 1 \end{aligned}$$

$$\Phi^{kr*} = \bar{\Phi}^{kr} C^{kr} + \mathfrak{N}_k \bar{c}^k C^{kr} \bar{D}^{rk} + \mathfrak{N}_r \bar{D}^{kr} C^{kr} \bar{c}^r, \quad 3 \text{ by } 3$$

$$\bar{\Delta}^k = \bar{\Phi}_k^T M^k (\Sigma_{0U} - \bar{r}_k \Sigma_{U0}), \quad N_k \text{ by } 3$$

$$\bar{P}^k = \Sigma_{U0}^T M^k \bar{\Phi}^k, \quad 3 \text{ by } N_k$$

( $\bar{\Phi}^{kr}$  does not include the effects of appendage deformation.)

As in Eqs. (32) and (33), substitutions have been made for  $\dot{h}^k$  and  $h^k$  based on restriction to three orthogonal axisymmetric rotors in  $\mathfrak{L}_k$ , with spin axes aligned to the unit vectors  $\{b^k\}$ , and the relations in Eqs. (43)–(45). Again, it is to be understood that any rotor's moments of inertia are to be included in  $\bar{J}^k$ , the undeformed substructure's inertia dyadic for  $o_k$ , and its mass is included in the substructure mass,  $\mathfrak{N}_k$ .

Operating on the appendage equation, Eq. (68), in a similar way provides

$$(k \in \mathfrak{F}) \quad A^{k0} \bar{\omega}^0 + \sum_{j \in \mathfrak{F}} A^{kj} \dot{y}_j + \sum_{r \in \mathfrak{F}} A^{rk} \ddot{\eta}^r = Q^k \quad (70)$$

where

$$A^{k0} = \bar{\Delta}^k C^{k0} - \bar{P}^{kT} \sum_{r \in \mathfrak{B}} C^{kr} \bar{D}^{rk} C^{r0}, \quad N_k \text{ by } 3$$

$$A^{kj} = \left( \bar{\Delta}^k C^{kj} \epsilon_{jk} - \bar{P}^{kT} \sum_{r \in \mathfrak{B}} C^{kr} \bar{D}^{rk} C^{rj} \epsilon_{jr} \right) g^j, \quad N_k \text{ by } 1$$

$$A^{rk} = -\bar{P}^{kT} C^{kr} \frac{\bar{P}^r}{\mathfrak{N}}, \quad (r \neq k); \quad N_k \text{ by } N_r$$

$$A^{rk} = U, \quad (r = k); \quad N_k \text{ by } N_k$$

$$\begin{aligned}
Q^k &= -2\bar{\xi}_k \bar{\sigma}_k \dot{\bar{\eta}}^k - \bar{\sigma}_k^2 \bar{\eta}^k - \bar{P}^{kT} C^{k0} \frac{F}{\mathcal{J}\mathcal{L}} + \bar{\phi}_k^T \lambda^k \\
&\quad - \sum_{j \in \mathcal{P}} \left( \bar{\Delta}^k C^{kj} \epsilon_{jk} - \bar{P}^{kT} \sum_{r \in \mathcal{B}} C^{kr} \bar{D}^{rk} C^{rj} \epsilon_{jr} \right) \bar{\omega}^j \dot{\bar{\gamma}}_j \\
&\quad - \bar{P}^{kT} \sum_{r \in \mathcal{B} - \mathcal{F}} C^{kr} \bar{\omega}^r \bar{\omega}^r D^{rk}, \quad N_k \text{ by } 1
\end{aligned}$$

where modal damping,  $\bar{\xi}_k$ , has been added (see discussion in Section IIIA).

The substructure and appendage equations may now be combined into a single matrix equation of the form  $A\dot{x} = B$ ,

$$\begin{bmatrix} A^{00} & | & A^{0j} & | & A^{0k} \\ \hline A^{i0} & | & A^{ij} & | & A^{ik} \\ \hline A^{k0} & | & A^{kj} & | & A^{rk} \end{bmatrix} \begin{bmatrix} \dot{\omega}^0 \\ \dot{\gamma}_j \\ \dot{\eta}^k \end{bmatrix} = \begin{bmatrix} \sum_{k \in \mathcal{B}} C^{0k} E^k \\ \hline g^{iT} \sum_{k \in \mathcal{P}} \epsilon_{ik} C^{ik} E^k + \tau_i \\ \hline Q^k \end{bmatrix} \quad (71)$$

Again the elements of  $A$  are, in general, time-variable because of substructure relative motion.  $A$  is also symmetric.

Very often, one can justify making the assumption that *all* the variables, i.e.,  $\omega^0$ ,  $\gamma_j$ ,  $\eta^k$ , and their derivatives are in some sense "small" and a complete linearization of Eq. (71) may be carried out. The computational benefits of a total linearization are quite substantial since the coefficient matrix,  $A$ , then becomes formally constant, allowing its inverse to be computed only once, in advance of numerical integration.

If each symbol in Eq. (71) is expanded into three parts, the first being free of the variables  $\omega^0$ ,  $\gamma_j$ ,  $\eta^k$ , and their derivatives (indicated by overbar), the second being linear in these variables (indicated by overcaret), and the third containing terms above the first degree in the variables (indicated by three dots), and if one then determines explicit expressions for the new barred and caret symbols from their definitions, the linearized form of Eq. (71) becomes

$$\bar{A}^{00} \dot{\omega}^0 + \sum_{j \in \mathcal{P}} \bar{A}^{0j} \dot{\gamma}_j + \sum_{k \in \mathcal{F}} \bar{A}^{0k} \dot{\eta}^k = \sum_{k \in \mathcal{B}} \left[ \bar{C}^{0k} (\bar{E}^k + \hat{E}^k) + \hat{C}^{0k} \bar{E}^k \right] \quad (72a)$$

$$\begin{aligned}
(i \in \mathcal{P}) \quad & \bar{A}^{i0} \dot{\omega}^0 + \sum_{j \in \mathcal{P}} \bar{A}^{ij} \dot{\gamma}_j + \sum_{k \in \mathcal{F}} \bar{A}^{ik} \dot{\eta}^k \\
& = g^{iT} \sum_{k \in \mathcal{P}} \epsilon_{ik} \left[ \bar{C}^{ik} (\bar{E}^k + \hat{E}^k) + \hat{C}^{ik} \bar{E}^k \right] + \bar{\tau}_i + \hat{\tau}_i \quad (72b)
\end{aligned}$$

$$(k \in \mathcal{F}) \quad \bar{A}^{k0} \dot{\omega}^0 + \sum_{j \in \mathcal{P}} \bar{A}^{kj} \dot{\gamma}_j + \sum_{r \in \mathcal{F}} \bar{A}^{rk} \dot{\eta}^r = \bar{Q}^k + \hat{Q}^k \quad (72c)$$

where

$$C^{i0} = \bar{C}^{i0} + \hat{C}^{i0} + \dots$$

$$\tau_i = \bar{\tau}_i + \hat{\tau}_i + \dots$$

$$A^{kj} = \bar{A}^{kj} + \hat{A}^{kj} + \dots$$

$$E^k = \bar{E}^k + \hat{E}^k + \dots$$

etc.,

and

$$\bar{C}^{ij} = \bar{C}^{jr} = U = 3 \text{ by } 3 \text{ identity matrix}$$

$$\hat{C}^{ij} = -\gamma_r \bar{g}_r \quad (r > j)$$

$$\hat{C}^{jr} = \gamma_r \bar{g}_r = (\hat{C}^{ij})^T$$

Specifically,

$$\bar{A}^{00} = \sum_{k \in \mathfrak{B}} \sum_{r \in \mathfrak{B}} \bar{\Phi}^{kr}$$

$$\bar{A}^{0j} = \sum_{k \in \mathfrak{B}} \sum_{r \in \mathfrak{Q}} \bar{\Phi}^{kr} \epsilon_{jr} g^j$$

$$\bar{A}^{0k} = \bar{\Delta}^{kT} + \sum_{r \in \mathfrak{B}} \bar{D}^{rk} \bar{P}^k$$

$$\bar{A}^{i0} = g^{iT} \sum_{k \in \mathfrak{Q}} \sum_{r \in \mathfrak{B}} \epsilon_{ik} \bar{\Phi}^{kr}$$

$$\bar{A}^{ij} = g^{iT} \sum_{k \in \mathfrak{Q}} \sum_{r \in \mathfrak{Q}} \epsilon_{ik} \epsilon_{jr} \bar{\Phi}^{kr} g^j$$

$$\bar{A}^{ik} = g^{iT} \left( \epsilon_{ik} \bar{\Delta}^{kT} + \sum_{r \in \mathfrak{B}} \epsilon_{ir} \bar{D}^{rk} \bar{P}^k \right)$$

$$\bar{E}^k = \bar{T}^k - \bar{\tau}_R^k + \sum_{r \in \mathfrak{B}} \bar{D}^{kr} \bar{F}^r$$

$$\hat{E}^k = \hat{T}^k - \hat{\tau}_R^k - \bar{\omega}^k g^k \psi_R^k + \sum_{r \in \mathfrak{B}} \bar{D}^{kr} \hat{C}^{kr} \bar{F}^r + \sum_{r \in \mathfrak{B}} \bar{D}^{kr} \hat{F}^r + \left[ \bar{F}^k - \left( \frac{\mathfrak{M}_k}{\mathfrak{M}} \bar{F} \right)^- \right] c^k$$

$$\bar{A}^{k0} = \bar{\Delta}^k - \bar{P}^{kT} \sum_{r \in \mathfrak{B}} \bar{D}^{rk}$$

$$\bar{A}^{kj} = \left( \bar{\Delta}^k \epsilon_{jk} - \bar{P}^{k\tau} \sum_{r \in \mathfrak{B}} \bar{D}^{rk} \epsilon_{jr} \right) g^j$$

$$\bar{A}^{rk} = -\bar{P}^{k\tau} \frac{\bar{P}^r}{\mathfrak{N}}, \quad (r \neq k)$$

$$\bar{A}^{rk} = U, \quad (r = k)$$

$$\bar{Q}^k = -\bar{P}^{k\tau} \frac{\bar{F}}{\mathfrak{N}} + \bar{\Phi}_k^T \bar{\lambda}^k$$

$$\hat{Q}^k = -2\bar{\xi}_k \bar{\sigma}_k \hat{\eta}^k - \bar{\sigma}_k^2 \hat{\eta}^k - \bar{P}^{k\tau} \frac{\hat{C}^{k0} \bar{F} + \hat{F}}{\mathfrak{N}} + \bar{\phi}_k^T \hat{\lambda}^k$$

It would remain then to determine  $\bar{T}^k$ ,  $\hat{T}^k$ ,  $\bar{F}^k$ ,  $\hat{F}^k$ ,  $\bar{F}$ ,  $\hat{F}$ ,  $\bar{\lambda}^k$ ,  $\hat{\lambda}^k$ ,  $\bar{\tau}_i$ ,  $\hat{\tau}_i$ , etc., for the particular system under study and to carry out the computations in Eq. (72). However, in constructing a subroutine to perform these computations, it was found to be more efficient to directly manipulate the combined form

$$\bar{A}^{00} \dot{\omega}^0 + \sum_{j \in \mathfrak{P}} \bar{A}^{0j} \ddot{y}_j + \sum_{k \in \mathfrak{F}} \bar{A}^{0k} \ddot{\eta}^k = \sum_{k \in \mathfrak{B}} \hat{C}^{0k} \hat{E}^k \quad (73a)$$

$$(i \in \mathfrak{P}) \quad \bar{A}^{i0} \dot{\omega}^0 + \sum_{j \in \mathfrak{P}} \bar{A}^{ij} \ddot{y}_j + \sum_{k \in \mathfrak{F}} \bar{A}^{ik} \ddot{\eta}^k = g^{i\tau} \sum_{k \in \mathfrak{P}} \epsilon_{ik} \hat{C}^{ik} \hat{E}^k + \hat{\tau}_i \quad (73b)$$

$$(k \in \mathfrak{F}) \quad \bar{A}^{k0} \dot{\omega}^0 + \sum_{j \in \mathfrak{P}} \bar{A}^{kj} \ddot{y}_j + \sum_{r \in \mathfrak{F}} \bar{A}^{rk} \ddot{\eta}^r = \hat{Q}^k$$

where

$$\hat{E}^k = \bar{E}^k + \hat{E}^k$$

$$\hat{C}^{ik} = \bar{C}^{ik} + \hat{C}^{ik}$$

$$\hat{\tau}_i = \bar{\tau}_i + \hat{\tau}_i$$

etc.

By avoiding the separation into the parts  $\bar{E}^k$ ,  $\hat{E}^k$ , etc., the computation becomes more efficient even though some second-order terms in the linearized variables are retained.

## B. Subroutines MBDYFN, MBDYFL

The Fortran V subroutines MBDYFN and MBDYFL were written to provide the solutions to Eqs. (71) and (73), respectively. As in the case of MBDYFR, these routines are also exercised by either of two call statements, the first of which initializes the program with the system constants.

### Initializing Call Statements

CALL MBDYFN(NC, H, MB, MS, PB, PS, G, PI,  
NF, F, EIG, REC, RF, WF, ZF)

or

CALL MBDYFL(NC, H, MB, MS, PB, PS, G, PI,  
NF, F, EIG, REC, RF, WF, ZF)

All the arguments in these call statements are defined exactly as given in IIIB, with the exception of the two new arguments, EIG and REC. Notice that the MBDYFR inputs ER, EI, SR, and MF no longer are used in these routines. The input arrays RF and EIG are used by the subroutine *only* if there are nonzero external forces and torques  $\lambda^k$  applied to an appendage.

EIG( $n, i, j$ ) = array of elements of  $\bar{\phi}_k^j$ ;  $n = 1, 2, \dots, n_f$ ;  $i = 1, 2, \dots, 6n_k$ ;  $k = F(n, 1)$ ;  $j = 1, 2, \dots, N_k$ . (Note! This array is not used by the routine if  $\lambda^k$ , for all  $k \in \mathcal{F}$ , is zero.)

REC( $n, i, j$ ) = array containing the "rigid-elastic coupling coefficients,"  $\bar{\Delta}^k$  and  $\bar{P}^k$ ;  $n = 1, 2, \dots, n_f$ ;  $i = 1, 2, \dots, 6$ ;  $k = F(n, 1)$ ;  $j = 1, 2, \dots, N_k$ . (For  $i = 1, 2, 3$ , the elements of REC are those of  $\bar{P}^k$ ; for  $i = 4, 5, 6$ , the elements are those of  $\bar{\Delta}^{kT}$ .)

In order to compute the angular accelerations  $\dot{\omega}^0, \ddot{\gamma}_1, \dots, \ddot{\gamma}_n$ , and the modal coordinate acceleration vectors  $\ddot{\eta}^k$  ( $k \in \mathcal{F}$ ) at every numerical integration step, the simulation must repeatedly enter the subroutine using the dynamic call statement.

### Dynamic Call Statement

CALL MRATE(NC, TH, TB, TS, FB, FS, TF, FF, GM,  
GMD, GMDD, ET, ETD, WO, WDOT, ETDD, HM)

where

ET( $n, i$ ) = array of appendage modal coordinates,  $\bar{\eta}^k$ ;  $n = 1, \dots, n_f$ ;  
 $k = F(n, 1), i = 1, \dots, N_k$ .

ETD( $n, i$ ) = array of modal coordinate rates,  $\dot{\bar{\eta}}^k$ ;  $n = 1, \dots, n_f$ ;  
 $k = F(n, 1), i = 1, \dots, N_k$ .

ETDD( $n, i$ ) = solution array for modal coordinate accelerations,  $\ddot{\bar{\eta}}^k$ ;  
 $n = 1, \dots, n_f; k = F(n, 1), i = 1, \dots, N_k$ .

and all other arguments are defined exactly as in IIIB.

Again, it should be noted that MBDYFN and MBDYFL do not incorporate the terms in  $E^k$  that describe rotor torques on  $\mathcal{A}_k$ . The user must include these terms, if rotors are present, in  $T^k$  (or  $\hat{T}^k$ ) as it is formed in the main program.

Also, if any of the  $\gamma_i$  are to be *prescribed*, appropriate values of  $\ddot{\gamma}_i$ , as well as  $\dot{\gamma}_i$  and  $\gamma_i$ , must be supplied to the subroutine by way of the MRATE dummy arguments GMDD, GM, and GMD, respectively.

When either the MBDYFN or the MBDYFL subroutine is used, the main calling program must contain Fortran "type" and storage allocation statements. The mandatory statements are:

*Required Specification Statements*

INTEGER NC, NF, H( $n_c, 2$ ), F( $n_f, 3$ ), PI( $n + 1$ )

REAL MB(7), MS( $n_c, 7$ ), PB( $n_c, 3$ ), PS( $n_c, n_c, 3$ ),

G( $n, 3$ ), TH( $n$ ), TB(3), TS( $n_c, 3$ ), FB(3), FS( $n_c, 3$ ),

GM( $n$ ), GMD( $n$ ), GMDD( $n$ ), EIG( $n_f, 6n_k, N_k$ ), REC( $n_f, 6, N_k$ ),

RF( $n_f, n_k, 3$ ), WF( $n_f, N_k$ ), ZF( $n_f, N_k$ ),

TF( $n_f, n_k, 3$ ), FF( $n_f, n_k, 3$ ), ET( $n_f, N_k$ ),

ETD( $n_f, N_k$ ), WO(3)

DOUBLE PRECISION WDOT( $n + 3$ ), ETDD( $n_f, N_k$ )

In order that storage allocation for arrays internal to MBDYFN and MBDYFL be minimized, the following statement must appear in the subroutine:

PARAMETER QH =  $n$ , QC =  $n_c$ , QF =  $n_f$ , NK =  $n_k$ , NKT =  $N_k$

The proper placement of this statement in MBDYFN and MBDYFL is shown in their listing (Appendices D and E).

**C. Sample Problems**

To illustrate the use of subroutines MBDYFN and MBDYFL, a sample problem suitable for computer simulation will be described. The test vehicle to be simulated has the configuration shown in Fig. 10—a rigid central body,  $\mathcal{B}_0$ , a rigid platform,  $\mathcal{B}_1$ , which is hinged to  $\mathcal{B}_0$  (2 degrees of freedom), and a flexible appendage,  $\mathcal{A}_0$ , also attached to  $\mathcal{B}_0$ .

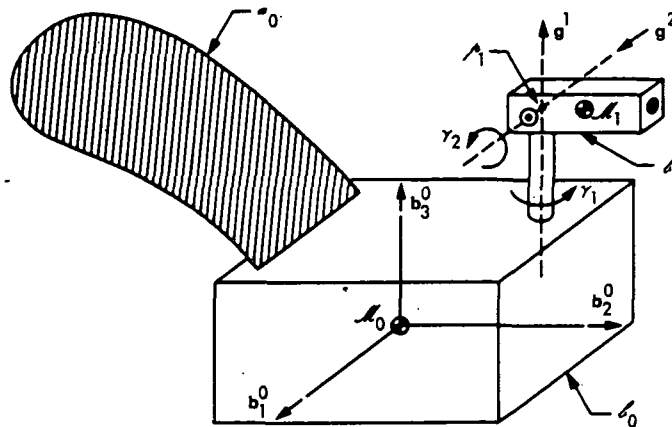


Fig. 10. MBDYFN, MBDYFL simulation test vehicle

For this example, the numbers used to describe the test vehicle's mass properties, including the appendage, were taken from an actual spacecraft design. The appendage model includes the characteristic vibration modes of four solar panels, a parabolic antenna, and several other structural members.

### Test Vehicle Constants

The following numerical constants are required for initializing the subroutines:

$$\mathcal{M}_0 = 79.0 \text{ kg}$$

$$\mathcal{M}_1 = 1.93 \text{ kg}$$

$$\bar{J}^0 = \begin{bmatrix} 1230. & 16.29 & 43.45 \\ & 1290. & -61.75 \\ \text{sym.} & & 1650. \end{bmatrix} \text{ kg-m}^2$$

$$\bar{J}^1 = \begin{bmatrix} 4.75 & 0. & 0. \\ & 5.53 & 0. \\ \text{sym.} & & 1.32 \end{bmatrix} \text{ kg-m}^2$$

Let the modal model for appendage  $\alpha_0$  ( $\mathcal{Q}_1$ ) be truncated to seven modes, i.e.,  $N_0 = 7$ . Thus,

$$\bar{P}^0 = \sum_{U_0}^T M^0 \bar{\phi}_0 =$$

$$\begin{bmatrix} .0338 & | & .0106 & | & .0023 & | & .0032 & | & -.6055 & | & -.3050 & | & -.0276 \\ .0017 & | & .0011 & | & -.0182 & | & .0010 & | & -.5381 & | & 1.753 & | & .1051 \\ -.8678 & | & -.00005 & | & 0. & | & 2.234 & | & 1.962 & | & .5585 & | & .3919 \end{bmatrix}$$

kg-m

$$\bar{\Delta}^{0T} = (\sum_{U_0}^T + \sum_{U_0}^T \bar{r}_0) M^0 \bar{\phi}_0 =$$

$$\begin{bmatrix} .0814 & | & .4236 & | & 21.30 & | & -.4081 & | & 7.577 & | & -4.320 & | & 2.032 \\ 17.17 & | & 12.30 & | & -.2386 & | & 5.930 & | & .4020 & | & -.1589 & | & -2.061 \\ .0080 & | & .0019 & | & .0009 & | & -.0521 & | & 2.520 & | & -.9205 & | & .2761 \end{bmatrix}$$

kg-m<sup>2</sup>

$$\bar{\sigma}_0 = 2\pi [.5756 \quad .6134 \quad .6134 \quad .6307 \quad 2.723 \quad 2.963 \quad 3.047]^T \text{ rad/s}$$

$$\bar{\xi}_0 = [.20 \quad .20 \quad .20 \quad .20 \quad .05 \quad .05 \quad .01]^T$$

Also, let

$$g^1 = [0. \quad 0. \quad 1.]^T$$

$$g^2 = [1. \quad 0. \quad 0.]^T$$

$$p^{01} = [0. \quad 0. \quad 0.]^T, \quad p^{10} = [0. \quad 0. \quad 0.]^T$$

Since no external forces or torques will be applied to the appendage, the eigenvector matrix  $\bar{\phi}_0$  is not needed, nor is the matrix  $\bar{r}_0$ . Finally,

$$n_c = 1, \quad n_f = 1, \quad n_0 = 1, \quad N_0 = 7$$

$$h_1 = 0, \quad d_1 = 2, \quad n = 2$$

The integer  $n_0$ , which indicates the number of sub-bodies in the appendage model and is only required if external forces and torques are applied to appendage  $\alpha_0$ , has been set to the smallest acceptable value that satisfies dimensioning requirements.

The initializing call statement arguments therefore become

$$NC = 1$$

$$H = [0 \quad 2]$$

$$MB = [1230. \quad 1290. \quad 1650. \quad -16.29 \quad -43.45 \quad 61.75 \quad 79.0]$$

$$MS = [4.75 \quad 5.53 \quad 1.32 \quad 0. \quad 0. \quad 0. \quad 1.93]$$

$$PB = 0$$

$$PS = 0$$

$$G = \begin{bmatrix} 0. & 0. & 1. \\ 1. & 0. & 0. \end{bmatrix}$$

$$PI = [0 \quad 0 \quad 1] \quad (\text{assuming no prescribed hinge motions})$$

$$NF = 1$$

$$F = [0 \quad 1 \quad 7]$$

$$EIG = 0$$

REC =

$$\begin{bmatrix} .0338 & | & .0106 & | & .0023 & | & .0032 & | & -.6055 & | & -.3050 & | & -.0276 \\ .0017 & | & .0011 & | & -.0182 & | & .0010 & | & -.5381 & | & 1.753 & | & .1051 \\ -.8678 & | & -.00005 & | & 0. & | & 2.234 & | & 1.962 & | & .5585 & | & .3919 \\ .0814 & | & .4236 & | & 21.30 & | & -.4081 & | & 7.577 & | & -4.320 & | & 2.032 \\ 17.17 & | & 12.30 & | & -.2386 & | & 5.930 & | & .4020 & | & -.1589 & | & -2.061 \\ .0080 & | & .0019 & | & .0009 & | & -.0521 & | & 2.520 & | & -.9205 & | & .2761 \end{bmatrix}$$



$$\begin{aligned} \text{WF} &= 2\pi [.5756 \quad .6134 \quad .6134 \quad .6307 \quad 2.723 \quad 2.963 \quad 3.047]^T \\ \text{ZF} &= [.20 \quad .20 \quad .20 \quad .20 \quad .05 \quad .05 \quad .01] \end{aligned}$$

### Test Vehicle Dynamics

As before, the platform hinge connections will be defined as being of the linear spring and viscous damper type, but the position commands will be deleted, so that

$$\tau_1 = -K_1\gamma_1 - B_1\dot{\gamma}_1$$

$$\tau_2 = -K_2\gamma_2 - B_2\dot{\gamma}_2$$

where

$$K_1 = 900. \text{ n-m/rad}$$

$$K_2 = 850. \text{ n-m/rad}$$

$$B_1 = 100. \text{ n-m-s/rad}$$

$$B_2 = 100. \text{ n-m-s/rad}$$

The vehicle response to be simulated in this example will be that due to an arbitrary sequence of force and torque pulses applied to the reference body,  $\mathcal{L}_0$ . A rectangular pulse of thrust will be applied in the  $b_3^0$  direction with magnitude 300 n and a duration of 2 s, starting at  $t = .5$  s. This will be followed by a 1-s torque pulse in the  $b_1^0$  direction of magnitude 10. n-m, starting at  $t = 3.5$  s. And the last disturbance will be a 1-s torque pulse in the  $b_2^0$  direction of magnitude 10. n-m, starting at  $t = 6.5$  s. The computer program for this dynamic simulation is given in Fig. 11.

Initially, the system is assumed to be completely at rest. Again, the CSSL III language function, "STEP," is used to construct the applied pulses. Only the angular rates of  $\mathcal{L}_0$  are calculated in this example; its inertial angular position is not computed. Appendage modal coordinate rates and positions are both provided, although only the rates are plotted in the system responses of Fig. 12. A sample of the printed output is shown in Fig. 13.

Notice that by far the greatest disturbing effect to both platform and flexible appendage is due to the applied force. However, the changes in  $\omega^0$  magnitude due to the torque disturbances are quite significant. It is not clear to what extent the platform vibrations are coupling with appendage vibrations and reference body motion, although the platform rotations are small in magnitude.

It is apparent that the applied force (fixed with respect to  $\mathcal{L}_0$ ) caused some slight accumulation of system angular momentum as the system mass center moved in response to platform and appendage vibrations. This small amount (.17 n-m-s) was dwarfed, however, by the next pulse of torque, so that after 4.5 s, the angular momentum should have been approximately 10 n-m-s. The last torque pulse, applied *orthogonally* to the preceding one, would then raise the total angular momentum magnitude to slightly more than  $\sqrt{(10)^2 + (10)^2} = 14.14$  n-m-s. The simulation printout shows a computed value of 14.25 n-m-s.

CSSL III JET PROPULSION LABORATORY 040374-A002H 021775-021424

\*\*\* START T(RUN)= 18.518 T(TASK)= .003 CTP P .544  
DT(TASK)= .003 DCTP S .544

PROGRAM 2-BODY VEHICLE WITH FLEX. APPENDAGE  
\*SC4020 BLDG/198,BOX/601,CAMERA/9IN,FRAMES/50

COMMENT

ARRAY MB(7),MS(1,7),PB(3),PS(1,1,3),G(2,3)  
ARRAY EIG(1,6,7),RF(1,1,3),REC(1,6,7),WF(1,7),ZF(1,7)  
ARRAY TB(3),TS(1,3),FB(3),FS(1,3),GM(2),GMD(2),GMDD(2)  
ARRAY TH(2),WO(3),TF(1,1,3),FF(1,1,3),ET(1,7),ETD(1,7)  
DOUBLE PRECISION WDOT(5),ETDD(1,7)  
INTEGER NC,NF,M(1,2),F(1,3),PI(3),L

DATA H(1,1)/0/H(1,2)/2/P1/0/0.1/

DATA F(1,1)/0/F(1,2)/1/F(1,3)/7/

DATA MB/1230.1290.1650.16.29.-43.45.61.75.79.0/

DATA MS/9.75.5.53.1.32.0.0.0.0.1.93/

DATA G(1,3)/1./G(2,1)/1./

DATA REC/.03375.001654.-.8478.00138.17.17.007955.00  
01055.001104.-.4608E-4.4236.12.3.001859.00  
002335.001818.4731E-5.21.3.-.2386.000918.00  
003244.001014.2.234.-.4081.5.930.-.05211.00  
06055.05381.1.942.7.577.4020.2.520.00  
03050.1.753.5585.-4.32.1.589.-.9205.00  
02762.1051.3919.2.032.2.061.2761/

DATA WF/.5756.061337.061337.63071.2.723.2.963.3.047/

DATA ZF/.20.20.20.20.05.05.01/

CONSTANT FINTIM=10. CLKTIM=900. PIE=3.14159265

CONSTANT KI=900. BI=100. K2=850. B2=100.

INITIAL

NC=1 S NF=1

DO 57 L=1,7

S7. WF(1,L)=WF(1,L)\*2.\*PIE

CALL MBDYFN(NC,M,MB,MS,PB,PS,G,PI,NF,F,EIG,REC,RF,WF,ZF)

END

DYNAMIC

IF(TIME.GT.FINTIM) GO TO FIN

STPCLK CLKTIM

OUTPUT IO,W1,W2,W3,NX,NY,FZ,ETA1,ETA2,ETA3,ETA4,ETA5,ETA6,ETA7,...

ETD1,ETD2,ETD3,ETD4,ETD5,ETD6,ETD7,ANGM,W1D,W2D,W3D,...

GM1,GMD,GH2,GH2D

PREPAR W1,W2,W3,NX,NY,FZ,ETD1,ETD2,ETD3,ETD4,ETD5,ETD6,ETD7,...

ANGM,GM1,GH2,GH1D,GH2D

DERIVATIVE BODY2F

VARIABLE TIME=0. S CINTERVAL CI=.01

XERROR W1=1.E-6 S MERROR W1=1.E-6

NOSORT

GMD(1)=GM1D S GM(1)=GM1

GMD(2)=GM2D S GM(2)=GM2

ET(1,1)=ETA1 S ET(1,2)=ETA2 S ET(1,3)=ETA3 S ET(1,4)=ETA4

ET(1,5)=ETA5 S ET(1,6)=ETA6 S ET(1,7)=ETA7

ETD(1,1)=ETD1 S ETD(1,2)=ETD2 S ETD(1,3)=ETD3 S ETD(1,4)=ETD4

ETD(1,5)=ETD5 S ETD(1,6)=ETD6 S ETD(1,7)=ETD7

WO(1)=W1 S WO(2)=W2 S WO(3)=W3 S ANGM=HM

COMMENT...

