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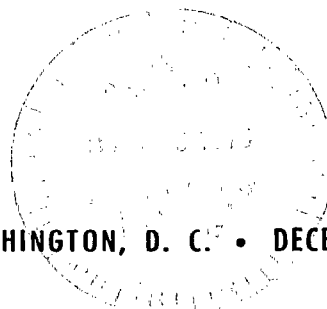
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**ADAPTATION OF THE THEODORSEN THEORY
TO THE REPRESENTATION OF AN AIRFOIL
AS A COMBINATION OF A LIFTING LINE
AND A THICKNESS DISTRIBUTION**

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ADAPTATION OF THE THEODORSEN THEORY TO THE REPRESENTATION
OF AN AIRFOIL AS A COMBINATION OF A LIFTING LINE
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SUMMARY

A representation of the Theodorsen airfoil theory as a combination of a lifting line and a thickness distribution is described. The approximations of thin-airfoil theory are avoided, since the full potential theory is used throughout. The theory provides a direct method for resolving an airfoil into a lifting line and a thickness distribution as well as a means of synthesizing thickness and lift components into a resultant airfoil and computing its aerodynamic characteristics. Specific applications of the technique are discussed.

INTRODUCTION

The thin-airfoil technique of representing an airfoil as a combination of a thickness distribution and a camber line is widely used because of the availability of a body of appropriate data (ref. 1) and because the method facilitates design, inasmuch as it enables the designer to treat the lifting and thickness components separately. On the other hand, the thin-airfoil theory itself is vulnerable to criticism on several grounds regarding both the consistency of the theory and the accuracy of the procedures. For example, the procedure of superimposing a thickness distribution normal to the camber line involves a nonuniform stretching of the basic thickness profile. Furthermore, the simple addition of velocities due to thickness, camber, and angle of attack is not always reliable. An even more serious difficulty is the failure of thin-airfoil theory to provide reasonable velocity values in the nose region. To improve the accuracy of the calculations in this region requires involved procedures. Finally, although the theory gives explicit equations for computing airfoil coordinates from thickness and camber distributions, it provides no analytic means for accomplishing the inverse task – that of resolving a given airfoil into its thickness and camber components.

The present report describes a method for representing the Theodorsen airfoil theory as a combination of a lifting line and a thickness distribution. In this approach the approximations of thin-airfoil theory are avoided, since the full potential theory is used throughout. Within the framework of the full potential theory, the new method not only

provides formulas for synthesizing an airfoil from a lifting line and a thickness distribution, but it also provides analytic means for resolving a given airfoil into its thickness and lifting-line components. This latter capability is especially useful in certain airfoil design procedures. Both the synthesis and the analysis procedures are simple in concept and convenient in application.

SYMBOLS

A_n, B_n	Fourier coefficients
a	$= \operatorname{Re}^{-\psi_0}$
C_p	pressure coefficient
c	airfoil chord length
R	radius of circle into which an airfoil is mapped by the Theodorsen transformation
r	radial coordinate in near-circle plane
V	undisturbed free-stream velocity
v	local velocity
x, y	airfoil coordinates
α	angle of attack
α_I	ideal angle of attack, $-\frac{\epsilon_N + \epsilon_{te}}{2}$
α_0	angle of attack at zero lift
ϵ	function relating angular coordinates of near-circle and exact-circle airfoil transformations
ϵ_N	value of ϵ at airfoil nose, $\epsilon(\theta)$ at $\theta = 0$
ϵ_{te}	value of ϵ at airfoil trailing edge, $\epsilon(\theta)$ at $\theta = \pi$

θ	angular coordinate in near-circle plane
ϕ	angular coordinate in exact-circle plane
ϕ^*	dummy variable of integration
ψ	function relating radial coordinates of near-circle and exact-circle airfoil transformations
ψ_0	average value of ψ

Subscripts:

a	antisymmetric
s	symmetric

Primes indicate derivatives with respect to θ .

BASIC CONCEPTS

Potential Theory

Before reviewing some thin-airfoil techniques, it is convenient first to state the basic formulas of the Theodorsen transformation theory for airfoil analysis. These formulas are useful in understanding the following discussion of thin-airfoil techniques, and they are required in the subsequent analysis section.

The Theodorsen airfoil theory (ref. 2) involves the Joukowski transformation of the airfoil into a shape approximating a circle. In the transformed plane, this near circle is described by polar coordinates (r, θ) . The approximate circle is then transformed into an exact circle having coordinates (R, ϕ) . The function ψ and the constant ψ_0 are defined by $r = ae^\psi$ and $R = ae^{\psi_0}$. Theodorsen shows that ψ_0 is the average value of ψ , that is,

$$\psi_0 = \frac{1}{2\pi} \int_0^{2\pi} \psi \, d\phi \quad (1)$$

and that the functions $\psi - \psi_0$ and $\epsilon \equiv \phi - \theta$ are related by the conjugate equations

$$\epsilon = -\frac{1}{2\pi} \int_0^{2\pi} \psi \cot \frac{\phi^* - \phi}{2} d\phi^* \quad (2)$$

$$\psi - \psi_0 = \frac{1}{2\pi} \int_0^{2\pi} \epsilon \cot \frac{\phi^* - \phi}{2} d\phi^* \quad (3)$$

The function ψ is directly related to the airfoil coordinates in the physical plane by

$$\left. \begin{aligned} x &= 2a \cosh \psi \cos \theta \\ y &= 2a \sinh \psi \sin \theta \end{aligned} \right\} \quad (4)$$

