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# THE RECOVERY OF MICROWAVE SCATTERING PARAMETERS FROM SCATTEROMETRIC MEASUREMENTS WITH SPECIAL APPLICATION TO THE SEA 

by J. P. Claassen and A. K. Fung

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## 16. Abstract (continued)

Within the simulations it is shown that there is little difficulty in recovering the dominant scattering coefficients with modest realizations of the polarization specifications. Retrieval of the weaker scattering parameters requires more careful observation of the polarization requirements. In the latter case, it is shown that more relaxed realizations of the polarization specifications can be tolerated for many of the measurements if the phase of the cross polarized leakage can be adjusted to an optimun value.

In general it is indicated that three real and three complex valued scattering coefficients can interact with the scatterometer antenna in an undesirable fashion when attempting recovery of any one coefficient. The measurement error arises either as a result of inadequate reatizations of the specified antenna polarization or as a result of the inherent mis-match between antenna and surface polarizations for small angtes.

This report was prepared by the Remote Sensing Laboratory of the University of Kansas Space Technology Laboratories under Contract NAST-10048. Under this coniract the principal investigator is Dr. R. K. Moore and the project engineer is Dr. A. K. Fung.

This document covers a particular task in an on-going effort between NASA Langley Research Center and the University of Kansas to demonstrate the value of the microwave satterometer as a remote sea wind sensor. Specifically the interaction between an arbitrarily polarized scatterometer antenna and a non-coherent distributive target is derived and applied to develop a measuring technique to recover all the scattering parameters. The results are helpful for specifying antenna polarization properties for accurate retrieval of the parameters not only for the sea but also for other distributive scenes.

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## GLOSSARY OF SYMBOLS

| A | = Area of radar illumination |
| :---: | :---: |
| $C_{s}$ | $=$ Coherence matrix for scattered field |
| $c_{t(r)}$ | $=$ Coherence matrix for transmit (receive) antenna |
| $\bar{E}_{5}$ | $=e_{v s} \overline{\bar{i}}_{\theta}+e_{h s} \overline{\bar{i}}_{\phi}$, scattered field intensity in voits/mefer per steradian |
| $\bar{E}_{T}$ | $=\mathrm{e}_{\mathrm{vt}} \overline{\mathrm{T}}_{\theta}+\mathrm{e}_{\mathrm{ht}} \overline{\mathrm{T}}_{\phi}$, incident field intensity in volts/meter |
| $G_{t(r)}$ | = Maximum directivity when tronsmitting (receiving) |
| $\mathrm{g}_{\mathrm{v}(\mathrm{h})}$ | $=$ Normalized vertically (horizontally) polarized pattern in the surface coordinate system |
| ${ }^{9}\left(\phi_{\text {( }}\right)$ | = As above, however, in the antenna coordinate system |
| $\overline{\mathrm{I}}$ | $=$ - Antenna input current |
| Im | = Imaginary part operator |
| ${ }^{i}{ }_{\theta\left(\theta^{\prime}\right)}$ | $=$ Unit spherical polar vector in the surface (antenna) coordinate system |
| $\bar{i}_{\text {¢ }}\left(\phi^{\prime}\right)$ | = Unit spherical azimuthal vector in the surface (antenna) coordinate system |
| k | = Propagation constant |
| L | = Complex effective height vector |
| $M_{s}$ | = Mutual coherence matrix for the scattered field |
| $M_{r}$ | = Mutual coherence matrix for the reception antenna |
| P | = Degree of polarization |
| $\mathrm{R}_{\mathrm{r}}$ | = Anfenna radiation resistance |
| R | = Radar range |
| Re | $=$ Real part operufor |
| $8{ }_{\text {ij }}$ | $=$ Scattering operator for the $j^{\text {th }}$ incident polarization and the $i^{\text {th }}{ }_{\text {scattered }}$ polarization ( $\phi_{i}^{\prime} \mathrm{j}^{\mathrm{R}^{2} / \mathrm{A} \cos \theta \text { ) }}$ |
| $8_{i j}^{\prime}$ | $=$ Scattering operator yielding resultant field (See $\beta_{i j}$ ) |


| $\left.\left\langle\mathrm{S}_{\mathrm{ij}} \mathrm{S}_{\mathrm{kl}}\right\rangle\right\rangle$ |  |
| :---: | :---: |
|  | $=$ Time |
| $<>$ | $=$ Time (spatial) average |
| T | = Polarization rotation matrix |
| ${ }_{\text {fr }}$ | $=$ Trace operator |
| ( $\mathrm{r}_{\mathrm{r}} \mathrm{y}, \mathrm{z}$ ) | = Surface coordinate system |
|  | = Antenna coordinare system |
| $W_{f}$ | = Transmit power |
| W | = Receive power |
| $\mathrm{Z}_{0}$ | = Free space impedance |
| $\beta$ | $=$ Relative phase between the vertical and horizental antenna polarizations defined with respect to the surface polarizations |
| $\beta^{\prime}$ | $=$ Relative phase between the vertical and horizontal antenna polarizations defined with respect to the antenna polarizations |
| $\theta_{0}$ | = Incident angle |
| $\lambda$ | = Radar wavelength |
| ${ }_{\circ}$ | = Free space permeability |
| $\psi$ | $=$ Angle between antenna and surface polarizations |
| $\Omega$ | $=(\theta, \phi)$, line of sight |
| $\mathrm{d} \Omega$ | $=\sin \theta \mathrm{d} \theta \mathrm{d} \phi$ |
| $\Omega_{0}$ | $=\left(\theta_{0}, \phi_{o}\right)$ antenna view angle |
| $\omega$ | $=$ Radian frequency |

### 1.0 SUMMARY

The non-coherent radar equation is derived within the framework of a generalized reception theory. For scenes satisfying reciprocity, the resulting equation confirms a previously derived theory [6]; this result, however, was extended to account for the difference beiweeit antenna and surface polarizations. The present theory permits one to interpret the radar return and its reception within the context of scattering and coherence theories (see Section 5.2). Under the reciprocity assumption it is shown that in addition to the three commonly known real valued scattering coefficients there are three complex valued coefficients (without reciprocity there are four real and six complex valued coefficients). As a result of the new coefficients, the definition of a scattering coefficient had to be extended. Specifically a descriptive definition was suggested, viz.,

$$
\begin{equation*}
\left\langle\dot{S}_{i j} S_{k T}^{*}\right\rangle=\frac{\left\langle 8_{i j}^{\prime} e_{j t^{\prime}}^{g_{k}^{\prime}} e_{1 t}^{*}\right\rangle}{e_{j t} e^{*} R^{2} A \cos \theta_{0}} \tag{1-1}
\end{equation*}
$$

where,

$$
\begin{array}{ll}
\mathcal{P}_{i j}^{\prime} e_{j \mathrm{jr}} & =\text { scattered field component } \\
\mathcal{8}_{\mathrm{ij}} & =\text { linear polarized scattering operator } \\
e_{\mathrm{jt}} & =\text { incident field component } \\
R & =\text { range to the illuminated area } \\
A & =\text { incremental area of illumination } \\
\theta_{0} & =\text { incident angle }
\end{array}
$$

The subscripts denote the polarization states of the incident and scattered fields, either vertical $v$ or horizontal $h$. The above definition encompasses the old as well as the new scattering coefficients. In the new notation $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}^{*}{ }_{\mathrm{vv}}\right\rangle$ and $\left\langle\mathrm{S}_{\mathrm{hh}} \mathrm{S}_{\mathrm{h} h}^{*}\right\rangle$ denote the polarized coefficients, $<\mathrm{S}_{\mathrm{Vh}^{5}}{ }^{*}{ }_{\mathrm{vh}}>$ is the cross polarized coefficient, and $\left\langle\mathrm{S}_{\mathrm{pv}} \mathrm{S}_{\mathrm{Hv}}{ }^{3}\right.$ $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hv}}^{*}\right\rangle$, and $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{h h}^{*}\right.$, are the new complexed valued coefficients. Other scattering coefficients participate in the scattering process as implied by Equation (ii) above; however, when reciprocity is satisfied the above set is sufficient (See Equations ( $4-26$ ) and (4-29).)

The complex valued coefficients account for the relative phase induced between the vertically and horizontally polarized components by the scattering surface. The phase characteristic of these scattering coefficients interact with the relative phase pro-
perties of the transmission and reception antennas to contribute an observed power com posed of real and complex valued scattering coefficients. This interaction occurs within the coherent radar equation also; however the inferaction must be inter preted differently for the non-coherent case. As a result of the spatial infegration to acquire an average return, the complex valued coefficient must, in general satisfy Schwartz ${ }^{\text {t }}$ inequalify

$$
\left.\left.\left|\left\langle S_{i j} S_{k l}\right\rangle\right|^{2} \leq\left.\langle | S_{i j}\right|^{2}\right\rangle\left.\langle | S_{k l}\right|^{2}\right\rangle
$$

For the coherent case, equality is always assured. However, for the non-coherent case strict inequality can occur. As a result of the strict inequality, one can aftribute a partially polarized character to non-coherent radar returns (See Section 5.4). Also as a result of the inequality, techniques for measuring the scattering matrix for coherent targets cannot be employed for non-coherent targets. To illustrate the character of these scattering coefficients, several scatfering theories applicable to sea returns were examined (See Section 5, 3).

On the basis of the above theory a measurement and inversion technique was developed to measure all six coefficients (nine parameters when the real and imaginary parts are considered). The fechnique is based on intensity measurements by narrow beam radar scatterometers (See Section 6.4). Inversions are proposed with and without regard to the distinction between antenna and surface polarizations (See Section 6.4 and Section 7.2, respectively). It is demonstrated that the distinction between polar izations is negligible for narrow beam antennas at all but small view angles 'See Sections 5.5 and 7.3). For small view angles, inversions based on surface polarizations are more accurate, in general, if the measurements are to bus compared with theory or with other experimenters. For example, a $50 \%$ error occurs at narir in inverting for $\left.\left.\langle | S_{v h}\right|^{2}\right\rangle$ (defined with respect to the surface polarizations) wher, ine inversion technique is based on anfenna polarizations. Comparison of the inversions with and without regard to the distiaction are shown in Figures (7.3) through (7.8) . Inversions based on antenna polarizations can, however, be performed at small view angles if very small beamwidths are employed. In this case nadir can only be probed in an asymptotic sense. The degree to which one can approach nadir and yet meet the constraint that the antenna polarizations across the main beam approximately match those of the surface is dependent on beamwidth. Figure (7.2) parametrically shows the beamwidth requirement as a function of view angle to minimize unwanted orthogonally polarized content in the measurement.

This latter technique is preferred in as much as the measurements may be restricted to a partial set of coefficients whereas when inversions are performed with respect to the surface polarizarions the entire set of measurements must be performed. It is also advaniageous to use inversions based on antenna polarizations and small beamwidth antennas when an anisotropic characteristic is to be measured at small view angles.

Computer simulations were conducted to determine the effect of deviations from the ideal antenna polarizations (required by the measurement technique) on the accurate recovering of all nine scattering parameters. The deviations, for example, can be introduced by the mis-match between surface and antenna polarizations presuming the scattering parameters are to be reported with respect to the surface polar izations. Also, deviations obviously occur because ideal antenna polarization speciifications cannot be realized by practical antennas. Within these simulations a scattering characteristic similar to that of the sea was employed as itustrated in Figure (7.1). All simulations were conducted with the assumption that the relative phase between the cross-polarizations was stationary across the main beam.

The simulations indicated that there is liftle difficulty in rscovering the three dominant scattering coefficients with off-the shelf antennas as illustrated by Figures (7.9) through (7.12) and Figure (7.15). Some difficulty can be anticipated when $\left.\left.\langle | S_{h h}\right|^{2}\right\rangle$ is more than 10 dB beneath $\left.\left.\langle | \mathrm{S}_{\mathrm{vv}}\right|^{2}\right\rangle$ as illustrated by Figures (7.11) and (7.12). In this case the cross polarized level must be better than 20 dB below the dominant ( h ) polar ization. On the other hand, the antenna polarization requirement must be more carefully observed when retrieving the six weak scatfering parameters as illustrated by Figures (7.13) through (7.16). In some cases an adjustment in the relative phase of the cross polarization (if possible) can relax the antenna requirement. When the relative phase cannot be controlled in the case of cross polarized measurements, a rule of thumb for the quality of the antenna was established. If the measurement is to be performed with a 0.5 dB accuracy and $\left.\left.\langle | \mathrm{S}_{v h}\right|^{2}\right\rangle$ is $X \mathrm{~dB}$ beneath the geometric mean of $\left.\left.\langle | \mathrm{S}_{\mathrm{wv}}\right|^{-}\right\rangle$ and $\left.\left.\langle | s_{h h}\right|^{2}\right\rangle$, then the one-way cross polarized patfern must be $X+16 \mathrm{~dB}$ beneath the dominent.

When the dynamic range of the scattering coefficients is large, it is clear from the simulation studies that the experimenter must carefully design his antenna to accurately retrieve the weaker coefficients. Certain types of antennas which have potential in achieving the ideal polarization states are suggested in

Chapter 6. The antenna specifications for observations over the sea or scenes having similar saaftering cinaracteristics can be based on the results reported in Chapter 7. However, for scenes having an entirely different characteristic, if is advisable to conduct simulations similar to those reported here. The simulation progrram, documented in Chapter 7 and Appendix $D$, may be easily modified for this purpose. These cbservations as well as others serve as a guide for designing meaningful and accurate scatterometer ${ }^{*}$ experiments.

[^0]
### 2.0 INTRODUCTION

Various resear ch programs have been proposed or are in progress to demonstrate the potential of monitoring, on a global basis, imporient geological, environmental, hydrological, oceanographic, meteoroloical, and ag rarian parameters. The usefulness of remotely sensing certain parameters has been repeatedly demonstrated with optical and infrared sensors. More recently however, satellite and microwave fechnologies have developed to a point where microwave sensors are also suitable candidates as remote sensing devices. The microwave radiometer and radar scatterometer are prime candidate sensors.

In remote sensing technology it is common knowledge that the retrieval of the remotely sensed parameters often entails compensation of the measurements for sensor and afmospheric effects. The antenna is one element of the sensor system that requires special consideration. An understanding of the antenna-scene interaction is essential to designing meaningful experiments and for specifying the antenna with which the experiments are to be conducted.

The radio astronomers, for example, have developed a rigorous theory involving the complex visiblity function to describe the interaction of a radiometer antenna with a small celestial scene.[1]. , Measurement techniques were based on the theory to derive complete emission properties of the scene. Recently Claassen and Fung [2] and Peake [3] have reported radiometer interaction relationships for nominally flat scenes having a simple partially polarized emission property. A measurement technique based on the relationship was developed by Claassen and Fung. Grody [4] has illustrated how the difference between antenna and surface polarizations impu ct radiometer experiments. To date little has been done to develop and use a comprehensive radar scatterometer antenna-scene interaction qelationship for non-coherent targets. Most efforts have freated only the spatial extent of the antenna pattern and have avoided general antenna and scene polarization properties [5]. An exception occurs in the theory developed by Williams, et al. [6]. Their characterization of the scene parameters was based on the coherent radar equation and no me surement technique was reported.

In this study a complete non-coherent radar equation is derived and interpreted. The resulfing expressions are valid for an arbitrary antenna. The result is also extended to consider the differences between antenna and surface polarizations. The distinction
is important when measurements are to be compared with theoretical predictions. It is shown that six differential scattering coefficients are required to describe the antennascene interaction when reciprocity applies. Three of the six coefficients are complex valued. The coefficients are interpreted within the context of scattering and coherence theories. A retrieval technique based on intensity measurements is proposed to measure all the scatfering coefficients, Computer simulations, based on the technique and a scattering characteristic similar to that of the sea, were conducted. The restits of the simulation were employed (1) to validate an approximation used in the inversion, (2) to demonstrate antenna requirements for accurate retrieval of the scattering coefficients, and (3) to evaluate whether the distinction between alftenna and surface polarizations is important.

The development of material in the subsequent chapters is accumulative. Chapter 3 develops the background theory relevant to the derivation and understanding of the complete non-coherent radar equation. An adequate number of references are cited so that the reader can fill in background more deeply if he so desires. The derivation of the non-coherent radar equation is presented in Chapter 4. In the latter section of this chapter the equation is altered to account for the difference between antenna and surface polarizations. Chapter 4 is strongly supported by the contents of Appendix $A$ and $B$. Chapter 5 is devoted to developing an understanding of the non-coherent radar equation and the polarization properties of radar returns. Certain scattering theories described in Appendices $A$ and $C$ are visited to illustrate the behavior of the complex valued coefficients. The difference between antenna and surface polarizations is also illustrafed, The measurement and inversion fechnique is presented in Chapter 6. The mathematical aspects of the inversion are treated in general and then specialized to the radar problem. Certain antenna properties which simplify the inversion are described. Antenna types capable of realizing these properties are suggested. The measurement and inversion technique is evaluated within Chapter 7. A computer program which simulates the measurement and refrieval of all nine scattering parameters is described briefly. Full documentation of the scatterometer simulation program is provided in Appendix D. The results of the simulation are employed to illustrate antenna polarization requirements to measure all nine scattering parameters. Other practical aspects in making radar scatterometer measurements are also discussed. The measurement of the pattern amplitudes is specifically treated. Appendix E describes a computer program which specifies the points
at which a pattern must be measured. The conclusions and recommendations are presented in Chapter 8. A summary of all significant results is presented in Chapter 1. It is advisable to read the summary before entering the technical chapters.

### 3.0 BACKGROUND

### 3.1 Introduction

The theory and measurement of radar cross sections have been well developed for discrete coherent targets. An excellent review on the measurement of radar cross sections is found in a special issue of the Proceedings of the IEEE [7]. The theory of measuring non-coherent radar cross sections, except for the isolated works of Williams, et al. [6] and to some degree Hagfors [8], is largely lacking. Williams, et al. simply extended the theory for coherent targets to a non-coherent scene. In doing so, they over-looked some subtle distinctions between coherent and non-coherent theories as shown in Chapter 5. No measurement technique was presented. Hagfors, on the other hand, related Stoke's parameters for the incident wave to Stoke's parameters for the scaftered wave in terms of the Mueller matrix [9]. In general, there are sixteen parameters in the Mueller matrix. However, as shown by Hagfors, fargets exhibiting reciprocity and circular symmetry can be characterized by five independent entries in the Mueller marrix. Hagfors related his measurements to some of the five independent entries but no attempt was made to isolate all five entries. By using "Gedanken Experimente" as Hagfors did, one can show that for a flat scene there are nine independent entries. At nadir there can conceivably be only five if the scene is cylindrical symmetric (isotropic). The fact that there are nine independert entries in the Mueller matrix for flat scenes implies that there should be nine scattering parameters. To date only fhree scattering coefficients have been reported by the earth resources community [10] [11] [12].

In preparing the background for this effort the author chooses to avoid the use of Stoke's parameters and Mueller matrices since the earth resource community is, for the large part, unfamiliar with them. Instead polarization coherency matrices, an entirely equivalent representation for the polarization state of the transverse wave, are employed. The relationship between the entries in the coherency matrix and the standard differential.scattering coefficients are clearer. To properly introduce the more general reception theory in terms of coherency matrices, the background for the reception of (polarized) monochromatic waves is first established. It is then employed to derive the coherent radar equation. In doing so the importance of reception theory in understanding the radar equation is clarified.

### 3.2 The Reception of Monochromatic Waves and the Radar Equation

### 3.2 Transmitfed Fields

Schelkunoff has shown that the far field of any antenna has a dipole field characteristic [13], viz.,

$$
\begin{equation*}
\vec{E}(\theta, \phi)=\frac{-j Z_{0} \vec{N}(\theta, \phi) e^{-j k r}}{2 \lambda r} \tag{3-1}
\end{equation*}
$$

where
N = radiation vector
$Z_{0}=$ intrinsic impedance of the medium
$\lambda=$ wave length
$5=$ distance to the far field point
k = propagation constant
and where the time factor $e^{j \omega t}$ is suppressed. The radiation vector $\hat{N}_{\text {, }}$ in general, has complex components and induces a relative phase between the far fieid exponents. As a consequence, the far field has an arbitrary elliptical polarization. Now since $\vec{N}$ is proportional to the antenna input current $I$, Sinclair [14] proposed that a complex effective height vector $t$ be introduced so that

$$
\begin{equation*}
\vec{L}(\theta, \phi)=\overrightarrow{\hat{N}}(\theta, \phi) / I \tag{3-2}
\end{equation*}
$$

The far field can therefore be expressed as

$$
\begin{equation*}
\vec{E}=\frac{-j \omega \mu_{0} I \vec{L} e^{-j k r}}{4 \pi r} \tag{3-3}
\end{equation*}
$$

where
$\omega$ = radian frequency
$1_{0}=$ permeability of free space
In general, $\mathbb{L}$ may have both $\theta$ and $\varnothing$ components in a spherical coordinate system and both may be complex. Spesifically to emphasize this property, we may write $亡$ in normalized form

$$
\begin{equation*}
\vec{L}=|\vec{L}|\left(\cos \delta \bar{i}_{\theta}+\sin \delta e^{j \beta_{\bar{i}}}\right) \tag{3-4}
\end{equation*}
$$

at 11"11" rempactly as

$$
\begin{equation*}
\vec{L}=|\vec{L}|\left(1_{v}{ }_{\theta}{ }_{\theta}+1_{h}^{T}{ }_{\phi}^{\top}\right) \tag{3-5}
\end{equation*}
$$

A Inllar ts the orientation of a linear polarization when $\beta=0$ and when $\beta \neq 0_{r}$ I: IA ilin selative phase between the components. With a little effort $\delta$ and $\beta$ calif his matuted to the axial ratio and orientation of a polarization ellipse [15].

### 3.7.7 line eiving Polarized Waves

Suppose the above antenna is used to receive a plane wave described by

$$
\vec{E}=\left(E_{v} \bar{i}_{\Theta}+E_{h} \bar{T}_{\phi}\right) e^{-j \vec{k} \cdot \vec{r}}
$$

whan $A$ is the propagation vector. $E_{v}$ and $E_{h}$ are the vertically and horizontally palurland amplitudes, respectively. Then it can be shown by the reciprocity theorem [la] thut tho open circuit valtage induced into the terminals of an anfenna having eflim, tlyn height $\bar{L}$ is given by [14]

$$
\begin{equation*}
V=\vec{E} \cdot \vec{L} \tag{3-7}
\end{equation*}
$$

The prown available at the antenna terminals under matched conditions is given by

$$
\begin{equation*}
P=|\vec{E} \cdot \vec{L}|^{2} / 8 R_{r} \tag{3-8}
\end{equation*}
$$

whal $K_{r}$ is the radiation resistance of the antenna.
3.?.il Murochromatic Reception and the Radar Equation

It has been shown by Sinclair [17] and by Kennaugh[18] that a radar farget
.- call unit is a polarization transformer. Sinclair expresses the transformation by a scallailuy matrix which can be incorporated in the radar equation. The scattering mothlu In defined by

$$
S=\left[\begin{array}{cc}
\sqrt{\sigma}_{v} e^{j \rho_{v v}} & {\sqrt{\sigma_{v h}}}^{e^{j \rho_{v h}}}  \tag{3-9}\\
{\sqrt{\sigma_{h}}} e^{j \rho_{h v}} & \sqrt{\sigma_{h h}} e^{j \rho_{h h}}
\end{array}\right]
$$

where

$$
\begin{aligned}
& \sigma_{p q}=\text { radar cross section for a q linearly polarized incident wave and ap } \\
& \quad \text { reflected wave } \\
& \rho_{p q}=\text { phase center for each component of the reflected wave } \\
& p, q=v \text { or } h
\end{aligned}
$$

If the incident field is denoted as

$$
\begin{equation*}
\vec{E}_{t}=\left(e_{v t} \vec{t}_{\theta}+e_{h t} \vec{t}_{\phi}\right) e^{-j k r} \tag{3-10}
\end{equation*}
$$

or

$$
\begin{equation*}
\vec{E}_{t}=\frac{-j \omega \mu_{0} I_{t} \vec{L}_{t}-j k r}{4 \pi r} \tag{3-II}
\end{equation*}
$$

then the scattered field $\vec{E}_{s}$ in component form is given by*

$$
\left[\begin{array}{l}
e_{v s}  \tag{3-12}\\
e_{h s}
\end{array}\right]=1 /(\sqrt{4 \pi} r) s\left[\begin{array}{l}
e_{v t} \\
e_{h t}
\end{array}\right]
$$

If $\tau_{r}$ denotes the complex effective height vector of the receiving antenna co-located with the transmit antenna, then the power received under matched conditions is given by

$$
\begin{equation*}
P=\left|\vec{E}_{S} \cdot \vec{L}_{r}\right|^{2} / 8 R_{r} \tag{3}
\end{equation*}
$$

or

$$
\begin{equation*}
P=\frac{\left(\omega \mu_{o} I_{t}\right)^{2}\left|L_{t}\right|^{2}\left|L_{r}\right|^{2}\left|T_{t} S 1_{r}\right|^{2}}{8(4 \pi) r^{4} R_{r}} \tag{3-14}
\end{equation*}
$$

where

$$
\begin{align*}
& t_{t, r}=\left|I_{t, r}\right|\left(1_{v t ; r} r_{\theta}+1_{h t, r} \tilde{T}_{\phi}\right)  \tag{3-15}\\
& 1_{t}=\left[\begin{array}{l}
J_{v t} \\
1_{h t}
\end{array}\right]
\end{align*}
$$

*Matrix notation for transverse wave components is frequently used throughout.

$$
{ }^{1} \mathrm{r}=\left[\begin{array}{l}
1_{\mathrm{vr}}  \tag{3-15}\\
\mathrm{l}_{\mathrm{hr}}
\end{array}\right]
$$

Now it is well known [18] that the antenna gain is given by

$$
\begin{equation*}
G_{t, r}=\frac{4 \pi\left|\vec{L}_{t, r}(\theta, \phi)\right|^{2}}{\int\left|\vec{L}_{t, r}\right|^{2} \mathrm{~d} \Omega} \tag{3-16}
\end{equation*}
$$

and the radiation resistance by

$$
\begin{equation*}
R_{t, r}=z_{0} /\left(4 \lambda^{2}\right) \int\left|L_{t, r}\right|^{2} \mathrm{~d} \Omega \tag{3-17}
\end{equation*}
$$

As a result, the received power can be written in more familiar form

$$
\begin{equation*}
W_{r}=\frac{\lambda^{2} G_{t}(\theta, \phi) G_{r}(\theta, \phi) W_{t} \sigma}{(4 \pi)^{3} r^{4}} \tag{3-18}
\end{equation*}
$$

where the radar cross-section has been identified as

$$
\begin{equation*}
\sigma=\left|1_{r} S l_{t}\right|^{2} \tag{3-19}
\end{equation*}
$$

The above expression for the radar cross section reduces to the linear polarized cases when both $1_{f}$ and $l_{r}$ contain a single non-zero component. For the coherent target the above formulation completely describes the interaction between three apertures, the transmitting and receiving antennas and the target*.

Methods for measuring the elements of the scattering matrix have been reviewed by Huynen [21]. Methods of measuring radar cross sections $\sigma$ have been reviewed by Blacksmith, et al. [22] and by Kell and Ross [23].

### 3.3 The Non-Coherent Radar Equation

Radar returns for a non "coherent scene have been defined in terms of a differential seattering coefficient $\sigma^{\circ}$ rather than a scattering cross section $\sigma . \quad \sigma^{\circ}$ is unitless
*It has been shown that the target actually acts like two coupled apertures [20].
and expresses the equivalent average radar cross section per unit area. Moore [5] has shown from elementary considerations that when $\sigma^{\circ}$ is employed the average return is given by

$$
\begin{equation*}
W_{r}=W_{t} /(4 \pi)^{3} \int G_{t} G_{r} \sigma^{\circ} / r^{4} d A \tag{3-20}
\end{equation*}
$$

where the integration is performed over the illuminated area. For linear polarizations the differential scattering coefficients have been defined in analogy to $\sigma$ for the Eoherent case [5] [24]

$$
\begin{equation*}
\sigma_{q p}^{0}=\frac{4 \pi r^{2}<\left|E_{s q}\right|^{2}>}{A\left|E_{i p}\right|^{2}} \tag{3-2l}
\end{equation*}
$$

where
A = illuminated area
r = distance between the illuminated area and the point of observarion
$\mathrm{E}_{\text {sq }}=$ scattered field intensity

- $E_{i p}=$ incident field intensity $\mathrm{p}, \mathrm{q}=\mathrm{vorh}$
Williams et al. [6] have shown that radar returns cannot be characterized by linear polarized scattering coefficients for an arbitrary antenna polarization and an arbitrary scene. They offer an expression for the differential power contribution by a small patch of the scene. Their formulation, however, is entirely identical to the radar equation for coherent targets, i.e., the effects of spatial averaging have not been considered.


### 3.4 The Reception of Quasi-Monochromatic, Partially Polarized Waves

### 3.4.1 General

The trearment of radar returns for non-coherent scenes to date has relied on infuitive exrensions of (polarized) monochromatic theory. Yet when one eontemplates how the measurement of the non-coherent scattering coefficients is acfually performed, one is acutely aware that the measurement invalves estimating the mean of a fading signal having a certain doppler bandwidth. The resulting refurns are, as a
consequence, quasi-monochromatic rather than monochromatic. Furthermore, it is presumptive to anticipate that average returns from a randomly rough target are completely polarized*. Indeed one should anticipate that the octual refurn will be a mixture of randomly polarized and polarized waves, $i, e .$, will be partially polarized. Hence ${ }_{r}$ any derivation of the non-coherent radar equation should include this possibility.

Ko [15] has developed a comprehensive reception theory for quasi-monochromatic partially polarized waves. This theory is reviewed below and will be employed in the succeeding chapter. Important to this theory are the notions of an analytic signal as defined by Gabor [26] and the polarization coherence matrix as originated by Wiener [27] and Perrin [28] and later developed by Wolf [29]. An excellent discussion of both topics appears in a text by Born and Wolf [30].

### 3.4.2 Quasi-Monochromaric Partially Polarized Waves

Wolf [29] has shown that a quasi-monochromatic wave whose bandwidth is small in comparison to the mean angular frequency $\bar{\omega}$ can be represented in analytic signal form of the type

$$
\begin{equation*}
\bar{E}(r, \theta, \phi, t)=e_{v}(r, \theta, \phi, t) T_{\theta}+e_{h}(r, \theta, \phi, t) \mathbb{i}_{\phi} \tag{3-22}
\end{equation*}
$$

where

$$
\begin{align*}
& e_{v}=a_{v}(r, \theta, \phi, t) e^{j\left(\bar{\omega} t-k r+\alpha_{v}(\theta, \phi, t)\right)}  \tag{3-23}\\
& \left.e_{h}=a_{h}(r, \theta, \Phi, t) e^{j\left(\bar{\omega} t-k r+\alpha_{h}\right.}(\theta, \phi, t)\right)
\end{align*}
$$

The actual signal may be isolated by taking the real part of the above expression. The elements of this analytic signal have properties such that $a_{v, h}(r, t) \geq 0$ and $\alpha_{v_{r}} h\left(\vec{r}_{r} t\right)$ is real. The correlation of the $\theta$ and $\varnothing \bar{\phi}$ components determines the state of polarization of the wave. W'olf [29] defines the correlation by the complex factor

$$
\begin{equation*}
\mu_{v h}=\frac{\left\langle e_{v} e_{\hat{H}}^{*}\right\rangle}{\sqrt{\left.\left.\left.\langle | e_{v}\right|^{2}\right\rangle\left.\langle | e_{h}\right|^{2}\right\rangle}} \tag{3-24}
\end{equation*}
$$

[^1]Where the angular bracket < > represents a time average. By Schwartz' inequality, $1 \mu_{\mathrm{vh}} \leqslant 1$. The absolute value of $\mu_{\mathrm{vh}}$ is a measure of the degree of correlation between $v$ and h components while the phase angle of $\mu_{\mathrm{vh}}$ reflects the relative phase between the two components. If $\left|\mu{ }_{\text {vh }}\right|=1$, the wave is said to be completely polarized. If $\left|\mu_{v h}\right|=0$ and if $\left.\left.\left.\langle | e_{v}\right|^{2}\right\rangle=\left.\langle | e_{h}\right|^{2}\right\rangle$, then the wave is randomly polarized. The wave is said to be partially polarized when $\left|\mu_{\mathrm{vh}}\right|$ is between zero and one. The state of polarization may be completely characterized by a coherency matrix

$$
c=\left[\begin{array}{cc}
\left\langle e_{v} e_{v}^{*}\right\rangle & \left\langle e_{v} e_{h}^{*}\right\rangle  \tag{3-25}\\
\left\langle e_{h} e_{v}^{*}\right\rangle & \left\langle e_{h} e_{h}^{*}\right\rangle
\end{array}\right]
$$

as shown by Wolf [29] (See also Born and Wolf [30]).
Following Ko [25] and Collin [9] we may now suppose that a quasi-monochromatic partially polarized wave with coherency matrix $C$ is incident on an antenna with effective height L. If the bandwidth of the wave or receiver is sufficiently narrow, then the open circuit voltage in analytic signal form is given by

$$
\begin{equation*}
V=\vec{E} \cdot \vec{L}(\theta, \phi, \bar{\omega}) \tag{3-26}
\end{equation*}
$$

where $\bar{\omega}$ is the mean frequency of the wave. If a coherency matrix is introduced for the antenna

$$
c_{r}=\left[\begin{array}{ll}
1_{v} 1_{v}^{*} & 1_{v} 1_{h}^{*}  \tag{3-27}\\
1_{h} 1_{v}^{*} & 1_{h} 1_{h}^{*}
\end{array}\right]
$$

where $\left|I_{v}\right|^{2}+\left|I_{h}\right|^{2}=I$, then as shown by $K o[25]$, the power observed at the antenna terminals under matched conditions is givia by

$$
\begin{equation*}
W(\theta, \phi)=\frac{\lambda^{2} G(\theta, \phi)}{4 \pi Z_{0}} \operatorname{tr} C_{r} C^{\dagger} \tag{3-28}
\end{equation*}
$$

where fr is the frace operator and $\dagger$ is the transpose operator. The coherency matrix for the impinging wave is the transpose of that defined by Ko . All coherency matrices employed within this work are defined with respect to a coordinate system located at the observing antenna. Further interpretation of this expression is deferred until Chapter 5 where a similar expression is discussed in the context of the scatterm efer equation.

### 4.0 DERIVATION OF THE SCATTEROMETER EQUATION

### 4.1 Introduction

A generalized reception theory [9] [25] and notions from scattering theory are combined to derive the compiete sacterometer equation for a scatterometer antenna having a specified but otherwise arbitrary transmit and receive property. The radar refurn is treated as a quasi-monochromatic-partially polarized wave. The quasimonochromatic character is induced into the return signal as the antenna linearily scans the scene. The scan is, of course, important in achieving a spatial average. The partially polarized assumption as well as the quasi-monochromatic characters permits one to derive the scatterometer equation eleguntly within the framework of the generalized reception theory. Intuitively, it is reasonable to assume that scatferometer returns are partially polarized since a spatial average constitutes the return. This interpretation will be illustrated in Section 5.4.

The scatterometer equation is initially derived assuming that the scatterometer antenna fransmission and reception properties are defined in terms of the surface polarizations. In the last section of the chapter the distinction between antenna polarizations and surface polarizations is infroduced and the impact of this distinction on the scatterometer equation is shown.

### 4.2 Derivation

To determine the average power return from a homogeneous randomly extensive target, we suppose that a narrow beam scatterometer linearily scans across the scene with its antenna pointed in direction $\Omega_{0}=\left(\theta_{0}, \varnothing_{0}\right)$. If the scene has an anisotropic character it is important that © be maintained constant during the scan (see Figure 4.1). The incident (transmitted) field $E_{f}$ may be related to the antenna complex effective height vector $L_{f r}$ a reception property, in the standard way [25]*

$$
E_{i}=\left[\begin{array}{l}
e_{v t}  \tag{4-1}\\
e_{h t}
\end{array}\right]=\frac{-j \omega \mu_{o} i_{t} L_{t} e^{j(\omega t-k r(t))}}{4 \pi r}
$$

[^2]

FIGURE 4.1 The Geometry of the Scatterometer AntennaScene Interaction
where $\mathrm{e}_{\mathrm{vt}}$ is the vertically polarized component, $\mathrm{e}_{\mathrm{ht}}$ is the horizontally polarized component and

$$
L_{t}=\left[\begin{array}{l}
1_{v t}  \tag{4-2}\\
1_{h t}
\end{array}\right]
$$

The subscript $v$ and $h$ are employed to denote the vector components aligning with the spherical polarized unit vectors $\overline{\mathrm{i}}_{\theta}$ and $\overline{\mathrm{i}} \not \underline{\text { r }}$, respectively, associated with the surface coordinate system of Figure 4.1. The backscattered field arriving with direction ( $\theta, \emptyset$ ) from a differential patch of the surface will be denoted

$$
E_{s}(\theta, \phi, t)=\left[\begin{array}{l}
e_{v s}  \tag{4-3}\\
e_{h s}
\end{array}\right]
$$

Only transverse components for each line of sight $(\theta, \varnothing)$ are admitted in the matrix. The field has the units of volts/meter per steradian. Each component of $\mathrm{E}_{\mathrm{s}}$ must be regarded as an analytic signal since the relative motion between the antenna and the rough scene induces a time varying response for each line of sight.

Now the antenna does not respond to the resultant field at the point of observation. Rather, if $L_{R}$ denotes the complex effective height vector during reception, the antenna integrates the field components arriving with different directions so that the open circuit voltage appearing at the antenna termincls is given by

$$
\begin{equation*}
v_{r}(\Omega)=\int E_{s}^{\dagger}(\Omega) L_{r}\left(\Omega, \Omega_{0}\right) d \Omega \tag{4-4}
\end{equation*}
$$

where $\Omega_{\mathrm{o}^{\prime}}$, as the reader will recall, denotes the look direction and where the symbol $\dagger$ denotes the transpose operator. For narrow beam scatterometers the integration may be limited to the main beam and under worst circumstances to the first side lobes. The average power observed at the terminals of the antenna under matched conditions is given by

$$
\begin{equation*}
W\left(\Omega_{0}\right)=\frac{\left.\left.\langle | V\left(\Omega_{0}\right)\right|^{2}\right\rangle}{8 R_{r}} \tag{4-5}
\end{equation*}
$$

where $R_{r}$ is the radiation resistance during reception (r) and < > denotes a time average or equivalently a spatial average since the scatterometer is scanning across the scene.

Expanded, the received power is given by

$$
\begin{equation*}
W\left(\Omega_{0}\right)=\frac{1}{8 R_{r}} \iint\left\langle E_{S}^{\dagger}(\Omega) L_{r}\left(\Omega, \Omega_{0}\right) E_{S}^{\dagger *}\left(\Omega^{\prime}\right) L_{r}^{*}\left(\Omega_{0}^{\prime} \Omega_{0}\right)\right\rangle d \Omega d \Omega^{\prime} \tag{4-6}
\end{equation*}
$$

Define a mutual (polarization) coherence matrix for the scattered fields as

Similarly a mutual coherence matrix, can be defined for the receiving antenna

$$
M_{r}=\left[\begin{array}{ll}
T_{v r}\left(\Omega, \Omega_{0}\right) 1_{v r}^{*}\left(\Omega^{t}, \Omega_{0}\right) & T_{v r}\left(\Omega, \Omega_{0}\right) 1_{\hat{h r}}^{*}\left(\Omega^{\prime}, \Omega_{o}\right)  \tag{4-8}\\
T_{h r}\left(\Omega_{0} \Omega_{0}\right) 1_{v r}^{*}\left(\Omega^{\prime}, \Omega_{0}\right) & T_{h r}\left(\Omega, \Omega_{0}\right) 1_{\hat{h r}}^{*}\left(\Omega^{*}, \Omega_{o}\right)
\end{array}\right]
$$

Then the average return can be written in compact form

$$
\begin{equation*}
W_{\operatorname{tr}}\left(\Omega_{0}\right)=\frac{1}{8 R_{r}} \iint \operatorname{tr} M_{r} M_{s}^{\dagger} d \Omega d \Omega^{\prime} \tag{4-9}
\end{equation*}
$$

where fr denotes the trace operator.
For a rondom scene it is reasonable to assume that the scatfered fields are angularly non-coherent, i.e..

$$
\begin{equation*}
\left\langle e_{i s}(\Omega) e_{j s}^{*}\left(\Omega^{1}\right)\right\rangle=\left\langle e_{i s}(\Omega) e_{j s}^{*}(\Omega)\right\rangle \delta\left(\Omega-\Omega^{1}\right) \tag{4-10}
\end{equation*}
$$

The pragmattic aspect of this assumption is established in Appendix A. There it is shown that for a finitely conducting-smoothly undulating surface the degree of coherency (correlation) defined by

$$
\begin{equation*}
D_{i j}=\left\langle e_{i s}(\Omega) e_{j 5}^{*}\left(\Omega^{\prime}\right)\right\rangle /\left\langle e_{i 5}(\Omega) e_{j 5}^{*}(\Omega)\right\rangle \tag{4-II}
\end{equation*}
$$

is given by

$$
\theta_{i j} \cong 2 \exp \left(-k^{2} \sin ^{2} \theta \sigma^{2} \Delta^{2} \theta / 2\right) \operatorname{Jinc}\left(k \cos \theta R_{0} \Delta \theta\right)
$$

where

$$
\begin{aligned}
& \sigma^{2}=\text { surface height variance } \\
& k=2 \pi / \lambda \\
& \theta=\text { incident angle } \\
& \Delta \theta=\text { small angular deviation from } \theta \\
& R_{0}=\text { radius of the illuminated area } \\
& i_{,}, j=v \text { or } h
\end{aligned}
$$

The delta function type character of the angular coherency $D_{i j}$ is illustrated in Figure 4.2 for a patch of rough surface having a radius of one meter and illuminated at 13.9 GHz . A close examination of $\mathrm{D}_{\mathrm{ij}}$ reveals that, in general, the size of the illuminated area rather than the surface roughness dominates the correlation property at all angles of incidence except for the very large incident angles. The above result is based on plane wave illumination. The degree of coherency is thought to have a stronger delta function character in the case of spherical wave illumination since returns arriving from different directions arise from different patches of the scene whose statistical characteristics are poorly correlated. A discussion of this latter point within the context of a scattering theory appears in Appendix B.

Under the above assumption the return power reduces to

$$
\begin{equation*}
W_{t r}\left(\Omega_{0}\right)=\frac{1}{8 R_{r}} \quad \int \operatorname{tr} C_{r} C_{s}^{\dagger} d \Omega \tag{4-13}
\end{equation*}
$$

where

$$
\begin{align*}
& C_{r}=M_{r}\left(\Omega, \Omega, \Omega_{0}\right) \\
& C_{s}=M_{s}(\Omega, \Omega) \tag{4-14}
\end{align*}
$$

are the coherency matrices for the receiving antenna and the scattered fields, respectively. As a resulf of the integration the units of the elements within $C_{s}$ become $(\mathrm{v} / \mathrm{m})^{2}$ per steradian. The change in units is clarified in Appendix $B$.

Now also under the non-coherent assumption, it is permissible to introduce the


FIGURE 4.2 ANGULAR COHERENCY OF BACKSCATTER FOR a circular patch with radius of one ileter Aidi FOR A FREDUEICY OF 13.9 Ghz
notion of a matrix of differential scattering operators so that for each arrival direction the backscattered field (coming from a differential patch of the surface) is related to the incident field in the following way

$$
E_{s}=\left[\begin{array}{ll}
8_{v v} & 8_{v h}  \tag{4-15}\\
8_{h v} & 8_{h h}
\end{array}\right] \quad E_{t}
$$

The second subscript indicates the polarization of the incident field and the first subscript denotes the polarization of the resulting backscattered field. The objective for introducing this operator is that it identifies the scattered field components for each component of the incident field. With the introduction of this matrix the coherence matrix associated with the scattered field may be written as

$$
\begin{aligned}
& \left.\left[C_{s}\right]_{v v}=\left\langle\beta_{v v^{\prime}} \delta_{v v}^{*} e_{v t} e_{v t}^{*}\right\rangle+2 R e<\delta_{v v} \delta_{v h}^{*} e_{v t} e_{h t}^{*}\right\rangle+ \\
& \left\langle 8_{v h^{8}} v^{*} h^{e} h t^{e}{ }_{h t}>\right.
\end{aligned}
$$

$$
\begin{aligned}
& +\left\langle g_{v h^{8}}^{h_{h}} e_{h t} e_{h t}^{*}\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
& { }^{8} h_{h h^{8}}^{8} h^{e}{ }_{h t}{ }^{e}{ }_{h t}{ }^{>} \\
& {\left[\mathrm{C}_{\mathrm{s}}\right]_{\mathrm{hv}}=\left[\mathrm{C}_{5}\right]_{\vee \mathrm{h}}^{*}}
\end{aligned}
$$

The action of the scattering operators on the incident fields is clearly evident in the above expression.

