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Produced by the NASA Center for Aerospace Information (CASI)

Interim Report
Grant No. NSG-il28
Submitted to:
NASA Scientific and Technical Information Facility
P. O. Bо; 8757

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## CHAPTER 1

## INTRODUCTION

This is a report of the initial phase of research under Grant NSG 1128 , awarded to the University in December 1974. The project is part of a program to design a receiver for aircraft ( $A / C$ ), which, as a component of the proposed Microwave Landing System (MLS), is capable of optimal performance in the mutipath environments found in air terminal areas [1]. This project focuses on the angle-tracking problem of the MLS receiver and deals with slgnal modeling, preliminary approaches to optimai design, suboptimal design and slmulation study. This effort was integrated in part with the work under contract NASI-I2754-1, which was summarized in a final report dated May 1975 [2].

CHAPTER II."
SIGNAL MODELING
The angular coordinate determination capability of the MLS has been tentatively es qbilished by a provision potentially for six (6) angle channels [l]. The signals in all channels have similar forms, however, and without loss of generality, this study has employed the azlmuth channel as a focus to provide concrete interpretations of the models and results developed. Parallel results for other channels would obtain from suitable adjustments of parameter values.

The coordinate system in use is the right-handed system shown in Figure 1-l with the origin located at the center of the azimuth transmitting antenna. The relevant variables $r, \theta$, $\phi$ are rage ( $n . m i$. ), azimuth (degrees, positive clockwise looking down) and elevation (degrees, positive up). Let $p(\cdot)$ and $\theta_{A}(\cdot)$, respectively the antenna selectivity and scanning functions, be defined as follows:
$p\left(\theta_{e}\right) \triangleq$ the far-field strength of the transmitted signal (field) relative to the boresight signal strength at the same range, where $\theta_{e}$ is the azimuth angle of the observer relative to the antenna boresight axis, and thus $p(0)=1$.
$\theta_{A}(t) \triangleq$ the angular orientation in azimuth of the antenna boresight axis as observed at time $t$ at the transmitting antenna site.

The definition of $p(\cdot)$ above does not recognize explicitly a dependency on observer elevation, which is present due to phased array radiation pattern properties. The effect is recognized implicitly however, with notation suppressed until cross-coupling of the various angular coordinate estimators (and the DME) is considered.

Further, letting
$\dagger_{k} \triangleq$ the time (as observed at the antenna site) that the kth TO-FRO antenna scan begins;
$T \triangleq$ the duration of the TO-FRO scan as observed by a stationary observer,


Figure 11-1 Coordinate System
then $\theta_{A}(\cdot)$ is spectifically defined by the antenna scanning motion as a function of its argument on the interval $\left[\dagger_{k}, t_{k}+T\right]$.

The coordlnates of the recelver (the $A / C)$ at time t are $r(t), \theta(t)$, $\phi(t)$. A direct-path-signal event arriving at the $A / C$ at time tis one that necessarlly left the transmltting antenna at time $+\frac{-r(t)}{c}$, where c is the propagation velocity of light. Hence, the component of the recelved signal observec at time $t$ and due to transmitter emissions derives from transmission occurring when $t_{k} \leq t-\frac{r(t)}{c} \leq t_{k}+T$. This component itself comprises two contributions $y_{D}(t), y_{R}(t)$, as follows, due respectively to direct-path and reflection-path propagation:

$$
\begin{align*}
& y_{D}(t)=\alpha(t) p\left[\theta_{A}\left(t-\frac{r(t)}{c}\right)-\theta(t)\right] \cos \left[\omega_{c}\left(t-\frac{r(t)}{c}\right)+\beta_{D}\right]  \tag{2-5}\\
& y_{R}(t)=\sum_{i} y_{R_{i}}(t) \tag{2-6}
\end{align*}
$$

where

$$
\begin{equation*}
y_{R_{i}}(t)=\alpha_{i}(t)_{P}\left[\theta_{A}\left(t-\frac{r_{T_{1}}{ }^{(t)}}{c}\right)-\theta_{R_{1}}(t)\right] \cos \left[\omega_{c}\left(t-\frac{r_{T_{i}}(t)}{c}\right)+\beta_{R_{1}}\right] \tag{2-7}
\end{equation*}
$$

and
$\alpha(t), \beta_{D} \quad=a m p l i t u d e$ and phase of the direct path component (2-8)

$$
\begin{align*}
\omega_{c} & =\text { radian frequency of propagating r-f energy } \\
r_{T_{i}}(t), \theta_{R_{i}}(t)= & \text { range thru and azimuth of the ith reflection point }(2-10) \\
\alpha_{i}(t), \beta_{R_{i}}= & \text { amplitude and phase of the component reflected by } \\
& \text { the ith reflection point (where, for each } i, \alpha_{i}(t)=0,  \tag{2-11}\\
& \text { if } \left.+-\frac{r_{1}(t)}{c}<t_{k}\right)
\end{align*}
$$

