A WALL-WAKE VELOCITY PROFILE FOR TURBULENT COMPRESSIBLE BOUNDARY LAYERS WITH HEAT TRANSFER

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SYMBOLS

a	=	A constant in the expression $\tau = \tau_w(1-\eta^a)$ (see Ref. 2)
A	=	$\{[(\gamma-1)/2]\ M_e^2/(T_w/T_e)\}^{\frac{1}{2}}$
A_1	=	$(Pr_t)^{\frac{1}{2}}\ \{[(\gamma-1)/2]\ M_e^2/(T_w/T_e)\}^{\frac{1}{2}}$
B	=	$\{(1+[(\gamma-1)/2]\ M_e^2)/(T_w/T_e)\}-1$
B_1	=	$\{(1+(Pr_t)^{\frac{1}{2}}\ [(\gamma-1)/2]\ M_e^2)/(T_w/T_e)\}-1$
C	=	a constant in Law of the Wall (usually 5.1)
C_f	=	Skin friction coefficient $\tau_w/(1/2)\rho_e\ u_e^2$
C_1	=	$5.1 - 0.614/aK + (1/K)\ \ln\ (\delta u_\tau/\nu_w)$
K	=	Constant in mixing length (usually 0.4)
M	=	Mach number
P	=	Pressure
Pr_t	=	Turbulent Prandtl number
T	=	Temperature
u	=	Velocity in streamwise direction
u*	=	(u_e/A) arc sin $\{[(2A^2u/u_e) - B]/(B^2+4A^2)^{\frac{1}{2}}\}$
u**	=	(u_e/A_1) arc sin $\{[(2A_1{}^2u/u_e) - B_1]/(B_1{}^2+ 4A_1{}^2)^{\frac{1}{2}}\}$
u_τ	=	Friction velocity $(\tau_w/\rho_w)^{\frac{1}{2}}$
U*	=	$u^* + (u_e/A)$ arc sin $\{B/(B^2+4A^2)^{\frac{1}{2}}\}$
W	=	Coles' universal wake function
y	=	Coordinate normal to wall
γ	=	Ratio of specific heat
δ	=	Boundary layer thickness
δ^*	=	Displacement thickness

δ^*_K	=	$\int_0^\delta (1-u/u_e)dy$
Δ	=	$\int_0^\delta [(u_e^{**}-u^{**})/u_\tau]\, dy$
θ	=	Momentum thickness
η	=	y/δ
ν	=	Kinematic viscosity
Π	=	Coefficient of Wake Function
ρ	=	Density
τ	=	Shear stress

Subscripts

e	=	Conditions at the edge of the boundary layer
o	=	Stagnation conditions
w	=	Conditions at the wall

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I. INTRODUCTION

The wall-wake velocity profile for isoenergetic compressible turbulent boundary layer flow developed by Mathews et al., [1], and modified by the present authors, [2], was based on the work of Maise and McDonald [3]. In their work, Maise and McDonald suggested that the defect of Van Driest's [4] generalized velocity u* has the same form for compressible turbulent boundary layer flow with zero pressure gradient as the velocity defect in incompressible flow given by Coles [5]. Mathews et al., [1], went further to suggest that a boundary layer profile similar in form to Coles' wall-wake profile, but the velocity replaced by u*, might be applicable for compressible flow with pressure gradient. The resulting profile and the subsequently modified form of it have indeed been found to provide a good representation of the mean velocity distribution for adiabatic flows with pressure gradient.

Since Van Driest's generalized velocity u* can take into account the effect of heat transfer, it was reasonable to investigate the use of profiles expressed in terms of u* for non-adiabatic flows. Maise and McDonald [3] attempted a u*-defect correlation for such flows using the same relationship they had used for adiabatic flat-plate flows. They examined a number of non-adiabatic velocity profiles but found an unacceptable level of scatter in the data.