If the incident wave were a plane wave, it is natural to define a scattering coefficient as

$$
\begin{equation*}
\left\langle S_{i j} j_{k 1}^{*}\right\rangle=\left\langle ళ_{i j} e_{j t} \delta_{k 1}^{*} e_{l t}^{*}\right\rangle / e_{i t} e_{1 t}^{*} \tag{4-17}
\end{equation*}
$$

If the above definition is employed for a spherical wave, the scattering operator will have to contend with a quadratic phase factor in the incident wave and with a varying intensity across the surface. The resulting scattering coefficient would depend on the geometry of the antenna pattern. However, under the non-coherent assumption the incident wave may be corisidered locally plane on each patch of the surface and the scattering action is then interpreted in accordance with the plane wave definition for the scatfering coefficient. In particular, the expectations in $\mathrm{C}_{\mathrm{s}}$ may be written as

$$
\begin{equation*}
\left\langle\hat{p}_{i j} e_{j t} f_{k 1}^{*} e_{1 t}^{*}\right\rangle=\left[\frac{\omega \mu_{0} i_{t}}{4 \pi r}\right]^{2}\left\langle S_{i j} S_{k 1}^{*}\right\rangle 1_{j t} T_{1 t}^{*} \tag{4-18}
\end{equation*}
$$

where Equation (4-1) has been employed. The scattering coefficient is now allowed to vary with $(\theta, \emptyset)$ across the illuminated area. The integrand of the equation can now equation can now be written as

$$
\begin{align*}
& \operatorname{tr} C_{r} C_{s}^{\dagger}=\frac{\left(\omega \mu_{0} i_{t}\right)^{2}}{(4 \pi r)^{2}}\left\{\left|1_{v r}\right|^{2}\left[\left.\langle | S_{v v}\right|^{2}\right\rangle\left|1_{v t}\right|^{2}+2 \operatorname{Re}\right. \\
& \left.\left\langle S_{v v} S_{v h}^{*}>1_{v t} 1_{h t}^{*}+\left.\langle | S_{v h}\right|^{2}\right\rangle\left|T_{h t}\right|^{2}\right\}+2 \operatorname{Re} \\
& 1_{v r}{ }^{1} \underset{h r}{ }\left[\left\langle S_{v v} S_{h v}^{*}\right\rangle\left|1_{v t}\right|^{2}+\left\langle S_{v v_{h}} S_{h}^{*}>1_{v t} 1_{h t}^{*}+\right.\right. \\
& \left.\left.\left\langle s_{v h} S_{h h}^{*}>\right| \tau_{h t}\right|^{2}\right]+\left|1_{h r}\right|^{2}\left[<\left|s_{h v}\right|^{2}>\left|1_{v t}\right|^{2}+\right. \\
& \left.\left.2 R e<s_{h v} S_{h h}^{*}>1_{v t} 1_{h t}^{*}+\left.\langle | s_{h h}|3| 1_{h t}\right|^{2}\right]\right\} \tag{4-19}
\end{align*}
$$

As a result of the non-coherent assumption and the infroduction of the scatfering coefficients, the fransmit antenna pattern parameters have been divorsed from the composite scattering operafors. The reader will observe that the scattering coefficient employed
here has the units of $\mathrm{m}^{2} / \mathrm{m}^{2}$ per steradian. This definition is natural to this derivation and is a direct consequence of the integrating action of the antenna about its observation point (Equation (4-4)). Further discussion of the scattering coefficients is deferred until Section 5.2. The above steps in the derivation are clarified in the context of a simple scattering theory in Appendix B. As illustrated there ${ }_{r}$ the above theory can be expressed as a continuum limit of an incremental theory which treats the backscatter on a patch by patch basis. Each patch is associated with an arrival direction.

Now the following identifications are helpful in re-formulating the results in more common ferminology:

$$
\begin{align*}
g_{p i}(\theta, \phi) & =\frac{\left|1_{p i}\right|^{2}}{\max _{\theta, \phi}\left\{\left|T_{v i}\right|^{2}+\left|l_{h i}\right|^{2}\right\}} \\
\beta_{i}(\theta, \phi) & =\tan ^{-1}\left(\operatorname{Im} 1_{v i} l_{h i}^{*} / \operatorname{Re} 1_{v i} l_{h i}^{*}\right) \\
G_{i}^{\prime}(\theta, \phi) & =\frac{4 \pi\left(\left|1_{v i}\right|^{2}+\left|T_{h i}\right|^{2}\right)}{\int\left[\left|1_{v i}\right|^{2}+\left|1_{h i}\right|^{2}\right] d \Omega}  \tag{4-22}\\
G_{i} & =\max _{\theta, \phi}\left\{G_{i}^{\prime}(\theta, \phi)\right\} \tag{4-23}
\end{align*}
$$

$$
\begin{equation*}
i_{t}^{2}=2 W_{t} / R_{t} \tag{4-24}
\end{equation*}
$$

$$
\begin{equation*}
R_{t} \quad=\left(z_{o} / 4 \lambda^{2}\right) \int\left[\left|1_{v i}\right|^{2}+\left|T_{h i}\right|^{2}\right] \mathrm{d} \Omega \tag{4-25}
\end{equation*}
$$

where $\mathbf{i}=t$ (transmit) or $r$ (receive) and $p=v$ or h polarization. Descriptively, during transmission $\mathrm{g}_{\mathrm{vt}}$ is the normalized gain of the vertically polarized pattern whereas $\mathrm{g}_{\mathrm{h}}$ is the accompanying horizontally polarized pattern. The relative phase between these fwo polarizations is denoted as $\beta_{f}$. In general all three are functions of the pattern coordinates. $G^{1}$ is the gain under a matched polarization condition and $G_{f}$ is the maximum gain (presumably on boresight). $W_{t}$ is the transmitted power and $R_{t}$ is the radiation resistance when the antenna is transmitting. Similar explanations apply to the reception parameters. They are identified with a subscript $r$. With the introduction of the above pattern parameters, the scatterometer equation can be writfen as.

$$
\begin{equation*}
W\left(\Omega_{0}\right)=(\lambda / 4 \pi)^{2} G_{t} G_{r} W_{t} \int I_{t r} / r^{2} d \Omega \tag{4-26}
\end{equation*}
$$

where

$$
\begin{align*}
I_{t r}= & g_{v r}\left(g_{v t}<\left|S_{v v}\right|^{2}\right\rangle+2 \sqrt{g_{v t} g_{h t}} R e<S_{v v} S_{v h}^{*}>e^{\left.j \beta_{t}\right)}+ \\
& 2 \sqrt{g_{v r} g_{h r}}\left[g_{v t} R e<S_{v v} S_{h v}^{*}>e^{j \beta_{r}}+g_{h t} R e<S_{v h} S_{h h}^{*}>e^{j \beta_{r}}\right. \\
& +\left(\sqrt { g _ { v t } g _ { h t } } \operatorname { R e } \left(\left\langle S_{v v} S_{h h}^{*} e^{j\left(\beta_{t}+\beta_{r}\right)}+\left\langle S_{v h} S_{h v}^{*}>e^{j\left(\beta_{t}-\beta_{r}\right)}\right)\right]\right.\right. \\
& +g_{h r}\left(\langle | S_{h h} i^{2} g_{h t}+2 \sqrt{g_{v t} g_{h t}} \operatorname{Re}\left\langle S_{h v} S_{h h}^{*}>e^{\left.j \beta_{t}\right)}\right.\right. \\
& \left.\left.+\left.g_{v r} g_{h t}\langle | S_{v h}\right|^{2}\right\rangle+\left.g_{h r} g_{v t}\langle | S_{h v}\right|^{2}\right\rangle \tag{4-27}
\end{align*}
$$

It is interesting to note at this point that there are ten scaffering coefficients. Additional simplification occurs when reciprocity applies. Under this assumption

$$
\begin{equation*}
f_{v h}=f_{h v} \tag{4-28}
\end{equation*}
$$

since field reciprocity implies that the operators must be identical. When the above property is applied to the definition of a scattering coefficient

$$
\begin{align*}
I_{t r}= & g_{v r} g_{v t}<\left|S_{v v}\right|^{2}>+2 R e\left(g_{v r} \sqrt{g_{v t} g_{h t}} e^{j \beta_{t}+}\right. \\
& \left.\left.g_{v t} \sqrt{g_{v r} g_{h r}} e^{j \beta_{r}}\right)<S_{v v} S_{v h}^{*}\right\rangle+2 R e\left(g_{h r} \sqrt{g_{v t} g_{h t}} .\right. \\
& e^{j \beta t}+g_{h t} \sqrt{g_{v r} g_{h r}} e^{\left.\left.j \beta_{r}\right\rangle<S_{v h} S_{h h}^{*}\right\rangle+2 \sqrt{g_{v r} g_{h r}} .} \\
& \sqrt{g_{v t} g_{h t}} R e<S_{v v} S_{h h}^{*}>e^{j\left(\beta_{t}+\beta_{r}\right)}+\left\langle g_{v r} g_{h t}+g_{h r} g_{v t}+\right. \\
& \left.\left.2 \sqrt{g_{v r} g_{h r} g_{v t} g_{h t}} R e e^{j\left(\beta_{t}-\beta_{r}\right)}\right)\left.\langle | S_{v h}\right|^{2}\right\rangle+ \\
& \left.g_{h r} g_{h t}<\left|S_{h h}\right|^{2}\right\rangle \tag{4-29}
\end{align*}
$$

When reciprocity applies the number of coefficients reduces to six,
The above result is the complete non-coherent radar equation under the reciprocity assumption. Although the equation was derived from the viewpoint of polar izations ascribable to the surface, the same equation would have resulted had the antenna and surface polarization states been defined with respect to the antenna. In the latfer case the scattering coefficients would not be comparable with those defined by the theorist who derives scattering coefficients with respect to the surface polarizations. In addition, the scattering coefficients for an arbifrarily line of sight would ${ }_{r}$ in general, be a function of antenna view angle also. Pragmatically, the antenna polar izations are referenced to a coordinate system rigidly bound to the physical antenna. The antenna polarization vectors, consequently, move with the antenna as it changes view angle. The surface polarization vectors on the otherhand, remain rigidly oriented with respect to the surface. The transformation between the two polarizations description is derived in the succeeding section. The distinction between antenna and surface polar izations on the scatterometer equation is treated simply by fransforming the transmission and reception coherency matrices, $\mathrm{C}_{\mathrm{f}}$ and $\mathrm{C}_{\mathrm{r}}$, from the anfenna coordinate system in which they were measured to ihe surface coordinate system in which the surface polar izations are naturally defined.

### 4.3 The Scatterometer Equation Including the Distinction Between Antenna and Surface Polarizations

Suppose that the antenna patterns, both polarized and cross-polarized patterns, are measured with the scatterameter antenna mounted on an azimuth-over-elevation positioner. To describe the antenna polarizations measured from such an antenna positioner, afix a primed coordinate system rigidly to the antennc. Let the $x^{\prime}$ axis denote the boresight axis and let the $z^{\prime}$ axis be oriented in a direction coinciding with the vertical polarization sense (with respect to the antenna) for an observer on the boresight axis. Then the antenna polarizations, vertical and horizontal, will coincide with the spherical polar unit vectors $\vec{i}_{\theta}$, and $\vec{i}_{\emptyset}$ r, respectively, of the afixed coordinate system. The antenna coordinate system is illustrated with respect to the pattern measuring antennas in Figure 4.3. Patterns are "cut" by incrementing the positioner in elevation when the $y^{\prime}$ and $y^{\prime \prime}$ axis coincide and then rotating the positioner about the $z^{\prime}$ axis. The measuring antennas are located on the $x^{\prime \prime}$ axis of the range coordinates ( $x^{\prime \prime}, y^{\prime \prime}, z^{\prime \prime}$ ).

Within the antenna coordinate system the transmifted fields will be denoted by $e_{\theta^{r} t}$ and ${ }_{\phi^{\prime} t}$ and the complex effective reception heights by $I_{\theta^{\prime} r}$ and $I_{\not D_{r}}{ }^{*}$. Both pairs of parameters are, in general, complex (to convey the relative phase between members within the pairs) and vary with $\theta^{\prime}$ and $\varnothing$ !

Now locate the antenna (primed) coordinate system so that its origin coincides with the origin of the surface coordinate system (Figure 4.1). Without loss in generality it is assumed that the antenna scans linearily in the $\times$ direction of the surface coordinate system and that observations are conducted in the $x-z$ plane. The antenna is so oriented that its vertical polarization sense coincides with the surface vertical polarization sense at the intersection of the boresight point with the surface. Within the $x z$ plane the antenna is pointed at an angle $\theta_{0}$ with respect to the local vertical ( $z$ axis). The geometry of the two coordinate systerns relative to one another is shown in Figure 4.4.

To develop the relationship between the antenna and surface coordinates consider any line of sight vector $\bar{i}_{r}$ which emanates from the common origin and whose extension intersects the surface (See Figure 4.4). By definition, the antenna polarization pair ( $\bar{i}_{\theta}{ }^{\prime},{ }^{\pi} \phi_{j}$ ) and the surface polarization pair ( $\left.\bar{i}_{\theta}, \bar{i}_{\phi}\right)$ are both perpendicular to $\tilde{i}_{r}$. It follows that the polarization pairs at every line of sight are related by a simple rotation,

[^3]

# FIGURE 4.3 gEOMETRY OF THE PATTERN MEASUREMENT COORDINATE SYSTEM 



HORIZONTAL POLARIZATION

$$
\bar{i}_{\phi}=\frac{\bar{i}_{2} \times \bar{i}_{r}}{\left|\bar{i}_{2} \times \bar{i}_{r}\right|}
$$

VERTICAL POLARIZATION

$$
\bar{i}_{\theta}=\bar{i}_{\phi} \times \bar{i}_{r} .
$$

FIGURE 4.4 COMPARISON OF ANTENNA AND SURFACE COORDINATE FRAMES WITH THE SURFACE POLARIZATIONS dEFined with respect to a general line of sight VECTOR
say $\psi$. Define $\psi_{p}$ so that*

$$
\begin{equation*}
\bar{i}_{\theta} \cdot \vec{i}_{\theta^{\prime}}=\bar{i}_{\phi} \cdot \bar{i}_{\phi^{\prime}}=\cos \psi \tag{4-30}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{i}_{\theta} \cdot \bar{i}_{\phi^{\prime}}=-\bar{i}_{\phi} \cdot \bar{i}_{\theta^{\prime}}=\sin \psi \tag{4-3~J}
\end{equation*}
$$

By noting the transformation between the coordinate systems, the reader can easily show that

$$
\begin{equation*}
\cos \psi=\cos \phi \cos \phi^{\prime}+\sin \phi \sin \phi^{\prime} \sin \theta_{0} \tag{4-32}
\end{equation*}
$$

and

$$
\begin{equation*}
\sin \psi=\cos \theta\left(\sin \phi \cos \phi^{\prime}-\cos \phi \sin \phi^{\prime} \sin \theta_{0}\right)+\sin \theta \cos \theta_{0} \sin \phi \tag{4-33}
\end{equation*}
$$

where $\emptyset^{1}$ is the spherical azimuthal angle in the primed coordinate system. Now $\emptyset^{1}$ can be eliminated by observing that

$$
\begin{equation*}
\tan \dot{\phi}^{\prime}=\left(\bar{i}_{r} \cdot \overline{\mathrm{i}}_{y^{\prime}}\right) /\left(\overline{\mathrm{i}}_{r} \cdot \overline{\mathrm{i}}_{x^{\prime}}\right) \tag{4-34}
\end{equation*}
$$

to get

$$
\begin{equation*}
\phi^{\prime}=\tan ^{-1}\left[\frac{\sin \theta \sin \phi}{\cos \theta \cos \theta_{0}+\sin \theta \sin \theta_{0} \cos \phi}\right] \tag{4-35}
\end{equation*}
$$

Finally from the above we have established the transform $T$ between the antenna and surface polarizotions, viz.r

$$
\begin{equation*}
\binom{\bar{i}_{\theta}}{\bar{i}_{\phi}}=T\binom{\bar{i}_{\theta^{\prime}}}{\bar{i}_{\phi^{\prime}}} \tag{4-36}
\end{equation*}
$$

where

$$
T=\left[\begin{array}{cc}
\cos \psi & \sin \psi  \tag{4-37}\\
-\sin \psi & \cos \psi
\end{array}\right]
$$

* Note: An alternate method of mounting the anfenna could have resulted in defining $\psi$ so that $\cos \psi={ }^{T} \theta^{\circ}{ }^{\circ} \sigma^{\prime \prime}$, etc. The difference between the two is discussed in Chapter

The entries in T are provided by Equations ( $4-32$ ) and ( $4-33$ ) with the assist of Equation (4-35). When the antenna pattern is finally introduced the following relationship

$$
\begin{equation*}
\cos \theta^{\prime}=\stackrel{\rightharpoonup}{\mathbf{T}}_{r} \cdot \stackrel{\rightharpoonup}{\mathbf{t}}_{z}! \tag{4-38}
\end{equation*}
$$

or

$$
\begin{equation*}
\cos \theta^{\circ}=\cos \theta \sin \theta_{0}-\sin \theta \cos \theta_{0} \cos \phi \tag{4-39}
\end{equation*}
$$

in addition to Equation (4-35) will be helpful in identifying the pattern coordinates when the surface coordinates are given.

Now from the preceding derivation (Equation (4-13)) we had

$$
\begin{equation*}
\ddot{W}_{t r}=\frac{1}{8 R_{r}} \iint \operatorname{tr} C_{r} C_{s}^{\dagger} d \Omega \tag{4-40}
\end{equation*}
$$

where

$$
\begin{align*}
& c_{s}=\left\langle\beta C_{t} \beta^{t^{*}}\right\rangle  \tag{4-47}\\
& c_{r}=\left[\begin{array}{ll}
\left|1_{v r}\right|^{2} & 1_{v r} 1_{h r}^{*} \\
1_{h r}{ }^{*}{ }_{v r}^{*} & \left|1_{h r}\right|^{2}
\end{array}\right] .  \tag{4-42}\\
& c_{t}=\left[\begin{array}{ll}
\left|e_{v t}\right|^{2} & e_{v t} e_{h t}^{*} \\
e_{h t} e_{v t}^{*} & \left|e_{h t}\right|^{2}
\end{array}\right] \tag{4-43}
\end{align*}
$$

The above coherency motrices are written in terms of the surface polarizations since the scattering operators are defined on the basis of these polarizations. When the antenna transmission and reception properties are, however, defined within another coordinate system (the primed coordinate system), these properties must be
appropriately transformed. It is easily shown that if $\mathrm{C}_{\boldsymbol{r}}{ }^{\mathbf{r}}$ and $\mathrm{C}_{\mathrm{f}}{ }^{\text {are }}$, the coherency matrices in the primed coordinate system, then in the surface coordinate system

$$
\begin{equation*}
C_{r}=T C_{r}^{\prime} T^{t} \tag{4-44}
\end{equation*}
$$

and

$$
\begin{equation*}
C_{t}=T C_{t}^{t} T^{\dagger} \tag{4-45}
\end{equation*}
$$

From the above expressions the following identities can be established

$$
\begin{aligned}
& \left|7_{v r}\right|^{2}=\cos ^{2} \psi\left|1_{v r}^{\prime}\right|^{2}+\sin 2 \psi \operatorname{Re} 1_{v r}^{1} J_{h r}^{\prime *}+\sin ^{2} \psi\left|T_{h r}^{\prime}\right|^{2} \\
& 1_{v r} 1_{h r}^{*}=\left[\left|1_{h r}^{\prime}\right|^{2}-\left|1_{v r}^{\prime}\right|^{2}\right] \sin \psi \cos \psi+\cos ^{2} \psi 1_{v r}^{1} 1_{h r}^{\prime *} \\
& -\sin ^{2} \psi 1_{h r}^{1 *} 1_{v r}^{t} \\
& \text { - } 1_{h r}^{*} \tau_{v h}=\left[1_{v r} 1_{\mathrm{hr}}^{*}\right]^{*} \\
& \left|1_{h r}\right|^{2}=\sin ^{2} \psi\left|1_{v r}^{\prime}\right|^{2}-\sin 2 \psi \operatorname{Re} 1_{v r} 1_{h r}^{1 *}+\cos ^{2} \psi\left|I_{h r}^{\prime}\right|^{2}
\end{aligned}
$$

A similar expression can be established for the elements of $C_{Y}$. It is noted that the coherency matrices reduce to those in the surface coordinate system when $\psi=0$.

Now let $g_{\theta^{\prime}+r} g_{\emptyset^{\prime} t^{\prime} r} \beta_{\ddagger}^{\prime}$ describe the antenna during transmission and $g_{\theta^{\prime} r}$ r $g_{\phi^{\prime} t}$ and $\beta_{r^{\prime}}$ during reception. When the transformed coherency matrices are incorporated into the scatterometer equation and relationships of the type as shown in Equations (4-20) through (4-25) are noted in the antenna coordinate system, the scotterometer equation can be written as

$$
\begin{equation*}
W_{\operatorname{tr}}\left(\theta_{0}\right)=\frac{\lambda^{2} G_{t} G_{r} W_{t}}{(4 \pi)^{2}} \int \frac{I_{t r}}{r^{2}} d \Omega \tag{4-47}
\end{equation*}
$$

provided that the following idenfities are understood .

$$
\begin{aligned}
& g_{v p}=g_{\theta^{\prime} p} \cos ^{2} \psi+\sqrt{g_{\theta^{\prime} p}^{g} \phi^{\prime} p} \sin 2 \psi \cos \beta_{p}^{\prime}+g_{\phi^{\prime} p} \sin ^{2} \psi \\
& g_{h p}=g_{\theta^{\prime} p} \sin ^{2} \psi-\sqrt{g_{\theta^{\prime} p}^{g} \phi^{\prime} p} \sin 2 \psi \cos \beta_{p}^{\prime}+g_{\phi^{\prime} p} \cos ^{2} \psi
\end{aligned}
$$

$$
\beta_{p}=\tan ^{-1} \frac{\sqrt{g_{\theta^{2}} p^{g} \phi^{\prime} p} \sin \beta_{p}^{\prime}}{\left(g_{\theta \cdot \prime^{\prime} p} g_{\phi^{\prime} p}\right) \sin \psi \cos \psi+\sqrt{g_{\theta^{\prime} p^{g} \phi^{\prime} p}} \cos \beta_{p}^{\prime}\left(\cos ^{2} \psi-\sin ^{2} \psi\right)}(4-48)
$$

where $p=+$ or $r$. The latter identities indicate how the common antenna parameters transform. If is noted that $\beta_{\mathrm{p}}, \mathrm{g}_{\mathrm{vp}}$ or $\mathrm{g}_{\mathrm{hp}}$ is each dependent on all three antenna parameters, $\beta_{p^{\prime}}^{\prime} g_{\theta p}^{\prime}$ and $g^{p}{ }_{\not q p}$. To appreciate the additional complexity in the scatterometer equation resulting from the transformation expand the integrand in the form

$$
\begin{align*}
I_{t r} & \left.\left.\left.=\left.I_{1}\langle | s_{v v}\right|^{2}\right\rangle+\left.I_{2}\langle | s_{h h}\right|^{2}\right\rangle+\left.I_{3}\langle | s_{v h}\right|^{2}\right\rangle \\
& \left.\left.+2 I_{4} R e<s_{v v} s_{h h}^{*}\right\rangle-2 I_{5} I m<s_{v v}^{*} s_{h h}^{*}\right\rangle \\
& \left.\left.+2 I_{6} R e<s_{v v} s_{h v}^{*}\right\rangle-2 I_{7} I m<s_{v v} s_{h v}^{*}\right\rangle  \tag{4-49}\\
& \left.+2 I_{8} R e<s_{v h} s_{h h}^{*}>-2 I_{9} I m<s_{v h} s_{h h}^{*}\right\rangle
\end{align*}
$$

Then it will be noted that

$$
\begin{aligned}
I_{1}= & {\left[g_{\theta^{\prime} r} \cos ^{2} \psi+\sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime} r}} \sin 2 \psi \cos \beta_{r}^{\prime}+g_{\phi^{\prime} r} \sin ^{2} \psi\right] \cdot } \\
& {\left[g_{\theta^{\prime} t} \cos ^{2} \psi+\sqrt{g_{\theta^{\prime} t} g_{\phi^{\prime} t}} \sin 2 \psi \cos \beta_{t}^{\prime}+g_{\phi^{\prime} t} \sin ^{2} \psi\right](4-50 a) } \\
I_{2}= & {\left[g_{\theta^{\prime} r} \sin ^{2} \psi-\sqrt{g_{\theta^{\prime} r^{\prime}} g_{\phi^{\prime} r}} \sin 2 \psi \cos \beta_{r}^{\prime}+g_{\phi^{\prime} r} \cos ^{2} \psi\right] \cdot(4-50 b) } \\
& {\left[g_{\theta^{\prime} t} \sin ^{2} \psi-\sqrt{g_{\theta^{\prime} t} g_{\phi^{\prime} r}} \sin 2 \psi \cos \beta_{t}^{\prime}+g_{\phi^{\prime} t} \cos ^{2} \psi\right] } \\
I_{3}= & {\left[g_{\theta^{\prime} r} \sin ^{2} \psi-\sqrt{g_{\theta^{\prime} r^{\prime}} g_{\phi^{\prime} r}} \sin 2 \psi \cos \beta_{r}^{\prime}+g_{\phi^{\prime} r} \cos ^{2} \psi\right] \cdot } \\
& {\left[g_{\theta^{\prime} t} \cos ^{2} \psi+\sqrt{g_{\theta^{\prime} t} g_{\phi^{\prime} t}} \sin 2 \psi \cos \beta_{t}^{\prime}+g_{\phi^{\prime} t} \sin ^{2} \psi\right]+(4-50 c) } \\
& {\left[g_{\theta}{ }^{i} r \cos ^{2} \psi+\sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime} r}} \sin 2 \psi \cos \beta_{r}^{\prime}+g_{\varphi^{\prime} r} \sin ^{2} \psi\right] \cdot } \\
& {\left[g_{\theta^{\prime} t} \sin ^{2} \psi-\sqrt{g_{\theta^{\prime} t} g^{\prime} t} \sin 2 \psi \cos \beta_{t}^{\prime}+g_{\phi^{\prime} r} \cos ^{2} \psi\right]+}
\end{aligned}
$$

$$
\begin{align*}
& 2\left[\left(g_{\phi^{\prime} r}-g_{\theta}{ }^{\prime}\right) \sin \psi \cos \psi+\sqrt{g_{\theta^{\prime} r^{\prime}} g^{\prime} t} \cos 2 \psi \cos \beta_{r}^{\prime \prime}\right] \text {. } \\
& {\left[\left(g_{\phi^{\prime} t}-g_{\theta^{\prime} t}\right) \sin \psi \cos \psi+\sqrt{g_{\theta^{\prime}} t g_{\phi^{\prime} t}} \cos 2 \psi \cos \beta_{t}^{\prime}\right]+} \\
& 2 \sqrt{g_{\theta^{\prime} t} g_{\phi^{\prime} t}} \sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime} r}} \sin \beta_{t}^{4} \sin \beta_{r}^{\prime} \\
& I_{4}=2\left[\left(g_{\phi^{\prime} r}-g_{\theta^{\prime} r}\right) \sin \psi \cos \psi+\sqrt{g_{\theta}{ }^{\prime} r g_{\phi^{\prime}} r} \cos 2 \psi \cos \beta_{r}^{\prime}\right] . \\
& {\left[\left(g_{\phi}{ }^{\prime} t-g_{\theta}{ }^{\prime} t\right) \sin \psi \cos \psi+\sqrt{g_{\theta} '^{\prime} g_{\phi}{ }^{\prime} t} \cos 2 \psi \cos \beta_{t}^{\prime}\right] .} \\
& \sqrt{g_{\theta^{\prime} t} g_{\phi}{ }^{\prime} r} \sqrt{g_{\theta}{ }^{\prime} t g_{\phi} g^{\prime} t} \sin \beta_{t}^{\prime} \sin \beta_{r}^{\prime} \\
& I_{5}=\left[\left(g_{\dot{\phi}^{\prime} r}-g_{\theta^{\prime} r}\right) \sin \psi \cos \psi+\sqrt{g_{\theta} r^{\prime} g_{\phi}{ }^{\prime} r} \cos 2 \psi \cos \beta_{r}^{\prime}\right] . \\
& \sqrt{g_{\theta}{ }^{\prime} t g_{\phi^{\prime} t}} \sin \beta_{t}^{\prime}+\left[\left(g_{\phi^{\prime} t}-g_{\theta^{\prime} r}\right) \sin \psi \cos \psi+\right.  \tag{4-50e}\\
& \left.\sqrt{g_{\theta}{ }^{\prime} g^{g} \phi^{\prime} t} \cos 2 \psi \cos \beta_{t}^{\prime}\right] \quad \sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime} r}} \sin \beta_{r}^{\prime} \\
& I_{6}=\left[g_{\theta^{\prime} r} \cos ^{2} \psi+\sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime}} r} \sin 2 \psi \cos \beta_{r}^{\prime}+g_{\phi^{\prime} r} \sin ^{2} \psi\right] . \\
& {\left[\left(g_{\phi^{\prime} t}-g_{\theta^{\prime} t}\right) \sin \psi \cos \psi+\sqrt{g_{\theta^{\prime} t} g_{\phi^{\prime}} t} \cos 2 \psi \cos \beta_{t}^{\prime}\right]+} \\
& {\left[g_{\theta}{ }^{\prime} t \cos ^{2} \psi+\sqrt{g_{\theta^{\prime}} t g_{\phi}{ }^{\prime} t} \sin 2 \psi \cos \beta_{t}^{\prime}+g_{\phi^{\prime} r} \sin ^{2} \psi\right] \text {. }} \\
& {\left[\left(g_{\phi^{\prime} r}-g_{\theta^{\prime} r}\right) \sin \psi \cos \psi+\sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime} r}} \cos 2 \psi \cos \beta_{r}^{\prime}\right]}  \tag{4-50f}\\
& I_{7}=\left[g_{\theta^{\prime} r} \cos ^{2} \psi+\sqrt{g_{\theta} \prime^{\prime} g_{\phi^{\prime} r}} \sin 2 \psi \cos \beta_{r}^{\prime}+g_{\phi^{\prime} r} \sin ^{2} \psi\right] . \\
& \sqrt{g_{\theta^{\prime}} t g_{\phi}{ }^{\prime} t} \sin \beta_{t}^{\prime}+\left[g_{\theta^{\prime} t} \cos ^{2} \psi+\sqrt{g_{\theta^{\prime}} t g_{\phi}{ }^{\prime} t} \sin 2 \psi \cos \beta_{t}^{\prime}\right. \\
& \left.g_{\phi^{\prime} t} \sin ^{2} \psi\right] \sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime} r}} \sin B_{r}^{\prime} \tag{4-50g}
\end{align*}
$$

$$
\begin{align*}
I_{8}= & {\left[g_{\theta^{\prime} t} \sin ^{2} \psi-\sqrt{g_{\theta^{\prime} t} g_{\phi^{\prime} t}} \sin 2 \psi \cos \beta_{t}^{\prime}+g_{\phi^{\prime} t} \cos ^{2} \psi\right] \cdot } \\
& {\left[\left(g_{\phi^{\prime} r}-g_{\theta^{\prime} r}\right) \sin \psi \cos \psi+\sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime} r}} \cos 2 \psi \cos \beta_{r}^{\prime}\right]+} \\
& {\left[g_{\theta^{\prime} r} \sin ^{2} \psi-\sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime} r}} \sin 2 \psi \cos \beta_{r}^{\prime}+g_{\phi^{\prime} r} \cos ^{2} \psi\right] \cdot } \\
& {\left[\left(g_{\phi^{\prime} t}-g_{\theta^{\prime} t}\right) \sin \psi \cos \psi+\sqrt{g_{e^{\prime} t^{\prime}} \phi^{\prime} t} \cos 2 \psi \cos \beta_{t}^{\prime}\right] } \tag{4-50h}
\end{align*}
$$

$$
\begin{align*}
I_{g}= & {\left[g_{\theta^{\prime} t} \sin ^{2} \psi-\sqrt{g_{\theta^{\prime}} t g_{\phi^{\prime}} t} \sin 2 \psi \cos \beta_{t}^{\prime}+g_{\phi^{\prime} t} \cos ^{2} \psi\right] . } \\
& \sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime} r}} \sin \beta_{r}^{\prime}+\left[g_{\theta^{\prime} r} \sin ^{2} \psi-\sqrt{g_{\theta^{\prime} r} g_{\phi^{\prime} r}} \sin 2 \psi \cos \beta_{r}^{\prime}+\right. \\
& \left.g_{\phi^{\prime} r} \cos ^{2} \psi\right] \cdot \sqrt{g_{\theta^{\prime} t} g_{\phi^{\prime} t}} \sin \beta_{t}^{\prime} \tag{4-50i}
\end{align*}
$$

When accurate measurements of the scattering coefficients, say the complete sef of nine parameters is desired, one must contend with inverting a system of integral equations of the type derived above. The scattering coefficients are rigorously defined in ferms of the surface polarization, a definition universally employed by the scatfering theorist. If comparisons with theory are necessary then the antenna properties must be transformed to conform with this definition. To date measurements have been reported without the recognition that Equations (4-47) through (4-50) governor the interaction between the scafterometer antenna and the scene. Yet reasonable agreement between measurements from targets with known statistics and theory have been reported [10] [31] for the polarized scattering coefficients. This indicates that the complexity of $I_{T}$ through $I_{9}$ may be avoidable under some circumstances. To resolve this problem and related ones, the polarization coordinate systems will be compared and the character of the scatterometer equation will also be examined in depth in succeeding chapters. Once the character of the scatterometer equation is established, a measurement fechnique to recover all six scatting coefficients is specified. Computer simulations based on the specified technique are then conducted to determine antenna requirements for accurate measurements.

### 5.0 DISCUSSION OF THE SCATTEROMETER EQUATION

### 5.1 Introduction

This chapter is devoted to developing an understanding of the scatterometer equation. The character of the scattering coefficients is established by reference to previous definitions, both coherent and non-coherent. It is shown that the noncoherent definition appearing in the literature must be extended to include new kinds of coefficients. The composition of the average return is examined from the standpoint of coherence theory and the complete set of scattering coefficients. The impartance of the phase characteristic of the wave and the receiving antenna in governing the observed power is described. It is also shown that certain properties of the coherent scattering coefficients cannot be extrapolated to the non-coherent case. Well known theories applicable to the sea are also employed to illustrate the behavior of the scattering coefficients naving a cross-correlation property. Other possibilities for the cross-correlation coefficients are also treated intuitively.

Within this chapter it is also shown that this formulation of the scatterometer equation admits partially polarized refurns. A previous formulation [6] failed in this respect. The degree of polarization of the average sea return is specifically illustrated using a simple scattering theory.

Finally the distinction between surface and antenna polarizations is illustrated. Certain aspects of this distinction are qualitaively applied to specifying antenna requirements.

### 5.2 General

### 5.2.1 The Scattering Coefficient

The scattering coefficients within the scatterometer equation may be partially identified with the differential scatering coefficients defined by Peake [24]. As the reader may recall, Peake defines

$$
\begin{equation*}
\gamma_{i j}=\frac{\left.\left.4 \pi r^{2}\langle | e_{i s}\right|^{2}\right\rangle}{A \cos \delta_{d}\left|e_{j t}\right|^{2}} \tag{5-1}
\end{equation*}
$$

where $\left|e_{j t}\right|^{2}$ is the polarized incident intensity $\left.{ }_{2}\langle | e_{i s}\right|^{2}>$ is the $i$ polarized backscatter intensity in volts ${ }^{2} / \mathrm{m}^{2}, \theta$ is the incident angle and $A$ is the illuminated area. The scattering coefficient employed in this formulation is simply related to $\gamma_{\mathrm{ij}}$ in the
following way

$$
\begin{equation*}
\left.\left.\langle | s_{i j}\right|^{2}\right\rangle=\gamma_{i j} / 4 \pi \tag{5-2}
\end{equation*}
$$

The difference by $4 \pi$ occurs since the scattered intensities were defined in terms of inverse steradians. It is clear that, in view of the three addition coefficients, it is more appropriate to define the coefficients in terms of the scattering operators
where $g_{i j}{ }^{j}{ }_{j t}$ yields a scattered field with units volis/meter. The operators in the derivation are related to those in the definition in the following way

$$
\begin{equation*}
\ddots f_{i} j^{\beta_{k}^{*}}=-s_{i j} \mathcal{P}_{\mathrm{k} k}^{* k} R^{2} / \Delta \mathrm{A} \cos \theta_{0} \tag{5-3b}
\end{equation*}
$$

To understand the function of these cross-correlation coefficients one must examine the coherence matrix for the scattered wave. Under the non-coherence and reciprocity assumptions the elements of $C_{s}$ are given by

$$
\begin{align*}
& \left.\left.\left[C_{s}\right]_{v v}=\left.\langle | S_{v v}\right|^{2}\right\rangle\left|e_{v t}\right|^{2}+2 R e<S_{v v} S_{v h}^{*}\right\rangle e_{v t} e_{h t}^{*}+ \\
& \left.\langle | s_{v h}\right|^{2}>\left|e_{h t}\right|^{2} \\
& \left.\left[C_{s}\right]_{v h}=\left\langle S_{v v} S_{h v}^{*}\right\rangle\left|e_{v t}\right|^{2}+\left\langle S_{v v} S_{h h}^{*}\right\rangle e_{v t} e_{h t}^{*}+\left.\langle | S_{v h}\right|^{2}\right\rangle \varepsilon_{h t} e_{v t}^{*} \\
& +\left.\left\langle s_{v h} S_{h h}^{*}>\right| e_{h t}\right|^{2} \\
& {\left[c_{s}\right]_{h v}=\left[c_{s}\right]_{v h}^{*}}  \tag{5-4}\\
& {\left[C_{s}\right]_{h h}=\left.\langle | S_{v h}\right|^{2}>\left|\epsilon_{v t}\right|^{2}+2 R e<S_{h v} S_{h h}^{*}>e_{v t} e_{h t}^{*}+} \\
& \left.\langle | s_{h 1}\right|^{2}>\left|e_{h t}\right|^{2}
\end{align*}
$$

The power in the scaftered wave is carried in the trace of $\mathrm{C}_{5}$ whereas the relative phase between the orthogonal components in an average sense is carried in the off diagonal elements. From the structure of the coherence matrix it is evident that the cross-correlation coefficients can be complex valued. The cross-correlation coefficients therefore alter the phase property of the scattered wave. Some of the relative phase is affributable to the incident wave and some to the surface, eg., $e_{\mathrm{vt}}$ $e_{h^{*}}$ and $\left\langle S_{w^{\prime}} S_{h h^{*}}\right\rangle$, respectively. The cross-correlation ferms also appear in the diagonal terms and consequently contribute to the total power available in the scattered wave when both polarizations appear in the incident wave.

During reception the coherency matrix of the antenna interacts with the coherency marrix for the wave. The inferaction is completely described by taking the trace of $\mathrm{C}_{\mathrm{r}} \mathrm{C}_{s}^{\dagger}$. The trace is given by

$$
\begin{gather*}
\left.\left.\operatorname{tr} C_{r} c_{s}^{\dagger}=\left|1_{v r}\right|^{2}\left[c_{s}\right]_{v v}+\right]_{v r} 1_{h r}^{*}\left[c_{s}\right]_{v h}+1_{h r}\right]_{v r}^{*}\left[C_{s} I_{h v}+\right. \\
\left|1_{h r}\right|^{2}\left[c_{s}\right]_{h h} \tag{5-5}
\end{gather*}
$$

(The expanded version of the trace is given in Equation (4-19) of Chapter 4). The phase interaction between the scattered wave and the antenna is described by the middle terms in the frace expression. These terms are complex conjugate pairs and consequently make a real contribution to the observed power.

To show that the phase properties of the antenna and the wave are important to the observed return it must be recalled that in the case of polarized waves, the antenna polarization states must be matched to the polarization state of the wave to observe maximum power [14]. This requirement in terms of coherency matrices implies that

$$
\begin{equation*}
c_{r} / \operatorname{tr} C_{r}=c_{s}^{*} / \operatorname{tr} C_{s} \tag{5-6}
\end{equation*}
$$

Under the polarized assumption $C_{s}$ takes the form

$$
C_{s} / \operatorname{tr} C_{s}=\left[\begin{array}{ll}
a & c  \tag{5-7}\\
c^{*} & b
\end{array}\right]
$$

where $a+b=1, c=\sqrt{a b} e^{j \alpha}$, and $\alpha$ is the relative phase between the $v$ and $h$ components. The observed power will be proportional to $(a+b)^{2} \operatorname{tr} \mathrm{C}_{\mathrm{r}} \operatorname{tr} \mathrm{C}_{5}$ or
$\operatorname{tr} \mathrm{C}_{\mathrm{r}}+\mathrm{C}_{\mathrm{s}}$. If the reception matrix had been given by

$$
c_{r}=\left[\begin{array}{cc}
b & -c^{x}  \tag{5-8}\\
-c & a
\end{array}\right]
$$

no power would be observed at the antenna terminals as can be easily demonstrated. In this case the anfenna polarization state is said to be orthogonal to the polarization state of the arriving wave.

If the wave is fiartially polarized Ko [25] has shown that the observed power may vary from a minimum of $\lambda^{2} G_{r}(1-P)+r C_{s} / 8 Z_{o}$ to a maximum of $\lambda^{2} G_{r}(1+P)+r C_{s} / 8 Z_{0}$. where P is the degree of polarization (See Section 4 of this chapter). To undersfand this result it must be noted that a partially polarized wave can be uniquely decomposed into a sum of a randomly polarized wave and a completely polarized wave [30]. As a consequence for an arbitrarily polarized backscattered wave, the decomposition can be wriften as

$$
C_{s}=\operatorname{tr}\left[C_{s}\right]\left\{(1-P)\left[\begin{array}{cc}
1 / 2 & 0 \\
0 & 1 / 2
\end{array}\right]+p\left[\begin{array}{cc}
\rho_{v v} & \rho_{v h} \\
\rho_{h v} & \rho_{h h}
\end{array}\right]\right\}(5-9)
$$

where $P$ is defined in Equation (5-30) and

$$
\begin{align*}
& \rho_{v v}=\frac{1}{p \operatorname{tr}\left[c_{s}\right]}\left[\left[c_{s}\right]_{v v}-1 / 2(1-P)\right] \\
& \rho_{v h}=\frac{\left[c_{j} s_{v h}\right.}{p \operatorname{tr}\left[c_{s}\right]}  \tag{5-10}\\
& \rho_{h v}=\rho_{v h}^{*} \\
& \rho_{h h}=\frac{1}{p \operatorname{tr}\left[c_{s}\right]}\left[\left[c_{s}\right]_{h h}-1 / 2(1-P)\right]
\end{align*}
$$

The first ferm is the randomly polarized component and fhe second is the completely polarized component. If the receiving antenna is orthogonal to the completely polarized part, then only the randomly polarized component is observed at the antenna terminals. If the antenna is matched to the completely polarized part maximum power is observed.

The extremes in the observable power are a positive indication of the imporiance of the phase interaction of the antenna and the wave. The cross-correlation scatiering .coefficients and the cross-polarized scattering coefficient can be effective in altering the phase property of the return.