HINGE TORQUES

Fig. 11. Simulation program for test vehicle dynamics using MBDYFN

```

COMMENT
  TH(1)=-K1*GM1 - B1*GM1D
  TH(2)=-K2*GM2 - B2*GM2D
COMMENT...
      FORCE EQUATION
COMMENT
  FZ=(STEP(1.5,TIME)-STEP(2.5,TIME))*300.
  FB(3)=FZ
COMMENT...
      ENGINE TORQUE
COMMENT
  NX=(STEP(3.5,TIME)-STEP(4.5,TIME))*10.
  NY=(STEP(6.5,TIME)-STEP(7.5,TIME))*10.
  TB(1)=NX  S  TB(2)=NY
COMMENT...
      SOLUTION FOR SYSTEM ACCELERATIONS
COMMENT
  CALL HRATE(INC,TH,TB,TS,FB,FS,TF,FF,GM,GMDD,ET,ETD,W0,WDOT,...
  ETDD,MM)
  W1D=WDOT(1)  S  W2D=WDOT(2)  S  W3D=WDOT(3)
COMMENT...
      SYSTEM RATES AND POSITIONS
COMMENT
  W1=INTEG(WDOT(1),0.)
  W2=INTEG(WDOT(2),0.)
  W3=INTEG(WDOT(3),0.)
  ETD1=INTEG(ETDD(1,1),0.)  S  ETA1=INTEG(ETD1,0.)
  ETD2=INTEG(ETDD(1,2),0.)  S  ETA2=INTEG(ETD2,0.)
  ETD3=INTEG(ETDD(1,3),0.)  S  ETA3=INTEG(ETD3,0.)
  ETD4=INTEG(ETDD(1,4),0.)  S  ETA4=INTEG(ETD4,0.)
  ETD5=INTEG(ETDD(1,5),0.)  S  ETA5=INTEG(ETD5,0.)
  ETD6=INTEG(ETDD(1,6),0.)  S  ETA6=INTEG(ETD6,0.)
  ETD7=INTEG(ETDD(1,7),0.)  S  ETA7=INTEG(ETD7,0.)
  GM1D=INTEG(WDOT(4),0.)  S  GM1=INTEG(GM1D,0.)
  GM2D=INTEG(WDOT(5),0.)  S  GM2=INTEG(GM2D,0.)

END
END
END
TERMINAL
FIN,, CONTINUE
END
END

```

Fig. 11 (contd)

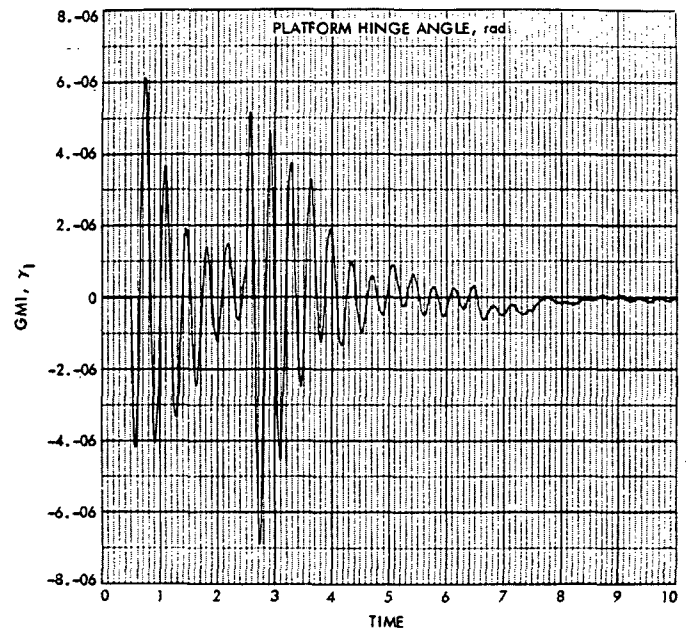
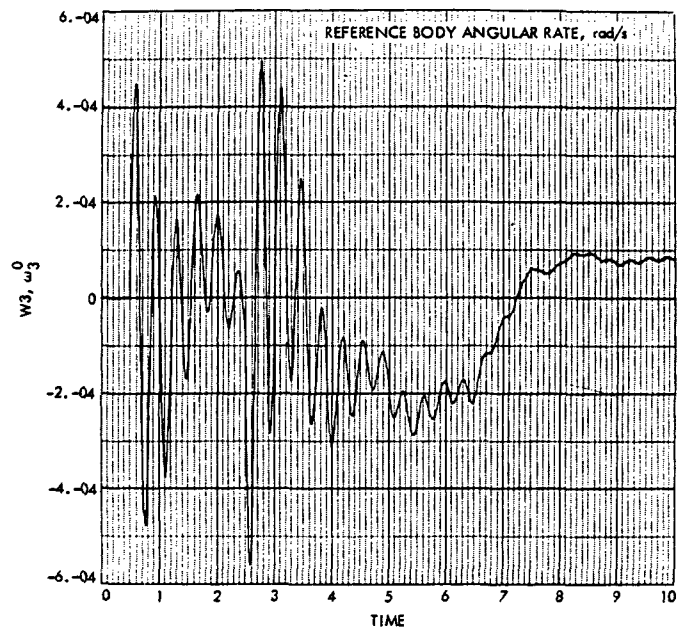
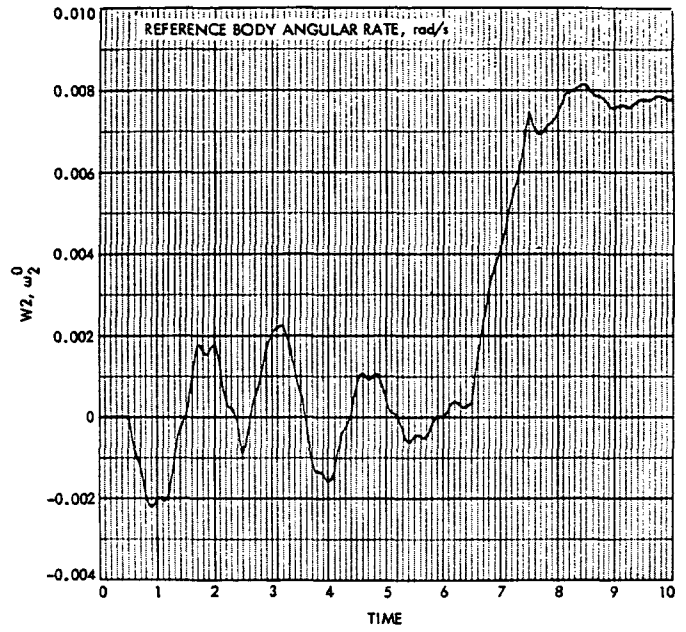
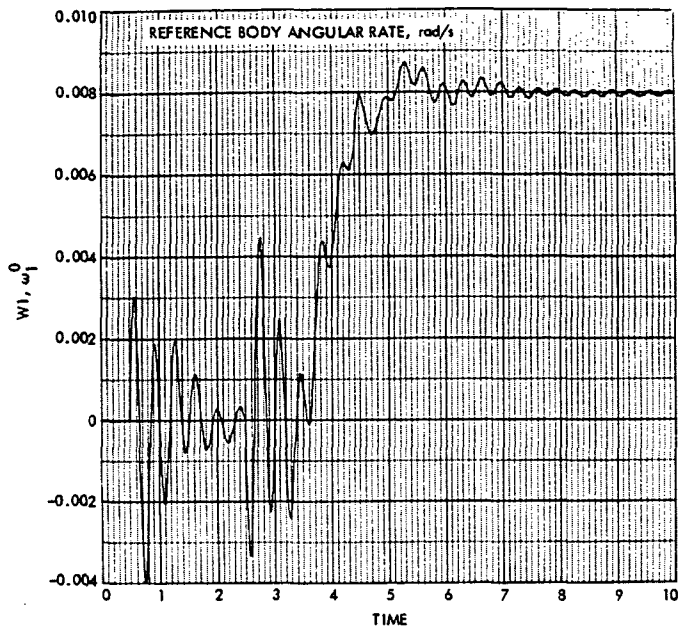


Fig. 12. Test vehicle simulation results using MBDYFN

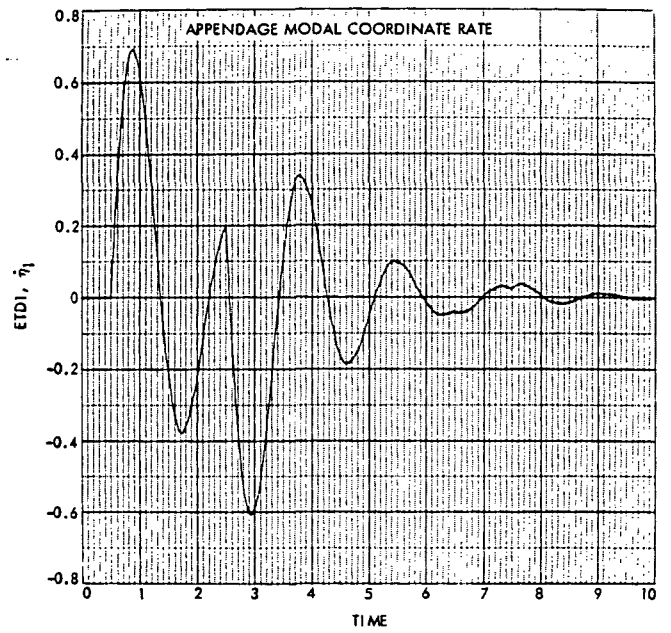
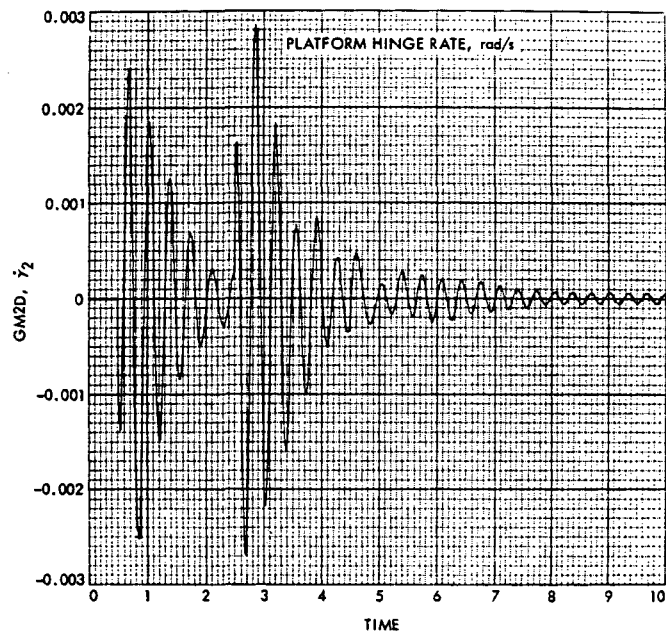
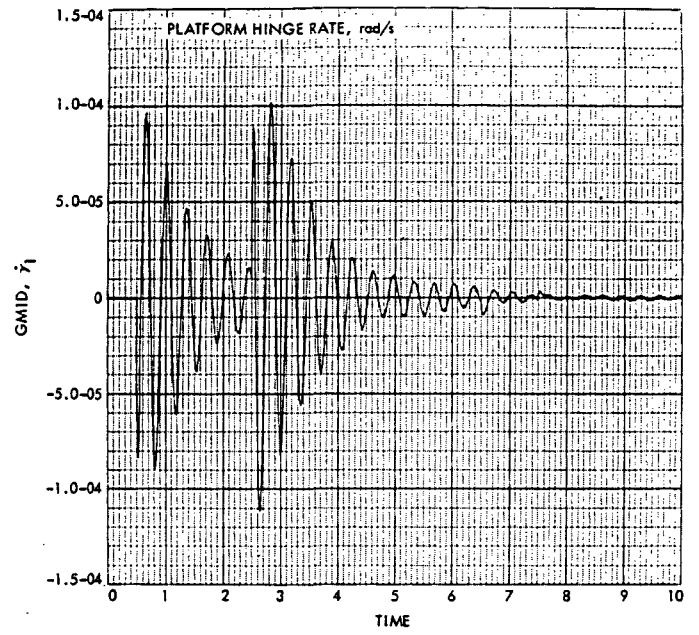
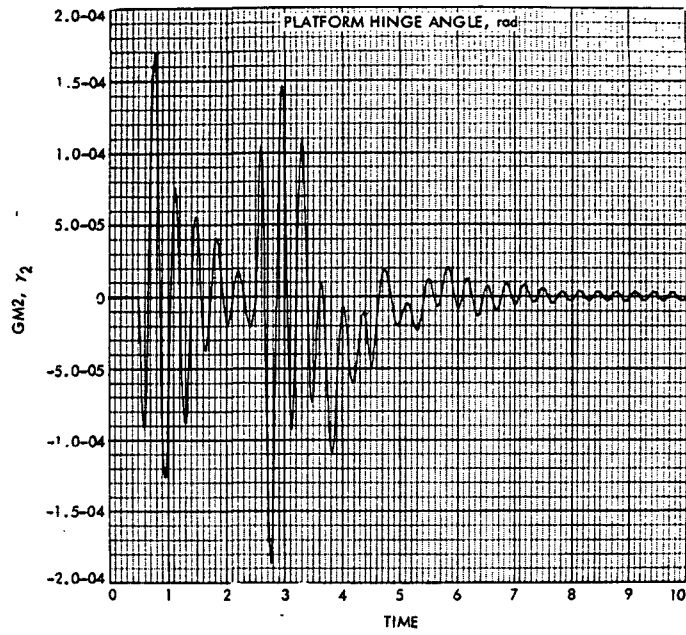


Fig. 12 (contd)

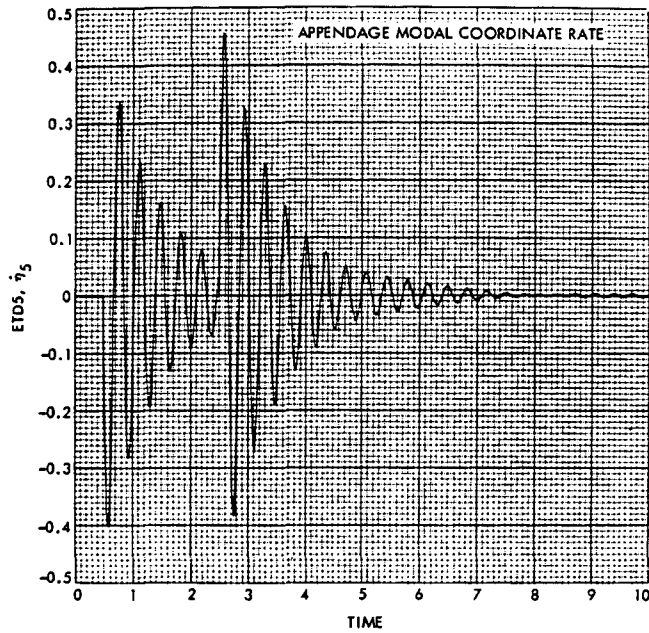
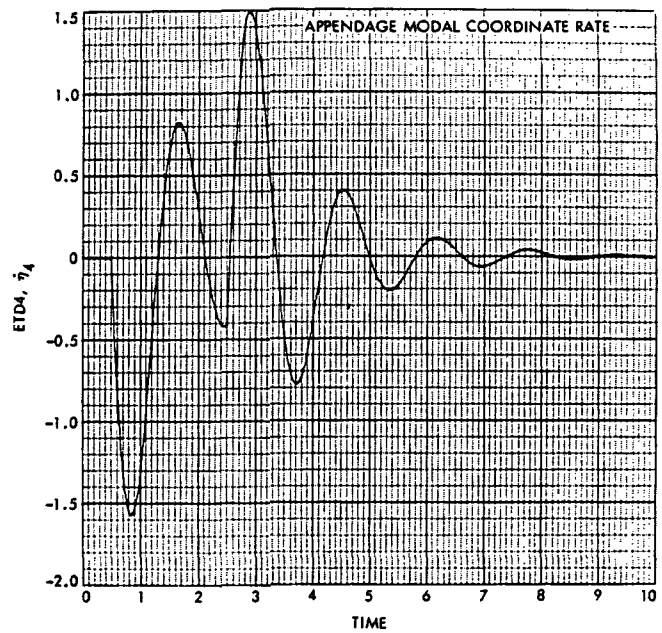
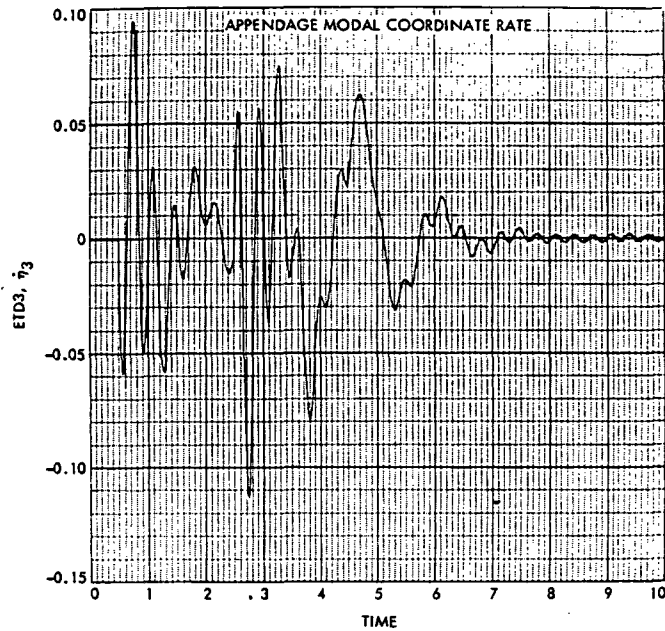
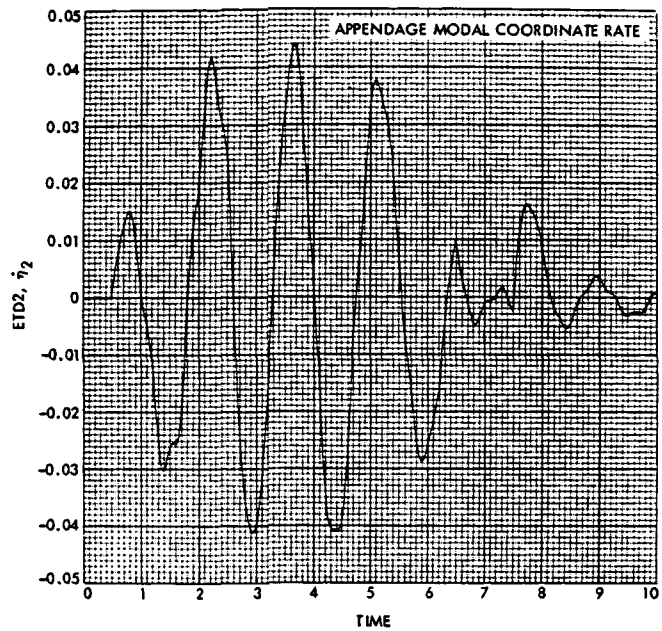


Fig. 12 (contd)

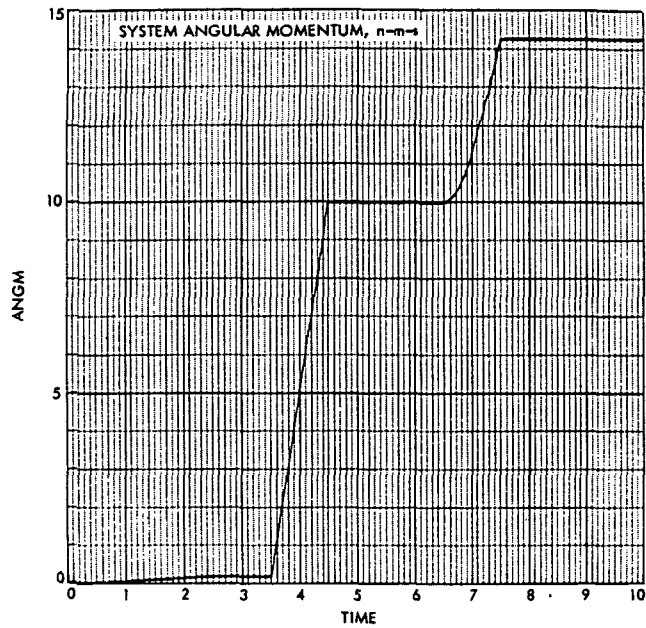
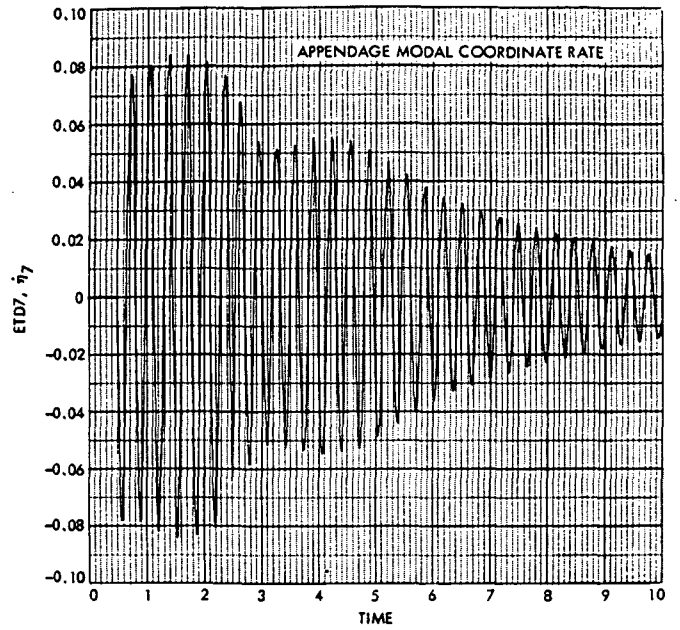
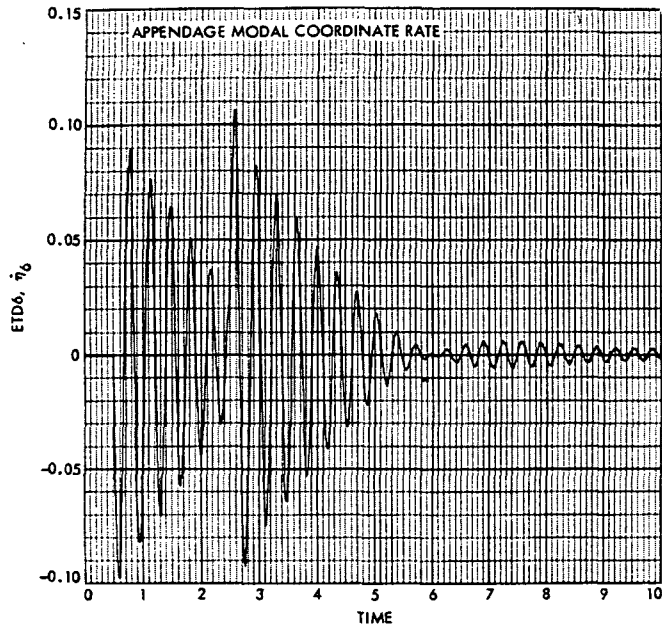


Fig. 12 (contd)

Exactly the same simulation can be made using the linearized subroutine version, MBDYFL. The only change necessary in the simulation program of Fig. 11 to allow the use of the linearized version is the change of "CALL MBDYFN(NC, . . .)" to "CALL MBDYFL(NC, . . .)" in the initialization section. This was done and, resulted in solutions for the system response which are virtually indistinguishable from those plotted in Fig. 12. However, some slight deviations are detectable in the printed output shown in Fig. 14 when compared with the MBDYFN results of Fig. 13. The major difference between the two simulations in this case is reflected in the computer running time. A total of 2 min of accountable central processor time (Univac 1108) was required by the program using MBDYFN as contrasted with only 1 min of central processor time used by the MBDYFL program. In addition, memory storage is considerably reduced by the use of MBDYFL, so that the overall cost of producing the desired solutions in this case is significantly reduced.

Another convenient method of reducing computation time and therefore cost under certain circumstances is to use these subroutines' prescribed variable option. By setting  $PI(i) = 1$ , the hinge angle variables  $\gamma_i$ ,  $\dot{\gamma}_i$ , and  $\ddot{\gamma}_i$  may be prescribed, i.e., defined by the user in the main program rather than computed within the subroutine. When this is done, any expression in the main program defining the hinge torque  $\tau_i(TH(i))$  is ignored by the subroutine. The equations normally solved by the subroutine to obtain  $\ddot{\gamma}_i$  are then deleted from consideration, thus reducing the system order and speeding up calculations.

For an example of this approach, we can return to the program of Fig. 11, using MBDYFN, and change PI so that  $PI(1) = 1$  and  $PI(2) = 1$  (leaving  $PI(3) = 1$  unchanged so that the angular momentum calculation is still performed), as shown in Fig. 15. This means that the platform hinge rotations are to be prescribed. However, by not defining any function for GMDD(1) and GMDD(2), these variables remain zero, as will their integrals. Thus, the simulation will proceed as before but with  $\ddot{\gamma}_i = \dot{\gamma}_i = \gamma_i = 0$  ( $i = 1, 2$ ); i.e., the platform will be "frozen" or rigidly connected to  $\phi_0$ .

The system response (with identical disturbances) in this configuration was simulated, and the plotted results were indistinguishable from those in Fig. 13. A sample of the simulation's printed output, shown in Fig. 16, indicates clearly that "freezing" the platform has had no significant effect on the dynamic response of the reference body or the appendage modal coordinates. However, some numerical differences are discernible in the printout.

Thus, prescribing the platform's "motion" in this case did not appreciably change the overall result and, as a matter of fact, took 15 s less computation time than the original run with no prescribed variables, a saving of  $\frac{1}{8}$ .

## V. Summary and Conclusions

N76 12095

In this report, detailed mathematical models have been developed, suitable for describing the attitude dynamics of vehicles that may be idealized as systems of interconnected rigid bodies with possible terminal flexible appendages. The resulting mathematical formulations apply to two kinds of system behavior: (1) generally arbitrary rigid-body rotations with the restriction that appendage base body deviations from some nominal constant spin rate are small, and (2) unrestrained rigid-body rotations with the restriction that appendage base motion deviations



TIME = 2.20000	W1 = -5.534412-04	W2 = 4.144790-04	W3 = -6.254046-05
	NX = 0.000000	NY = 0.000000	FZ = 300.000
	ETA1 = .175147	ETA2 = -1.585589-03	ETA3 = 3.189027-03
	ETA4 = -.388847	ETA5 = -2.479597-02	ETA6 = -5.228946-03
	ETA7 = -4.007270-03	ETD1 = -1.014957-02	ETD2 = 4.175284-02
	ETD3 = 1.388555-02	ETD4 = -.153645	ETD5 = 7.823737-02
	ETD6 = 3.289947-02	ETD7 = -7.956042-02	ANGM = .147617
	W1D = 5.287033-04	W2D = -5.007478-03	W3D = -2.201360-04
	GM1 = 1.434090-06	GM1D = -5.751964-06	GM2 = 1.725330-05
	GM2D = 5.170688-05		
TIME = 2.30000	W1 = -1.194999-04	W2 = 2.035237-04	W3 = 9.543952-06
	NX = 0.000000	NY = 0.000000	FZ = 300.000
	ETA1 = .178812	ETA2 = 2.452690-03	ETA3 = 3.883458-03
	ETA4 = -.412792	ETA5 = -2.078913-02	ETA6 = -4.532183-03
	ETA7 = -7.834672-03	ETD1 = 7.930726-02	ETD2 = 3.725216-02
	ETD3 = -1.929868-03	ETD4 = -.315038	ETD5 = -1.657669-02
	ETD6 = -2.085271-02	ETD7 = 2.710963-02	ANGM = .154926
	W1D = 6.476792-03	W2D = -1.162214-03	W3D = 1.115572-03
	GM1 = -7.351207-08	GM1D = -1.607631-05	GM2 = 2.053021-06
	GM2D = -2.875026-04		
TIME = 2.40000	W1 = 3.141630-04	W2 = -1.896092-04	W3 = 4.590424-05
	NX = 0.000000	NY = 0.000000	FZ = 300.000
	ETA1 = .190410	ETA2 = 5.807521-03	ETA3 = 2.901173-03
	ETA4 = -.449504	ETA5 = -2.620123-02	ETA6 = -7.185827-03
	ETA7 = -1.473263-03	ETD1 = .150234	ETD2 = 3.053111-02
	ETD3 = -1.494873-02	ETD4 = -.406566	ETD5 = -6.151531-02
	ETD6 = -1.660914-02	ETD7 = 5.842808-02	ANGM = .162645
	W1D = 5.194020-04	W2D = -7.042102-03	W3D = -4.531757-04
	GM1 = -4.531444-07	GM1D = 9.725387-06	GM2 = -1.994955-05
	GM2D = -6.297991-05		
TIME = 2.50000	W1 = -2.551416-05	W2 = -8.965070-04	W3 = -7.507707-05
	NX = 0.000000	NY = 0.000000	FZ = 300.000
	ETA1 = .208125	ETA2 = 8.598550-03	ETA3 = 1.576923-03
	ETA4 = -.491649	ETA5 = -2.791420-02	ETA6 = -6.540928-03
	ETA7 = -1.978658-03	ETD1 = .198350	ETD2 = 2.431256-02
	ETD3 = -9.086693-03	ETD4 = -.425067	ETD5 = 3.388802-02
	ETD6 = 2.364968-02	ETD7 = -6.403697-02	ANGM = .171715
	W1D = -5.547869-03	W2D = -4.766604-03	W3D = -1.194857-03
	GM1 = 9.270155-07	GM1D = 1.021694-05	GM2 = -6.263739-06
	GM2D = 2.760264-04		

Fig. 13. Simulation printout for program using MBDYFN

TIME	=	2.20000	W1	=	-5.534418-04	W2	=	4.144801-04	W3	=	-6.254079-05		
			NX	=	0.000000			Ny	=	0.000000	FZ	=	300.000
			ETA1	=	.175147	ETA2	=	-1.585590-03	ETA3	=	3.189027-03		
			ETA4	=	-.388847	ETA5	=	-2.479597-02	ETA6	=	-5.228946-03		
			ETA7	=	-4.007270-03	ETD1	=	-1.014957-02	ETD2	=	4.175283-02		
			ETD3	=	1.388555-02	ETD4	=	-.153645	ETD5	=	7.823734-02		
			ETD6	=	3.289947-02	ETD7	=	-7.956043-02	ANGM	=	.147618		
			WID	=	5.287010-04	W2D	=	-5.007477-03	W3D	=	-2.201367-04		
			GM1	=	1.433806-06	GM2D	=	-5.756394-06	GM2	=	1.725269-05		
			GM2D	=	5.171269-05								
TIME	=	2.30000	W1	=	-1.195008-04	W2	=	2.035250-04	W3	=	9.543577-06		
			NX	=	0.000000			Ny	=	0.000000	FZ	=	300.000
			ETA1	=	.178812	ETA2	=	2.452688-03	ETA3	=	3.883458-03		
			ETA4	=	-.412792	ETA5	=	-2.078914-02	ETA6	=	-4.532183-03		
			ETA7	=	-7.834672-03	ETD1	=	7.930725-02	ETD2	=	3.725216-02		
			ETD3	=	-1.929866-03	ETD4	=	-.315038	ETD5	=	-1.657669-02		
			ETD6	=	-2.085271-02	ETD7	=	2.710963-02	ANGM	=	.154927		
			WID	=	6.476788-03	W2D	=	-1.142213-03	W3D	=	1.115672-03		
			GM1	=	-7.378356-08	GM2D	=	-1.607390-05	GM2	=	2.052887-06		
			GM2D	=	-2.874996-04								
TIME	=	2.40000	W1	=	3.141622-04	W2	=	-1.896078-04	W3	=	4.590388-05		
			NX	=	0.000000			Ny	=	0.000000	FZ	=	300.000
			ETA1	=	.190410	ETA2	=	5.807519-03	ETA3	=	2.901173-03		
			ETA4	=	-.449504	ETA5	=	-2.620123-02	ETA6	=	-7.185827-03		
			ETA7	=	-1.473263-03	ETD1	=	.150234	ETD2	=	3.053111-02		
			ETD3	=	-1.494873-02	ETD4	=	-.406566	ETD5	=	-6.151830-02		
			ETD6	=	-1.660914-02	ETD7	=	5.842808-02	ANGM	=	.162647		
			WID	=	5.194041-04	W2D	=	-7.042101-03	W3D	=	-6.531752-04		
			GM1	=	-4.530906-07	GM2D	=	9.730052-06	GM2	=	-1.994953-05		
			GM2D	=	-6.297931-05								
TIME	=	2.50000	W1	=	-2.551484-05	W2	=	-8.965056-04	W3	=	-7.507740-05		
			NX	=	0.000000			Ny	=	0.000000	FZ	=	300.000
			ETA1	=	.208125	ETA2	=	8.598547-03	ETA3	=	1.576924-03		
			ETA4	=	-.491649	ETA5	=	-2.791420-02	ETA6	=	-6.540928-03		
			ETA7	=	-1.978658-03	ETD1	=	.198350	ETD2	=	2.431256-02		
			ETD3	=	-9.086695-03	ETD4	=	-.425067	ETD5	=	3.388802-02		
			ETD6	=	2.364968-02	ETD7	=	-6.403697-02	ANGM	=	.171716		
			WID	=	-5.547869-03	W2D	=	-4.746604-03	W3D	=	-1.194857-03		
			GM1	=	9.273785-07	GM2D	=	1.021708-05	GM2	=	-6.263727-06		
			GM2D	=	2.760250-04								