Under the assumption of isotrople reflection at the ith reflection point, located at $r_{T_{i}}(t), \theta_{R_{i}}(t), \phi_{R_{j}}(t)$, the range through reflection, $r_{T_{i}}{ }^{(t)}$, is easily shown (see Figure (1-2) to be

$$
\begin{equation*}
r_{T i}(t)=r_{R_{i}}(t)+r_{A R_{i}}(t) \tag{2-12}
\end{equation*}
$$



Figure 11-2 Reflection Geometry
where

$$
\begin{equation*}
r_{A R_{t}}(t)=\left[r^{2}(t)-2 r(t) r_{R_{i}}(t) \zeta(t)+r_{R_{i}}^{2}(t)\right]^{1 / 2} \tag{2-13}
\end{equation*}
$$

and

$$
\begin{equation*}
\zeta(t)=\cos \phi(t) \cos \phi_{R_{i}}(t) \cos \left[\theta(t)-\theta_{R_{i}}(t)\right]+\sin \phi(t) \sin \phi_{R_{i}}(t) \tag{2-14}
\end{equation*}
$$

In view of the generally independent motions of the $\mathrm{A} / \mathrm{C}$ and a reflector, the equations (2-7) and (2-12) through (2-14) are clearly approximations. A more exact expression for, say ' ${ }_{x}$, the time of transmission, given receipt at time $t$ (i.e., $t_{x}$ would be the argument of $\theta_{A}()$ in $y_{R_{1}}{ }^{(t)}$ generally) would have the form

$$
\begin{equation*}
t_{x}=t-\frac{r_{A R_{1}}(t)}{c}-\frac{r_{R_{1}}}{c}\left(t-\frac{r_{A R_{i}}(t)}{c}\right), \tag{2-14a}
\end{equation*}
$$

based on an argument similar to that preceeding (2-5) above. On the basis of the linear approximation

$$
r_{R_{i}}\left(t-\frac{r_{A R_{i}(t)}}{c}\right) \simeq r_{R_{i}}(t)-\dot{r}_{R_{i}}(t) \frac{r_{A R_{i}(t)}}{c}
$$

the expression for $t_{x}$ becomes

$$
\begin{aligned}
t_{x} & =t-\frac{r_{A R_{1}}(t)}{c}-\frac{r_{R_{i}}(t)}{c}+\frac{\dot{r}_{R_{i}}(t) r_{A R_{1}}(t)}{c^{2}} \\
& =t-\frac{r_{T_{1}}(t)}{c}+\frac{\dot{r}_{R_{i}}(t) r_{A R_{1}}(t)}{c^{2}}
\end{aligned}
$$

The original approximation for $t_{x}(\operatorname{in}(2-7))$ results when the term $\dot{r}_{R_{i}} r_{A R} / c^{2}$ is dropped, which seems reasonable under the circumstances. In the strict sense the expression in (2-14a) does not give the exact value for $t_{x}$, but is adequate to show the approximate magnitude of error in the original approximation (in (2-7)).

The basic model has now been presented in terms of a number of parameters potentlally time-varying within a scan interval In an unspecified manner. The update rate of the MLS (13.3 Hz in Azimuth) is sufficiently high relatlve to the dynamics of the env:ronment to allow simple approximations for these parametric functions within a scan interval. For $t_{k} \leq t \leq \dagger_{k+1}$ (which contains the $k$ th scan interval, $t_{k} \leq t-\frac{r(t)}{c} \leq t_{k}+\bar{T}$, at the $\left.A / C\right)$, the approximations used are the following, which seem reasonable:

$$
\begin{align*}
\alpha(t) & =\alpha\left(t_{k}\right)  \tag{2-15}\\
\theta(t) & =\theta\left(t_{k}\right)  \tag{2-16}\\
r(t) & =r\left(t_{k}\right)+\dot{r}\left(t_{k}\right)\left[t-t_{k}\right]  \tag{2-17}\\
\alpha_{i}(t) & =\alpha_{i}\left(t_{k}\right)  \tag{2-18}\\
\theta_{R_{i}}(t) & =\theta_{R_{i}}\left(t_{k}\right)  \tag{2-19}\\
r_{T_{i}}(t) & =r_{T_{i}}\left(t_{k}\right)+\dot{r}_{T_{i}}\left(t_{k}\right)\left[t-t_{k}\right] \tag{2-20}
\end{align*}
$$

It is emphasized these approximations are limited to a given scan. Clearly, the result in (2-12) through (2-14) generally invalidates (2-20) in any but a loca! sense, with $r_{T_{1}}\left(t_{k}\right)$ determined even then by (2-12) through (2-14), (and $\dot{r}_{T_{1}}\left(t_{k}\right)$ by the derivative of the expression in (2-12), as was done in the simulation study; see Section IV).