The cross-correlation terms have their analogues in scattering theory for coherent targets [2t]. In the theory for discrete targets the complex scattering matrix is commonly employed to define scattering properties. The elements of this matrix have the property that

$$
\begin{equation*}
\left|s_{i j} s_{j k} *\right|=\left|s_{i j}\right|\left|s_{j k}\right| \tag{5-11}
\end{equation*}
$$

However for a statistical target this property is not necessarily true. Since the scatfering coefficients can be considered as a inner product of the form

$$
\begin{equation*}
\left\langle S_{i j} S_{j k}^{*}\right\rangle=\left\langle\delta_{i j} e_{j t} \delta_{j k}^{*} e_{k t}^{*-}\right\rangle \tag{5-12}
\end{equation*}
$$

where $e_{i t} e_{k t}^{*}=1$, it is concluded by Schwartz ${ }^{1}$ inequality that

$$
\begin{equation*}
\left|\left\langle s_{i i} s_{j k}{ }^{*}\right\rangle\right| \leqslant \sqrt{\left.\left.\left.\langle | s_{i j}\right|^{2}\right\rangle\left.\langle | s_{j k}\right|^{2}\right\rangle} \tag{5-13}
\end{equation*}
$$

As a consequence the magnifudes of the scattering coefficients may not be simply related as suggested by William, et al. [6]. The inequality is an admission that the amplitudes or phase centers between scattered field components can be correlated.

One can identify two scattering parameters with each complex valued scattering coefficient, viz., its real and imaginary parts. As a result one may attribute nine scattering parameters to Equation (4-29) where reciprocity has been applied. Similarly from Equation (4-27) where reciprocity has not been applied, sixteen scattering parameters can be idenfified. These observations are in agreement with the "Gedanken Experimente" cited in Chapter 3.

### 5.22 Special Cases

An examination of the scatterometer equation indicates that the equation under appropriate conditions reduces to the classical cases. For example, when vertically polarized measurements are conducted, i.e., $g_{h t}=g_{\mathrm{hr}}=0$, the integrand factor $\mathrm{I}_{\mathrm{fr}}$ of the scatterometer equation becomes

$$
\begin{equation*}
I_{t r}=g_{v r} g_{v t}<\left|s_{v v}\right|^{2}> \tag{5-14}
\end{equation*}
$$

Similarly when horizontaliy polarized measurements are conducted, $i_{i} e, . g_{v r}=g_{v r}=0$,

$$
\begin{equation*}
\left.I_{t r}=g_{h r} g_{h t}<\left|S_{h h}\right|^{2}\right\rangle \tag{5-15}
\end{equation*}
$$

and when cross-polarized measurements are conducted, i.e., $g_{\mathrm{ht}}=\mathrm{g}_{\mathrm{vr}}=0$,

$$
\begin{equation*}
\left.I_{t r}=g_{v t} g_{h r}<\left|s_{v h}\right|^{2}\right\rangle \tag{5-16}
\end{equation*}
$$

It should be noted that the reductions result from highly idealized representations of practical antennas. Invariably antennas have cross polarized leakage; and when leakage is present other scattering coefficients, both auto-correlation and cross-correlation fypes, will be excited. As shown in the last section of Chapter 4, even if the leakage is not present, the difference between antenna and surface polarizations can introduce, in effect, cross-polarized components in the incident wave and in the reception antenna. The impact of undesirable antenna properties and polarization mis-match on the measurement of isolated scatfering parameters will be treated in Chapter 7.

An understanding of the cross-correlation coefficients from theories applicable to sea returns is developed in the succeeding section.

### 5.3 Characteristics of the Correlation Terms

Several scattering theories are examined to disclose the character of the cross" correlation terms in the scatterometer equation. Approximate backscatter solutions to the small perturbation theory [32] [33] [34] [35] and the Kirchhoff theory [36] [37] are specifically examined. These theories with some slight alterations are thought to apply to ocean backscatter and have shown reasonable agreement with measured results. The selection of these theories by no means exhausts the possibilities. Physically intuitive arguments are given at the end of this section to further enhance our understanding.

When returns are considered from o surface having a small roughness, satisfying $k^{2} \sigma^{2} \cos ^{2} \theta \leq 1$, where $\sigma$ is the rms surface height and $k$ is the wave number, it can be shown that (see Appendix C)*

$$
\begin{equation*}
<S_{v v} S_{h h^{*}}>=\frac{k^{4}}{\pi^{2}} \cos ^{3} \theta R_{v} R_{h} W(2 k \sin \theta, 0) \tag{5-17}
\end{equation*}
$$

[^4]In the above equation

$$
\begin{align*}
& R_{y}=\frac{\left(\epsilon_{r}-i\right)\left[\epsilon_{r}\left(1+\sin ^{2} \theta\right)-\sin ^{2} e\right]}{\left(\epsilon_{r} \cos \theta+\sqrt{\epsilon_{r}-\sin ^{2} \theta}\right)^{2}}  \tag{5-18}\\
& R_{h}=\frac{\cos \theta-\sqrt{\epsilon_{r}-\sin ^{2} \theta}}{\cos \theta+\sqrt{\epsilon_{r}-\sin ^{2} \theta}} \tag{5-19}
\end{align*}
$$

Comparison of the magnitude of this term with the polarized scattering soefficients shows that

$$
\begin{equation*}
1\left\langle s_{v v} s_{h h} *\right\rangle \mid=\sqrt{\left.\left.\left.\langle | s_{v v}\right|^{2}\right\rangle\left.\langle | s_{h h}\right|^{2}\right\rangle} \tag{5-20}
\end{equation*}
$$

The equality is true of least to the order to which these soiutions are valid. The magnitude of this term is illustrated in Figure 5.1 wherein it is also compared with the polarized coefficients. The computations were based on a slightly rough sea. The phase of the cross correlation, defined by

$$
\begin{equation*}
\dot{\Phi}=\tan ^{-1}\left(\operatorname{Im}<S_{v v} S_{h h}^{*}>/ R e<S_{v v} S_{h h}^{\star}>\right) \tag{5-21}
\end{equation*}
$$

was computed and is shown in Figure 5.2 for three different water temperatures. The sea water temperature alters the complex dielectric constant of the surface and consequently influences $Q_{i,}$ and $R_{h}$. It is observed from the graph that the imaginary part of $\left\langle\mathrm{S}_{\mathrm{wv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ is small in comparison to its real part and only tends to become significant at the larger angles when compared with the real part.

An examination of the integrand of the scatterometer equation (4-29) indicates that the above cross-correlation term can make a significant contribution to a radar refurn when like and cross antenna polarizations are present during fransmission and reception.

figure 5.1 scattering characteristics based on SMALL PERTURBATION THEORY


FIGURE 5.2 CROSS-CORRELATION PHASE PROPERTY BASED ON SMALL PERTURBATION THEORY

The contribution can be positive or negative depending upon the value of $\left(\beta_{t}+\beta_{r}\right)$ and can be comparable to the sum of the contributions arising from the polarized scattering coefficients. To illustrate the above statement it is sufficient to observe that the polarized contributions are proportional to $g_{v t} g_{v r}\langle | S_{v v} \left\lvert\, \frac{2}{s}\right.$ and $\left.\left.g_{h t} g_{h r}\langle | S_{h h}\right|^{2}\right\rangle$. On the otherhand, if the imaginary part of $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ is small so that $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle \cong \bar{\equiv}\left\langle\mathrm{S}_{\mathrm{vv}}{ }^{2}\right\rangle$. $<\bar{S}_{h h}{ }^{1 /}$ then the contribution by the cross-correlation coefficient is given by

$$
\sqrt[2]{q_{v t} g_{v r} g_{h t} g_{h r}} \sqrt{\left.\left.\left.\langle | s_{v v}\right|^{2}\right\rangle\left.\langle | s_{h h}\right|^{2}\right\rangle} \operatorname{Re} e^{j\left(\beta_{r}+\beta_{t}\right)}
$$

If right circular polarization* is transmitted and reneived, then $\cos \left(\beta_{f}+\beta_{r}\right)=-1$ and $g_{\mathrm{vt}}=g_{\mathrm{ht}}=g_{\mathrm{vr}}=g_{\mathrm{hr}}$. It is apparent that when $\left\langle\left.\mathrm{s}_{\mathrm{vv}}\right|^{2} \cong\left\langle\left.\mathrm{~S}_{\mathrm{hh}}\right|^{1 /}{ }^{\mathrm{T}}\right.\right.$, the magnitude of the cross-correlation contribution is identical to the sum of the polarized terms. The sign of the contribution is, in this case, negative. However, had the wave been received with a LC polarized antenna, the sign of the contribution would have been positive. The contribution by this scatfering coefficient can also be very effective when attempting measurement of a weak scattering coefficient such as $\left\langle\left. S_{\mathrm{vh}}\right|^{2}{ }^{2}\right.$ with a "linearily" polarized antenna having some cross polarized leakage. This will be illustrated in Chapter 7.

When the cross-correlation terms of the type $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{vh}}{ }^{*}\right\rangle$ and $\left\langle\mathrm{S}_{\mathrm{hv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right.$, are examined in the context of small perturbation theory, it is easily shown that these coefficients vanish at the lowest order where $\left.\left\langle\left.\mathrm{S}_{\mathrm{vv}}\right|^{2}{ }^{2}\right\rangle,\left.\langle | S_{h h^{\prime}}\right|^{2}\right\rangle$ and $\left.\langle | \mathrm{S}_{\mathrm{vh}}\right|^{\left.1^{2}\right\rangle}$ are non-zero (see Appendix C)**. The lack of correlation is physically reasonable since it is believed that the cross-polarized fields result. from multiple scatter. When higher order solutions are included these cross-correlation terms will not vanish; however, their magnitudes will be extremely small.

The above theory is thought to apply with some modification to the sea for angles of observation between 30 and 80 degrees [37]. At smaller angles Kirchhoff theory [37] has predicted sea returns reasonably well. When the theory reported by Fung [36] is employed to explain near vertical returns it can be shown that (see Appendix A) for on isotropic stationary gaussian surface

$$
\begin{equation*}
\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{~S}_{\mathrm{hh}}{ }^{*}\right\rangle=\frac{R_{1} R_{2} \exp \left(-\tan ^{2} \theta / 2 \mathrm{~m}^{2}\right)}{8 \pi \mathrm{~m}^{2} \cos ^{3} \theta} \tag{5-22}
\end{equation*}
$$

* Circular polarization is defined with respect to the antenna; but for narrow beams and sufficiently large angles, the circular polarization sfate transforms to the surface without significant alteration. This will be clarified at the end of this chapter and within Chapter 7.
** Recall $\left\langle\left.\mathrm{S}_{\text {vur }}\right|^{2}\right\rangle$ and $\left\langle\mathrm{I}_{\text {hh }}{ }^{1 \frac{2}{3}}\right.$ order solution.
are first order solutions and $\left\langle/\left.S_{\text {vh }}\right|^{\frac{2}{3}}\right.$ is a second
where

$$
\begin{align*}
& R_{1}=R_{v}\left[\cos \theta+\sin \theta+T_{v} \cos \theta\right]  \tag{5-23}\\
& R_{2}=R_{h}\left[\cos \theta+\sin \theta+T_{h} \cos \theta\right]  \tag{5-24}\\
& T_{v}=\frac{2 \epsilon_{r} \sin \theta}{\sqrt{\epsilon_{r}-\sin ^{2} \theta}\left(\epsilon_{r} \cos ^{2} \theta-\sin ^{2} \theta\right)}  \tag{5-26}\\
& T_{h}=\frac{-2 \cdot \sin \theta}{\sqrt{\epsilon_{r}-\sin ^{2} \theta}} \tag{5-27}
\end{align*}
$$

Comparison of the magnitude of this cross"correlation terms with the magnitudes of the polarized scattering coefficients again shows that (Appendix A)

$$
\begin{equation*}
\left|<s_{w v} S_{h h}{ }^{*}\right\rangle \mid=\sqrt{\left.\left.\left.\langle | s_{v v}\right|^{2}\right\rangle\left.\langle | s_{h h}\right|^{2}\right\rangle} \tag{5-28}
\end{equation*}
$$

The equality is valid to at least first order in corrections to the reflection coefficient for the local slope. The magnitude and phase of the above result is illustrated in Figures 5.3 and 5.4 , respectively, for an isotropically rough sea surface having a moderate rms surface slope. It is noted that the phase property may be attributed to the linear corrections of the reflection coefficients for the local slope. The resulting reflection coefficients compare favorably with that for normal incidence.

When cross"correlations involving $S_{w h}$ or $S_{h v}$ are considered within the Kirchhoff approximation litfle can be said regarding their character. A fypical cross-correlation


FIGURE 5.3 THE SCATTERING CHARACTERISTIC OF $1<\mathrm{S}_{\mathrm{VV}} \mathrm{S}_{\mathrm{HH}}^{*}>\mid$ BASED ON KIRCHHOFF THEORY


FIGURE 5.4 CROSS-CORRELATION PHASE PROPERTY BASED Dil KIRCHHOFF THEORY
between field components is given by

$$
\begin{align*}
\left\langle E_{v v} E_{v h} *\right\rangle= & 4|K|^{2} R_{v}(\theta)\left[\cos \theta+\left(\sin \theta+T_{v} \cos \theta\right) \tan \theta\right] \\
& \iiint \int\left\langle\frac{Z_{x}\left(R_{v}+R_{h}\right)\left(\cos \theta z_{y}-\sin \theta\right)}{Z_{x}^{2}+\left(\sin \theta-\cos \theta z_{y}\right)^{2}} e^{-j 2 \bar{x} \cdot\left(\bar{F}_{1}-\bar{F}_{2}\right\rangle}>d x_{1} d y_{1} d x_{2} d y_{2}\right. \tag{5-29}
\end{align*}
$$

where integration by parts has simplified $E_{v v}$ and the reflection coefficient $R_{v}$ has been linearily approximated. Since the expectation involves higher order slope terms it is anticipated that the correlation will be weak. The stationary phase technique for solving the infegral, for example, would cause the integrand to vanish.

On the basis of the above simple scattering theories, it is clear that the crosscorrelation coefficien $\left\langle\mathrm{S}_{\mathrm{VV}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right.$ " can contribute to a radar retum when both like and cross polarizations are present during transmission and reception. The theory for the slightly rough surface indicates that the phase of the correlation product is dependent on the relative phase between the so-called Rice reflection coefficients. The phase factor is somewhat significant at the larger angles. Correlation products containing $S_{v h}$ or $S_{h v}$ vanish for the slightly rough surface and appear to be negligible for Kirchhoff type surfaces also. These two theories by no means exhaust the possibilities.

Consider, for example, radar returns from a strongly de-polarizing scene such as a tenuous vegetated terrain in which the depolarization is attributable to linear reradiation. Intuitively, one would expect sizeable contributions from the cross-correlation products containing $S_{v h}$ or $S_{h v}$. The correlation contributions will likely arise from a single scatter process, particularly at the canopy. The contributions, as an examination of $\mathrm{C}_{5}$ shows, will arise if like and cross antenna polarizations are present during transmission or reception or both. When $\left\langle/\left.S_{\mathrm{vh}^{\prime}}\right|^{\frac{2}{3}}\right.$ is somewhat less than $\left.\langle | S_{v i f}\right|^{2}$, and $\left.\langle | S_{h h}\right|^{\frac{2}{2}}$ and one attempts retrieval of $\left.\left.\langle | S_{v h}\right|^{2}\right\rangle$ with a "linearily" polarized antenna possessing a cross leakage pattern one can anticipate contamination not only by $\left.\left.\langle | S_{v v}\right|^{2}\right\rangle,\langle | S_{h h}| \rangle$ and $\left\langle S_{v v} S_{h h}{ }^{*}\right\rangle$ but also by $\left\langle S_{v v} S_{h v}{ }^{*}\right\rangle$ and $\left\langle S_{v h} S_{h h}{ }^{*}\right\rangle$.

### 5.4 The Degree of Polarization of Radar Returns

Another important aspect of the scatterometer equation derived here is that the backscattered fields can be considered partially polarized. The partially polarized character is induced by measuring average returns from a non-coherent scene. This permits us to consider a statistical coherency matrix as a suitable representation for the return. It is well known that the degree of polarization of a wave is given by [30]

$$
\begin{equation*}
P=\sqrt{1-\frac{4\left\|c_{5}\right\|}{\left(\operatorname{tr} c_{s}\right)^{2}}} \tag{5-30}
\end{equation*}
$$

where $\mathrm{C}_{s}$ is the coherency matrix of the wave and $\left\|\mathrm{C}_{s}\right\|$ denotes its determinant. If $\mathrm{P}=1$ the wave is said to be completely polarized. This occurs if and only if $\left\|\mathrm{C}_{5}\right\|=0$. To show that the present formulation admits partially polarized waves (possibly randomly polarized, $P=0$ ), it is sufficient to show that $\left\|C_{s}\right\| \neq 0$. Now $C_{s}$ is given in Equation (4-16) and with a little tedious effort one can show

$$
\begin{align*}
\left\|c_{s}\right\|= & \left.\left.\left.\left|e_{v}\right|^{4}\left[\left.\langle | s_{v v}\right|^{2}\right\rangle\langle | s_{h v}\right|^{2}\right\rangle-\left|\left\langle s_{v v} s_{h v}^{*}\right\rangle\right|^{2}\right]+\left|e_{v}\right|\left|e_{h}\right|^{2}\left[\left.\langle | s_{v v}\right|^{2}\right\rangle \\
& \left.\left.\left.\langle | s_{h h}\right|^{2}\right\rangle-\left|\left\langle s_{v v} s_{h h} *\right\rangle\right|^{2}\right]+\left|e_{h}\right|^{4}\left[\left.\langle | s_{h h}\right|^{2}\right\rangle\langle | s_{h v}| \rangle-\mid\left\langle\left. s_{h v} s_{h h} *\right|^{2}\right] \\
& +2\left|e_{v}\right|^{2} \operatorname{Ree}_{v} e_{h} *\left[\left\langle\left. s_{v v}\right|^{3}\right\rangle\left\langle s_{h v} s_{h h}^{*}\right\rangle-\left\langle s_{v v} s_{h h} *\right\rangle\left\langle s_{v v} * s_{h v}\right\rangle\right]+2 \operatorname{Re} \\
& \left.\left(e_{v} e_{h}^{*}\right)^{2}\left[\left\langle s_{v v} s_{v h}^{*}\right\rangle\left\langle s_{h v} s_{h h} *\right\rangle-\left.\left\langle s_{v v} s_{h h} *\right\rangle\langle | s_{h v}\right|^{2}\right\rangle\right]+2\left|e_{h}\right|^{2} \operatorname{Re} \\
& \left.\left\langle e_{v h} e_{h}^{*}\right\rangle\left[\left.\left\langle s_{v v} s_{v h}^{*}\right\rangle\langle | s_{h h}\right|^{2}\right\rangle-\left\langle s_{v v} s_{h h} *\right\rangle\left\langle s_{v h} * s_{h h}\right\rangle\right] \tag{5-31}
\end{align*}
$$

where the subscript $t$ has been dropped. If the determinant is to vanish independent of the transmifted fields then each difference term in the above expression must vanish.

A non-statistical farget having a scattering matrix

$$
S=\left[\begin{array}{ll}
i s_{v v} \mid e^{j \alpha} & \left|s_{v h}\right| e^{j \delta}  \tag{5-32}\\
\left|s_{v h}\right| e^{j \delta} & \left|s_{h h}\right| e^{j \beta}
\end{array}\right]
$$

will obviously meet the requirement. However, the general result indicates that the backscattered wave is partially polarized as the examples below illustrate.

For the case where terms of the type $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{vh}}{ }^{*}\right\rangle$ are assumed negligibly small, as we suspect they are over the sea, we have

$$
\begin{align*}
\left\|c_{s}\right\|= & \left.\left.\left.\left.\left|e_{v}\right|^{4}\langle | s_{v v}\right|^{2}\right\rangle\left.\langle | s_{h v}\right|^{2}\right\rangle+\left.\left|e_{v}\right|^{2}\left|e_{h}\right|^{2}\left[\left.\langle | s_{v v}\right|^{2}\right\rangle\langle | s_{h h}\right|^{2}\right\rangle-\mid\left\langle\left. s_{v v} s_{h h} *\right|^{2}\right] \\
& \left.\left.\left.+\left.\left|e_{h}\right|^{4}\langle | s_{h h}\right|^{2}\right\rangle\left.\langle | s_{h v}\right|^{2}\right\rangle- \text { PRe }\left.\left(e_{v} e_{h}^{*}\right)^{2}\left\langle s_{v v} s_{h h}^{*}\right\rangle\langle | s_{h v}\right|^{2}\right\rangle \tag{5-33}
\end{align*}
$$

At moderate to large incident angles over the ocean it is anticipated that the above term can be significantly different from zero. Sépecifically if'ahorizontally polarized wave is transmitted, we have

$$
\begin{equation*}
\left.\left.\left\|c_{s}\right\|=\left.\left|e_{h}\right|^{4}\langle | s_{h h}\right|^{2}\right\rangle\left.\langle | s_{h v}\right|^{2}\right\rangle \tag{5-34}
\end{equation*}
$$

The corresponding degree of polarization is given by

$$
\begin{equation*}
P_{h .}=\frac{\left.\left.\left.\langle | s_{h h}\right|^{2}\right\rangle-\left.\langle | s_{v h}\right|^{2}\right\rangle}{\left.\left.\left.\langle | s_{h h}\right|^{2}\right\rangle+\left.\langle | s_{v h}\right|^{2}\right\rangle} \tag{5-35}
\end{equation*}
$$

Similarly when a vertically polarized wave is transmitted, the degree of polarization is given by

$$
\begin{equation*}
P_{v}=\frac{\left.\left.\left.\langle | s_{v v}\right|^{2}\right\rangle-\left.\langle | s_{v h}\right|^{2}\right\rangle}{\left.\left.\left.\langle | s_{v v}\right|^{2}\right\rangle+\left.\langle | s_{v h}\right|^{2}\right\rangle} \tag{5-36}
\end{equation*}
$$

If a circularly polarized wave is transmitted, then

$$
\begin{align*}
\left\|c_{s}\right\|= & \left.\left.\left.\left.\left.\left.\langle | s_{h v}\right|^{2}\right\rangle\left[\left.\langle | s_{w v}\right|^{2}\right\rangle+\left.\langle | s_{h h}\right|^{2}\right\rangle+2 R_{e}\left\langle s_{v v} s_{h h}^{*}\right\rangle\right]+\left.\langle | s_{v v}\right|^{2}\right\rangle\left.\langle | s_{h h}\right|^{2}\right\rangle \\
& -\left|\left\langle s_{v v} s_{h h}{ }^{*}\right\rangle\right|^{2} \tag{5-37}
\end{align*}
$$

and the degree of polarization is given by Equation (5-30).
The above cases were evaluated as a function of incident angle for the scattering characteristic of Figures 5.1 and 5.2. The results are shown in Figure 5.5. It is apparent that the partially polarized character is an important factor when the non-coherent scatterometer equation is appropriately interpreted.

### 5.5 Visualization of the Polarization Properties of the Antenna and Scene

Within the latter section of Chapter 4 the scatterometer equation accounting for the difference between antenna and surface polarizations was derived. It was shown that the polarization mis-alignment could be characterized by a simple rotation of either orthogonal polarization pair through an angle $\psi$. To show this mis-alignment character, rather than study the functional behavior of $\psi$ on $(\theta, \varnothing)$, it is more convenient to fall back on the properties of the spherical polar vectors $\bar{i}_{\theta}$ and $\bar{i}_{\phi}$.

Regardless of whether one considers the antenna or surface coordinate system, the projection of the polar vector $\bar{i}_{\theta}$ and the azimuthal vector $\bar{i}_{g}$ on any sphere whose center is located af the point of observation can be depicted, respectively, by longitudes and latitudes on that sphere. For any line of sight emanating from the origin of the sphere the longitude and latitude lines intersecting the line of sight on the sphere will correspond to the orientation of vertical and horizontal polarization, respectively, for that line of sight. We can therefore employ spheres marked with longitudes and latitudes to visualize the antenna or the surface polarizations.

To compare the alignment between antenna and surface potarizations choose the radit of both polarization spheres so that the spheres are tangent to the scattering surface at the sub-observational point as illustrated in Figures 5.6 and 5.7. A pole of the surface polarization sphere will be afixed to the sub-observational point. This polar axis will correspond to the $z$ axis of Figure 4.4 The xy plane coincides with the equatorial plane and is parallel the surface. The antenna boresight axis lies in the $x z$ plane and points at an angle of $\theta_{0}$ with respect to the $z$ axis (Figure 5.6). Now, on the otherhand, the equatorial plane of the antenna polarization sphere coincides with the


FIgure 5.5 TIIE degree of polarization behavior as predicted from SmALL PERTURBATIOM THIDRY


FIGURE 5.6 SURFACE POLARIZATION SPHERE
plane containing the boresight and the $y$ axis. This plane corresponds to the $x^{\prime} y^{\prime}$ plane in the antenna coordinate frame (Figure 5.7). The polar axis of antenna polarization sphere aligns with the $z^{t}$ coordinate. Comparison of the orientations of the latitudes and longitudes for any common line of sight will indicate the polarizafion mis-alignment property (Compare Figures 5.6 and 5.7). Within the plane of observation (the $x z$ or $x^{\prime} z^{\prime}$ plane) regardiess of view angles the polarizations $\mathrm{co}^{-}$ incide. For any other line of sight there will be a difference in alignment. The misalignment is greatest in the polar regions of the antenna or surface polarization spheres. When the antenna is pointed toward the horizon the alignment is everywhere perfect (one must mentally rotate the sphere in Figure 5.7 so that the $x^{1}$ axis points to the horizon). When the antenna is pointed at the sub-observational point, the misalignment is severe everywhere in the vicinity of the sub-observational point. Within the nadir region the scattering coefficients defined with respect to the surface as compared to those one may define with respect to the antenna differ radically. For example, if a significant anisotropic scattering behavior occurs at nadir, any finife beam scatterometer would tend to integrate this behavior. The measurement, as a consequence, would be difficult to refer to the surface polarizations. The surface polarization character at nadir indicates that infinitesimai beamwidths must be used if the nadir region is to be probed and if scattering coefficients defined with respect to the surface are to be reported. This is clearly true if there is a difference in $\left.\left.\langle | S_{\mathrm{vv}}\right|^{2}\right\rangle$ and $\left.\langle | S_{h h}\right|^{2}>$ scatfering properties as viewed with respect to the surface.

As pointed out in Section 4.3, there is an alternate method of mounting the antenna which will produce a different polarization character. Suppose the antenna had been mounted so that its horizontal polarization vector ( $\bar{i}_{\phi}{ }^{\prime}$ ) on the boresight axis ( $x$ ') aligned with the surface vertical polarization at that line of sight. The polar axis of the antenna polarization sphere ( $z^{\prime}$ ) would coincide with the $-y$ axis of the surface coordinate sysfem. The corresponding polarization sphere is illustrated in Figure 5.8. Comparison of the polarization property with that of the surface indicates that the misalignment is invariant with view angle and the polarizations do not align globally for any view angle. The polarizations continue to align in the plane of observation; however, the same mis-alignment in the nadir region remains a problem.

Regardless of the mounting position it is evident that for non-zero beamwidth antennas the discrepancy between antenna and surface polarizations prevails in the


FIGURE 5.7 ANTENNA POLARIZATION SPHERE


FIGURE 5.8 ALTERNATE ANTENNA POLARIZATION SPHERE
nadir region. It is clear that nadir is a forbidden region when one views it from the point of surface polarizations. At all but small view angles the polarization discrepancy over the main beam of narrow beamwidth antennas is generally small (how small will be shown in Chapter 7). At these angles as the beamwidth becomes narrower for a linearily polarized antenna, the percent anfenna power occurring in the orthogonal surface polarization becomes smaller. As one approaches nadir the beamwidth must become increasingly narrower for the same degree of mis-alignment.

### 6.0 THE INVERSION OF SCATTEROMETER MEASUREMENTS FOR THE SCENE PARAMETERS

### 6.1 Introduction

The recovery of the scene scattering parameters entails an appropriate set of measurements and the inversion of a corresponding system of scatterometer equations of the type derived in Chapter 4. Within this chapter a measurement and an inversion technique is derived to recover a complete set of scattering coefficients. The technique is also specialized to the case where the scattered fields may be regarded as completely polarized. To assure that the technique is both as simple and as accurate as possible, certain antenna properties are specified. The consequences of not employing a suitably chosen antenna is illustrated in Chaper 7. The mathematical and physical aspects of inverting scatterometer measurements are treated in the following sections. Certain antenna properties which are helpful in approximating the measurements by a system of algebraic. equations are identified. In this chapter the distinction between surface and antenna polarization is appropriately discarded to simplify the presentation. The consequence of this action is treated in Chapter 7.

### 6.2 Mathematical and Physical Aspects

The inversion of scatterometer measurements falls into the same mathematical category as do many remote sensing problems. Typically, the observational relationship reduces to solving a Fredholm integral equation of the first kind, viz.,

$$
\begin{equation*}
g(y)=\int K(y, x) f(x) d x \tag{6-1}
\end{equation*}
$$

where $K(y, x)$ is usually a continuous function over a rectangular domain atributable to a sensor, $f(x)$ is the unknown sensor stimulus and $g(y)$ is the observed sensor response. The scatterometer equation si a generalization of the above expression. Since there are nine unknown scatfering parameters, it is clear that there must be at least nine different kinds of measurements to retrieve all the parameters. If each kind of measurement is identified by a subscript $i$ and if the scattering parameters are denoted by $\xi_{j}(\Omega)$ where $\Omega=(\theta, \phi)$, then the system of measurements can be written as

$$
\begin{array}{r}
W_{i}\left(\Omega_{0}\right)=\left(\frac{\lambda}{4 \pi}\right)^{2} G_{t} G_{r} W_{t} \int \sum_{j=1}^{g} K_{i j}\left(\Omega, \Omega \partial \xi_{j}(\Omega) d \Omega\right. \\
i=1,2, \ldots, 9
\end{array}
$$

where

$$
\begin{align*}
& k_{i \underline{1}}=\left(g_{v r} g_{v t}\right)_{i} / r^{2} \\
& K_{i 2}=\left(g_{h r^{g} h t}\right)_{i} r^{2} \\
& K_{i 3}=\left(g_{v t} g_{h r}+g_{v r} g_{h t}+2 \sqrt{9_{v t} g_{h t} g_{v r} g_{h r}} \cos \left(\beta_{t}-\beta_{r}\right)\right)_{i} / r^{2} \\
& K_{i 4}=2\left(\sqrt{g_{v t} g_{h t} g_{v r} g_{h r}} \cos \left(\beta_{t}+\beta_{r}\right)\right)_{i} / r^{2} \\
& K_{i 5}=-2\left(\sqrt{g_{v t}{ }^{g} h t^{g} v r g^{g} r} \cos \left(\beta_{t}+\beta_{r}\right)\right)_{i} / r^{2} \\
& k_{i 6}=2\left(g_{v r} \sqrt{g_{v t} g_{h}} \cos \beta_{t}+g_{v t} \sqrt{g_{v r} g_{h r}} \cos \beta_{r}\right)_{i} / r^{2} \\
& K_{i 7}=-2\left(g_{v r} \sqrt{g_{v t} g_{h t}} \cos \beta_{t}+g_{v t} \sqrt{g_{v r} g_{h r}} \cos \beta_{r}\right)_{i} / r^{2} \\
& K_{i 8}=2\left(g_{h r} \sqrt{g_{v t} g_{h t}} \cos \beta_{t}+g_{h t} \sqrt{g_{v r} g_{h r}} \cos \beta_{r}\right)_{i} / r^{2} \\
& K_{i 9}=-2\left(g_{h r} \sqrt{g_{v t} g_{h t}} \cos \beta_{t}+g_{h t} \sqrt{g_{v r^{g}} g_{r}} \cos \beta_{r}\right)_{i} / r^{2} \tag{6-3}
\end{align*}
$$

are the kernel functions with respect to an integration on a sphere and where

$$
\begin{align*}
& \left.\xi_{1}=\left.\langle | S_{V V}\right|^{2}\right\rangle \\
& \left.\xi_{2}=\left.\langle | S_{h h}\right|^{2}\right\rangle \\
& \left.\xi_{3}=\left.\langle | S_{v h}\right|^{2}\right\rangle \\
& \xi_{4}=\operatorname{Re}\left\langle S_{V V^{\prime}} S_{h h^{*}}^{*}\right. \\
& \dot{\dot{\xi}}_{5}=-I m\left\langle S_{V V} S_{h h}^{*}\right\rangle \\
& \xi_{G}=\operatorname{Re}\left\langle S_{v v} S_{v h}^{*}\right\rangle \\
& \left.\xi_{7}=I m<S_{V v} S_{V h}^{*}\right\rangle \\
& \xi_{g}=\operatorname{Re}\left\langle S_{v h} S_{h h}^{*}\right\rangle \\
& \xi_{g}=I m\left\langle S_{v h} S_{h h}^{*}\right\rangle \tag{6-4}
\end{align*}
$$

are the unknown scattering parameters. The parameters leading the integral are constants.

For each i one must ipecify receive and transmit antenna polarization states and patterns such that the resulting system of equations can be solved approximately. There are undoubtedly many such specifications. However, there are certain physical considerations which make the search for the appropriate kernel function (untenna polarizations) simpler.

It has been shown, for example, that in the measurement of a auto-correlation coefficient, the kernel function can be approximated by a delta function if the antenna beam is sufficiently narrow to resolve the angular behavior of the coefficient [ 33$]$ ]. The method assumes that the scattering parameter is constant across the significant portion of the kernel function. The unknown parameter is withdrawn from the integral and the resulting integral expression evaluated. The solution then becomes algebraic. This, in effect, is equivalent to assuming that the kernel is a delta function with a weight corresponding to the evaluation of the integral expression. The method is feasible since the kernel function is sharpened by a product of pattern terms as indicated in Equation 6-2. The two-way sharpening effect is illustrated in Figure 6.1 where both g and $\mathrm{g}^{2}$ are plotted. It should be noted that the ordinate scale has been transformed logarithmically to dB . The kernel function is consequently significant only over a very small domain of $\{(\theta, \dot{\phi}), \theta<\pi, 0 \angle \phi \leq 2 \pi\}$.

It would be helpful if the delta function approximation could also be used to recover the cross-correlation scattering paramete. . An examination of Equations 6-2 and 6-3 indicates that the two-way sharpening effect is present in the gain functions. However, there is no guarantee that $\beta_{f}$ and $\beta_{r}$ will remain constant across the significant domain of the gain functions. Generally the antenna phase factors are functions of $(\theta, 0)$. On the other hand, if these factors are stationary on the main beam, then the delta function approximation can be employed for these parameters also (see Equation 6-6). The ability to realize the stationary condition is treated in the subsequent section.


FIGURE 6.I INCREASED RESOLUTION OF THE KERNEL FUNCTION CAUSED BY THE TWO-WAY SHARPENING OF THE PATTERN

### 6.3 Desirable Antenna Properties

If the delta function approximation is to be employed, then it is desirable to have the relative phase $\beta_{r}$ and $\beta_{f}$ constant across the main beam (See Equation (4-29)). This objective is equivalent to requiring that the gain and polarization be staitionary across the main beam. Chu and Kouyoumjian [39] have derived the conditions under which stationary gain and polarizations can be achieved. Coincident stationarity, they state, can be realized by any planar aperture distribution which is symmetric with respect to two orthogonal axes in the aperture plane. An aperture is planar only if the excitation lies in the aperture plane and not orthogonal to it.

For some center fed parcboloids the above requirement can be met; however not all feeds result in a planar distribution even though the symmetry property is observed. This is illustrated for the case of a dipole feed. Although the distribution in the aperture plane is symmetric, it contains excitation components orthogonal to the plane, The orthogonal components are induced by the depolarization property of the paraboloid. The far field of such a dish is illustrated in Figure 6.2. The computation was based on a -10 dB taper, a $\mathrm{f} / \mathrm{D}^{*}$ ratio of 0.36 and a wavelength of 2.16 cm .

- The introduction of variable cross polarized content can clearly destroy the stationary polarization requirement. Admittedly the cross-polarized content in the illustrated case is small; however, as will be shown later in Section 7.4, retrieval of the cross polarized scattering coefficient can be affected by weak cross polarized pattern levels. Furthermore, dipole fed paraboloids with smaller f/D ratios will have a larger cross polarized level than iliustrated here [44].

Recently, corrugated horns [40] [41] and dual mode horns [42] [43] with circularly symmetric patterns have been recognized as capable of eliminating crosspolarization in center fed paraboloids. The feed pattern of these horns are said to be balanced. Mathematically their radiation takes the form

$$
\vec{E}_{f}=F\left(\theta^{\prime}, \phi^{\prime}\right)\left[\binom{\cos \phi^{\prime}}{\sin \phi^{\prime}} \bar{L}_{\theta^{\prime}} \mp\binom{\sin \phi^{\prime}}{\cos \phi^{\prime}} \bar{\zeta}_{\phi^{\prime}}\right] \frac{\exp \left(-i k p^{\prime}\right)}{\rho^{\prime}}(6-5)
$$

where the $z^{1}$ axis is directed along the axis of the paraboloid. Chu and Turrin [59] have shown that center fed paraboloids with balanced illumination exhibit no crosspolarized content in the aperture plane. The far-fields of the above described parar

[^5]

FIGURE 6.2 FAR FIELDS OF A PARABOLOID WITH A DIPOLE FEED
boloid with a balanced feed was computed and are shown in Figure 6.3. The crosspolarized field was totally absent in the numerical computations.

Balanced fed paraboloids are suitable candidates for scatterometry work when a complete set of scene parameters are desired. Since support struts and aperture blockage, in general, introduce cross-polarized radiation it is important to minimize blockage in addition to choosing an appropriate feed. The Cufler type feed with balance radiation may be a suitable approach.

Alternatively, an array of broadly directional radiators is also a suitable candidate. . If the interaction between elements is weak, then the pattern of the array is the product of the array factor and the pattern of one of the elements. The polar ization property in the main labe will be dictated by the polarization property of the central segment of the elementary pattern. The polarization will generally be stationary across a smail segment of the elementary pattern; and, therefore, the array polar izafion will also be stationary there.

### 6.4 The Inversion of Scatterometer Measurements

When a narrow beam scatferometer anfenna with a coincident stationary gain and polarization property is employed, the scatterometer equation may be approximated by*

$$
\begin{align*}
W_{f r}(\Omega)= & K\left\{I_{1}<\left|S_{v v}\right|^{2}>+I_{2}<\left|S_{h h}\right|^{2}>+\left[I_{3}+2 I_{4} \cos \left(\beta_{r}-\beta_{t}\right)\right]<\left|S_{v h}\right|^{2}>+\right. \\
& 2 I_{4} \operatorname{Re}\left[e^{i\left(\beta_{t}+\beta_{r}\right)}<S_{v v} S_{h}>\right]+2 \operatorname{Re}\left[I_{5} e^{j \beta_{t}}+I_{6} e^{j \beta_{r}}\right]<S_{v v} S_{h v}^{*}>+ \\
& 2 \operatorname{Re}\left[I_{7} e^{j \beta_{t}}+I_{8} e^{j \beta_{r}} 1<S_{v h} S_{h h^{*}}>\right\} \tag{6-6}
\end{align*}
$$

where

[^6]

FIGURE 6.3 FAR FIELDS OF A PARABOLOID WITH A BALANCED FEED

$$
\begin{align*}
I_{1} & =\iint g_{v t} g_{v r} \cos ^{2} \theta d \Omega \\
I_{2} & =\iint g_{h t} g_{h r} \cos ^{2} \theta d \Omega \\
I_{3} & =\iint\left(g_{h t} g_{v r}+g_{v t} t_{h r}\right) \cos ^{2} \theta d \Omega \\
I_{4} & =\iint \sqrt{g_{v t^{g}} h t} \sqrt{g_{v r} g_{h r}} \cos ^{2} \theta d \Omega \\
I_{5} & =\iint \sqrt{g_{v t}{ }^{g} h t} g_{v r} \cos ^{2} \theta d \Omega  \tag{6-7}\\
I_{6} & =\iint g_{v t} \sqrt{g_{v r} g^{2} r r} \cos ^{2} \theta d \Omega \\
I_{7} & =\iint \sqrt{g_{v t}{ }^{g} h t}{ }^{g_{h r}} \cos ^{2} \theta d \Omega \\
I_{8} & =\iint g_{h t} \sqrt{g_{v r} g_{h r}} \cos ^{2} \theta d \Omega \\
K & =\lambda^{2} w_{t} G_{t} G_{r} /(4 \pi z)^{2} \tag{6-8}
\end{align*}
$$

It has been assumed that observations are conducted over a planar earth so that $\mathrm{r}=\mathrm{z} / \mathrm{cose}$. It has also been assumed that the kernel function has sufficient resolution that the scattering coefficient may be considered constant in the domain where the kernel function is significant. Now suppose that the scatterometer is equipped with a dual linearily polarized feed or if necessary two antenna with orthogonal linear polarizations to assure good isolation. The amplitude and phase of each feed channel is assumed controllable. Then as will be shown below, a series of fifteen intensity measurements with different polarization combinations is capable of extracting a complete set of nine scattering parameters,viz.,
$\left.\left.\left.\left.\langle | S_{v v}\right|^{2}\right\rangle,\left.\langle | S_{h h}\right|^{2}\right\rangle,\left.\langle | S_{v h}\right|^{2}\right\rangle, \operatorname{Re}\left\langle S_{w v} S_{h h^{*}}{ }^{*}, \operatorname{Im}^{2}\left\langle S_{v v} S_{h h}{ }^{*}\right\rangle, \operatorname{Re}\left\langle S_{v v} S_{h v^{*}}{ }^{*}\right\rangle\right.$, $\operatorname{Im}\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hv}}{ }^{*}\right.$, , $\operatorname{Re}\left\langle\mathrm{S}_{\mathrm{vh}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$, and $\operatorname{Im}\left\langle\mathrm{S}_{\mathrm{vh}} \mathrm{S}_{h h}{ }^{*}\right\rangle$. A pair of measurements is required to isolate the real or imaginary part of the complex valued coefficients. The transmif-receive polarization states are indicated for each measurement:

1) WV

$$
\begin{align*}
& \left(g_{v t}=g_{v r}=g, g_{h t}=g_{h r}=0\right) \\
& \left.H_{t r}=\left.K I\langle | s_{v v}\right|^{2}\right\rangle \tag{6-9a}
\end{align*}
$$

2) $\mathrm{H}-\mathrm{H}$

$$
\left(g_{v t}=g_{v r}=0, g_{h t}=g_{h r}=g\right)
$$

$$
\begin{equation*}
\left.\mathrm{H}_{\mathrm{tr}}=\left.\mathrm{KI}\langle | \mathrm{s}_{\mathrm{hh}}\right|^{2}\right\rangle \tag{6-9b}
\end{equation*}
$$

3) $\mathrm{V} \circ \mathrm{H}$

$$
\begin{align*}
& \left(g_{v t}=g_{h r}=g, g_{h t}=g_{v r}=0\right) \\
& \left.w_{t r}=\left.k I\langle | s_{v h}\right|^{2}\right\rangle \tag{6-9c}
\end{align*}
$$

Aa) LC-RC $\quad\left(g_{v t}=g_{h t}=g_{v r}=g_{h r}=\frac{1}{2} g ; \beta_{t}=-90^{\circ} ; \beta_{r}=90^{\circ}\right)$

$$
\begin{equation*}
\left.\left.W_{t r}=K I\left[\left.\frac{1}{4}\langle | s_{v v}\right|^{2}\right\rangle+\left.\frac{1}{4}^{2}\langle | s_{h h}\right|^{2}\right\rangle+\frac{1}{2} r e\left\langle s_{v v} S_{h h}^{*}\right\rangle\right] \tag{6-9d}
\end{equation*}
$$

Ab) Cross-Linear $\left(g_{v t}=g_{h t}=g_{v r}=g_{h r}=\frac{1}{2} g, \beta_{t}=0, \beta_{r}=180^{\circ}\right)$

$$
\begin{equation*}
\left.\left.w_{t r}=K I\left[\left.\frac{1}{4}\langle | s_{v v}\right|^{2}\right\rangle+\left.\frac{1}{4}\langle | s_{h h}\right|^{2}\right\rangle-\frac{1}{2} R e\left\langle s_{v v} s_{h h}^{*}\right\rangle\right] \tag{6-9e}
\end{equation*}
$$

5a) Elifiptical $\left(g_{v t}=g_{h t}=g_{v r}=g_{h r}=\frac{1}{2} g, \beta_{t}=-45^{\circ}, \beta_{r}=135^{\circ}\right)$

$$
\left.\left.W_{\mathrm{tr}}=\mathrm{KI}\left[\left.\frac{1}{4}\langle | \mathrm{s}_{\mathrm{vv}}\right|^{2}\right\rangle+\frac{1}{4}<\left|\mathrm{s}_{\mathrm{hh}}\right|^{2}\right\rangle+\frac{1}{2} \mathrm{Im}\left\langle\mathrm{~s}_{\mathrm{vv}} \mathrm{~s}_{\mathrm{hh}}^{*}\right\rangle\right]
$$

5c) Elliotical $\left(g_{v t}=g_{h t}=g_{v r}=g_{h r}=\frac{1}{2} g, B_{t}=45^{\circ}, B_{r}=-135^{\circ}\right)$

$$
\begin{equation*}
\left.\left.w_{t r}=K I\left[\left.\frac{1}{4}\langle | s_{v v}\right|^{2}\right\rangle+\left.\frac{1}{4}\langle | s_{h h}\right|^{2}\right\rangle-\frac{1}{2} I m\left\langle s_{w v} s_{h h}^{*}\right\rangle\right] \tag{6-9~g}
\end{equation*}
$$

6a) $v$ - Diagonal Linear ( $\left.g_{v t}=g, g_{h t}=0, g_{v r}=g_{h r}=\frac{1}{2} g, \beta_{r}=0\right)$

$$
\left.\left.H_{t r}=K I\left[\left.\frac{1}{2}\langle | s_{v v}\right|^{2}\right\rangle+\left.\frac{1}{2}\langle | s_{v h}\right|^{2}\right\rangle+\operatorname{Re}\left\langle s_{v v} S_{h v}^{*}\right\rangle\right]
$$

$$
(6-9 h)
$$

6b) $v$ - Diagonal Linear $\left(g_{v t}=g, g_{h t}=0, g_{v r}=g_{h r}=\frac{1}{2} g, B_{r}=180^{\circ}\right)$

$$
\left.\left.W_{t r}=K I\left[\left.\frac{1}{2}\langle | s_{v v}\right|^{2}\right\rangle+\left.\frac{1}{2}\langle | S_{v h}\right|^{2}\right\rangle-R e\left\langle S_{v v} S_{h v}^{*}\right\rangle\right]
$$

$$
\left(g_{v t}=g, g_{h t}=0, g_{v r}=g_{h r}=\frac{i}{2} g, \beta_{r}=90^{\circ}\right)
$$

$$
\left.\left.W_{t r}=K I\left[\left.\frac{1}{2}\langle | s_{\cdot v v}\right|^{2}\right\rangle+\left.\frac{1}{2}\langle | S_{v h}\right|^{2}\right\rangle+\operatorname{Im}\left\langle S_{v v^{\prime}} s_{h v}^{*}\right\rangle\right]
$$

7b) V-LC

$$
\begin{aligned}
& \left(g_{v t}=g, g_{h t}=0, g_{v r}=g_{h r}=\frac{1}{2} g, \quad(6-9 j)\right. \\
& \left.\beta_{r}=-90^{\circ}\right) \\
& \left.H_{t r}=K I\left[\frac{1}{2}<\left|s_{v v}\right|^{2}>+\frac{1}{2}<\left|s_{v h}\right|^{2}>-I m<s_{v v} s_{h v}^{*}\right\rangle\right] \\
& (\delta-9 k)
\end{aligned}
$$

8a) H-Diagonal Linear ( $g_{v t}=0, g_{h t}=g, g_{v r}=g_{h r}=\frac{1}{2} g, \beta_{r}=0$ )

$$
\left.\left.H_{t r}=K I\left[\left.\frac{1}{2}\langle | s_{h h}\right|^{2}\right\rangle+\left.\frac{1}{2}\langle | s_{v h}\right|^{2}\right\rangle+R e\left\langle S_{v h} s_{h h}^{*}\right\rangle\right]
$$

8b) H-Diagonal Linear ( $\left.g_{v t}=0, g_{h t}=g, g_{v r}=g_{h r}=\frac{1}{2} g, \beta_{r}=18-90^{\circ}\right)$

$$
\left.\left.H_{t r}=K I\left[\left.\frac{1}{2}\langle | s_{h h}\right|^{2}\right\rangle+\left.\frac{1}{2}\langle | s_{v h}\right|^{2}\right\rangle-R e\left\langle s_{v h} \mathrm{~s}_{h h^{\prime}}^{*}\right\rangle\right]
$$

9a) $\mathrm{H}-\mathrm{RC}$

9b) $H-L C$

$$
\begin{aligned}
& \left(g_{v t}=0, g_{h t}=g, g_{v r}=g_{h r}=\frac{1}{2} g, \beta_{r}=90^{\circ}\right) \\
& \left.\left.W_{t r}=K I\left[\left.\frac{1}{2}\langle | s_{h h}\right|^{2}\right\rangle+\left.\frac{1}{2}\langle | s_{v h}\right|^{2}\right\rangle+I m\left\langle s_{v h} s_{h h}^{*}\right\rangle\right] \\
& \left(g_{v t}=0, g_{h t}=g, g_{v r}=g_{h r}=\frac{1}{2} g, \beta_{r}=\begin{array}{c}
\left(6-90^{\circ}\right)
\end{array}\right. \\
& \left.\left.W_{t r}=K I\left[\left.\frac{1}{2}\langle | s_{h h}\right|^{2}\right\rangle+\left.\frac{I}{2}\langle | s_{v h}\right|^{2}\right\rangle-I m\left\langle S_{v h} s_{h h}^{*}\right\rangle\right] \\
& (6-90)
\end{aligned}
$$

where

$$
\begin{equation*}
I=\iint g^{2} \cos ^{2} \theta d \Omega \tag{6-10}
\end{equation*}
$$

The transmit and receive polarizations may be inferchanged without affecting the above equations. The abcy : set of equations assumes that the scattering coefficients are defined with respect $t$ : the antenna frame. As will be shown for narrow beam antenna, the above polarization states will retrieve scattering coefficiants defined with respect to the surface at all but very small incident angles. The above polarization states may be incorporated in the equati m which distinguish surface and antenna polarizations fo develop an inversion technique based on the distinction. These equations are developed in the succeerling chapter.