Fig. 14. Simulation printout for program using MBDYFL

```

CSSL III JET PROPULSION LABORATORY 040374-002H 021775-225736

*** START T(RUN)= 17.986 T(TASK)= .003 CTP = .536
DT(TASK)= .003 DCTP = .536

PROGRAM 2=BODY VEHICLE WITH FLEX. APPENDAGE (FROZEN PLATFORM)
*SC4020 BLDG/198,BOX/601,CAHARA/91N,FRAMES/50
COMMENT
  ARRAY MB(7),MS(1,7),PB(3),PS(1,1,3),G(2,3)
  ARRAY EIG(1,6,7),RF(1,1,3),REC(1,6,7),WF(1,7),ZF(1,7)
  ARRAY TB(3),TS(1,3),FB(3),FS(1,3),GM(2),GMD(2),GMDD(2)
  ARRAY TH(2),WO(3),TF(1,1,3),FF(1,1,3),ET(1,7),ETD(1,7)
  DOUBLE PRECISION WDOT(5),ETDD(1,7)
  INTEGER NC,NF,H(1,2),F(1,3),PI(3),L
DATA H(1,1)/U/H(1,2)/Z/PI(1,1,1)/
DATA F(1,1)/U/F(1,2)/1/F(1,3)/7/
DATA MB/1230.1290.1650.16.29,-43.45.61.75.79.0/
DATA MS/4.75.5.53.1.32.0.0.0.0.1.93/
DATA G(1,3)/1./G(2,1)/1./
DATA REC/.03375.001654,-.8678,.08135.17.17.007955,....
      .01055.001104,-.4608E-4,.4236.12.3.001859,....
      .002335,-.01818,-.4731E-5.21.3,-.2386.000918,....
      .003244.001014.2.234,-.4081.5.9301-.05211,....
      -.6055,-.5381.1.962.7.577.4020.2.520,....
      .3060.1.753.5585-4.32.1589.9205,....
      .02762.1051.3919.2.032-2.061.2761/
DATA WF/.5756.61337.61337.63071.2.723.2.963.3.047/
DATA ZF/.20.20.20.20.05.05.01/
  CONSTANT FINTIM=10. CLKTIM=900. PIE=3.14159265
  CONSTANT K1=900. B1=100. K2=850. B2=100.

INITIAL
  NC=1 S NF=1
  DO 57 L=1,7
S7.. WF(1,L)=WF(1,L)*2.*PIE
  CALL MBUYFN(NC,H,MB,MS,PB,PS,G,PI,NF,F,EIG,REC,RF,WF,ZF)

END
DYNAMIC
  IF(TIME.GT.FINTIM) GO TO FIN
  STPCLK CLKTIM
  OUTPUT IO,W1,W2,W3,NX,NY,FZ,ETA1,ETA2,ETA3,ETA4,ETA5,ETA6,ETA7,....
      ETD1,ETD2,ETD3,ETD4,ETD5,ETD6,ETD7,ANGH,W1D,W2D,W3D,....
      GM1,GMD,GM2,GM2D
  PREPAR W1,W2,W3,NX,NY,FZ,ETD1,ETD2,ETD3,ETD4,ETD5,ETD6,ETD7,....
      ANGH,GM1,GM2,GM1D,GM2D

DERIVATIVE BODYZF
  VARIABLE TIME=0. S CINTERVAL CI=.01
  XERROR W1=1.E-6 S MERROR W1=1.E-6

NOSORT
  GMD(1)=GM1D S GM(1)=GM1
  GMD(2)=GM2D S GM(2)=GM2
  ET(1,1)=ETA1 S ET(1,2)=ETA2 S ET(1,3)=ETA3 S ET(1,4)=ETA4
  ET(1,5)=ETA5 S ET(1,6)=ETA6 S ET(1,7)=ETA7
  ETD(1,1)=ETD1 S ETD(1,2)=ETD2 S ETD(1,3)=ETD3 S ETD(1,4)=ETD4
  ETD(1,5)=ETD5 S ETD(1,6)=ETD6 S ETD(1,7)=ETD7
  WO(1)=W1 S WO(2)=W2 S WO(3)=W3 S ANGH=MH

COMMENT...
  HINGE TORQUES

```

Fig. 15. Simulation program for test vehicle with prescribed platform motion using MBDYFN

```

COMMENT
  TH(1)=-K1*GM1 - B1*GM1D
  TH(2)=-K2*GM2 - B2*GM2D
COMMENT...
      FORCE EQUATION
COMMENT
  FZ=(STEP(1.5,TIME)-STEP(2.5,TIME))*300.
  FB(3)=FZ
COMMENT...
      ENGINE TORQUE
COMMENT
  NX=(STEP(3.5,TIME)-STEP(4.5,TIME))*10.
  NY=(STEP(6.5,TIME)-STEP(7.5,TIME))*10.
  TB(1)=NX  S  TB(2)=NY
COMMENT...
      SOLUTION FOR SYSTEM ACCELERATIONS
COMMENT
  CALL MRATE(NC,TH,TB,TS,FB,FS,TF,FF,GM,GM0,GMDD,ET,ETD,W0,WDOT,...
  ETDD,MM)
  W1D=WDOT(1)  S  W2D=WDOT(2)  S  W3D=WDOT(3)
COMMENT...
      SYSTEM RATES AND POSITIONS
COMMENT
  W1=INTEG(WDOT(1),0.)
  W2=INTEG(WDOT(2),0.)
  W3=INTEG(WDOT(3),0.)
  ETD1=INTEG(ETDD(1,1),0.)  S  ETA1=INTEG(ETD1,0.)
  ETD2=INTEG(ETDD(1,2),0.)  S  ETA2=INTEG(ETD2,0.)
  ETD3=INTEG(ETDD(1,3),0.)  S  ETA3=INTEG(ETD3,0.)
  ETD4=INTEG(ETDD(1,4),0.)  S  ETA4=INTEG(ETD4,0.)
  ETD5=INTEG(ETDD(1,5),0.)  S  ETA5=INTEG(ETD5,0.)
  ETD6=INTEG(ETDD(1,6),0.)  S  ETA6=INTEG(ETD6,0.)
  ETD7=INTEG(ETDD(1,7),0.)  S  ETA7=INTEG(ETD7,0.)
  GM1D=INTEG(WDOT(4),0.)  S  GM1=INTEG(GM1D,0.)
  GM2D=INTEG(WDOT(5),0.)  S  GM2=INTEG(GM2D,0.)
END
END
END
TERMINAL
FIN.. CONTINUE
END
END

```

Fig. 15 (contd)

TIME	2.2000	W1	-5.544373-04	W2	4.139068-04	W3	-6.283581-05
		NX	0.000000	NY	0.000000	FZ	300.000
		ETA1	.175147	ETA2	-1.585691-03	ETA3	3.192810-03
		ETA4	-.388847	ETA5	-2.478928-02	ETA6	-5.240584-03
		ETA7	-3.998999-03	ETD1	-1.013886-02	ETD2	4.176088-02
		ETD3	1.390250-02	ETD4	-.153641	ETD5	7.847859-02
		ETD6	3.297854-02	ETD7	-7.973409-02	ANGM	.147619
		W1D	5.576106-04	W2D	-5.012866-03	W3D	-2.146401-04
		GM1	0.000000	GM1D	0.000000	GM2	0.000000
		GM2D	0.000000				
TIME	2.3000	W1	-1.164182-04	W2	2.034345-04	W3	9.839955-06
		NX	0.000000	NY	0.000000	FZ	300.000
		ETA1	.178813	ETA2	2.453164-03	ETA3	3.885903-03
		ETA4	-.412792	ETA5	-2.077761-02	ETA6	-4.525558-03
		ETA7	-7.845880-03	ETD1	7.930792-02	ETD2	3.725211-02
		ETD3	-1.960251-03	ETD4	-.315037	ETD5	-1.672914-02
		ETD6	-2.068424-02	ETD7	2.702194-02	ANGM	.154925
		W1D	6.476257-03	W2D	-1.150093-03	W3D	1.118775-03
		GM1	0.000000	GM1D	0.000000	GM2	0.000000
		GM2D	0.000000				
TIME	2.4000	W1	3.138598-04	W2	-1.889694-04	W3	4.603753-05
		NX	0.000000	NY	0.000000	FZ	300.000
		ETA1	.190410	ETA2	5.807416-03	ETA3	2.901913-03
		ETA4	-.449504	ETA5	-2.620992-02	ETA6	-7.179536-03
		ETA7	-1.473728-03	ETD1	.150223	ETD2	3.052319-02
		ETD3	-1.494876-02	ETD4	-.406570	ETD5	-6.166167-02
		ETD6	-1.677170-02	ETD7	5.865821-02	ANGM	.162643
		W1D	5.074690-04	W2D	-7.044410-03	W3D	-6.582883-04
		GM1	0.000000	GM1D	0.000000	GM2	0.000000
		GM2D	0.000000				
TIME	2.5000	W1	-2.521466-05	W2	-8.968403-04	W3	-7.534413-05
		NX	0.000000	NY	0.000000	FZ	300.000
		ETA1	.208125	ETA2	8.598212-03	ETA3	1.577519-03
		ETA4	-.491649	ETA5	-2.792129-02	ETA6	-6.549905-03
		ETA7	-1.967483-03	ETD1	.198357	ETD2	2.431720-02
		ETD3	-9.100753-03	ETD4	-.425064	ETD5	3.404423-02
		ETD6	2.359918-02	ETD7	-6.410951-02	ANGM	.171715
		W1D	-5.530066-03	W2D	-4.777549-03	W3D	-1.195949-03
		GM1	0.000000	GM1D	0.000000	GM2	0.000000
		GM2D	0.000000				

Fig. 16. Simulation printout for program using MBDYFN with prescribed platform motion

from a nominally *zero* angular rate are small. The second approach was then further restricted to the often very useful assumption that *all* system rotations are small, permitting a formal linearization with respect to hinge and reference body rotations. Of course, appendage deformations are assumed small in every case.

Three FORTRAN subroutines were then described which solve the equations of motion for these three cases, namely, MBDYFR (for spinning appendages), MBDYFN (for nonspinning appendages), and MBDYFL (linearized for small rotations). Each of the routines has much the same functional appearance as those programs described in Ref. 6, i.e., an initializing entry and a dynamic entry point, with the only differences being the addition of *appendage-related* parameters, variables, and forcing functions. The routines also retain the option of user-prescribed rotations at selected hinge connections. However, an additional option provided in these programs is that of calculating angular momentum magnitude, which at times provides a valuable check on computational accuracy.

In applying MBDYFR, one can conclude that the mathematical difficulties introduced by spin have forced not only a first-order transformation to obtain uncoupled coordinates but, as a consequence, *two* coordinates per mode must be solved for in the subroutine. However, what appears to be a computational disadvantage in this case may well be softened by the necessity to consider fewer modes. Some other difficulties are also introduced by this particular modal transformation. The presence of *both* the modal coordinate position and rate in the expressions for appendage deformation and deformation rate can lead to significant error if modal damping is inserted (thus disturbing eigenvector orthogonality) *and* large steady-state appendage deformations are present. The user must ensure that any appendage deformations in the damped case remain essentially oscillatory about a nominally zero mean. MBDYFR, as it now stands, also forces the user, regardless of which appendages are spinning or not spinning, to formulate each appendage's modal description using only the first-order transformation, i.e., as if it were subject to spin. While it was much more convenient to program MBDYFR in this way, future requirements for improved computational efficiency may make a modification of MBDYFR desirable. Still, in spite of these particular characteristics, it is felt that MBDYFR can be successfully employed in a wide variety of applications because of its inherent generality and versatility. In addition to the prescribed variable and angular momentum calculation options, the user may also choose to use MBDYFR to directly calculate the steady-state deformations due to centrifugal forces. This is accomplished by setting SR, the nominal appendage spin rate, to zero even though, in the simulation, the appendage is spinning. Setting SR to zero *restores* the centrifugal force terms to the equations, and appropriate deformations will appear in the solution. However, as indicated before, the greater the modal damping under these circumstances, the larger the numerical error will be in the steady-state deformations due to spin.

The routines MBDYFN and MBDYFL are of more immediate utility at JPL since current spacecraft designs here are three-axis-stabilized. They represent a generalization of the hybrid-mode concept, developed in Ref. 2, to the rigid-body-tree approach. As a result, it is no longer necessary to add special terms and re-derive equations of motion in order to accommodate discrete rigid-body rotations (or translations) in the system (as was done, for example, in Ref. 9 for the Viking Orbiter with flexible appendages and rigid propellant slosh masses). Even translational dampers can be reasonably well approximated within the hinge-connected tree system. Because of its speed advantages and because it usually

provides acceptable solution accuracy even when rotations are not strictly small, the completely linearized version, MBDYFL, will offer the greatest utility among the three programs at JPL for routine control design studies.

To make these subroutines more easily available to the aerospace industry, they have been submitted to COSMIC (Computer Software Management and Information Center), University of Georgia, Athens, Georgia, for evaluation and dissemination to interested agencies and institutions.

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## Appendix A

### Effects of Damping on Rotating Appendage Equations

In Section IIIA, it was pointed out that the addition of viscous damping-like terms to the already transformed appendage equations, particularly for the case of a nominally rotating appendage/base, is *mathematically* not justified. However, the insertion of modal damping terms is usually thought to be justified on the practical basis that it reasonably and more conveniently represents the *physical* response of systems as determined from actual test data.

However, it may be useful to illustrate how and to what extent the mathematical inconsistencies so introduced may affect computational results. For example, one can show that the insertion of modal damping into Eq. (26) introduces errors in the *steady-state* values of  $\bar{\delta}^k$ ,  $\bar{\eta}^k$ , and therefore the deformations  $q^k$  and  $\dot{q}^k$ . This can be seen from the following. Repeating Eqs. (26) and (27), we have

$$\dot{\bar{\delta}}^k = -\bar{\sigma}^k \bar{\eta}^k - \bar{\sigma}^k \bar{\Gamma}_k^T L'_k - \bar{\xi}^k \bar{\sigma}^k \bar{\delta}^k \quad (\text{A-1})$$

$$\dot{\bar{\eta}}^k = \bar{\sigma}^k \bar{\delta}^k - \bar{\sigma}^k \bar{\Psi}_k^T L'_k - \bar{\xi}^k \bar{\sigma}^k \bar{\eta}^k \quad (\text{A-2})$$

$$q^k = 2(\bar{\Psi}_k \bar{\delta}^k - \bar{\Gamma}_k \bar{\eta}^k) \quad (\text{A-3})$$

$$\dot{q}^k = -2(\bar{\Gamma}_k \bar{\sigma}^k \bar{\delta}^k + \bar{\Psi}_k \bar{\sigma}^k \bar{\eta}^k) \quad (\text{A-4})$$

$$\ddot{q}^k = -2(\bar{\Gamma}_k \bar{\sigma}^k \dot{\bar{\delta}}^k + \bar{\Psi}_k \bar{\sigma}^k \dot{\bar{\eta}}^k) \quad (\text{A-5})$$

If we now examine  $q^k$  and  $\dot{q}^k$  when  $\bar{\delta}^k$  and  $\bar{\eta}^k$  have reached a steady-state condition, i.e., when  $\dot{\bar{\delta}}^k = \dot{\bar{\eta}}^k = 0$ , we have, from (A-1),

$$\bar{\sigma}^k \bar{\eta}^k = -\bar{\sigma}^k \bar{\Gamma}_k^T L'_k - \bar{\xi}^k \bar{\sigma}^k \bar{\delta}^k$$

and from (A-2),

$$\bar{\sigma}^k \bar{\delta}^k = \bar{\sigma}^k \bar{\Psi}_k^T L'_k + \bar{\xi}^k \bar{\sigma}^k \bar{\eta}^k$$

Substituting from (A-2) into (A-1),

$$\bar{\eta}^k = -\bar{\Gamma}_k^T L'_k - \bar{\sigma}^{k-1} \bar{\xi}^k \bar{\sigma}^k [\bar{\Psi}_k^T L'_k + \bar{\sigma}^{k-1} \bar{\xi}^k \bar{\sigma}^k \bar{\eta}^k]$$

or

$$\bar{\eta}^k = -\bar{\Gamma}_k^T L'_k - \bar{\xi}^k \bar{\Psi}_k^T L'_k - \bar{\xi}^k \bar{\xi}^k \bar{\eta}^k$$



or

$$\bar{\eta}_{ss}^k = (U + \bar{\xi}^k \bar{\xi}^k)^{-1} (-\bar{\Gamma}_k^T - \bar{\xi}^k \bar{\psi}_k^T) L'_{k_s} \quad (\text{A-6})$$

Substituting from (A-1) into (A-2),

$$\bar{\delta}^k = \bar{\psi}_k^T L'_k + \bar{\sigma}^{k-1} \bar{\xi}^k \bar{\sigma}^k [-\bar{\Gamma}_k^T L'_k - \bar{\sigma}^{k-1} \bar{\xi}^k \bar{\sigma}^k \bar{\delta}^k]$$

or

$$\bar{\delta}^k = \bar{\psi}_k^T L'_k - \bar{\xi}^k \bar{\Gamma}_k^T L'_k - \bar{\xi}^k \bar{\xi}^k \bar{\delta}^k$$

or

$$\bar{\delta}_{ss}^k = (U + \bar{\xi}^k \bar{\xi}^k)^{-1} (\bar{\psi}_k^T - \bar{\xi}^k \bar{\Gamma}_k^T) L'_{k_s} \quad (\text{A-7})$$

From (A-3), (A-6), and (A-7),

$$\begin{aligned} q_{ss}^k &= 2 \bar{\psi}_k (U + \bar{\xi}^k \bar{\xi}^k)^{-1} (\bar{\psi}_k^T - \bar{\xi}^k \bar{\Gamma}_k^T) L'_k - 2 \bar{\Gamma}_k (U + \bar{\xi}^k \bar{\xi}^k)^{-1} (-\bar{\Gamma}_k^T - \bar{\xi}^k \bar{\psi}_k^T) L'_k \\ q_{ss}^k &= 2 \left[ \bar{\psi}_k U_\xi^{-1} (\bar{\psi}_k^T - \bar{\xi}^k \bar{\Gamma}_k^T) + \bar{\Gamma}_k U_\xi^{-1} (\bar{\Gamma}_k^T + \bar{\xi}^k \bar{\psi}_k^T) \right] L'_{k_s} \end{aligned} \quad (\text{A-8})$$

where

$$U_\xi = (U + \bar{\xi}^k \bar{\xi}^k)$$

From (A-4), (A-6), and (A-7),

$$\dot{q}_{ss}^k = 2 \left[ \bar{\psi}_k \bar{\sigma}^k U_\xi^{-1} (\bar{\Gamma}_k^T + \bar{\xi}^k \bar{\psi}_k^T) - \bar{\Gamma}_k \bar{\sigma}^k U_\xi^{-1} (\bar{\psi}_k^T - \bar{\xi}^k \bar{\Gamma}_k^T) \right] L'_{k_s} \quad (\text{A-9})$$

Notice that from (A-9),  $\dot{q}_{ss}^k \neq 0$  in general! However, as  $\xi^k$  becomes infinitesimally small, (A-8) and (A-9) approach

$$q_{ss}^k = 2 \left[ \bar{\psi}_k \bar{\psi}_k^T + \bar{\Gamma}_k \bar{\Gamma}_k^T \right] L'_k$$

and

$$\dot{q}_{ss}^k = 2 \left[ \bar{\psi}_k \bar{\sigma}^k \bar{\Gamma}_k^T - \bar{\Gamma}_k \bar{\sigma}^k \bar{\psi}_k^T \right] L'_k \equiv 0$$

due to orthogonality relations between  $\bar{\psi}_k$  and  $\bar{\Gamma}_k$ .

The discovery above that, in general,  $\dot{q}_{ss}^k \neq 0$  when modal damping is introduced is rather disconcerting. It is further disturbing to realize that if the appendage deformation rates  $\dot{q}^k$  are not zero when the modal coordinates appear to indicate an appendage *at rest*, then the angular momentum calculations of the subroutines, based on  $\dot{q}^k$ , will be in error as well.

Fortunately, we have assumed that the appendage deformations,  $q^k$ , and their derivatives are small *and* represent only the oscillatory component of the total possible deformation. This tends to imply that  $L'_k$  must be very small to begin with and that the steady-state levels of  $q^k$  (or its derivatives) after damping are "small" compared to its *transient* oscillatory amplitudes. Therefore the errors introduced in (A-8) and (A-9) should be of relatively little significance. However, one should be aware of their existence and that they can add to other computational errors.

## Appendix B

### System Angular Momentum Computation

In Ref. 5, Hooker shows that for a dynamical system of the type considered here, namely, a topological tree of rigid bodies any one of which may carry a flexible appendage, the equations are of the general form

$$A\dot{x} = B$$

where

$$A = \begin{bmatrix} a_{00} & | & a_{0k} & | & b_0 \\ a_{0k}^T & | & a & | & b \\ b_0^T & | & b^T & | & c \end{bmatrix}, \quad \text{and} \quad x = \begin{bmatrix} \omega^0 \\ \dot{\gamma} \\ \dot{\eta} \end{bmatrix}$$

and Hooker proves that the angular momentum of this system about its mass center is the product of the first row of  $A$  with  $x$ :

$$H = a_{00}\omega^0 + a_{0k}\dot{\gamma} + b_0\dot{\eta} \tag{B-1}$$

and that the 3 by 3 matrix  $a_{00}$  represents the instantaneous system inertia. The relation (B-1) is precisely that implemented in each of the subroutines MBDYFR, MBDYFN, and MBDYFL to calculate  $H$  (3 by 1).  $H$  is a 3 by 1 vector matrix whose elements are the components of the system angular momentum vector *in the reference body frame*. These three elements are available within the subroutine if the user wishes to extract them. He may also wish to transform them to an inertial reference frame in certain situations as a check on his simulation accuracy. However, the normal subroutine function as shown here in the examples and listings is to supply the user with only the *magnitude* of  $H$ , i.e.,

$$|H| = (h_1^2 + h_2^2 + h_3^2)^{\frac{1}{2}}$$

where

$$H = [h_1 h_2 h_3]^T$$

## Appendix C

### Subroutine MBDYFR Listing and User Requirements

#### *Subroutine Entry Statements*

```
CALL MBDYFR(NC, H, MB, MS, PB, PS, G, PI, NF, F,  
            ER, EI, SR, MF, RF, WF, ZF)  
CALL MRATE(NC, TH, TB, TS, FB, FS, TF, FF, GM, GMD,  
            GMDD, DT, ET, WO, WDOT, DTD, ETD, HM)
```

#### *Input / Output Variable Type and Storage Specifications*

```
INTEGER NC, NF, H( $n_c$ , 2), F( $n_f$ , 3), PI( $n + 1$ )  
  
REAL MB(7), MS( $n_c$ , 7), PB( $n_c$ , 3), PS( $n_c$ ,  $n_c$ , 3), G( $n$ , 3),  
      TH( $n$ ), TB(3), TS( $n_c$ , 3), FB(3), FS( $n_c$ , 3), GM( $n$ ),  
      GMD( $n$ ), GMDD( $n$ ), ER( $n_f$ ,  $6n_k$ ,  $N_k$ ), EI( $n_f$ ,  $6n_k$ ,  $N_k$ ),  
      MF( $n_f$ ,  $n_k$ , 7), RF( $n_f$ ,  $n_k$ , 3), WF( $n_f$ ,  $N_k$ ), ZF( $n_f$ ,  $N_k$ ),  
      TF( $n_f$ ,  $n_k$ , 3), FF( $n_f$ ,  $n_k$ , 3), DT( $n_f$ ,  $N_k$ ), ET( $n_f$ ,  $N_k$ ),  
      WO(3), SR( $n_f$ , 3)  
  
DOUBLE PRECISION WDOT( $n + 3$ ), DTD( $n_f$ ,  $N_k$ ),  
      ETD( $n_f$ ,  $N_k$ )
```

#### *External Subroutines Called*

CHOLD—double precision subroutine for solving matrix equations of the form

$$Ax = B$$

where  $A$  is a square, symmetric, positive-definite matrix (see statement 1291).

#### *Subroutine Setup*

Insert the Fortran statement

```
PARAMETER QC =  $n_c$ , QH =  $n$ , QF =  $n_f$ , NK =  $n_k$ , NKT =  $N_k$ 
```

(If more than one appendage is present, use the *largest*  $n_k$  and  $N_k$  for the PARAMETER statement to provide sufficient storage.)

### Data Restrictions

$$n > 1, n_f > 1, n_c > 1, n_k > 1, N_k > 1$$

### Core Storage Required

Code: 6500 words

Data: ~500 words (minimum; increases with  $n, n_f$ , etc.)

### Listing

```
10 SUBROUTINE MBDYFR(NC,C,MB,MA,PB,PA,G,PI,NF,F,ER,EI,SR,MF,RF,WF,ZF)
20 C
30 C ADJUSTABLE DIMENSIONS
40 C
50 INTEGER PI(1),C(NC,2)
60 REAL MB(1),MA(NC,7),PB(NC,3),PA(NC,NC,3)
70 PARAMETER QC=2,QH=3,QF=2,NK=1,NKT=2
80 PARAMETER NAK=6*NK,S=QC+1,V=QH+3,V4=4*V,S3=3*S,Q=QH,NH=QH
90 PARAMETER ST=V+2*QF*NKT,S4=4*ST
100 C
110 C ADDITIONAL DIMENSIONED VARIABLES
120 C
130 DOUBLE PRECISION A(ST,ST),BMASS(S)
140 INTEGER EPS(Q,S),CPS(QC,S),H(Q),HI(S),FI(S),F(NF,3)
150 REAL ADFR(QF,QH,NKT),ADFR(QF,3,NKT),AKFR(QF,QH,NKT)
160 S),AKFI(QF,QH,NKT),AC(3,3),AS(Q,Q),AV(Q,3),AIS(3),B(QF,NK,3),BD(QF,
170 SNK,3),CE(3),CL(3),CK(QF,3),CKD(QF,3),CDU(QF,3),CQ(3),CWD(S,3),CV(
180 S3),CW(S,3),DX(S,S),DY(S,S),DZ(S,S),DXO(S,S),DYO(S,S),DZO(S,S),DDSO
190 S(QF,3),DLKR(QF,3,NKT),DLKI(QF,3,NKT),DLKRO(QF,3,NKT),DLKIU(QF,3,NK
200 ST),DUR(3,NKT),DUI(3,NKT),DUXO(QF),DUZO(QF),EA(3),ER(NF,N6
210 SK,NKT),EIN(NF,N6K,NKT),FEXO(S),FEYO(S),FEZO(S),FS(S,3),GO(Q,3),GG(Q
220 S,3),G(Q,3),GK(QF,3,NKT),GPSO(QF,3),GKOS(QF,3,NKT),I1,I2,I3,I12,
230 S13,I23,IX(S),IYY(S),IZZ(S),IXY(S),IXZ(S),IYZ(S),LX(S,S),LY(S,S),
240 SLZ(S,S),MSB(S),MS,MF(NF,NK,7),MCK(QF,3),MCKD(QF,3),PH(S,3,3),PSG(S
250 S,S,3),PS(S,S,3,3),PK(QF,3,NKT),PGSO(QF,3),PSF(S,S,3,3),PKOS(QF,3,N
260 SKT),RF(NF,NK,3),SR(QF,3),TXO(S),TYO(S),TZO(S),T(G,3,3),TS(S,3),U(Q
270 SF,NK,3),UD(QF,NK,3),VJ(3,3),VJD(3,3),VJDO(QF,3,3),VE(QF,3),VB(QF,N
280 S6K),WF(NF,NKT),WDE(QF,3),WGJ(QH,3),ZF(NF,NKT),ZSR(QF,NKT),ZSI(QF,
290 SKT),W(3),HH(3)
300 EQUIVALENCE (A,PS),(LX,DXO),(LY,DYU),(LZ,DZO)
310 NB=NC+1
320 C
330 C DEFINE EPS(K,J) USING C
340 C
350 DO 86 K=1,NC
360 DO 86 J=2,NB
370 IF(K.EQ.(J-1)) CPS(K,J)=1
380 IF(K.LT.(J-1)) GO TO 87
390 GO TO 86
400 87 CONTINUE
410 J0=K+1
420 J1=J-1
430 DO 89 L=J0,J1
440 IF(K.GT.(L-1)) GO TO 89
450 IF((CPS(K,L).EQ.1).AND.(C(J-1,1).EQ.(L-1))) CPS(K,J)=1
460 89 CONTINUE
470 86 CONTINUE
480 L=0
490 DO 1 J=1,NC
500 KK=C(J,2)
510 DO 1 K=1,KK
520 L=L+1
```

```

53° DO 1 I=1,NB
54° I EPS(L,I)=CPS(J,I)
55° C
56° C COMPUTE HI(I)=C, WHERE I=HINGE LABEL AND C=CONNECTION LABEL
57° C
58° I=0
59° DO 8 J=2,NB
60° KK=C(J-1,2)
61° DO 8 K=1,KK
62° I=I+1
63° 8 H(I)=J-1
64° C
65° C COMPUTE HI(I)=J, WHERE I=BODY LABEL+1 AND J=NEAREST HINGE LABEL
66° C
67° HI(I)=1
68° HI(NB)=NM
69° DO 47 I=NM,1
70° IF(I.EQ.1) GO TO 47
71° K1=HI(I)
72° K2=HI(I-1)
73° IF(K1.EQ.K2) GO TO 47
74° HI(K2+1)=I-1
75° 47 CONTINUE
76° C
77° C DEFINE FI(J)=K, WHERE J=BODY-LABEL+1 AND K IS APPENDAGE-LABEL
78° C (IF K=0, BODY HAS NO FLEX. APPENDAGE)
79° C
80° DO 239 N=1,NB
81° 239 FI(N)=0
82° DO 242 K=1,NF
83° JN=F(K,1)+1
84° 242 FI(JN)=K
85° NF=NF
86° NB=NB
87° C
88° C DEFINE SUBSTRUCTURE MASSES
89° C
90° MSB(1)=MB(7)
91° DO 248 N=2,NB
92° 248 MSB(N)=MA(N-1,7)
93° C
94° C TOTAL NUMBER OF FLEX. APPENDAGE MODES TO BE RETAINED
95° C
96° NTHO=0
97° DO 461 K=1,NF
98° 461 NTHO=NTHO+F(K,3)
99° NTZ=2*NTHO
100° C
101° C INITIAL CALCULATION OF BARYCENTER VECTORS W.R.T. BODY C.G.S
102° C AND HINGE POINTS
103° C
104° IXX(1)=MB(1)
105° IYY(1)=MB(2)
106° IZZ(1)=MB(3)
107° IXY(1)=MB(4)
108° IXZ(1)=MB(5)
109° IYZ(1)=MB(6)
110° BMASS(1)=MB(7)
111° TM=BMASS(1)
112° DO 35 J=2,NB
113° IXX(J)=MA(J-1,1)
114° IYY(J)=MA(J-1,2)
115° IZZ(J)=MA(J-1,3)
116° IXY(J)=MA(J-1,4)
117° IXZ(J)=MA(J-1,5)
118° IYZ(J)=MA(J-1,6)
119° BMASS(J)=MA(J-1,7)
120° 35 TM=TM+BMASS(J)
121° DO 149 I=1,NB

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122*      I1=I-1
123*      DO 149 J=1,NB
124*      J1=J-1
125*      IF(I,EQ,J) GO TO 163
126*      IF(I,GT,J) GO TO 70
127*      IF(I,EQ,1) GO TO 80
128*      IF(CPS(I1,J),EQ,1) GO TO 400
129* 70    LX(I,J)=PA(I1,I1,1)
130*      LY(I,J)=PA(I1,I1,2)
131*      LZ(I,J)=PA(I1,I1,3)
132*      GO TO 149
133* 400   CONTINUE
134*      DO 600 K=I,J1
135*      IF(CPS(K,J),EQ,1) GO TO 500
136* 600   CONTINUE
137*      GO TO 149
138* 500   LX(I,J)=PA(I1,K,1)
139*      LY(I,J)=PA(I1,K,2)
140*      LZ(I,J)=PA(I1,K,3)
141*      GO TO 149
142* 80    DO 90 L=1,J1
143*      IF(CPS(L,J1),EQ,1) GO TO 101
144* 90    CONTINUE
145*      GO TO 149
146* 101   LX(I,J)=PB(L,1)
147*      LY(I,J)=PB(L,2)
148*      LZ(I,J)=PB(L,3)
149*      GO TO 149
150* 163   LX(I,J)=0.
151*      LY(I,J)=0.
152*      LZ(I,J)=0.
153* 149   CONTINUE
154*      DO 13 N=1,NB
155*      DO 13 J=1,NB
156*      DX(N,J)=LX(N,J)
157*      DY(N,J)=LY(N,J)
158*      DZ(N,J)=LZ(N,J)
159*      DO 13 K=1,NB
160*      DX(N,J)=DX(N,J)-(BMASS(K)/TM)*LX(N,K)
161*      DY(N,J)=DY(N,J)-(BMASS(K)/TM)*LY(N,K)
162* 13    DZ(N,J)=DZ(N,J)-(BMASS(K)/TM)*LZ(N,K)
163*  C
164*  C    NOMINAL SPIN RATE CENTRIFUGAL FORCES
165*  C
166*      DO 736 K=1,NF
167*      I=F(K,1)+1
168*      R1=SR(K,1)
169*      R2=SR(K,2)
170*      R3=SR(K,3)
171*      D1=DX(I,1)
172*      D2=DY(I,1)
173*      D3=DZ(I,1)
174*      WUDE(K,1)=-R3*(R3*D1-R1*D3)+R2*(-R2*D1+R1*D2)
175*      WUDE(K,2)=R3*(-R3*D2+R2*D3)-R1*(R1*D2-R2*D1)
176* 736   WUDE(K,3)=-R2*(-R3*D2+R2*D3)+R1*(R3*D1-R1*D3)
177*  C
178*  C    CALCULATION OF AUGMENTED INERTIA DYADICS FOR EACH BODY
179*  C
180*      DO 31 N=1,NB
181*      PH(N,1,1)=IXX(N)
182*      PH(N,1,2)=-IXY(N)
183*      PH(N,1,3)=-IXZ(N)
184*      PH(N,2,2)=IYY(N)
185*      PH(N,2,3)=-IYZ(N)
186*      PH(N,3,3)=IZZ(N)
187*      DO 30 J=1,NB
188*      PH(N,1,1)=PH(N,1,1)+BMASS(J)*(DY(N,J)**2+DZ(N,J)**2)
189*      PH(N,1,2)=PH(N,1,2)-BMASS(J)*DX(N,J)*DY(N,J)
190*      PH(N,1,3)=PH(N,1,3)-BMASS(J)*DX(N,J)*DZ(N,J)