Lewis, Gran and Kubota [6], working with Van Driest's transformed velocity, U*, which differs from u* by a constant for a given profile, were able to correlate low-speed and compressible turbulent boundary layer data for adiabatic flow with pressure gradients through the use of a pressure gradient parameter $\beta_K = (\delta^*_K/\tau_w)(dp/dx)$ suggested by Alber and Coats [7]. Subsequently Gran et al., [8] used the wall-wake profile proposed in Reference 6 to represent boundary layer profiles for cold-wall flows subjected to pressure gradients. They found that the pressure gradient parameter correlated low-speed data with both adiabatic and cold-wall high-speed data.

The velocity profile used in References 6 and 8 has a non-zero velocity gradient at the boundary layer edge. The purpose of this note is to suggest a modified form of the wall-wake profile which is applicable to flows with heat transfer, and for which $\partial u/\partial y = 0$ at $y = \delta$. The modified profile also takes into account the effect of turbulent Prandtl number. It has been found to provide a good representation of the experimental data from several sources and the C_f values which are determined by a least squares fit of the profile to the data agree well with values which were measured by the floating element technique.

II. VELOCITY PROFILE

For incompressible flow the wall-wake velocity profile as proposed by Coles [5] may be written as,

$$u/u_\tau = (1/K) \ln (yu_\tau/\nu) + C + \Pi W(y/\delta)/K \qquad (1)$$

Setting $u = u_e$ and $W(y/\delta) = 2$ at $y/\delta = 1$ in Eq. (1) and subtracting the resulting equation from Eq. (1) leads to an expression for the velocity defect,

$$(u_e - u)/u_\tau = - (1/K) \ln (y/\delta) + (\Pi/K) [2 - W(y/\delta)] \qquad (2)$$

Maise and McDonald [3] suggested that for compressible flow the generalized velocity defect also assumes a unique universal functional relationship such as that given by Eq. (2), i.e.,

$$(u_e^* - u^*)/u_\tau = -(1/K) \ln (y/\delta) + (\Pi/K) [2 - W(y/\delta)] \tag{3}$$

They found that this expression provided a good representation of adiabatic, flat-plate, boundary layer profiles with Π/K set equal to 1.25.

Mathews et al., suggested for isoenergetic flow a profile similar in form to that given by Eq. (1) but with u replaced by u*, i.e.,

$$u^*/u_\tau = (1/K) \ln (yu_\tau/\nu_w) + C + (\Pi/K) W(y/\delta) \tag{4}$$

This profile was found to be applicable to adiabatic flows with pressure gradient for a range of Mach numbers and flow configurations. The value of Π/K, which was determined through a least squares fit of the wall-wake profile to experimental data, was different for different values of pressure gradient.

With both Eq. (1) and Eq. (4) the velocity gradient at the boundary layer edge has a non-zero value. This shortcoming is avoided with the modified profile proposed by Sun and Childs [2]:

$$\frac{u}{u_e} = \frac{(B^2+4A^2)^{\frac{1}{2}}}{2A^2} \sin \{[\text{arc} \sin (\frac{2A^2-B}{(B^2+4A^2)^{\frac{1}{2}}})] \times [1+ \frac{1}{K} \frac{u_\tau}{u_e^*} (\ln n + \frac{2}{a} (1-n^a)^{\frac{1}{2}} - \frac{2}{a} \ln (1+(1-n^a)^{\frac{1}{2}})) - \frac{\Pi}{K} \frac{u_\tau}{u_e^*} (2-W(n))]\} + \frac{B}{2A^2} \tag{5}$$

where

$$\frac{\Pi}{K} = \frac{1}{2} [\frac{u_e^*}{u_\tau} - \frac{1}{K} \ln \frac{\delta u_\tau}{\nu_w} - 5.1 + \frac{0.614}{aK}] \tag{6}$$

and

$$\frac{u_e^*}{u_\tau} = \frac{u_e}{u_\tau}\frac{1}{A} \text{ arc sin } [\frac{2A^2-B}{(B^2+4A^2)^{\frac{1}{2}}}] \tag{7}$$

with the value of a taken to be 1.

Equation (5) has been tested against a large number of experimental boundary layer profiles on adiabatic walls for a wide range of Mach numbers and has been found to provide a good representation of the velocity profile. However, for flow on a non-adiabatic wall, the applicability of Eq. (5) deteriorates as the ratio of the temperature on the wall to the free stream total temperature decreases.