In practice, ψ is obtained directly as a function of θ from equations (4), and then the function $\epsilon(\theta)$ is obtained by taking the conjugate of ψ . To be more precise, $\epsilon(\phi)$ should be obtained by iteration by means of the relation $\phi = \epsilon + \theta$, and then $\epsilon(\theta)$ should be calculated from $\epsilon(\theta) = \epsilon[\phi(\theta)]$. However, for practical airfoil shapes the additional precision gained by this step is slight (ref. 3), and so it is often omitted. The velocity distribution on the airfoil is computed from the equation

$$\frac{v}{V} = \frac{\left[\sin(\alpha + \epsilon + \theta) + \sin(\alpha + \epsilon_{te}) \right] (1 + \epsilon') e^{\psi_0}}{\sqrt{(\sinh^2 \psi + \sin^2 \theta)(1 + \psi'^2)}} \quad (5)$$

Comments on Thin-Airfoil Methods

The thin-airfoil theory, which affords a simple approximate procedure of synthesizing and analyzing an airfoil by combining lifting-line and thickness elements, does not provide a means for separating a given airfoil into its camber line and thickness distribution. Furthermore, the thin-airfoil calculation of the velocity, by a distribution of sources and sinks along the axis, gives a poor approximation, especially in the nose region. Attempts to improve the accuracy of the basic thin-airfoil theory have led to complicated procedures. (See sec. 4.5 of ref. 1, and ref. 4.)

Another difficulty is that the thin-airfoil method of obtaining a profile by superimposing a thickness distribution (fig. 1(a)) normal to a camber line, as illustrated in figure 1(b), is an approximate procedure. The arc length of the camber line is longer than the chord of the original thickness profile, and therefore the thickness form undergoes a distortion, the stretching being greater for the upper surface than for the lower surface. The simpler thin-airfoil procedure, which attributes the lift to a mean line with the thick-

ness distribution superimposed on it in a vertical direction (fig. 1(c)), is an even poorer approximation. With this procedure, not only is the thickness distribution distorted, but the effective camber line is changed also.

A theory that avoids some of the difficulties of the thin-airfoil methods is described in the following section.

ANALYSIS AND APPLICATIONS

Airfoil Synthesis From Thickness and Lift Components

This section describes the theory and procedure for synthesizing an airfoil by combining an arbitrary thickness distribution with an arbitrary lifting line within the context of the full potential theory. The aerodynamic relationships between the resulting airfoil and its thickness and lifting-line components are also discussed.

In equations (a) and (b) of reference 2, it is shown that (if θ is substituted for ϕ , as previously discussed) ϵ and ψ can be expressed by

$$-\epsilon = \sum_{n=0}^{\infty} \frac{1}{R^n} (B_n \cos n\theta - A_n \sin n\theta) \quad (6a)$$

$$\psi - \psi_0 = \sum_{n=0}^{\infty} \frac{1}{R^n} (A_n \cos n\theta + B_n \sin n\theta) \quad (6b)$$

From its definition, ϵ must be a continuous function of θ , and therefore,

$$\epsilon(0) = \epsilon(2\pi) \equiv \epsilon_N \quad (7)$$

It follows from equation (6a) that

$$\int_0^{2\pi} \epsilon(\theta) d\theta = 0 \quad (8)$$

Now consider first the case for which ϵ is antisymmetric about π (fig. 2(a)); that is, $\epsilon(\pi-\theta) = -\epsilon(\pi+\theta)$. Then $B_n = 0$ for all n , and therefore the conjugate function $\psi - \psi_0$ is symmetric about π (fig. 2(a)). Since ψ_0 is constant, $\psi(\pi-\theta) = \psi(\pi+\theta)$. Consequently $\cosh \psi(\pi-\theta) = \cosh \psi(\pi+\theta)$ and $\sinh \psi(\pi-\theta) = \sinh \psi(\pi+\theta)$. Thus in equations (4) for the airfoil coordinates, $x(\pi+\theta) = x(\pi-\theta)$; that is, x is symmetric with

respect to π . But because the $\sin \theta$ factor is an odd function, y is antisymmetric; that is, $y(\pi+\theta) = -y(\pi-\theta)$. In other words, since $\theta = \pi$ corresponds to the trailing edge, the airfoil is symmetric, and so it represents a thickness distribution (fig. 2(b)) with the same pressure distribution for upper and lower surfaces (fig. 2(c)). The maximum thickness is controlled by the average value ψ_0 of ψ .

It is easily verified that for symmetric airfoils (thickness distributions) the conditions expressed by equations (7) and (8) are assured by the antisymmetry of the ϵ -function together with the continuity conditions. Furthermore it is seen from figure 2(a) that, in this case, both ϵ_N and ϵ_{te} vanish. Thus the angle of attack at zero lift $\alpha_0 = -\epsilon_{te}$ and the ideal angle of attack $\alpha_I = -\frac{\epsilon_N + \epsilon_{te}}{2}$ are both zero, as expected for a symmetric airfoil. These are important aerodynamic parameters because ϵ_{te} is proportional to the lift at $\alpha = 0$ and because α_I is useful in locating the bucket of the drag curve (see pt. III of ref. 5). When the converse situation, for which ϵ is symmetric about π (fig. 3(a)), is considered, ψ is antisymmetric about π (fig. 3(a)), provided that $\psi_0 = 0$. In that case, $x(\pi+\theta) = x(\pi-\theta)$ because \cosh is an even function of its argument. Also $y(\pi+\theta) = y(\pi-\theta)$ because the product of the two odd functions $\sinh \psi$ and $\sin \theta$ is even. In other words the upper and lower surfaces are identical, and the airfoil consists of a single line (fig. 3(b)). Its pressure distribution is shown in figure 3(c).