```

```

191. PH(N,2,2)=PH(N,2,2)+BMASS(J)*(DX(N,J)**2+DZ(N,J)**2)
192. PH(N,2,3)=PH(N,2,3)-BMASS(J)*DY(N,J)*DZ(N,J)
193. 30 PH(N,3,3)=PH(N,3,3)+BMASS(J)*(DX(N,J)**2+DY(N,J)**2)
194. PH(N,2,1)=PH(N,1,2)
195. PH(N,3,1)=PH(N,1,3)
196. 31 PH(N,3,2)=PH(N,2,3)
197. C
198. C COMPUTE PK AND GK (3 X NKT ARRAYS)
199. C
200. DO 201 K=1,NF
201. LN=F(K,2)
202. JNT=F(K,3)
203. DO 201 I=1,3
204. DO 201 J=1,JNT
205. PK(K,I,J)=0.
206. GK(K,I,J)=0.
207. DO 202 L=1,LN
208. LL=6*(L-1)+1
209. PK(K,I,J)=PK(K,I,J)+MF(K,L,7)*ER(K,LL,J)
210. 202 GK(K,I,J)=GK(K,I,J)+MF(K,L,7)*EI(K,LL,J)
211. PK(K,I,J)=2.*PK(K,I,J)
212. GK(K,I,J)=2.*GK(K,I,J)
213. 201 CONTINUE
214. C
215. C COMPUTE DLKR-AND DLKI-TRANSPOSE MATRICES (3 X NKT ARRAYS)
216. C
217. DO 203 K=1,NF
218. LN=F(K,2)
219. JNT=F(K,3)
220. DO 203 J=1,JNT
221. DO 204 I=1,3
222. 204 DLKR(K,I,J)=0.
223. DLKI(K,I,J)=0.
224. DO 205 L=1,LN
225. L1=6*(L-1)+1
226. L2=L1+1
227. L3=L2+1
228. L4=L3+1
229. L5=L4+1
230. L6=L5+1
231. DLKR(K,I,J)=DLKR(K,I,J)+MF(K,L,7)*(EI(K,L3,J)*RF(K,L,2)
232. S -EI(K,L2,J)*RF(K,L,3))+MF(K,L,1)*EI(K,L4,J)
233. S -MF(K,L,4)*EI(K,L5,J) - MF(K,L,5)*EI(K,L6,J)
234. DLKR(K,2,J)=DLKR(K,2,J)+MF(K,L,7)*(EI(K,L1,J)*RF(K,L,3)
235. S -EI(K,L3,J)*RF(K,L,1))+MF(K,L,2)*EI(K,L5,J)
236. S -MF(K,L,4)*EI(K,L4,J) - MF(K,L,6)*EI(K,L6,J)
237. DLKR(K,3,J)=DLKR(K,3,J)+MF(K,L,7)*(EI(K,L2,J)*RF(K,L,1)
238. S -EI(K,L1,J)*RF(K,L,2))+MF(K,L,3)*EI(K,L6,J)
239. S -MF(K,L,5)*EI(K,L4,J) - MF(K,L,6)*EI(K,L5,J)
240. DLKI(K,I,J)=DLKI(K,I,J)+MF(K,L,7)*(ER(K,L3,J)*RF(K,L,2)
241. S -ER(K,L2,J)*RF(K,L,3))+MF(K,L,1)*ER(K,L4,J)
242. S -MF(K,L,4)*ER(K,L5,J) - MF(K,L,5)*ER(K,L6,J)
243. DLKI(K,2,J)=DLKI(K,2,J)+MF(K,L,7)*(ER(K,L1,J)*RF(K,L,3)
244. S -ER(K,L3,J)*RF(K,L,1))+MF(K,L,2)*ER(K,L5,J)
245. S -MF(K,L,4)*ER(K,L4,J) - MF(K,L,6)*ER(K,L6,J)
246. DLKI(K,3,J)=DLKI(K,3,J)+MF(K,L,7)*(ER(K,L2,J)*RF(K,L,1)
247. S -ER(K,L1,J)*RF(K,L,2))+MF(K,L,3)*ER(K,L6,J)
248. S -MF(K,L,5)*ER(K,L4,J) - MF(K,L,6)*ER(K,L5,J)
249. 205 CONTINUE
250. DO 206 I=1,3
251. DLKR(K,I,J)=-2.*DLKR(K,I,J)*WF(K,J)
252. 206 DLKI(K,I,J)=-2.*DLKI(K,I,J)*WF(K,J)
253. 203 CONTINUE
254. RETURN
255. ENTRY MRATE(NC,TH,TB,TA,FB,FA,TF,FF,GM,GMD,GHDD,DT,ET,WQ,WDOT,
256. S DTD,ETD,HM,U,UD)
257. REAL TF(QF,NK,3),FF(QF,NK,3),DT(QF,NKT),ET(QF,NKT),TB(3),TA(NC,3),
258. S FB(3),FA(NC,3),GM(1),GMD(1),GHDD(1),TH(1),WQ(3),WXS(S),WYO(S),WZO(
259. S S),E(S3,1)

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260•      DOUBLE PRECISION EC(IST),DTD(QF,NKT),ETD(QF,NKT),WDOT(V)
261•      C
262•      C      BODY-TO-BODY COORDINATE TRANSFORMATION MATRICES
263•      C
264•      DO 335 J=1,NH
265•      MM=J-1
266•      N=M(J)+1
267•      SGM=5IN(GM(J))
268•      CGM=COS(GM(J))
269•      CGM1=1.-CGM
270•      G1=CGM1*G(J,1)
271•      G2=CGM1*G(J,2)
272•      G3=CGM1*G(J,3)
273•      SG1=SGM*G(J,1)
274•      SG2=SGM*G(J,2)
275•      SG3=SGM*G(J,3)
276•      G15=G1*G(J,1)
277•      G25=G2*G(J,2)
278•      G35=G3*G(J,3)
279•      G12=G1*G(J,2)
280•      G13=G1*G(J,3)
281•      G23=G2*G(J,3)
282•      AB(1,1)=CGM+G15
283•      AB(1,2)=SG3+G12
284•      AB(1,3)=-SG2+G13
285•      AB(2,1)=-SG3+G12
286•      AB(2,2)=CGM+G25
287•      AB(2,3)=SG1+G23
288•      AB(3,1)=SG2+G13
289•      AB(3,2)=-SG1+G23
290•      AB(3,3)=CGM+G35
291•      IF(J.EQ.1) GO TO 3350
292•      DO 321 L=MM,1
293•      IF(EPS(L,N).EQ.1) GO TO 322
294•      321 CONTINUE
295•      GO TO 3350
296•      322 K=L
297•      DO 334 L=1,3
298•      DO 334 M=1,3
299•      T(J,L,M)=0.
300•      DO 334 I=1,3
301•      334 T(J,L,M)=T(J,L,M)+AB(L,I)*T(K,I,M)
302•      GO TO 335
303•      3350 CONTINUE
304•      DO 3351 L=1,3
305•      DO 3351 M=1,3
306•      3351 T(J,L,M)=AB(L,M)
307•      335 CONTINUE
308•      C
309•      C      COORD. TRANSFORMATION OF G VECTORS (TO REF. BODY FRAME)
310•      C
311•      DO 362 I=1,NH
312•      DO 362 J=1,3
313•      GO(I,J)=0.
314•      DO 362 K=1,3
315•      G0(I,J)=G0(I,J)+T(I,K,J)*G(I,K)
316•      362 CONTINUE
317•      C
318•      C      ANG. VELOCITY COMPONENTS OF EACH BODY (IN REF. BODY FRAME)
319•      C
320•      DO 366 K=1,NH
321•      GG(K,1)=GM0(K)*GO(K,1)
322•      GG(K,2)=GM0(K)*GO(K,2)
323•      366 GG(K,3)=GM0(K)*GO(K,3)
324•      DO 361 J=1,NB
325•      KV=HT(J)
326•      WXO(J)=WO(1)
327•      WYO(J)=WO(2)
328•      WZO(J)=WO(3)

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329. DO 36 K=1,KV
330. IF(EPS(K,J),EQ.0) GO TO 36
331. WXO(J)=WXO(J)+GG(K,1)
332. WYO(J)=WYO(J)+GG(K,2)
333. WZO(J)=WZO(J)+GG(K,3)
334. 36 CONTINUE
335. 361 CONTINUE
336. C
337. C ANG. VELOCITY COMPONENTS AT EACH HINGE (IN REF. BODY FRAME)
338. C
339. DO 3666 M=1,NH
340. M1=M+1
341. MC=M(M)+1
342. N1=M1(MC)
343. WHXO=WXO(MC)
344. WHYO=WYO(MC)
345. WHZO=WZO(MC)
346. IF(INI.EQ.M) GO TO 3667
347. DO 3668 N=M1,N1
348. WHXO=WHXO-GG(N,1)
349. WHYO=WHYO-GG(N,2)
350. 3668 WHZO=WHZO-GG(N,3)
351. 3667 CONTINUE
352. WGJ(M,1)=GG(M,3)*WHYO-GG(M,2)*WHZO
353. WGJ(M,2)=GG(M,1)*WHZO-GG(M,3)*WHXO
354. WGJ(M,3)=GG(M,2)*WHXO-GG(M,1)*WHYO
355. 3666 CONTINUE
356. C
357. C TRANSFORM PK AND GK MATRICES TO REFERENCE BODY BASIS-MULT. BY FREQ.
358. C
359. DO 468 K=1,NF
360. KK=F(K,1)+1
361. JNT=F(K,3)
362. IF(KK.EQ.1) GO TO 4720
363. M=M1(KK)
364. DO 472 I=1,3
365. DO 472 J=1,JNT
366. DLKRO(K,I,J)=0.
367. DLKIO(K,I,J)=0.
368. PKOS(K,I,J)=0.
369. GKOS(K,I,J)=0.
370. DO 469 L=1,3
371. DLKRO(K,I,J)=DLKRO(K,I,J)+T(M,L,1)*DLKR(K,L,J)
372. DLKIO(K,I,J)=DLKIO(K,I,J)+T(M,L,1)*DLKI(K,L,J)
373. PKOS(K,I,J)=PKOS(K,I,J)+T(M,L,1)*PK(K,L,J)
374. 469 GKOS(K,I,J)=GKOS(K,I,J)+T(M,L,1)*GK(K,L,J)
375. PKOS(K,I,J)=PKOS(K,I,J)*WF(K,J)
376. 472 GKOS(K,I,J)=GKOS(K,I,J)*WF(K,J)
377. GO TO 468
378. 4720 CONTINUE
379. DO 4721 I=1,3
380. DO 4721 J=1,JNT
381. DLKRO(K,I,J)=DLKR(K,I,J)
382. DLKIO(K,I,J)=DLKI(K,I,J)
383. PKOS(K,I,J)=PK(K,I,J)*WF(K,J)
384. 4721 GKOS(K,I,J)=GK(K,I,J)*WF(K,J)
385. 468 CONTINUE
386. C
387. C COMPUTE TOTAL EXTERNAL FORCE ON EACH SUBSTRUCTURE (IN REF. COORD.)
388. C
389. FEXO(1)=FB(1)
390. FEYO(1)=FB(2)
391. FEZO(1)=FB(3)
392. IF(F1(1).EQ.0) GO TO 254
393. IL=F1(1)
394. JN=F(IL,2)
395. DO 253 J=1,JN
396. FEXO(1)=FEXO(1)+FF(IL,J,1)
397. FEYO(1)=FEYO(1)+FF(IL,J,2)

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398. 253 FEZO(1)=FEZO(1)*FF(IL,J,3)
399. 254 CONTINUE
400. FS(1,1)=FEXO(1)
401. FS(1,2)=FEYO(1)
402. FS(1,3)=FEZO(1)
403. DO 246 N=2,NB
404. K=N-1
405. DO 246N L=1,3
406. 2460 FS(N,L)=FA(K,L)
407. IF(FI(N).EQ.0) GO TO 246
408. IL=FI(N)
409. JN=F(IL,2)
410. DO 245 J=1,JN
411. DO 245 I=1,3
412. 245 FS(N,I)=FS(N,I)*FF(IL,J,I)
413. 246 CONTINUE
414. C
415. C COMPUTE TRANSL. AND ROTAT. DISPLACEMENTS OF APPENDAGE SUB-BODIES
416. C
417. DO 232 K=1,NF
418. JN=F(K,2)
419. LK=F(K,3)
420. DO 233 J=1,JN
421. DO 233 I=1,3
422. U(K,J,I)=0.
423. B(K,J,I)=0.
424. UD(K,J,I)=0.
425. BD(K,J,I)=0.
426. ID=(J-1)*6+1
427. IR=ID+3
428. DO 233 L=1,LK
429. U(K,J,I)=U(K,J,I)+2.*ER(K,ID,L)*DT(K,L)-2.*EI(K,ID,L)*ET(K,L)
430. B(K,J,I)=B(K,J,I)+2.*ER(K,IR,L)*DT(K,L)-2.*EI(K,IR,L)*ET(K,L)
431. UD(K,J,I)=UD(K,J,I)-2.*ER(K,ID,L)*ET(K,L)*WF(K,L)
432. S -2.*EI(K,ID,L)*DT(K,L)*WF(K,L)
433. 233 BD(K,J,I)=BD(K,J,I)-2.*ER(K,IR,L)*ET(K,L)*WF(K,L)
434. S -2.*EI(K,IR,L)*DT(K,L)*WF(K,L)
435. 232 CONTINUE
436. C
437. C COMPUTE C.M. PERTURBATION (FROM NOM. UNDEFORMED LOCATION) ON EACH
438. C SUBSTRUCTURE WITH AN APPENDAGE (LOCAL COORDS.)
439. C
440. DO 262 K=1,NF
441. IK=F(K,1)+1
442. JN=F(K,2)
443. DO 263 I=1,3
444. MCKD(K,I)=0.
445. 263 MCK(K,I)=0.
446. DO 265 J=1,JN
447. DO 265 I=1,3
448. MCKD(K,I)=MCKD(K,I)-UD(K,J,I)*MF(K,J,7)
449. 265 MCK(K,I)=MCK(K,I)-U(K,J,I)*MF(K,J,7)
450. DO 264 I=1,3
451. CKD(K,I)=MCKD(K,I)/MSB(IK)
452. 264 CK(K,I)=MCK(K,I)/MSB(IK)
453. 262 CONTINUE
454. C
455. C COMPUTE TOTAL EXTERNAL TORQUE ON EACH SUBSTRUCTURE W.R.T. ITS
456. C INSTANTANEOUS C.M. (IN LOCAL COORD.)
457. C
458. DO 268 L=1,3
459. 268 TS(L)=TB(L)
460. DO 267 N=2,NB
461. K=N-1
462. DO 267 L=1,3
463. 267 TS(N,L)=TA(K,L)
464. DO 2670 N=1,NB
465. IL=FI(N)
466. IF(IL.EQ.0) GO TO 2670

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467*      JN=F(IL,2)
468*      DO 2671 J=1,JN
469*      DO 2671 L=1,3
470* 2671  TS(N,L)=TS(N,L)+TF(IL,J,L)
471* 2670  CONTINUE
472*      DO 269 N=1,NB
473*      K=FI(N)
474*      IF(K.EQ.0) GO TO 269
475*      TS(N,1)=TS(N,1)+CK(K,2)*FS(N,3)-CK(K,3)*FS(N,2)
476*      TS(N,2)=TS(N,2)+CK(K,3)*FS(N,1)-CK(K,1)*FS(N,3)
477*      TS(N,3)=TS(N,3)+CK(K,1)*FS(N,2)-CK(K,2)*FS(N,1)
478* 269   CONTINUE
479*      DO 271 N=1,NB
480*      K=FI(N)
481*      IF(K.EQ.0) GO TO 271
482*      JN=F(K,2)
483*      DO 272 J=1,JN
484*      RUX=RF(K,J,1)+U(K,J,1)
485*      RUY=RF(K,J,2)+U(K,J,2)
486*      RUZ=RF(K,J,3)+U(K,J,3)
487*      TS(N,1)=TS(N,1)+RUY*FF(K,J,3)-RUZ*FF(K,J,2)
488*      TS(N,2)=TS(N,2)+RUZ*FF(K,J,1)-RUX*FF(K,J,3)
489* 272   TS(N,3)=TS(N,3)+RUX*FF(K,J,2)-RUY*FF(K,J,1)
490* 271   CONTINUE
491*      C
492*      C      TRANSFORM VECTORS TO REF. BODY FRAME
493*      C
494*      TXO(1)=TS(1,1)
495*      TYO(1)=TS(1,2)
496*      TZO(1)=TS(1,3)
497*      DO 17 I=2,NB
498*      M=MI(I)
499*      K=I-1
500*      L=C(K,1)+1
501*      FEXO(1)=T(M,1,1)*FS(1,1)+T(M,2,1)*FS(1,2)+T(M,3,1)*FS(1,3)
502*      FEYO(1)=T(M,1,2)*FS(1,1)+T(M,2,2)*FS(1,2)+T(M,3,2)*FS(1,3)
503*      FEZO(1)=T(M,1,3)*FS(1,1)+T(M,2,3)*FS(1,2)+T(M,3,3)*FS(1,3)
504*      TXO(I) =T(M,1,1)*TS(1,1)+T(M,2,1)*TS(1,2)+T(M,3,1)*TS(1,3)
505*      TYO(I) =T(M,1,2)*TS(1,1)+T(M,2,2)*TS(1,2)+T(M,3,2)*TS(1,3)
506*      TZO(I) =T(M,1,3)*TS(1,1)+T(M,2,3)*TS(1,2)+T(M,3,3)*TS(1,3)
507*      DXO(I,1)=T(M,1,1)*DX(1,1)+T(M,2,1)*DY(1,1)+T(M,3,1)*DZ(1,1)
508*      DYO(I,1)=T(M,1,2)*DX(1,1)+T(M,2,2)*DY(1,1)+T(M,3,2)*DZ(1,1)
509*      DZO(I,1)=T(M,1,3)*DX(1,1)+T(M,2,3)*DY(1,1)+T(M,3,3)*DZ(1,1)
510*      DXO(I,L)=T(M,1,1)*DX(I,L)+T(M,2,1)*DY(I,L)+T(M,3,1)*DZ(I,L)
511*      DYO(I,L)=T(M,1,2)*DX(I,L)+T(M,2,2)*DY(I,L)+T(M,3,2)*DZ(I,L)
512*      DZO(I,L)=T(M,1,3)*DX(I,L)+T(M,2,3)*DY(I,L)+T(M,3,3)*DZ(I,L)
513*      DO 17 J=1,NB
514*      IF(I.EQ.J) GO TO 17
515*      IF(CPS(K,J).EQ.1) GO TO 177
516*      IF(C(K,1).EQ.(J-1)) GO TO 17
517*      OXO(I,J)=OXO(I,L)
518*      DYO(I,J)=DYO(I,L)
519*      DZO(I,J)=DZO(I,L)
520*      GO TO 17
521* 177   DXO(I,J)=T(M,1,1)*DX(I,J)+T(M,2,1)*DY(I,J)+T(M,3,1)*DZ(I,J)
522*      DYO(I,J)=T(M,1,2)*DX(I,J)+T(M,2,2)*DY(I,J)+T(M,3,2)*DZ(I,J)
523*      DZO(I,J)=T(M,1,3)*DX(I,J)+T(M,2,3)*DY(I,J)+T(M,3,3)*DZ(I,J)
524* 17   CONTINUE
525*      DO 367 I=1,NB
526*      DXO(1,1)=DX(1,1)
527*      DYO(1,1)=DY(1,1)
528* 367   DZO(1,1)=DZ(1,1)
529*      C
530*      C      COMPUTE TOTAL EXTERNAL FORCE ON VEHICLE (IN REF. COORD.)
531*      C
532*      FTXO=0.
533*      FTYO=0.
534*      FTZO=0.
535*      DO 247 N=1,NB

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536•      FTXO=FTXO+FEYO(N)
537•      FTYO=FTYO+FEYO(N)
538•      FTZO=FTZO+FEZO(N)
539•      C
540•      C      ADDITIONAL AUGMENTED INERTIA DYADICS (IN REF. BODY FRAME)
541•      C
542•      DO 37 I=1,NB
543•      DO 37 J=1,NB
544•      IF(I,GE,J) GO TO 37
545•      DX2=DXO(I,J)*DXO(J,I)
546•      DY2=DYO(I,J)*DYO(J,I)
547•      DZ2=DZO(I,J)*DZO(J,I)
548•      PS(I,J,1,1)=-TM*(DY2+DZ2)
549•      PS(I,J,1,2)=TM*DXO(J,I)*DYO(I,J)
550•      PS(I,J,1,3)=TM*DXO(J,I)*DZO(I,J)
551•      PS(I,J,2,1)=TM*DYO(J,I)*DXO(I,J)
552•      PS(I,J,2,2)=-TM*(DX2+DZ2)
553•      PS(I,J,2,3)=TM*DYO(J,I)*DZO(I,J)
554•      PS(I,J,3,1)=TM*DZO(J,I)*DXO(I,J)
555•      PS(I,J,3,2)=TM*DZO(J,I)*DYO(I,J)
556•      PS(I,J,3,3)=-TM*(DX2+DY2)
557•      DO 378 M=1,3
558•      DO 378 N=1,3
559•      PS(J,I,M,N)=PS(I,J,N,M)
560•      378 CONTINUE
561•      DO 751 J=1,NB
562•      DO 751 M=1,3
563•      DO 751 N=1,3
564•      751 PS(J,J,M,N)=PH(J,M,N)
565•      C
566•      C      COMPUTE VARIABLE PART OF APPENDAGE INERTIA (IN SUBSTR. COORDS.)
567•      C
568•      DO 236 K=1,NF
569•      KK=F(K,1)+1
570•      M=M(K)
571•      JN=F(K,2)
572•      DO 235 I=1,3
573•      DO 235 J=1,3
574•      VJ(I,J)=0.
575•      235 VJD(I,J)=0.
576•      DO 234 J=1,JN
577•      I11=MF(K,J,1)
578•      I22=MF(K,J,2)
579•      I33=MF(K,J,3)
580•      I12=-MF(K,J,4)
581•      I13=-MF(K,J,5)
582•      I23=-MF(K,J,6)
583•      MS=MF(K,J,7)
584•      R1=RF(K,J,1)
585•      R2=RF(K,J,2)
586•      R3=RF(K,J,3)
587•      U1=U(K,J,1)
588•      U2=U(K,J,2)
589•      U3=U(K,J,3)
590•      B1=B(K,J,1)
591•      B2=B(K,J,2)
592•      B3=B(K,J,3)
593•      VJ(1,1)=VJ(1,1)+2.*(MS*(R2*U2+R3*U3)-I12*B3+I13*B2)
594•      VJ(2,2)=VJ(2,2)+2.*(MS*(R1*U1+R3*U3)-I23*B1+I12*B3)
595•      VJ(3,3)=VJ(3,3)+2.*(MS*(R1*U1+R2*U2)-I13*B2+I23*B1)
596•      VJ(1,2)=VJ(1,2)-MS*(R1*U2+R2*U1)-I13*B1+I23*B2-B3*(I22-I11)
597•      VJ(1,3)=VJ(1,3)-MS*(R1*U3+R3*U1)+I12*B1-I23*B3-B2*(I11-I33)
598•      VJ(2,3)=VJ(2,3)-MS*(R2*U3+R3*U2)-I12*B2+I13*B3-B1*(I33-I22)
599•      U1=UD(K,J,1)
600•      U2=UD(K,J,2)
601•      U3=UD(K,J,3)
602•      B1=BD(K,J,1)
603•      B2=BD(K,J,2)
604•      B3=BD(K,J,3)

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605*      VJD(1,1)=VJD(1,1)+2.*(MS*(R2*U2+R3*U3)-I12*B3+I13*B2)
606*      VJD(2,2)=VJD(2,2)+2.*(MS*(R1*U1+R3*U3)-I23*B1+I12*B3)
607*      VJD(3,3)=VJD(3,3)+2.*(MS*(R1*U1+R2*U2)-I13*B2+I23*B1)
608*      VJD(1,2)=VJD(1,2)-MS*(R1*U2+R2*U1)-I13*B1+I23*B2-B3*(I22-I11)
609*      VJD(1,3)=VJD(1,3)-MS*(R1*U3+R3*U1)+I12*B1-I23*B3-B2*(I11-I33)
610* 234   VJD(2,3)=VJD(2,3)-MS*(R2*U3+R3*U2)-I12*B2+I13*B3-B1*(I33-I22)
611*      VJ(2,1)=VJ(1,2)
612*      VJ(3,1)=VJ(1,3)
613*      VJ(3,2)=VJ(2,3)
614*      DO 495 I=1,3
615*      DO 495 J=1,3
616* 495   PS(KK, KK, I, J)=PS(KK, KK, I, J)+VJ(I, J)
617*      VJD(2,1)=VJD(1,2)
618*      VJD(3,1)=VJD(1,3)
619*      VJD(3,2)=VJD(2,3)
620*      C
621*      C      CONVERT INERTIA MATRIX TO REF. BODY COORDS.
622*      C
623*      IF(KK.EQ.1) GO TO 2370
624*      DO 237 J=1,3
625*      DO 237 I=1,3
626*      AC(J, I)=0.
627*      DO 237 L=1,3
628*      AC(J, I)=AC(J, I)+VJD(J, L)*T(M, L, I)
629* 237   CONTINUE
630*      DO 238 J=1,3
631*      DO 238 I=1,3
632*      VJDO(K, J, I)=0.
633*      DO 238 L=1,3
634*      VJDO(K, J, I)=VJDO(K, J, I)+T(M, L, J)*AC(L, I)
635* 238   CONTINUE
636*      GO TO 236
637* 2370  CONTINUE
638*      DO 2371 J=1,3
639*      DO 2371 I=1,3
640* 2371  VJDO(K, J, I)=VJD(J, I)
641* 236   CONTINUE
642*      C
643*      C      TRANSFORM AUGMENTED BODY INERTIA DYADICS TO REF. BODY FRAME
644*      C
645*      DO 363 I=2, NB
646*      M=H(I)
647*      DO 364 J=1,3
648*      DO 364 K=1,3
649*      AB(J, K)=0.
650*      DO 364 L=1,3
651*      AB(J, K)=AB(J, K)+PS(I, I, J, L)*T(M, L, K)
652* 364   CONTINUE
653*      DO 365 J=1,3
654*      DO 365 K=1,3
655*      PS(I, I, J, K)=0.
656*      DO 365 L=1,3
657*      PS(I, I, J, K)=PS(I, I, J, K)+T(M, L, J)*AB(L, K)
658* 365   CONTINUE
659* 363   CONTINUE
660*      C
661*      C      COMPUTE THE PG50, GP50, AND D50 VECTORS FOR EACH FLEX. APPEND.
662*      C
663*      DO 207 K=1, NF
664*      KK=F(K, 1)+1
665*      M=H(KK)
666*      JNT=F(K, 3)
667*      DO 207 I=1,3
668*      CV(I)=0.
669*      DO 207 J=1, JNT
670* 207   CV(I)=CV(I)+DLKR(K, I, J)*DT(K, J)+DLKI(K, I, J)*ET(K, J)
671*      IF(KK.EQ.1) GO TO 2090
672*      DO 209 I=1,3
673*      PG50(K, I)=0.

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674•      GPSO(K,1)=0.
675•      DDSO(K,1)=0.
676•      DO 209 J=1,3
677•      PGSO(K,1)=PGSO(K,1)+T(M,J,1)*(-MCK(K,J))
678•      GPSO(K,1)=GPSO(K,1)+T(M,J,1)*(-MCKO(K,J))
679•      209  DDSO(K,1)=DDSO(K,1)+T(M,J,1)*CV(J)
680•      GO TO 208
681•      2090 CONTINUE
682•      DO 2091 I=1,3
683•      PGSO(K,1)=-MCK(K,I)
684•      GPSO(K,1)=-MCKD(K,I)
685•      2091  DDSO(K,1)=CV(I)
686•      208  CONTINUE
687•      C
688•      C      VECTOR CROSS PRODUCTS DESCRIBING SYSTEM ROTATIONAL COUPLING.
689•      C      (QUADRATIC TERMS INVOLVING THE CONNECTING BODY ANGULAR
690•      C      VELOCITIES AND THE MUTUAL BARYCENTER-HINGE VECTORS)
691•      C
692•      DO 261 K=1,NF
693•      I=F(K,1)+1
694•      DUX=WZO(I)*PGSO(K,2)-WYO(I)*PGSO(K,3)
695•      DUY=WXO(I)*PGSO(K,3)-WZO(I)*PGSO(K,1)
696•      DUZ=WYO(I)*PGSO(K,1)-WXO(I)*PGSO(K,2)
697•      DUXO(K)=WYO(I)*(DUZ-2.*GPSO(K,3))-WZO(I)*(DUY-2.*GPSO(K,2))
698•      DUYO(K)=WZO(I)*(DUX-2.*GPSO(K,1))-WXO(I)*(DUZ-2.*GPSO(K,3))
699•      261  DUZO(K)=WXO(I)*(DUY-2.*GPSO(K,2))-WYO(I)*(DUX-2.*GPSO(K,1))
700•      DO 230 N=1,NB
701•      I=F(I,N)
702•      DO 476 J=1,3
703•      476  CWNW(N,J)=0.
704•      CPX=0.
705•      CPY=0.
706•      CPZ=0.
707•      CPFX=0.
708•      CPFY=0.
709•      CPFZ=0.
710•      DCPX=0.
711•      DCPY=0.
712•      DCPZ=0.
713•      DO 2301 L=1,NB
714•      IL=F(IL)
715•      IF(IL.EQ.0) GO TO 2303
716•      DCPX=DCPX+DYO(N,L)*DUZO(IL)-DZO(N,L)*DUYO(IL)
717•      DCPY=DCPY+DZO(N,L)*DUXO(IL)-DXO(N,L)*DUZO(IL)
718•      DCPZ=DCPZ+DXO(N,L)*DUYO(IL)-DYO(N,L)*DUXO(IL)
719•      2303 CONTINUE
720•      WDX=WYO(L)*DZO(L,N)-WZO(L)*DYO(L,N)
721•      WDY=WZO(L)*DXO(L,N)-WXO(L)*DZO(L,N)
722•      WDX=WXO(L)*DYO(L,N)-WYO(L)*DXO(L,N)
723•      WWFDX=WYO(L)*WDZ-WZO(L)*WDY
724•      WWFDY=WZO(L)*WDX-WXO(L)*WDZ
725•      WWFDZ=WXO(L)*WDY-WYO(L)*WDX
726•      IF(I.EQ.0) GO TO 482
727•      CWNW(N,1)=CWNW(N,1)+WWFDX
728•      CWNW(N,2)=CWNW(N,2)+WWFDY
729•      CWNW(N,3)=CWNW(N,3)+WWFDZ
730•      482 CONTINUE
731•      CPFX=CPFX+WWFDX
732•      CPFY=CPFY+WWFDY
733•      CPFZ=CPFZ+WWFDZ
734•      IF(N.EQ.L) GO TO 2301
735•      WNDX=TN+WWFDX+FEYO(L)
736•      WNDY=TN+WWFDY+FEYO(L)
737•      WNDZ=TN+WWFDZ+FEZO(L)
738•      DWNWX=DYO(N,L)*WNDZ-DZO(N,L)*WNDY
739•      DWNWY=DZO(N,L)*WNDX-DXO(N,L)*WNDZ
740•      DWNWZ=DXO(N,L)*WNDY-DYO(N,L)*WNDX
741•      CPX=CPX+DWNWX
742•      CPY=CPY+DWNWY

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743*      CPZ=CPZ+D*WZ
744* 2301 CONTINUE
745*      DFX=DYO(N,N)+FEZO(N)-DZO(N,N)+FEYO(N)
746*      DFY=DZO(N,N)+FEXO(N)-DXO(N,N)+FEZO(N)
747*      DFZ=DXO(N,N)+FEYO(N)-OYO(N,N)+FEXO(N)
748*      HX=PS(N,N,1,1)*WXO(N)+PS(N,N,1,2)*WYO(N)+PS(N,N,1,3)*WZO(N)
749*      HY=PS(N,N,2,1)*WXO(N)+PS(N,N,2,2)*WYO(N)+PS(N,N,2,3)*WZO(N)
750*      HZ=PS(N,N,3,1)*WXO(N)+PS(N,N,3,2)*WYO(N)+PS(N,N,3,3)*WZO(N)
751*      IF(I.EQ.0) GO TO 243
752*      HXD=VJDO(1,1,1)*WXO(N)+VJDO(1,1,2)*WYO(N)+VJDO(1,1,3)*WZO(N)
753*      HYD=VJDO(1,2,1)*WXO(N)+VJDO(1,2,2)*WYO(N)+VJDO(1,2,3)*WZO(N)
754*      HZD=VJDO(1,3,1)*WXO(N)+VJDO(1,3,2)*WYO(N)+VJDO(1,3,3)*WZO(N)
755*      FACT=MSB(N)/TM
756*      FTXM=FTXO*FACT
757*      FTYM=FTYO*FACT
758*      FTZM=FTZO*FACT
759*      PGFX=(PGSO(1,2)*(FEZO(N)-FTZM)-PGSO(1,3)*(FEYO(N)+FTYM))/MSB(N)
760*      PGFY=(PGSO(1,3)*(FEXO(N)-FTXM)-PGSO(1,1)*(FEZO(N)-FTZM))/MSB(N)
761*      PGFZ=(PGSO(1,1)*(FEYO(N)-FTYM)-PGSO(1,2)*(FEXO(N)-FTXM))/MSB(N)
762*      PWDX=PGSO(1,2)*CPFZ-PGSO(1,3)*CPFY
763*      PWDY=PGSO(1,3)*CPFZ-PGSO(1,1)*CPFZ
764*      PWDZ=PGSO(1,1)*CPFY-PGSO(1,2)*CPFZ
765*      WDSXO=WYO(N)*DDSO(1,3)-WZO(N)*DDSO(1,2)
766*      WDSYO=WZO(N)*DDSO(1,1)-WXO(N)*DDSO(1,3)
767*      WDSZO=WXO(N)*DDSO(1,2)-WYO(N)*DDSO(1,1)
768*      GO TO 244
769* 243 CONTINUE
770*      HXD=0.
771*      HYD=0.
772*      HZD=0.
773*      PGFX=0.
774*      PGFY=0.
775*      PGFZ=0.
776*      PWDX=0.
777*      PWDY=0.
778*      PWDZ=0.
779*      WDSXO=0.
780*      WDSYO=0.
781*      WDSZO=0.
782* 244 CONTINUE
783*      K = 3*(N-1)
784*      E(K+1,1)=HY*WZO(N)-HZ*WYO(N)+IXO(N)*CPX+DFX-HXD+PGFX-PWDX-WDSXO
785*      S=DCPX
786*      E(K+2,1)=HZ*WXO(N)-HX*WZO(N)+IYO(N)*CPY+DFY-HYD+PGFY-PWDY-WDSYO
787*      S=DCPY
788*      E(K+3,1)=HX*WYO(N)-HY*WXO(N)+IZO(N)*CPZ+DFZ-HZD+PGFZ-PWDZ-WDSZO
789*      S=DCPZ
790* 230 CONTINUE
791* C
792* C      ADD MATRIX ELEMENT COMPUTATION (3X3)
793* C
794*      DO 3001 I=1,3
795*      DO 3001 J=1,3
796* 3001 ADD(I,J)=0.
797*      DO 3 I=1,NB
798*      DO 3 J=1,NB
799*      ADD(1,1)=ADD(1,1)+PS(I,J,1,1)
800*      ADD(1,2)=ADD(1,2)+PS(I,J,1,2)
801*      ADD(1,3)=ADD(1,3)+PS(I,J,1,3)
802*      ADD(2,2)=ADD(2,2)+PS(I,J,2,2)
803*      ADD(2,3)=ADD(2,3)+PS(I,J,2,3)
804*      ADD(3,3)=ADD(3,3)+PS(I,J,3,3)
805*      CONTINUE
806*      ADD(2,1)=ADD(1,2)
807*      ADD(3,1)=ADD(1,3)
808*      ADD(3,2)=ADD(2,3)
809* C
810* C      FLEX. APPEND. CONTRIBUTION TO ADD MATRIX COMPUTATION (3X3)
811* C

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812•      DO 210 K=1,NB
813•      KK=F1(K)
814•      DO 210 L=1,NB
815•      IF(K.GT.L) GO TO 210
816•      DO 2103 I=1,3
817•      DO 2103 J=1,3
818•      2103 PSF(K,L,I,J)=0.
819•      LL=F1(L)
820•      IF(KK.EQ.0) GO TO 2101
821•      DP1=PGSO(KK,1)*DXO(L,K)
822•      DP2=PGSO(KK,2)*DYO(L,K)
823•      DP3=PGSO(KK,3)*DZO(L,K)
824•      PSF(K,L,1,1)=-DP2-OP3
825•      PSF(K,L,2,2)=-DP1-OP3
826•      PSF(K,L,3,3)=-DP1-OP2
827•      PSF(K,L,1,2)=PGSO(KK,2)*DXO(L,K)
828•      PSF(K,L,1,3)=PGSO(KK,3)*DXO(L,K)
829•      PSF(K,L,2,1)=PGSO(KK,1)*DYO(L,K)
830•      PSF(K,L,2,3)=PGSO(KK,3)*DYO(L,K)
831•      PSF(K,L,3,1)=PGSO(KK,1)*DZO(L,K)
832•      PSF(K,L,3,2)=PGSO(KK,2)*DZO(L,K)
833•      2101 CONTINUE
834•      IF(LL.EQ.0) GO TO 210
835•      IF(K.EQ.L) GO TO 2102
836•      PD1=PGSO(LL,1)*DXO(K,L)
837•      PD2=PGSO(LL,2)*DYO(K,L)
838•      PD3=PGSO(LL,3)*DZO(K,L)
839•      PSF(K,L,1,1)=PSF(K,L,1,1)-PD2-PD3
840•      PSF(K,L,2,2)=PSF(K,L,2,2)-PD1-PD3
841•      PSF(K,L,3,3)=PSF(K,L,3,3)-PD1-PD2
842•      PSF(K,L,1,2)=PSF(K,L,1,2)+DYO(K,L)*PGSO(LL,1)
843•      PSF(K,L,1,3)=PSF(K,L,1,3)+DZO(K,L)*PGSO(LL,1)
844•      PSF(K,L,2,1)=PSF(K,L,2,1)+DXO(K,L)*PGSO(LL,2)
845•      PSF(K,L,2,3)=PSF(K,L,2,3)+DZO(K,L)*PGSO(LL,2)
846•      PSF(K,L,3,1)=PSF(K,L,3,1)+DXO(K,L)*PGSO(LL,3)
847•      PSF(K,L,3,2)=PSF(K,L,3,2)+DYO(K,L)*PGSO(LL,3)
848•      GO TO 210
849•      2102 CONTINUE
850•      DO 214 I=1,3
851•      DO 214 J=1,3
852•      214 AB(I,J)=PSF(K,L,I,J)
853•      DO 215 I=1,3
854•      DO 215 J=1,3
855•      215 PSF(K,L,I,J)=AB(I,J)+AB(J,I)
856•      210 CONTINUE
857•      DO 2151 K=1,NB
858•      DO 2151 L=1,NB
859•      IF(K.LE.L) GO TO 2151
860•      DO 2141 I=1,3
861•      DO 2141 J=1,3
862•      2141 PSF(K,L,I,J)=PSF(L,K,J,I)
863•      2151 CONTINUE
864•      DO 3004 K=1,NB
865•      KK=F1(K)
866•      DO 3004 L=1,NB
867•      LL=F1(L)
868•      IF((KK.EQ.0).AND.(LL.EQ.0)) GO TO 3004
869•      DO 3003 I=1,3
870•      DO 3003 J=1,3
871•      A00(I,J)=A00(I,J)-PSF(K,L,I,J)
872•      3003 CONTINUE
873•      3004 CONTINUE
874•      C
875•      C      AGK VECTOR ELEMENT COMPUTATION (3x1)
876•      C
877•      C      AKM SCALAR ELEMENT COMPUTATION
878•      C
879•      DO 14 M=1,NH
880•      IQ=M(M)+1

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881*      AV(M,1)=0.
882*      AV(M,2)=0.
883*      AV(M,3)=0.
884*      DO 7 J=1,NB
885*      DO 7 I=IQ,NB
886*      DO 11 N=1,3
887*      IF(EPS(M,I).EQ.0) GO TO 7
888*      PSG(J,I,N)=0.
889*      DO 10 L=1,3
890*      10  PSG(J,I,N)=PSG(J,I,N)+(PS(J,I,N,L)-PSF(J,I,N,L))*GO(M,L)
891*      11  AV(M,N)=AV(M,N)+PSG(J,I,N)
892*      7   CONTINUE
893*      DO 14 K=1,NH
894*      IF(K.GT.M) GO TO 14
895*      JQ=H(K)+1
896*      AIS(1)=0.
897*      AIS(2)=0.
898*      AIS(3)=0.
899*      DO 15 J=JQ,NB
900*      DO 15 I=IQ,NB
901*      IF((EPS(K,J).EQ.0).OR.(EPS(M,I).EQ.0)) GO TO 15
902*      DO 18 N=1,3
903*      18  AIS(N)=AIS(N)+PSG(J,I,N)
904*      15  CONTINUE
905*      AS(K,M)=GO(K,1)*AIS(1)+GO(K,2)*AIS(2)+GO(K,3)*AIS(3)
906*      14  CONTINUE
907*      C
908*      C      ADFI AND ADFR MATRIX COMPUTATION (3XNKT)
909*      C
910*      DO 219 K=1,NF
911*      JK=F(K,3)
912*      JQ=F(K,1)+1
913*      DO 222 I=1,3
914*      DO 222 J=1,3
915*      222  AB(I,J)=0.
916*      DO 221 L=1,NB
917*      AB(1,2)=AB(1,2)-DZO(L,JQ)
918*      AB(1,3)=AB(1,3)+DYO(L,JQ)
919*      221  AB(2,3)=AB(2,3)-DXO(L,JQ)
920*      AB(2,1)=-AB(1,2)
921*      AB(3,1)=-AB(1,3)
922*      AB(3,2)=-AB(2,3)
923*      DO 220 I=1,3
924*      DO 220 J=1,JK
925*      ADFR(K,I,J)=DLKRO(K,I,J)
926*      ADFI(K,I,J)=DLKIO(K,I,J)
927*      DO 220 L=1,3
928*      ADFR(K,I,J)=ADFR(K,I,J) - AB(I,L)*GKOS(K,L,J)
929*      220  ADFI(K,I,J)=ADFI(K,I,J) - AB(I,L)*PKOS(K,L,J)
930*      219  CONTINUE
931*      C
932*      C      AKFR VECTOR COMPUTATION (1XNKT) (FLEX,COUPLING WITH RIGID SUBSTRUCTURE)
933*      C
934*      C      AKFI VECTOR COMPUTATION (1XNKT) (FLEX,COUPLING WITH RIGID SUBSTRUCTURE)
935*      C
936*      DO 224 K=1,NF
937*      JK=F(K,3)
938*      JQ=F(K,1)+1
939*      DO 2245 J=1,JK
940*      ZSR(K,J)=0.
941*      2245 ZSI(K,J)=0.
942*      DO 224 M=1,NH
943*      DO 231 I=1,3
944*      DO 231 J=1,3
945*      231  AB(I,J)=0.
946*      DO 226 L=1,NB
947*      IF(EPS(M,L).EQ.0) GO TO 226
948*      AB(1,2)=AB(1,2)-DZO(L,JQ)
949*      AB(1,3)=AB(1,3)+DYO(L,JQ)

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950•      AB(2,3)=AB(2,3)-DXO(L,JQ)
951• 226  CONTINUE
952•      AB(2,1)=-AB(1,2)
953•      AB(3,1)=-AB(1,3)
954•      AB(3,2)=-AB(2,3)
955•      DO 228 I=1,3
956•      DO 228 J=1,JK
957•      DUR(I,J)=DLKRO(K,I,J)
958•      DUI(I,J)=DLKID(K,I,J)
959•      IF(EPS(M,K),EQ,0) DUR(I,J)=0.
960•      IF(EPS(M,K),EQ,0) DUI(I,J)=0.
961•      DO 228 L=1,3
962•      DUR(I,J)=DUR(I,J)-AB(I,L)*GKOS(K,L,J)
963• 228  DUI(I,J)=DUI(I,J)-A4(I,L)*PKOS(K,L,J)
964•      DO 2241 J=1,JK
965•      DO 2241 I=1,3
966•      ZSR(K,J)=ZSR(K,J)+DUR(I,J)*WGJ(M,I)
967• 2241 ZSI(K,J)=ZSI(K,J)+DUI(I,J)*WGJ(M,I)
968•      DO 229 J=1,JK
969•      AKFR(K,M,J)=0.
970•      AKFI(K,M,J)=0.
971•      DO 229 I=1,3
972•      AKFR(K,M,J)=AKFR(K,M,J)+GO(M,I)*DUR(I,J)
973• 229  AKFI(K,M,J)=AKFI(K,M,J)+GO(M,I)*DUI(I,J)
974• 224  CONTINUE
975•      C
976•      C   COMPUTE CORRECTION ELEMENTS FOR (E) VECTOR
977•      C
978•      DO 41 J=2,NB
979•      JK=MI(J)
980•      DO 411 M=1,3
981• 411  CW(J,M)=0.
982•      DO 42 K=1,JK
983•      IF(EPS(K,J),EQ,0) GO TO 42
984•      CW(J,1)=CW(J,1)+WGJ(K,1)
985•      CW(J,2)=CW(J,2)+WGJ(K,2)
986•      CW(J,3)=CW(J,3)+WGJ(K,3)
987• 42  CONTINUE
988• 41  CONTINUE
989•      DO 40 I=1,NB
990•      EA(I)=0.
991•      EA(2)=0.
992•      EA(3)=0.
993•      DO 401 J=2,NB
994•      DO 4507 M=1,3
995•      DO 4507 L=1,3
996• 4507 EA(M)=EA(M)+(PS(I,J,M,L)-PSF(I,J,M,L))*CW(J,L)
997• 401  CONTINUE
998•      KI=3*(I-1)
999•      E(KI+1,1)=E(KI+1,1)-EA(1)
1000•      E(KI+2,1)=E(KI+2,1)-EA(2)
1001•      E(KI+3,1)=E(KI+3,1)-EA(3)
1002• 40  CONTINUE
1003•      DO 55 MI=1,3
1004• 55  EC(MI)=E(MI,1)
1005•      DO 52 J=2,NB
1006•      DO 52 M=1,3
1007•      KI=3*(J-1)+M
1008• 52  EC(M)=EC(M)+E(KI,1)
1009•      I=0
1010•      DO 60 K=1,NB
1011•      JK=M(K)+1
1012•      IF(P1(K),NE,0) GO TO 60
1013•      I=I+1
1014•      EC(I+3)=0.
1015•      DO 601 M=1,3
1016• 601  CE(M)=0.
1017•      DO 61 J=JK,NB
1018•      IF(EPS(K,J),EQ,0) GO TO 61

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1019•      DO 65 M=1,3
1020•      J1=3*(J-1)+M
1021• 65   CE(M)=CE(M)+E(J1,1)
1022• 61   CONTINUE
1023•      DO 66 L=1,3
1024• 66   EC(I+3)=EC(I+3)+GO(K,L)*CE(L)
1025•      EC(J+3)=EC(J+3)+TH(K)
1026• 60   CONTINUE
1027•      DO 610 I=1,3
1028•      DO 610 J=1,NH
1029•      IF(P1(J).EQ.0) GO TO 610
1030•      EC(I)=EC(I)-AV(J,1)*GMDD(J)
1031• 610  CONTINUE
1032•      K=0
1033•      IV=3
1034•      DO 612 I=1,NH
1035•      IF(P1(I).NE.0) GO TO 612
1036•      K=K+1
1037•      IV=IV+1
1038•      DO 611 J=1,NH
1039•      IF(P1(J).EQ.0) GO TO 611
1040•      IF(I.GT.J) AS(I,J)=AS(J,I)
1041•      EC(K+3)=EC(K+3)-AS(I,J)*GMDD(J)
1042• 611  CONTINUE
1043• 612  CONTINUE
1044• C
1045• C   COMPUTE RT. HAND SIDE OF APPENDAGE EQUATIONS (IN APPEND. COORDS.)
1046• C
1047•      DO 477 K=1,NF
1048•      DO 479 I=1,3
1049• 479  CDU(K,I)=0.
1050•      DO 478 L=1,NF
1051•      IF(K.EQ.L) GO TO 478
1052•      CDU(K,2)=CDU(K,2)+DUYO(L)
1053•      CDU(K,1)=CDU(K,1)+DUXO(L)
1054•      CDU(K,3)=CDU(K,3)+DUZO(L)
1055• 478  CONTINUE
1056• 477  CONTINUE
1057•      DO 483 K=1,NF
1058•      I=F(K,1)+1
1059•      M=HI(I)
1060•      CQ(1)=(FTXO+CDU(K,1))/TH + CWWO(I,1)
1061•      CQ(2)=(FTYO+CDU(K,2))/TH + CWWO(I,2)
1062•      CQ(3)=(FTZO+CDU(K,3))/TH + CWWO(I,3)
1063•      IF(I.EQ.1) GO TO 4840
1064•      DO 484 J=1,3
1065•      VE(K,J)=-WWE(K,J)
1066•      DO 484 L=1,3
1067• 484  VE(K,J)=VE(K,J)+T(M,J,L)*CQ(L)
1068•      GO TO 483
1069• 4840  CONTINUE
1070•      DO 4841 J=1,3
1071• 4841  VE(K,J)=CQ(J)-WWE(K,J)
1072• 483  CONTINUE
1073•      DO 485 K=1,NF
1074•      NL=F(K,2)
1075•      I=F(K,1)+1
1076•      M=HI(I)
1077•      R1=SR(K,1)
1078•      R2=SR(K,2)
1079•      R3=SR(K,3)
1080•      IF(I.EQ.1) GO TO 4870
1081•      DO 487 J=1,3
1082• 487  WW(J)=T(M,J,1)*WXO(I)+T(M,J,2)*WYO(I)+T(M,J,3)*WZO(I)
1083•      GO TO 4872
1084• 4870  CONTINUE
1085•      WW(1)=WXO(I)
1086•      WW(2)=WYO(I)
1087•      WW(3)=WZO(I)
1088• 4872  CONTINUE