Van Driest [5] developed a compressible law-of-the-wall from the concept of mixing length in a manner similar to that followed by Prandtl for incompressible flow. It may be written as,

$$\frac{u^*}{u_\tau} + \frac{u_e}{u_\tau}\frac{1}{A} \text{ arc sin } \frac{B}{(B^2+4A^2)^{\frac{1}{2}}} = \frac{1}{K} \ln \frac{yu_\tau}{\nu_w} + C \tag{8}$$

A comparison of Eq. (8) with Eq. (1) suggests that a boundary layer profile for compressible flow with heat transfer should be written as,

$$\frac{u^*}{u_\tau} + \frac{u_e}{u_\tau}\frac{1}{A} \text{ arc sin } \frac{B}{(B^2+4A^2)^{\frac{1}{2}}} = \frac{1}{K} \ln \frac{yu_\tau}{\nu_w} + C + \frac{\Pi}{K} W(\frac{y}{\delta}) \tag{9}$$

The left-hand side of this expression is the transformed velocity, U*, which is used in References 6 and 8.

In Van Driest's development of Eq. (8), he assumed a turbulent Prandtl number of unity, which leads to the following temperature and density distributions across the boundary layer,

$$\frac{T}{T_w} = 1 + B\frac{u}{u_e} - A^2 \left(\frac{u}{u_e}\right)^2 \quad (10)$$

and

$$\frac{\rho}{\rho_w} = \frac{1}{1+B\left(\frac{u}{u_e}\right)-A^2\left(\frac{u}{u_e}\right)^2} \quad (11)$$

In addition, he assumed a constant shear stress near the wall. Thus, with Eq. (9) as written, the velocity gradient at the boundary layer edge will have a non-zero value. If we assume, as in Reference 2, that τ may be expressed as $\tau = \tau_w (1-\eta^a)$, and if for $Pr_t \neq 1$, the temperature and density distributions may be written as (cf. Schlichting [9]),

$$\frac{T}{T_w} = 1 + B_1 \frac{u}{u_e} - A_1^2 \left(\frac{u}{u_e}\right)^2 \quad (12)$$

and

$$\frac{\rho}{\rho_w} = \frac{1}{1+B_1\left(\frac{u}{u_e}\right)-A_1^2\left(\frac{u}{u_e}\right)^2} \quad (13)$$

we may follow the same procedure as is used in Reference 2 to obtain a boundary layer velocity profile of the following form,

$$\frac{u^{**}}{u_\tau} + \frac{u_e}{u_\tau}\frac{1}{A_1} \text{arc sin} \frac{B_1}{(B_1^2+4A_1^2)^{1/2}} = \frac{1}{K}\{\ln \eta + \frac{2}{a}(1-\eta^a)^{1/2} - \frac{2}{a}\ln[1+(1-\eta^a)^{1/2}]\} + C_1 + \frac{\Pi}{K}W(\eta) \quad (14)$$

or in defect form,

$$\frac{u_e^{**} - u^{**}}{u_\tau} = -\frac{1}{K}\{\ln \eta + \frac{2}{a}(1-\eta^a)^{1/2} - \frac{2}{a}\ln[1+(1-\eta^a)^{1/2}]\} + \frac{\Pi}{K}[2-W(\eta)] \quad (15)$$

For $a \to \infty$, which corresponds to the assumption of a linear shear stress distribution in the derivation of the compressible law-of-the-wall, and for $Pr_t \to 1$, $u^{**} \to u^*$ as in Eq. (9). Setting $\eta = 1$ in Eq. (14) yields the following expression for Π/K,

$$\frac{\Pi}{K} = \frac{1}{2} \left\{\frac{u_e^{**}}{u_\tau} + \frac{u_e^{**}}{u_\tau} \frac{u_e}{u_e^{**}} \frac{1}{A_1} \text{arc sin} \frac{B_1}{(B_1{}^2+4A_1{}^2)^{1/2}} - \frac{1}{K} \ln \frac{\delta u_\tau}{\nu_w} - 5.1 + \frac{0.614}{aK}\right\} \quad (16)$$