From the above set of equations it is noted that $\left.\left.\left.\langle | S_{v v}\right|^{2}\right\rangle,\left.\langle | S_{h h}\right|^{2}\right\rangle$ and $\left.\left.\langle | S_{v h}\right|^{2}\right\rangle$ are each derived from a single measurement $i_{0} e$. measurements (1), (2), and $(3)$, respectively. The remaining parameters are isolated by differencing pairs of equàtions. If is clear that if a complefe sef of scaftering parameters is desired, the measurement set is over-specified. If a minimal set of equations is required, then all measurement pairs can be reduced to one of the members. It is advisable, however ${ }_{r}$ to work with an overspecified set of measurements to reduce the sensitivity fo measurement errors if all the coefficients are desired. It is a distinct advantage to specify equatic: pairs if a particular complex valued coefficient is to be isolated. The above fechnique does not pre-suppose that the scattered fields are completely polarized.

If one further assumes that the scattered fields are completely polarized, then the inversion problem reduces to that for non-statistical targets. Various measurement schemes have been reviewed for this case by Huynen [21]. One of these schemes is based on field amp! itude and phase measurements of a pair of orthogonal returns from each of the two orthogonal illum nating polarizations. Another scheme involves amplitude measurements at different polarizations. The latter technique yields a set of target invariant parameters which must be transformed to a scattering matrix, Inrensity measurements as described above will, of course, also wark. The set of measurements may be solved subject to the constraints

$$
\begin{align*}
& \left\langle s_{v v} s_{h h}^{*}\right\rangle \mid=\sqrt{\left.\left.\left.\langle | s_{v v}\right|^{2}\right\rangle\left.\langle | s_{h h}\right|^{2}\right\rangle} \\
& \left.K_{v v} s_{v h}^{*}\right\rangle 1=\sqrt{\left.\left.\left.\langle | s_{v v}\right|^{2}\right\rangle\left.\langle | s_{v h}\right|^{2}\right\rangle}  \tag{6-11}\\
& \mid\left\langle s_{h v} s_{h h}^{*}\right\rangle 1=\sqrt{\left.\left.\left.\langle | s_{h v}\right|^{2}\right\rangle\left.\langle | s_{h h}\right|^{2}\right\rangle}
\end{align*}
$$

for additional accuracy. Nori-linear regression techniques as described in reference [45] or 146] may be employed to solve the sysfem of measuremenis subject to these constraints.

In retrospect one can alsa use correlation and cross-correlation techniques to isolate some of the non-coherent scattering coefficients. For example to measure $\left\langle S_{v v} S_{h v}{ }^{*}\right\rangle, e_{v t}$ is transmitted. During reception both $\epsilon_{v s}$ and $e_{h s}$ are cross-correlated without and with $90^{\circ}$ phase shift injected into one of the channels to isolate the real and imaginary parts, respectively.

Either correlation techniques or intensity techniques as proposed will suffer from poor realizations of the desired antenna properties. Since infensity measurements are commonily made, this investigation will restrict its aftention to the infensity technique.

### 7.0 PRACTICAL CONSIDERATIONS IN RETRIEVING THE SCATTERING COEFFICIENTS

### 7.1 Introduction

In attempting to retrieve the scattering coefficients by the method developed in Chapter 6, one is immediately confronted with the fact that the ideal antenna polar ization states specified in each measurement are seldom achieved in practice. To defermine the sensitivity of the measurement to deviations from these ideal states, computer simulations were conducted. Measurements were simulated on the basis of the complete scatterometer equation as developed in Chapter 4 and a scattering characteristic similar to that of the sea under low wind conditions. The scattering coefficients were expressed with respect to the surface polarizations; and, consequently, all simulated power refurns involve fransforming the pattern information to the surface polarization states to compute accurafe power returns, Measurements were computed based on known deviations from the ideal antenna polarization requirements and were inverted on the basis of the ideal antenna specifications. The sensitivity in retrieving each coefficient was thus established, namely, by comparing the actual coefficient with the estimated coefficient.

The computer simulation was designed not only to derermine the sensitivity of the measurement to non-ideal anfenna poiarization states, but was designed to establish the beamwidth limitation to realize the delta function approximation for the integrand in the scatterometer equation. It was also designed to defermine whether the distinction between surface polarizations and antenna polarization is important; and if so, under what conditions it is important.

Within the latter portion of this chapter special consideration is given to the sampling requirements when measuring an antenna pattern. The simulations described above were based on idealized functional representations for antenna patterns. In reality these ideal symmetric representations are seldom achieved (See Figures 6.2 and 6.3 , for examples of non-symmetric patferns). As a consequence, to accurately specify the scatterometer integrand recourse to pattern measurements is necessary. The laiter section of this chapter develops the theory which specifies the density of points at which the pattern must be measured to teniquely represent the pattern. This section of the chapter is important in numerically evaluating the inversion parameters in the scatterometer equation.

### 7.2 Description of the Scatterometer Simulation Program

The reader will recall that the inversion technique developed in Chapter 6 was derived without regard to the distinction between surface and antenna polarizations. As a consequence to compute the return power accurately from scattering coefficients defined with respect to the surface polarizations, the scatterometer simulation program was specifically designed to compute the return power on the basis of Equation (4-50) of Chapter 4 rather than Equation (4-29), i.e., with the pattern transformation included. For an antenna pattern and a view angle selected externally to the program, the exact return power is computed for all fiffeen measurements described in Chapter 6.

The inversion of the resulting measurements is performed in two ways. In the first method, called the approximate method, the inversion is performed without regard to the distinction between antenna and surface polarizatians. It is (erroneously) assumed, as in Chapter 6, that the scattering coefficients are expressed in the antenna coordinate system. Equation ( $6-19$ ) served as the inversion model. Since the return power was computed on the basis of the difference between surface and antenna polarization and the inversion was performed without regard to the difference, the distinction befween surface and antenna polarizations could be evaluated. The second method, called the exact method, does not ignore the difference between antenna and surface polarizations. The inversion is based on antenna weights that are computed by transforming the pattern polarization states to the surface polarization states for each of the fifteen measurements. The transformation, in general, "excites" additional scattering coefficients above those recognized in the approximate method (See, for example, Equation (4-49).)

A delta function approximation was also employed in the matrix inversion model. The model was based on an approximation of Equation (4-47) and takes the form

$$
\begin{aligned}
W_{t r}= & \lambda^{2} G_{t} G_{r} W_{t} /(4 \pi z)^{2}\left\{\left.\langle | S_{v v}\right|^{2}\right\rangle \int I_{1} \cos \theta^{2} d \Omega+ \\
& \left.\left.<\left|S_{h h}\right|^{2}\right\rangle \int I_{2} \cos ^{2} d \Omega+\left.\langle | S_{v h}\right|^{2}\right\rangle \int I_{3} \cos \theta^{2} d \Omega+
\end{aligned}
$$

$$
\begin{align*}
& 2 R e<S_{v v^{\prime}} S_{v h}^{*}>\int I_{6} \cos \theta^{2} d \Omega-2 \operatorname{Im}<S_{v v^{\prime}} S_{v h}^{*}>\int I_{7} \cos \theta^{2} d \Omega+ \\
& \left.2 R e<S_{h v} S_{h h}^{*}>\int I_{8} \cos \theta^{2} d \Omega-I m<S_{h v} S_{h h}^{*}>\int I_{g} \cos \theta^{2} d \Omega\right\} \tag{7-1}
\end{align*}
$$

where the Is are defined in Equations (4-50). The resulting fifteen equations are employed in a least squares estimation technique fo recover the scattering coe: icients. The matrix technique was developed to test whether the fifteen measurements were sufficient to invert for the coefficients when the difference in polarizations is recognized.

In addition to specifying the choice of antenna view angle, the program user may, through the use of the input control card, introduce cross pattern amplitude bias and relative phase bias into those measurements employing vertically or horizontally polarized transmissions or receptions. The return power is accurately computed for all fifteen mecsurements with the biases included. The inversions, both approximate and exact methode, are performed, however, without regard to the biases, i.e., they are based on ideal antenna states. The sensitivity of the inversions to paitern deviations from ideal conditions could thus be studied.

Cross pattern amplitude and phase biases have precise meanings for vertically and horizontally polarized transmissions or receptions. However, for those measurements requiring simulationeous vertically polarized and horizontally polarized patterns (eg., LC, RC, linear $\pm 45^{\circ}$ ), it was more meaningful to conduct Monte Carlo studies on amplitude and phase. This technique requires many simulations to be conducted. Each simulation is based on a different set of deviations in amplifude and/or in phase. First and second order error statistics are accumulated from all the experiments conducted in this fashion. In the measurements requiring simultaneous cross patterns, it is evident from Chapter 6 ihat balanced patterns are required, i.e.,

[^7]$g_{v}=g_{h}=1 / 2 \mathrm{~g}$. So it was appropriate to specify the pattern amplitude perturbation in the Monte Carlo studies as a deviation from a balanced condition. For each experiment the amplitude and phase are randomly perturbed within bounds specified by the user. The deviations are based on samples from a uniform distribution so that the perturbed gain and phase satisfy
\[

$$
\begin{align*}
& g_{h}^{1}=1 / 2 g_{h}+A \rho_{g} \\
& g_{v}^{\prime}=1-g_{h}^{\prime}  \tag{7-2}\\
& \beta^{\prime}=\beta+B p_{B}
\end{align*}
$$
\]

where $A<1 / 2,0<\beta<\pi$ and $\rho_{g}$ and $\rho_{\beta}$ are random samples from a population distribute uniformly over $[-1,1], 1 / 2 g_{h}$ and $\beta$ are the ideal gain and phase requirements. Both approximate and exact inversions are performed for each experiment. The error statistics are formed independently for each. The above studies are initiated by specifying $2 A$ and $2 \overline{5}$ on the input control card.

This program allows the selection of one four symmetric antenna patterns. For any selection it is assumed that both dominant and cross patterns have identical functional forms. The relative phase between the patterns (if both exist) was assumed stationary. When amplitude error is introduced into any one of the fifteen measurements, the deviation is applied so that the normalized gains satisfy $\mathrm{g}_{\mathrm{v}}(0)+\mathrm{g}_{\mathrm{h}}(0)=1$ on the boresight axis. The specific pattern options are given by the following functions:

$$
\begin{align*}
& p_{1}=(\sin x / x)^{2} \\
& p_{2}=\left(J_{1}(x) / x\right)^{2} \\
& p_{3}=\left(\frac{3}{x^{2}}\left(\frac{\sin x}{x}-\cos x\right)\right)^{2}  \tag{7-3}\\
& p_{4}=\frac{J_{2}(x)}{x^{2}}
\end{align*}
$$

where

$$
\begin{aligned}
& x=k a \sin \theta^{s} \\
& a=\text { aperiure radius } \\
& k=2 \pi / \lambda
\end{aligned}
$$

The above pattern functions correspond to one-way patferns having respective side lobe levels of $-13.2,-17.6,-20.6$ and -24.6 dB . In addition to providing a choice in paifern functions, the program requires an input parameter denoted as ka to control the beamwidth. The beamwidth for the respective patferns are related to ka by the following expressions:

$$
\begin{align*}
& \Delta \theta_{1}=0.88 \pi / \mathrm{ka} \\
& \Delta \theta_{2}=1.02 \pi / \mathrm{ka}  \tag{7-4}\\
& \Delta \theta_{3}=1.15 \pi / \mathrm{ka} \\
& \Delta \theta_{4}=1.27 \pi / \mathrm{ka}
\end{align*}
$$

For a fuller understanding of the pattern functions the reader is referred to pages 9.149.21 of reference [5].

The scattering characteristics on which the simulations were conducted are illustrated in Figure 7.1. The coefficients except for the real and imaginary parts of $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\text {hv }}{ }^{*}\right\rangle$ are based on theoretical results reported in reference [10] . The magrifude of $\left\langle\mathrm{S}_{\mathrm{vV}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ was set at the geometric mean of $\left.\left.\langle | \mathrm{S}_{\mathrm{vv}}\right|^{2}\right\rangle$ and $\left.\left.\langle | S_{h h}\right|^{2}\right\rangle$ in accordance with the results of Chapter 5. The phase characteristic of $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ was assigned to be that for small perturbation theory for a sea water temperature of $293^{\circ}$. The characteristios are similar to that of the sea under low wind conditions. In accordance with small perturbation theory the coefficients $\left\langle\mathrm{S}_{\mathrm{Vv}} \mathrm{S}_{\mathrm{hv}}{ }^{*}\right\rangle$ and $\left\langle\mathrm{S}_{\mathrm{vh}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ are extremely small. For the sake of the simulations weak but identical characteristics were arbitrarily assigned to the real and imaginary parts of these coefficients. All characteristics were assumed isofropic。

For a complefe description of the scatterometer simulation program the reader is referred to Appendix D.


### 7.3 Resolution Requirement

### 7.3.1 General

Angular resolution in scatterometry has been achieved either by employing a narrow beam or by doppler filtering or by a combination of both. Angular resolution is clearly required to search the scattering characteristic. It is also required to realize the delta function approximation in the inversion technique. It has been common practice to specify the resolution on the basis of some notion of the scattering character istic. However, when the difference between surface and antenna polarizations is an important consideration, a resolution guideline can also be established to assure that the antenna polarization coincides with the surface polarization over the significant portion of the beam. An expression is developed showing the percent power incident on the surface in the orthogonal surface polarization for an antenna whose polarization is pure with respect to the antenna frome. The results can be inferpreted in terms of resolution (beamividth).

Resolution requirements are also established for the assumed scattering character istics by employing the simulation program. The result expresses the measurement accuracy achieved by the delfa function approximation with ideal antenna polarization specifications.

### 7.3.2 Polarization Decomposition of the Incident Beam

Suppose a scatterometer transmits a horizontally polarized wave $\mathrm{E}_{\not 01}$ when pointed in direction $\theta_{0}$. The total power incident on the surface is given by

$$
\begin{equation*}
P^{\prime}=\frac{1}{2 Z_{o}} \int\left|E_{\phi^{\prime}}\right|^{2} \mathrm{~d} \Omega \tag{7-5}
\end{equation*}
$$

When $\mathrm{E}_{\emptyset}$ is decomposed into orthogonal surface components, the above expressions can be written by

$$
P=\frac{1}{2 Z_{0}} \int\left|E_{\phi^{\prime}}\right|^{2}\left[\left|i_{\theta} \cdot i_{\phi^{\prime}}\right|^{2}+\left|i_{\phi} \cdot i_{\phi^{\prime}}\right|^{2}\right] d \Omega
$$

The percent. power appearing in the orthogonal surface polarization is given by

$$
\begin{equation*}
\left(\frac{P_{\theta}}{\rho}\right) \%=100 \int \frac{\left|E_{\phi^{\prime}}\right|^{2}\left|i_{\theta} \cdot i_{\phi^{\prime}}\right|^{2}}{P} d \Omega \tag{7-7}
\end{equation*}
$$

or

$$
\left(\frac{p_{\theta}}{p}\right) \%=100\left[1-\int \frac{\left|E_{\phi}\right|^{2}\left|i_{\phi} \cdot i_{\phi^{\prime}}\right|^{2}}{p} d \Omega\right]
$$

The lafter expression is simpler to evaluate numerically.
The above expression was evaluated as a function of view-angle for various beamwidths. A Jinc pattern function was employed in the computation. The results of the evaluation are shown in the graphs of Figure 7.2. The polarization mis-match as anticipated from Chapter 5 is greatest at nadir regardless of beamwidth. It is evident that small beamwidths are able to probe closer to nadir without introducing significant orthogonally polarized components. The permissable level of orthogonal polarization will be treated in a subsequent section. Although the above results were based on a horizontally polarized incident wave, a similar result could heve been computed for a vertically polarized incident wave.

If one chooses to avoid transforming the pattern polarization states to the sur face and accurate measurements of the surface scattering coefficient are desired near nadir, then the graphs of Figure 7.2 are helpful in choosing the proper beamwidth. If the experiment requires that the cross polarized content be less than, say, -20 dB , then the $1 \%$ ordinant will specify how close one can probe nadir with various beamwidihs.

An alternative to the above procedure is to employ the exact inversion model based on the differences between antenna and surface polarizations. The delta function accuracy of this technique for small angles is developed in the succeeding section.

### 7.3.3 An Evaluation of the Delta Function Approximation

To determine the beamwidth (resolution) requirement to realize the delfa function approximation,scatterometer simulations were conducted in the vicinity of nadir where angular resolution is required to search the rapidly varying scattering characteristics. The ability of the delta function approximation to retrieve each scattering coefficient was established at incident angles of $0^{\circ}, 4^{\circ}$ and $8^{\circ}$. Bearnwidths from 1 degree to 12 degrees were considered. The results are illustrated in the graphs of Figures 7.3 and Figures 7.5 through 7.8 for both the approximate and exact methods.

The performance of the delta function approximation at nadir is shown in Figure 7.3 for the approximate method. It is evident that there is liftle difficulty in retrieving $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{vv}}{ }^{*}\right\rangle,\left\langle\mathrm{S}_{\mathrm{hh}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ and $\operatorname{Re}\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ except for beamwidths in excess of 10 degrees. The degradation at large beamwidths is, of course, the result of


FIGURE 7.2 DEPOLARIZATION OF THE INCIEDNT BEAM as induced by the difference between the antenna AND SURFACE POLARIZATIONS


FIGURE 7.3 ACCURACY OF THE DELTA FUNCTION APPROXIMATION FOR THE APPROXIMATE INVERSION MODEL FOR $\theta=0^{\circ}$
the antenna beamw dth inferacting with the scattering surface "beamwidth". The beamwidth interaction problem is clearly evident in the error characteristic of $\operatorname{Im}\left\langle\mathrm{S}_{\mathrm{Vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$. Since this coefficient has a notch character at nadir, it is impossible for any non-zero beamwidth antenna to retrieve this paramefer.

Unusual error performances are apparent in retrieving $\left.\left.\langle | S_{v h}\right|^{2}\right\rangle, \operatorname{Re}\left\langle S_{v v} S_{h v}{ }^{*}\right\rangle$ and $\left.\mathrm{Re}<\mathrm{S}_{\mathrm{vh}} \mathrm{S}_{\mathrm{hh}^{*}}{ }^{*}\right\rangle$. A constant $50 \%$ error occurs for $\left\langle\mid \mathrm{S}_{\mathrm{vh}}{ }^{2}\right\rangle$ regardless of beamwidth; whereas a $100 \%$ error occurs for the léter two parameters. An explanation for the error in $\left.\left.\langle | \mathrm{S}_{\mathrm{vh}}\right|^{2}\right\rangle$ can be construcied soiely on the basis of the difference between antenna and surface polarizations. A similar explanation is thought to apply to the other two parameters, although no quantitative argument could be constructed. The error in $\left.\left.\langle | \mathrm{S}_{\mathrm{vh}}\right|^{2}\right\rangle$ ean be best understood when the antenna and surface polarizations are projected on the surface. The surface polarizations will project as a polar grid whereas the antenna polarizations will project roughly as a rectangular grid as illustrated in Figure 7.4. From these diagrams it is understood that when a vertically polarized spherical wave is incident on the surface, half the pawer appears in the surface vertical polarization and the other half in the surface horizontal polarization. As shown in the accompanying decomposition diagram both incident components are depolarized by the surface and upon their return to the anfenna each depolarized component is transformed (T) back to the antenna polarizations. Upon fransforming back to the antenna polar izotions, one half of each depolarized component is transformed into the antenna horizontally polarized state. As a result, the inversion based on the antenna polarizations is $50 \%$ low. This result indicates that it is futile to recover $\left.\left.\langle | S_{\mathrm{vh}}\right|^{2}\right\rangle$ as defined with respect to surface polarizations with a recovery technique based on the antenna polar izarions.

It is also informative to examine the power structure for a cress-polarized measurement. The third row of Table 7.1 shows how the returi power is distribuied among the scatfering coefficients for a cross-polarized measurement. A sizeable contribution arises from the polarized coefficients (columns 1 and 2); however, the sum of those components is fortunately cancelled by the contribution from $\mathrm{Re}\left\langle S_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{\text {h }}\right.$ (column 4). The cancellation is assured by the isotropic character assumed for the surface.

For the same incident angle no results can be reported for the exact method. For the nadir angle the observation marrix is singular. The singularity is plainly evident in the observation matrix as shown in Table 7.2. The reader will observe that the

A) PROJECTION OF AND ANTENNA AND SURFACE POLARIZATIONS

$$
\begin{aligned}
& \rightarrow 1 / 2 \mathrm{H}_{s} \longrightarrow 1 / 2\left\langle\mathrm{~S}_{\mathrm{h}} \mathrm{l}^{2} \mathrm{H}_{\mathrm{s}} \longrightarrow 1 / 4\left\langle\mathrm{~S}_{\mathrm{hv}}\right|{ }^{2} \mathrm{H}_{\mathrm{s}}=\mathrm{V}_{\mathrm{a}}\right.
\end{aligned}
$$

B) DECOMPOSITION DIAGRAM

FIGURE 7.4 COMPARISION OF ANTENNA AND SURFACE POLARIZATIONS AT NADIR WITH DECOMPOSITION DIAGRAM TO EXPLAIN CROSS POLARIZED MEASUREMENTS


TABLE 7.1 POWER COMPOSITION MATRIX FOR A NADIR MEASUREMENT

| HEAS/COEF | $F$ W | HH | UH | V VHITR | VUHHI | UWUHR | VUVHI | HyHHR | HUHHT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 0.1654E-01 | 0.1654E-01 | 0.2205E-01 | 0.1103E-01 | 0. | 0. | 0. | 0. | 0. |
| 2 | $0.1654 E-01$ | 0.1654F-01 | 0.2205E-01 | 0.1103E-01 | 0. | 0. | 0. | 0. | 0. |
| 3 | $0.5513 \mathrm{E}=02$ | 0.5513E-02 | 0.2205E-02 | -0.1103E-01 | 0. | 0. | 0. | 0. | 0. |
| 4 | 0.110.3E-01 | 0.1103E-01 | 0. $5725 \mathrm{~F}-08$ | $0.2205 E=01$ | -0.2968E-I | -0.E856E-15 | 0. | -D.5f: E-15 | 0. |
| 5 | 0.55 $13 \mathrm{E}-02$ | 0.E513E-02 | 0.2205E-01 | -0.1103E-01 | 0.5491E=15 | 0. | -0.4376E=10 | 0. | -0.4376E-10 |
| 6 | $0.8270 \mathrm{E}-02$ | 0.8270E-02 | B.1103E-01 | $0.5513 E-02$ | -0.2969E-06 | 0. | 0. | 0. | 0. |
| 7 | 0.8270E-02 | 0. $2270 E=02$ | 0.1103E-01 | 0.5513E-02 | $0.2969 \mathrm{E}-06$ | 0. | 0. | 0. | 0. |
| ${ }^{6}$ | $0.1103 \mathrm{E}=01$ | $0.1103 \mathrm{E}-01$ | 0.2205E-01 | 0. | 0. | -0.2978ㄷ-06 | 0. | -0.2924E-06 | 0. |
| 9 | $0 \cdot 1103 \mathrm{E}-31$ | 0.1103E-01 | 0.2205F-01 | 0. | 0. | $0.297 \mathrm{dE}-06$ | $-4.4376 \mathrm{E}-10$ | D. 2924 E -06 | -0.4376F-10 |
| 10 | $0.1103 \mathrm{E}-01$ | 0.1103E-01 | 0.2205F-02 | 0. | 0. | -0.2940E-15 | $0.2205 E-01$ | -0.2886F-15 | $0.2205 \mathrm{E}-01$ |
| 11 | $0.1103 \mathrm{E}-01$ | $0.1103 F=01$ | 0.2205E-01 | 0. | 0. | -0.2971F-15 | -0.2205E-01 | -0.2915E-15 | -0.2205E-01 |
| 12 | F. 1103E-01 | 0.1104E-01 | 0.2205E-01 | 0. | 0. | -0.2924E-06 | 0. | -0.2978E-06 | 0. |
| 13 | 0.1103E-01 | 0.1103E-01 | 0.2205F-01 | 0. | 0. | 0.2924E-06 | -0.4376E-10 | 0. $2578 \mathrm{BE}-06$ | -0.4376E-10 |
| 14 | $0.1103 \mathrm{E}-0.1$ | 0.1103E-01 | 0.2205E-0: | 0. | 0. | -0.2886E-15 | 0.2205E-01 | -0.2940E-15 | 0.2205E-01 |
| 15 | 0.1103E-01 | $0.1103 \mathrm{E}=01$ | 0.2205E-0 | 0. | 0. | -0.2425E-15 | -0.2205E-01 | -0.2971E-15 | -0.2205E-01 |

TABLE 7.2 OBSERVATION MATRIX BASED ON SURFACE POLARIZATIONS
following pairs of observations are identical: 1 and 2,3 and 5, 6 and 7,8 and 12, 9 and 13,10 and 14 und 11 and 15 . The rank of the matrix is consequently 8 . For isotropic scenes the singularity may be removed by solving the system of measurements subject to the constraints: $\left.\left.\left.\langle | \mathrm{S}_{\mathrm{Vv}}\right|^{2}\right\rangle=\left.\langle | \mathrm{S}_{h h}\right|^{2}\right\rangle,\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hv}}{ }^{*}\right\rangle=\left\langle\mathrm{S}_{\mathrm{vh}} \mathrm{S}_{h h^{*}}\right\rangle$, and $\left\langle S_{v v} S_{h h}{ }^{*}\right\rangle=\sqrt{\left.\left.\left.\langle | S_{v v}\right|^{2 \cdot}\right\rangle\left.\langle | S_{h h}\right|^{2}\right\rangle}$. As a result of the constraint there are only five independent parameters. This result was anticipated from the "Gedanken Experimente ${ }^{\text {" }}$ referenced in Chaprer 3.

In general, retrieval of the scattering coefficients at nadir is a difficult task, it is merely coincidence that the approximate method yielded as many accurafe estimates as it did. If a scene is anisotropic or if if has a peculiar character where $\left|<S_{v v} S_{h h}{ }^{*}\right|^{2}$ $\left\langle\left.\langle | S_{v v^{\prime}}\right|^{2}\right\rangle\left\langle\left. S_{h h}\right|^{2}\right\rangle$, then there is no assurance that either method will work.
 important role in forming the polarized measurements. Careful investigations at nadir will require very narrow beams to search nadir asymptotically if the coefficients are to be reported with respect to the surface polarizations.

The accuracies of the delta function approximation for the approximate and exact models at a view ongle of $4^{\circ}$ are shown, Figures 7.5 and 7.6 , respectively. From Figure 7.6 it is apparent that the approximate method can be employed with reasonable accuracy $(0.5 \mathrm{~dB})$ to retrieve all coefficients if the beamwidth is less than $3^{\circ}$. A beamwidth as much as $10^{\circ}$ can be tolerated for the recovery is restricted to the polarized coefficients. The exact inversion method will permit beamwidths up to $9^{\circ}$ in retrieving all the scattering coefficients. Similar results are apparent in the error characteristics for a view angle of $8^{\circ}$ (See Figures 7.7 and 7.8).

### 7.4 Antenng Requirements for the Accurate Recovery of the Scattering Coefficients

### 7.4.1 General

A number of simulations were conducted at various incident angles with and without biases and also with and without random perturbations introduced into the measurement. These simulations served as a training set to identify the particular scattering coefficient or coefficients which primarily contributed to the error character istic for each scattering coefficient. Invariably the best single parameter to which the error could be attributed was the magnitude of $\left\langle S_{\mathrm{vv}^{\prime}} \mathrm{S}_{\mathrm{h}}{ }^{*}\right\rangle$. The magnitude of this parameter conveys a notion of the size of $\left.\left.\langle | S_{v v}\right|^{2 v}\right\rangle$ and $\left\langle\mid S_{h} h^{2}\right\rangle a s$ well. These three coefficients generally interacted to introduce an error in the measurement when


FIGURE 7.5 ACCURACY OF THE DELTA FUICTION APPROXIMATION FOR THE APPROXIMATE INVERSION MODEL FOR $\theta=4^{\circ}$


FIGURE 7.6 ACCURACY OF THE DELTA FUNCTION APPROXIMATION FOR THE EXACT INVERSION MODEL
FOR $\theta=4^{\circ}$


FIGURE 7.7 ACCURACY OF THE DELTA FUNCTION APPROXIMATION FOR THE APPROXIMATE INVERSION
FOR $\theta=8^{\circ}$


FIGURE 7.8 ACCURACY OF THE DELTA FUNCTION APPROXIMATION FOR THE EXACT INVERSION MODEL
FOR $\theta=8^{\circ}$.
the antenna transmission and reception properties deviated from the ideal state specified in Chapter 6.

The training set also made it apparent that the error characteristics were primarily governed by the level of the cross'polarized leakage for those measurements involving linearily polarized inansmission or reception. The level of the leakage is expressed in terms of one-way depression relative to the dominant pattern. The relative phase between the dominant and leakage patterns was treated as an independent error parameter. A bias error study was, therefore, applied to the retrieval of $\left.\left\langle S_{v v} S_{h h}{ }^{*}\right\rangle,\left.\langle | S_{h h}\right|^{2}\right\rangle$, $\left\langle S_{v v} S_{h v}{ }^{*}\right\rangle$ and $\left\langle S_{\mathrm{vh}^{\prime}} S_{h h}{ }^{*}{ }^{*}\right.$. Although the latter two coefficients involve balanced cross patterns during reception, studies showed that the error performance was largely insensitive to small deviations from a balanced condition small deviations from the required phase condition in comparison to leakage appearing in the linearily polarized transmission. Now in the case of $\left\langle\mathrm{S}_{\left.\mathrm{vv} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle \text {, it is more meaningful to consider Monte }}\right.$ Carlo studies since both transmissions and receptions involve balanced cross patterns. All simulations were conducted for a one degree beam having a $\left(2 J_{1}(x) / x\right)^{2}$ paftern. The resulting error characteristics apply equally as well to approximate or exact inversion methods. When translating the performance to small incident angles where the antenna and surface polarizations differ significantly across the beam, then one must assume that the inversions had been performed by the exact method. The graphs of Figure 7.2 serve as a guide as to when the matrix method must be used. Simulations with the other pattern functions yielded similar results and so are not reported.

### 7.4.2 Error Characteristics

The error characteristics for the recovery of $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{vv}}{ }^{*}\right\rangle$ are shown in Figures 7.9 and 7.10. The results are shown for two phase conditions in which $\beta_{t}=\beta_{5}=0^{\circ}$ and $\beta_{t}=\beta_{r}=90^{\circ}$, respectively. These two conditions result in extremal error characteristics in which the maximum error results from one phase condition and a minimum error from the other condition. The extremes are induced by a sign change in the contribution from $\operatorname{Re}<\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}>$, a dominant parameter. As shown by Figure 7.9 there is no difficulty in retrieving the dominant scattering coefficient $\left.\left.\langle | S_{\mathrm{vv}}\right|^{2}\right\rangle$ except for a cross polarized pattern less than 10 dB beneath the vertical polarized patfern and a large separation between $\left.\left.\langle | S_{v}\right|^{2}\right\rangle$ and $\left.\langle | S_{h h}\right|^{2}$. The weakened dominant pattern results in less refurn power from $\left.\langle | S_{\mathrm{VV}}\right|^{2}$. A further reduction occurs when $\left.\langle | S_{h h^{\prime}}\right|^{2}>$ and $<\mathrm{S}_{\mathrm{w}} \mathrm{S}_{h h^{*}}{ }^{*}$ are significantly weaker than $\left.\left.\langle | \mathrm{S}_{\mathrm{w}}\right|^{2}\right\rangle$. A similar result occurs when $\beta_{t}=\beta_{r}=90^{\circ}$ (Figure 7.10). The error is slightly larger because the coefficient


FIGURE 7.9 ERROR CHARACTERISTICS OF $\left.\left.\langle | \mathrm{S}_{\mathrm{vv}}\right|^{2}\right\rangle$
FOR VARIOUS ANTENNA CROSS POLARIZATION ISOLATIONS
WITH PHASE CONDITION $\beta_{T}=\beta_{R}=0^{\circ}$.


FIGURE 7.10 ERROR CHARACTERISTICS OF $\left.\left.\langle | S_{\mathrm{Vv}}\right|^{2}\right\rangle$ FOR various fintenna cross polarization isllations with A.PFASE CONIITION OF $\beta_{T}=\beta_{R}=90^{\circ}$.
$\mathrm{Re}\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{Sh}{ }^{*}\right\rangle$ causes a "negative" power contribution, resulting in an even smaller resultant power. Recall that $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ responds to the product pattern $\sqrt{g_{\mathrm{v}} \mathrm{g}_{\mathrm{h}}}$ and its sign is controlled by the sum $\beta_{\dagger}+\beta_{\mathrm{r}}$ (See Equation 4-29).

The retrieval performance, when attempting measurements of $\left.\left.\langle | S_{h h}\right|^{2}\right\rangle$, is illustrated in Figures 7.11 and 7.12 for pattern phase conditions corresponding to $\beta_{\dagger}=$ $\beta_{r}=0^{\circ}$ and $\beta_{t}=\beta_{r}=90^{\circ}$, respectively. Since $\left.\left.\langle | S_{h h}\right|^{2}\right\rangle$ is generally less than or equal to $\left.\langle | S_{\mathrm{yv}}\right|^{2}>$, one can anticipate a poorer error characteristic. For the case where $\beta_{t}=\beta_{r}=0^{\circ}$, the ability to recover $\left.\left.\langle | S_{h h}\right|^{2}\right\rangle$ is shown to be strongly dependent on its separai. . on from $\left|\left\langle S_{v v} S_{\text {hh }}{ }^{*}\right\rangle\right|$. Positive power contributions are made by both $\left.\left.\langle | S_{\mathrm{vv}}\right|^{2}\right\rangle$ and $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$. The resultant power in this case is excessive. When $\beta_{\mathrm{f}}=$ $\beta=90^{\circ}$, the contributions by $\left\langle S_{v v} \mathrm{~S}_{\mathrm{hh}}{ }^{*}\right\rangle$ is negative and partially cancels the $\left\langle\bar{S}_{\mathrm{vv}}{ }^{5}{ }_{\mathrm{vv}}{ }^{*}\right\rangle$ contribution. As a consequence, one may suspect that the letter phase condition yields a slightly better error characteristic. Comparison of Figures 7.11 and 7.12 demonstrates that this is the case. From either graph it is observed that when $\left.\left.\langle | S_{h h}\right|^{2}\right\rangle$ is 10 dB lower than $\left|\left\langle S_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle\right|$, the antenna cross polarization level must be less than -30 dB for a error less than 0.5 dB . When the separation is 5 dB , the antenna cross polarization level must be better than $\mathbf{- 2 0}$ dB. The latter is probably representative of the sea for angles of incidence up to $70^{\circ}$.

The error characteristics for retrieving $\left.\left.\langle | S_{\mathrm{vh}}\right|^{2}\right\rangle$ for the same two relative phase conditions are shown in Figures 7.13 and 7.14. From Figure 7.13 it is apparent that the weakness of the scattering coefficient in the presence of cross-leakage makes it very difficult to isolate. The ability to measure $\left.\left.\langle | S_{v h}\right|^{2}\right\rangle$ is shown to depend strongly on its depression from the polarized scattering coefficients as conveyed parametrically by $\left|<S_{v v} S_{h h}{ }^{*}\right\rangle \mid$. Figure 7.13 represents a worst case situation in which all the dominant coefficients to include $\operatorname{Re}\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\text {hh }}{ }^{*}\right\rangle$ make positive contributions to the return power. This situation is consequently useful for formulating a criteria for accurate measurement of $\left.\left.\langle | \mathrm{s}_{\mathrm{vh}}\right|^{2}\right\rangle$. It has been common practice to judge the ability of an anfenna to measure cross-polarized coefficients by its one-way and in some cases by its two-way isolation in comparison to the separation between the polarized and crosspolarized coefficients. The graphs of Figure 7.13 show explicit the antenna requirement. If a 0.5 dB accuracy is desired and if $\left.\left.\langle | \mathrm{S}_{\mathrm{vh}}\right|^{\mid}\right\rangle$lies $X \mathrm{~dB}$ beneath $\left|\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle\right|$, then approximately $X+16 \mathrm{~dB}$ one-way isolation is required. The above result indicates that one must not only consider the polarized coefficients in making a judgement on an antenna but a complex valued coefficient must also be considered.


FIGURE 7.11 ERROR CHARACTERISTICS FOR $\left.\left.\langle | S_{H H}\right|^{2}\right\rangle$ AT VARIOUS LEVELS OF PATTERN CROSS POLARIZATION FOR AN ANTENNA PHASE CONDITION $\beta_{T}=\beta_{R}=0^{\circ}$.


FIGURE 7:12 ERROR CHARACTERISTICS FOR < $\left|\mathrm{S}_{\mathrm{HH}}\right|^{2}>$ AT various levels of pattern cross polarization for Ais ANTENMA PHASE CONDITION $\beta_{T}=\beta_{R}=90^{\circ}$


FIGURE 7:13 ERROR CHARACTERISTICS FOR $\left\langle\left\langle\left.\mathrm{S}_{\mathrm{VH}}\right|^{2}\right\rangle\right.$ AT Various levels of pattern cross polarization for an ANTENNA PHASE CONDITION $\beta_{T}=\beta_{R}=0^{\circ}$


FIGURE 7:14 ERROR CHARACTERISTICS FOR $\left.\langle | \mathrm{S}_{\mathrm{VH}}\right|^{2}>$ AT VARIOUS LEVELS OF PATTERN CROSS POLARIZATION FOR AN ANTENNA PHASE CONDITION $\beta_{T}=\beta_{R}=90^{\circ}$

It is apparent from the graphs of Figure 7.14 that if the phase of the leakage pattern can be adjusted to $90^{\circ}$ during transmission and reception, the level of leakage is almost immaterial. In this case contributions by $\left.\left.\langle | \mathrm{S}_{\mathrm{vv}}\right|^{2}\right\rangle$ and $\left.\left.\langle | \mathrm{s}_{\mathrm{hh}}\right|^{2}\right\rangle$ are almost entirely cancelled by the confribution from $\left.\mathrm{Re}<\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$. These results show that if the phase of the cross leakage can be adjusted for $\beta_{t}=\beta_{r}=90^{\circ}$, the stringent requirements on the cross pattern amplitude can be relaxed.

The error characteristics for $\mathrm{Re}\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{h \mathrm{~h}}{ }^{*}>\right.$ and $\mathrm{Im}\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{h h}{ }^{*}\right\rangle$ are shown in Figure 7.15. Monte Carlo studies were performed to construct this characteristic. The random deviations in amplitude (from balance) and in phase were uniformly distributed. Maximum deviations are indicated on the graphs. It is apparent that the real part of $<\mathrm{S}_{\mathrm{wv}} \mathrm{S}_{\mathrm{hh}}{ }^{*}$ > is easy to recover. Phase perturbations have little effect on the accuracy. The recovery of the imaginary part appears to be more difficult; but this is mainly a result of its weak response in comparison to $\left.\left.\langle | s_{\mathrm{vv}}\right|^{2}\right\rangle$ and $\left.\left.\langle | \mathrm{s}_{\mathrm{hh}}\right|^{2}\right\rangle$.