For such a lifting line the ϵ -function must satisfy

$$\int_0^\pi \epsilon(\theta) d\theta = 0$$

in order to satisfy equation (8). The magnitude of the lift at $\alpha = 0$ is determined by $\alpha_0 = -\epsilon_{te}$. Usually ϵ_N has a somewhat larger negative value than α_0 , so that $\alpha_I = -\frac{\epsilon_N + \epsilon_{te}}{2}$ is positive.

Now consider the ϵ -function that results from adding the ϵ -function for the lifting line to that of the symmetric airfoil. If neither ϵ -function is identically zero, the sum is neither symmetric nor antisymmetric about π . (See fig. 4(a).) Since both component ϵ -functions are continuous and satisfy the conditions expressed by equations (7) and (8), the resultant ϵ -function also satisfies these conditions, and so ϵ corresponds to a real airfoil. Also, since ϵ_{te} and ϵ_N are both zero for the symmetric airfoil, these parameters have the same values in the resulting ϵ -function as for the lifting line. This means that the airfoil corresponding to the resultant ϵ -function has the same angle of attack at zero lift and the same design lift coefficient as that of the lifting line.

In order to construct an airfoil corresponding to the ϵ -function obtained by adding those of a lifting line and of a thickness distribution, it is necessary to have the corresponding $\psi - \psi_0$ function. This function can be obtained as the conjugate of ϵ , or, if the ψ -functions for both component airfoils are known, it is obtained simply by adding the $\psi - \psi_0$ functions for the components. The parameter ψ_0 should be assigned the same value as that for the symmetric airfoil since the value of ψ_0 for the lifting line is zero. Then the airfoil coordinates are determined by equations (4), and the velocity distribution is computed from equation (5). Results for the sample case are shown in figures 4(b) and 4(c).

The constant ψ_0 is basically a thickness parameter, but this does not necessarily mean that the maximum thickness of the resultant airfoil will be exactly the same as that of the original thickness distribution, although it should be very nearly the same. However, the important factor, e^{ψ_0} , in the velocity equation (5) will be exactly the same as that for the thickness distribution. This factor is not a function of position on the airfoil, and so it determines the level of the velocity curves independently of the details of the shape of the airfoil.

Thus it is seen that this method of combining a lifting line and a thickness distribution to synthesize an airfoil has a more precise aerodynamic significance than the thin-airfoil methods. The resultant airfoil has the same angle of attack at zero lift and ideal angle of attack as the lifting line and the same velocity-amplitude factor e^{ψ_0} as the original thickness profile. Its pitching moment is not exactly the same as that of the lifting line, because the pitching moment is not entirely independent of the thickness distribution according to the full potential theory. The procedure may be summarized as follows:

- (1) Determine the ψ -functions for the thickness and lifting-line components from their coordinates. Compute the average value ψ_0 of ψ for the thickness distribution and then compute the conjugate ϵ -functions from equation (2).
- (2) Add the ϵ -functions and the ψ -functions to obtain those for the resultant airfoil.
- (3) Obtain the resultant airfoil coordinates from equations (4) and the velocity distribution for this airfoil from equation (5).

If one intends to maintain a catalog of thickness distributions and lifting lines, he should also maintain a file of the corresponding ϵ - and ψ -functions. The simple addition of a pair of these functions yields the resultant transformation functions from which both the coordinates and the velocity distribution for this airfoil can be obtained.

Airfoil Resolution Into Thickness and Lift Components

In order to find the lifting line and thickness distribution corresponding to a specified arbitrary airfoil (fig. 5), it is necessary to determine its transformation functions and to

resolve them into their symmetric and antisymmetric parts (fig. 6). The formulas (for the ϵ -function) are

$$\epsilon_s = \frac{\epsilon(\theta) + \epsilon(2\pi - \theta)}{2}$$

$$\epsilon_a = \frac{\epsilon(\theta) - \epsilon(2\pi - \theta)}{2}$$

It is readily verified that $\epsilon_s(\pi - \theta) = \epsilon_s(\pi + \theta)$ and $\epsilon_a(\pi - \theta) = -\epsilon_a(\pi + \theta)$.

The first of these functions, ϵ_s , corresponds to the lifting line, and ϵ_a corresponds to the thickness distribution. Conversely, when $\psi - \psi_0$ is resolved into its components, the symmetric part corresponds to the thickness distribution, and ψ_0 is set equal to that of the original airfoil. For the antisymmetric part of $\psi - \psi_0$, corresponding to the lifting line, $\psi_0 = 0$. The ψ - and ϵ -functions for each of these components can then be used to compute the profiles from equations (4) (figs. 5(b) and 5(c)) and the velocity distributions from equation (5) (fig. 7).

Application of the theory to altering certain properties of an airfoil can be demonstrated by a rather extreme example. The airfoil of figure 5(a) is taken as the basic shape. Its lift at zero angle of attack is increased by 50 percent by adding to the ϵ -function of its lifting line (fig. 6(c)) a bilinear function that increases from $-0.5\epsilon_{te}$ at $\theta = 0$ to $+0.5\epsilon_{te}$ at $\theta = \pi$ and then decreases linearly to $-0.5\epsilon_{te}$ at $\theta = 2\pi$. The thickness of the airfoil is increased from 10.7 percent to 15 percent by changing the value of ψ_0 for the thickness distribution (fig. 6(b)) from 0.10 to 0.137. The new ϵ - and ψ -functions are then combined, and the resulting airfoil and its pressure distribution are shown in figure 8.