For mathematical convenience [1-3] we may replace $[2-W(\eta)]$ in Eq. (15) by $[1 + \cos \eta\pi]$ and write the wall-wake velocity profile as,

$$\frac{u}{u_e} = \frac{(B_1{}^2+4A_1{}^2)^{1/2}}{2A_1{}^2} \sin \left\{\left[\text{arc sin} \frac{2A_1{}^2-B_1}{(B_1{}^2+4A_1{}^2)^{1/2}}\right] \times \left[1+ \frac{1}{K} \frac{u_\tau}{u_e^{**}} \left(\ln \eta + \frac{2}{a} (1-\eta^a)^{1/2} - \frac{2}{a} \ln (1+(1-\eta^a)^{1/2})\right) - \frac{\Pi}{K} \frac{u_\tau}{u_e^{**}} (1+ \cos \eta\pi)\right]\right\} + \frac{B_1}{2A_1{}^2} \quad (17)$$

with Π/K given by Eq. (16). A least squares fit of the profile to experimental data yields values of δ and u_τ or C_f. As $Pr_t \to 1$, $u^{**} \to u^*$, so that Eq. (17) reduces to Eq. (5).

The value of Π/K may be calculated from the conditions at the wall and at the boundary layer edge by Eq. (16). However, it may also be obtained from velocity profile data by integrating Eq. (15) from $\eta = 0$ to $\eta = 1$. For $a = 1$ the integration yields,

$$\frac{\Pi}{K} = \frac{u_e^{**}}{u_\tau} \int_0^1 \left(1 - \frac{u^{**}}{u_e^{**}}\right) d\eta - \frac{2}{3K} \tag{18}$$

III. DISCUSSION OF RESULTS

The method of least squares has been used to fit the wall-wake profile, Eq. (17), to experimental velocity profiles reported by Hopkins and Keener [10], Voisinet et al., [11, 12], Horstman and Owen [13] and Kilburg [14] for zero pressure gradient flows with heat transfer. The wall-wake velocities were obtained under the assumption of $Pr_t = 0.8$ and $a = 1$. K was taken as 0.4 and C as 5.1. Data in the sublayer were excluded. The results shown here are from studies for which directly measured values of wall shear stress were reported [10, 11, 13]. The data reported by Voisinet et al., and Hopkins and Keener were obtained on wind tunnel walls. The Horstman-Owen data are for flow along the cylindrical surface of a cone-cylinder model whose axis was aligned with the primary wind tunnel flow direction.

Comparisons of the experimental and wall-wake profiles are given in Figure 1. Also shown on the figures are values of δ^*, θ, C_f and Π/K. The wall-wake profile is seen to provide a good representation of the experimental velocity profiles. In addition, the C_f values determined by the fit of the proposed profile agree reasonably well with the directly measured values. For reasons which are not known at this time the wall-wake profile does not provide as good an overall fit to the data reported by Voisinet et al., as it does to the data of Hopkins and Keener or Horstman and Owen. As Figure 1 shows, the wall-wake profile agrees quite well with the Voisinet data between the wall and the value of δ determined by the least squares fit. However, at the value of δ determined by the fit, $u/u_e = 0.965$ rather than 0.995 which is sometimes taken to correspond to the boundary layer edge in high speed flows. For the $M_{\mathcal{L}} = 4.92$ flow examined by Voisinet, $P_o/P_{o_e} = 0.56$ and $T_o/T_{o_e} = 0.938$ at $u/u_e = 0.965$. On the other hand, for the Hopkins-Keener and the Horstman-Owen data the values of u/u_e, P_o/P_{o_e}, T_o/T_{o_e} are essentially unity at the value of δ determined by the fit. Values of δ were also determined for $a \rightarrow \infty$ in Eq. (17). As would be expected, they are lower than for $a = 1$ and u/u_e, P_o/P_{o_e} and T_o/T_{o_e} at δ are correspondingly lower.