Under the assumptions (2-15) through (2-20) the received signal can be expressed in terms of a time varlable local to the scan beling received at the $A / C$. First, note that $t-\frac{r(t)}{C}$, the argument of $\theta_{A}(\cdot)$ in the recelved direct path component, $y_{D}(t),(2-5)$, as a direct result of (2-17), can be written as

$$
\begin{equation*}
+-\frac{r(t)}{c}=+\left(1-\frac{\dot{r}\left(t_{k}\right)}{c}\right)-\frac{r\left(t_{k}\right)}{c}+t_{k} \frac{\dot{r}\left(t_{k}\right)}{c} . \tag{2-21}
\end{equation*}
$$

Consequently, the "recelved scan interval," defined herefofore by $t_{k} \leq t-\frac{r(t)}{c} \leq t_{k}+T$, can be described as follows:

$$
\begin{equation*}
\left.\left(t_{k}+\frac{r\left(t_{k}\right) / c}{1-r\left(t_{k}\right) / c}\right) \leq t^{-r\left(t_{k}\right.}+\frac{r\left(t_{k}\right) / c}{1-\Gamma\left(t_{k}\right) / c}\right)+\frac{T}{1-r\left(t_{k}\right) / c} \tag{2-22}
\end{equation*}
$$

and thus on the $k$ th scan interval we may write in general

$$
\begin{equation*}
t=\left(t_{k}+\frac{r\left(t_{k}\right) / c}{1-r_{k}\left(t_{k}\right) / c}\right)+\tau=t_{R_{k}}+\tau \tag{2-23}
\end{equation*}
$$

where t is a local time variable on the $k$ th recelved scan interval, varyIng as follows:

$$
\begin{equation*}
0 \leq \tau \leq T \frac{T}{-\Gamma\left(\varphi_{k}\right) / c} \tag{2-24}
\end{equation*}
$$

with the same scaling as $t$, i.e., $d t / d \tau=1$. (note a change in the duration of the received scan period in (2-22) and (2-24) due to A/C motion). The quantity

$$
\begin{equation*}
t_{R_{k}} \triangleq t_{k}+\frac{r\left(t_{k}\right) / c}{1-r\left(t_{k}\right) / c} \tag{2-25}
\end{equation*}
$$

is the time of reception at the $A / C$ of a signal event transmitted at time $t_{k}$, in particular the start of the $k$ th scan. Errors in determining the arrival of this event at the $A / C$ translate into errors in starting the local $\tau$-clock. It is the function of the Barker code, however, to keep these errors small.

As a consequence of the above development

$$
\begin{equation*}
t-\frac{r(t)}{c}=t_{k}+\left(1-\frac{\dot{r}\left(t_{k}\right)}{c}\right) \tau, \tag{2-26}
\end{equation*}
$$

and, using approximations (2-15) and (2-16), the direct path component $y_{D}$ can be redefined as a function of a global discrete-time variable $\dagger_{k}$ and a local continuousmtime variable $\tau$, as follows:

$$
\begin{equation*}
y_{D}\left(t_{k}, \tau\right)=\alpha\left(t_{k}\right) p\left[\theta_{A}\left(t_{k}+\tau\right)-\theta\left(t_{k}\right)\right] \cos \left[\omega_{0}\left(t_{k}\right) \tau+\beta\left(t_{k}\right)\right] \tag{2-27}
\end{equation*}
$$

in which the $-\dot{r}\left(t_{k}\right) \tau / c$ term of (2-26) has been dropped from the argument of $\theta_{A}(\cdot)$ because of its relative insigniftcance, and $\omega_{0}\left(t_{k}\right), \beta\left(t_{k}\right)$ are defined as follows:

$$
\begin{align*}
& \omega_{0}\left(t_{k}\right) \triangleq\left\{\begin{array}{l}
\omega_{c}\left(1-\dot{r}\left(t_{k}\right) / c\right), \text { as the apparent r-f carrier frequency } \\
\omega_{l F}-\omega_{c} \dot{r}\left(t_{k}\right) / c, \text { as the apparent i-f carrier frequency } \\
(2-28 b)
\end{array}\right. \\
& \beta\left(t_{k}\right)=\beta_{D}+\omega_{c} \dagger_{k}, \text { inttial phase on the } k+h \text { scan period }  \tag{2-29}\\
& (2-29)
\end{align*}
$$