The error characteristics for the cross-correlation coefficients $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hv}}{ }^{*}>\right.$ and $\left\langle\mathrm{vh} \mathrm{S}_{\mathrm{hh}}{ }^{*}{ }^{*}\right.$ are all shown in the graphs of Figure 7.16. Both extremal phase conditions are superimposed on the same plot. The graphs show that the real parts of the coefficients are difficult to retrieve if $\beta_{t}=\beta_{r}=0$. Similarly, the imaginary parts are difficult to retrieve if $\beta_{q}=\beta_{r}=90^{\circ}$. On the otherhand, the imaginary part and the real parts are easily recovered if $\beta_{t}=\beta_{r}=0^{\circ}$ and $\beta_{t}=\beta_{r}=90^{\circ}$, respectively. The graphs also indicate that the ability to retrieve the coefficients is dependent upon the separation of the coefficients from the real or imaginary part of $\left\langle\mathrm{S}_{\mathrm{vv}}{ }^{5} \mathrm{hh}^{*}\right\rangle$. It is again evident if the correct phase property is employed that a reasonable accuracy can be anticipared.

### 7.4.3 Alternatives

When the ideal antenna states as specified in Chs er 6 cannot be approximated ressonably and if as a consequence significant error is introduced into the measurements, the experimenter has recourse to specifying the complete antenna polarization states he is able to achieve. As long as he approaches the desired states and performs an adequate number of measurements, he can be reasonably assured that inversions based on the complete scatterometer equation will yield improvements in the estimates of the coefficients. The inversion model should be tested to determine whether his system of mecsurements is well conditioned. At least nine measurements must be performed unless one has prior knowledge that some of the coefficients are negligible. In this technique one must reconcile with making at least nine measurements; whereas if the beamwidth constraint


TIGURE 7.15 ERROR CHARACTERISTICS FOR $\left\langle S_{V V^{*}} S_{H H}^{*}\right\rangle$ AS IIF.PENDENT ON UNCERTAINTY IN PATTERN BALANCE AND PHASE

is met, a single seattering parameter can be recovered with at most two measurements if the antenna specification can be realized.

### 7.5 Evaluation of the Inversion Parameters

### 7.5.1 Introduction

Essential to accurate inversion of scatterometric measurements is the knowledge of the actual antenna pattern. To form the integral weights for each scattering $\mathrm{co}^{-}$ efficient, the pattern and phase functions must be numerically integrated over the main beam and perhaps the first side lobes. Since pattern information is seldom available in functional form, one is dependent on measurements. In measuring the patfern, the question arises as to what sampling density is required to adequately specify the pattern. The sample requirement is derived on the basis of simple aperture theory. The results of the theory are applied to the SKYLAB S-193 antenna fo illustrate the sampling requirement.

### 7.5.2 Derivation of the Pattern Spectrum

. It is well known that the far field E of an aperture type anfenna is related to the aperture illumination function, $A(x, y)$, through an inverse Fourier transform relationship [19]

$$
E(r, \theta, \phi)=k_{0} \iint_{-\infty}^{\infty} A(x, y) \exp \left[j\left(k_{x} x+k_{y} y\right)\right] d x d y
$$

where

$$
\begin{aligned}
& K_{o}=(j / \lambda r) \exp (-j k r) \\
& k_{x}=k \sin \theta \cos \phi \\
& k_{y}=k \sin \theta \sin \phi \\
& k=2 \pi / \lambda
\end{aligned}
$$

The relationship is considered valid for spherical polar angles $\theta$ satisfying $\cos \theta \geq 0.9$. $A$ is assumed to be a real function* so that the main beam of the antenna is located about the positive $z$ axis. Now it is convenient to rewrite the above expression in the form

$$
\begin{equation*}
E\left(r, f_{\xi}, f_{\eta}\right)=K_{1} \iint A(\xi, \eta) \exp j 2 \pi\left(f_{\xi} \xi+f_{\eta} \eta\right) d \xi d \eta \tag{7-1i}
\end{equation*}
$$

[^8]where
\[

$$
\begin{align*}
& \mathrm{f}_{\xi}=\sin \theta \cos \emptyset \\
& \mathrm{f}_{\eta}=\sin \theta \sin \phi  \tag{7-12}\\
& \xi=x / \lambda \\
& \dot{\eta}=y / \lambda \\
& K_{1}=(j \lambda / r) \exp (-j k r)
\end{align*}
$$
\]

The far field power pattern $P$ is given by

$$
\begin{equation*}
P\left(f_{\xi}, f_{\eta}\right)=K_{2} E E^{*} \tag{7-13}
\end{equation*}
$$

where $K_{2}$ is a suitable constant. The Fourier spectrum of ${ }^{\beta}$ is given by

$$
\begin{equation*}
\nexists[P]=K_{2} \nexists\left[E E^{*}\right] \tag{7-14}
\end{equation*}
$$

or

$$
\begin{equation*}
=K_{2} A * A \tag{7-15}
\end{equation*}
$$

where * is the autocorrelation operator. Specifically

$$
\begin{equation*}
7[P]=K_{2} \iint A(\xi, \eta) A(\xi+\alpha, \eta+\beta) d \xi d \eta \tag{7-16}
\end{equation*}
$$

and implies that the spectrum of the power pattern is proportional to the autocorrelation of the aperture distribution and is therefore band limited for finite apertures.

For circularly symmetric aperture distribution a similar theory could have been derived if the initial expression had been transformed to the Bessel-Fourier integral. However, seldom are aperfure distribution circularly symmefric, as a consequence, we have a more general result.

### 7.5.3 Sampling Requirement

Now suppose that an aperture has maximum length $x_{0}$ and maximum height $\gamma_{0}$. From the above result and the illustration in Figure 7.17 , it is clear that the spectrum of $P$ is restricted to the product domain $\left(x_{\sigma} / \lambda,-x_{0} / \lambda\right) \times\left(y_{\sigma} / \lambda_{i}-y_{\sigma} / \lambda\right)$. By the sampling theorem, the pattern can be specified uniquely if samples are taken at

$$
\begin{equation*}
\left(f_{\xi}, f_{\eta}\right)=\left(\frac{m \lambda}{2 x_{0}}, \frac{n \lambda}{2 y_{0}}\right) \tag{7-17}
\end{equation*}
$$



FIGURE 7.17 THE DOMAIN OF INTEGRATION FOR THE AUTOCORRELATION FUNCTION
where $m_{r} n \in \ldots,-2,-1,0,1,2, \ldots$. The above result can be written in terms of $\theta$ and $\emptyset$ by means of (Equation 7-14). Specifically

$$
\begin{equation*}
\sin \theta \cos \phi=\frac{m \lambda}{2 x_{0}} \tag{7-18}
\end{equation*}
$$

and

$$
\begin{equation*}
\sin \theta \sin \phi=\frac{n \lambda}{2 y_{0}} \tag{7-19}
\end{equation*}
$$

As can be easily shown the above relationships require that the antenna paftern be sampled at points ( $\theta_{\mathrm{mn}}{ }^{\prime} \phi_{\mathrm{mn}}^{\prime}$ ) satisfying

$$
\begin{align*}
\theta_{m n} & =\sin ^{-1}\left[\frac{\lambda}{2}\left(\frac{m^{2}}{x_{0}^{2}}+\frac{n^{2}}{y_{0}^{2}}\right)^{\frac{1}{2}}\right]  \tag{7-20}\\
\phi_{\operatorname{mn}} & =\tan ^{-1} \frac{n}{m} \frac{x_{0}}{y_{0}} \tag{7-21}
\end{align*}
$$

In the principal planes the above sampling requirements reduce to

$$
\begin{equation*}
\theta_{m 0}=\sin ^{-1} \frac{m \lambda}{2 x_{0}} \tag{7-22}
\end{equation*}
$$

in the " $x$ " plane and

$$
\begin{equation*}
\theta_{\text {on }}=\sin ^{-1} \frac{n \lambda}{2 y_{0}} \tag{7-23}
\end{equation*}
$$

in the " $y$ " plane. Between the planes in the pattern must be sampled in accord with Equations (7-22) and (7-23)。.

### 7.5.4 Illustration

To develop an understanding of the sample requirement, Equations (7-20) and ( $7-21$ ) were evaluated for an aperture having a maximum dimension of 1.12 meters in the $x$ as well as the $y$ dimension and illuminated at 13.9 GHz . The sampling points for one quadrant out to approximately seven degrees in theta is illustrated in Figure 7.18. Sampling in the remaining quadrants is performed in an identical fashion. It is noted that the sampling array forms a square matrix in polar coordinates where theta is represented as the polar radius and phi as the polar angle.


FIgure 7.18 SAMPLing points for a square aperture 1.1 METERS BY 1.1 METERS OPERATING AT A WAVELENGTH OF 2.16 CENTIMETERS

The above results are representative of the sampling requirement for the S-193 antenna [12]. Although, since the physice! aperture was under-illuminated, the above result is a very conservative sampling density. A description of a program which computes the sampling points when given the aperture dimensions appears in Appendix E .

### 8.0 CONCLUSIONS AND RECOMMENDATIONS

### 8.1 General

The scatterometer equation was derived for scenes whose mean plane is flat and for an antenna having an arbitrary polarization. Ten scattering coefficients were identified for scenes not satisfying reciprocity and six were identified for scenes satisfying reciprocity. Some of the scattering coefficients were demonsirated to be complex valued and were shown to impart a relative phase between the vertically and horizontally polarized scattered components. As a result of the complex valued coefficients, the definition of a scattering coefficient had to be generalized. A new descriptive notation for the coefficients was suggested.

As a consequence of linearily scanning the scene to obtain a spatial avercge, it was demonstrated that the scattering coefficients must satisfy Schwartz' inequality

$$
\begin{equation*}
\left.\left.\left|\left\langle S_{i j} S_{k 1}^{*}\right\rangle\right|^{2} \leqq\left.\langle | s_{i j}\right|^{2}\right\rangle\left.\langle | \bar{s}_{k l}\right|^{2}\right\rangle \tag{8-1}
\end{equation*}
$$

where $i, j, k$ or $I=v$ orh. This naturally implies that equality is assured for the polarized and cross-polarized coefficients. However, equality should, in general, not be anticipated for the cross-correlation scattering coefficients. It is this feature which distinguishes coherent and non-coherent scattering coefficients. As a result of this inequality, scatterometer returns can be partially polarized. Furthermore, the inequality also implies that one cannot employ the properties of the (coherent) scattering matrix to describe non-coherent measurements. For a coherent target five independent parameters (from the scattering matrix) are required to describe its scattering coefficients. However, a non-coherent scene requires as many as nine independent paramefers.

The scatterometer equation under the reciprocity assumption was extended to account for the difference between antenna and surface polarizations. It was illustrated that the difference in polarizations was significant only a. nall view angles for narrow beam radars. The effect of misalignment can be minimized by reducing the beamwidth as one approaches nadir as illustrated in Figure (7.2). Minimizing the misalignment is imporfant if an experimenfer wishes to compare his measurements with theoretical predictions which are invariably reported with respect to the surface polarizations. It is shown, for example, that a cross polarized measurement af nadir
cannot be interpreted as an attempt to retrieve $\left\langle\left\langle\mathrm{Sh}_{\mathrm{vh}}{ }^{2}\right\rangle\right.$ as defined with respect to the surface polarizations. In view of the difficulty in interpreting measurements at small angles with respect to the surface polarizations, it is recommended that the nadir region be probed in an asymptotic sense with a very narrow beam antenna when scattering parameters are to be reported with respect to the surface polarizations. When a scene has an anisotropic behavior at small incident angles, it is particularly advantageous to report parameters in this fashion.

A measurement and inversion technique was proposed to measure all nine scattering parameters. The technique was formulated without regard to the distinction between antenna and surface polarizations. Since the difference between the polar izations is negligible for narrow beam radars at a'll but the small view angles, the formulation without alteration is valid there. In addition, it was shown that the system of measurements (antenna polarization states) is sufficient to retrieve all the parameters at small incident angles under an isotropic surface assumption if the inversion is based on the extended formulation, i.e., accounting for the difference between antenna and surface polarizations.

The computer simulations based on the above fechnique demonstrated that the dominant scattering parameters could be recovered with modest realizations of the antenna polarization requirements. However, retrieval of the weaker scattering parameters, as shown by the simulations, requires more careful adherence to the antenna polarization requirements. Cross polarized leakage in the case of linearily polarized transmissions or receptions causes the antenna to couple to the dominant scattering parameters. The leakage results not only in coupling to the real valued coefficients but also to the complex valued coefficients. The degree of coupling depends strongly on the relative phase of the leakage as well as on its amplitude. For scattering characteristics similar to that of the sea (where $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hv}}{ }^{*}\right\rangle$ and $\left\langle\mathrm{S}_{\mathrm{vh}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ are considered weak), strong undesirable contributions can be anticipated from $\left.\left.\left.\langle | s_{v v}\right|^{2}\right\rangle,\left.\langle | s_{h h}\right|^{2}\right\rangle$, and $\left\langle\mathrm{S}_{\mathrm{Vv}} \mathrm{S}_{h h^{*}}{ }^{*}\right.$, as demonstrated by the simulations. For a scene having randomly oriented linear re-radiators such as vegetation one can anticipate not only strong contributions from the above coefficients but also from $\left\langle\mathrm{S}_{\mathrm{vp}} \mathrm{S}_{\mathrm{hv}}{ }^{*}\right\rangle,\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{wh}}{ }^{*}\right\rangle,\left\langle\mathrm{S}_{\mathrm{vh}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ and $\left\langle S_{h v} S_{h h}{ }^{*}\right\rangle$. All four scatter ng coefficients have been cited to emphasize that the scattering processes are differe it although under the reciprocity assumption there are only two independent coefficients.

It is evident from the simulations that whan only the amplitude of the orthogonal leakage is known and not its phase, stringent specifications on the amplitude
are required to achieve, say, an accuracy of 0.5 dB for the weaker coefficients. This is illustrated when $\left\langle\left. S_{\mathrm{vh}}\right|^{2}\right\rangle$ is to be recovered from scenes having weak cross-correlation coefficients $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hv}}{ }^{*}\right\rangle$ and $\left\langle\mathrm{S}_{\mathrm{vh}} \mathrm{Shh}^{*}\right\rangle$. When it is suspected that $\left.\left.\langle | \mathrm{S}_{\mathrm{vh}}\right|^{2}\right\rangle$ is $X$ $d B$ beneath the geometric mean of $\left.\left.\langle | S_{\mathrm{vv}}\right|^{2}\right\rangle$ and $\left.\left.\langle | S_{h h}\right|^{2}\right\rangle$, then the permissible level in the cross leakage is $-(x+16) \mathrm{dB}$. On the otherhand if the phase of the leakage can be adjusted so that it is at or near $90^{\circ}$ (or it is known to be near $90^{\circ}$ ) during transmission and reception, then the amplitude specification can be relaxed as demonstrated by Figure 7.14. For scenes in which $\operatorname{Re}\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{h v^{*}}\right\rangle$ and $\mathrm{Re}\left\langle\mathrm{S}_{\mathrm{hv}} \mathrm{S}_{h h^{*}}{ }^{*}\right\rangle$ are dominant, this same phase condision can minimize contributions by these terms in the case of cross-polarized measurements. This may be concluded by an examination of Equation (4-29).

Although the assumed scattering characteristics reflected a wide latitude of conditions, the error characteristics generated here are by no means exhaustive. The retrieval accuracy to some degree is dependent on the assumed scattering characteristics. For example, $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{vv}}{ }^{*}\right\rangle$ and $\left\langle\mathrm{S}_{\mathrm{vh}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ were assumed weak and their error character istics reflected this weakness. It is recommended that simulations similar to those reporied in this effort be conducted whenever significantly different scattering behaviors are encountered. The antenna specifications can be established on the basis of these simulations. One may employ the program described in Appendix $D$ in which case subroutine SIGMA must be replaced with a subroutine that will compute the scattering parameters of interest.

It is evident from these efforts that there is a fine opportunity to extend the three standard measurements to nine measurements. When the distinction between polarizazions is nof important,* any combinations of measurements can be selected to isolafe particular coefficients. It is intriguing to consider certain combinations of measurements to observe soil moisture, crop maturity, etc. From small perturbation theory there is evidence that the cross-correlation coefficients may contain additional information on the dielectric property of the scene when compared with the autocorrelation coefficients. The comparison of like and cross-correlated coefficients may be the key to distinguishing dielectric effects, say in agrarian scenes, from volume roughness effects.

[^9]The above results also have an impact on emission theory and radiometer measurements. It is clear tnat the backscatter coefficients employed within this effort can be exfended to the bi-static case. As a consequence, we may address emission theory from the aspects of bi-static coefficients as Peake [24] did. Generalizing Kirchoff's radiation law, Peake has shown that the definition of emissivity, when assigned standard surface polarizations, may be related to the bistatic differential scattering coefficients in the following way

$$
\begin{equation*}
\left.\left.\varepsilon_{p}=1-\int\left(\left.\langle | s_{p p}\right|^{2}\right\rangle+\left.\langle | S_{p q}\right|^{2}\right\rangle\right) d \Omega \tag{8-2}
\end{equation*}
$$

where the integration is performed over all incident angles. The corresponding brightness temperatures were given as

$$
\begin{equation*}
T_{p}=\varepsilon_{p} T_{s} \tag{8-3}
\end{equation*}
$$

where $T_{s}$ is the physical femperature of the emitting surface. Peake's formulation for brightness ignores the possibility of correlation between emitted components. When correlation exists between the components, the concept of brightness temperature must be exfended as shown by Ko [47]. Ko had shown that an emission of total brighiness (intensity) $B_{o}$ and with a normalized coherency matrix $\rho$ can be regarded as a unique superposition of two coherent oppositely polarized emissions, i.e.,

$$
B_{0}\left[\begin{array}{ll}
\rho_{11} & \rho_{12} \\
\rho_{21} & \rho_{22}
\end{array}\right]=B_{1}\left[\begin{array}{ll}
\rho_{11}^{\prime} & \rho_{12}^{\prime} \\
\rho_{21}^{\prime} & \rho_{22}^{\prime}
\end{array}\right]+B_{2}\left[\begin{array}{ll}
\rho_{11}^{\prime} & \rho_{12}^{\prime \prime} \\
\rho_{21}^{\prime} & \rho_{22}^{\prime}
\end{array}\right]
$$

where

$$
\begin{align*}
& \rho_{11}^{\prime}=\rho_{22}^{\prime} \hat{1}  \tag{8-4}\\
& \rho_{12}^{\prime}=-\rho_{21}^{\prime} \\
& \rho_{21}^{\prime}=-\rho_{12}^{\prime} \\
& \rho_{22}^{\prime}=\rho_{11}^{1}  \tag{8-5}\\
& \rho_{11}^{\prime}+\rho_{22}^{\prime}=1 \\
& \rho_{11}^{\prime} \overrightarrow{1}+\rho_{22}^{\prime}=1
\end{align*}
$$

Temperatures are assigned to the brightness according to the Rayleigh-Jeans law

$$
\begin{equation*}
T_{i}=B_{i} \lambda^{2} / k \tag{8-6}
\end{equation*}
$$

Arbitrary measurement of this emitted field, say, with any two orthogonal polarizations will not necessarily result in any unique temperatures. The correlation between emitted components plays an important role in defining the brightness temperatures. Within the context of bi-static theory, the cross-corralation coefficients $\left\langle S_{w v} S_{h h}{ }^{*}\right\rangle,\left\langle S_{v v} S_{v h}{ }^{*}\right\rangle$, $\left\langle S_{h v} S_{h h}{ }^{*}\right\rangle,\left\langle S_{v v} S_{h v}{ }^{*}\right\rangle$ and $\left\langle S_{v h} S_{h h}{ }^{*}\right\rangle$ establish this correlation. For some surfaces, the first three coefficients are not important unless the emissions within the radiating body are correlared. Under this circumstance the correlation is governed by $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hv}}{ }^{*}\right.$ > and $\left\langle S_{\mathrm{wh}} \mathrm{S}_{\mathrm{hh}}{ }^{*}{ }^{\text {, }}\right.$, i.e., by the correlarions which the emitting surface induces. These cross-correlations for the sea are assumed negligibly small. The brightness temperatures are, therefore, given by the vertically and horizontally polarized emissions and the corresponding decomposition into coherency matrices is aiven by

$$
\mathrm{B}_{0}\left[\begin{array}{ll}
\rho_{11} & \rho_{12}  \tag{8-7}\\
\rho_{21} & \rho_{22}
\end{array}\right]=k T_{v} / \lambda^{2}\left[\begin{array}{ll}
1 & 0 \\
0 & 0
\end{array}\right]+\mathrm{kT}_{\mathrm{h}} / \lambda^{2}\left[\begin{array}{ll}
0 & 0 \\
0 & 1
\end{array}\right]
$$

For an agrarian scene the above simple decomposition may nof occur af all, since multiple reflections are likely to induce correlations info the emissions internal to the radiating boundary and because $\left\langle\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hv}}{ }^{*}\right\rangle$ and $\left\langle\mathrm{S}_{\mathrm{vh}} \mathrm{S}_{\mathrm{hh}}{ }^{*}\right\rangle$ are not negligible. Therefore the brightness temperature concept must be altered for agrarian scenes.

### 8.2 Final Remarks

The above observations as well as the developments in the earlier chapters indicate the importance of having derived the complete scotteremeter equation and in particular having derived it in the framework of coherency theory. The inferaction of the fransmitted fields with the scattering surface was expressed as a fransformation of a coherence matrix (Equation (4-41)). Interpretation of the scattered fields from its coherence matrix imparted meaning to the ctoss-correlation scattering coefficients. The complete scattering action of the surface, when interpreted in the context of emission theory also enables one to interpret the coherency properties of microwave emissions. The reception of scattered or emiffed fields is also expressed as the product of two coherence matrices.

Althouah a fuiler interpretational basis lies in coherency theory, practical application of the theory has led to a technique for measuring all six scattering coefficients. The measuring technique was evaluated for practical antennas. As a result of this evaluation it becomes apparent that measurement staridards or standardized reporting procedures or both should be instituted. There is also a clear need to distinguish scattering parameters reported with respect to the antenna polarizations from those reported with respect to the surface polarizations. The measurement of weak scattering coefficients requires stringent realization of the antenna polarization requirements. The documentation of the antenna transmission and reception property must be complete, to include amplitude and phase properties, to validate a measurement. Until such a procedure is followed it could be erroneous to report, for example, "cross" polsrized" measurements as cross-polarized scattering coefficients. To assist the experimenter it is also clear that a program should be initiated to develop a scatterometer antenna which is capable of meeting the antenna specifications for most if not all the scattering parameters for a variety of scenes.

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## APPENDIX A Sorrelation and Cross-Correlation <br> Products from Kirchhoff Theory

### 1.0 INTRODUCTION

The scattering and coherency properties of a finitely conducting random surface satisfying the Kirchhoff approximation are investigated within this appendix. The expressions for the polarized scattered fields in the plane of incidence gre specifically derived for both vertically polarized and horizontally polarized incident plane waves. The seattered fields are derived under the assumption that the surface slopes are small. (Only zero order and first order slope terms are retained within the derivation). The resulting expression is specialized to the backscatter case to derive the seif-correlation and cross-correlation scattering coefficients. The angular coherency of the scattered fields about the backscatter direction is also considered.

### 2.0 THEORY

### 2.1 General

For a plane wave $\bar{E}_{0}$ incident with direction $\bar{n}_{i}$ on a gently undulating finitely conducting bounded surface Fung [36] has shown that the far field scattered in direction $\bar{n}_{s}$ is given by

$$
\bar{E}_{s}=k \bar{n}_{s} \times \iint\left[\bar{n} \times \bar{E}-\eta \bar{n}_{s} \times(\bar{n} \times \bar{H})\right] e^{j k\left(\bar{n}_{s}-\bar{n}_{i}\right) \cdot \bar{\rho}} d S(A-1)
$$

where

$$
\begin{equation*}
K=\frac{-j k e^{-j k R}}{4 \pi R} \tag{A-2}
\end{equation*}
$$

$R=$ distance from the surface to the far field point
$\vec{\rho}=$ position vector from an origin local to the surface to a point on that surface

$$
\begin{aligned}
& \bar{n} \times \bar{E}=\left[\left(1+R_{n}\right)\left(\bar{a} \cdot \bar{t}_{\mathbf{i}}\right)\left(\bar{n} \times \bar{t}_{i}\right)-\left(1-R_{v}\right)\left(\bar{n} \cdot \bar{n}_{\mathbf{j}}\right)\left(\bar{a} \cdot \bar{d}_{i}\right) \bar{t}_{i}\right]\left|\bar{E}_{0}\right|(A-3) \\
& \vec{n} \times \bar{H}=\left[-\left(1+R_{V}\right)\left(\bar{a}_{a} \cdot \bar{d}_{i}\right)\left(\bar{n} \times \bar{t}_{i}\right)-\left(1-R_{n}\right)\left(\bar{a} \cdot \bar{t}_{i}\right) \bar{t}_{i}\right]\left|\bar{E}_{0}\right| \\
& E=E \bar{a} e^{j\left(\omega t-\bar{k}_{i} \cdot \bar{p}\right)} \\
& \bar{k}_{i}=k \bar{n}_{i} \\
& k=2 \pi / \lambda \\
& \bar{t}_{i}=\frac{\bar{n}_{i} \times \bar{n}}{\left|\vec{n}_{i} \times \bar{n}\right|} \\
& \bar{d}_{i}=\bar{n}_{i} \times \bar{t}_{i} \\
& \eta=\text { intrinsic impedance } \\
& \mathrm{R}_{\mathbf{V}_{z} h}=\text { Fresnel coefficient for horizontal (vertical) polarization }
\end{aligned}
$$

The geometry associated with the scattering problem is illustrated in Figure A-1. $\left\{\hbar_{i} ; f_{i}, d_{i}^{\prime}\right\}$ forms an orthogonal triad of vectors at each point of the random surface. The plane of incidence coincides with the $y=z$ plane. Now expanding the terms

> Figure A-1. Scattering Geometry

within the integrand we have

$$
\begin{align*}
\bar{n}_{s} \times(\bar{n} \times \bar{E})= & \left|\bar{E}_{0}\right|\left\{\left(1+R_{h}\right)\left(\bar{a} \cdot \bar{t}_{i}\right)\left[\left(\bar{n}_{s} \cdot \bar{t}_{i}\right) \bar{n}-\left(\bar{n}_{s} \cdot \bar{n}_{n}\right) \bar{t}_{i}\right]-\right. \\
& \left.\left(1-R_{v}\right)\left(\bar{n} \cdot \bar{n}_{i}\right)\left(\bar{a} \cdot \bar{d}_{i}\right)\left(\bar{n}_{s} \times \bar{t}_{i}\right)\right\} \tag{A-6}
\end{align*}
$$

and

$$
\begin{align*}
-\eta \bar{n}_{s} \times\left(\bar{n}_{s} \times(\bar{n} \times \bar{H})\right)= & \left|E_{0}\right|\left\{( 1 + R _ { v } ) ( \overline { a } \cdot \overline { d } _ { i } ) \left[\left(\bar{n}_{s} \cdot \bar{t}_{i}\right) \bar{n}_{s} \times \bar{n}-\left(\bar{n}_{s} \cdot \bar{n}_{n} \bar{n}_{s} \times \bar{t}_{i}\right]-\right.\right. \\
& \left.\left(\bar{n}_{n} \cdot \bar{n}_{i}\right)\left(1-F_{h}\right)\left(\bar{a} \cdot \bar{t}_{i}\right) \bar{t}_{i}\right\} \tag{A-7}
\end{align*}
$$

where a radial component was dropped under the far-field assumption. Now restrict observations to the plane of incidence so that

$$
\begin{equation*}
\bar{n}_{s}=\sin \theta_{s} \sin \phi_{s} \bar{i}_{y}+\cos \theta_{s} \bar{i}_{z} \tag{A-8}
\end{equation*}
$$

with $\emptyset_{s}=\pi / 2$ or $-\pi / 2$, depending whether the forward or back scatter quadrant is, respectively, chosen. Denote

$$
\begin{equation*}
\bar{n}_{i}=\sin \theta_{i} \bar{i}_{y}-\cos \theta_{i} \bar{i}_{z} \tag{A-9}
\end{equation*}
$$

where $\theta_{i}$ is the incident angle. Now it is easily shown that [36]

$$
\overline{\mathrm{i}}_{i}=\left[\left(\sin \theta_{i}-\cos \theta_{i} z_{y}\right) \bar{i}_{x}+\cos \theta_{i} z_{x} \bar{i}_{y}+\sin \theta_{i} z_{x} \bar{i}_{z}\right] / D_{1}(A-10)
$$

and

$$
\bar{d}_{i}=\left[Z_{x} \bar{i}_{x}+\cos \theta_{i}\left(\cos \theta_{i} z_{y}-\sin \theta_{i}\right) \bar{i}_{y}+\sin \theta_{i}\left(\cos \theta_{i} z_{y}-\sin \theta_{i}\right) \bar{i}_{z}\right]_{(A-11)}^{/ D_{1}}
$$

where

$$
\begin{align*}
& {D_{i}^{2}}_{2}=z_{x}^{2}+\left(\sin \theta_{i}-\cos \theta_{i} z_{y}\right)^{2}  \tag{A-12}\\
& z=z(x, y) \\
& z_{x, y}=\frac{\partial z}{\partial x, y}
\end{align*}
$$

Then

$$
\begin{aligned}
\bar{n}_{s} \cdot \bar{t}_{i}= & \left(\sin \theta_{s} \sin \phi_{s} \cos \theta_{i}+\cos \theta_{s} \sin \theta_{i}\right) Z_{x} / D_{1} \\
\bar{n}^{\prime} \cdot \bar{n}_{i}= & -\left(\sin \theta_{i} Z_{y}+\cos \theta_{i}\right) / D_{z} \\
\bar{n}_{s} \cdot \bar{n}_{=}= & \left(\cos \theta_{s}-\sin \theta_{s} \sin \phi_{s} Z_{y}\right) / \theta_{2} \\
\bar{n}_{s} \bar{x} \bar{\epsilon}_{i}= & {\left[\left(\sin \theta_{s} \sin \phi_{s} \sin \theta_{i}-\cos \theta_{s} \cos \theta_{i}\right) Z_{x} \bar{i}_{x}+\cos \theta_{s}\left(\sin \theta_{i}-\cos \theta_{i} Z_{y}\right) \bar{i}_{y}\right.} \\
& \left.-\sin \theta_{s} \sin \phi_{s}\left(\sin \theta_{i}-\cos \theta_{i} Z_{y}\right) \bar{i}_{z}\right] / D_{1} \\
\bar{n}_{s} x \bar{n}= & {\left[\left(\sin \theta_{s} \sin \theta_{s}+\cos \theta_{s} Z_{y}\right) \bar{i}_{x}-\cos \theta_{s} Z_{x} \bar{i}_{y}+\sin \theta_{s} \sin \phi_{s} Z_{x} \overline{\bar{i}}_{z}\right] / D_{1} }
\end{aligned}
$$

where

$$
\begin{equation*}
D_{2}^{2}=1+z_{x}^{2}+z_{y}^{2} \tag{A-14}
\end{equation*}
$$

### 2.2 Horizontally Polarized Incident Wave

Suppose $a=\bar{i}_{x}, \mathrm{i}, \mathrm{e}$, , the incident wave is horizontally polarized. Then

$$
\begin{equation*}
\bar{a} \cdot \bar{t}_{i}=-\left(\sin \theta_{i}-\cos \theta_{i} z_{y}\right) / D_{i}^{1 / 2} \tag{A-15}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{a} \cdot \bar{d}_{I}=-Z_{x} / D_{1}^{I / 2} \tag{A-16}
\end{equation*}
$$

The horizontally polarized field scattered in the plane of incidence may be shown to be given by

$$
\begin{equation*}
\dot{\bar{E}}_{S} \cdot \overline{\mathbf{i}}_{\phi_{S}}=K \iint I_{h h} \exp \left[i k\left(\bar{n}_{s}-\bar{n}_{i}\right) \cdot \bar{\rho}\right] d x d y \tag{A-17}
\end{equation*}
$$

where

$$
\begin{gathered}
\overrightarrow{\mathbf{i}}_{\phi_{s}}=-\sin \phi_{s} \overline{\mathfrak{i}}_{x} \\
I_{h h}=\sin \phi_{s}\left[-\left(1+R_{h}\right)\left(\cos \theta_{s}-\sin \theta_{s} \sin \phi_{s} Z_{y}\right)+\left(1-R_{h}\right)\left(\cos \theta_{i}+\sin \theta_{i} Z_{y}\right)\right]\left|E_{0}\right|
\end{gathered}
$$

Only terms to first order in $Z_{x}$ and $Z_{y}$ have been retained. To the same order it may be shown that the depolarized component is zero, i.e.,

$$
\begin{equation*}
\bar{E}_{s} \cdot \bar{i}_{\theta}=0 \tag{A-20}
\end{equation*}
$$

### 2.3 Vertically Polarized Incidenf Wave

Suppose $\vec{a}=-\cos \theta_{i} \bar{T} y-\sin \theta_{i} \bar{i} z, ~ i . e{ }_{0}$, a verfically polarized wave is incident on the $x$ - $y$ plane. Then

$$
\begin{equation*}
\overline{\mathrm{a}} \cdot \overline{\mathrm{t}}_{\mathrm{i}}=-Z_{\mathrm{x}} / \mathrm{D}_{1} \tag{A-21}
\end{equation*}
$$

and

$$
\begin{equation*}
\overline{\mathrm{a}} \cdot \overline{\mathrm{~d}}_{i}=\left(\sin \theta_{i}-\cos \theta_{i} z^{\prime} y\right) / D_{1} \tag{A-22}
\end{equation*}
$$

The vertically polarized scottered field may be shown to be given by

$$
\begin{equation*}
\overline{\mathrm{E}}_{\mathrm{s}} \cdot \overline{\mathrm{i}}_{\theta_{\mathrm{S}}}=k \iint \mathrm{I}_{\mathrm{vv}} \exp \left[\mathrm{jk}\left(\bar{n}_{\mathrm{s}}-\bar{n}_{\mathrm{i}}\right) \cdot \bar{\rho}\right] \mathrm{ds} \tag{A-23}
\end{equation*}
$$

where

$$
\begin{aligned}
& \bar{i}_{\theta_{s}}=\cos \theta_{s} \sin \phi_{s} \bar{i}_{y}-\sin \theta_{s} \bar{i}_{z} \\
& I_{w v}=\sin \phi_{s}\left[-\left(1+R_{v}\right)\left(\cos \theta_{s}-\sin \theta_{s} \sin \phi_{s} z_{y}\right)+\left(1-R_{v}\right)\left(\cos \theta_{i}+\sin \theta_{i} z_{y}\right)\right]\left|\overrightarrow{E_{0}}\right|
\end{aligned}
$$

Only ferms to first order in $Z_{x}$ and $Z_{y}$ have been retained in $I_{v v}$. To the same order it may be shown that the depolarized component is zero, i.e.,

$$
\begin{equation*}
\vec{E}_{s} \cdot \vec{t}_{\phi}=0 \tag{A-26}
\end{equation*}
$$

### 2.4 Linear Approximations for the Reflection Coefficients

It is necessary to understand that the reflection coefficients are functions of the local incident angle and are therefore functions of the local slopes, $Z_{x}$ and $Z_{y}$. For small slopes we may tpproximate $R_{h}$ and $R_{v}$ linearily by

$$
\begin{equation*}
R_{h, v}\left(z_{x}, z_{y}\right)=R_{h, v}(0,0)+\frac{\partial R_{h, v}(0,0)}{\partial z_{x}} z_{x}+\frac{\partial R_{h, v}(0,0)}{\partial z_{y}} z_{y} \tag{A-27}
\end{equation*}
$$

Now the relation between the local incident angle $\theta^{\prime}$ and the Incal slopes is

$$
\begin{equation*}
\cos \theta^{\prime}=-\bar{n}_{i} \cdot \bar{n} \tag{A-28}
\end{equation*}
$$

or

$$
\begin{equation*}
=\left[z_{y} \sin \theta_{i}+\cos \theta_{\mathbf{i}}\right] / D_{2} \tag{A-29}
\end{equation*}
$$

The derivatives within the linearily approximated reflection coefficients can then by the chain rule be written

$$
\begin{equation*}
\frac{\partial R_{v, h}}{\partial z_{x, y}}=\frac{\partial R_{v, h}}{\partial \theta^{\prime}} \frac{\partial \theta^{\prime}}{\partial z_{x, y}} \tag{A-30}
\end{equation*}
$$

An evaluation of the derivatives yields

$$
\left.\frac{\partial R_{h}}{\partial \theta^{\theta}}\right|_{Z_{x}=Z_{y}=0}=\frac{2 \sin \theta_{h}\left(\theta_{i}\right)}{\sqrt{\epsilon_{r}-\sin ^{2} \theta_{i}}}
$$

$$
\begin{align*}
& \left.\frac{\partial R_{v}}{\partial \theta^{\prime}}\right|_{Z_{x}=Z_{y}=0}=\frac{-2 \epsilon_{r} \sin \theta_{i} R_{v}\left(\theta_{i}\right)}{\sqrt{\epsilon_{r}-\sin ^{2} \theta_{i}}\left(\epsilon_{r} \cos ^{2} \theta_{i}-\sin ^{2} \theta_{i}\right)} \\
& \left.\frac{\partial \theta^{\prime}}{\partial z_{x}}\right|_{Z_{x}=z_{y}=0}=0  \tag{A-31}\\
& \left.\frac{\partial \theta^{\prime}}{\partial Z_{y}}\right|_{Z_{x}=Z_{y}=0}=-1
\end{align*}
$$

It has been recognized in the above expressions that $\theta^{\prime}=\theta_{i}$ when $Z_{x}=Z_{y}=0$. We finally have the approximate expressions

$$
\begin{equation*}
R_{v}\left(Z_{r}, z_{y}\right) \cong R_{v}\left(\theta_{i}\right)+\frac{2 \epsilon_{r} \sin \theta_{i} R_{v}\left(\theta_{i}\right) z_{y}}{\sqrt{\epsilon_{r}-\sin ^{2} \theta_{i}}\left(\varepsilon_{r} \cos ^{2} \theta_{i}-\sin ^{2} \theta_{i}\right)} \tag{A-32}
\end{equation*}
$$

and

$$
\begin{equation*}
R_{h}\left(Z_{x}, Z_{y}\right) \cong R_{h}\left(\theta_{i}\right)-\frac{2 \sin \theta_{i} R_{h}\left(\theta_{i}\right) z_{y}}{\sqrt{\epsilon_{r}-\sin ^{2} \theta_{i}}} \tag{A-33}
\end{equation*}
$$

### 2.5 Partial Evaluation of the Field Integrals

The evaluation of the polarized field expressions requires that integrals of the type

$$
\begin{equation*}
\text { Intg }=\iint Z_{y} \exp \left[j k\left(\bar{n}_{s}-\bar{n}_{i}\right) \cdot \bar{p}\right] d x d y \tag{A-34}
\end{equation*}
$$

be considered. By employing an integration by parts technique, specifically by letting

$$
\begin{equation*}
d v=\exp \left[j k\left(\cos \theta_{s}+\cos \theta_{i}\right) z\right] z_{y} d y \tag{A-35}
\end{equation*}
$$

and

$$
\begin{equation*}
u=\exp \left[j k\left(\sin \theta_{s} \sin \phi_{s}-\sin \theta_{i}\right) y\right] \tag{A-36}
\end{equation*}
$$

we get

$$
\begin{equation*}
\text { Intg }=\int v u_{\text {boundary }} d x-\iint \frac{s}{c} \exp \left[j k\left(\bar{n}_{s}-\bar{n}_{i}\right) \cdot \vec{\rho}\right] \tag{A-37}
\end{equation*}
$$

where

$$
\begin{equation*}
\frac{s}{c}=\frac{\sin \theta_{s} \sin \phi_{s}-\sin \theta_{i}}{\cos \theta_{s}+\cos \theta_{i}} \tag{A-38}
\end{equation*}
$$

The first ferm is identified as the edge effect and may be neglected.
When the above results are incorporated in the field integrals we can write a unifying expression

$$
\begin{equation*}
E_{j}\left(\theta_{s}, \theta_{s}\right)=-K B_{j} I\left(\bar{n}_{s}, \bar{n}_{i}\right) E_{0} \tag{A-39}
\end{equation*}
$$

where

$$
\begin{gathered}
I=\iint \exp \left[j k\left(\bar{n}_{s}-\bar{n}_{i}\right) \cdot \bar{\rho}\right] d x d y \\
B_{j}=\sin _{\phi_{s}}\left\{\left(1+R_{j}-\frac{s}{c} \frac{\partial R_{j}}{\partial Z_{y}}\right) \cos \theta_{s}-\left(1-R_{j}+\frac{s}{c} \frac{\partial R_{j}}{\partial Z_{y}}\right) \cos \theta_{i}-\left[\left(1+R_{j}\right) \sin \theta_{s} \sin \phi_{s}\right.\right. \\
\left.\left.+\left(1-R_{j}\right) \sin \theta_{j}\right] \frac{s}{c}\right\}
\end{gathered}
$$

$E_{j}(j=v$ or $h)$ denotes the like polarized field component when a $j$ th polarized wave of amplitude $E_{0}$ illuminates the surface. The reflection coefficient and its derivative are evaluated at the incident angle.