If it is desired to change the lifting-line component of an airfoil in order to redistribute the loading, the calculations should be carried out at the ideal angle of attack, at which the leading-edge velocity is not infinite for the lifting line. For example, the pressure distributions shown in figure 9(a) (for the airfoil) and in figure 9(b) (for its lifting line), which are computed at α_I , may be compared with those at $\alpha = 0$ for the same airfoil and for its lifting line (figs. 4(c) and 3(c), respectively).

CONCLUDING REMARKS

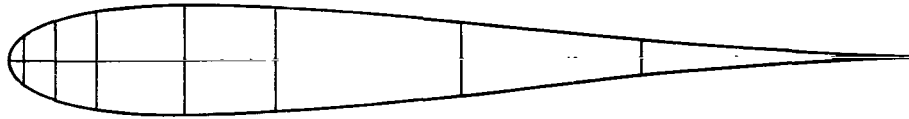
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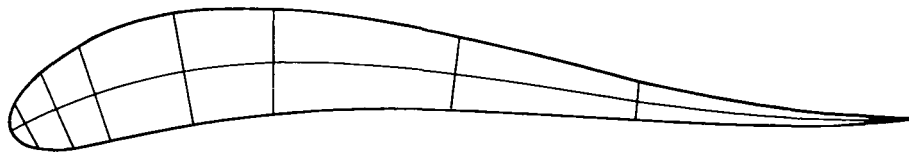
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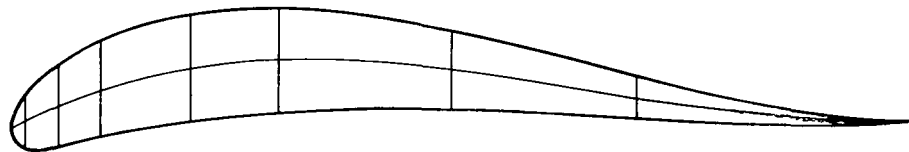
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5. Theodorsen, Theodore: On the Theory of Wing Sections With Particular Reference to the Lift Distribution. NACA Rep. 383, 1931.



(a) Thickness distribution.

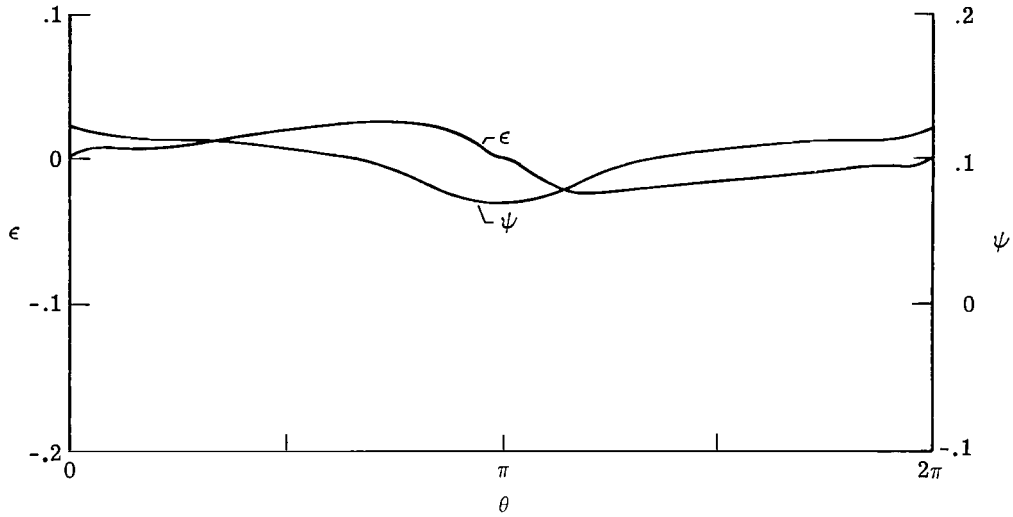


(b) Lifting line used as camber line with thickness superimposed normal to the line.



(c) Lifting line used as mean line with thickness superimposed vertically.

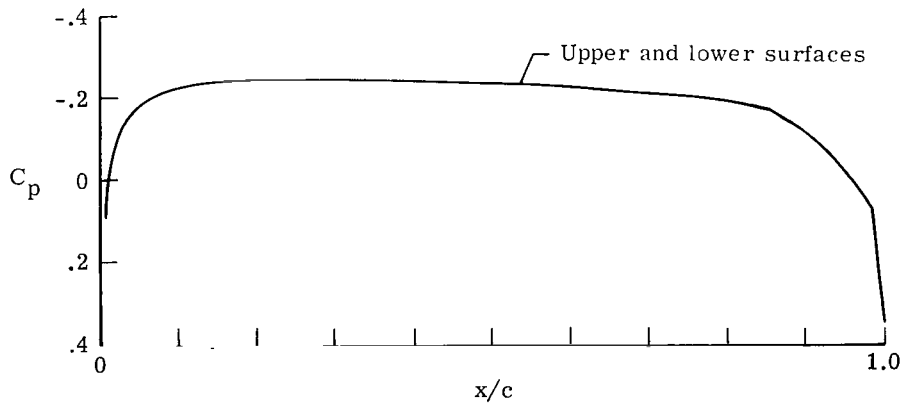
Figure 1. - Thin-airfoil procedures for combining a thickness distribution with a lifting line.