The expected doppler shift (neglecting relativity) is present in (2-28).
The phase parameter $\beta_{D}$ in (2-5) and (2-8) was modeled originally as an independent random variable uniformly distributed on $[-\pi, \pi]$. Equation (2-29) above for $B\left(t_{k}\right)$ suggests that $B\left(t_{k}\right)$ then is also a random variable but one possible highly correlated with $B_{D}$. As a practical matter, however, the increment $\omega_{c}\left(\dagger_{k}-\dagger_{k-1}\right)$ is a large multiple of $2 \pi$ radians and can never be determined accurately enough to make velld use of the correlation implied by (2-29) above, for two reasons:

1. The $t_{k}$ are determined from the Barker decoder output via (2-25), and with a noisy input the uncertainty may become a significant part of a bit pulse width (approximately $67 \mu 5$, also many periods of the 5000 MHs r-f carrier).
2. The quantity $t_{k}-t_{k-1}$ is not constant from scan-to-scan, and may even be jittered to suppress propeller modulation effects.

On this basis the $\beta\left(t_{k}\right)$ also were modeled as independent random variables uniformly distributed on $[-\pi, \pi]$, which seems reasonable.

The ith reflection component, $\mathrm{y}_{\mathrm{R}_{1}}(\dagger),(2-7)$, can also be expressed in terms of the local recelver time variable $\tau$, associated above with the direct path signal. Under the approximation of $r_{T_{i}}(t),(2-20)$, we may write

$$
\begin{equation*}
t-\frac{r_{T_{1}}{ }^{(t)}}{c}=+\left(1-\frac{\dot{r}_{T_{i}}\left(t_{k}\right)}{c}\right)-\frac{r_{T_{i}}{ }^{(+)}}{c}+t_{k} \frac{\dot{r}\left(t_{k}\right)}{c} . \tag{2-30}
\end{equation*}
$$

Substituting from (2-33) for $t$ and (2-25) for $\dagger_{R_{k}}$ gives

$$
\begin{align*}
t-\frac{r_{T_{1}}{ }^{(t)}}{c} & =t_{R_{k}}-\frac{\left.{ }_{r_{1}}{ }^{\left({ }^{(t} R_{k}\right.}\right)}{c}+\tau\left(1-\dot{r}_{T_{1}}{ }^{\left.\left(t_{k}\right) / c\right)}\right.  \tag{2-31}\\
& =t_{k}-\frac{\Delta r_{1}\left(t_{k}\right)}{c}-\frac{\Delta \dot{r}_{i}\left(t_{k}\right)}{c} \cdot \frac{r\left(t_{k}\right) / c}{1-\dot{r}\left(t_{k}\right) / c}+\tau\left(1-\dot{r}_{T}\left(t_{k}\right) / c\right) \tag{2-32}
\end{align*}
$$

where, in general

$$
\begin{align*}
& \Delta r_{i}(t) \triangleq r_{T_{i}}(t)-r(t)  \tag{2-33}\\
& \Delta \dot{r}_{1}(t) \triangleq \dot{r}_{T_{i}}(t)-\dot{r}(t) \tag{2-34}
\end{align*}
$$

Scan data is received through the reflection path, clearly, when

$$
\tau \geq \frac{1}{1-\dot{r}_{T}\left(t_{k}\right) / c}\left[\frac{\Delta r_{1}\left(t_{k}\right)}{c}+\left(\frac{\Delta \dot{r}_{i}\left(\dagger_{k}\right)}{c}\right)\left(\frac{r\left(t_{k}\right) / c}{1-\beta\left(t_{k}\right) / c}\right)\right]=\frac{\Delta r_{i}\left(\dagger_{k}\right)}{c} .
$$

Neither of the terms involving the time derivatives $\Delta \dot{r}_{i}\left(\dagger_{k}\right)$ and $\dot{r}_{T_{j}}\left(t_{k}\right)$ in (2-32) is important in the argument of $\theta_{A}(\cdot)$, hence, using the approxImations (2-18, 2-19) for $\alpha_{1}(t), \theta_{R_{1}}(t)$ respectively, the ith reflection component $y_{R_{i}}$ can be redefined with bivariate argument $\left(t_{k}, \tau\right)$ as follows, for $0 \leq \tau \leq \frac{T}{1-r\left(t_{h}\right) / C}$ :