The values of Π/K shown in Figure 1 were determined from Eq. (16), assuming $Pr_t = 0.8$ and $a = 1$ and with u_τ and δ determined from the wall-wake fit. Although the pressure gradient was zero for the flows under consideration, the value of Π/K varies considerably from profile to profile and is generally higher than for low-speed or compressible adiabatic flows. Note that, for the modified profile used here, the expression for Π/K differs from that used in References 1, 3, 6 and 8 through the inclusion of the Prandtl number effect, through the term $0.614/aK$, and through the differences in u_τ and δ which are obtained with the modified profile.

Figure 2 shows a comparison of the experimental data from References 10, 11 and 13 with the u**-defect relationship given by Eq. (15) for $Pr_t = 0.8$ and with the value of K taken as 0.4. The values of Π/K which are listed were calculated from Eq. (18) using the measured values of shear stress to evaluate u_τ. The experimental values of u** have been calculated from the reported values of Mach number and total temperature. As is shown, Eq. (15) provides a reasonably good representation of the u**-defect for each profile.

The results may also be plotted in the form $(u_e^{**} - u^{**})/u_\tau$ vs. y/Δ, where Δ is analogous to the defect thickness used by Gran et al., [8] in their comparison of cold-wall data with low-speed data [15]. Such a plot is shown in Figure 3. It is seen that the data of Hopkins and Keener agree quite closely with the low-speed results. The Horstman-Owen data, for which Π/K is about thirty percent higher than for the Hopkins Keener profile, deviates from the low-speed results in the outer part of the boundary layer. The Voisinet data, which Π/K is considerably higher than for either of the other two profiles, are quite different from the low-speed data, even though the wall-wake profile fits the experimentally determined velocity distribution quite closely between the wall and the value of δ determined by the least squares fit.

The values of δ^* and θ which were determined from the wall-wake profile are listed on Figure 1, along with the experimental values reported in the references. The results agree quite well for the Horstman-Owen profile, but only moderately well for the other two. In the case of the data by Voisinet, the difference appears to result from the fact that the velocity, total temperature and total pressure continue to increase well beyond the value of δ determined by the least squares fit. For the data of Hopkins and Keener on the other hand, the difference is due to differences between the experimental density profile and the profile implied by Eq. (13). Data reported by Schlichting [16] suggests that the turbulent Prandtl number for high-speed flow on a flat plate is on the order of 0.9 and that for flow on the wall of a plane channel is on the order of 0.8. Recent work of Meier et al., [17] also suggests that Pr_t is near 0.8 for flow on the wall of a plane channel. The total temperature distribution reported by Hopkins and Keener imply a considerably lower Prandtl number, and consequently, a considerably different density distribution than is obtained when $Pr_t = 0.8$ is assumed in Eq. (13). This in turn leads to differences between the experimental and the wall-wake values of δ^* and θ.

IV. CONCLUSION

An extension of the wall-wake velocity profile of Reference 2 to compressible turbulent boundary layer flow on non-adiabatic walls has been tested against experimental data covering a wide range of Mach number and heat transfer and has been found to provide a good representation of experimental velocity profiles. The profile considers the effect of turbulent Prandtl number and allows for variation of the shear stress near the wall. Values of C_f determined by a least squares fit of the wall-wake profile to experimental data agree reasonably well with directly measured values. The values of δ determined by the fit are closer to those based on u/u_e = 0.995 than are found with earlier versions of the wall-wake profile which were developed under the assumption of a constant shear stress near the wall. The values of the integral properties δ^* and θ as determined with the wall-wake profile deviate more, in some instances, from experimentally determined values than was found to be the case for adiabatic flows. This occurs because the density profiles given by Eq. (13) and those computed from the experimental total temperature and Mach number profiles differ considerably.

The wake function coefficient Π/K, varies considerably more than has been reported for zero-pressure-gradient non-adiabatic flows in earlier references, and, for most of the data examined, is higher than has been reported for adiabatic flows.