```

```

1089•      W11=WW(1)*2-R1**2
1090•      W22=WW(2)*2-R2**2
1091•      W33=WW(3)*2-R3**2
1092•      W12=WW(1)*WW(2)-R1*R2
1093•      W13=WW(1)*WW(3)-R1*R3
1094•      W23=WW(2)*WW(3)-R2*R3
1095•      DO 486 N=1,NL
1096•      N6=6*(N-1)
1097•      DO 488 J=1,3
1098•      JN=N6+J
1099•      JM=JN+3
1100•      VB(K,JN)=FF(K,N,J)
1101• 488   VB(K,JM)=TF(K,N,J)
1102•      VB(K,N6+1)=VB(K,N6+1)-MF(K,N,7)*(-RF(K,N,1)*(W33+W22)+RF(K,N,2)*W1
1103•      S2+RF(K,N,3)*W13)
1104•      VB(K,N6+2)=VB(K,N6+2)-MF(K,N,7)*(-RF(K,N,2)*(W33+W11)+RF(K,N,1)*W1
1105•      S2+RF(K,N,3)*W23)
1106•      VB(K,N6+3)=VB(K,N6+3)-MF(K,N,7)*(-RF(K,N,3)*(W11+W22)+RF(K,N,1)*W1
1107•      S3+RF(K,N,2)*W23)
1108•      CE(1)=MF(K,N,1)*WW(1)-MF(K,N,4)*WW(2)-MF(K,N,5)*WW(3)
1109•      CE(2)=-MF(K,N,4)*WW(1)+MF(K,N,2)*WW(2)-MF(K,N,6)*WW(3)
1110•      CE(3)=-MF(K,N,5)*WW(1)-MF(K,N,6)*WW(2)+MF(K,N,3)*WW(3)
1111•      CL(1)=MF(K,N,1)*R1-MF(K,N,4)*R2-MF(K,N,5)*R3
1112•      CL(2)=-MF(K,N,4)*R1+MF(K,N,2)*R2-MF(K,N,6)*R3
1113•      CL(3)=-MF(K,N,5)*R1-MF(K,N,6)*R2+MF(K,N,3)*R3
1114•      VB(K,N6+4)=VB(K,N6+4)-(WW(2)*CE(3)-WW(3)*CE(2))
1115•      S+(R2*CL(3)-R3*CL(2))
1116•      VB(K,N6+5)=VB(K,N6+5)-(WW(3)*CE(1)-WW(1)*CE(3))
1117•      S+(R3*CL(1)-R1*CL(3))
1118•      VB(K,N6+6)=VB(K,N6+6)-(WW(1)*CE(2)-WW(2)*CE(1))
1119•      S+(R1*CL(2)-R2*CL(1))
1120• 486   CONTINUE
1121• 485   CONTINUE
1122•      NV=IV
1123•      DO 491 K=1,NF
1124•      JN=F(K,3)
1125•      NL=F(K,2)
1126•      NL6=6*NL
1127•      DO 492 J=1,JN
1128•      IL=NV+J
1129•      IO=IL+NTHO
1130•      VV1=(ET(K,J)+ZF(K,J)*DT(K,J))*2.
1131•      VV2=(DT(K,J)-ZF(K,J)*ET(K,J))*2.
1132•      DO 493 N=1,NL6
1133•      VV1=VV1+EI(K,N,J)*VB(K,N)*2.
1134• 493   VV2=VV2-ER(K,N,J)*VB(K,N)*2.
1135•      DO 494 N=1,3
1136•      VV1=VV1-GK(K,N,J)*VE(K,N)
1137• 494   VV2=VV2+PK(K,N,J)*VE(K,N)
1138•      VV1=-WF(K,J)*VV1-ZSR(K,J)
1139•      VV2=-WF(K,J)*VV2-ZSI(K,J)
1140•      EC(IL)=VV1
1141•      EC(IO)=VV2
1142•      DO 4920 L=1,NH
1143•      IF(PI(L).EQ.0) GO TO 4920
1144•      EC(IL)=EC(IL)-AKFR(L,K,J)*GMDD(L)
1145•      EC(IO)=EC(IO)-AKFI(L,K,J)*GMDD(L)
1146• 4920  CONTINUE
1147• 492   CONTINUE
1148• 491   NV=NV+JN
1149•      C
1150•      C   ENTER CONSTANTS INTO FLEX. BODY PORTION OF COEFF. MATRIX A
1151•      C
1152•      NV=IV
1153•      DO 462 K=1,NF
1154•      NL=F(K,3)
1155•      DO 463 I=1,NL
1156•      IL=NV+I
1157•      IO=IL+NTHO

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```

1158*      DO 463 J=1,NL
1159*      JL=NV+J
1160*      JO=JL+NTMO
1161*      A(IL,JL)=0.
1162*      A(IL,JO)=0.
1163*      A(IO,JL)=0.
1164*      A(IO,JO)=0.
1165*      IF(I.EQ.J) A(IL,JL)=2.
1166*      IF(I.EQ.J) A(IO,JO)=2.
1167*      463  CONTINUE
1168*      462  NV=NV+NL
1169*      C
1170*      C      ENTER COEFF. WHICH COUPLE REF. BODY AND FLEX. APPENDAGES INTO A
1171*      C
1172*      NV=IV
1173*      DO 464 K=1,NF
1174*      NL=F(K,3)
1175*      DO 465 J=1,3
1176*      DO 465 I=1,NL
1177*      IL=NV+I
1178*      IO=IL+NTMO
1179*      A(IL,J)=AOF(K,J,I)
1180*      A(J,IL)=A(IL,J)
1181*      A(IO,J)=AOFI(K,J,I)
1182*      465  A(J,IO)=A(IO,J)
1183*      464  NV=NV+NL
1184*      C
1185*      C      ENTER COEFF. WHICH COUPLE SUBSTR. BODIES AND FLEX. APPEND. INTO A
1186*      C
1187*      NV=IV
1188*      DO 466 K=1,NF
1189*      NL=F(K,3)
1190*      JI=0
1191*      DO 467 J=1,NH
1192*      IF(P1(J).NE.0) GO TO 467
1193*      JI=JI+1
1194*      DO 467 I=1,NL
1195*      IL=NV+I
1196*      IO=IL+NTMO
1197*      A(IL,JI+3)=AKFR(K,J,I)
1198*      A(IO,JI+3)=AKFI(K,J,I)
1199*      A(JI+3,IL)=A(IL,JI+3)
1200*      A(JI+3,IO)=A(IO,JI+3)
1201*      467  CONTINUE
1202*      467  CONTINUE
1203*      466  NV=NV+NL
1204*      C
1205*      C      CALCULATE FLEX. BODY COUPLING COEFF. AND ENTER INTO A MATRIX
1206*      C
1207*      NCO=IV
1208*      DO 473 L=1,NF
1209*      NL=F(L,3)
1210*      NRO=IV
1211*      DO 474 K=1,NF
1212*      NR=F(K,3)
1213*      IF(K.EQ.L) GO TO 474
1214*      DO 475 I=1,NR
1215*      IK=NRO+I
1216*      IO=IK+NTMO
1217*      DO 475 J=1,NL
1218*      JK=NCO+J
1219*      JO=JK+NTMO
1220*      A(IK,JK)=0.
1221*      A(IO,JK)=0.
1222*      A(IK,JO)=0.
1223*      A(IO,JO)=0.
1224*      DO 475 N=1,3
1225*      A(IK,JK)=A(IK,JK)-GKOS(K,N,I)*GKOS(L,N,J)/TM
1226*      A(IO,JK)=A(IO,JK)-PKOS(K,N,I)*GKOS(L,N,J)/TM

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1227•      A(IK,JO)=A(IK,JO)-GKOS(K,N,I)*PKOS(L,N,J)/TM
1228•      A(IO,JO)=A(IO,JO)-PKOS(K,N,I)*PKOS(L,N,J)/TM
1229•  9750  CONTINUE
1230•      A(JK,IK)=A(IK,JK)
1231•      A(JK,IO)=A(IO,JK)
1232•      A(JO,IK)=A(IK,JO)
1233•      A(JO,IO)=A(IO,JO)
1234•  475   CONTINUE
1235•  474   NRO=NRO+NR
1236•  473   NCO=NCO+NL
1237•      C
1238•      C      LOAD SYSTEM MATRIX (A) WITH ADD,ADK,AKM ELEMENTS
1239•      C
1240•      DO 23 I=1,3
1241•      DO 23 J=1,3
1242•  23     A(I,J)=A00(I,J)
1243•      DO 24 I=1,3
1244•      K=0
1245•      DO 24 J=1,NH
1246•      IF(P1(J).NE.0) GO TO 24
1247•      K=K+1
1248•      A(K+3,I)=AV(J,I)
1249•      A(I,K+3)=AV(J,I)
1250•  24     CONTINUE
1251•      K=0
1252•      DO 250 I=1,NH
1253•      IF(P1(I).NE.0) GO TO 250
1254•      K=K+1
1255•      L=0
1256•      DO 25 J=1,NH
1257•      IF(P1(J).NE.0) GO TO 25
1258•      L=L+1
1259•      IF(K.GT.L) GO TO 26
1260•      A(K+3,L+3)=AS(I,J)
1261•      GO TO 25
1262•  26     A(K+3,L+3)=A(L+3,K+3)
1263•  25     CONTINUE
1264•  250    CONTINUE
1265•      C
1266•      C      ANGULAR MOMENTUM OF THE SYSTEM
1267•      C
1268•      IF(P1(NH+1).NE.0) GO TO 8752
1269•      DO 5451 I=1,3
1270•      MH(I)=0.
1271•      DO 5451 J=1,3
1272•  5451   MH(I)=MH(I)+A(I,J)*W0(J)
1273•      DO 5452 I=1,3
1274•      DO 5452 J=1,9M
1275•  5452   MH(I)=MH(I)+AV(J,I)*GMD(J)
1276•      DO 5453 I=1,3
1277•      DO 5453 K=1,NF
1278•      NL=F(K,3)
1279•      DO 5454 J=1,NL
1280•  5454   MH(I)=MH(I)+AOPR(K,I,J)*DT(K,J)+AOFI(K,I,J)*ET(K,J)
1281•  5453   CONTINUE
1282•      MH=SQRT(MH(1)**2 + MH(2)**2 + MH(3)**2)
1283•  8752   CONTINUE
1284•      C
1285•      C      SOLVE SYSTEM MATRIX FOR REFERENCE BODY ANG. ACCELERATION AND HINGE
1286•      C      (RELATIVE) ROTATIONAL ACCELERATIONS
1287•      C
1288•      NT=V+NT2
1289•      IT=IV+NT2
1290•      KV=IV
1291•      CALL CHOLD(92,A,ST,IT,EC,0.,1.0-7)
1292•      DO 910 J=NT,4,-1
1293•      IF(J.LE.V) GO TO 913
1294•      JV=J-(V-IV)
1295•      EC(J)=EC(JV)

```

```

1296*      GO TO 910
1297* 913  CONTINUE
1298*      K=J-3
1299*      IF (PI(K).NE.0) GO TO 911
1300*      EC(J)=EC(KV)
1301*      KV=KV-1
1302*      GO TO 910
1303* 911  EC(J)=GMDD(K)
1304* 910  CONTINUE
1305*      DO 9003 I=1,V
1306* 9003 #OOT(I)=EC(I)
1307*      I=V
1308*      DO 9001 K=1,NF
1309*      NL=F(K,3)
1310*      DO 9002 N=1,NL
1311*      IO=I+N
1312*      IL=IO+NTMO
1313*      DTD(K,N)=EC(IO)
1314* 9002 ETD(K,N)=EC(IL)
1315* 9001 I=I+NL
1316* 92   CONTINUE
1317*      RETURN
1318*      END

```

DIAGNOSTICS

ATION TIME = 44.48 SUPS

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## Appendix D

### Subroutine MBDYFN Listing and User Requirements

#### *Subroutine Entry Statements*

```
CALL MBDYFN(NC, H, MB, MS, PB, PS, G, PI,  
            NF, F, EIG, REC, RF, WF, ZF)  
CALL MRATE(NC, TH, TB, TS, FB, FS, TF, FF, GM, GMD,  
            GMDD, ET, ETD, WO, WDOT, ETD, HM)
```

#### *Input / Output Variable Type and Storage Specifications*

```
INTEGER NC, NF, H( $n_c$ , 2), F( $n_f$ , 3), PI( $n + 1$ )  
REAL MB(7), MS( $n_c$ , 7), PB( $n_c$ , 3), PS( $n_c$ ,  $n_c$ , 3),  
      G( $n$ , 3), TH( $n$ ), TB(3), TS( $n_c$ , 3), FB(3), FS( $n_c$ , 3),  
      GM( $n$ ), GMD( $n$ ), GMDD( $n$ ), EIG( $n_f$ ,  $6n_k$ ,  $N_k$ ),  
      REC( $n_f$ , 6,  $N_k$ ), RF( $n_f$ ,  $n_k$ , 3), WF( $n_f$ ,  $N_k$ ),  
      ZF( $n_f$ ,  $N_k$ ), TF( $n_f$ ,  $n_k$ , 3), FF( $n_f$ ,  $n_k$ , 3),  
      ET( $n_f$ ,  $N_k$ ), ETD( $n_f$ ,  $N_k$ ), WO(3).  
DOUBLE PRECISION WDOT( $n + 3$ ), ETDD( $n_f$ ,  $N_k$ )
```

#### *External Subroutines Called*

CHOLD—(see Appendix C and statement 1013)

#### *Subroutine Setup*

Insert the Fortran statement

```
PARAMETER QC =  $n_c$ , QH =  $n$ , QF =  $n_f$ , NK =  $n_k$ , NKT =  $N_k$ 
```

(If more than one appendage is present, use the *largest*  $n_k$  and  $N_k$  for the PARAMETER statement to provide sufficient storage.)