$$
Y_{R_{i}}\left(\dagger_{k}, \tau\right)=\left\{\begin{array}{l}
\alpha_{i}\left(\dagger_{k}\right) P\left[\theta_{A}\left(\dagger_{k}+\tau-\frac{\Delta r_{1}\left(t_{k}\right)}{c}\right)-\theta_{R_{1}}\left(t_{k}\right)\right]  \tag{2-35a}\\
\cos \left[\omega_{0}\left(\dagger_{k}\right) \tau+\beta_{1}\left(t_{k}\right)\right], \frac{\Delta r_{i}\left(t_{k}\right)}{c} \leq \tau \leq \frac{T}{1-\tilde{r}\left(t_{k}\right) / c} \\
0,0 \leq \tau<\Delta r_{i}\left(t_{k}\right) / c
\end{array}\right.
$$

where

$$
\omega_{0_{i}}\left(\dagger_{k}\right)= \begin{cases}\omega_{c}\left(1-\dot{r}_{T_{i}}\left(t_{k}\right) / c\right), & \text { as } r-f \text { carrier frequency }  \tag{2-36a}\\ \omega_{I F}-\omega_{c} \dot{r}_{T_{i}}\left(t_{k}\right) / c, & \text { as } 1-f \text { carrier frequency }\end{cases}
$$

$$
\begin{align*}
\beta_{i}\left(t_{k}\right) & =\beta_{R_{i}}+\omega_{c}\left[t_{R_{k}}-r_{T_{i}}\left(t_{R_{k}}\right) / c\right]  \tag{2-37}\\
& =\beta_{R_{i}}+\omega_{c}\left[t_{k}-\frac{\Delta r_{i}\left(t_{R_{k}}\right)}{c}\right]  \tag{2-37a}\\
& \left.=\beta_{R_{i}}+\omega_{c}\left[\dagger_{k}-\frac{\Delta r_{i}\left(t_{k}\right)}{c}+\frac{\Delta \dot{r}_{i}\left(t_{k}\right)}{c} \cdot \frac{r\left(t_{k}\right) / c}{1-r\left(t_{k}\right) / c}\right)\right] \tag{2-37b}
\end{align*}
$$

A more compact form of (2-35), which will be useful, employs a bivariate argument in $\alpha_{j}$, as follows:
where

$$
\begin{align*}
\gamma_{R_{i}}\left(t_{k}, \tau\right)= & \alpha_{i}\left(\dagger_{k}, \tau\right) p\left[\theta_{A}\left(\dagger_{k}+\tau-\frac{\Delta r_{i}\left(\dagger_{k}\right)}{c}\right)-\theta_{R_{i}}\left(\dagger_{k}\right)\right] \\
& \cos \left[\omega_{O_{i}}\left(t_{k}\right) \tau+\beta_{i}\left(t_{k}\right)\right] \tag{2-38}
\end{align*}
$$

$$
\alpha_{i}\left(t_{k}, \tau\right)=\left\{\begin{array}{l}
\alpha_{i}\left(t_{k}\right), \Delta r_{i}\left(t_{k}\right) / c \leq \tau \leq \frac{T}{1-\tilde{r}\left(t_{k}\right) / c}  \tag{2-39a}\\
0,0 \leq \tau<\Delta r_{i}\left(t_{k}\right) / c
\end{array}\right.
$$

It was established, following equation (2-29) for the $\beta\left(t_{k}\right)$ in the direct path component, that the $\beta\left(t_{k}\right)$ were independent random variables. The $\beta\left(+_{k}\right)$, however, are functions of properties of the transmitted signal which the $\beta_{i}\left(t_{k}\right)$ in the reflection components share. For each 1 , the initlal value of $\beta_{i}\left(t_{k}\right)$, say $\beta_{i}\left(t_{1}\right)$, was modeled as an independent random varlable unlformly distributed on $[-\pi, \pi]$, but as a consequence of the above, it was felt that the scan-to-scan change in $\beta_{1}$, e.g., $\beta_{i}\left(t_{k+1}\right)-\beta_{i}\left(\dagger_{k}\right)$, should be related to the corresponding change in $\beta$, i.e. $\beta\left(t_{k+1}\right)-\beta\left(t_{k}\right)$. Use of (2-29) and (2-37) gave the relations

$$
\begin{align*}
{\left[\beta_{i}\left(\dagger_{k+1}\right)-\beta_{i}\left(t_{k}\right)\right] } & =\left[\beta\left(t_{k+1}\right)-\beta\left(t_{k}\right)\right]-\frac{\omega_{c}}{c}\left[\Delta r_{i}\left(t_{R_{k+1}}\right)-\Delta r_{i}\left(t_{R_{k}}\right)\right] \\
& \simeq\left[\beta\left(\dagger_{k+1}\right)-\beta\left(t_{k}\right)\right]-\frac{\omega_{c}}{c}\left(t_{k+1}-t_{k}\right) \Delta \dot{r}_{i}\left(\dagger_{k}\right)
\end{align*}
$$

in the latter of which relatively insignificant terms corresponding to
the last in (2-37b) were dropped. In the simulation this relation assured the smooth evolution of the simulated multipath interference phenomena.