### 3.0 THE BACKSCATTER COEFFICIENTS

Now specialize to the backscatfer case. Specifically, lef $\theta_{s}=\theta_{i}$ and $\theta_{5}=-\pi / 2$.
Then

$$
\begin{equation*}
E_{j}\left(e_{i},-\frac{\pi}{2}\right)=K B_{j}^{\prime} I\left(-\bar{n}_{i}, \bar{n}_{i}\right) E_{0} \tag{A-41}
\end{equation*}
$$

where

$$
\begin{equation*}
B_{j}^{\prime}=B_{j}\left(\theta_{s}=\theta_{i}, \quad \Phi=-\Pi / 2\right) \tag{A-42}
\end{equation*}
$$

The differential scattering coefficient employed in this effort is given by

$$
\begin{equation*}
\left\langle S_{j j} s_{k k}^{*}\right\rangle=\frac{\left\langle E_{j} E_{k}^{*}\right\rangle R^{2}}{\left|E_{j}\right|^{2} A \cos \theta_{i}} \tag{A-43}
\end{equation*}
$$

where $A$ is the illuminated area. Consider the ratio of the intensities

$$
\begin{equation*}
\frac{\left\langle E_{j} E_{k}^{*}\right\rangle}{\left|E_{0}\right|^{2}}=|K|^{2} B_{j} B_{k}\left\langle I I^{*}\right\rangle \tag{A-44}
\end{equation*}
$$

where

$$
\begin{align*}
&\langle I I *\rangle\left.=\iiint \int \exp \left[-j 2 k_{y}\left(y_{1}-y_{2}\right)\right]<\exp \left[j 2 k_{z}\left(z_{1}-z_{2}\right)\right]\right\rangle \mathrm{dx}_{1} d x_{2} \mathrm{dx}{ }_{2} \mathrm{dy} \\
& 2 \tag{A-45}
\end{align*}
$$

Suppose that $Z_{1}$ and $Z_{2}$ are joint gaussian variables with zero mean, variance $\sigma^{2}$ and correlation $\Lambda\left(x_{1}, x_{2}, y_{1}, y_{2}\right)$. Then it is easily shown from the characteristic function properties of gaussian variables that

$$
\begin{equation*}
\left\langle\exp \left[j 2 k_{2}\left(Z_{1}-Z_{2}\right)\right]\right\rangle=\exp \left[-4 k_{z}^{2} \sigma^{2}(1-\Lambda)\right] \tag{A-47}
\end{equation*}
$$

Now transform the resulting integral to the center of mass coordinates. Lef

$$
\begin{align*}
& u=x_{1}-x_{2} \\
& v=y_{1}-y_{2} \\
& x_{2}=x_{2}  \tag{A-48}\\
& y_{2}=y_{2}
\end{align*}
$$

The integral then can be written as

$$
\begin{equation*}
\left\langle I I^{*}\right\rangle=\iiint \int \exp \left[-j 2 k_{y} v\right] \exp \left[-4 k_{z}^{2} \sigma^{2}(1-\Lambda)\right] d u d v d x_{2} d y_{2} \tag{A-49}
\end{equation*}
$$

Further transform the integral to cylindrical coordinates where

$$
\begin{align*}
u & =\rho \cos \xi \\
y & =\rho \sin \xi  \tag{A-50}\\
x_{2} & =\rho^{\prime} \cos \zeta \\
y_{2} & =\rho^{\prime} \sin \zeta
\end{align*}
$$

If it is further assumed that the surface is statistically stationary and isotropic, then

$$
\langle I I *\rangle=\iiint \int \mathrm{G}(\rho) \mathrm{G}\left(\rho^{\prime}\right) \exp \left[-\mathrm{j} 2 \mathrm{k}_{z} \sin \xi \rho\right] \exp \left[-4 \mathrm{k}_{z}{ }^{2} \sigma^{2}(1-\Lambda(\rho)] \rho \mathrm{d} \rho \mathrm{~d} \xi \rho^{\prime} \mathrm{d} \rho^{\prime} \mathrm{d} \xi\right.
$$

where $G(\rho)$ is a gate function describing the limits of the illuminated area. Specifically

$$
G(\rho)= \begin{cases}1 & \text { if } \rho \leq A / \pi  \tag{A-52}\\ 0 & \text { if } \rho \geq A / \pi .\end{cases}
$$

where $A$ is the area of illumination in the mean plane of the surface. Now recall that

$$
\begin{equation*}
\int_{0}^{2 \pi} \exp ( \pm j \alpha \sin \xi) d \xi=2 \pi J_{0}(\alpha) \tag{A-53}
\end{equation*}
$$

The integral will then reduce to

$$
\begin{equation*}
\langle I I *\rangle=2 \pi A \int_{0}^{\infty} G(\rho) \exp \left[-4 k_{z}^{2} \sigma^{2}(1-\Lambda(\rho)) I J_{0}\left(2 k \sin \theta_{i} p\right) \rho d \rho\right. \tag{4-54}
\end{equation*}
$$

Ain asymptotic evaluation of the above integral for large $\mathrm{k}_{\mathrm{z}}^{2}{ }_{\mathrm{o}}{ }^{2}$ yields

$$
\begin{equation*}
\langle I I *\rangle=\frac{\pi A e^{-\tan ^{2} \theta_{j} / 2 m^{2}}}{2 k^{2} m^{2} \cos ^{2} \theta_{i}} \tag{A-55}
\end{equation*}
$$

where $\sigma^{2} / \rho^{\prime \prime}(0)$ has been identified as the slope variance $m^{2}$.
Combining the above results it is clear that

$$
\begin{equation*}
\left\langle S_{j j} S_{k k}^{*}\right\rangle=\frac{B_{j}^{\prime} B_{k}^{*} e^{-\tan ^{2} \theta_{i} / 2 m^{2}}}{32 \pi^{2} \cos ^{3} \theta_{j}} \tag{A-56}
\end{equation*}
$$

where $j, k=v$ or $h$.

### 4.0 THE ANGULAR COHERENCY OF THE SCATTERED FIELDS

At this point consider the mutual coherence function

$$
\begin{equation*}
\varepsilon_{i j}\left(\theta_{i}, \theta_{2}, \dot{\phi}_{5}\right)=\left\langle E_{i}\left(\theta_{1}, \phi_{5}\right) E_{j}^{*}\left(\theta_{2}, \phi_{5}\right)\right\rangle \tag{A-57}
\end{equation*}
$$

where $i(j)=v$ or $h$. The cohereace function denotes the cross-correlation between two field components scattered in the plane of incidence at scattering angles $\theta_{1}$ and $\theta_{2}$ with a common range $R$. The expectation is an ensemble average over all random surfaces satisfying the Kirchhoff approximation. Now in view of (A-39) the coherence function can be written in the form

$$
\begin{equation*}
\varepsilon_{i j}=\left|E_{0} k\right|^{2} c_{i j} \iiint \int \operatorname{ess}\left[j k\left(\bar{n}_{1}-\bar{n}_{i}\right) \cdot{\left.\overline{p_{1}}-\left(\bar{n}_{2}-\bar{n}_{i}\right) \cdot \bar{\rho}_{2}\right]>d x_{1} d y_{1} d x_{2} d y_{2}}_{(A)}\right. \tag{A-58}
\end{equation*}
$$

where

$$
\begin{equation*}
\dot{c}_{j k}=B_{j} B_{k}^{*} \tag{A-59}
\end{equation*}
$$

Now transform the center of mass coordinate system where

$$
\begin{align*}
& u=x_{1}-x_{2}  \tag{A-60}\\
& v=y_{1}-y_{2}
\end{align*}
$$

Also let $\theta_{1}=\theta_{i}$ and $\theta_{2}=\theta_{i}+\Delta \theta$ where $\Delta \theta$ is a small deviation from the backscatter direction. We have

$$
\begin{align*}
& \left(\bar{n}_{1}-\bar{n}_{i}\right) \cdot \bar{\rho}_{1}=-2\left(\sin \theta_{i} y_{1}+\cos \theta_{i} z_{1}\right) .  \tag{A-61}\\
& \left(\bar{n}_{2}-\bar{n}_{i}\right) \cdot \bar{\rho}_{2}=\left(-2 \sin \theta_{i}+\Delta \theta \cos \theta_{i}\right) y_{2}+\left(2 \cos \theta_{i}-\Delta \theta \sin \theta_{i}\right) z_{2}
\end{align*}
$$

Now for a gaussian random surface having a surface height characteristic with zero mean, variance $\sigma^{2}$ and correlation function $\Lambda$, the expectation within the integral becomes

$$
<>=\exp \left[-j 2 k_{y}{ }^{v+j k_{z} \Delta \theta y_{2}}\right] \exp \left[-k_{y}^{2} \sigma^{2} \Delta \theta^{2} / 2\right] \exp \left[-4 k_{z}{ }^{2} \sigma^{2}(1-A)\right]
$$

The infegral then can be written as

$$
\begin{align*}
\text { Intg }=\exp \left[-k_{y}^{2} \sigma^{2} \Delta \theta^{2} / 2\right] \iiint \int & \exp \left[-j 2 k_{y}^{v}+j k_{z} \Delta \theta y_{2}\right]  \tag{A-63}\\
& \exp \left[4 \ldots_{z}^{2} \sigma^{2}(1-A)\right] d u d v d x_{2} d y_{2}
\end{align*}
$$

Now transform the integral expression to cylindrical coordinates by letting

$$
\begin{align*}
& u=\rho \cos \xi \\
& v=\rho \sin \xi  \tag{A,-64}\\
& x_{2}=\rho^{\prime} \cos \xi \\
& y_{2}=\rho^{\prime} \sin \xi
\end{align*}
$$

Then

$$
\begin{equation*}
\varepsilon_{j k}\left(\theta_{1}, \theta_{2}\right)=|K|^{2} C_{j k}\left|E_{o}\right|^{2} \exp \left(-k_{j}^{2} \sigma^{2} \Delta \theta^{2} / 2\right) \text { Intg } \tag{A-65}
\end{equation*}
$$

where

$$
\begin{array}{r}
\text { Intg }=\iiint \int_{G}(\rho) G\left(\rho^{\prime}\right) \exp \left[-j 2 k_{y} \sin \xi \rho\right] \exp \left[j k_{z^{\Delta}} \sin \sin \rho^{\prime}\right] \\
\exp \left[2 k_{z}{ }^{2} \sigma^{2}(1-A(\rho))\right] \rho \mathrm{d} \rho \mathrm{~d}_{\xi} \rho^{\prime} \mathrm{d} \rho \mathrm{~d} \zeta
\end{array}
$$

and where it has been assumed that $z$ is stationary and isotropic. $G(\rho)$ is a gate function defining the region of uniform illumination on the mean plane. Using Equation A-53 twice, we can write the integral as

$$
\begin{equation*}
\text { Intg }=(2 \pi)^{2} I_{1} I_{2} \tag{A-67}
\end{equation*}
$$

where

$$
\begin{align*}
& I_{I}=\int G(\rho) J_{0}\left(2 k_{y} \rho\right) \exp \left[4 k_{z}^{2} \sigma^{2}(1-A)\right] \rho d \rho  \tag{A-68}\\
& I_{2}=\int G\left(\rho^{\prime}\right) J_{0}\left(k_{z} \Delta \theta \rho^{\prime}\right) \rho^{\prime} d \rho^{\prime}
\end{align*}
$$

For a circularly illuminated area of radius $R_{0}$ the latter integral can be evaluated to get

$$
\begin{equation*}
I_{2}=R_{0}^{2} \operatorname{Jinc}\left(k_{z} R_{0} \Delta \theta\right) \tag{A-69}
\end{equation*}
$$

where $\operatorname{Jinc}(x)=J_{1}(x) / x$. As a consequence, for a circular region of area $A$ we have

$$
\begin{equation*}
\varepsilon_{j k}=4 A|k|^{2} C_{j k} \exp \left(-\dot{k}_{y}^{2} \sigma^{2} \Delta \theta^{2} / 2\right) \operatorname{jinx}\left(k_{z} R_{o} \Delta \theta\right) I_{1}\left|E_{o}\right|^{2} \tag{A-70}
\end{equation*}
$$

Now normalize the mutual coherence function in the following fashion

$$
\begin{equation*}
\Gamma_{j k}=\varepsilon_{j k} 4 \pi k_{o}^{2} i\left(\left|E_{o}\right|^{2} A \cos \theta_{j}\right) \tag{A-71}
\end{equation*}
$$

Now recognize that for small $\Delta \theta$

$$
\Sigma_{j k} \cong 2 \exp \left(-k_{y}^{2}{ }_{\sigma}{ }^{2} \Delta \theta^{2} / 2\right) \operatorname{Jinc}(x) 4 \pi<s_{j j} s_{k k}^{*}>
$$

where $x=k_{z} R_{o} \Delta \theta$. (See equations $A-43$ and A-54). The degree of coherence or correlation is consequently related to the mutual coherency function by

$$
\begin{equation*}
D=\Gamma_{j k} /\left(4 \pi<S_{j j} S_{k k}^{*}>\right) \tag{A-72}
\end{equation*}
$$

or

$$
=2 \exp \left(-k_{y}^{2} \sigma^{2} \Delta \theta^{2} / 2\right) \operatorname{Jinc}\left(k_{z} R_{0} \Delta \theta\right)
$$

Consider the character of the degree of coherency. Except for extremely large kovalues the exponential ferm confributes negligibly to $D$ at small incident angles. The decorrelation is consequently largely governed by the Bessel function for small incident angles. D vanishes at the zeroes of Jinc. The first zero occurs where

$$
\begin{equation*}
\Delta \theta k R_{0} \cos \theta=3.832 \tag{A-73}
\end{equation*}
$$

The corresponding angular separation is given by

$$
\begin{equation*}
\Delta \theta=3.832 / k R_{0} \cos \theta \tag{A-74}
\end{equation*}
$$

Suppose $k=291(f=13.9 \mathrm{GHz}), R_{o}=10$ meters and $\theta=25^{\circ}$; then decorrelation occurs when $\Delta \theta=0.00146$ radians or at .08 degrees, it is concluded that radar returns decorrelate rapidly with changes in view angles.

Af large incident angle (grazing angles) the exponential factor will predominate. This result is physically reasonable since the surface roughness predominates the view; whereas at small angles the area of illumination as conveyed in Jinc is the dominant factor.

# APPENDIX B <br> The Scafterometer Equation Within the Context of a Scattering Theory 

### 1.0 INTRODUCTION

The scatterometer equation is once again derived within the context of a specific scattering theory. The structurs and meaning of the formulation, as a consequence, readily becomes apparent. Specifically, the angular correlation assumption is shown to be equivalent to the non-ccherent property of scattering; the relation between scattering operator and the scattering coefficient is clarified also.

### 1.1 Derivation and Discussion

Silver [19] has shown that the far field radiated in the direction $\bar{n}_{s}$ from a bounded closed surface $S$ having surface excitation $\vec{E}$ and $\vec{H}$ is given by

$$
\begin{equation*}
\bar{E}_{s}=\frac{-j k}{4 \pi R} e^{-j k R} \bar{n}_{s} \times \iint_{S}\left[\bar{n} \times \bar{E}-\eta \bar{n}_{s} \times(\bar{n} \times \bar{H})\right] e^{j k \bar{p} \cdot \bar{n}_{s}} d s \tag{B-1}
\end{equation*}
$$

where
$R=$ distance from the surface to the far field point
$\bar{\rho}=$ position vector from an origin local to the surface to a point on the surface
$\overline{\mathrm{n}}=$ surface unit normal
$k=2 \pi / \lambda$
$\eta=$ intrinsic impedance
The geometrical entries of the above expression are illustrated in Figure $\mathrm{B}-1$. Suppose that the surface is smoothly undulating and perfectly conducting. Then under the Kirchhoff approximation the tangent surface fields are given by

$$
\begin{equation*}
\bar{n} \times \bar{E}=0 \tag{B-2}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{n} \times \bar{H}=2 \bar{n} \times \bar{H}_{t} \tag{8-3}
\end{equation*}
$$



FIGURE B-1 GEOMETRY FOR SCATTERING INTEGRAL
where $\bar{H}_{t}$ is transmitted field incident on the surface. The scatfered field, therefore, simplifies to

$$
\begin{equation*}
\bar{E}_{s}=\frac{j k n}{2 \pi R} \cdot e^{-j k R} \bar{n}_{s} \times \iint \bar{n}_{s} \times\left(\bar{n} \times \bar{H}_{t}\right) e^{j k \bar{p} \cdot \bar{n}_{i}} d s \tag{B-4}
\end{equation*}
$$

Now when specializing to the backscatter case we can write

$$
\begin{equation*}
\bar{n}_{s} \times\left(\bar{n}_{s} \times\left(\bar{n} \times \bar{H}_{t}\right)\right)=-\left(\bar{n}_{s} \cdot \bar{n}\right) \bar{n}_{s} \times \bar{H}_{t} \tag{B-5}
\end{equation*}
$$

so that

$$
\begin{equation*}
\bar{E}_{s}=\frac{-j k e^{-j k R}}{2 \pi R} \iint\left(\bar{n}_{s} \cdot \bar{n}\right) \bar{E}_{t} e^{j k \bar{p} \cdot \bar{n}_{s}} d s \tag{B-6}
\end{equation*}
$$

where for the backscatter case

$$
\begin{equation*}
\bar{E}_{\mathrm{t}}=\eta \bar{n}_{\mathrm{s}} \times \bar{H}_{\mathrm{t}} \tag{B-7}
\end{equation*}
$$

In beginning with the far field expression we have in effect anticipated the use of a noncoherent assumption since in scatterometry one would not necessarily be in the far field if the entire scattering aperture were coherent.

Now the spherical incident field is denoted as
where

$$
\begin{equation*}
\bar{E}_{t}=-j k \cdot \bar{L}_{t} e^{j(\omega t-k R)} \tag{B-8}
\end{equation*}
$$

$$
\begin{align*}
K & =w \mu_{0} i_{t} / 4 \pi R  \tag{B-9}\\
i_{t} & =\text { antenna input current } \\
\mathrm{L}_{t} & =\text { complex effective height vector of the antenna }
\end{align*}
$$

The range $R$ is measured from the antenna as illustrated in Figure $\mathrm{B}-1$. The spherical wave can be approximated by segments of plane waves each illuminating a pafch of the surface. In addition, the incident field amplitude components as conveyed by $\overrightarrow{\mathrm{L}}_{f}$ may be considered constant on a given patch. Suppose the entire illuminated surface is segmented into N patches. Then the incident field on the mith patch may be approximated by

$$
\begin{equation*}
\bar{E}_{t m}=-j k_{m} \bar{L}_{t m} e^{j\left(\omega t-k R_{m}+k \bar{\rho}_{m} \cdot \bar{n}_{s m}\right)} \tag{B-10}
\end{equation*}
$$

where
$R_{m}=$ range to the centroid of the mih patch
$\bar{P}_{\mathrm{m}}=$ position vector from the centroid to a surface point in the moh patch
$\bar{n}_{s m}=$ unit vector in the backscatter direction for the mth patch

$$
K_{m}=w \mu_{0} i_{t} / 4 R_{m}
$$

Now from Equation ( $B-6$ ) we note that the backscattered field for the $m$ th patch is given by

$$
\begin{equation*}
\bar{E}_{s m}=\frac{k k_{m} e^{-j 2 k R_{m}}}{2 \pi R_{m}} \bar{L}_{t m} \iint_{S_{m}}\left(\bar{n} \cdot \bar{n}_{s m}\right) e^{j 2 k \bar{\rho} \cdot \bar{n}_{s m}} d s \tag{B-11}
\end{equation*}
$$

The integration is performed on the surface within patch $m$.
Now define the scattered field per differential steradian subtended about the antenna as.

$$
\begin{equation*}
\dot{\varepsilon}_{\mathrm{sm}}=\frac{\dot{E}_{\mathrm{sm}} R_{m}^{2}}{\Delta A_{\mathrm{m}} \cos \theta_{\mathrm{m}}} \tag{B-12}
\end{equation*}
$$

where $\Delta A_{m}$ is the area of the $m$ th patch in the mean plane of the surface and $\theta_{m}$ is the inciden: angle on the mth parch. The scattering operator employed within Chapter 4 may be identified with the above expression, viz.,

$$
\begin{align*}
& \delta_{v v}\left(\theta_{m}, \phi_{m}\right)=\lim _{\Delta A_{m} \rightarrow 0} \frac{\vec{\varepsilon}_{s m} \cdot \bar{i}_{\theta m} 4 \pi R}{-j K_{m} 1_{v t f^{-j}} e^{-j K R} m}  \tag{B-13}\\
& \delta_{h h}\left(\theta_{m}, \phi_{m}\right)=\lim _{\Delta A_{m} \rightarrow 0} \frac{\vec{\varepsilon}_{s m} \cdot \bar{i}_{\phi m} 4 \pi R}{-j k_{m} l_{h t m} e^{-j k R_{m}}}
\end{align*}
$$

where

$$
\begin{align*}
& 1_{v \operatorname{tm}}=\vec{L}_{t} \cdot \bar{i}_{\theta \mathrm{in}} \\
& 1_{\mathrm{htm}_{\mathrm{m}}}=\vec{L}_{t} \cdot \bar{i}_{\phi m} \tag{B-14}
\end{align*}
$$

The inner products above isolate the vertically and horizontally polarized components as defined with respect to surface. It is appropriate to denote $\mathcal{Z}_{\mathrm{Pp}}$ as an operator since it must recognize the phase of the incident field relative to the centroid of the patch. Whether these differential scattering operators exist is not important to the development within the appendix. However, within the main text they are assumed to exist af least in approximate form.

Within the context of this theory $\mathscr{V}_{\mathrm{vh}}=\ell_{\mathrm{hv}}=0$. This is simply a statement of a well known result that a smoothly undulating perfectly conducting surface does not depolarize the incident field. In general, the latter operators are not zero. The above increment al field is defined so that the total field at the antennc is given by

$$
\begin{equation*}
\bar{E}_{a}=\sum_{m=1}^{N} \bar{\varepsilon}_{s m} \Delta \Omega_{m} \tag{B-15}
\end{equation*}
$$

or in the continuum limit

$$
\begin{equation*}
\bar{E}_{a}=\int \vec{E}_{s} d \Omega \tag{B-16}
\end{equation*}
$$

It must be recognized that the antenna does not respond to the fotal field, a quantity often computed by the theorist. Instead the anfenna responds so that the open circuit voltage induced into the antenna terminals by the mth patch is given by

$$
\begin{equation*}
\Delta V_{\mathrm{oc}}=\bar{\varepsilon}_{\mathrm{sm}} \cdot \tau_{\mathrm{rm}} \Delta \Omega_{\mathrm{m}} \tag{B-17}
\end{equation*}
$$

where $\bar{L}_{\mathrm{rm}}$ is the complex effective height vector in the direction of the mth patch during reception. The fotal induced voltage is clearly approximated by

$$
\begin{equation*}
v_{\mathrm{oc}}=\sum_{\mathrm{m}=1}^{N} \bar{\varepsilon}_{\mathrm{sm}} \cdot \bar{L}_{\mathrm{rm}} \Delta \Omega_{\mathrm{m}} \tag{B-18}
\end{equation*}
$$

The average power available under matched condifions at the antenna terminals is given by the ensemble average
or

$$
\begin{equation*}
\left.W_{r}=\left.\frac{1}{8 R_{r}}\langle | V_{o c}\right|^{2}\right\rangle \tag{B-19}
\end{equation*}
$$

$$
\begin{equation*}
W_{r}=\frac{1}{8 R_{r}} \sum_{m=1}^{N} \sum_{n=1}^{N}<\vec{\varepsilon}_{s m} \cdot \bar{L}_{r m} \vec{\varepsilon}_{s n}^{*} \cdot \bar{L}_{r n}^{*}>\Delta \Omega_{m} \Delta \Omega_{n} \tag{B-20}
\end{equation*}
$$

where $R_{r}$ is the radiation resistance of the antenna during reception. Now the scattered fields and effective heights can be decomposed into polarization components coinciding with the surface polarizations, viz. .
ayd

$$
\begin{equation*}
\stackrel{\rightharpoonup}{\varepsilon}_{\mathrm{sm}}=\varepsilon_{\mathrm{smv}} \overline{\mathrm{i}}_{\theta \mathrm{m}}+\varepsilon_{\mathrm{smh}} \overline{\bar{i}}_{\phi m} \tag{B-2I}
\end{equation*}
$$

$$
\begin{equation*}
\bar{L}_{r m}=1_{v r m} \bar{i}_{\theta m}+T_{h r m} \bar{i}_{\phi m} \tag{B-22}
\end{equation*}
$$

The expectation can then be writfen in the form

$$
\begin{align*}
\left\langle\vec{\varepsilon}_{\mathrm{sm}} \cdot \bar{L}_{r m} \vec{\varepsilon}_{\mathrm{sm}}^{*} \cdot \bar{L}_{\mathrm{rn}}^{*}\right\rangle & =\left\langle( \varepsilon _ { \mathrm { vsm } } 1 _ { \mathrm { vrm } } + \varepsilon _ { \mathrm { hsn } } 1 _ { \mathrm { hrn } } ) \left(\varepsilon_{\mathrm{vsn}}^{*} 1_{\mathrm{vrn}}^{*}+\underset{\substack{(B-23) \\
(B-24)}}{\left.\varepsilon_{\mathrm{hsn}}^{*} 1_{\mathrm{hsn}}^{*}\right)>}\right.\right. \\
& =\operatorname{tr} M_{\mathrm{rmn}} M_{\mathrm{smn}}^{\dagger} \quad
\end{align*}
$$

where

$$
M_{\mathrm{smn}}=\left[\begin{array}{lll}
<\varepsilon_{\mathrm{vsm}} \varepsilon_{\mathrm{vsn}}^{*}> & <\varepsilon_{\mathrm{vsm}} & \varepsilon_{\mathrm{hsn}}^{*}>  \tag{B-25}\\
<\varepsilon_{\mathrm{hsm}} \varepsilon_{\mathrm{vsn}}^{*}> & <\varepsilon_{\mathrm{hsm}} & \varepsilon_{\mathrm{hsn}}^{*}>
\end{array}\right]
$$

and

$$
M_{\mathrm{rmn}}=\left[\begin{array}{cccc}
I_{\mathrm{vrm}} & 1_{\mathrm{vrn}}^{*} & I_{\mathrm{vrm}} & I_{\mathrm{hrn}}^{*}  \tag{B-26}\\
1_{\mathrm{hrm}} & I_{\mathrm{vrn}}^{*} & & 1_{\mathrm{hrm}} \\
1_{\mathrm{hrn}}^{*}
\end{array}\right]
$$

are identified as mutual coherence matrices. The mutual coherence matrix for the scattered field is composed of elements correlating fields arriving from patches $m$ and $n$ or equivalently from different angular directions $\left(\theta_{m} \phi_{m}\right)$ and $\left(\theta_{n^{2}}, \theta_{n}\right)$. Now within Appendix A it is shown it is reasonable to assume, on pragmatic grounds, that returns arriving at different view angles from the same parch are uncorrelated. The assumption is exact in the geometric-optics limit. It is even more reasonable to assume here that fields arriving from different angles are uncorrelafed since they arise from different patches. As a consequence the mutual coherence matrices have the special property

$$
\begin{equation*}
M_{s j k}=N_{s j j} \delta_{j k} \tag{B-27}
\end{equation*}
$$

for every $\mathrm{jk} . \delta_{\mathrm{jk}}$ is the Kronecker delta and

$$
N_{s j j}=\left[\begin{array}{cc}
\left.\left.\langle | \varepsilon_{v s j}\right|^{2}\right\rangle & \left\langle\varepsilon_{v s j} \varepsilon_{h s j .}^{*}\right\rangle  \tag{B-28}\\
\left\langle\varepsilon_{h s j} \varepsilon_{v s j}^{*}\right\rangle & \left.\left.\langle | \varepsilon_{h s j}\right|^{2}\right\rangle
\end{array}\right]
$$

The non-coherent assumption, consequently, allows us to write the received power in the form

$$
\begin{equation*}
W_{r}=\frac{1}{8 R_{r}} \operatorname{tr} \sum_{j=1}^{N}\left(\sum_{k=1}^{N} M_{r j k} \delta_{j k} \Delta \Omega_{k}\right) N_{s j j}^{\dagger} \Delta \Omega_{j} \tag{B-29}
\end{equation*}
$$

or

$$
\begin{equation*}
=\frac{1}{8 R_{r}} \sum \operatorname{tr} C_{r j}\left(N_{s j j}^{\dagger} \Delta \Omega_{j}\right) \Delta \Omega_{j} \tag{B-30}
\end{equation*}
$$

where

$$
\begin{equation*}
C_{r j}=M_{r j j} \tag{B-3i}
\end{equation*}
$$

is the coherency matrix for the receiving antenna in direction $\left(\theta_{j}, \emptyset_{j}\right)$. Now expanding $N_{\mathrm{sjj}} \Delta \Omega_{\mathrm{j}^{\prime}}$ we have

$$
N_{s j j} \Delta \Omega_{j}=\frac{4 k^{2}\left|K_{j}\right|^{2} R_{j}^{2}}{(4 \pi)^{2} \Delta A_{j} \cos \theta_{j}} \quad\left[\begin{array}{ll}
B_{j v v} & B_{j v h}  \tag{B-32}\\
B_{j h v} & B_{j h h}
\end{array}\right]
$$



$$
S_{j}
$$

From the above expression we note that the incident field complex amplifude components ${ }^{1} \mathrm{v}_{\mathrm{t}}$ and $\mathrm{I}_{\text {ht }}$ have been separated from the scatfering integrals. The relative phase befween incident amplitude components is retained in the products $I_{p}{ }^{f}{ }_{j} I_{q t j}{ }_{j}^{*}$. Now the non-coherent differential scaftering coefficients per unit steradian for the jth patch is defined as

$$
\left\langle S_{p p} S_{q q}^{*}\right\rangle=\frac{4 k^{2}}{(4 \pi)^{2}} \iint_{S_{j}}\left\langle\left(\bar{n} \cdot \bar{n}_{s j}\right)\left(\bar{n}^{\prime} \cdot \bar{n}_{s j}\right) e^{j 2 k\left[\bar{\rho}^{\prime} \cdot \vec{n}_{s j}-\bar{\rho}^{\prime} \cdot \vec{n}_{s j}\right]}>d S d S^{\prime}\right.
$$

(See Equation (A-43), Appendix A). As a consequence we can write the coherency matrix for the scatfered fields based on intensities per unit steradian in notation similar to that of Equation (4-16) of Chapter 4. We have
or

$$
\begin{equation*}
C_{s j}=N_{s j j} \Delta \Omega_{j} \tag{B-35}
\end{equation*}
$$

$$
=\left|k_{j}\right|^{2}\left[\begin{array}{cc}
\left.\left.\langle | s_{v v}\right|^{2}\right\rangle\left|1_{v t j}\right|^{2} & \left\langle s_{v v} S_{h h}^{*}>1_{v t j} 1_{h t j}^{*}\right. \\
<s_{v v}^{*} S_{h h}>1_{v t j}^{*} 1_{h t j} & \left.\left.\langle | s_{h h}\right|^{2}\right\rangle\left|1_{h t j}\right|^{2}
\end{array}\right]
$$

We have shown above that the elements of the coherency matrix change units after the non-coherent assumption has been applied to the double summation. The return power can now be wiitten as

$$
\begin{equation*}
W=\frac{1}{8 R_{r}} \sum_{j=1}^{N} \operatorname{tr} C_{r j} C_{s j}^{\dagger} \Delta \Omega_{j} \tag{B-37}
\end{equation*}
$$

or upon taking the limit of the sum as $\Delta \Omega_{k} \rightarrow 0$, we get an integral approximation

$$
\begin{equation*}
W=\frac{1}{8 R_{r}} \int \operatorname{tr} c_{r} c_{s}^{\dagger} d \Omega \tag{B-88}
\end{equation*}
$$

where the above equation has a form identical to that of Equation (4-13) of Chapter 4 when $\delta_{\mathrm{yh}}=0$.

## APPENDIX C <br> Correlation and Cross-Correlation

Scaftering Properties of a Slightly Rough Surface.

### 1.0 THEORETICAL DEVELOPMENT

For a plane wave incident with angle $\theta_{i}$ in the $x-z$ plane on slightly rough surface, satisfying the requirement

$$
\begin{equation*}
\left(k \sigma \cos \theta_{i}\right)^{2} \ll I \tag{C-1}
\end{equation*}
$$

where $\sigma^{2}=$ surface height variance and $k=2 \pi / \lambda$, the solution for the scattered fields is expressed in a perturbation expansion of spatial Fourier components [32] [33] [34] [35] . The Furrier components are interpreted as an angular spectrum of plane waves. Suppose, the spectral components are denoted as $A_{p}^{v}\left(k_{x}, k_{y}\right)$ where

$$
\begin{align*}
& k_{x}=k \sin \theta_{s} \cos \phi_{s} \\
& k_{y}= k \sin \theta_{s} \sin \phi_{s} .  \tag{C-2}\\
& p=x, y, z
\end{align*}
$$

Figure $C-1$. Scattering Geometry

when a vertically polarized plane wave illuminates the surface and as $A_{p}^{h}(x, y, z)$ when a horizontally polarized plane illuminates the surface (See Figure C-1). For either case the electric field $E_{p}{ }^{q}$ af point ( $x, y, z$ ) is given by an inverse Fourier transform relarionship

$$
E_{p}^{r}=\frac{1}{(2 \pi)^{2}} \iint A_{D}^{r}\left(k_{x}, k_{y}\right) e^{j\left(k_{x} x+k_{y} y-k_{z} z\right)} d k_{x} d k_{y}(C-3)
$$

where $k_{z}^{2}=k^{2}-k_{x}^{2}-k_{y}^{2}$. The superscripts denote the incident polarization wheres; the subscripts denote the scattered cartesian components $\left(E_{x}{ }^{r}, E_{y}^{r}, E_{z}^{r}\right)$. If we suppose that a highly directional antenna points in the direction $\left(\theta_{5},,_{5}\right)$ with beam-volume $\Delta \ominus \Delta \emptyset$, then only certain spectral components near

$$
\begin{align*}
& k_{x}=k \sin \theta_{s} \cos \phi_{s} \\
& k_{y}=k \sin \theta_{s} \sin \phi_{s}  \tag{C-4}\\
& k_{z}=k \cos \theta_{s}
\end{align*}
$$

will be observed. The angular spectral space surrounding $\left(\theta_{\mathrm{s}}, \phi_{\mathrm{s}}\right)$ with angular volume $\sin \theta_{s} \Delta \theta \Delta y$ is given by

$$
\begin{equation*}
\Delta k_{x} \Delta k_{y}=|j| \Delta \theta \Delta \phi \tag{C-5}
\end{equation*}
$$

where

$$
|J|=\left|\begin{array}{ll}
\frac{\partial k_{x}}{\partial \theta_{s}} & \frac{\partial k_{v}}{\partial \theta_{s}}  \tag{c-6}\\
\frac{\partial k_{x}}{\partial \phi_{s}} & \frac{\partial k_{v}}{\partial \phi_{s}}
\end{array}\right|=k^{2} \sin \theta_{s} \cos \theta_{s}
$$

Now the cross-correlation of electric field components observed within the antenna beamwidth is given by

$$
\begin{aligned}
& \left\langle\Delta E_{p}^{r} \Delta E_{q}^{s *}>\cong \frac{1}{(2 \pi)^{4}} \iiint \int \mathcal{K A}_{p}^{r_{A}}{ }^{s *}>\right. \\
& e^{j\left(k_{x} x+k_{y} y\right)} e^{-j\left(\kappa_{x} x+\kappa_{y} y\right)} d k_{x} d k_{y} d \kappa_{x} d \kappa_{y} \\
& (C-7)
\end{aligned}
$$

where:
$\Delta_{x} \Delta=\left(\Delta k_{x} \Delta k_{y}\right) \times\left(\Delta k_{x} \Delta k_{y}\right)$ denotes the cartesian domain of integration. The solution for the spectral components are given by the form [35]

$$
\begin{equation*}
A_{p}^{r}\left(k_{x}, k_{y}\right)=A_{o_{0}}{ }^{r}\left(\bar{k}, \bar{k}^{\prime}\right) z\left(k_{x}+k \sin \theta_{0}, k_{v}\right) \tag{C-8}
\end{equation*}
$$

for the first order solutions and by the forms

$$
\begin{align*}
B_{x, y}^{r}\left(k_{x}, k_{y}\right)= & B_{o x, y}^{r} \iint A_{x, y}^{r} Z\left(k_{x}-\alpha, k_{y}-\beta\right) d \alpha d \beta+ \\
& B_{1_{x, y}^{r}}^{r} \iint f_{1}\left(\bar{k}, \bar{k}^{\prime}\right) Z(\alpha+k \sin \theta, \beta) Z\left(k_{x}-\alpha, k_{y}-\beta\right) d \alpha d \beta+ \\
& B_{2_{x, y}}^{r} \iiint_{2}\left(\bar{k}, \bar{k}^{\prime}\right) Z(\alpha+k \sin \theta, \beta) Z\left(k_{x}-\alpha, k_{y}-\beta\right) d \alpha d \beta+ \\
& B_{3_{x, y}} \iint A_{y, x}^{r} Z\left(k_{x}-\alpha, k_{y}-\beta\right) d \alpha d \beta \tag{C-9}
\end{align*}
$$

and

$$
\begin{equation*}
B_{z}^{r}\left(k_{x}, k_{y}\right)=\frac{k_{x}}{k_{z}} \exists_{x}^{r}+\frac{k_{y}}{k_{z}} B_{y}^{r} \tag{C-10}
\end{equation*}
$$

for the second order fields. $Z\left(k_{x}, k_{y}\right)$ is a random variable dascribing the Fourier spectral heights of the rough surface and $k_{z}=\sqrt{k^{2}-k_{x}{ }^{2}-k_{y}{ }^{2}}$. The coefficients $A_{o p}^{r} B_{o x, y}^{r}$ $B_{2 x, y}^{r}$ and $B_{3 x_{f} y}^{r}$ are determinstic functions depending on the propagation constants in the upper and lower media. For the purposes of this appendix it is sufficient to observe that the first order fields are dependent on $Z\left(k_{x}, k_{y}\right)$ whereas the second order fields are dependent on $Z\left(\alpha+k \sin \theta_{0}\right.$ : $\left.\beta\right) Z\left(k_{x}-\alpha, k_{y}-\beta\right)_{\text {. . The functional form of the } A \text { and }}$ $B$ coefficients is af this point immaterial.

### 2.0 CORRELATION PRODUCTS FROM THE FIRST ORDER FIEIDS - CASE I

Now consider a typical correlation producf from the first order solutions. We have from the integrand of (C-5)

$$
\begin{align*}
A_{p}\left(k_{x}, k_{y}\right) A_{p}^{*}\left(\kappa_{x}, k_{y}\right)= & A_{o p}^{r}\left(\bar{k}, \bar{k}^{\prime}\right) A_{o p}^{s}\left(\bar{\kappa}, \bar{\kappa}^{\prime}\right) \\
& \quad<\bar{L}\left(k_{x}+k \sin \theta_{0}, k_{y}\right) Z^{*}\left(k_{x}+k \sin \theta_{0}, \kappa_{y}\right)> \tag{C-11}
\end{align*}
$$

The expectation can be written as

$$
\begin{equation*}
\left\langle z(u, v) z^{*}\left(u^{\prime}, v^{\prime}\right)\right\rangle=\iiint \int_{e^{-j(u x+v y)}<}^{<z(x, y) z\left(x^{\prime}, y^{\prime}\right)>} e^{j\left(u^{\prime} x^{\prime}+v^{\prime} y^{\prime}\right)} d x d y d x^{\prime} d y^{\prime} \tag{C-12}
\end{equation*}
$$

where $z$ and $Z$ are Fourier transform pairs. If the random process is stationary, then

$$
\begin{equation*}
\left\langle z(x, y) z\left(x^{\prime}, v^{\prime}\right)\right\rangle=R\left(x-x^{\prime}, v-y^{\prime}\right) \tag{C-13}
\end{equation*}
$$

where $R$ is the auto-correlation function of the surface heights. Now transform the variable to the center of mass coordinates, i.e., let $\tau_{x}=x-x^{\prime}, \tau_{y}=y=y^{1}, x^{\prime}=x$ and $y^{\prime}=y^{\prime}$. Then

$$
\begin{array}{r}
\left\langle Z(u, v) Z^{*}\left(u^{\prime}, v^{\prime}\right)\right\rangle=\iiint \int R\left(\tau_{x}, \tau_{y}\right) e^{-j\left(\tau_{x} u+\tau^{\prime} v\right)} d \tau_{x} d \tau_{y} \\
\cdot e^{j\left[\left(u^{\prime}-u\right) x^{\prime}+\left(v^{\prime}-v\right) y^{\prime}\right]} d x^{\prime} d y^{\prime}
\end{array}
$$

or

$$
=\iint W(u, v) e^{-j\left[\left(u-u^{\prime}\right) x^{\prime}+\left(v-v^{\prime}\right) y^{\prime}\right]_{d x} d y^{\prime}}
$$

where $W(u, v)$ is the spectral density of the surface heights. Finally we recognize that

$$
\begin{equation*}
\left.<Z(u, v) Z^{*}\left(u^{\prime}, v^{\prime}\right)\right\rangle=(2 \pi)^{2} W(u, v) \delta\left(u-u^{\prime}, v-v^{\prime}\right) \tag{C-15}
\end{equation*}
$$

When the above results is substituted into the correlation integral we get

$$
<\Delta E_{p}^{r} \Delta E_{q}^{*} S=\frac{1}{(2 \pi)^{2}} \iint_{\Delta x \Delta} A_{o_{p}}^{r} A_{o q}^{*} S_{q} W\left(k_{x}+k \sin \theta_{o}, k_{y}\right) d k_{x} d k_{y}(C-16)
$$

When $\Delta$ is sufficiently small, the incremental complex intensity may be approximated by

$$
\begin{equation*}
<\Delta E_{p}^{r} \Delta E_{q}^{*} s>=\frac{A_{o p}^{r} A_{o}^{*}{ }_{q}^{*}}{(2 \pi)^{2}} W\left(k_{x}+k \sin \theta_{o}, \dot{k}_{y}\right) \Delta k_{x} \Delta k_{y} \tag{C-17}
\end{equation*}
$$

The cross-correlation per unit steradian is therefore given by

$$
\begin{equation*}
\frac{\left\langle\Delta E_{p}^{r} \Delta E_{q}^{* s}\right\rangle}{\sin \theta_{s} \Delta \theta_{s} \Delta \phi_{s}}=\frac{k^{2} \cos \theta_{s} A_{o p}^{r} A_{o q}^{* s}}{(2 \pi)^{2}} W\left(k_{x}+k \sin \theta, k_{y}\right) \tag{C-18}
\end{equation*}
$$

where $(C-5)$ has been used. The total cross-correlation in direction $\left(\theta_{5}, \varnothing_{5}\right)$ is given by

$$
\left\langle E_{p}^{r} E_{q}^{S^{*}}\right\rangle=\frac{k^{2} \cos ^{2} \theta_{s} A_{o p}^{r} A_{o q}^{S^{*}} W\left(k_{x}+k \sin \theta_{o}, k_{y}\right)}{(2 \pi)^{2}} \frac{\Delta A}{R^{2}(C-19)}
$$

where $\Delta A$ is the illuminated area in the $x-y$ plane and $R$ is the range to the element of area.

To consider the horizonfally and vertically polarized backscaffered components we must realize that the horizontal polarized component is relafed to the cartesian components by

$$
A_{h}^{r}\left(k_{x}, k_{y}\right)=-A_{x}^{r}\left(k_{x}, k_{y}\right) \sin \phi_{s}+A_{y}^{r}\left(k_{x}, k_{y}\right) \cos \phi_{s}(C-20)
$$

and the vertical component by

$$
\begin{align*}
A_{v}^{r}\left(k_{x}, k_{y}\right)= & A_{x}^{r}\left(k_{x}, k_{y}\right) \cos \theta_{s} \cos \phi_{s}+A_{y}^{r}\left(k_{x}, k_{y}\right) \cos \theta_{s} \sin \phi_{s}= \\
& A_{z}^{r}\left(k_{x}, k_{y}\right) \sin \theta_{s} \tag{C-21}
\end{align*}
$$

When we specialize to the backscatter direction $\left(\theta_{s}=\theta_{o}, \emptyset_{s}=\pi\right)$, we see that

$$
\begin{equation*}
A_{h}^{r}\left(k_{x}, k_{y}\right)=-A_{y}^{r}\left(k_{x}, k_{y}\right) \tag{C-22}
\end{equation*}
$$

and

$$
\begin{equation*}
A_{v}^{r}\left(k_{x}, k_{y}\right)=A_{x}^{r}\left(k_{x}, k_{y}\right) \cos \theta_{0}-A_{z}^{r}\left(k_{x}, k_{y}\right) \sin \theta_{0} \tag{C-23}
\end{equation*}
$$

with $k_{x}=k \sin \theta_{0^{\prime}} k_{y}=0, k_{z}=k \cos \theta_{0^{\circ}}$. It is well known that the first order backscatter fields do not involve depolarized components [34]. From reference [35] we now identify

$$
\begin{equation*}
A_{h}^{h}\left(k_{x}, k_{y}\right)=-j 2 k \cos \theta_{0} R_{h} Z\left(2 k \sin \theta_{0}, 0\right) \tag{C-24}
\end{equation*}
$$

and

$$
\begin{equation*}
A_{v}^{v}\left(k_{x}, k_{y}\right)=-j 2 k \cos \theta_{0} T_{v}\left[\epsilon_{r}\left(1+\sin ^{2} \theta_{0}\right)-\sin ^{2} \theta_{0}\right] z\left(2 k \sin \theta_{0}, 0\right) \tag{C-25}
\end{equation*}
$$

where
$R_{h}=$ Fresnel reflection coefficien $\dagger$ for horizontal polarization
$T_{v}=$ Fresnel transmission coefficient for vertical polarization
$\varepsilon_{r}=$ relative (complex) dielectric constant .
and where a unit amplitude incident wave has been assumed. Therefore the correlation components in the backscatter direction become

$$
\begin{gather*}
\left\langle E_{v}^{v} E_{v}^{V^{*}}\right\rangle=\frac{4 k^{4} \cos ^{4} \theta_{0}}{(2 \pi)^{2}}\left|T_{v}\left[\epsilon_{r}\left(1+\sin ^{2} \theta_{0}\right)-\sin ^{2} \theta_{0}\right]\right|^{2} W\left(2 k \sin \theta_{0}, 0\right) \frac{\Delta A}{R^{2}} \\
\left\langle E_{V}^{v} E_{h}^{h^{*}}\right\rangle=\frac{4 k^{4} \cos ^{4} \theta_{0}}{(2 \pi)^{2}} T_{v}\left[\epsilon_{r}\left(1+\sin ^{2} \theta_{0}\right)-\sin ^{2} \theta_{0}\right] R_{h}^{*} W\left(2 k \sin \theta_{0}, 0\right) \frac{\Delta A}{R^{2}} \\
\quad<E_{h}^{h} E_{h}^{h^{*}>}=\frac{4 k^{4} \cos ^{4} e_{0}}{(2 \pi)^{2}}\left|R_{h}\right|^{2} W\left(2 k \sin \theta_{0}, 0\right) \frac{\Delta A}{R^{2}} \quad \therefore \tag{C-28}
\end{gather*}
$$

The corresponding generalized differential scattering coefficient per unit intensity per steradian is given by

$$
\begin{equation*}
\left\langle S_{p p}^{r} S_{q q}^{*} S^{\prime}\right\rangle=\left\langle E_{p}^{r} E_{q}^{s}\right\rangle \frac{R^{2}}{\Delta A \cos \theta_{0}} \tag{C-29}
\end{equation*}
$$

(See the generalized definition of the scattering coefficients in Chapter 4). So

$$
\left.\left.\langle | S_{v v}\right|^{2}\right\rangle=\frac{k^{4}}{\pi^{2}} \cos ^{3} \theta_{0}\left|T_{v}\left[\epsilon_{r}\left(1+\sin ^{2} \theta_{0}\right)-\sin ^{2} \theta_{0}\right]\right|^{2} \quad W\left(2 k \sin \theta_{0}, 0\right)
$$

$$
\begin{align*}
& \left.\left\langle S_{W} S_{h h}^{*}\right\rangle=\frac{k^{4} \cos ^{3} \theta}{\pi^{2}}\left[T_{v}\left[\epsilon_{r}\left(1+\sin ^{2} \theta_{0}\right)-\sin ^{2} \theta_{0}\right]\right]\right]_{h}^{*} W\left(2 k \sin \theta_{0}, 0\right) \\
& \left.\left.\langle | S_{h h}\right|^{2}\right\rangle=\frac{k^{4} \cos ^{3} \theta}{\pi^{2}}\left|R_{h}\right|^{2} W\left(2 k \sin \theta_{0}, 0\right) \tag{C-32}
\end{align*}
$$

### 3.0 CORRELATION BETWEEN FIRST AND SECOND ORDER FIELDS - CASE II

To develop the correlations between first and second order fields it is sufficient to note that the correlations involve expectations of the type $<Z\left(k_{x}, k_{y}\right) Z^{*}(\alpha, \beta) Z^{*}\left(K_{x}\right.$ $-\alpha_{0} K_{y}-\beta$ " and of type $Z\left(k_{x}, k_{y}\right) Z^{*}(\alpha+k \sin \theta, \beta) Z^{*}\left(K_{x}-\alpha, K_{y}-\beta\right)$. These expectations involve independent gaussian random variables with zero meari and consequently vanish. The first and second order fields are therefore uncorrelated. It is concluded that $\left\langle S_{V v} S_{v h}{ }^{*}\right\rangle=0$ and $\left\langle S_{h v} S_{h h}{ }^{*}\right\rangle=0$ at the lowest order.