#### *Data Restrictions*

```
 $n > 1$ ,  $n_f > 1$ ,  $n_c > 1$ ,  $n_k > 1$ ,  $N_k > 1$ 
```

Core Storage Required

Code: 4500 words

Data: ~ 500 words (minimum; increases with  $n, n_f$ ).

Listing

```

10 SUBROUTINE MBDYFN(NC,C,MB,MA,PB,PA,G,PI,NF,F,EIG,REC,RF,WF,ZF)
20 C
30 C ADJUSTABLE DIMENSIONS
40 C
50 INTEGER PI(1),C(NC,2)
60 REAL MB(1),MA(NC,7),PB(NC,3),PA(NC,NC,3)
70 PARAMETER QC=1,QH=2,QF=1,NK=1,NKT=7
80 PARAMETER NOK=6*NK,S=QC+1,V=QH+3,V4=4*V,S3=3,S,Q=QH,NH=QM
90 PARAMETER ST=V+QF*NKT,S4=4*ST
100 C
110 C ADDITIONAL DIMENSIONED VARIABLES
120 C
130 DOUBLE PRECISION A(ST,ST),BMASS(S)
140 INTEGER EPS(Q,S),CPS(QC,S),H(Q),MI(S),F(S),F(NF,3)
150 REAL ADD(3,3),AB(3,3),AOF(QF,3,NKT),AKF(QF,QH,NKT),AS(Q,Q),AV(Q,3)
160 S,A(S,3),CE(3),CK(QF,3),CQ(3),CWD(S,3),CN(S,3),DX(S,S),DY(S,S),DZ(
170 S,S),DXO(S,S),DYO(S,S),DZO(S,S),DLK(QF,3,NKT),DLKO(QF,3,NKT),DUR(3
180 S,NKT),EA(3),EIG(NF,NOK,NKT),FEXO(S),FEYO(S),FEZO(S),FS(S,3),GQ(Q,3
190 S),GG(Q,3),G(Q,3),IX(S),IYY(S),IZZ(S),IXY(S),IXZ(S),IYZ(S),LX(S,S)
200 S,LY(S,S),LZ(S,S),MSB(S),MCK(QF,3),PH(S,3,3),PSG(S,S,3),PS(S,S,3,3)
210 S,PK(QF,3,NKT),PGSO(QF,3),PSF(S,S,3,3),PKO(QF,3,NKT),RF(NF,NK,3),RE
220 SC(NF,6,NKT),TXO(S),TYO(S),TZO(S),T(Q,3,3),TS(S,3),U(QF,NK,3),VE(QF
230 S,3),VB(QF,NOK),WF(NF,NKT),WGU(QH,3),ZF(NF,NKT),ZSR(QF,NKT),HM(3)
240 EQUIVALENCE (A,PS),(LX,DXO),(LY,DYO),(LZ,DZO)
250 NB=NC+1
260 C
270 C DEFINE EPS(K,J) USING C
280 C
290 DO 86 K=1,NC
300 DO 86 J=2,NB
310 IF(K.EQ.(J-1)) CPS(K,J)=1
320 IF(K.LT.(J-1)) GO TO 87
330 GO TO 86
340 87 CONTINUE
350 JO=K+1
360 JI=J+1
370 DO 89 L=JO,J1
380 IF(K.GT.(L-1)) GO TO 89
390 IF((CPS(K,L).EQ.1).AND.(C(J-1,1).EQ.(L-1))) CPS(K,J)=1
400
410 89 CONTINUE
420 86 CONTINUE
430 L=0
440 DO 1 J=1,NC
450 KK=C(J,2)
460 DO 1 K=1,KK
470 L=L+1
480 DO 1 I=1,NB
490 EPS(L,1)=CPS(J,1)
500 C
510 C COMPUTE H(I)=C, WHERE I=HINGE LABEL AND C=CONNECTION LABEL
520 C
530 I=0
540 DO 8 J=2,NB
550 KK=C(J-1,2)
560 DO 8 K=1,KK
570 I=I+1
580 8 H(I)=J-1

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59° C
60° C COMPUTE MI(I)=J, WHERE I=BODY LABEL+1 AND J=NEAREST HINGE LABEL
61° C
62° MI(I)=1
63° MI(NB)=NH
64° DO 47 I=NH,1
65° IF(I.EQ.1) GO TO 47
66° K1=H(I)
67° K2=H(I)-1
68° IF(K1.EQ.K2) GO TO 47
69° MI(K2+1)=I-1
70° 47 CONTINUE
71° C
72° C DEFINE F1(I,J)=K, WHERE J=BODY-LABEL+1 AND K IS APPENDAGE-LABEL
73° C (IF K=0, BODY HAS NO FLEX, APPENDAGE)
74° C
75° DO 239 N=1,NB
76° 239 F1(N)=0
77° DO 242 K=1,NF
78° JN=F(K,1)+1
79° 242 F1(JN)=K
80° NF=NF
81° NB=NB
82° C
83° C DEFINE SUBSTRUCTURE MASSES
84° C
85° MSB(1)=MB(7)
86° DO 248 N=2,NB
87° 248 MSB(N)=MA(N-1,7)
88° C
89° C TOTAL NUMBER OF FLEX, APPENDAGE MODES TO BE RETAINED
90° C
91° NTHO=0
92° DO 461 K=1,NF
93° 461 NTHO=NTHO+F(K,3)
94° C
95° C INITIAL CALCULATION OF BARYCENTER VECTORS W.R.T. BODY C.G.S
96° C AND HINGE POINTS
97° C
98° IX(1)=MB(1)
99° IY(1)=MB(2)
100° IZ(1)=MB(3)
101° IX(1)=MB(4)
102° IX(1)=MB(5)
103° IY(1)=MB(6)
104° BMASS(1)=MB(7)
105° TH=BMASS(1)
106° DO 35 J=2,NB
107° IX(J)=MA(J-1,1)
108° IY(J)=MA(J-1,2)
109° IZ(J)=MA(J-1,3)
110° IX(J)=MA(J-1,4)
111° IX(J)=MA(J-1,5)
112° IY(J)=MA(J-1,6)
113° BMASS(J)=MA(J-1,7)
114° 35 TH=TH+BMASS(J)
115° DO 149 I=1,NB
116° I1=I-1
117° DO 149 J=1,NB
118° J1=J-1
119° IF(I1.EQ.J1) GO TO 163
120° IF(I1.GT.J1) GO TO 70
121° IF(I1.EQ.1) GO TO 80
122° IF(CPS(I1,J1).EQ.1) GO TO 400
123° 70 LX(I,J)=PA(I1,I1,1)
124° LY(I,J)=PA(I1,I1,2)
125° LZ(I,J)=PA(I1,I1,3)
126° GO TO 149
127° 400 CONTINUE