Specializing more concretely now to the signal in the (linear) i-f channel of the receiver, where the total signal, $y$, comprises direct path, reflection path and receiver noise components, we assume the i-f bandpass characteristic is approprlately symmetrical about the nominal bandcenter frequency, $\omega_{1 F}$. The noise then, a process generated at the recelver and hence modeled with sample function $r(\tau)$, is assumed to be a stationary, bandpass, zero-mean Gaussian procesc, with variance $s_{n}{ }^{2}$. A quadrature expansion of the noise $n(\tau)$ is fossible, therefore, and the following formulation of the composite signal $y$ with bivariate argument $\left(t_{k}, \tau\right)$ results:

$$
y\left(\dagger_{k}, \tau\right)=y_{D}\left(\dagger_{k}, \tau\right)+y_{R}\left(\dagger_{k}, \tau\right)+n(\tau)
$$

where

$$
\begin{align*}
& Y_{D}\left(\dagger_{k}, \tau\right)=\alpha\left(\dagger_{K}\right) p\left[\theta_{A}\left(\dagger_{k}+\tau\right)-\theta\left(\dagger_{K}\right)\right] \cos \left[\omega_{1} F^{\left.\tau+\left(\beta\left(\dagger_{K}\right)-\omega_{c} \dot{\tau}\left(\dagger_{K}\right) / c\right)\right]}\right.  \tag{2-42}\\
& =y_{D_{C}}\left(\dagger_{k}, \tau\right) \cos \omega_{1} F^{\tau-y_{D_{S}}}\left(\dagger_{k}, \tau\right) \sin \omega 1 F^{\tau}  \tag{2-43}\\
& y_{R}\left(\dagger_{k}, \tau\right)=\sum_{i} \alpha_{i}\left(\dagger_{k}, \tau\right) p\left[\theta_{A}\left(\dagger_{k}+\tau-\frac{\Delta r_{i}\left(\dagger_{k}\right)}{c}\right)-\theta_{R_{i}}\left(\dagger_{k}\right)\right] \\
& \cos \left[\omega_{1 F} \tau+\left(\beta_{i}\left(\dagger_{k}\right)-\omega_{c} \tau \dot{r}_{T_{i}}\left(\dagger_{k}\right) / c\right)\right]  \tag{2-44}\\
& =\gamma_{R_{c}}\left(\dagger_{k}, \tau\right) \cos \omega 1 F^{\tau-\gamma_{R_{s}}\left(\dagger_{K}, \tau\right) \sin \omega} I F^{\tau}  \tag{2-45}\\
& n(\tau)=n_{c}(\tau) \cos \omega 1 F^{\tau}-n_{s}(\tau) \sin \omega 1 F^{\tau} \tag{2-46}
\end{align*}
$$

in which

$$
\begin{equation*}
\gamma_{D_{C}}\left(t_{k}, \tau\right)=\alpha\left(t_{k}\right) p\left[\theta_{A}\left(t_{k}+\tau\right)-\theta\left(t_{k}\right)\right] \cos \left[\beta\left(t_{n}\right)-\omega_{c} \tau \dot{r}\left(t_{k}\right) / c\right] \tag{2-47}
\end{equation*}
$$

$$
\begin{align*}
\gamma_{D_{s}}\left(\dagger_{k}, \tau\right)= & \alpha\left(\dagger_{k}\right) p\left[\theta_{A}\left(\dagger_{k}+\tau\right)-\theta\left(\dagger_{k}\right)\right] \sin \left[\beta\left(\dagger_{n}\right)-\omega_{c} \tau \dot{r}\left(\dagger_{k}\right) / c\right] \\
y_{R_{c}}\left(\dagger_{k}, \tau\right)= & \sum_{i} \alpha_{i}\left(\dagger_{k}, \tau\right) \rho\left[\theta_{A}\left(\dagger_{k}+\tau-\frac{\Delta r_{i}\left(\dagger_{k}\right)}{c}\right)-\theta_{R_{i}}\left(\dagger_{k}\right)\right]  \tag{2-48}\\
& \cos \left[\beta_{i}\left(\dagger_{k}\right)-\omega_{c} \tau \dot{r}_{T_{i}}\left(\dagger_{k}\right) / c\right]  \tag{2-49}\\
y_{R_{s}}\left(\dagger_{k}, \tau\right)= & \sum_{i} \alpha_{i}\left(\dagger_{k}, \tau\right) p\left[\theta_{A}\left(\dagger_{k}+\tau-\frac{\Delta r_{i}\left(\dagger_{k}\right)}{c}\right)-\theta_{R_{i}}\left(\dagger_{k}\right)\right] \\
& \sin \left[\beta_{i}\left(\dagger_{k}\right)-\omega_{c} \tau \dot{r}_{T_{i}}\left(\tau_{k}\right) / c\right] \tag{2-50}
\end{align*}
$$

and
$n_{c}(\tau), n_{s}(\tau)$, the quadrature components of the noise, are independent, stationary low-pass Gaussian processes, each with mean zero, variance $\sigma_{n}{ }^{2}$, and two-sided bandwidth equal to the i-f bandwidth. (2-51)