(a) Transformation functions.

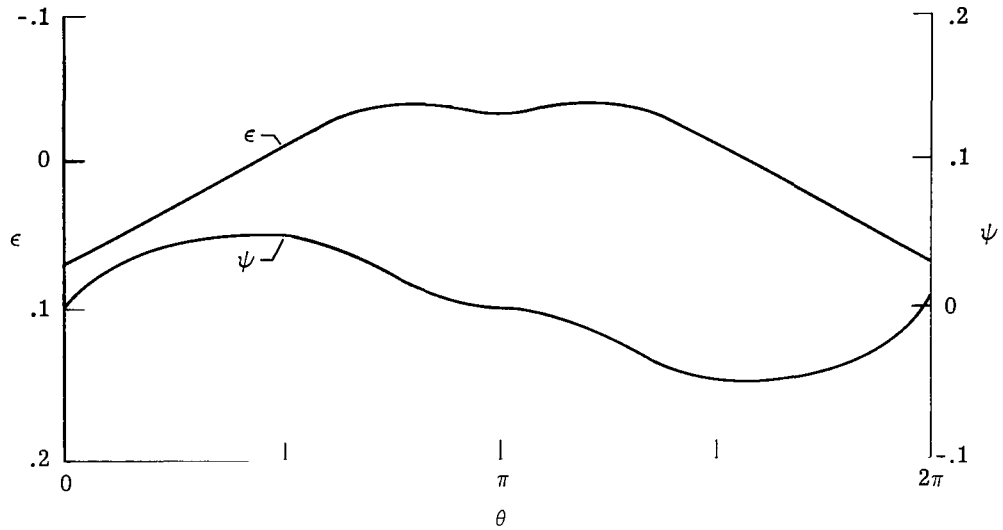


(b) Thickness profile.

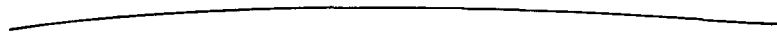


(c) Pressure distribution. $\alpha = 0$.

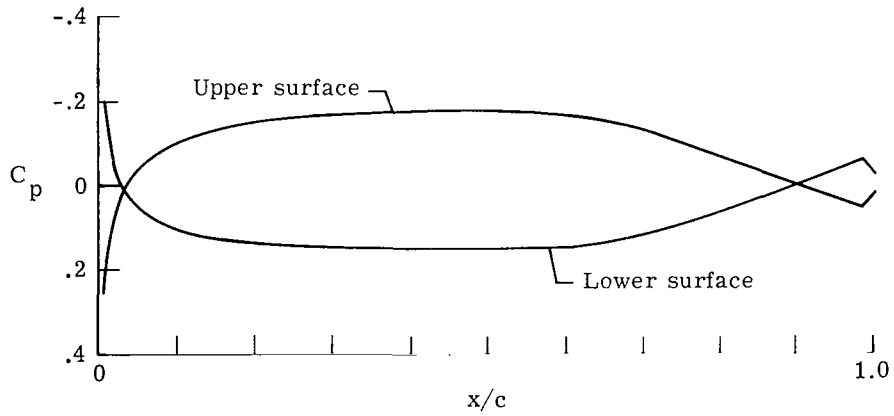
Figure 2.- Calculations for symmetric airfoil (thickness distribution).



(a) Transformation functions.

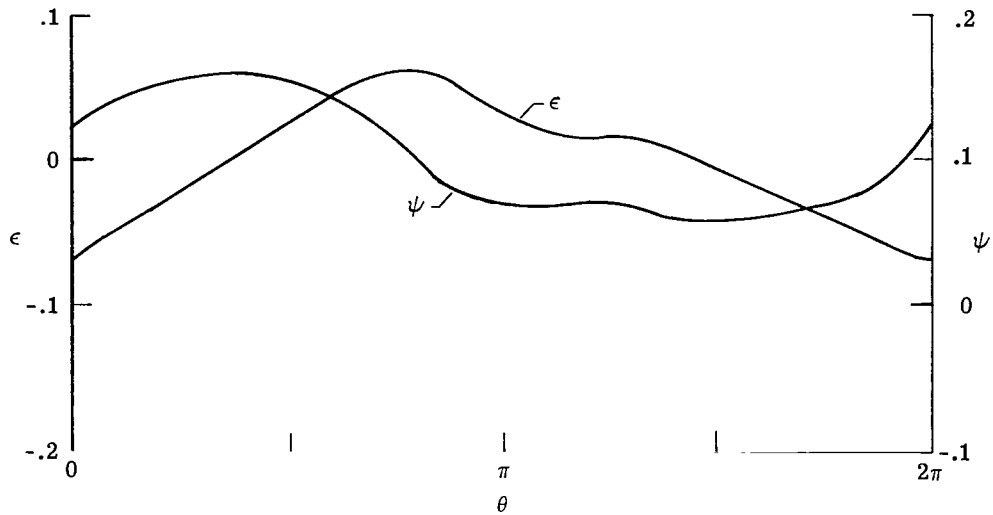


(b) Lifting-line airfoil shape.



(c) Pressure distribution. $\alpha = 0$.