```

```

1280      DO 600 K=1,J1
1290          IF(CPS(K,J)*EQ.1) GO TO 500
1300      600  CONTINUE
1310          GO TO 149
1320      500  LX(I,J)=PA(I,K,1)
1330          LY(I,J)=PA(I,K,2)
1340          LZ(I,J)=PA(I,K,3)
1350          GO TO 149
1360      80   DO 90 L=1,J1
1370          IF(CPS(L,J)*EQ.1) GO TO 101
1380      90   CONTINUE
1390          GO TO 149
1400      101  LX(I,J)=PB(L,1)
1410          LY(I,J)=PB(L,2)
1420          LZ(I,J)=PB(L,3)
1430          GO TO 149
1440      163  LX(I,J)=0.
1450          LY(I,J)=0.
1460          LZ(I,J)=0.
1470      149  CONTINUE
1480          DO 13 N=1,NB
1490          DO 13 J=1,NB
1500          DX(N,J)=LX(N,J)
1510          DY(N,J)=LY(N,J)
1520          DZ(N,J)=LZ(N,J)
1530          DO 13 K=1,NB
1540          DX(N,J)=DX(N,J)+(BMASS(K)/TM)*LX(N,K)
1550          DY(N,J)=DY(N,J)+(BMASS(K)/TM)*LY(N,K)
1560          DZ(N,J)=DZ(N,J)+(BMASS(K)/TM)*LZ(N,K)
1570      C
1580      C   CALCULATION OF AUGMENTED INERTIA DYADICS FOR EACH BODY
1590      C
1600          DO 31 N=1,NB
1610          PH(N,1,1)=IX(N)
1620          PH(N,1,2)=-IXY(N)
1630          PH(N,1,3)=-IXZ(N)
1640          PH(N,2,2)=IYY(N)
1650          PH(N,2,3)=-IYZ(N)
1660          PH(N,3,3)=IZZ(N)
1670          DO 30 J=1,NB
1680          PH(N,1,1)=PH(N,1,1)+BMASS(J)*(DY(N,J)**2+DZ(N,J)**2)
1690          PH(N,1,2)=PH(N,1,2)+BMASS(J)*DX(N,J)*DY(N,J)
1700          PH(N,1,3)=PH(N,1,3)+BMASS(J)*DX(N,J)*DZ(N,J)
1710          PH(N,2,2)=PH(N,2,2)+BMASS(J)*(DX(N,J)**2+DZ(N,J)**2)
1720          PH(N,2,3)=PH(N,2,3)+BMASS(J)*DY(N,J)*DZ(N,J)
1730      30  PH(N,3,3)=PH(N,3,3)+BMASS(J)*(DX(N,J)**2+DY(N,J)**2)
1740          PH(N,2,1)=PH(N,1,2)
1750          PH(N,3,1)=PH(N,1,3)
1760          PH(N,3,2)=PH(N,2,3)
1770      C
1780      C   DEFINE PK(3 X NKT ARRAY)
1790      C   DEFINE DLK=TRANSPOSE MATRIX (3 X NKT ARRAY)
1800      C
1810          DO 201 K=1,NF
1820          JNT=FK(K,3)
1830          DO 201 I=1,J
1840          DO 201 J=1,JNT
1850          PK(K,I,J)=REC(K,I,J)
1860      201  DLK(K,I,J)=REC(K,I+3,J)
1870          RETURN
1880          ENTRY HRATE(NC,TH,TB,TA,FB,PA,TF,FF,GM,GMO,GMOO,ET,ETO,NG,NOOT,ETO
1890          SD,MM)
1900          REAL TF(QF,NK,3),FF(QF,NK,3),ET(QF,NKT),ETD(QF,NKT),TB(3),TA(NC,3)
1910          S,FB(3),FA(NC,3),GH(1),GMO(1),GMOO(1),TM(1),NO(3),WXS(S),WYS(S),WZ
1920          S(S),E(S,1)
1930          DOUBLE PRECISION EC(1),ETDD(QF,NKT),WDOT(V)
1940      C
1950      C   BODY-TO-BODY COORDINATE TRANSFORMATION MATRICES
1960      C

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197* DO 335 J=1,NH
198* MM=J-1
199* N=M(J)+1
200* SGH=SIN(GH(J))
201* CGH=COS(GH(J))
202* CGH1=1.-CGH
203* G1=CGH1*G(J,1)
204* G2=CGH1*G(J,2)
205* G3=CGH1*G(J,3)
206* SG1=SGH*G(J,1)
207* SG2=SGH*G(J,2)
208* SG3=SGH*G(J,3)
209* G1S=G1*G(J,1)
210* G2S=G2*G(J,2)
211* G3S=G3*G(J,3)
212* G12=G1*G(J,2)
213* G13=G1*G(J,3)
214* G23=G2*G(J,3)
215* AB(1,1)=CGH*G1S
216* AB(1,2)=SG3+G12
217* AB(1,3)=-SG2+G13
218* AB(2,1)=-SG3+G12
219* AB(2,2)=CGH*G2S
220* AB(2,3)=SG1+G23
221* AB(3,1)=SG2+G13
222* AB(3,2)=-SG1+G23
223* AB(3,3)=CGH*G3S
224* IF(J.EQ.1) GO TO 3350
225* DO 321 L=MM+1
226* IF(EPS(L,N).EQ.1) GO TO 322
227* 321 CONTINUE
228* GO TO 3350
229* 322 K=L
230* DO 334 L=1,3
231* DO 334 M=1,3
232* T(J,L,M)=0.
233* DO 334 I=1,3
234* 334 T(J,L,M)=T(J,L,M)+AB(L,I)*T(K,I,M)
235* GO TO 335
236* 3350 CONTINUE
237* DO 3351 L=1,3
238* DO 3351 M=1,3
239* 3351 T(J,L,M)=AB(L,M)
240* 335 CONTINUE
241* C
242* C COORD. TRANSFORMATION OF G VECTORS (TO REF. BODY FRAME)
243* C
244* DO 362 I=1,NH
245* DO 362 J=1,3
246* GO(I,J)=0.
247* DO 362 K=1,3
248* GO(I,J)=GO(I,J)+T(I,K,J)*G(I,K)
249* 362 CONTINUE
250* C
251* C ANG. VELOCITY COMPONENTS OF EACH BODY (IN REF. BODY FRAME)
252* C
253* DO 366 K=1,NH
254* GG(K,1)=GMD(K)*GO(K,1)
255* GG(K,2)=GMD(K)*GO(K,2)
256* 366 GG(K,3)=GMD(K)*GO(K,3)
257* DO 361 J=1,NB
258* KV=M(J)
259* WXO(J)=WO(1)
260* WYO(J)=WO(2)
261* WZO(J)=WO(3)
262* DO 36 K=1,KV
263* IF(EPS(K,J).EQ.0) GO TO 36
264* WXO(J)=WXO(J)+GG(K,1)
265* WYO(J)=WYO(J)+GG(K,2)

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266*      WZO(J)=WZO(J)+GG(K,J)
267* 36   CONTINUE
268* 361  CONTINUE
269*  C
270*  C   ANG. VELOCITY COMPONENTS AT EACH HINGE (IN REF. BODY FRAME)
271*  C
272*      DO 3666 M=1,NH
273*      NI=M+1
274*      MC=M(NI)+1
275*      NI=M1(MC)
276*      WHXO=WXO(MC)
277*      WHY0=WYO(MC)
278*      WHZO=WZO(MC)
279*      IF(N1.EQ.M) GO TO 3667
280*      DO 3668 N=M1,NI
281*      WHXO=WHXO-GG(N,1)
282*      WHY0=WHY0-GG(N,2)
283*      WHZO=WHZO-GG(N,3)
284* 3668  CONTINUE
285*      WGJ(M,1)=GG(M,3)*WHY0-GG(M,2)*WHZO
286*      WGJ(M,2)=GG(M,1)*WHZO-GG(M,3)*WHXO
287*      WGJ(M,3)=GG(M,2)*WHXO-GG(M,1)*WHY0
288* 3666  CONTINUE
289*  C
290*  C   TRANSFORM PK AND DLK TO REF. BODY BASIS
291*  C
292*      DO 468 K=1,NF
293*      KK=F(K,1)+1
294*      JNT=F(K,3)
295*      IF(KK.EQ.1) GO TO 4720
296*      M=M1(KK)
297*      DO 469 I=1,3
298*      DO 469 J=1,JNT
299*      DLK0(K,I,J)=0.
300*      PK0(K,I,J)=0.
301*      DO 469 L=1,3
302*      DLK0(K,I,J)=DLK0(K,I,J)+T(M,L,I)*DLK(K,L,J)
303*      PK0(K,I,J)=PK0(K,I,J)+T(M,L,I)*PK(K,L,J)
304*      GO TO 468
305* 4720  CONTINUE
306*      DO 4721 I=1,3
307*      DO 4721 J=1,JNT
308*      DLK(K,I,J)=DLK0(K,I,J)
309*      PK(K,I,J)=PK0(K,I,J)
310* 468   CONTINUE
311*  C
312*  C   COMPUTE TOTAL EXTERNAL FORCE ON EACH SUBSTRUCTURE (IN REF. COORD.1)
313*  C
314*      FEXO(1)=FB(1)
315*      FEYO(1)=FB(2)
316*      FEZO(1)=FB(3)
317*      IF(F1(1).EQ.0) GO TO 254
318*      IL=F1(1)
319*      JN=F(IL,2)
320*      DO 253 J=1,JN
321*      FEXO(1)=FEXO(1)+FF(IL,J,1)
322*      FEYO(1)=FEYO(1)+FF(IL,J,2)
323*      FEZO(1)=FEZO(1)+FF(IL,J,3)
324* 253   CONTINUE
325*      FS(1,1)=FEXO(1)
326*      FS(1,2)=FEYO(1)
327*      FS(1,3)=FEZO(1)
328*      DO 246 N=2,NB
329*      K=N-1
330*      DO 2460 L=1,3
331*      FS(N,L)=FA(K,L)
332*      IF(F1(N).EQ.0) GO TO 246
333*      IL=F1(N)
334*      JN=F(IL,2)

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335. DO 245 J=1,JN
336. DO 245 I=1,3
337. 245 FS(N,I)=FS(N,I)*FF(IL,J,I)
338. 240 CONTINUE
339. C
340. C COMPUTE TRANSL. AND ROTAT. DISPLACEMENTS OF APPENDAGE SUB-BODIES
341. C
342. DO 232 K=1,NF
343. JN=F(K,2)
344. LK=F(K,3)
345. DO 233 J=1,JN
346. DO 233 I=1,3
347. U(K,J,I)=U
348. ID=(J-1)*6+I
349. DO 233 L=1,LK
350. 233 U(K,J,I)=U(K,J,I)+EIG(K,ID,L)*ET(K,L)
351. 232 CONTINUE
352. C
353. C COMPUTE C.M. PERTURBATION (FROM NOM. UNDEFORMED LOCATION) ON EACH
354. C SUBSTRUCTURE WITH AN APPENDAGE (LOCAL COORD.)
355. C
356. DO 262 K=1,NF
357. IK=F(K,1)*I
358. JN=F(K,3)
359. DO 263 I=1,3
360. 263 MCK(K,I)=0.
361. DO 265 J=1,JN
362. DO 265 I=1,3
363. 265 MCK(K,I)=MCK(K,I)-PK(K,I,J)*ET(K,J)
364. DO 266 I=1,3
365. 266 CK(K,I)=MCK(K,I)/MSB(IK)
366. 262 CONTINUE
367. C
368. C COMPUTE TOTAL EXTERNAL TORQUE ON EACH SUBSTRUCTURE W.R.T. ITS
369. C INSTANTANEOUS C.M. (IN LOCAL COORD.)
370. C
371. DO 268 L=1,3
372. 268 TS(I,L)=TB(L)
373. DO 267 N=1,NB
374. K=N-1
375. DO 267 L=1,3
376. 267 TS(N,L)=TA(K,L)
377. DO 2670 N=1,NB
378. IL=F1(N)
379. IF(IL.EQ.0) GO TO 2670
380. JN=F(IL,2)
381. DO 2671 J=1,JN
382. DO 2671 L=1,3
383. 2671 TS(N,L)=TS(N,L)+TF(IL,J,L)
384. 2670 CONTINUE
385. DO 269 N=1,NB
386. K=F1(N)
387. IF(K.EQ.0) GO TO 269
388. TS(N,1)=TS(N,1)+CK(K,2)*FS(N,3)+CK(K,3)*FS(N,2)
389. TS(N,2)=TS(N,2)+CK(K,3)*FS(N,1)+CK(K,1)*FS(N,3)
390. TS(N,3)=TS(N,3)+CK(K,1)*FS(N,2)+CK(K,2)*FS(N,1)
391. 269 CONTINUE
392. DO 271 N=1,NB
393. K=F1(N)
394. IF(K.EQ.0) GO TO 271
395. JN=F(K,2)
396. DO 272 J=1,JN
397. RUX=RF(K,J,1)+U(K,J,1)
398. RUY=RF(K,J,2)+U(K,J,2)
399. RUZ=RF(K,J,3)+U(K,J,3)
400. TS(N,1)=TS(N,1)+RUY*FF(K,J,3)-RUZ*FF(K,J,2)
401. TS(N,2)=TS(N,2)+RUZ*FF(K,J,1)-RUX*FF(K,J,3)
402. 272 TS(N,3)=TS(N,3)+RUX*FF(K,J,2)-RUY*FF(K,J,1)
403. 271 CONTINUE

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404 C
405 C   TRANSFORM VECTORS TO REF. BODY FRAME
406 C
407 TXO(1)=TS(1,1)
408 TYO(1)=TS(1,2)
409 TZO(1)=TS(1,3)
410 DO 17 I=2,NB
411 M=H(I)
412 K=I-1
413 L=C(K,1)+L
414 FEYO(1)=T(M,1,1)*FS(1,1)+T(M,2,1)*FS(1,2)+T(M,3,1)*FS(1,3)
415 FEZO(1)=T(M,1,2)*FS(1,1)+T(M,2,2)*FS(1,2)+T(M,3,2)*FS(1,3)
416 FEZO(1)=T(M,1,3)*FS(1,1)+T(M,2,3)*FS(1,2)+T(M,3,3)*FS(1,3)
417 TXO(I)=T(M,1,1)*TS(1,1)+T(M,2,1)*TS(1,2)+T(M,3,1)*TS(1,3)
418 TYO(I)=T(M,1,2)*TS(1,1)+T(M,2,2)*TS(1,2)+T(M,3,2)*TS(1,3)
419 TZO(I)=T(M,1,3)*TS(1,1)+T(M,2,3)*TS(1,2)+T(M,3,3)*TS(1,3)
420 DXO(1,1)=T(M,1,1)*DX(1,1)+T(M,2,1)*DY(1,1)+T(M,3,1)*DZ(1,1)
421 DYO(1,1)=T(M,1,2)*DX(1,1)+T(M,2,2)*DY(1,1)+T(M,3,2)*DZ(1,1)
422 DZO(1,1)=T(M,1,3)*DX(1,1)+T(M,2,3)*DY(1,1)+T(M,3,3)*DZ(1,1)
423 DXO(1,L)=T(M,1,1)*DX(1,L)+T(M,2,1)*DY(1,L)+T(M,3,1)*DZ(1,L)
424 DYO(1,L)=T(M,1,2)*DX(1,L)+T(M,2,2)*DY(1,L)+T(M,3,2)*DZ(1,L)
425 DZO(1,L)=T(M,1,3)*DX(1,L)+T(M,2,3)*DY(1,L)+T(M,3,3)*DZ(1,L)
426 DO 17 J=1,NB
427 IF(I,EQ,J) GO TO 17
428 IF(CPS(K,J)*EQ,1) GO TO 177
429 IF(C(K,1)*EQ,(J=1)) GO TO 17
430 DXO(I,J)=DXO(I,L)
431 DYO(I,J)=DYO(I,L)
432 DZO(I,J)=DZO(I,L)
433 GO TO 17
434 177 DXO(I,J)=T(M,1,1)*DX(I,J)+T(M,2,1)*DY(I,J)+T(M,3,1)*DZ(I,J)
435 DYO(I,J)=T(M,1,2)*DX(I,J)+T(M,2,2)*DY(I,J)+T(M,3,2)*DZ(I,J)
436 DZO(I,J)=T(M,1,3)*DX(I,J)+T(M,2,3)*DY(I,J)+T(M,3,3)*DZ(I,J)
437 17 CONTINUE
438 DO 367 I=1,NB
439 DXO(1,1)=DX(1,1)
440 DYO(1,1)=DY(1,1)
441 367 DZO(1,1)=DZ(1,1)
442 C
443 C   COMPUTE TOTAL EXTERNAL FORCE ON VEHICLE (IN REF. COORD.)
444 C
445 FTXO=0.
446 FTYO=0.
447 FTZO=0.
448 DO 247 N=1,NB
449 FTXO=FTXO+FEYO(N)
450 FTYO=FTYO+FEZO(N)
451 247 FTZO=FTZO+FEZO(N)
452 C
453 C   ADDITIONAL AUGMENTED INERTIA DYADICS (IN REF. BODY FRAME)
454 C
455 DO 37 I=1,NB
456 DO 37 J=1,NB
457 IF(I,GE,J) GO TO 37
458 DX2=DXO(1,J)*DXO(J,1)
459 DY2=DYO(1,J)*DYO(J,1)
460 DZ2=DZO(1,J)*DZO(J,1)
461 PS(1,J,1,1)=-TM*(DY2+DZ2)
462 PS(1,J,1,2)=TM*DXO(J,1)*DYO(1,J)
463 PS(1,J,1,3)=TM*DXO(J,1)*DZO(1,J)
464 PS(1,J,2,1)=TM*DYO(J,1)*DXO(1,J)
465 PS(1,J,2,2)=-TM*(DX2+DZ2)
466 PS(1,J,2,3)=TM*DYO(J,1)*DZO(1,J)
467 PS(1,J,3,1)=TM*DZO(J,1)*DXO(1,J)
468 PS(1,J,3,2)=TM*DZO(J,1)*DYO(1,J)
469 PS(1,J,3,3)=-TM*(DX2+DY2)
470 DO 378 M=1,3
471 DO 378 N=1,3
472 378 PS(J,1,M+N)=PS(1,J,M,N)

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473. 37 CONTINUE
474. DO 751 M=1,3
475. DO 751 N=1,3
476. 751 PS(I,1,M,N)=PH(I,M,N)
477. C
478. C TRANSFORM AUGMENTED BODY INERTIA DYADICS TO REF. BODY FRAME
479. C
480. DO 363 I=2,NB
481. M=H1(I)
482. DO 364 J=1,3
483. DO 364 K=1,3
484. AB(J,K)=0.
485. DO 364 L=1,3
486. AB(J,K)=AB(J,K)+PH(I,J,L)*T(M,L,K)
487. 364 CONTINUE
488. DO 365 J=1,3
489. DO 365 K=1,3
490. PS(I,1,J,K)=0.
491. DO 365 L=1,3
492. PS(I,1,J,K)=PS(I,1,J,K)+T(M,L,J)*AB(L,K)
493. 365 CONTINUE
494. 363 CONTINUE
495. C
496. C COMPUTE THE PGSO VECTORS FOR EACH FLEX. APPENDAGE
497. C
498. DO 208 K=1,NF
499. KK=F(K,1)+1
500. M=H1(KK)
501. JNT=F(K,3)
502. IF(KK.EQ.1) GO TO 2090
503. DO 209 I=1,3
504. PGSO(K,I)=0.
505. DO 209 J=1,3
506. 209 PGSO(K,I)=PGSO(K,I)+T(M,J,I)*(-MCK(K,J))
507. GO TO 208
508. 2090 CONTINUE
509. DO 2091 I=1,3
510. 2091 PGSO(K,I)=MCK(K,I)
511. 208 CONTINUE
512. C
513. C VECTOR CROSS PRODUCTS DESCRIBING SYSTEM ROTATIONAL COUPLING.
514. C (QUADRATIC TERMS INVOLVING THE CONNECTING BODY ANGULAR
515. C VELOCITIES AND THE MUTUAL BARYCENTER-HINGE VECTORS)
516. C
517. DO 230 N=1,NB
518. I=F1(N)
519. DO 476 J=1,3
520. 476 C*WD(N,J)=0.
521. CPX=0.
522. CPY=0.
523. CPZ=0.
524. C*PFX=0.
525. C*PFY=0.
526. C*PFZ=0.
527. DCPX=0.
528. DCPY=0.
529. DCPZ=0.
530. DO 2301 L=1,NB
531. IL=F1(L)
532. IF(IL.NE.0) GO TO 7149
533. WDX=WY0(L)*DZO(L,N)-WZO(L)*DYO(L,N)
534. WDY=WZO(L)*DXO(L,N)-WXO(L)*DZO(L,N)
535. Wdz=WXO(L)*DYO(L,N)-WYO(L)*DXO(L,N)
536. W*FDX=WY0(L)*WDZ-WZO(L)*WDY
537. W*FDY=WZO(L)*WDX-WXO(L)*WDZ
538. W*FDZ=WXO(L)*WDY-WYO(L)*WDX
539. GO TO 7148
540. 7149 CONTINUE
541. W*FDX=0.

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542°      WFFDY=0.
543°      WFFDZ=0.
544°  7148 CONTINUE
545°      IF(I.EQ.0) GO TO 482
546°      CWNW(N,1)=CWNW(N,1)+WFFDX
547°      CWNW(N,2)=CWNW(N,2)+WFFDY
548°      CWNW(N,3)=CWNW(N,3)+WFFDZ
549°  482 CONTINUE
550°      CPFY=CPFY+WFFDX
551°      CPFZ=CPFZ+WFFDY
552°      CPFX=CPFZ+WFFDZ
553°      IF(N.EQ.L) GO TO 2301
554°      WWDY=TH*WFFDX+FEYO(L)
555°      WWDZ=TH*WFFDY+FEZO(L)
556°      WWDZ=TH*WFFDZ+FEZO(L)
557°      DWWDY=DYO(N,L)*WWDZ-DZO(N,L)*WWDY
558°      DWWDZ=DZO(N,L)*WWDY-DXO(N,L)*WWDZ
559°      DWWDZ=DXO(N,L)*WWDY-DYO(N,L)*WWDZ
560°      CPX=CPX+DWWDY
561°      CPY=CPY+DWWDZ
562°      CPZ=CPZ+DWWDZ
563°  2301 CONTINUE
564°      DFY=DYO(N,N)*FEZO(N)-DZO(N,N)*FEYO(N)
565°      DFZ=DZO(N,N)*FEXO(N)-DXO(N,N)*FEZO(N)
566°      DFZ=DXO(N,N)*FEYO(N)-DYO(N,N)*FEXO(N)
567°      IF(I.NE.0) GO TO 7147
568°      HX=PS(N,N,1,1)*WXO(N)+PS(N,N,1,2)*WYO(N)+PS(N,N,1,3)*WZO(N)
569°      HY=PS(N,N,2,1)*WXO(N)+PS(N,N,2,2)*WYO(N)+PS(N,N,2,3)*WZO(N)
570°      HZ=PS(N,N,3,1)*WXO(N)+PS(N,N,3,2)*WYO(N)+PS(N,N,3,3)*WZO(N)
571°      GO TO 7146
572°  7147 CONTINUE
573°      HX=0.
574°      HY=0.
575°      HZ=0.
576°  7146 CONTINUE
577°      IF(I.EQ.0) GO TO 243
578°      FACT=MSB(N)/TM
579°      FTXM=FTXO*FACT
580°      FTYM=FTYO*FACT
581°      FTZM=FTZO*FACT
582°      PGFY=(PGSO(1,2)*(FEZO(N)-FTZM)-PGSO(1,3)*(FEYO(N)-FTYM))/MSB(N)
583°      PGFY=(PGSO(1,3)*(FEXO(N)-FTXM)-PGSO(1,1)*(FEZO(N)-FTZM))/MSB(N)
584°      PGFZ=(PGSO(1,1)*(FEYO(N)-FTYM)-PGSO(1,2)*(FEXO(N)-FTXM))/MSB(N)
585°      PWDY=PGSO(1,2)*CPFZ-PGSO(1,3)*CPFY
586°      PWDY=PGSO(1,3)*CPFX-PGSO(1,1)*CPFZ
587°      PWDZ=PGSO(1,1)*CPFY-PGSO(1,2)*CPFX
588°      GO TO 244
589°  243 CONTINUE
590°      PGFY=0.
591°      PGFZ=0.
592°      PWDY=0.
593°      PWDZ=0.
594°      PWDZ=0.
595°      PWDZ=0.
596°  244 CONTINUE
597°      K = 3*(N-1)
598°      E(K+1,1)=HY*WZO(N)-HZ*WYO(N)+TXO(N)+CPX+DFY+PGFY-PWDY
599°      E(K+2,1)=HZ*WXO(N)-HX*WZO(N)+TYO(N)+CPY+DFZ+PGFZ-PWDZ
600°      E(K+3,1)=HX*WYO(N)-HY*WXO(N)+TZO(N)+CPZ+DFZ+PGFZ-PWDZ
601°  230 CONTINUE
602°  C
603°  C      ADD MATRIX ELEMENT COMPUTATION (3x3)
604°  C
605°      DO 3001 I=1,3
606°      DO 3001 J=1,3
607°  3001 A00(I,J)=0.
608°      DO 3 I=1,NB
609°      DO 3 J=1,NB
610°      A00(I,J)=A00(I,J)+PS(I,J,1,1)

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ORIGINAL PAGE 13  
OF POOR QUALITY

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611°      AOO(1,2)=A00(1,2)+PS(1,J,1,2)
612°      AOO(1,3)=A00(1,3)+PS(1,J,1,3)
613°      AOO(2,2)=A00(2,2)+PS(1,J,2,2)
614°      AOO(2,3)=A00(2,3)+PS(1,J,2,3)
615°      AOO(3,3)=A00(3,3)+PS(1,J,3,3)
616°      J      CONTINUE
617°      AOO(2,1)=A00(1,2)
618°      AOO(3,1)=A00(1,3)
619°      AOO(3,2)=A00(2,3)
620°      C
621°      C      FLEX. APPEND. CONTRIBUTION TO AOO MATRIX COMPUTATION (JXJ)
622°      C
623°      DO 210 K=1,N6
624°      KK=F1(K)
625°      DO 210 L=1,N8
626°      IF(K.GT.L) GO TO 210
627°      DO 2103 I=1,3
628°      DO 2103 J=1,3
629°      2103 PSF(K,L,I,J)=0.
630°      LL=F1(L)
631°      IF(KK.EQ.U) GO TO 2101
632°      DP1=PGSO(KK,1)*DXO(L,K)
633°      DP2=PGSO(KK,2)*DYO(L,K)
634°      DP3=PGSO(KK,3)*DZO(L,K)
635°      PSF(K,L,1,1)=-DP2-DP3
636°      PSF(K,L,2,2)=-DP1-DP3
637°      PSF(K,L,3,3)=-DP1-DP2
638°      PSF(K,L,1,2)=PGSO(KK,2)*DXO(L,K)
639°      PSF(K,L,1,3)=PGSO(KK,3)*DXO(L,K)
640°      PSF(K,L,2,1)=PGSO(KK,1)*DYO(L,K)
641°      PSF(K,L,2,3)=PGSO(KK,3)*DYO(L,K)
642°      PSF(K,L,3,1)=PGSO(KK,1)*DZO(L,K)
643°      PSF(K,L,3,2)=PGSO(KK,2)*DZO(L,K)
644°      2101 CONTINUE
645°      IF(LL.EQ.O) GO TO 210
646°      IF(K.EQ.L) GO TO 2102
647°      PD1=PGSO(LL,1)*DXO(K,L)
648°      PD2=PGSO(LL,2)*DYO(K,L)
649°      PD3=PGSO(LL,3)*DZO(K,L)
650°      PSF(K,L,1,1)=PSF(K,L,1,1)-PD2-PD3
651°      PSF(K,L,2,2)=PSF(K,L,2,2)-PD1-PD3
652°      PSF(K,L,3,3)=PSF(K,L,3,3)-PD1-PD2
653°      PSF(K,L,1,2)=PSF(K,L,1,2)+DYO(K,L)*PGSO(LL,1)
654°      PSF(K,L,1,3)=PSF(K,L,1,3)+DZO(K,L)*PGSO(LL,1)
655°      PSF(K,L,2,1)=PSF(K,L,2,1)+DXO(K,L)*PGSO(LL,2)
656°      PSF(K,L,2,3)=PSF(K,L,2,3)+DZO(K,L)*PGSO(LL,2)
657°      PSF(K,L,3,1)=PSF(K,L,3,1)+DXO(K,L)*PGSO(LL,3)
658°      PSF(K,L,3,2)=PSF(K,L,3,2)+DYO(K,L)*PGSO(LL,3)
659°      GO TO 210
660°      2102 CONTINUE
661°      DO 214 I=1,3
662°      DO 214 J=1,3
663°      214 AB(I,J)=PSF(K,L,I,J)
664°      DO 215 I=1,3
665°      DO 215 J=1,3
666°      215 PSF(K,L,I,J)=AB(I,J)+AB(J,I)
667°      210 CONTINUE
668°      DO 2151 K=1,N8
669°      DO 2151 L=1,N8
670°      IF(K.LE.L) GO TO 2151
671°      DO 2141 I=1,3
672°      DO 2141 J=1,3
673°      2141 PSF(K,L,I,J)=PSF(L,K,J,I)
674°      2151 CONTINUE
675°      DO 3004 K=1,N8
676°      KK=F1(K)
677°      DO 3004 L=1,N8
678°      LL=F1(L)
679°      IF((KK.EQ.O).AND.(LL.EQ.U)) GO TO 3004

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```

680°      DO 3003 I=1,3
681°      DO 3003 J=1,3
682°      A00(I,J)=A00(I,J)-PSF(K,L,I,J)
683° 3003 CONTINUE
684° 3004 CONTINUE
685° C
686° C   AOK VECTOR ELEMENT COMPUTATION (3x1)
687° C
688° C   AKH SCALAR ELEMENT COMPUTATION
689° C
690°      DO 14 M=1,NH
691°      IQ=M(M)+1
692°      AV(M,1)=0.
693°      AV(M,2)=0.
694°      AV(M,3)=0.
695°      DO 7 J=1,NB
696°      DO 7 I=IQ,NB
697°      DO 11 N=1,3
698°      IF(EPS(M,I).EQ.0) GO TO 7
699°      PSG(J,I,N)=0.
700°      DO 10 L=1,3
701° 10   PSG(J,I,N)=PSG(J,I,N)+(PS(J,I,N,L)-PSF(J,I,N,L))*GO(M,L)
702° 11   AV(M,N)=AV(M,N)+PSG(J,I,N)
703° 7    CONTINUE
704°      DO 14 K=1,NH
705°      IF(K.GT.M) GO TO 14
706°      JQ=M(K)+1
707°      AIS(1)=0.
708°      AIS(2)=0.
709°      AIS(3)=0.
710°      DO 15 J=JQ,NB
711°      DO 15 I=IQ,NB
712°      IF((EPS(K,J).EQ.0).OR.(EPS(M,I).EQ.0)) GO TO 15
713°      DO 18 N=1,3
714° 18   AIS(N)=AIS(N)+PSG(J,I,N)
715° 15   CONTINUE
716°      AS(K,M)=GO(K,1)*AIS(1)+GO(K,2)*AIS(2)+GO(K,3)*AIS(3)
717° 14   CONTINUE
718° C
719° C   AQF MATRIX (3 X NKT) (REF. BODY/FLEX. APPENDAGE COUPLING)
720° C
721°      DO 219 K=1,NF
722°      JK=F(K,3)
723°      JQ=F(K,1)+1
724°      DO 222 I=1,3
725°      DO 222 J=1,3
726° 222  AB(I,J)=0.
727°      DO 221 L=1,NB
728°      AB(1,2)=AB(1,2)+DZO(L,JQ)
729°      AB(1,3)=AB(1,3)+DYO(L,JQ)
730° 221  AB(2,3)=AB(2,3)+DXO(L,JQ)
731°      AB(2,1)=-AB(1,2)
732°      AB(3,1)=-AB(1,3)
733°      AB(3,2)=-AB(2,3)
734°      DO 220 I=1,3
735°      DO 220 J=1,JK
736°      AQF(K,I,J)=OLKO(K,I,J)
737°      DO 220 L=1,3
738° 220  AQF(K,I,J)=AQF(K,I,J)-AB(I,L)*PKO(K,L,J)
739° 219  CONTINUE
740° C
741° C   AKF VECTOR (1 X NKT) (FLEX. COUPLING WITH RIGID SUBSTRUCTURES)
742° C
743°      DO 224 K=1,NF
744°      JK=F(K,3)
745°      JQ=F(K,1)+1
746°      DO 2245 J=1,JK
747° 2245 ZSR(K,J)=0.
748°      DO 224 M=1,NH
749°      DO 231 I=1,3

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```

750*      DO 231 J=1,3
751* 231  AB(I,J)=0.
752*      DO 226 L=1,NB
753*      IF(EPS(M;L)*EQ.0) GO TO 226
754*      AB(1,2)=AB(1,2)+DZO(L,JQ)
755*      AB(1,3)=AB(1,3)+DYO(L,JQ)
756*      AB(2,3)=AB(2,3)+DXO(L,JQ)
757* 22*  CONTINUE
758*      AB(2,1)=-AB(1,2)
759*      AB(3,1)=-AB(1,3)
760*      AB(3,2)=-AB(2,3)
761*      DO 228 I=1,3
762*      DO 228 J=1,JK
763*      DUR(I,J)=DLKO(K,I,J)
764*      IF(EPS(M,K)*EQ.0) DUR(I,J)=0.
765*      DO 228 L=1,3
766* 22*  DUR(I,J)=DUR(I,J)+AB(I,L)*PKO(K,L,J)
767*      DO 2241 J=1,JK
768*      DO 2241 I=1,3
769* 2241 ZSR(K,J)=ZSR(K,J)+DUR(I,J)*RGJ(M,I)
770*      DO 229 J=1,JK
771*      AKF(K,M,J)=0.
772*      DO 229 I=1,3
773* 229  AKF(K,M,J)=AKF(K,M,J)+GO(M,I)*DUR(I,J)
774* 224  CONTINUE
775*      C
776*      C      COMPUTE CORRECTION ELEMENTS FOR (E) VECTOR
777*      C
778*      DO 41 J=2,NB
779*      JK=M1(J)
780*      DO 411 M=1,3
781* 411  CW(J,M)=0.
782*      DO 42 K=1,JK
783*      IF(EPS(K,J)*EQ.0) GO TO 42
784*      CW(J,1)=CW(J,1)+WGJ(K,1)
785*      CW(J,2)=CW(J,2)+WGJ(K,2)
786*      CW(J,3)=CW(J,3)+WGJ(K,3)
787* 42  CONTINUE
788* 41  CONTINUE
789*      DO 40 I=1,NB
790*      EA(1)=0.
791*      EA(2)=0.
792*      EA(3)=0.
793*      DO 401 J=2,NB
794*      DO 4507 M=1,3
795*      DO 4507 L=1,3
796* 4507 EA(M)=EA(M)+(PS(I,J,M,L)-PS(I,J,M,L))*CW(J,L)
797* 401  CONTINUE
798*      K1=3*(I-1)
799*      E(K1+1,1)=E(K1+1,1)-EA(1)
800*      E(K1+2,1)=E(K1+2,1)-EA(2)
801*      E(K1+3,1)=E(K1+3,1)-EA(3)
802* 40  CONTINUE
803*      DO 55 M1=1,3
804* 55  EC(M1)=E(M1,1)
805*      DO 52 J=2,NB
806*      DO 52 M=1,3
807*      K1=3*(J-1)+M
808* 52  EC(M)=EC(M)+E(K1,1)
809*      I=0
810*      DO 60 K=1,NM
811*      JK=M(K)+1
812*      IF(P1(K)*NE.0) GO TO 60
813*      I=I+1
814*      EC(I+3)=0.
815*      DO 601 M=1,3
816* 601  CE(M)=0.
817*      DO 61 J=JK,NB
818*      IF(EPS(K,J)*EQ.0) GO TO 61

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```

819°      DO 65 M=1,3
820°      JI=3*(J-1)+M
821°      65  C(E(M)=CE(M)+E(JI,1)
822°      61  CONTINUE
823°      DO 66 L=1,3
824°      66  EC(I+3)=EC(I+3)+GO(K,L)*CE(L)
825°      EC(I+3)=EC(I+3)+TH(K)
826°      60  CONTINUE
827°      DO 610 I=1,3
828°      DO 610 J=1,NH
829°      IF(PI(J).EQ.0) GO TO 610
830°      EC(I)=EC(I)-AV(J,I)*GMDD(J)
831°      610  CONTINUE
832°      K=0
833°      IV=3
834°      DO 612 I=1,NH
835°      IF(PI(I).NE.0) GO TO 612
836°      K=K+1
837°      IV=IV+1
838°      DO 611 J=1,NH
839°      IF(PI(J).EQ.0) GO TO 611
840°      IF(I.GT.J) AS(I,J)=AS(J,I)
841°      EC(K+3)=EC(K+3)-AS(I,J)*GMDD(J)
842°      611  CONTINUE
843°      612  CONTINUE
844°      C
845°      C      COMPUTE RT, HAND SIDE OF APPENDAGE EQUATIONS (IN APPEND. COORDS.)
846°      C
847°      DO 483 K=1,NF
848°      IF(K,1)+1
849°      M=HI(I)
850°      CQ(1)=FTXO/TH + CWD(1,1)
851°      CQ(2)=FTYO/TH + CWD(1,2)
852°      CQ(3)=FTZO/TH + CWD(1,3)
853°      IF(I.EQ.1) GO TO 4840
854°      DO 484 J=1,3
855°      VE(K,J)=0.
856°      DO 484 L=1,3
857°      484  VE(K,J)=VE(K,J)+T(M,J,L)*CQ(L)
858°      GO TO 483
859°      4840  CONTINUE
860°      DO 4841 J=1,3
861°      4841  VE(K,J)=CQ(J)
862°      483  CONTINUE
863°      DO 485 K=1,NF
864°      NL=F(K,2)
865°      DO 486 N=1,NL
866°      N6=6*(N-1)
867°      DO 488 J=1,3
868°      JN=N6+J
869°      JM=JN+3
870°      VB(K,JN)=FF(K,N,J)
871°      488  VB(K,JM)=TF(K,N,J)
872°      486  CONTINUE
873°      485  CONTINUE
874°      NV=IV
875°      DO 491 K=1,NF
876°      JN=F(K,3)
877°      NL=F(K,2)
878°      NL6=6*NL
879°      DO 492 J=1,JN
880°      IL=NV+J
881°      VVJ=-WF(K,J)+(2.*ZF(K,J)*ETD(K,J)+WF(K,J)*ET(K,J))
882°      DO 493 N=1,NL6
883°      493  VVJ=VVJ+EIG(K,N,J)*VB(K,N)
884°      DO 494 N=1,3
885°      494  VVJ=VVJ-PK(K,N,J)*VE(K,N)
886°      VVJ=VVJ-ZSR(K,J)
887°      EC(IL)=VVJ

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888° DO 4920 L=1,NH
889° IF(P1(L)*EQ*0) GO TO 4920
890° EC(IL)=EC(IL)-AKF(L,K,J)*GMDD(L)
891° 4920 CONTINUE
892° 492 CONTINUE
893° 491 NV=NV+JN
894° C
895° C ENTER CONSTANTS INTO FLEX. BODY PORTION OF COEFF. MATRIX A
896° C
897° NV=IV
898° DO 462 K=1,NF
899° NL=F(K,3)
900° DO 463 I=1,NL
901° IL=NV+I
902° DO 463 J=1,NL
903° JL=NV+J
904° A(IL,JL)=0.
905° IF(I*EQ*J) A(IL,JL)=1.
906° 463 CONTINUE
907° 462 NV=NV+NL
908° C
909° C ENTER COEFF. WHICH COUPLE REF. BODY AND FLEX. APPENDAGES INTO A
910° C
911° NV=IV
912° DO 464 K=1,NF
913° NL=F(K,3)
914° DO 465 J=1,3
915° DO 465 I=1,NL
916° IL=NV+I
917° A(IL,J)=AJF(K,J,I)
918° 465 A(J,IL)=A(IL,J)
919° 464 NV=NV+NL
920° C
921° C ENTER COEFF. WHICH COUPLE SUBSTR. BODIES AND FLEX. APPEND. INTO A
922° C
923° NV=IV
924° DO 466 K=1,NF
925° NL=F(K,3)
926° JI=0
927° DO 467 J=1,NH
928° IF(P1(J)*NE*0) GO TO 467
929° JI=JI+1
930° DO 467 I=1,NL
931° IL=NV+I
932° A(IL,JI+3)=AKF(K,J,I)
933° A(JI+3,IL)=A(IL,JI+3)
934° 467 CONTINUE
935° 467 CONTINUE
936° 466 NV=NV+NL
937° C
938° C CALCULATE FLEX. BODY COUPLING COEFF. AND ENTER INTO A MATRIX
939° C
940° NCO=IV
941° DO 473 L=1,NF
942° NL=F(L,3)
943° NRO=IV
944° DO 474 K=1,NF
945° NR=F(K,3)
946° IF(K*EQ*L) GO TO 474
947° DO 475 I=1,NR
948° IK=NRO+I
949° DO 475 J=1,NL
950° JK=NCO+J
951° A(IK,JK)=0.
952° DO 4750 N=1,3
953° A(IK,JK)=A(IK,JK)-PKO(K,N,I)*PKO(L,N,J)/TM
954° 4750 CONTINUE
955° A(JK,IK)=A(IK,JK)
956° 475 CONTINUE
957° 474 NRO=NRO+NR

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958° 473 NCO=NCO+NL
959° C
960° C LOAD SYSTEM MATRIX (A) WITH A00,A0K,AKM ELEMENTS
961° C
962° DO 23 I=1,J
963° DO 23 J=1,J
964° 23 A(I,J)=A00(I,J)
965° DO 24 I=1,J
966° K=0
967° DO 24 J=1,NH
968° IF(P1(J).NE.0) GO TO 24
969° K=K+1
970° A(K+3,I)=AV(J,I)
971° A(I,K+3)=AV(J,I)
972° 24 CONTINUE
973° K=0
974° DO 250 I=1,NH
975° IF(P1(I).NE.0) GO TO 250
976° K=K+1
977° L=0
978° DO 25 J=1,NH
979° IF(P1(J).NE.0) GO TO 25
980° L=L+1
981° IF(K.GT.L) GO TO 26
982° A(K+3,L+3)=A5(I,J)
983° GO TO 25
984° 26 A(K+3,L+3)=A(L+3,K+3)
985° 25 CONTINUE
986° 250 CONTINUE
987° C
988° C ANGULAR MOMENTUM OF THE SYSTEM
989° C
990° IF(P1(NH+1).NE.1) GO TO 8752
991° DO 5651 I=1,J
992° MH(I)=0
993° DO 5651 J=1,J
994° 5651 MH(I)=MH(I)+A(I,J)*W0(J)
995° DO 5652 I=1,J
996° DO 5652 J=1,QH
997° 5652 MH(I)=MH(I)+AV(J,I)*GM0(J)
998° DO 5653 I=1,J
999° DO 5653 K=1,NF
1000° NL=F(K,3)
1001° DO 5654 J=1,NL
1002° 5654 MH(I)=MH(I)+A0F(K,I,J)*ETD(K,J)
1003° 5653 CONTINUE
1004° MH=SQRT(MH(1)**2 + MH(2)**2 + MH(3)**2)
1005° 8752 CONTINUE
1006° C
1007° C SOLVE SYSTEM MATRIX FOR REFERENCE BODY ANG. ACCELERATION AND HINGE
1008° C (RELATIVE) ROTATIONAL ACCELERATIONS
1009° C
1010° NT=V+NTM0
1011° IT=IV+NTM0
1012° KV=IV
1013° CALL CHOLD(392,A,ST,IT,EC,0,.1,0*7)
1014° DO 910 J=NT+4,-1
1015° IF(J.LE.V) GO TO 913
1016° JV=J-(V-IV)
1017° EC(J)=EC(JV)
1018° GO TO 910
1019° 913 CONTINUE
1020° K=J-3
1021° IF(P1(K).NE.0) GO TO 911
1022° EC(J)=EC(KV)
1023° KV=KV-1
1024° GO TO 910
1025° 911 EC(J)=GMDD(K)
1026° 910 CONTINUE

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```
1027*      DO 9003 I=1,V
1028* 9003  WOOT(I)=EC(I)
1029*      I=Y
1030*      DO 9001 K=1,NF
1031*      NL=F(K,3)
1032*      DO 9002 N=1,NL
1033*      IO=I+N
1034* 9002  ETDD(K,N)=EC(IO)
1035* 9001  I=I+NL
1036* 92    CONTINUE
1037*      RETURN
1038*      END
```

DIAGNOSTICS

ATIUM TIME = 30.73 SUPS

CSSL=TRAN,CSSL

## Appendix E

### Subroutine MBDYFL Listing and User Requirements

#### *Subroutine Entry Statements*

Same as MBDYFN (see Appendix D)

#### *Input / Output Variable Type and Storage Specifications*

Same as MBDYFN (see Appendix D)

#### *External Subroutines Called*

AINVD—double precision matrix inversion subroutine. Inverts any real square, nonsingular matrix,  $A$ , and leaves the result in  $A$  (see statement 419).

#### *Subroutine Setup*

Same as MBDYFN (see Appendix D)

#### *Data Restrictions*

Same as MBDYFN

#### *Core Storage Required*

Code: 3500 words

Data: ~ 500 words (minimum; varies with  $n$ ,  $n_f$ )