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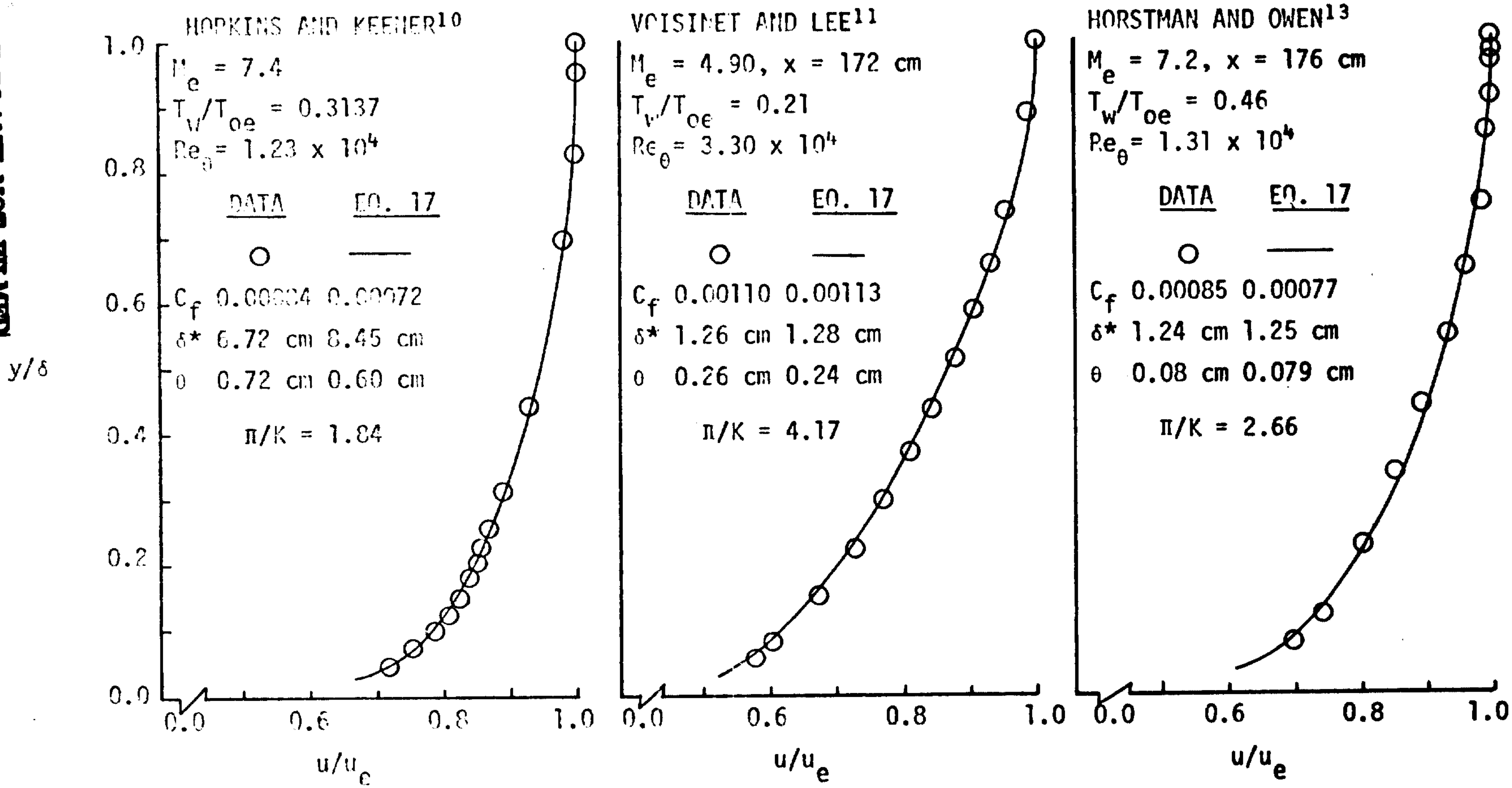
Fig. 1 Velocity distributions in boundary layer with heat transfer.

Fig. 2 Defect of generalized velocity profiles.

Fig. 3 Comparison of generalized velocity defect for low-speed flow and compressible flow with heat transfer.