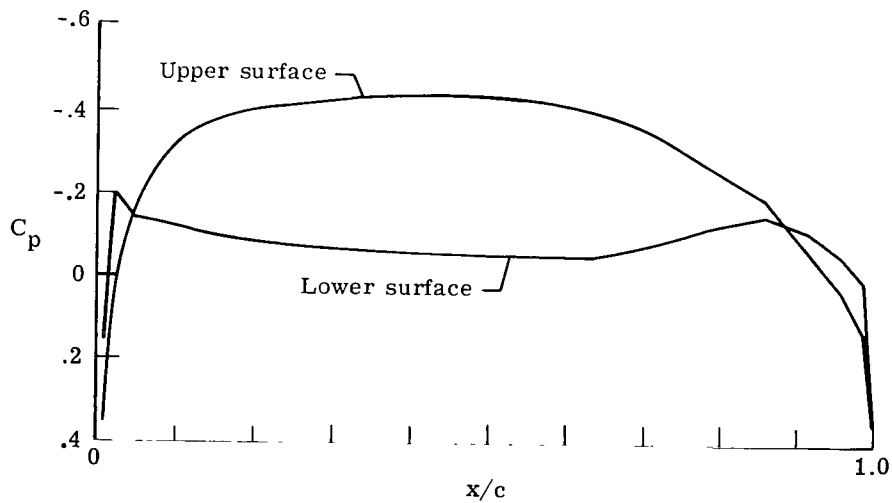
Figure 3. - Calculations for lifting-line airfoil.



(a) Transformation functions obtained by adding functions of figure 2(a) to corresponding functions of figure 3(a).

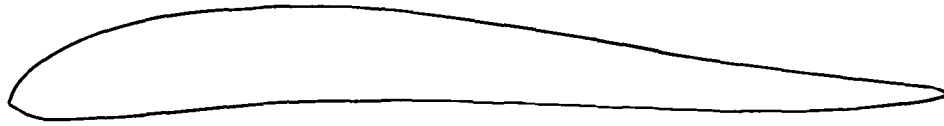


(b) Airfoil profile.

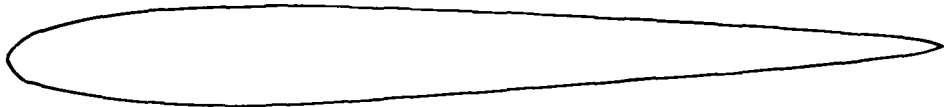


(c) Pressure distribution. $\alpha = 0$.

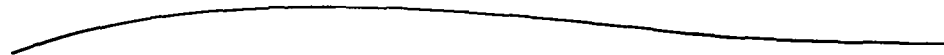
Figure 4.- Theoretical results for airfoil synthesized from thickness distribution of figure 2 and lifting line of figure 3.



(a) Given airfoil.

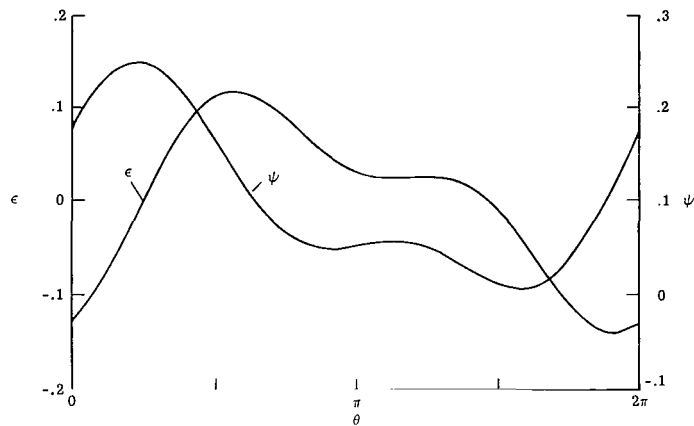


(b) Thickness distribution.

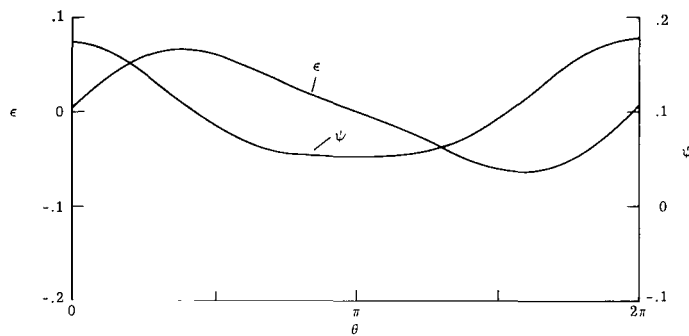


(c) Lifting line.

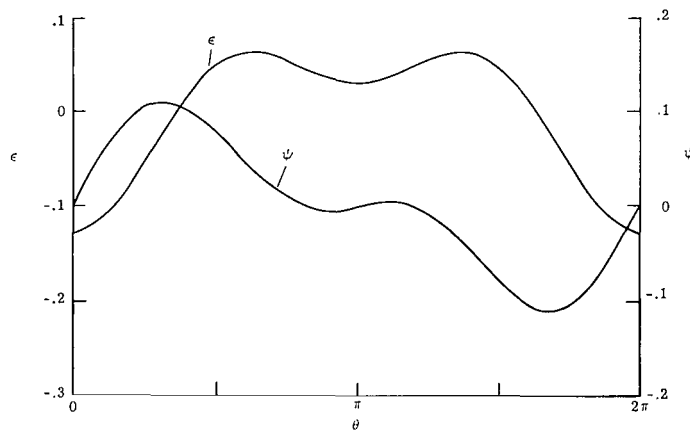
Figure 5.- Resolution of an airfoil into thickness and lifting-line components.



(a) Functions corresponding to given airfoil.