## APPENDIX D <br> Scatterometer Simulation Program (SCATSIM)

### 1.0 INTRODUCTION

The theory and operation of the scatterometer simulation program is described within this appendix. The following section shows how the scatterometer equation of Chapter 4 was implemenfed with ideal and non-ideal antenna parameters. The compufation of the inversion models with and withouf recognition of the difference between surface and antenna polarizations is also described. Finally the operation of the program is treated by means of a flow chart. A source listing and a sample output is also presented. Additional program documentation is provided by comments within the program.

### 2.0 THEORY

### 2.1 Simulation of the Scatterometer Equation

Since the scattering characteristics were based on surface polarizations, Equations ( $6-47$ ), $(6-49)$ and ( $6-50 a$ ) through ( $6-50 i$ ) were implemented for use on the computer. The equation was simulated using identical functional forms for the vertically and horizontally polarized patterns (if they are both present during a transmission or reception). Recall that the normalized patterns are given by

$$
\begin{equation*}
g_{v, h}=\frac{\left|1_{v, h}\left(\theta^{\prime}, \phi^{\prime}\right)\right|^{2}}{\left|I_{v}(0,0)\right|^{2}+\left|1_{h}(0,0)\right|^{2}} \tag{D-1}
\end{equation*}
$$

where $\left(\theta^{\prime}, \phi^{\prime}\right)=(0,0)$ is the boresight point. As a consequence, we require

$$
\begin{equation*}
g_{v}(0,0)+g_{h}(0,0)=1 \tag{D-2}
\end{equation*}
$$

Now if $g$ denotes the functional form for the pattern and has the property $g(0,0)=1$ and if $g_{h}$ is assigned' the value ag where $a \leq 1$, then we require that $g_{v}=(1-a) g_{0}$. The scatterometer equation under the above assumption an be written as

$$
\begin{equation*}
W\left(\theta_{0}\right)=\frac{\lambda^{2} G_{t} G_{r}}{(4 \pi z)^{2}} \int I_{t r}(g \cos \theta)^{2} d \Omega \tag{D-3}
\end{equation*}
$$

where

$$
\begin{aligned}
& \left.\left.I_{t r}=\left.I_{1}\langle | S_{V V}\right|^{2}+\left.I_{2}\langle | S_{h h}\right|^{2}\right\rangle^{2}+\left.I_{3}\langle | S_{v h}\right|^{2}\right\rangle .+ \\
& 2 I_{4} R_{e}<S_{v v} S_{h h}^{*}>-2 I_{5} I_{m}<S_{v v} S_{h h}^{*}>+2 I_{6} R e<S_{v v} S_{h v}^{*}>- \\
& 2 I_{7} I^{m}<S_{v v} S_{h v}^{*}>+2 I_{8} R e<S_{v h} S_{h h}^{*}>-2 I_{g} I_{m}<S_{v h}^{(D-4)} S_{h h_{h}}^{*}>
\end{aligned}
$$

where

$$
\begin{align*}
I_{1}= & \left(1-a_{r}\right)\left(1-a_{t}\right) \cos ^{4} \psi+a_{r} a_{t} \sin ^{4} \psi+\left(\left(1-a_{r}\right) a_{t} \underset{(D-5)}{+}\right. \\
& \left.\left(1-a_{t}\right) a_{r}+4 c_{t} c_{r}\right) \sin ^{2} \psi \cos ^{2} \psi  \tag{D-5}\\
I_{2}= & a_{t} a_{r} \cos ^{4} \psi+\left(1-a_{t}\right)\left(1-a_{r}\right) \sin ^{4} \psi+\left(\left(1-a_{r}\right) a_{t}+\right. \\
& \left.\left(1-a_{t}\right) a_{r}+4 c_{t} c_{r}\right) \sin ^{2} \psi \cos ^{2} \psi  \tag{D-6}\\
I_{3}= & \left(\left(1-a_{t}\right) a_{r}+\left(1-a_{r}\right) a_{t}-2 c_{t} c_{r}\right)\left(\cos ^{4} \psi+\sin ^{4} \psi\right)+ \\
& 2\left(\left(1-a_{r}\right)\left(1-a_{t}\right)+a_{t} a_{r}-6 c_{t} c_{r}+\left(2 a_{r}-1\right)\left(2 a_{t}-1\right)\right) \\
& \sin ^{2} \psi \cos ^{2} \psi+2 s_{t} s_{r} \tag{D-7}
\end{align*}
$$

$$
\begin{align*}
& I_{A}=c_{t} c_{r}\left(\cos ^{4} \psi+\sin ^{4} \psi\right)+\left(\left(2 a_{r}-1\right)\left(2 a_{t}-1\right)-2 c_{t} c_{r}\right) \cdot  \tag{D-8}\\
& \sin ^{2} \psi \cos ^{2} \psi-s_{r} t \\
& I_{5}=\left(c_{r} s_{t}+c_{t} s_{r}\right)\left(\cos ^{2} \psi-\sin ^{2} \psi\right)  \tag{D-9}\\
& I_{6}=\left(\left(1-a_{r}\right) c_{t}+\left(1-a_{t}\right) c_{r}\right) \cos ^{4} \psi-\left(a_{r} c_{t}+a_{t} c_{r}\right) \\
& \sin ^{4} \psi+3\left(\left(2 a_{r}-1\right) c_{t}+\left(2 a_{t}-1\right) c_{r}\right) \sin ^{2} \psi \cos ^{2} \psi  \tag{D-10}\\
& I_{7}=\left(\left(1-a_{r}\right) s_{t}+\left(1-a_{t}\right) s_{t}\right) \cos ^{2} \psi+\left(a_{r} s_{t}+a_{t} s_{r}\right) \sin ^{2} \psi  \tag{D-11}\\
& I_{8}=\left(a_{t} c_{r}+a_{r} c_{t}\right) \cos ^{4} \psi-\left(\left(1-a_{r}\right) c_{t}+\left(1-a_{t}\right) c_{r}\right) \\
& \sin ^{4} \psi-3\left(\left(2 a_{r}-1\right) c_{t}+\left(2 a_{t}-1\right) c_{r}\right) \cos ^{2} \psi \sin ^{2} \psi(D-12) \\
& I_{9}=\left(a_{t} s_{r}+a_{r} s_{t}\right) \cos ^{2} \psi+\left(\left(1-a_{r}\right) s_{t}+\left(1-a_{t}\right) s_{r}\right) \\
& \sin ^{2} \psi  \tag{D-13}\\
& c_{t}=\sqrt{a_{t}\left(1-a_{t}\right)} \cos \beta_{t} \\
& c_{r}=\sqrt{a_{r}\left(1-a_{r}\right)} \cos \beta_{t} \\
& s_{t}=\sqrt{a_{t}\left(1-a_{t}\right)} \sin \beta_{t}  \tag{D-14}\\
& s_{r}=\sqrt{a_{r}\left(I-a_{r}\right)} \sin \beta_{r}
\end{align*}
$$

The integration is performed in the surface coordinate system (See Figure 4.1). In the above expression all odd powers in $\sin \psi$ have been pragmatically dropped. These factors are odd functions of $\rho$ and will not contribute to the integral (See Equation 4-33).

Now since the scattering characteristic was assumed isotropic, the return power can be approximated by

$$
\begin{aligned}
& \left.W\left(\theta_{0}\right)=\left.\frac{\lambda^{2} G_{t} G_{r}}{(4 \pi z)^{2}} \sum_{\omega=1}^{N}\langle | s_{v v}\right|^{2}\right\rangle_{\omega} \int_{\Omega_{\omega}} I_{1}(g \cos \theta)^{2} d \Omega \\
& \left.+\left.\langle | s_{h h}\right|^{2}\right\rangle_{\omega} \int_{\Omega} I_{2}(g \cos \theta)^{2} d \Omega \\
& \left.+\left.\langle | s_{v h}\right|^{2}\right\rangle_{\omega} \int_{\Omega_{\omega}}^{\omega} I_{3}(g \cos \theta)^{2} d \\
& +2 \operatorname{Re}<\mid s_{v v^{5}}{ }_{h h^{*}}{ }_{\omega} \int_{\Omega_{b}} I_{4}(g \cos \theta)^{2} d \Omega \\
& \text { - } 2 \mathrm{Im}<\left|\mathrm{s}_{V V^{\prime}}{ }_{h h}^{*}\right\rangle_{\omega} \int_{\Omega_{\omega}}^{\omega} I_{5}(G \cos \theta)^{2} d \Omega \\
& +2 \operatorname{Re}\left\langle s_{v v^{s}}{ }^{*} v^{*}\right\rangle_{\omega} \int_{\Omega_{\omega}}^{\Omega_{6}}(g \cos \theta)^{2} d \Omega \\
& -2 \operatorname{Im}\left\langle s_{v v^{\prime}} s_{h v^{*}}\right\rangle_{\omega} \int_{\Omega \omega}^{\omega} I_{7}(g \cos \theta)^{2} d \Omega
\end{aligned}
$$

$$
\begin{align*}
& -2 \operatorname{Im}\left\langle s_{v h^{s} h^{\prime}{ }_{\omega} \int_{\Omega_{\omega}}^{\Omega_{\theta}} I_{g}(g \cos \theta)^{2} d \Omega}\right. \tag{D-15}
\end{align*}
$$

and where $\left\{\Omega_{\omega}, \omega=1,2, \ldots, N\right\}$ is a set of half degree annuli centered about the subobservation point. $<>_{\omega}$ denotes an evaluation of the scattering coefficient on the $\Omega_{\omega}$ annulus. Function subroutine SIGMA contains the functional representations for all nine scattering coefficients. The integrations are performed, of course, only over those annuli where the pattern function $g$ is significant. The above approximation reduces the computation to integrals of the following kinds:
$J_{1}(\omega)=\int_{\Omega_{\omega}} g^{2} \cos ^{2} \theta \cos ^{4} \psi d \Omega \quad J_{3}(\omega)=\int_{\Omega_{\omega}} g^{2} \cos ^{2} \theta \sin ^{2} \psi \cos ^{2} \psi d \Omega$
$U_{2}(\omega)=\int_{\Omega_{\omega}} g^{2} \cos ^{2} \theta \sin ^{4} \psi d \Omega \quad J_{4}(\omega)=\int_{\Omega_{\omega}} g^{2} \cos ^{2} \theta \sin ^{2} \psi d \Omega$

$$
\mathrm{J}_{5}(\omega)=\int_{\Omega_{\omega}} \mathrm{g}^{2} \cos ^{2} \theta \cos ^{2} \psi \mathrm{~d} \Omega \quad \mathrm{~J}_{6}(\omega)=\int_{\Omega_{\omega}} g^{2} \cos ^{2} \theta \mathrm{~d} \Omega
$$

These integrals are evaluated in subroutine DINTEG using a two-dimensional GaussianLegendre quadrature fechnique [48]. For a selected antenna pattern and a selected view angle the refurn power is computed in accord with the above expression. The antenna gains $G_{f}$ and $G_{r}$ are formed in a separate computation. These factors are based on an evaluation of the expression

$$
\begin{equation*}
G_{t}=G_{r}=\frac{2}{\int g \sin \theta^{\prime} d \theta^{\prime}} \tag{D-17}
\end{equation*}
$$

The evaluation of the above integral is performed in subroutine SOLID, which employs a single dimension Gaussian-Legendre quadrature. The numerical evaluation of the paitern functions is provided by subroutine LAMBDA. All of the above integrations are executed from the mainline of SCATSIM.

Since the relative phases $\beta_{f}$ and $\beta_{r}$ were assumed stationary over the main beam and first side lobe, the return power could be evaluated for various combinations of $a_{r}$, $a_{t}, \beta_{t}$ and $\beta_{r}$ without re-evaluating the double integrals. As a consequence, an arbitrary pattern condition within the above constraints $\left(g_{v}=(I-a) g\right.$ and $\left.g_{h}=a g\right)$ could be established. The combination of relative amplitudes and phases for the fifteen prescribed measurements are shown in Table D.1. Subroutine ANTENNA, when addressed with zero arguments, generates those prescribed values. When amplitude and phase biases and/or perturbations are entered as arguments, subroutine ANTENNA will apply biases of the prescribed value to all measurements in which $a_{r}$ or $a_{f}$ is zero or unity. Random perturbations are applied to the remaining cases if the perturbation arguments are nonzero. In this fashion either measurements based on 15 ideal or 15 deviated anfenna conditions can be generated. The actual coefficients required in the integrand factors $\left\{I_{i}, i=1,9\right\}$ are computed in subroutine COEF. COEF fills a $15 \times 9 \times 6$ array with the appropriate yalues so that the return power can be computed for each of the 15 measurements. Let $C_{i j k}$ denote the array. The $i$ subscript designates the measurement number, the j subscript identifies one of the nine scattering coefficients within the integrand, and the $k$ subscript identifies une of the six kinds of infegrands $\left(J_{k}(\omega)\right)$. See Table D. 2 for the entries in $C_{i j k}$. Lef $\gamma_{i}, i=1,9$ denote the nine scattering coefficients and

TABLE D. 1

| MEASUREMENT NO. | COEF | $a_{i}$ | $\beta_{t}$ | ${ }^{\text {r }}$ | $\mathrm{Br}_{\mathrm{r}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\left\langle\mid S_{\mathrm{vv}}{ }^{2}\right\rangle$ | 0 | - | 0 | - |
| 2 | $\left\langle\mid s_{h h^{\prime}}{ }^{2}\right\rangle$ | 1 | - | 1 | - |
| 3 | $\left\langle\mid S_{\left.v\right\|^{\prime}}{ }^{2}\right\rangle$ | 0 | - | 1 | - |
| 4 | $\mathrm{Re}<\mathrm{S}_{\mathrm{vv}} \mathrm{S}_{\mathrm{hh}}{ }^{\text {P }}$ > | 0.5 | $-90^{\circ}$ | 0.5 | $90^{\circ}$ |
| 5 | $\operatorname{Re}<\mathrm{S}_{\mathrm{wv}} \mathrm{Shh}^{*}>$ | 0.5 | $0^{\circ}$ | 0.5 | $180^{\circ}$ |
| 6 | Im $<\mathrm{S}_{\mathrm{wv}} \mathrm{Shh}^{*}>$ | 0.5 | $45^{\circ}$ | 0.5 | $-135^{\circ}$ |
| 7 | Im $<\mathrm{S}_{\mathrm{yv}} \mathrm{Shh}^{*}>$ | 0.5 | $-45^{\circ}$ | 0.5 | $135^{\circ}$ |
| 8 | $\mathrm{Re}<\mathrm{S}_{\mathrm{wv}} \mathrm{Shv}^{*}>$ | 0 | -- | 0.5 | $0^{\circ}$ |
| 9 | $\mathrm{Re}<\mathrm{S}_{\mathrm{ww}} \mathrm{Shv}^{*}>$ | 0 | - | 0.5 | $180^{\circ}$ |
| 10 | $\mathrm{Im}<\mathrm{S}_{\mathrm{w}} \mathrm{Shv}^{*}>$ | 0 | - | 0.5 | $90^{\circ}$ |
| 11 | Im $<S_{\text {vv }} S_{\text {hv }}{ }^{*}>$ | 0 | - | 0.5 | $-90^{\circ}$ |
| 12 | $\mathrm{Re}<\mathrm{S}_{\mathrm{vh}} \mathrm{Shh}^{*}$ > | 1 | - | 0.5 | $0^{\circ}$ |
| 13 | $\mathrm{Re}<\mathrm{S}_{\text {vh }} \mathrm{Sh}^{*}{ }^{*}>$ | 1 | - | 0.5 | $180^{\circ}$ |
| 14 | $\mathrm{Im}^{<} \mathrm{S}_{\text {vh }} \mathrm{Shh}^{*}>$ | 1 | - | 0.5 | $90^{\circ}$ |
| 15 | $\mathrm{Im}<\mathrm{S}_{\mathrm{vh}} \mathrm{Sh}^{\text {\% }}$ * $>$ | 1 | - | 0.5 | $-90^{\circ}$ |

the Coefficient matrix $\mathrm{C}_{\mathrm{ijk}}$

|  | ROW/COL. | $1{ }^{\circ}$ | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | $\left(1-a_{t}\right)\left(1-a_{r}\right)$ | $\mathrm{ar}_{r}{ }_{\text {t }}$ | $\begin{aligned} & \left(1-a_{r}\right) a_{r}+\left(1-a_{t}\right) a_{r} \\ & \quad+4 c_{c_{1}} c_{r} \end{aligned}$ | 0 | 0 | 0 |
|  | 2 | $\mathrm{a}_{\mathrm{r}}{ }^{\text {+ }}$ | $\left(1-a_{t}\right)\left(7-a_{r}\right)$ | $\begin{gathered} \left(1-a_{p} a_{+}+\left(1-a_{p}\right) a_{r}\right. \\ +4 c_{r} c_{r} \end{gathered}$ | 0 | 0 | 0 |
|  | 3 | $\begin{gathered} \left(1-a_{t}\right) a_{r}+\left(1-a_{r}\right) a_{t} \\ -2 c_{t} \varepsilon_{r}{ }_{r} \end{gathered}$ | $\begin{gathered} \left(1-a_{t}\right) a_{r}+\left(1-a_{r}\right) a_{t} \\ -2 c_{t} c_{r} \end{gathered}$ | $\begin{aligned} & \left(1-a_{r}\right)\left(1-\sigma_{t}\right)+\sigma_{t} a_{r}+ \\ & \left(2 a_{r}-1\right)\left(2 a_{r}-1\right)-6 c_{t} c_{r} \end{aligned}$ | 0 | 0 | $25_{\text {f }}^{5}$ |
|  | 4 | ${ }^{2 c} c_{r}{ }^{\text {r }}$ r | $2 c_{f} c_{r}$ | $\left.2!\left(2 a_{r}^{\prime}-1\right)\left(2 a_{f}-1\right)-2 c_{r} c_{r}\right]$ | 0 | 0 | $-2 s_{\text {r }} \mathrm{c}_{\mathrm{r}}$ |
| ज | 5 | 0 | 0 | 0 | ${ }^{-2\left(c_{r^{5}}{ }^{5}+c_{4}{ }^{5}\right)}$ | $2\left(c_{r^{3}+c^{3}+c_{r}^{5}}\right)$ | 0 |
|  | 6 | $\left.2!\left(1-a_{r}\right) c_{r}+\left(1-a_{r}\right) c_{r}\right]$ | $-2\left[a_{r} c_{+}+o_{f} c_{r}\right]$ | $6\left[\left(2 a_{r}-1\right) c_{t}+\left(2 a_{t}-1\right) c_{r}\right]$ | 0 | 0 | 0 |
|  | 7 | 0 | 0 | 0 | $\begin{array}{r} -2\left[\left(1--_{r_{1}} s_{\mathrm{t}}+\right.\right. \\ \left(1-\mathrm{a}_{\mathrm{f}} \mathrm{~s}_{\mathrm{r}}\right] \end{array}$ | $-2\left[a_{r_{t}}+a_{r_{5}^{5}}\right]$ | 0 |
|  | 8 | $2\left[a_{t} c_{r}+a_{r} c_{t}\right]$ | $-2\left[\left(1-a_{r}\right) c_{t}+\left(1-a_{t}\right) c_{r}\right]$ | $-6\left[\left(2 a_{r}-1\right) c_{t}+\left(2 a_{t}-1\right) c_{r}\right]$ | 0 | 0 | 0 |
|  | 9 | 0 | 0 | 0 | $-2\left(a_{r_{r}}^{s}+a_{r^{s}}{ }^{s}\right)$ | $\begin{gathered} -2\left[\left(1-o_{r} s_{\mathrm{s}}+\right.\right. \\ \left.\left(1-a_{\mathrm{t}}\right)_{\mathrm{r}}\right] \end{gathered}$ | 0 |
| NOTE: The different plares of $\mathrm{C}_{\mathrm{ijk}}, i=1,2, \ldots, 15$, are formed by substituting values for $a_{r}, \beta_{r}, a_{r}$ and $\beta_{r}$ from Toble D.2. |  |  |  |  |  |  |  |

let $\gamma_{i}(\omega)$ denote its evaluation on the $\Omega_{\omega}$ annulus. Furthermore let

$$
\begin{equation*}
K_{k 1}=\sum_{\omega=1}^{N} \gamma_{T}(\omega) J_{k}(\omega) \tag{D-18}
\end{equation*}
$$

where $J_{k}(\omega)$ denotes the evaluation of $k_{k}$ on the $\Omega_{\omega}$ annulus. Then the return power is given by

$$
W_{j}\left(\theta_{0}\right)=\left[\frac{\lambda^{2} G_{t} G_{r}}{(4 \pi z)^{2}}\right] \operatorname{tr}\left[\sum_{k=1}^{6} c_{i j k} K_{k J}\right]_{j \tau}(D-19)
$$

Subroutine EXACT performs the above computation for $i=1,2, \ldots . .15$. For each measurement (i) the contribution by each $\gamma_{j}$ is isolated by EXACT and stored in its second argument. The power matrix is clearly given by

$$
\begin{equation*}
P_{i j}=\frac{\lambda^{2} G_{t} G_{r}}{(4 \pi z)^{2}} \sum_{k=1}^{6} C_{i j k} K_{k j} \tag{D-20}
\end{equation*}
$$

The strucfure of the return can thus be examined.
Another routine, called IDEAL, also estimates the return power but without regard to the distinction between surface and antenna polarizations. The computation follows the above scheme, however, it recognizes that $\psi=0$. In this case all the necessary information is carried in $J_{6}(\omega)$. Again IDEAL isolafes the power contributions by each scatter irg zoefficient and consequently forms a power matrix also.

## 2.n The Inversion Models

The above formulation of the return power was designed so that the exact or approximate inversion model paramefers could be isolated from intermediafe steps. Ether model assumes that the measurement can be approximated by

$$
\begin{equation*}
\sum_{j=1}^{\sum} \|_{i j} \gamma_{j}\left(\theta_{0}\right)=W_{j}\left(\theta_{0}\right) \tag{D-21}
\end{equation*}
$$

where $M_{i j}$ is a $15 \times 9$ matrix. Each row of $M$ corresponds to one of the fifteen measurements. When the distinction between surface and untenna polarizations is required, the elements of $M_{i j}$ are simply filled by forming

$$
\begin{equation*}
M_{i j}=\sum_{k=1}^{6} c_{i j k} \sum_{\omega=1}^{N} j_{k}(\omega) \tag{D-22}
\end{equation*}
$$

$M_{i j}$ is constructed in subroutine EXACT. Once $M_{i j}$ has been constructed the inverse model is computed by forming the normal equations $M_{i j} M_{i k}$ and computing its inverse (HEMINV in the mainline). The inversion for this model is performed in subroutine MATRIX. Within MATRIX a least squares solution is executed, viz.,

$$
\begin{equation*}
\gamma_{i}^{\prime}\left(\theta_{0}\right)=\left\langle M^{\dot{T}} M\right)_{i j}^{-1} \sum_{k=1}^{15} M_{j k}^{\dagger} W_{k}\left(\theta_{o}\right) \tag{D-23}
\end{equation*}
$$

This subroutine also accumulates the first and second order error statistics during a Monte Carlo study. A call to a secondary entry MATSHOW will display the statistical results.

Similarly when the distinction between polarization frames is not required, an approximate inversion model may be formed from Equation D-22 above by simply setting $\psi=0$ in each $J_{K}(\omega)$ (See Equation D-16). Symbolically we have

$$
\begin{equation*}
M_{i j}=\sum_{k=1}^{6} C_{i j k} \sum_{\omega=1}^{N} j_{k}(\omega, \psi=0) \tag{D-24}
\end{equation*}
$$

The integrals need not be re-evaluated since all the desired information is contained in $J_{6}(\omega)$. Particularly $J_{1}\left(\omega_{5} \psi=0\right)=J_{5}\left(\omega_{4} \psi=0\right)=J_{6}(\omega)$. The remaining $J$ are identically zero. These special properties were recognized and accordingly a routine (IDEAL) was prepared to evaluate the elements of $M_{i j}$ for this case. The inversion of this model is performed as suggested in Chapter 6. Recall that the $\left.\langle | S_{v}\right|^{2},\left.\langle | S_{h h}\right|^{2}$ and $\left.\left.\langle | S_{v h}\right|^{2}\right\rangle$ are each computed from a single observation (a row of $M$ ). The remaining coefficients are computed by differencing pairs of equations (rows). The inversion for this model is performed in subroutine DIFFER. Again first and second order error sfafistics are accumulated. They are displayed by calling the secondary entry DIFSHOW.

The reader should note that routines EXACT and IDEAL play dual roles. Either can form their respective inversion models or they can compute the return powers for the fifteen kinds of measurements. Only EXACT computes the exact return power since she scaffering coefficients are defined with respect to the surface polarizations. The option to use IDEAL to compute return power exists to compare the two polarization frames.

### 2.3 Documentation for SCATSIM

A macro-flow chart of program SCATSIM is shown in Figures D. 1 through D.4. The program is organized into roughly four functions. Each figure covers one of the program functions. Figure D. 1 documents the part of the program which reads the instruction card and initializes various parameters for use in the actual simulations. This portion of the program, once validating the instruction, prepares various descripi:ye antenna paramefers such as beamwidfh and gain. These parameters and the input parameters are displayed to document the case study. The second zero in the pattern function is employed to establish the domain of integration. The domain is broken into N half degree annuli. On each annulus $J{ }^{\prime}(\omega), k=1,2, \ldots 6$ and $\omega=1,2, \ldots, N$ is computed. Once $J_{k}(\omega)$ are formed, $K_{k}, \sum_{w=1}^{N} J_{k}(\omega)$ and $\sum_{w=1}^{N} J_{k}(\omega, \psi=0)$ are computed and scaled for the antenna gain effect.

Following the above initialization, the program simulates the fifteen measurements under ideal antenna specifications. The strucfure of this program portion is illustrated in Figure D.2. Subroutine ANTENNA is called with zero arguments to prepare an ideal antenna parameter set $\left\{a_{t}, \beta_{t}, a_{r}, \beta_{t}\right\}_{i}, i=1,2, \ldots, 15$. Subroutine IDEAL forms the approximate (ideal) inversion model and a power matrix which ignores the distinction beiween surface and anfenna polarizations. Both the inversion model and the power matrix are displayed. The inversion model is stored for subsequent use by DIFFER. Subroutine COEF forms $C_{i j k}$ from the ideal antenna parameter set, in furn, EXACT then uses $C_{i j k}$ to compute the exact inversion model, $M_{i j}$. It is again called to form the exact power matrix. The normal equations are prepared from the moitel and then is inverted by HEMINV. If the system is singular, a flag (ISING) is set true. All matrix inversions are subsequently by-passed by an appropriate test. The exact inversion model and the inverse of the normal equations are stored for subsequent use by MATRIX. Once the return power is computed, both difference and marrix inversions are performed and the statistical (accuracies) results are shown for the ideal antenna.

If the bias parameter ABIAS is non-zero, a bias error study is performed. This portion of the program is illustrated in Figure D.3. The processing follows, for the most part, that performed in characterization of the ideal antenna; however, the two bias parameters ABIAS and BBIAS are employed in the arguments of ANTENNA to infroduce pattern deviations from the ideal case. The inversions are performed using the ideal antenna models.

If the perturbation parameter AMAX is non-zero a Monte-Carlo study is performed.


Figure D.la - MACRO-FLOW CHART FOR SCATSIM PROGRAM INITIALIZATION AND ANTENNA PARAMETERIZATIONS


Figure D. $1 \mathrm{~b} \sim \mathrm{MACRO}-\mathrm{FLOW}$ CHART FOR SCATSIM PROGRAM INITIALIZATION AND ANTENNA PARAMETERIZATIONS


Figure D. 2a - MACRO-FLOW CHART OF SCATSIM - COMPUTATION OF THE IDEAL ANTENNA RESPONSE


Figure D. 2 b - MACRO-FLOW CHART OF SCATSIM - COMPUTATION OF THE IDEAL ANTENNA RESPONSE


Figure D. 3 - MACRO FLOW CHART FOR SCATSIM COMPUTATION OF THE BIAS RESPONSE


Figure D. 4 - MACRO FLOW CHART FOR SCATSIM COMPUTATION OF THE MONTE CARLO STUDY

The documentation for this portion of the program is illustrated in Figure D:4. The course of this portion of the program is identical to the bias study except that many cases are examined. The number of cases is specified on the instruction card. For each case the antenna paramefer set is perturbed randomly within subroutine ANTENNA.

### 2.4 Program Listing and Sample Output

The source listing of SCATSIM is shown in Figures D. 5 through D. 18. Sufficient comments have been inserted to identify variables with the theory and to track the operation of the program. A sample output is shown in Figures D.19a through D.19F.


## 

162

## scat simulation proghah

THIS PPOGFAH ENIANLE= THE USER TO STUDY THE PERFGRHAHCE OF HIS SCATTEFOHETER ONTEMNA WHEN IT POSSESSES LESKAGE PROALEHS FFOM THE ORTHOGONAL PQLARIZATION OR HMEM ITS POLL IZATICH PRGPERIIES AREN T KHOHN HITH CIL ATTEPT AHVONE DF THE FIFTEEN HEASUREMENTS AS GESCRTGED IM the REFOPT GY J.P. CLAASSEN ENTITLED
(CR

1. thf useg has foge ehoices of antenea ATTEPHS 4S SPECIFIEJ OY ITYPE $=1,2,3$ OR $4 *$ THE TYPES CONGESROAJ TO THE LAHEC A PATTEYNS UF TYPE =-1/2.0,1/2,1 AS CESCRIGED IN THE GAOCQ HLNOYODK, GOVEOMOPED 日Y ThL IAPUT PagAHETEY KA. THE GEAMHIDTH 15 PELATL? TO MA ifi jiatement NC. $\qquad$ OF THE F MHOGTH THE VITH AHGLE AT HFTCH THE USER HISHFS TO COHDUGT -IS STUOY IS SPECIFIED IN TNOT. THE OUTCOHE OF THE IHULATIOR IS JASED ON A SCATTERING CHARACTEQTSTL ©IMILAP TD THAT OF TRE SEA. EY EFDLACING SUZPCUTIHE SIGHA, THE USER MAY INTRCJUEE AHOTHER CHAFACIEPISTIC, MOTE THAT THE ROUTINE MISI COMPUTE



VETTOA POLAGTZATION ANC IS COHTROLLED HIT
 THEE OUTCCHE OF THE SCAT NEASUREMENTS YHEN SHALLISN CFRTAIUTIES IN THE AMPLTTUDE ANE PHASE PFOPERTIES OE IHE AHIEMHA EXISI, INFUT PARMETEOS AHAX AND BMAX Way DE SFECIFIEJ TO TE OTHER THAV ZERO. HHEN AHAX=0 SILD RHLY=O+ IT IS ASSUHEC THAT NO SLSH STUDY IS OEEIPEL. THE COHTRAIMTS DH THE OIAS AND RANOOH PaRAHETFRE ARE DESCRIBE [ IH SUOROUTINE ANTENNA. KHEH BIASFS fRE mDt-7ERO JHE MOHTE CARLO studtes are tomougifu hith elases insertio.

POE = THTEGFAL (PATTERN*COS (THETAB * * 2

POBS = ICFAL OSSENVATION HATFIX
PACT $=$ ACTUAL ORSEPVATIOH MATRIX HITH PERTUREATIOHS
PINV
PINV = INVERSE OF THE NCRHAL EQUAT IONS FORHED
from poes
SC = agrual gCatteping coefficients at thot
THOT = VTEH AHGLE
ITYPE = FNTENNA TYPE
KA $=$ ANTEHAA HOFRALIZEG RACIUS
AT $=$ RELATIVE GAIM OF HORI ZONTAL PATTERH
DURING XHISEIOM
ar = RELATIVE GAIN OF HORIZONTAL PATTERN

00000070
00000880
0000199
00000100
0000110
00080120
00000130
00000130
30000150
0000150
00000170
00009180
00009180
00000190
03000190
3000200
0060210
0000220
00090220
00090230
0000240
0000250
000002 EO
00000270
00000280 000 030290 0000300 00000310 00004320 00010 00070750 0000036 00060370 00000 בAD 03000290 00000400 00000410 $000[420$ 0000530 09000440 0000450 $0 C O C O 4 E$ 00000470 00000480 0.0000490 0000500 00000510 00000520 0000 O30 0000540 0000560 00000570 0.0000580

Figure D.5a - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - MAINLINE


Figure D. 5b - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - MAINLINE
3000 FORHATCIM1/'* SCAT STUDY FOR VIEH AHGLE OF*.*
C COMPUTE THETASHAX:*
C IFIJUMPIt GO TO 30
GINTH = SNCOCITYPEI /KA
CoSTM = SGNT(1.0 = SInTM SInTH:
\#HAK = ATAHG SINTH,COSTH 1/EEG
\#HAX = ATAH( SINTH,COSTH 1/EEGG, 2,0

```

```

C CCHIUIL LITESHA PARAMETERS.
GAIU\#\#2*OGS

```

```

        HEAH = HIOTH{ITYPE)*PI/IKA*CEGG
    30
CoIAF = 10* =\&LOGIDIANIAS*1*OE-2A)
COIAR = 10**GLOGIDIANIAS*1,OEE-2G)

```




```

    44.1,1x,7F10.2/1/
        GHAX = GFPASE*DEG
        GHAX = GFPASE*DEG
    C
HRITEC6,5000)
IF(JUHP1, ANO, JUHP2) EU TO 295
HRITEC6,E000!

```

```

        -PRECISICN'//I
    compute humber of sampling annul.*.
A) PIGHT OF EORESIGHT.
IHCR = 2.0 * TMAX
B% LEFT DF GORESIGHT.
35
IF| THOT - IHAX 135.40.40
IHCL m 2.0 +THOT
50 T0 50
IHCL x IMCR
C
C
F }\begin{array}{l}{105}<br>{206}<br>{107}
log
107
110
141
112
1i\#
114
115

```