#### *Listing*

```

10      SUBROUTINE MBDYFL(NC,C,MB,MA,PB,PA,G,PI,NF,F,EIG,REC,HF,WF,ZF)
11      C
12      C
13      C      ADJUSTABLE DIMENSIONS
14      C
15      C      INTEGER PI(1),C(1:NC,2)
16      C      REAL MB(1),MA(1:NC,7),PB(1:NC,3),PA(1:NC,NC,3)
17      C      PARAMETER QC=1,QH=2,QF=1,NK=1,NKT=7
18      C      PARAMETER NOK=6*NK,S=QC*1,V=QH*3,V4=4*V,S3=3,S,Q=QH,NH=QH
19      C      PARAMETER ST=V+QF*NKT,S4=4*ST
20      C
21      C      ADDITIONAL DIMENSIONED VARIABLES
22      C
23      C      DOUBLE PRECISION A(1:ST,1:ST),WRK(54),BMASS(5)
24      C      INTEGER EPS(Q,S),CPS(QC,S),M(Q),M1(S),F1(S),F1NF,3)
25      C      REAL A00(3,3),AB(3,3),AOF(QF,3,NKT),AKF(QF,QH,NKT),AS(Q,Q),AV(Q,3)
26      C      S,A1S(3),CE(3),CK(QF,3),CQ(3),DA(S,S),DY(S,S),DZ(S,S),DXO(S,S),DYO(
27      C      S,S),DZO(S,S),DLK(QF,3,NKT),DUR(3,NKT),EIG(NF,NOK,NKT),FEXU(S),FEY
28      C      S(S),FEZO(S),FS(S,3),GO(Q,3),G(Q,3),LXX(S),LYT(S),LZZ(S),LXY(S),LX
29      C      SZ(S),LYZ(S),LX(S,S),LY(S,S),LZ(S,S),MSB(S),MCK(QF,3),PH(S,3,3),PSG
30      C      S(S,S,3),PS(S,S,3,3),PK(QF,3,NKT),PGSO(QF,3),RF(NF,NK,3),REC(NF,4,N
31      C      SKT),TXO(S),TYO(S),TZO(S),TIQ(3,3),TS(S,3),U(QF,NK,3),VE(QF,3),VB(Q
32      C      SF,NOK),WF(NF,NKT),ZF(NF,NKT),HM(3)

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230      EQUIVALENCE (A,PS),(LX,DAX),(LY,DYO),(LZ,DZO)
240      NB=NC+1
250      C
260      C      DEFINE EPS(K,J) USING C
270      C
280      DO 86 K=1,NC
290      DO 86 J=2,NB
300      IF(K,EW.(J-1)) CPS(K,J)=1
310      IF(K,LT.(J-1)) GO TO 87
320      GO TO 86
330      87      CONTINUE
340      JU=K+1
350      JI=J-1
360      DO 89 L=JU,J1
370      IF(K,GT.(L-1)) GO TO 89
380      IF((CPS(K,L).EQ.1).AND.(C(J-1,L).EQ.(L-1))) CPS(K,J)=1
390
400      89      CONTINUE
410      86      CONTINUE
420      L=0
430      DO 1 J=1,NC
440      KK=C(J,2)
450      DO 1 K=1,KK
460      L=L+1
470      DO 1 I=1,NB
480      EPS(L,I)=CPS(J,I)
490      C
500      C      COMPUTE HI(I)=C, WHERE I=HINGE LABEL AND C=CONNECTION LABEL
510      C
520      I=0
530      DO 8 J=2,NB
540      KK=C(J-1,2)
550      DO 8 K=1,KK
560      I=I+1
570      8      HI(I)=J-1
580      C
590      C      COMPUTE HI(I)=J, WHERE I=BODY LABEL+1 AND J=NEAREST HINGE LABEL
600      C
610      HI(1)=1
620      HI(NB)=NH
630      DO 47 I=NH,1
640      IF(I,EQ.1) GO TO 47
650      K1=HI(I)
660      K2=HI(I-1)
670      IF(K1.EQ.K2) GO TO 47
680      HI(K2+1)=I-1
690      47      CONTINUE
700      C
710      C      DEFINE FI(J)=K, WHERE J=BODY-LABEL+1 AND K IS APPENDAGE-LABEL
720      C      (IF K=0, BODY HAS NO FLEX. APPENDAGE)
730      C
740      DO 239 N=1,NB
750      239      FI(N)=0
760      DO 242 K=1,NF
770      JN=F(K,1)+1
780      242      FI(JN)=K
790      NF=NF
800      NB=NB
810      C
820      C      DEFINE SUBSTRUCTURE MASSES
830      C
840      MSB(1)=MB(7)
850      DO 248 N=2,NB
860      248      MSB(N)=MAIN(1,7)
870      C
880      C      TOTAL NUMBER OF FLEX. APPENDAGE MODES TO BE RETAINED
890      C
900      NTHO=0
910      DO 461 K=1,NF

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920 461 NTMO=NTMO*F(K,3)
930 C
940 C INITIAL CALCULATION OF BARYCENTER VECTORS B.R.T.O BODY C.G.S
950 C AND HINGE POINTS
960 C
970 IX(1)=MB(1)
980 IY(1)=MB(2)
990 IZ(1)=MB(3)
1000 IX(1)=MB(4)
1010 IXZ(1)=MB(5)
1020 IYZ(1)=MB(6)
1030 BMASS(1)=MB(7)
1040 TM=BMASS(1)
1050 DO 35 J=2,NB
1060 IX(J)=MA(J-1,1)
1070 IY(J)=MA(J-1,2)
1080 IZ(J)=MA(J-1,3)
1090 IXZ(J)=MA(J-1,4)
1100 IYZ(J)=MA(J-1,5)
1110 IYZ(J)=MA(J-1,6)
1120 BMASS(J)=MA(J-1,7)
1130 35 TM=TM+BMASS(J)
1140 DO 149 I=1,NB
1150 II=I-1
1160 DO 149 J=1,NB
1170 JI=J-I
1180 IF(I,EQ,J) GO TO 163
1190 IF(I,GT,J) GO TO 70
1200 IF(I,EQ,1) GO TO 80
1210 IF(CPS(II,J)*EQ,1) GO TO 400
1220 70 LX(I,J)=PA(II,II,1)
1230 LY(I,J)=PA(II,II,2)
1240 LZ(I,J)=PA(II,II,3)
1250 GO TO 149
1260 400 CONTINUE
1270 DO 600 K=1,J1
1280 IF(CPS(K,J)*EQ,1) GO TO 500
1290 600 CONTINUE
1300 GO TO 149
1310 -500 LX(I,J)=PA(II,K,1)
1320 LY(I,J)=PA(II,K,2)
1330 LZ(I,J)=PA(II,K,3)
1340 GO TO 149
1350 80 DO 90 L=1,J1
1360 IF(CPS(L,J)*EQ,1) GO TO 101
1370 90 CONTINUE
1380 GO TO 149
1390 101 LX(I,J)=PB(L,1)
1400 LY(I,J)=PB(L,2)
1410 LZ(I,J)=PB(L,3)
1420 GO TO 149
1430 163 LX(I,J)=0.
1440 LY(I,J)=0.
1450 LZ(I,J)=0.
1460 149 CONTINUE
1470 DO 13 N=1,NB
1480 DO 13 J=1,NB
1490 DX(N,J)=LX(N,J)
1500 DY(N,J)=LY(N,J)
1510 DZ(N,J)=LZ(N,J)
1520 DO 13 K=1,NB
1530 DX(N,J)=DX(N,J)+(BMASS(K)/TM)*LX(N,K)
1540 DY(N,J)=DY(N,J)+(BMASS(K)/TM)*LY(N,K)
1550 13 DZ(N,J)=DZ(N,J)+(BMASS(K)/TM)*LZ(N,K)
1560 C
1570 C CALCULATION OF AUGMENTED INERTIA DYADICS FOR EACH BODY
1580 C
1590 DO 31 N=1,NB
1600 PH(N,1,1)=IX(N)

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1610 PH(N,1,2)=-1*XY(N)
1620 PH(N,1,3)=-1*XZ(N)
1630 PH(N,2,2)=1*YY(N)
1640 PH(N,2,3)=-1*YZ(N)
1650 PH(N,3,3)=1*ZZ(N)
1660 DO 30 J=1,NB
1670 PH(N,1,1)=PH(N,1,1)+BMASS(J)*(DY(N,J)**2+DZ(N,J)**2)
1680 PH(N,1,2)=PH(N,1,2)+BMASS(J)*DX(N,J)*DY(N,J)
1690 PH(N,1,3)=PH(N,1,3)+BMASS(J)*DX(N,J)*DZ(N,J)
1700 PH(N,2,2)=PH(N,2,2)+BMASS(J)*(DX(N,J)**2+DZ(N,J)**2)
1710 PH(N,2,3)=PH(N,2,3)+BMASS(J)*DY(N,J)*DZ(N,J)
1720 30 PH(N,3,3)=PH(N,3,3)+BMASS(J)*(DX(N,J)**2+DY(N,J)**2)
1730 PH(N,2,1)=PH(N,1,2)
1740 PH(N,3,1)=PH(N,1,3)
1750 31 PH(N,3,2)=PH(N,2,3)
1760 C
1770 C ADDITIONAL AUGMENTED INERTIA DYADICS (IN REF. BODY FRAME)
1780 C
1790 DO 751 J=1,NB
1800 DO 751 M=1,3
1810 DO 751 N=1,3
1820 751 PS(J,J,M,N)=PH(J,M,N)
1830 DO 37 I=1,NB
1840 DO 37 J=1,NB
1850 IF(I,GE,J) GO TO 37
1860 DX2=DX(I,J)*DX(J,I)
1870 DY2=DY(I,J)*DY(J,I)
1880 DZ2=DZ(I,J)*DZ(J,I)
1890 PS(I,J,1,1)=-TM*(DY2+DZ2)
1900 PS(I,J,1,2)=TM*DX(J,I)*DY(I,J)
1910 PS(I,J,1,3)=TM*DX(J,I)*DZ(I,J)
1920 PS(I,J,2,1)=TM*DY(J,I)*DX(I,J)
1930 PS(I,J,2,2)=-TM*(DX2+DZ2)
1940 PS(I,J,2,3)=TM*DY(J,I)*DZ(I,J)
1950 PS(I,J,3,1)=TM*DZ(J,I)*DX(I,J)
1960 PS(I,J,3,2)=TM*DZ(J,I)*DY(I,J)
1970 PS(I,J,3,3)=-TM*(DX2+DY2)
1980 DO 378 M=1,3
1990 DO 378 N=1,3
2000 378 PS(J,I,M,N)=PS(I,J,N,M)
2010 37 CONTINUE
2020 C
2030 C ADD MATRIX ELEMENT COMPUTATION (3X3)
2040 C
2050 DO 3001 I=1,3
2060 DO 3001 J=1,3
2070 3001 A00(I,J)=0.
2080 DO 3 I=1,NB
2090 DO 3 J=1,NB
2100 A00(I,1)=A00(I,1)+PS(I,J,1,1)
2110 A00(I,2)=A00(I,2)+PS(I,J,1,2)
2120 A00(I,3)=A00(I,3)+PS(I,J,1,3)
2130 A00(2,2)=A00(2,2)+PS(I,J,2,2)
2140 A00(2,3)=A00(2,3)+PS(I,J,2,3)
2150 A00(3,3)=A00(3,3)+PS(I,J,3,3)
2160 3 CONTINUE
2170 A00(2,1)=A00(1,2)
2180 A00(3,1)=A00(1,3)
2190 A00(3,2)=A00(2,3)
2200 C
2210 C AOK VECTOR ELEMENT COMPUTATION (3X1)
2220 C
2230 C AKM SCALAR ELEMENT COMPUTATION
2240 C
2250 DO 14 M=1,NH
2260 IQ=M(M)+1
2270 AV(M,1)=G.
2280 AV(M,2)=0.
2290 AV(M,3)=0.

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230*      DO 7 J=1,NB
231*      DO 7 I=1,Q,NB
232*      DO 11 N=1,3
233*      IF(EPS(M,I).EQ.0) GO TO 7
234*      PSG(J,I,N)=0.
235*      DO 10 L=1,3
236*      PSG(J,I,N)=PSG(J,I,N)+PS(J,I,N,L)*G(M,L)
237* 10    AV(M,N)=AV(M,N)+PSG(J,I,N)
238*      7    CONTINUE
239*      DO 14 K=1,NM
240*      IF(K.GT.M) GO TO 14
241*      JQ=H(K)+1
242*      AIS(1)=0.
243*      AIS(2)=0.
244*      AIS(3)=0.
245*      DO 15 J=JQ,NB
246*      DO 15 I=1,Q,NB
247*      IF((EPS(K,J).EQ.0).OR.(EPS(M,I).EQ.0)) GO TO 15
248*      DO 18 N=1,3
249*      AIS(N)=AIS(N)+PSG(J,I,N)
250* 15    CONTINUE
251*      AS(K,M)=G(K,1)*AIS(1)+G(K,2)*AIS(2)+G(K,3)*AIS(3)
252* 14    CONTINUE
253*      C
254*      C      DEFINE PK(3 X NKT ARRAY)
255*      C      DEFINE DLK=TRANSPPOSE MATRIX (3 X NKT ARRAY)
256*      C
257*      DO 201 K=1,NF
258*      JNT=F(K,3)
259*      DO 201 I=1,3
260*      DO 201 J=1,JNT
261*      PK(K,I,J)=REC(K,I,J)
262* 201    DLK(K,I,J)=REC(K,I+3,J)
263*      C
264*      C      AOF MATRIX (3 X NKT) (REF. BODY/FLEX. APPENDAGE COUPLING)
265*      C
266*      DO 219 K=1,NF
267*      JK=F(K,3)
268*      JQ=F(K,1)+1
269*      DO 222 I=1,3
270*      DO 222 J=1,3
271* 222    AB(I,J)=0.
272*      DO 221 L=1,NB
273*      AB(1,2)=AB(1,2)+DZ(L,JQ)
274*      AB(1,3)=AB(1,3)+DY(L,JQ)
275* 221    AB(2,3)=AB(2,3)+DX(L,JQ)
276*      AB(2,1)=-AB(1,2)
277*      AB(3,1)=-AB(1,3)
278*      AB(3,2)=-AB(2,3)
279*      DO 220 I=1,3
280*      DO 220 J=1,JK
281*      AOF(K,I,J)=DLK(K,I,J)
282*      DO 220 L=1,3
283* 220    AOF(K,I,J)=AOF(K,I,J) - AB(I,L)*PK(K,L,J)
284* 219    CONTINUE
285*      C
286*      C      AKF VECTOR (1 X NKT) (FLEX. COUPLING WITH RIGID SUBSTRUCTURES)
287*      C
288*      DO 224 K=1,NF
289*      JK=F(K,3)
290*      JQ=F(K,1)+1
291*      DO 224 M=1,NM
292*      DO 231 I=1,3
293*      DO 231 J=1,3
294* 231    AB(I,J)=0.
295*      DO 226 L=1,NB
296*      IF(EPS(M,L).EQ.0) GO TO 226
297*      AB(1,2)=AB(1,2)+DZ(L,JQ)
298*      AB(1,3)=AB(1,3)+DY(L,JQ)

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299°      AB(2,3)=AB(2,3)-DX(L,JW)
300°      226  CONTINUE
301°      AB(2,1)=-AB(1,2)
302°      AB(3,1)=-AB(1,3)
303°      AB(3,2)=-AB(2,3)
304°      DO 228 I=1,3
305°      DO 228 J=1,JK
306°      DUR(I,J)=ULK(K,I,J)
307°      IF(EPS(M,K)*EQ*U) DUR(I,J)=0.
308°      DO 228 L=1,3
309°      220  DUR(I,J)=DUR(I,J)-AB(I,L)*PK(K,L,J)
310°      DO 229 J=1,JK
311°      AKF(K,M,J)=0.
312°      DO 229 I=1,3
313°      229  AKF(K,M,J)=AKF(K,M,J)+G(M,I)*DUR(I,J)
314°      224  CONTINUE
315°      C
316°      C   ENTER CONSTANTS INTO FLEX. BODY PORTION OF COEFF. MATRIX A
317°      C
318°      IV=3
319°      DO 6129 I=1,NH
320°      IF(P1(I).NE.0) GO TO 6129
321°      IV=IV+1
322°      6129  CONTINUE
323°      NV=IV
324°      DO 462 K=1,NF
325°      NL=F(K,3)
326°      DO 463 I=1,NL
327°      IL=NV+I
328°      DO 463 J=1,NL
329°      JL=NV+J
330°      A(IL,JL)=0.
331°      IF(I.EQ.J) A(IL,JL)=1.
332°      463  CONTINUE
333°      464  NV=NV+NL
334°      C
335°      C   ENTER COEFF. WHICH COUPLE REF. BODY AND FLEX. APPENDAGES INTO A
336°      C
337°      NV=IV
338°      DO 464 K=1,NF
339°      NL=F(K,3)
340°      DO 465 J=1,3
341°      DO 465 I=1,NL
342°      IL=NV+I
343°      A(IL,J)=AJF(K,J,I)
344°      465  A(J,IL)=A(IL,J)
345°      464  NV=NV+NL
346°      C
347°      C   ENTER COEFF. WHICH COUPLE SUBSTR. BODIES AND FLEX. APPEND. INTO A
348°      C
349°      NV=IV
350°      DO 466 K=1,NF
351°      NL=F(K,3)
352°      JI=0
353°      DO 467 J=1,NH
354°      IF(P1(J).NE.0) GO TO 467
355°      JI=JI+1
356°      DO 467 I=1,NL
357°      IL=NV+I
358°      A(IL,JI+3)=AKF(K,J,I)
359°      A(JI+3,IL)=A(IL,JI+3)
360°      467  CONTINUE
361°      467  CONTINUE
362°      460  NV=NV+NL
363°      C
364°      C   CALCULATE FLEX. BODY COUPLING COEFF. AND ENTER INTO A MATRIX
365°      C
366°      NCO=IV
367°      DO 473 L=1,NF

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368*      NL=F(L,3)
369*      NRO=IV
370*      DO 474 K=1,NF
371*      NR=F(K,3)
372*      IF(K.EQ.L) GO TO 474
373*      DO 475 I=1,NR
374*      IK=NRO+I
375*      DO 475 J=1,NL
376*      JK=NCO+J
377*      A(IK,JK)=0.
378*      DO 4750 N=1,3
379*      A(IK,JK)=A(IK,JK)-PK(K,N,1)*PK(L,N,2)/TM
380* 4750 CONTINUE
381*      A(JK,IK)=A(IK,JK)
382* 475 CONTINUE
383* 474 NRO=NRO+NR
384* 473 NCO=NCO+NL
385* C
386* C      LOAD SYSTEM MATRIX (A) WITH A00,A0K,AKM ELEMENTS
387* C
388*      DO 23 I=1,3
389*      DO 23 J=1,3
390* 23 A(I,J)=A00(I,J)
391*      DO 24 I=1,3
392*      K=0
393*      DO 24 J=1,NH
394*      IF(P1(I).NE.0) GO TO 24
395*      K=K+1
396*      A(K+3,I)=AV(J,I)
397*      A(I,K+3)=AV(J,I)
398* 24 CONTINUE
399*      K=0
400*      DO 250 I=1,NH
401*      IF(P1(I).NE.0) GO TO 250
402*      K=K+1
403*      L=0
404*      DO 25 J=1,NH
405*      IF(P1(J).NE.0) GO TO 25
406*      L=L+1
407*      IF(K.GT.L) GO TO 26
408*      A(K+3,L+3)=AS(I,J)
409*      GO TO 25
410* 26 A(K+3,L+3)=A(L+3,K+3)
411* 25 CONTINUE
412* 250 CONTINUE
413* C
414* C      SOLVE SYSTEM MATRIX FOR REFERENCE BODY ANG. ACCELERATION AND HINGE
415* C      (RELATIVE) ROTATIONAL ACCELERATIONS
416* C
417*      NT=V+NTMO
418*      IT=IV+NTMO
419*      CALL AINVU(A,ST,IT,51095,WRK)
420* 1095 CONTINUE
421*      RETURN
422*      ENTRY HRATE(NC,TH,TB,TA,FB,FA,TF,FF,GH,GMD,GDD,ET,ETD,WO,WOT,ETD
423*      SO,MM)
424*      REAL TF(QF,NK,3),FF(QF,NK,3),ET(QF,NKT),ETD(QF,NKT),TB(3),TA(NC,3)
425*      S,FB(3),FA(NC,3),GH(1),GMD(1),GDD(1),TH(1),WO(3),E(S,1)
426*      DOUBLE PRECISION EC(ST),ETDD(QF,NKT),WOT(V),E(S,1)
427* C
428* C      BODY-TO-BODY COORDINATE TRANSFORMATION MATRICES
429* C
430*      DO 335 J=1,NH
431*      MM=J-1
432*      N=H(J)+1
433*      AB(1,1)=1.
434*      AB(1,2)=GH(J)*G(J,3)
435*      AB(1,3)=-GH(J)*G(J,2)
436*      AB(2,1)=-AB(1,2)

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437*      AB(2,2)=1.
438*      AB(2,3)=GM(J)*G(J,1)
439*      AB(3,1)=-AB(1,3)
440*      AB(3,2)=-AB(2,3)
441*      AB(3,3)=1.
442*      IF(J.EQ.1) GO TO 3350
443*      DO 321 L=MM,1
444*      IF(EPS(L,N).EQ.1) GO TO 322
445* 321  CONTINUE
446*      GO TO 3350
447* 322  K=L
448*      DO 334 L=1,3
449*      DO 334 M=1,3
450*      T(J,L,M)=0.
451*      DO 334 I=1,3
452* 334  T(J,L,M)=T(J,L,M)+AB(L,I)*T(K,I,M)
453*      GO TO 335
454* 3350  CONTINUE
455*      DO 3351 L=1,3
456*      DO 3351 M=1,3
457* 3351  T(J,L,M)=AB(L,M)
458* 335  CONTINUE
459*  C
460*  C   COORD. TRANSFORMATION OF G VECTORS (TO REF. BODY FRAME)
461*  C
462*      DO 362 I=1,NH
463*      DO 362 J=1,3
464*      GO(I,J)=0.
465*      DO 362 K=1,3
466*      GO(I,J)=GO(I,J)+T(I,K,J)*G(I,K)
467* 362  CONTINUE
468*  C
469*  C   COMPUTE TOTAL EXTERNAL FORCE ON EACH SUBSTRUCTURE (IN REF. COORD.)
470*  C
471*      FEX0(1)=FB(1)
472*      FEY0(1)=FB(2)
473*      FEZ0(1)=FB(3)
474*      IF(F1(1).EQ.0) GO TO 254
475*      IL=F1(1)
476*      JN=F(IL,2)
477*      DO 253 J=1,JN
478*      FEX0(1)=FEX0(1)+FF(IL,J,1)
479*      FEY0(1)=FEY0(1)+FF(IL,J,2)
480* 253  FEZ0(1)=FEZ0(1)+FF(IL,J,3)
481* 254  CONTINUE
482*      FS(1,1)=FEX0(1)
483*      FS(1,2)=FEY0(1)
484*      FS(1,3)=FEZ0(1)
485*      DO 246 N=2,NB
486*      K=N+1
487*      DO 2460 L=1,3
488* 2460  FS(N,L)=FA(K,L)
489*      IF(F1(N).EQ.0) GO TO 246
490*      IL=F1(N)
491*      JN=F(IL,2)
492*      DO 245 J=1,JN
493*      DO 245 I=1,3
494* 245  FS(N,I)=FS(N,I)+FF(IL,J,I)
495* 246  CONTINUE
496*  C
497*  C   COMPUTE TRANSL. AND ROTAT. DISPLACEMENTS OF APPENDAGE SUB-BODIES
498*  C
499*      DO 232 K=1,NF
500*      JN=F(K,2)
501*      LK=F(K,3)
502*      DO 233 J=1,JN
503*      DO 233 I=1,3
504*      U(K,J,I)=0.
505*      ID=(J-1)*6+1

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506*      DO 233 L=1,LK
507* 233  U(K,J,1)=U(K,J,1)+EIG(K,10,L)*ET(K,L)
508* 232  CONTINUE
509* C
510* C      COMPUTE C.M. PERTURBATION (FROM NOM. UNDEFORMED LOCATION) ON EACH
511* C      SUBSTRUCTURE WITH AN APPENDAGE (LOCAL COORDS.)
512* C
513*      DO 262 K=1,NF
514*      IK=F(K,1)+1
515*      JN=F(K,3)
516*      DO 263 I=1,3
517* 263  MCK(K,I)=0.
518*      DO 265 J=1,JN
519*      DO 265 I=1,3
520* 265  MCK(K,I)=MCK(K,I)+PK(K,I,J)*ET(K,J)
521*      DO 266 I=1,3
522* 266  CK(K,I)=MCK(K,I)/MSB(IK)
523* 262  CONTINUE
524* C
525* C      COMPUTE TOTAL EXTERNAL TORQUE ON EACH SUBSTRUCTURE W.R.T. ITS
526* C      INSTANTANEOUS C.M. (IN LOCAL COORD.)
527* C
528*      DO 268 L=1,3
529* 268  TS(1,L)=TB(L)
530*      DO 267 N=2,NB
531*      K=N+1
532*      DO 267 L=1,3
533* 267  TS(N,L)=TA(K,L)
534*      DO 2670 N=1,NB
535*      IL=F1(N)
536*      IF(IL.EQ.0) GO TO 2670
537*      JN=F(IL,2)
538*      DO 2671 J=1,JN
539*      DO 2671 L=1,3
540* 2671 TS(N,L)=TS(N,L)+TF(IL,J,L)
541* 2670 CONTINUE
542*      DO 269 N=1,NB
543*      K=F1(N)
544*      IF(K.EQ.0) GO TO 269
545*      TS(N,1)=TS(N,1)+CK(K,2)*FS(N,3)-CK(K,3)*FS(N,2)
546*      TS(N,2)=TS(N,2)+CK(K,3)*FS(N,1)-CK(K,1)*FS(N,3)
547*      TS(N,3)=TS(N,3)+CK(K,1)*FS(N,2)-CK(K,2)*FS(N,1)
548* 269  CONTINUE
549*      DO 271 N=1,NB
550*      K=F1(N)
551*      IF(K.EQ.0) GO TO 271
552*      JN=F(K,2)
553*      DO 272 J=1,JN
554*      RUX=RF(K,J,1)+U(K,J,1)
555*      RUY=RF(K,J,2)+U(K,J,2)
556*      RUZ=RF(K,J,3)+U(K,J,3)
557*      TS(N,1)=TS(N,1)+RUY*FF(K,J,3)-RUZ*FF(K,J,2)
558*      TS(N,2)=TS(N,2)+RUZ*FF(K,J,1)-RUX*FF(K,J,3)
559*      TS(N,3)=TS(N,3)+RUX*FF(K,J,2)-RUY*FF(K,J,1)
560* 272  CONTINUE
561* 271  CONTINUE
562* C
563* C      TRANSFORM VECTORS TO REF. BODY FRAME
564* C
565*      TX0(1)=TS(1,1)
566*      TY0(1)=TS(1,2)
567*      TZ0(1)=TS(1,3)
568*      DO 17 I=2,NB
569*      M=H1(I)
570*      K=1+1
571*      L=C(K,1)+1
572*      FEYO(I)=T(M,1,1)*FS(1,1)+T(M,2,1)*FS(1,2)+T(M,3,1)*FS(1,3)
573*      FEYO(I)=T(M,1,2)*FS(1,1)+T(M,2,2)*FS(1,2)+T(M,3,2)*FS(1,3)
574*      FEZO(I)=T(M,1,3)*FS(1,1)+T(M,2,3)*FS(1,2)+T(M,3,3)*FS(1,3)
575*      TX0(I)=T(M,1,1)*TS(1,1)+T(M,2,1)*TS(1,2)+T(M,3,1)*TS(1,3)

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575° TYO(I) =T(M,1,2)*TS(I,1)+T(M,2,2)*TS(I,2)+T(M,3,2)*TS(I,3)
576° TZO(I) =T(M,1,3)*TS(I,1)+T(M,2,3)*TS(I,2)+T(M,3,3)*TS(I,3)
577° DXO(I,1)=T(M,1,1)*DX(I,1)+T(M,2,1)*DY(I,1)+T(M,3,1)*DZ(I,1)
578° DYO(I,1)=T(M,1,2)*DX(I,1)+T(M,2,2)*DY(I,1)+T(M,3,2)*DZ(I,1)
579° DZO(I,1)=T(M,1,3)*DX(I,1)+T(M,2,3)*DY(I,1)+T(M,3,3)*DZ(I,1)
580° DXO(I,L)=T(M,1,1)*DX(I,L)+T(M,2,1)*DY(I,L)+T(M,3,1)*DZ(I,L)
581° DYO(I,L)=T(M,1,2)*DX(I,L)+T(M,2,2)*DY(I,L)+T(M,3,2)*DZ(I,L)
582° DZO(I,L)=T(M,1,3)*DX(I,L)+T(M,2,3)*DY(I,L)+T(M,3,3)*DZ(I,L)
583° DO 17 J=1,NB
584° IF(I.EQ.J) GO TO 17
585° IF(CPS(K,J).EQ.1) GO TO 177
586° IF(C(K,1).EQ.(J-1)) GO TO 17
587° DXO(I,J)=DXO(I,L)
588° DYO(I,J)=DYO(I,L)
589° DZO(I,J)=DZO(I,L)
590° GO TO 17
591° 177 DXO(I,J)=T(M,1,1)*DX(I,J)+T(M,2,1)*DY(I,J)+T(M,3,1)*DZ(I,J)
592° DYO(I,J)=T(M,1,2)*DX(I,J)+T(M,2,2)*DY(I,J)+T(M,3,2)*DZ(I,J)
593° DZO(I,J)=T(M,1,3)*DX(I,J)+T(M,2,3)*DY(I,J)+T(M,3,3)*DZ(I,J)
594° 17 CONTINUE
595° DO 367 I=1,NB
596° DXO(I,1)=DX(I,1)
597° DYO(I,1)=DY(I,1)
598° 367 DZO(I,1)=DZ(I,1)
599° C
600° C COMPUTE TOTAL EXTERNAL FORCE ON VEHICLE (IN REF. COORD.)
601° C
602° FTXO=0.
603° FTYO=0.
604° FTZO=0.
605° DO 247 N=1,NB
606° FTXO=FTXO+FEZO(N)
607° FTYO=FTYO+FEYO(N)
608° 247 FTZO=FTZO+FEZO(N)
609° C
610° C COMPUTE THE PGSO VECTORS FOR EACH FLEX. APPENDAGE
611° C
612° DO 208 K=1,NF
613° KK=F(K,1)+1
614° M=M1(KK)
615° JNT=F(K,3)
616° IF(KK.EQ.1) GO TO 209G
617° DO 209 I=1,3
618° PGSO(K,I)=0.
619° DO 209 J=1,3
620° 209 PGSO(K,I)=PGSO(K,I)+T(M,J,1)*(-MCK(K,J))
621° GO TO 208
622° 209G CONTINUE
623° DO 209I I=1,3
624° 209I PGSO(K,I)=-MCK(K,I)
625° 208 CONTINUE
626° C
627° C VECTOR CROSS PRODUCTS DESCRIBING SYSTEM ROTATIONAL COUPLING.
628° C (QUADRATIC TERMS INVOLVING THE CONNECTING BODY ANGULAR
629° C VELOCITIES AND THE MUTUAL BARYCENTER-HINGE VECTORS)
630° C
631° DO 230 N=1,NB
632° I=F1(N)
633° CPX=0.
634° CPY=0.
635° CPZ=0.
636° DO 230I L=1,NB
637° CPX=CPX+DYO(N,L)*FEZO(L)-DZO(N,L)*FEYO(L)
638° CPY=CPY+DZO(N,L)*FEXO(L)-DXO(N,L)*FEZO(L)
639° CPZ=CPZ+DXO(N,L)*FEYO(L)-DYO(N,L)*FEXO(L)
640° 230I CONTINUE
641° IF(I.EQ.0) GO TO 243
642° FACT=MSB(N)/TH
643° FIXM=FTXO*FACT

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644°      FTYM=FTYO*FACT
645°      FTZM=FTZO*FACT
646°      PGFX=(PGSO(1,2)*(FEZO(N)-FTZM)-PGSO(1,3)*(FEVO(N)-FTYM))/MSB(N)
647°      PGFY=(PGSO(1,3)*(FEXO(N)-FTXM)-PGSO(1,1)*(FEZO(N)-FTZM))/MSB(N)
648°      PGFZ=(PGSO(1,1)*(FEYO(N)-FTYM)-PGSO(1,2)*(FEXO(N)-FTXM))/MSB(N)
649°      GO TO 244
650° 243  CONTINUE
651°      PGFX=0.
652°      PGFY=0.
653°      PGFZ=0.
654° 244  CONTINUE
655°      K = 3*(N-1)
656°      E(K+1,1)=TXO(N)*CPX+PGFX
657°      E(K+2,1)=TYO(N)*CPY+PGFY
658°      E(K+3,1)=TZO(N)*CPZ+PGFZ
659° 230  CONTINUE
660°  C
661°  C   COMPUTE CORRECTION ELEMENTS FOR (E) VECTOR
662°  C
663°      DO 55 M=1,3
664° 55  EC(M)=E(M,1)
665°      DO 52 J=2,NB
666°      DO 52 M=1,3
667°      K1=3*(J-1)+M
668° 52  EC(M)=EC(M)*E(K1,1)
669°      I=0
670°      DO 60 K=1,NH
671°      JK=M(K)+1
672°      IF(P1(K).NE.0) GO TO 60
673°      I=I+1
674°      EC(I+3)=0.
675°      DO 601 M=1,3
676° 601  CE(M)=0.
677°      DO 61 J=JK,NB
678°      IF(EPS(K,J).EQ.0) GO TO 61
679°      DO 65 M=1,3
680°      J1=3*(J-1)+M
681° 65  CE(M)=CE(M)*E(J1,1)
682° 61  CONTINUE
683°      DO 66 L=1,3
684° 66  EC(I+3)=EC(I+3)+G0(K,L)*CE(L)
685°      EC(I+3)=EC(I+3)+TH(K)
686° 60  CONTINUE
687°      DO 610 I=1,3
688°      DO 610 J=1,NH
689°      IF(P1(J).EQ.0) GO TO 610
690°      EC(I)=EC(I)+AV(J,1)*GMDD(J)
691° 610  CONTINUE
692°      K=0
693°      IV=3
694°      DO 612 I=1,NH
695°      IF(P1(I).NE.0) GO TO 612
696°      K=K+1
697°      IV=IV+1
698°      DO 611 J=1,NH
699°      IF(P1(J).EQ.0) GO TO 611
700°      IF(I.GT.J) AS(I,J)=AS(J,I)
701°      EC(K+3)=EC(K+3)+AS(I,J)*GMDD(J)
702° 611  CONTINUE
703° 612  CONTINUE
704°  C
705°  C   COMPUTE RT. HAND SIDE OF APPENDAGE EQUATIONS (IN APPEND. COORDS.)
706°  C
707°      DO 483 K=1,NF
708°      IF(K,1)+1
709°      M=H1(1)
710°      CQ(1)=FTXO/TH
711°      CQ(2)=FTYO/TH
712°      CQ(3)=FTZO/TH
713°      IF(I.EQ.1) GO TO 484

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714. DO 484 J=1,3
715. VE(K,J)=0.
716. DO 484 L=1,3
717. 484 VE(K,J)=VE(K,J)+Y(N,J,L)*CQ(L)
718. GO TO 483
719. 484. CONTINUE
720. DO 484 J=1,3
721. 484.1 VE(K,J)=C4(J)
722. 483 CONTINUE
723. DO 485 K=1,NF
724. NL=F(K,2)
725. DO 486 N=1,NL
726. N6=6*(N-1)
727. DO 488 J=1,3
728. JN=N6+J
729. JH=JN+3
730. VB(K,JN)=FF(K,N,J)
731. 488 VB(K,JH)=TF(K,N,J)
732. 486 CONTINUE
733. 485 CONTINUE
734. NV=IV
735. DO 491 K=1,NF
736. JN=F(K,3)
737. NL=F(K,2)
738. NL6=6*NL
739. DO 492 J=1,JN
740. IL=NV+J
741. VV1=-WF(K,J)*(2.0*ZF(K,J)*ETD(K,J)+NF(K,J)*ET(K,J))
742. DO 493 N=1,NL6
743. 493 VV1=VV1+EIG(K,N,J)*VB(K,N)
744. DO 494 N=1,3
745. 494 VV1=VV1-PK(K,N,J)*VE(K,N)
746. EC(IL)=VV1
747. DO 4920 L=1,NH
748. IF(P1(L).EQ.0) GO TO 4920
749. EC(IL)=EC(IL)-AKF(L,K,J)*GMDD(L)
750. 492. CONTINUE
751. 492 CONTINUE
752. 491 NV=NV+JN
753. C
754. C ANGULAR MOMENTUM OF THE SYSTEM
755. C
756. IF(P1(NH+1).NE.1) GO TO 8752
757. DO 5651 I=1,3
758. MH(I)=0.
759. DO 5651 J=1,3
760. 5651 MH(I)=MH(I)+ADD(I,J)*W(J)
761. DO 5652 I=1,3
762. DO 5652 J=1,QH
763. 5652 MH(I)=MH(I)+AV(J,I)*GMD(J)
764. DO 5653 I=1,3
765. DO 5653 K=1,NF
766. NL=F(K,3)
767. DO 5654 J=1,NL
768. 5654 MH(I)=MH(I)+AQF(K,I,J)*ETD(K,J)
769. 5653 CONTINUE
770. MH=SQRT(MH(1)**2 + MH(2)**2 + MH(3)**2)
771. 8752 CONTINUE
772. C
773. C SOLVE SYSTEM MATRIX FOR REF. BODY ANG. ACCEL., SUBSTRUCTURE
774. C HINGE ANGLE ACCEL., AND FLEX. BODY MODE ACCEL.
775. C
776. DO 671 I=1,IT
777. EQ(I)=0.
778. DO 671 J=1,IT
779. 671 EQ(I)=EQ(I)+A(I,J)*EC(J)
780. KV=IV
781. DO 910 J=NT+4,-1
782. IF(J.LE.V) GO TO 913

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783•      JV=J-(V-IV)
784•      EC(J)=EQ(JV)
785•      GO TO 910
786•  913  CONTINUE
787•      K=J-3
788•      IF(P1(K).NE.0) GO TO 911
789•      EC(J)=EQ(KV)
790•      KV=KV-1
791•      GO TO 910
792•  911  EC(J)=GH00(K)
793•  910  CONTINUE
794•      DO 6710 I=1,3
795•  6710  EC(I)=EQ(I)
796•      DO 9003 I=1,IV
797•  9003  WDOT(I)=EC(I)
798•      I=V
799•      DO 9001 K=1,NF
800•      NL=F(K,3)
801•      DO 9002 N=1,NL
802•      IO=I+N
803•  9002  ETDD(K,N)=EC(IO)
804•  9001  I=I+NL
805•  92   CONTINUE
806•      RETURN
807•      END

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