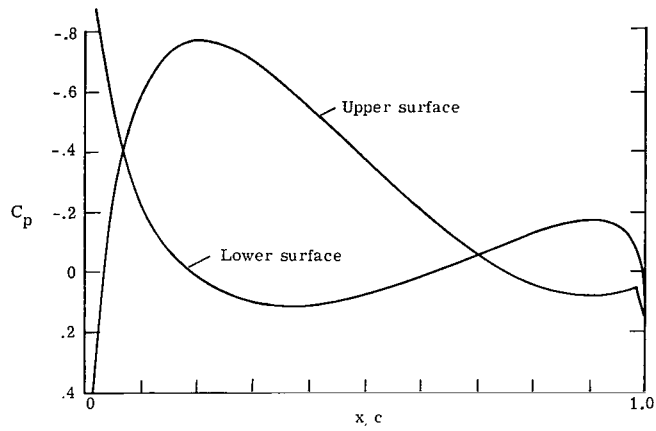


(b) Symmetric part of ψ and antisymmetric part of ϵ (thickness component).

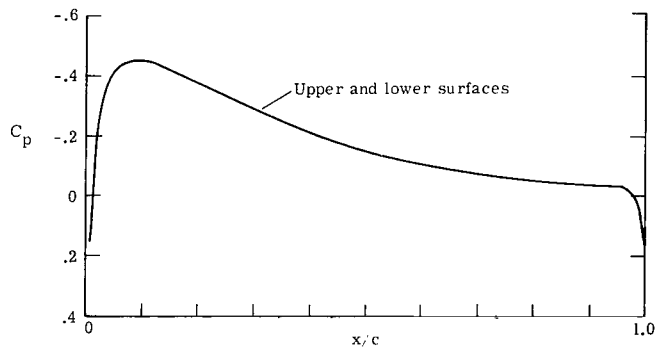


(c) Symmetric part of ϵ and antisymmetric part of ψ (lifting-line component).

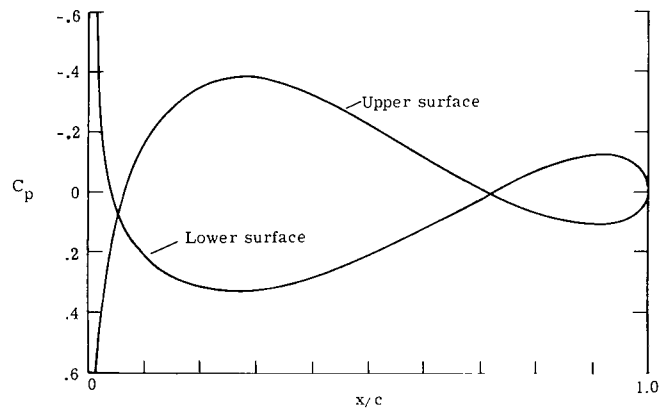
Figure 6.- Transformation functions corresponding respectively to airfoils of figures 5(a), 5(b), and 5(c).



(a) For given airfoil.

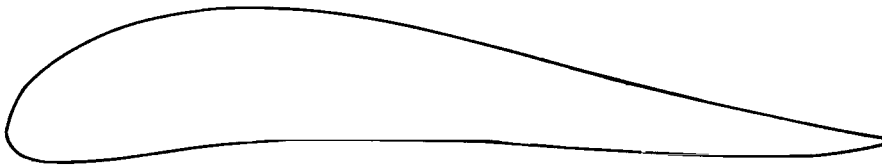


(b) For thickness distribution.