```

4000

```
00001110
09001120
00001130
0001140
00001150
\(00001 \pm 60\)
\(0001 \pm 60\)
001180
0001180
00001190
0041190
00001220
00001220
40091230
00051240
00001250
00001260
0001270
0001280
00001290
00001300
04001310
\(0 \operatorname{coc} 3 \pm 10\)
00001370
09001330
00011340
00001750
00001360
0001370
00001380
00001390
0001390
0001400
00001410
00014120
00001420
09001430
00001440
00081440
00001450
00001450
00091460
\(00014+460\)
00001470
00001470
00001400
00004500
0001490
00021500
00021500
00001515
30001512
00001520
00001520
00001530
00001540
00001540
00001540
00001550
000150
00001550
00004500
00001560
00001570
00001570
00001580
00001599
0001600
0001600
00002610
00001620

Figure D. 5 c - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - MAINLINE
\begin{tabular}{|c|c|c|c|c|c|}
\hline & T014401 & 02－15－75 & 23．0．99 SCAT StMUlattok prograh & & 4．4日E \\
\hline & 157 & C & & & 00001630 \\
\hline & 156 & c & COAPUTE HIOPOINTS CF SAhPlINT STRIPS． & & 00001640 \\
\hline & 159 & c & & & 00001650 \\
\hline & 160 & &  & & \(00001 \in 60\) \\
\hline & 161 & & ITEMP＝INCL＋ 1 & & 00001670 \\
\hline & 162 & & co ma Imitincl & & OOODJE日G \\
\hline & 16.3 & & THETA（I）＝THOT－（ItEhP－I ）2．al & & 00091690 \\
\hline & 164 & 64 & continue & & 00001700 \\
\hline & 165 & 70 & \(0080 \quad I=1 . I N C R+1\) & & 00001720 \\
\hline & 156 & &  & & 00001720 \\
\hline & 167 & 80 & continue & －． & 0.0001730 \\
\hline & 159 & c & & & 00001740 \\
\hline & 159 & c & clear accumulators & & 00061750 \\
\hline & 170 & c & & & dodob 1760 \\
\hline & 171 & & Do 85 \(\mathrm{I}=1.9\) & & 00001770 \\
\hline & 172 & & FSCIIII \(=0.0\) & & 900017月0 \\
\hline & 173 & & PBIII）\(=0.0\) & & 00001790 \\
\hline & 174 & & \(0085 \mathrm{Jxi.6}\) & & 000．thoo \\
\hline & 175 & & PCEdJ， \(11=0.0\) & & 00001810 \\
\hline & 176 & & FSCE（J，t \(=0.0\) & & 00001820 \\
\hline & 177 & － 85 & COhtitue & & 00021830 \\
\hline & 170 & & CO 250 II \(=1\) \％ITOTAL & & 00097840 \\
\hline & 1.9 & & I＝II & & 00001150 \\
\hline & 189 & & & & 00001860 \\
\hline & 181 & C & A）LIMITS ON COS（Thetaj & & 00001870 \\
\hline & 102 & c & & & 00001880 \\
\hline & 183 & & THETAR＝THETA（t） 4 GEG & & 00001890 \\
\hline & 184 & & \(\times 2=\cos\)（THETAR－DELTA） & & 00008900 \\
\hline & 145 & & IF（THETAR ．LT， 0.00011 K2 \(\times 1.0\) & & 00901910 \\
\hline & 186 & &  & & 00001420 \\
\hline O & 187 & \({ }_{6}^{C}\) & & & 00001930 \\
\hline & 188 & 6 & 日1 LIMITS OH PHI． & & 00001940 \\
\hline & 189 & c & & & 00001450 \\
\hline & 190 & & CENOM＝SINTTHETAR）\({ }^{\text {S S INTM }}\) & & 00001960 \\
\hline & 191 & &  & & 00001570 \\
\hline & 192
193 & &  & & 00001980
00009990 \\
\hline & 194 & 90 & PHI \(=\) PI \({ }^{\text {ctat }}\) & & 00002400 \\
\hline & 195 & & GOTO 110 & & 00002010 \\
\hline & 176 & 160 & FHI＝atanze SORT（ ABSEA，O－GOSPHI＊COSPHI） 1 ， & & 00002 c \\
\hline & 197 & 4 & CoSphi） & & \(00072 C 30\) \\
\hline & 198 & C & & & 00002040 \\
\hline & 199 & c & c）SET NO．Of IMTEGAATION DCMAL & & 00002050 \\
\hline & 200 & C & & & 00002060 \\
\hline & 201 & 110 & INCT \(=\) PHI／OEG＊ 1 & 1 & 00002070 \\
\hline & 202 & & \[
\text { IF(INCY GT: 32) INCY }=32
\] & 1 & \[
00002080
\] \\
\hline & 203
204 & C & c）InItIalize convergence testing paraheters． & & 0.0022900
00002100 \\
\hline & 205 & c & & & 00002110 \\
\hline & 206 & & 如 \(114 \mathrm{~J}=1,6\) & & 00002120 \\
\hline & 207 & & 0（J）\(=0.0\) & & 00002130 \\
\hline & 200 & 114 & COhttave & & 00002140 \\
\hline
\end{tabular}

Figure D． 5 d －FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM－MAINLINE


Figure D. 5e - FORTRAN LISTING FOR THE SCATTEROMETER SIMLLATION PROGRAM - MAINLINE
\begin{tabular}{|c|c|c|}
\hline 7014t 00 & 02－15－75 & 23．0ng SCAT SIMLLATION PROGFAH \\
\hline 261 & &  \\
\hline 262 & & P㫙（J，I）＝PGE（J，I）\％FACTOR \\
\hline 263 & 160 & cont inue \\
\hline 264 & C & HAITE（6，SiOQ）FACJOF，POI，PSCI \\
\hline 26.5 & C &  \\
\hline 255 & c & CALL HA\＃CUT（PSCE， \(6,9.6,9,4 \mathrm{PSCE}, 3 \mathrm{HSCA})\) \\
\hline 267 & c & forh ideal pehitl gean heights ano poner hatrix \\
\hline 269 & & WRITE（G， 7 ODO \\
\hline \(0^{257}\) & 7000 & FOPMATITH1，＂TOEAL ARTEIMA HEIGHTS AND POHER HATRIX＊： \\
\hline P\％ 270 & & CALL ANTFNAA（0．0，0．0，0．0，0．0） \\
\hline E2271 & & CALE IUFAL（PGI，POBSI！ \\
\hline \(7{ }^{7}\) Q272 & & CALL ISEALSPSCI，PACt） \\
\hline \％ 273 & C & GPPFMO POHER VEGTOR TO PAGT FOR OISPLAY \\
\hline 2274 & & \(0170 \mathrm{I}=1.15\) \\
\hline 275 & & 口ACT（Ifics＝HIT） \\
\hline 275 & 170 & comithue \\
\hline \(0^{277}\) & &  \\
\hline Ery 270 & & CALL HATCUTPACT，15， \(50,15,10\)－5HPOHER，GHHAIRIKI \\
\hline 2 279 & c & FIDPH EXACT PENCIL HEIGHTS AND POHER HATRIX \\
\hline 9280 & & HPITf（6，7500） \\
\hline 2－1 24 & 7500 & FgPMatilhi，ExACT ANTEHNA HETGMTS ANO POHER RETURNS＊ \\
\hline 282 & c &  \\
\hline \(6{ }^{683}\) & C 7100 & FOPHAT（／／1514E12．4／1） \\
\hline 28.4 & & Call（daf（c） \\
\hline 235 & \(C\) &  \\
\hline 276 & C 6100 & FOFHAT（／f＇（EE12．4／1） \\
\hline 287 & & CALL EXACTIPPE，POSSE，CiH） \\
\hline 2月8 & & CALL HATCUT（POESE，15，9，15，9，5HEELTA，6FHEIGHT） \\
\hline 289 & C & frepare hormal enis ． \\
\hline － 270 & & co 180 I \(=1.9\) \\
\hline 291 & & ［0 1月0 J＝1．1 \\
\hline 292 & & PIHV（I．J）\(=0.0\) \\
\hline 293 & & \(00175 \mathrm{~K}=1.15\) \\
\hline 294 & &  \\
\hline 295 & 175 & COMTIUUE \\
\hline 236 & &  \\
\hline 297 & 180 & continue \\
\hline 298 & \(c\) & COMFUTE AND CISPLAY INUERSION HATRIX \\
\hline 299 & &  \\
\hline 300 & C & FOFM EXACT POHEQ HATQIX \\
\hline 391 & 1.15 & CALL EXACT（PSCE，PACT，CiSh \\
\hline 302 & c & APPEND POHEF VEGTOR tO PAGT for oisflay \\
\hline 303 & & C0 \(190 \mathrm{I}=1,15\) \\
\hline 304 & & PaCtitaiol \(=\mathrm{HIT}\) \\
\hline 305 & 190 & contituve \\
\hline 306 & & CALL．HATCUT（FACT＋15，10，15，10，5HPOHER，GHHATRIXI \\
\hline 307 & c & COMPUTE SGATTERIHG COEFFICIENTS \\
\hline 304 & C & Al by the difference method． \\
\hline 309 & & HRITE（6，6500） \\
\hline 310 & 6500 & FOhHET（1－1） \\
\hline 311 & & CALL DIFFER \\
\hline 312 & & CALL EIFSHOH \\
\hline
\end{tabular}

\section*{00002670}

00002680
00002690
00002700
00002710
00002720
00002730
00002750
00002760
00102770
00002780
00002790
000 C 2 HOO

\(00002 A 10\)
\(00002 月 20\)
00002420
\(00022 A 50\)
00002140
00902050
00002 F 60
03002870
000021580
\(00002^{\circ} 90\)
01002900
00002910
00002920
00002530
00002940
00002950
00012960
00002580
00002990
00002990
00003000
00003010
0 003010
00003020
0000320
00003030
00003030
00013040
00003050
00 CO 3060
00007070
0 ODCFOBO
000 O 3990
000 E 100
00003110
00003120
00003130
00003140
00003150
00003160
00003170
\(00003 i 30\)

Figure D． 5 f －FORTRAN LISTINGS FOR THE SCATTEROMETER SIMULATION PROGRAM－MAINLINE


Figure D. 5 g - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - MAINLINE
CAhtenna subpout me antenna
subroutine antennatamax, abias, bhax, beiass
gubpout ine antemna
subroutine antentatamax,abias, bmax, beias
this sugpoutine syathestzes the relative ahplytudes ano phases of the thn orthoignally polarizeo antenna fatterns guring transhisssion t ano recepition r rod the fifteen stamaifo measurements. the input arGUFATS PEEMIT THE FRECISE ANTENNA FEDUIREMENTS FOR Each of the fifteen measurement conditions to be perturues hith giases eithef fixeo di randon or both amaximaxiyum random perturgaticn introduceo into the rafzontally polapizeg pattern
abtasebias introduged into the horizontally polagiled pattepa

EHAXFMAXIMUM RLNJOH PERTURBATION I TRODUCED THTO THE felative phase ietheen oqthogchaley polarized FATTEPNS
gias=eins introjuced into the relative phase eEtheen opthigohally polarized pattern

If The IHPUT afguments are set to zero, prectse antranta reoulrements are estálishel in the output UECTORS:

Artf: =cELATIVE ampitituae of tre horizontally polar GEZED PATTERH DURIMG THANSHIESIGN(RECEPTION)
etips=pelative phase eetheen ohthozonal polarization CUPING TPAHGNTSSEOAREGEPTICNI

OTMPKISE, PFRTUPGATIONS ARE IBTROQUCED IN AGCORD WIYH ALGORITHMS EELOH. BIASES APE EFFECTIUE ONLY HFTHEFIFTELN MEASUREHENTS. IT IS ASSUHEO THAT DE EAKAGE OR CE-POLATIZATION IS THE CAUSE OF THE eiases. the user mist agserve that

11AHAX GT. D.
cragas .gT: 0.
3) AHAX 4 AEIAS LEE. 1.0
5)-PI.GF. BMAX LE. PI
the randon perturgations are oistrieuten uniformily VEP (0, AHAX/2)
IF AT OR AR IS ZERO IN THE UNPERTURRED GASE, AND

00003730
00003740
00.d03750

00003760
00003770
00003760
\(0000=790\)
00003400
00003 F 10
00003820
00023850
00003240
000 hso
00003 60

00903990
00003900
00003910
00003920
00003930
\(0000 \div 940\)
00003950
ancol960
0009470
\(000 \geq 980\)
07003490
00004000
00094510
00004020
9074030
00004040
00004559
00304970
00004080
00004990
00094100
\(009041: 0\)
0000120
00074130
00004140
00034150
00004160
0004770
00004180
00014190
0004200
0004220
00004220
00004230
00004240

\section*{CVEF [1-AMAK/211}
IF AT DP AR IS GHE IN THE UNPERTURBEO GASE, ANO CVEP 1.5-AHAX/2.,5+AHAK/2)
IF AT Oq AK IS.:5 IA THL UNPERTLIROEC CASE. RANDOH PRISES ADE DISTRIBUTEO UNIFGRMILY OVER. (ET (R)-gHAX/Z.ET(R) +BHAX/Z)

00004250
00004260
00094270
00094270
00004980
00004.80
0004290

08004290
00004300
00004310
00004320 0 100 4330 00004340 00004350 00004360 00074370 00004380 00004390 00004400 00004410 00104420 \(00-104430\) 000044450 00074450 00004460 00004470 00804480
00004490 00004500 00004510 00004520 00004530 00004540 00004550 00004560 00004570 00904580 00004590 00004500 TOOO4E10 \(00004 E 20\) \(09004 \in 30\) 00004650 00004 E60 \(00004 E 60\) 00004670
\(00074 E 80\) \(00094 E R D\)
\(00004 E 90\) \(00004 E 90\) 00004700 00004710
00004720 00004720 00004740 00004750 00004760

Figure D. 6 b - FORTRAN LISTING FOR THE SCATTEROMETER SINULATION PROGRAM - SUBROUTINE ANTENNA


Figure D. \(6 \mathrm{c}-\mathrm{FORTRAN}\) LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - SUBROUTINE ANTENNA


SUBROUTINE COUTITE Coumour Cre culc
COMHOH／CNE／H（15），POBSIC45，9），POESE（15．9），PTHU（9，9），SC（9）
LAEEL（10），AT（15），AP（15）．ET（251，BP（15）；SINTN．COSTN，
KA，ITYPE
DIMEHSICH C（E．f．15）
THIS ROUTINE PREPARES THE ANTEANA PATTERN
FAGTDPE FBOH THE ANTENNA GAIHS ANO FHASES SET
\(C\)
\(C\)
\(C\)
\(c\)
EY SUBPOUTINE AMTENNA
\(0101=1,15\)
ati＝1．0－atit）
SA＝2．0＂SqFT\｛AT（I）FATI）
cot＝cnccoster（I）


二a＝zecesjotitalil－aril

CBF＝SA＊CGS（RFif）
COTGPP＝CETFC日R
\(C(4+1+I)=C 口 t c e r / 2.0\)
\(4 R 2=2.0\) AR \(11 /-1+0\)

C
\(\mathbf{C}\)
C
SIGMA－VU
C\｛1，1，If＝ARI＊ATI
\(C(1+1+1)=A R I * A T I\)
\(C(1+2,1\}=A S I I+A T(I)\)
\(C(1, \pm, I)=A P_{1} \psi A T(I)+A T I * A R(I) * C B T C B R\)

SIGHA－HH
\(C(2,1, I)=C(1,2, I)\)
\(C(2,2, I)=C(1,1, I)\)

\(C(3,1, I)=C(1,3, I)-C(4, I+I)\)
C（3．2．IIIC（3．1．I）

 CaLL FXOP \(467+1+0+0)\)

00005140
00005150
00005160
00405170
00405170
00005180
07005198
00045200
00005220
0005230
00015240
00005250
000052E0
00005270
00005280
00005790
\(0005=200\)
00005310
00005220
0005330
00005340
00 CF 550
09075370
09075370
00005390
0005340
00095400
00005410
00005420
0000 C4． 30
00005440
0907550
09005460
00005470
000054100
00005490
00005500
9000510
\(01005=20\)
001553

09005550
3005550
0－10．55 70
00005540
00055500
00005590
00005600
DORO5E10
ORDOSE20
C（4，1，\(I)=\operatorname{cetcse/2.0~}\)
00056310
\(\therefore\{4,2, I \|=C\{4,1, I)\)
C\｛4， \(3,11=2,0 * A T 2 A R 2-C E T C B R\)
00005640
00005650

Figure D．7a－FORTRAN LISTING FOR THE
SCATTEROMETER SIMUULATION PROGRAM－SUBROUTINE COEF
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline 10144 01 & 02-15-75 & 23.093 Sugroutine coef & & label coef & page & 2 \\
\hline 53 & c & & & 00005660 & & \\
\hline 54 & c & imagimary sigha vyh & & 00005670 & & \\
\hline 55 & c & & & 00005680 & & \\
\hline 55 & & \(\mathrm{C}(5,4 * \mathrm{I})=\) CCBT*SBP4CER*SET \(1 / 2.0\) & & 00005590 & & \\
\hline 57 & & \(\mathrm{C}(5,5.1)=-\mathrm{C} 5,4.11\) & + & 00005700 & & \\
\hline 58
59 & \({ }_{c}^{\text {c }}\) & heal sigha yuhy & & 000005710 & & \\
\hline 69 & \({ }_{c}\) & & & 00005730 & & \\
\hline 81 & &  & & 00005740 & & \\
\hline 62 & &  & & 00005750 & & \\
\hline 63 & & \(\mathrm{C}(6,2,1)=-\mathrm{C}(\mathrm{B}, 1, \mathrm{I})\) & & 00005760 & & \\
\hline 64 & &  & - & 00005770 & & \\
\hline 65 & & & & 00005780 & & \\
\hline 65 & c & jhaginary sigha yuhy & . & 0.0005790 & & \\
\hline 67 & c & & & 00005800 & & \\
\hline \(6{ }^{6}\) & &  & & 00005810 & & \\
\hline 69 & &  & & 10005920 & & \\
\hline 70 & c & & & 00005830 & & \\
\hline 72 & c & feal sigma uhhe & & 00005040 & & \\
\hline 72 & c & & : & 00005950 & & \\
\hline 73 & c &  & & 07005960 & & \\
\hline 74
75 & & \(C(8,2,1)=-C(6,1 * I)\)
\(C(R, 3=1)=-C(6,3,1)\) & & 00505470 & & \\
\hline 75 & &  & & 00005840 & & \\
\hline 76 & \({ }_{c}^{c}\) & Ihagimary sigha vhih & & 00005990
0005900 & & \\
\hline 78 & \({ }_{c}\) & Imaginary sigha vimu & & 00005910 & & \\
\hline 79 & & C(9,4, 1 ) \(=\mathbf{C}\) (7,5,11 & & \(000055{ }^{2}\) & & \\
\hline 80 & & ( \(69,5,1)=C(7+4,1)\) & & 00005930 & & \\
\hline 81 & 10 & contimue & & 02005940 & & \\
\hline 02
83 & & patuch
end & & 00005950
00005960 & & \\
\hline 83 & & ENO & - & 00005960 & & \\
\hline
\end{tabular}

Figure D. 7 b- FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - SUBROUTINE COEF

Figure D. 8 - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - SUBROUTINE MATOUT


Figure D. 9 - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PRQGRAM - SUBROUTINE EXACT
```

80144 01 02-15-75 23,090

```
chatal:
SUBROUTINE HATPIX SUBROUTINE MATRTX
```



```
00006540
00006550
00006560
00006570
00006580
00006590
00006600
00006610
00006620 00006630 00095640 fodateso 00006660 \(0003 E E T O\) DOOTiEESO 000 OF 90 0000 E 70 00006710 00096720 00006740 00006740 00000750
\(0000 \in 7 E 0\) 00 EPE 00006770 \(0000 F 780\) 00006790 00005 SA 00005 A10
\(0.7005 月 20\) 00075月30 00906440 0000 0850 00096 B60 00006470 ODODEABD 0000EA90 0000Fgen \(0000 F 900\)
\(0005 E G 10\) \(00016 G 10\)
00006920

``` PSUA(I) FRHS(I), T = \(1, g\) ) 0.0096930 00004940 \(000 C E 50\)
1000 FOFHAT \(/ / / 1 \mathrm{X}\), 'STATISTICS FGR THE MATRIX * 00096960
```



```
00006970
0000Eego
```




```
00005490
00097000
06007 E 10
00007020
00007020
CLEAR SUAMING VARIAELES
00007040
RHEII) \(=0.0\)
00007050
50 COMtINuE 0000706
IODS = 0
RETUP: 00007070 00007080
```

Figure D. 10 - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - SUBROUTINE MATRIX

000070.90

00007100
00087110
00007:?0 00007130 00007140 00007150 00001160 00007170 00007180 00017190 000720 00007220 00077230 00007240 ค00.17250 00007260 00007260
08097270
 00097290 00007300 00007310 00097320 00007330 00017740 00097350 00097360 00007370 00007200 DOOC $7: 90$ 00007490 00907410 00077430 00007440 00017450 00017460 00077450 0.017470
00007450
 00057490 $00 \mathrm{CJ75CO}$ 00007510
0005550 000750
$00007 \div 30$ 00007540 00007550 00007560 00007570 00007EBO 00007590 00007600

Figure D. 11a-FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - SUBROUTINE DINTEG

```
02-35-75 23.097
    MY E HHY
    MP = MHF
cRHPUFE LENGFII OF CELG STDES
M14={x2f%\1p},0.5
DELX = %2P-X1P
HDELH= LELY*D.5
DELY = 'Y2P-Y1PI/FLOAT 6HY
HDELY = CELY00.5
RJACOE = MDELH*HDELV
FORft SAHPLE FAGTOR fOR H
    00 20 T = 1,NP
```



```
    CONTINEE
            FORH SAHPEE FAGTOR FOR Y
    00 30 I = 1,44P
    SY(I) = SAMPLE{#PPI}OHDELY
    cont Imue
            FORH GAUSSIAN HEIGHTS
        CO 50 = = 1,NP
        DO 40 J=1,MP
```



```
        continue
        comiIMUE
            INTEGRATE IN STGIP OF DELH
    C0.90 I = 1,NP
    co 90 I = 1;N
    cos: = 5x(I
    sink = scorif.0-\operatorname{cosx+cosx)}
                Imtegrate al cng y
```

00007513
00007623
0007830
0007640
$0007 \leq 50$
$0007 E 50$
00007570
00007.00
00007690
00007690
00007700
00007700
00007710
0.007710
09007720
09007720
00007730
00007740
00107750
00007760
00007770
00077 Ba
00017790
00007000
00007000
00007810
00007810
00007820
0.007820
00007930
00.007930
00097840
0 0007a50
00097860
00007870
$0 \mathrm{CaC7ROD}$
00007 ag
00007900
00007910
0 D 007 O 20
00007930
00007930
00007940
00007450
$000079 E 0$
00007960
00007970
00007910
00007990
0000 ac 90
0000 el 17
$0000 \pi 020$
0000 AD 30
0000 BO 40
00008040
00004350
00074150
OCOD4070
000e日ceo
dogeaceo
00000090
00008090
00000110
00008110
00008120

Figure D. 1 lb - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - SUBROUTUNE DINTEG


```
SUBroutine OIFFER
COIFFER SUEFOUTIME CIFFER
this RDUTIHE EStImeTES ThE MEASURED SCATTERI
OEFFICIENTS BY THE SO-CALLED OIFFERENCE
HTHOD. THE ESTIHATES AFE CCHPARED HITH*THE ASTUAL
COEFFICIEMTS Amj ThF STATISIICS ARE ACCUnULATED.
```



```
    & LABEL(10), ATII5I, AR(15), ET&15I, EP(151, SINTH, COSTHa
        Ma. Itype
        GIHEHSICR SU&(9),RHS(9),PSUH(9),PRHS(9:
```





```
        IOBS = IOMS+1
```



```
            SUH(I) x SUH(I) + EFF
            SUH(I) = SUH{I) +EFR
        conTIHEE
        gozeI=49
            II=2"I-4
            |um(II = SUM(I)*ERR
            EMStI)=RHS(I)*ERK*ERR
    20 CONTInUE
        RETUPA
        ENTRY OIFSHOH
    C FECONOARY ENTR:
        co 50 1 = 1.9
                z1&9 = rum+ty/Iags
                Cum(t)= Sum(I)/IORS
                -5jM(I) = 10t.0*SUM(I)/SC(I)
                IFIIOBS =LT. 2%GOTG 25
                MS(I) = SQRT(AOS(RMS(T)/IOOS.SUHIT)*SUH(I)I)
                FOMEII)= =200.D*RHSItI/SG(I)
    25
```





```
        4 'hftmon fajfr On"*I4** OISERVATIONS*/f
```




```
        < (2x,A6,2x,3Eiz,3,2F:3,31)
            2x,3Eiz.3,2Fi2.3II
            CLEAR S
        40 40I = IFG
        40 40I = IFG
        conithue
        IOBS=0
        OETUFH
        OETU
\begin{tabular}{|c|c|c|c|}
\hline T814401 & 02＊15－75 & 23．5u0 SUPROUTIME IOEAL & LABEL IDEAL \\
\hline 1 & croeal & SUDPOUT 1NE IOEAI． & 00006890 \\
\hline 2 & & SUAGOUTIAE IOEALIPTH，PGTS & 00000900 \\
\hline 3 & c & this routine phepares an antenna observation hataiz & 00000920 \\
\hline 4 & \(c\) & and ifs doh suhj for the antehna as pheyiousiy & 00008920 \\
\hline 5 & C & SPECIFIED BY SUQPOUTITE ANTENHA & 00006330 \\
\hline 6 & \(c\) & & 00000940 \\
\hline 7 & &  & 00008950 \\
\hline 0 & 4 &  & 00008960 \\
\hline 9 & 4 & KA＊tTVPE & 00006970 \\
\hline 40 & &  & 00008940 \\
\hline 11 & c & PIHzUTGTOR COntaining fattern or pattekn and scatter & 00006970 \\
\hline 12 & c & COEFFICIENT FFFECT \(1=\) UU， \(2=\mathrm{HH}, 3=\) UH， 4 E YUHHR， \(5=\) UUHHI & 00009000 \\
\hline 13 & c &  & 00009010 \\
\hline 14 & 6 & PDT＝VECTOU CONTAIHIHG MEASUREHEST COMPOFENTS & 00009020 \\
\hline 15 & C & H＝TUY OF POH ELEHEHTS IN POT & 00009030 \\
\hline 16 & c & InITIAL SOME PARAHETEPS & 00009040 \\
\hline 17 & & \(00101=1,25\) & 00009050 \\
\hline 18 & & CATEI．0－AT（I） & 00009060 \\
\hline 19 & &  & 00009070 \\
\hline 20 & & SDT＝2．0．jnRt（CATFAT（I） & 09009000 \\
\hline 21 & & SPR＝2．0＊SQRT（CAP＊AP IIM & 0 ncogego \\
\hline 22 & & SF＝5FT＊SCP／z．0 & 00009100 \\
\hline 23 & & COERR＝G「こtoplill & 00009110 \\
\hline 24 & & cospt costat（I） & 00009120 \\
\hline 25 & &  & 00009130 \\
\hline 26 & & Stherasincoitil） & 02009140 \\
\hline 27 & C & COAPUTE THE HINE CONTPIEUTICNS & 00009150 \\
\hline 2月 & & POT（I－1）\(=\) CAT＊CAQ Fik（1） & 00009160 \\
\hline 29 & & POT（İ己） & 00009170 \\
\hline 40 & &  & 0009918t \\
\hline \＄1 & 4 &  & 00009190 \\
\hline 5 & &  & 00004200 \\
\hline 5 5 & &  & ondorge 20 \\
\hline 34 & &  & 00009220 \\
\hline 35 & & FOT（I； 7 ：\(=-1\) CAR＊SRT＊SINET＋CAT＊SRR＊SISGRI＊PIN（7） & 00009230 \\
\hline 36 & &  & 00009240 \\
\hline 17 & &  & 00009250 \\
\hline 38 & c & comhute the total ogservations & 00009260 \\
\hline 39 & 10 & COMTINUE & 00000270 \\
\hline 40 & & 0020 Ixi．15 & 00009280 \\
\hline 41 & & HIII＝0．0 & 00009290 \\
\hline 42 & & \[
6020 \mathrm{Jx}=9
\] & 00009300 \\
\hline 43 & & H（I）की（I）\＆POT（I；J） & 00009310 \\
\hline 44 & 20 & & 00009320 \\
\hline 45 & & certurn & － 00009330 \\
\hline 46 & ＊ & EHD & 100009340 \\
\hline
\end{tabular}

Figure D． 13 －FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM－SUBROUTINE IDEAL
fuic eviton sithtine sigha
OIMEHEION CIGABI
00009370
OATA TEN. DEG 110.0, 0.0174532925/ 00009390 CATA (tC(I,JI, J=1; Bi, \(I=1 ; 4)\) ) 00009400
 -0.17226f 01, - \(0.41559 E\) 01, 0.13382E 01. 0.16147E 02 \(-0.22504 E-02 . \quad 0.59789 E-01,-0.59855 E 00,0.27115 E 01\).
 0.0 , \(0.0 \quad, 0.48\) B97E-02, \(0.75403 E-010\)



\(c\)
\(c\)
\(c\)
\(c\)
\(c\)
\(c\)
THIS POUTIPE COHPUTES SLATTEPING COEFFICIENTS
CF FIVE KINDE, I=1,2,3,4,5, CCFRESPCNDING TO
 COEFFICITHTS AkE ASSUMEO TERO.
0009410
00009420
00009430
06019440
0009450
\(000094 f\)
00009470
00009470
00004480
\(0 C 004480\)
00009490
00009500
\(00000+20\)
0009
00009540
00009550
0000956
000075
        \(A=\) erten
        go roito.10, 10,20.20,40,40,40,401,



\section*{I}

 SIGHA \(=\) TEN*-SIGYA PETtidy



 दFIG. LT. 12.0160 10 25

t \(C\{2+51)+A+C(2,61)+A+C(2,71)+A+C(2, B 2) / T E N / 2-0+5 I G H A / 2+0\) IFII-GT. 41 GO ro 30
 PETURA

EETUFN
 FHO
0009 E

00009590
00009590
\(00009 E 00\)
00009620
00009620
00000630
00009 F 40
\(00009 F 50\)
00009660
000017670
00009660
00009690
00009700
00009720
00009720
0.0009730
00009750
00009740
00009750

Figure D. 14 - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - SUBROUTINE SIGMA
\(\square\)


LABEL 5alid FAGE 8

00008820
00008000
00008900
00006940
00004920 01000930 00006940
00006950 00006050 0.005960 00005970 00006900
00008990 00000990
00007000 00007000 00007020 00007030 00007040 00007050 00007040 00007050 00007080 00007090 00007100 00007110 00007120 00007130 00007140 00007100 00007170 00007170
00007180 00007180
00007190 00007190
00007200 00007200 00007220 00007250 00007240 00007250 00007260 00007270 00007240 00007290 00007100 00007310 00007320 00007340 000730 0007300 00007370 0007370 00007300

Figure D. 15a - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - SUBROUTINE SOLID 00007510
00007520
00001530
09007540
09007540
00007550
00007560
00007560
00007570
40007580
00007800
00007810
00007620
00007630
00007640
00007650
00007660
10007660
00007670
00007670
01007680

Figure D. 15b - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - SUBROUTINE SOLID
```

        77674 0.4 01:25075 96,390 SUGROUTINE LAHBDA
            Label lamboa Pate
    

```
00007600
0000700
00007750
00007770
00007730
00007740
000770
0007770
0007740
00007790
00007800
00007810
00007810
00007820
00007830
00007840
0007850
00007080
0007Ta70
0007980
100.07890
00007900
00007910
00107920
00007930
00007940
00007930
90007980
0007970
00007980
00007900
40006000
00006010 00008020
```

Figure $\mathrm{D}_{\mathrm{g}} 16$ - FORTRAN LISTING FOR THE SCATTEROMETER SIMULATION PROGRAM - SUBROUTINE LAMBDA
7797909
9
$\frac{1}{2}$
3
4
3
3
7
7
9
9
20
11
12
13
24
15
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e - cOHPUTES THE EESSEL FUNGTIOH OF INPGEER ORbER ZERO. uSES A polynomtal hethid,
0000040
000000040
00000050
00004050
00008070
00006080
FFAL $X, T_{1}, T$ T
00004090
00000100
CHECK TO SEE HHIEH APPROXIHAYION IS NEEDED.
0000 H 110

000 e 12
000013
11 : 333333353
0000014 j
71 14591
$000 \mathrm{P15}$

 RETURN
00017
90000140
0008100
00008200
$C$
$C$
$C$
$x>3,0$
000820
00008210
000 O 22
${ }^{C} 100$

00008230

000824
0.0008250
00006260

 BJZERO P BJZERU GOS\{42)/SORY\{X\} 00008280 0000824 RETUR END

Figure D. 17 - FORTRAN LISTING FOR THE SCATTERONETER SIMULATTON PROGRAM - SUBROUTINE BJZERO



## goeal antehna getghts anp poher hatri



Figure D.19b - SAMPLE OUTPUT FOR SCATTEROMETER SIMULATION PROGRAM

|  | MEAS／CEEF | F UV | H\％ | VH | UWHHP | WUHEI | UWVHR | UVUHI |  | HVHHE |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 10 | 0．4336E－01 | 0．1045E－06 | $0.1454 \mathrm{E}=05$ | $0.7473 \pm-04$ | 0. | － 0. | 0 ， | 0. | 0. |  |
|  | 2 | 0． $1075 \mathrm{E}-06$ | 0．6．736E－01 | $0.14545-03$ | $0.74725-14$ | 0. | 0. | 0. | 0. | 0. |  |
|  | 3 | $0.373555-04$ | 0.373 Ef － 04 | 0．4328F－0！ | －0．7472E－64 | 0. | 0. | 0. | 0. | 0. |  |
|  | 4 | 0．1046E－01 | 0．1046E－01 | －0．73135－29 | $0.2172 \mathrm{E}-01$ | 0．2460E－12 | $0.4502 E-1.0$ | 0. | 0．4302E－20 | 0. |  |
|  | 50 | 0.1042 E －01 | 0．1042E－04 | 0．14945－63 | －0．2164E－01 | －0．4302E－10 | 0.0 | －0．4309E－10 | 0． | －0．4309E－20 |  |
|  | 60 | 0．1504E－01 | $0.10 \mathrm{CH}_{4} \mathrm{~F}-\mathrm{DI}$ | 0．74．72F－04 | 0．3736E－04 | 0．2168E－01 | 0. | 0 ． | 0. | 0. |  |
|  | 70 | 0．1684E－01 | 0．1094E－01 | 0．7472［－04 | $0.3736 \mathrm{E}-04$ | －0．216EE－01 | 0. | 0. | 0. | 0. |  |
|  | 30 | $0.21708-01$ | 0．ta．73E－0．4 | 0．2t72E－01 | 0. | 0. | 0．432t5－01 | 0. | 0．1120E－03 | 0. |  |
|  | 90 | 0．2179F－71 | 0．18735－74 | 0．2172E－01 | 0. | 1. | －0．4324E－01 | －0．861iE－10 | －0．11245－03 | －0．74：5E－13 |  |
|  | 100 | 0.2177 －01 | 0．1ETTE－g． | 0．2172E－01 | 0. | 0. | 0．4269E－10 | D．4339E－01 | $0.11050-12$ | $0.37475-14$ |  |
|  | 110 | 0．2179t－02 | 0．1973E－04 | $0 \cdot 21725-01$ | 0. | 0. | $0.4312 E-10$ | －0．4339E－02 | 0．11175－12 | －0．374．7E－04 |  |
|  | 12 l | 9．147才t－04 | 0．21705－01 | 0．21725－01 | 0. | 0. | $0.1120 \mathrm{E}-03$ | 「． | 0．4324E－81 | 0. |  |
|  | 13 0 | $0.1873 \mathrm{E}-14$ | 0．2170E－01 | 0．71725－0i | 0. | 0 ＊ | －0．1120E－03 | － $10.7435 \mathrm{E}-13$ | －0．4324E－01 | －0．06148－10 |  |
|  | 140 |  | 0．2170E－0： | 0．2172¢－01 | 0. | 0. | 0．1105E－12 | 9．3747E－04 | $0.4269 E=10$ | $0.43 .39 E-0.1$ |  |
|  | 15 0 | $0.10735-104$ | 0．2170E－01 | 0.2172 E －02 | 0. | 0. | 0．1117E－12 | －10．3747E－04 | 0．4512E－10 | －0．4339E－08 |  |
|  |  |  |  | POHE | R HATRIX |  |  |  |  |  |  |
|  | MEAS／CEEF | F V | HH | UH | V UHHE | V UHHI | UWVHR | UVVHI | H SHEA | HVHHI | POHER |
|  |  | O．A316E 00 | 0．2113E－05 | 0．月577F－05 | 0．145EE－02 | 0. | 0. | 0. | t． | 0. | O．M3き1E 00 |
|  | 20 | 0．215 5E－05 | 0． 0316 E 03 | 0．8677F－05 | $0.1436 E-02$ | 0. | 0. | 0. | 0. | 0. | 0.9331200 |
|  | 30 | 0．717fe－03 | $0.717 \mathrm{AE}-03$ | $0.2513 E-12$ | $-0.143 \mathrm{bE}-02$ | 0. | 0. | 0. | 0. | 0. | 0．2513t－02 |
|  | 40 | 0.2043500 | 0．2593E 00 | －0．4366F－19 | $0.4165 E$ a | $0.1808 \mathrm{C}=13$ | 0．4223E－12 | 0. | $0.4223 E-12$ | 0． | $0.9331 E 00$ |
|  | 50 | 0.2075900 | $0.2075 \mathrm{O}^{0} 00$ | 0．85775－05 | －0．4154E 00 | －0．374EE－11 | 0 ． | －0．4230E－12 | 0. | －0．4230E－12 | 0．1295E－04 |
| 5 | 60 | 0.2575 E 90 | O．2074E 00 | 0．4534－95 | 0．7170E－03 | 0．160日E－02 | 0. | 0. | 0. | 0. | 0．4174500 |
|  | 70 | 0.2579 Ca |  | 0．4336F－05 | $0.7178 \mathrm{E}-03$ | －0．188日E－02 | 0. | 0. | 0. | 0. | 0.416 EE Of |
|  | 0 － 0 | O．4LECE 00 | $0.35995-03$ | $0.12615-02$ | 0. | 0. | 0．4245E－03 | 0. | 0．1049E－85 | 0. | $0.4142 c 90$ |
|  | 90 | 0.6152 E 00 | 0．35995－0．3 | $0 \cdot 12 ち 15-02$ | 0 ． | 0. | －0．4245E－03 | －0．8453E－12 | －0．1099t－05 | －0．7299E－15 | $0.4176 \% 20$ |
|  | 10 0 | $0.41 E E E 00$ | 9． 5599 E －33 | $0 \cdot 1261 \overrightarrow{-02}$ | 0. | 0. | 0．4290E－12 | 0．4259E－03 | 0．1085E－14 | 0．367 HE－36 | 0．4182E 00 |
|  | 110 | 0.415 EE 00 | 0． 5 579E－03 | 0．1251E－02 | 0. | 0. | $0.4233 \mathrm{E}-12$ | －0．4259E－03 | 0．1096E－14 | －0．3678E－06 | 0.4174500 |
|  | 120 |  | 0.4162500 | $0.1261 E-02$ | 0. | 0 ＊ | $0.1099 \mathrm{E}-05$ | 0. | 0．4245E－03 | 0. | 0．4182E 0J |
|  | 130 | 0．3539E－03 | 0．4．522E 00 | 0．1261E－02 | 0 ＊ | 0. | －0．1099E－05 | －0．7299E－15 | －0．4245E－03 | －0．84535－22 | 0．4174E 00 |
|  | 140 | $0.3599 \mathrm{~F}-03$ | 0．4ヶ6？ 00 | 0．1261E－02 | 0. | 0 ． | $0.1085 E-14$ | 0．3678E－06 | 0．4190E－12 | $0.4259 E=03$ | Q．4182E 00 |
|  | 15 D | $0.3599 \mathrm{E}-03$ | 0.4162 CO | 0．1261E－02 | 0. | 0. | $0.1096 E=14$ | －0， $3678 \mathrm{BE}=06$ | O．4233E－12 | －0．4254E－03 | 0．4174E 00 |

Figure D：19c－SAMPLE OUTPUT FOR SCATTEROMETER SIMULATION PROGRAM

Statisitcs fon thf orfference hethon easer on fobservatichs.

| scat cogr | Whlue | ¢FAR | Fus | zhedn | GRES |
| :---: | :---: | :---: | :---: | :---: | :---: |
| B4 | 0.192802 | 6.276E-02 | 0.752E-05 | 0.014 | $0 ;$ |
| H | 0.192 E - | 0. 27 fe 02 | 2.762E-05 | 0.014 | 0. |
| \% ${ }^{\text {H }}$ | 4. Sftemi | -0. $207 \mathrm{~F}=03$ | $0.429 \mathrm{C}-07$ | -0.357 | 0. |
| VWHHR | 4.-才25 02 | 0.235E-02 |  | 0.012 | 0. |
| WYHHT | $0.47 z^{2-01}$ | -0. 216 E - -13 | 0.4685-07 | -0.240 | 0. |
| UVVMP | 0. SE2E-02 | -9.173E-14 | - है795-09 | -0.176 | 0. |
| VVYHI | $0.9425-02$ | - $0.3095-06$ | $0.956-13$ | -0.003 | 0. |
| HVHHFt | 0.902E-92 | -0.173E-04 | 0.249E-09 | -0.176 | 0. |
| NVMEI | $0.302 r-02$ | -0.309F-06 | 0.95bE-13 | -0.005 | 0. |
| spatistres | Fog the mateix rejhod baseg on i observaitcns |  |  |  |  |
| Scat coef | value | MFAH | FHL | YHEAN | 4R ${ }^{\text {HS }}$ |
| v4 | 0.:92E 02 | 0.256F-02 | 0.656E-05 | 0.013 | 0. |
| +H | 0.197502 | 0.256L-02 | C.E5才E-05 | 0.013 | 0. |
| UH | - 5815-01 | -0.756F-05 | 0.c19E-10 | -0.014 | 0. |
| VUHHP | 0.193602 | 0.256E-02 | C. E5GE-05 | 0.013 | 0. |
| UUHHI | $0.272 E-01$ | -0.E62E-34 | C. 23 AE-00 | -0.0.75 | 0. |
| Wivina | 0 ¢ 5 ¢2E-02 | -0.330E-06 | 0.139E-12 | -0.003 | 0. |
| vuwht | 0.982E-0? | -0.294E-06 | 0.B65E-13 | -0.003 | 0. |
| Furink | $0.9 \mathrm{q} 2 \mathrm{2E-02}$ | -0.359E-06 | $0.129 E-12$ | -0.004 | 0. |
| HVHH? | 0.982E-02 | -0.3i7E-06 | 0.10tE-12 | -0.003 | 0. |

Figure D.19d - SAMPLE OUTPUT FOR SCATTEROMETER SIMULATION PROGRAM

## MORTE CASLO GTUGY



STATISTIGS FOR THE OIFFERENCE METHOO GASEO OA 150 OBSERUARIONS

|  | SGAT CCEF | Value | PEAN | FHS | ZHEAM | SRHS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | W4 | 0.192502 | $0.317 E-02$ | 0．113E－05 | 0.017 | 0.000 |
|  | H H | 0.192 E 02 | 0．31．15－02 | 0．119E－05 | 0.017 | 0.000 |
|  | VH | 0．cete－01 | 0．7A5E－02 | 0．305E－05 | 13．E30 | 0.005 |
|  | VWHHR | 0.192 OL | －0．675E－01 | 0．55．5E－01 | －0．352 | 0.285 |
|  | WUHTI | 0．A72E－02 | －0． $235 \mathrm{E}-\mathrm{O} 2$ | 0．644E－01 | －2．701 | 51.418 |
|  | WVYTE | 0．942E－02 | 0.171600 | 0.362 E 0 | 1739.784 | 3691．119 |
|  | YUVHE | 0．9月2ご02 | 0．344F－01 | 0.299708 | 350.417 | 4062．628 |
|  | HVHHP． | C． 5 H2E－02 | $0.162 E 00$ | $0.377 E 00$ | 4653．317 | 3839.045 |
|  | FVHMI | 0．982rio02 | － $0.224 E=02$ | 0.359800 | －64．299 | 3654．711 |
|  | STATISTICS FOD THE MATRIX HFTHOD BASED On 150 OBSERVATIONS |  |  |  |  |  |
|  | scat ceef | valle | FEALI | fris | ZHEAM | YRrS |
|  | vV | 0.192502 | －0．192E－01 | $0.222 e d 0$ | －0． 0.00 | 1.155 |
|  | H＋4 | 0.197502 | 0．15 EE－01 | 0.272 E 00 | 0.681 | 1．156 |
|  | UH | 0．541E－01 | －0．065F－02 | 0．226E Da | －14．889 | 389．728 |
|  | WYHMR | 0.15 CE 02 | －0．675E－01 | 0．554E－01 | －0．35？ | 0.289 |
|  | WVHHI | 0．e725－01 | －0．22IE－02 | 0．44．9E－01 | －2．533 | 51.506 |
| $\bar{\circ}$ | WVUHE | $0.98 P=02$ | 0.171500 | 0.364500 | 1743．189 | 4794．870 |
|  | WUVHI | $0.9425-02$ | $0.345 \mathrm{E}-01$ | 0．3992 06 | 351．193 | 4066.049 |
|  | PUHH0 | 0．9A2E－02 | 0.163 E 0 | 0.5 de ac | 1656．123 | 3n53．521 |
|  | н才ннI | 0．582E－02 | －0．e31E－02 | 0.359 ELG | －84．675 | 3657.770 |

Figure D． 19 e －SAMPLE OUTPUT FOR SCATTEROMETER SIMULATION PROGRAM

BHTLKHA BITM RIASES GFLY


## PGuER mater．



STATIStics foo the oifference methoo based on1 OBSERUATIONS
scat coef value phs rean zhean zrps

| Wv | 0．1425 0 ？ | 0．3175－02 | ＇，101E－04 | 0.017 | 0. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| HK | 0.152502 | $0.31 \mathrm{AE}-02$ | 91E－04 | 9．017 | 0. |
| Wh | －．5\＃5E－di | $9.785 \mathrm{Cl} \mathrm{Cl}_{2}$ | 0．017E－04 | 13.530 | 0. |
| W VMHR | G．15PE O2 | 0，235E－02 | 0． $0.54 \mathrm{fi}-05$ | 0.012 | 0. |
| VVHMI | 0．3725－31 | － $0.315 \mathrm{E}+73$ | 0．4645－07 | － 0.248 | 0 |
| WUWH | 0．3F2E－02 | 0．1925 30 | 0．370E－01 | 1959．587 | 0. |
| GUVME | 0．98ここ－02 | 0． 25 E － 73 | 0．755E－06 | 0.652 | 0. |
| Hy Hea | $0.7425-02$ | $0.17 \mathrm{c}=00$ | $0.370 \mathrm{E}-\mathrm{BI}$ | 1959.604 | 0. |
| WWrthy | 0．98さを－02 | G．P6＇IE－03 | 0．755E－06 | 8.852 | 0. |


| Scat caef | valle | Hean | R4： | XHEAN | ERHS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| vV | 0.152502 | 0．139E－02 | 0．199E－05 | 0.007 | 0. |
| HH | 0.142502 | 0．138E－02 | 0.170 E－05 | 0.0017 | 0. |
| YH | Qusese－01 | 0．374E－D2 | 0．14DE－64 | 6.422 | 0. |
| บVHㅐㄱ | －1．1025 02 | 0．2575－02 | 0．554E－65 | 0.714 | 0. |
| UVHht | 0．9775－01 | －0．EEZE－04 | $0.43 \mathrm{AE*OH}$ | －0．0．76 | 0. |
| YYWHa | $0.942 \mathrm{E}-02$ | 0.195900 | 0．371E－14 | 1963.14 \％ | 0. |
| UVUME | 0．942E－02 | 0． 0 C69E－03 | 0．755E－bE | 0.352 | 0. |
| ИVHHR | 0．932E－02 | 0．EI3E 00 | 0.371 E－02 | 1963．164 | 0. |
| HUHHI | O－982E－02 | 0．859E－．93 | 0．7555－06 | g．05z | 0. |

Figure D． 19 F －SAMPLE OUTPUT FOR SCATTEROMETER SIMULATION PROGRAM

## APPENDIX E

## Routine WHERE

### 1.0 PROGRAM DESCRIPTION

Fortran program WHERE was developed to compute the sampling points for apertures having maximum dimensions $x_{0}$ and $y_{0}$ across the $x$ and $y$ axis, respectively. The program will compute and list $\left(\theta_{m n^{\prime}} \varnothing_{m n}\right)$ for $m \geqslant 0$ and $n \geqslant 0$ out to values of $m$ and $n$ restricted by

$$
\begin{equation*}
0.9 \leq \cos \theta \leq 1.0 \tag{E-1}
\end{equation*}
$$

If the volve of $m$ or $n$ exceeds 48 the value is restricted to 48 to limit the storage and printed estput to a reasonable amount. The listing of the program is shown in Figure $E=1$.

### 2.0 EXAMPLERUN

The maximum aperture dimensions ( $x_{0}, y_{0}$ ) and operating wavelength ( $\lambda$ ) form the program input requirement. These must be dimensionally in the same units. An input data card containing these parameters must be prepared in accord with the read statement and its accompanying format statement.

An example output of the program is illustrated in Table E. 1 for an aperture having a maximum dimension of 1.1760 meters and illuminated at a .02158 meter wavelength.

```
6440゙5
```



```
ROUT:ME HMENE
```






```
SPECEFIES LN SAME !MTTS.
```



```
    FEAL tameqa
DATA THT*2X, DEE/O.45&9. S.C174572935/
0
C
SPECIFIET APEDPUEE OIMENSIONS AR3 HAVELENTH
GDHEEUSIANS ODOEEED IH PIGHT-HAGOED GOORDIKATE
SYSTEM &-Y-Z%
QEAD [5,IOOGI XNOT, YPCT,L_qMGCA
10E!
i500
C
C
GSTAPLISH VALID JOMRAH OE SAMPLIVG
```




```
IF (\muH&x , CT, 4%) N4*स=49
```




```
E
DETERUINE SAMPLITG EOTNTS
```



```
    10 10 I=1,4max
            qP n #-1
```



```
            RN = $ $ 1
C
FORH SIN(THETATE,J)
```



```
            IF SSINTH,FE. &.FI FO TO 1:
```



```
            IF II .EQ. 1 esmDe J EED. I) GO TC 5
```



```
            60to 20
            G070 20, %HITI.J) = u=0
            conT IHIE
                    DIS'LGY SAMPLTHG fü#TS
```



```
            A= F+7
            IF 4H वCT, MMAY; N-HMaH
        00 15 K=?%H
            KK:(K) =K-1
    15 CONTTNUE
            WPITE (6, 20O\) YHOT, YaOT, LAHEDER, (KK(K), K=I*N)
```





```
        00 20 J=1;4%4A
            H=N-1
            N=揞1
```



```
    3nnc
```



```
        COnTIEJE
        STDP
        EnD
```

FIGURE E-1. FORTRAN LISTING OF PROGRAM WHERE,


[^0]:    * The ferm scafterometer was introduced by R. K. Moore of the University of Kansas, A scatterometer is a radar designed to accurately measure the scattering properties of non-coherent scenes. The term scatferometer and non-coherent radar will be used interchangeably.

[^1]:    *Monochromatic waves are completely polarized.

[^2]:    * Reception and scattering relationships in the far field adapt well to the matrix notation. Capital letters will denote matrices and lower case letters will denote their elements.

[^3]:    * Not normalized as in Chapter 3.

[^4]:    * The reader should be aware that in constructing the scattering coefficients from scattering theory and identifying them with measured coefficients involves an ergodic assumption, i.e., an ensemble average is equated with a spatial average.

[^5]:    * focal length $\div$ paraboloid diameter

[^6]:    * The degree of accuracy will be demonstrated in Chapter 7.

[^7]:    * Assuming the antenna is not simply rotated.

[^8]:    * A uniform phase distribution across the aperture.

[^9]:    * This can always Le achiseved at all view angles except nadir if the beam is sufficiently small.