Hence

$$
\begin{align*}
y\left(\dagger_{k}, \tau\right) & =m_{c}\left(\dagger_{k}, \tau\right) \cos \omega_{1 F} \tau-m_{s}\left(\dagger_{k}, \tau\right) \sin \omega \mid F^{\tau}  \tag{2-52}\\
& =M\left(\dagger_{k}, \tau\right) \cos \left[\omega_{1} \mid F^{\tau}-\Gamma\left(\dagger_{k}, \tau\right)\right] \tag{2-53}
\end{align*}
$$

where

$$
\begin{align*}
& m_{c}\left(\dagger_{k}, \tau\right)=y_{D_{c}}\left(\dagger_{k}, \tau\right)+y_{R_{c}}\left(\dagger_{k}, \tau\right)+n_{c}(\tau)  \tag{2-54}\\
& m_{s}\left(\dagger_{k}, \tau\right)=y_{D_{s}}\left(\dagger_{k}, \tau\right)+y_{R_{s}}\left(\dagger_{k}, \tau\right)+n_{s}(\tau)  \tag{2-55}\\
& M\left(\dagger_{k}, \tau\right)=\sqrt{m_{c}^{2}\left(\dagger_{k}, \tau\right)+m_{s}^{2}\left(\dagger_{k}, \tau\right)}  \tag{2-56}\\
& \Gamma\left(\dagger_{k}, \tau\right)=\arctan \left[m_{s}\left(\dagger_{k}, \tau\right) / m_{c}\left(\dagger_{k}, \tau\right)\right] \tag{2-57}
\end{align*}
$$

The principal signal modeling results to date have been presented. These results have been used in the digital simulation studies described in Chapter IV and in the optimal receiver design analysis of Chapter 111. in the simulation the local time variable $\tau$ was discretized, of course, a sampling rate equal to the $i-f$ bandwidth ( 160 kHz ) being chosen.

Other small refinements, such as dropping certain $\dot{r} / \mathrm{c}$ terms, etc., were made also where permissable; reference Is made to Chapter IV for a more detalled description of the simulation study.

The optimal design studies undertaken to date and reported in Chapter III were speciallzed to a signal model involving only the direct-path and receiver nolse components. This initial focus on the multipath-free case was justifled on several counts:

1. The study establishes for reference purposes the optimum level of performance obtainable in a multipath-free environment.
2. It provides insight into the structural properties of MLS receivers resulting from the requirement for optimal performance in a very basic and generally applicable disruptive environment.
3. The algorithms that resuit under this specialization represent lower bounds in computational complexity in relation to those optimal processors associated with the more complex environment of interest.

The first point cannot be overemphasized; the MLS receiver will operate most of the time without significant multipath interference, and its performance during such periods is not outside our concern in this study.

In re-extending our concern to the muitipath-corrupted signal, the results obtained above wlll probably not be used directly because of the very high dimension of the parameter space associated with the total reflected signal

$$
\begin{equation*}
y_{R}\left(t_{k}, \tau\right)=\sum_{i} y_{R_{i}}\left(t_{k}, \tau\right) \tag{2-58}
\end{equation*}
$$

Where $y_{R_{1}}$ is given In equation (2-38). Instead a stocastic process model (really a random field model) with associated sample function $y_{R}\left(\tau_{k}, \tau\right)$ is being considered. This is done in conjuntion with considerations of the estimation algorithm to be used, since the statistics of interest
depend to some extent on the latter. Second-order statistics on the scan interval and local to the arrival time of the direct-path pulse are expected to be important; variations of these statistics with $\dagger_{k}$ are the basis for an adaptive approach to the suppression of multipath interference. If first- and second-order statistics turn out to be sufficient, a unique Gaussian random field model is implied and the possibility of an adaptive algorithm of manageable dimensions.