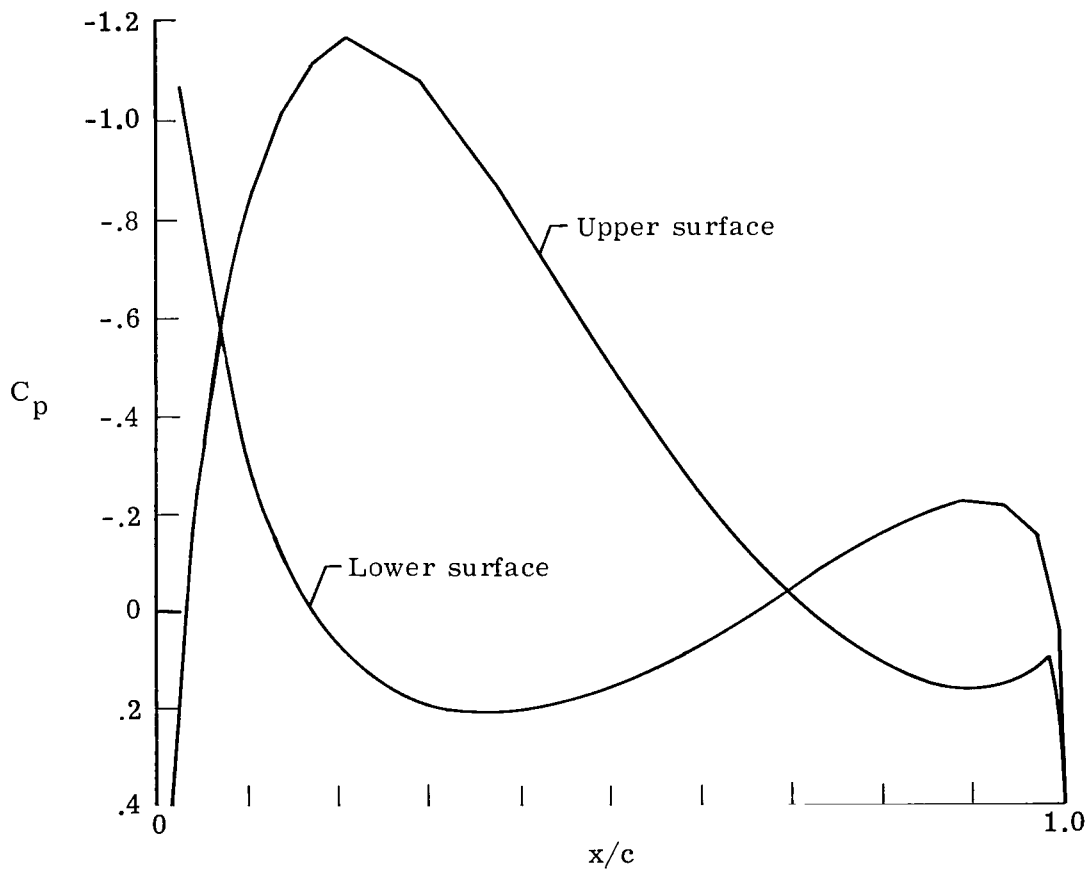


(c) For lifting line.

Figure 7.- Pressure distributions corresponding respectively to airfoil shapes of figures 5(a), 5(b), and 5(c).

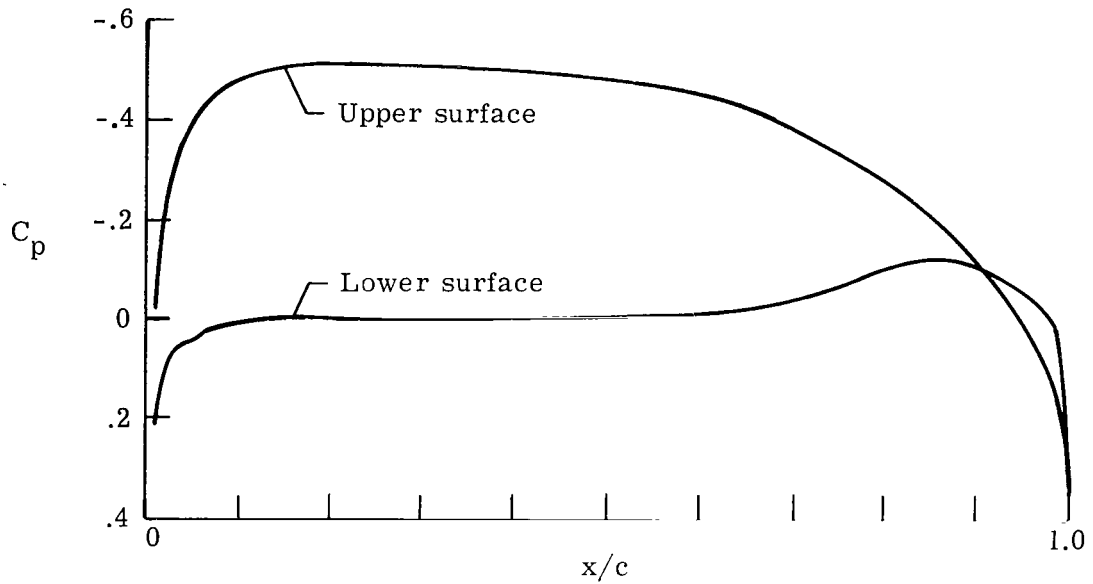


(a) Revised airfoil shape.

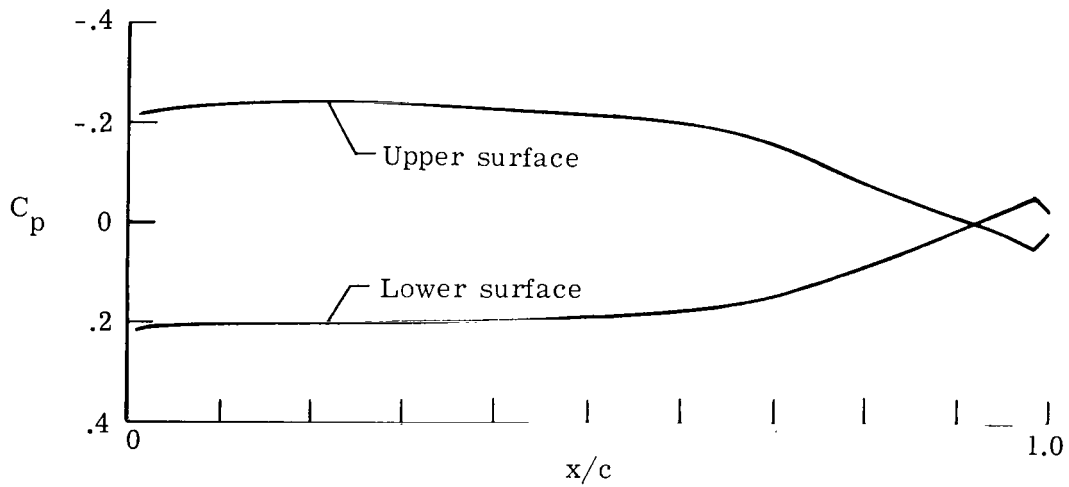


(b) Pressure distribution. $\alpha = 0$.

Figure 8.- Redesign of airfoil of figure 5(a) to increase lift at zero angle of attack by 50 percent and to increase thickness from 10.7 percent to 15 percent.



(a) For airfoil of figure 4.



(b) For lifting line of airfoil of figure 4. (See fig. 3.)

Figure 9.- Pressure distributions at ideal angle of attack.

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