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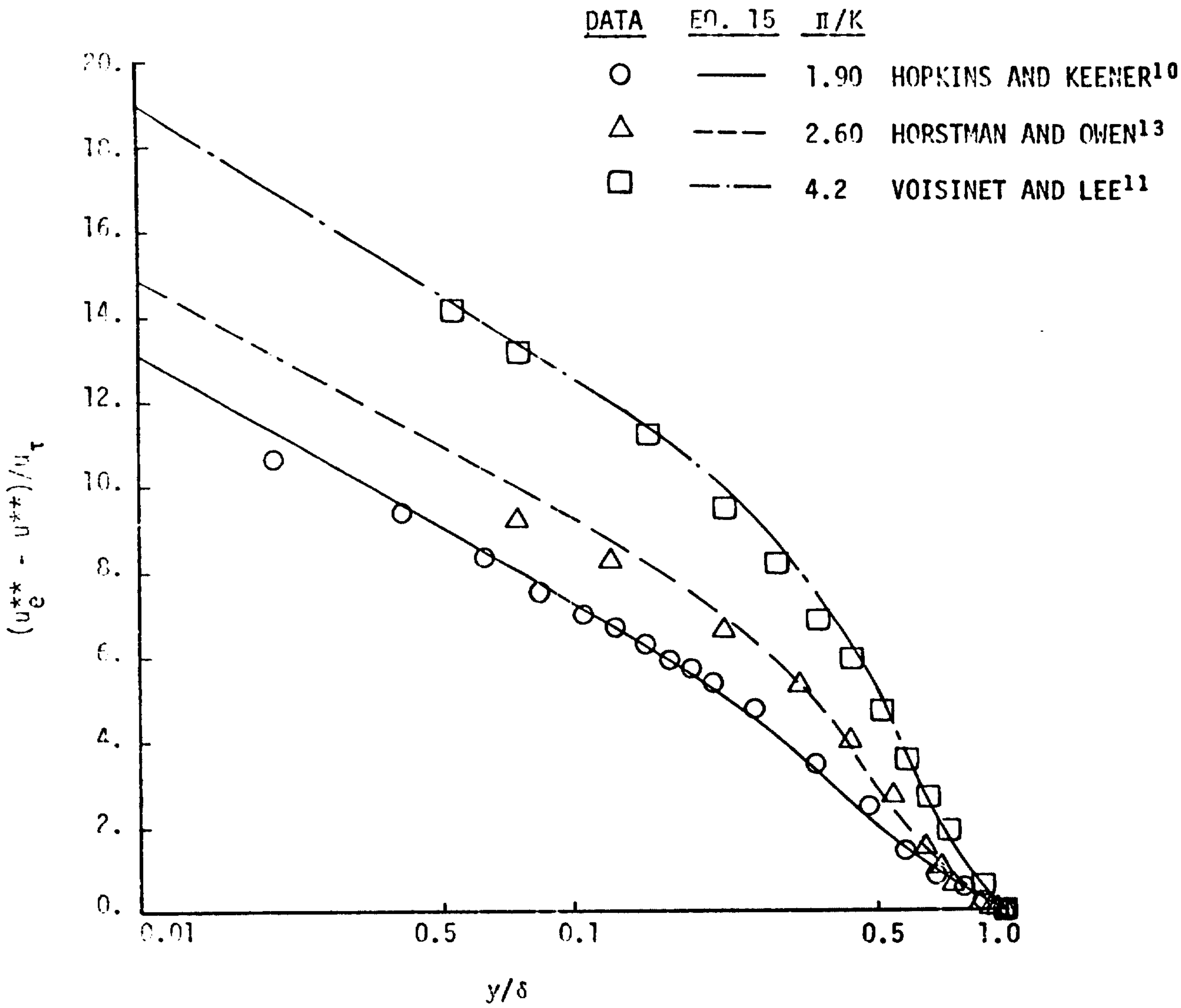
DATA EQ. 15 Π/K
1.90 HOPKINS AND KEENER[10]
2.60 HORSTMAN AND OWEN[13]
4.2 VOISINET AND LEE[11]
(u*e* - u**)/uτ
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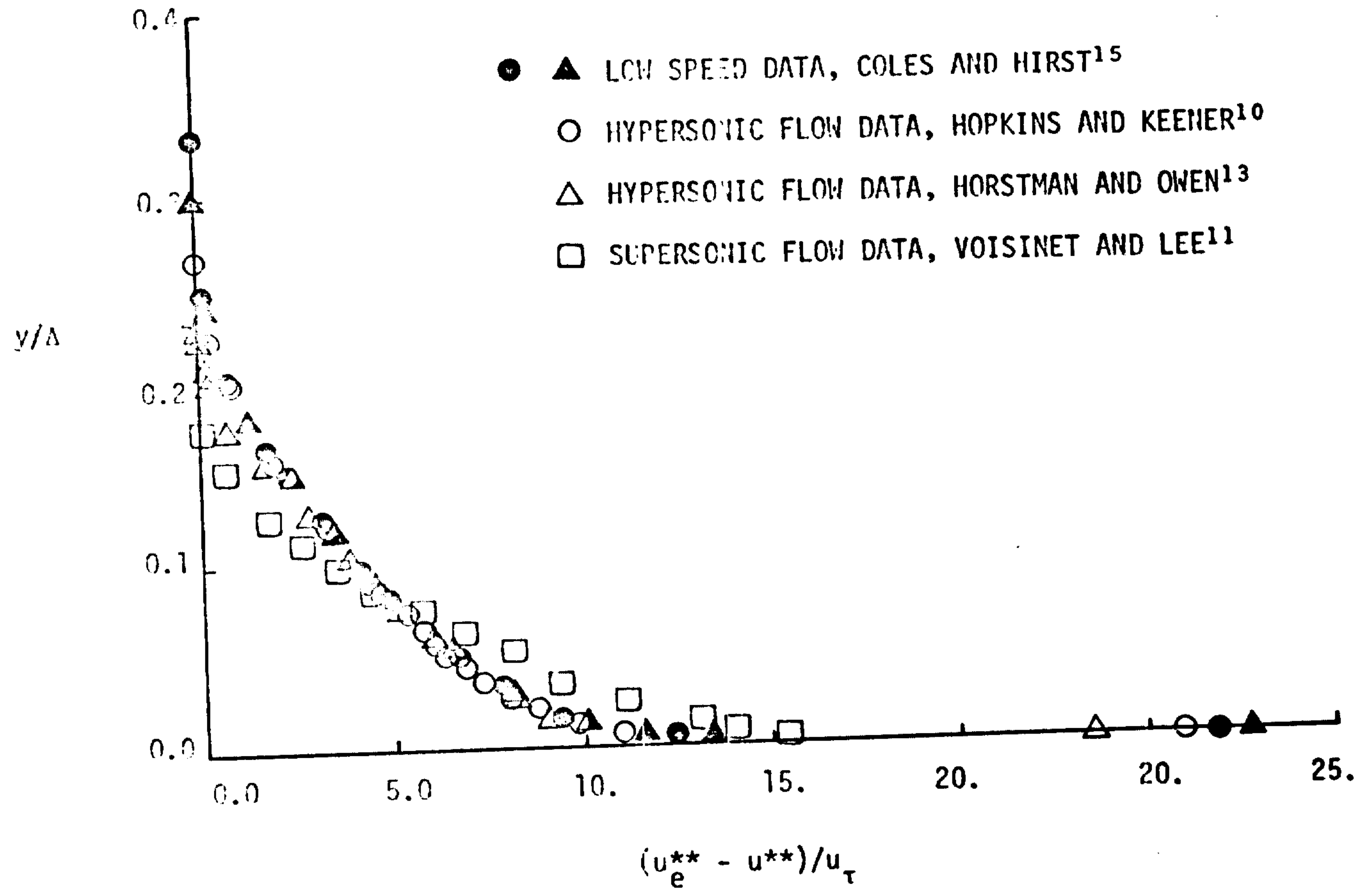
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HYPERSONIC FLOW DATA, HOPKINS AND KEENER[10]
HYPERSONIC FLOW DATA, HORSTMAN AND OWEN[13]
SUPERSONIC FLOW DATA, VOISINET AND LEE[11]
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$(u_e^{**} - u^{**})/u_\tau$