Lastly, a state-variable model is needed as a prerequisite to the application of modern recursive estimation techniques to this problem. Such a model generally would describe the (short-term) evolution of the $A / C$ angular coordinate and other parameters of interest and relate these to the observed receiver signal. In the desired model the "state" should include the $A / C$ angular coordinate, of course, but also all unknown parameters whose estimates are needed for use in the angular coordinate estimate calculation. In the locally optimum estimation scheme to be described in Chapter 111, for example, the latter parameter set included the signal strength parameter $\alpha(t)$ and the apparent i-f carrier frequency $w_{0}(t)$ (Note: the latter estimate may not be needed if the nominal i-f carrier frequency is sufficiently great with respect to the doppler shift (which is about $800 \mathrm{~Hz} / 100 \mathrm{kts}$ speed, for a 5000 MHz r-f carrier frequency)). In addition, in an adaptive design an "augmented state" may appear and include some parameters associated with evolution of the "state-proper," thus allowing some estimation of the model as well as the variables of prime interest. The forms of models needed in the recursive estimation study will be determined later when recursive estimation algorithms are considered in greater detail.

## CHAPTER 111

## OPTIMAL MLS RECEIVER DESIGN

Under the optlmal tracking criterion two approaches to recelver design were considered:

1. Locally optimum estimation [3] in which it is assumes the tracking error is always small and constant on the interval of observation (the scan interval), and thus can be estimated as a parameter wlthout requiring a knowledge of the a priori statistics of the $A / C$ angular coordinate of interest.
2. Recursive state estimation [4, Chapters 7,8, and 9] in which a state space model is assumed and used to provide smoothing of the evolving estimetes and also extrapolation into and possible through periods of signal fade.

Quadrature detection of the I-F signal arises in these algorithns, and to preserve the option for analog implementation of these stages con-tinuous-time formulatlons were used mostly. The principal signal model used in the initial studies was the direct-path-component-plusnolse specialization.

## Locally Optimum Estimation [3]

in this formulation the recelved signal on the present scan interval is modeled as an observations process $\{y(t), t \in[0, T]\}$ with sample function of the general form

$$
\begin{equation*}
y(t)=c \cdot s(t, m+\theta, a)+n(t), \dagger \varepsilon[0, T] \tag{3-1}
\end{equation*}
$$

where
$t=$ local receiver time on the present scan interval
$[0, T]=$ the present scan interval wrt local receiver time $s(t, \cdot, \cdot)=$ a sample function of a process or a sure function with which the modulation or parameter of interest is assoclated. (3-4) $n(t)=$ a sample function of a white Gaussian noise process $[n(t)$, $t e[0, T]\}$ with known power density $N_{0}$.
m+e $=$ true present value of the unknown parameter (or parameter vector)
-to be estimated, assumed constant for $\ddagger \in[0, T]$
$m=$ estimate of the unknown parameter, based on observations
through the last, but not the present, scan interval
(possible extrapolated to the present, however)
$e=$ error in the last estimate, $m$
$a=a \operatorname{random}$ parameter (or parameter vector) in which we have no estimation interest

To avold technical difficulty with mathematical processing of white nolse the integrated observations process $\{Y(t), t \varepsilon[0, T]\}$ is considered instead of $\{y(t),+\varepsilon[0, T]\}$, where

$$
\begin{equation*}
Y(t)=\int_{0}^{\dagger} s(\tau, m+e, a) d \tau+N(t) \tag{3-10}
\end{equation*}
$$

and $N(t)$ is a Wiener process for which

$$
\begin{equation*}
\langle N(t) N(\tau)\rangle=N_{0} \min (t, \tau) \tag{3-11}
\end{equation*}
$$

(and where $\rangle$ denotes mathematical expectation).
The objective is to estimate the (unobservable) error e and to refine the parameter estimate $m$ using the error estimate. Following Murphy [3], probability measures relevant to this objective in a locally optimum estimation context are the following:

$$
\begin{align*}
Q & =\text { measure corresponding to }\{N(t), \dagger \varepsilon[0, T]\}  \tag{3-12}\\
S & =\text { measure corresponding to }\{s(t, m+e, a), \dagger \varepsilon[0, T]\}  \tag{3-13}\\
Y_{s} & =\text { measure corresponding to }\{Y(t), t \varepsilon[0, T] \mid s\}  \tag{3-1.4}\\
P_{e} & =\text { measure corresponding to }\{Y(t), \dagger \varepsilon[0, T]\} \tag{3-15}
\end{align*}
$$

An estimate $\hat{e}$ of $e$ is said to be locally optimum [3] at $e=\theta_{0}$ if two conditions, stated below, are satistied; all integrations are taken over the underlying elementary event space (universe), denoted $\Omega$ :
1.) The estimate $\hat{e}$ is locally unbiased at $e_{0}$, i.e.,

$$
\begin{equation*}
\text { a) } \int_{\Omega}\left(\hat{\theta}-e_{0}\right) d P_{e_{0}}=0 \text { (a vector), and } \tag{3-16}
\end{equation*}
$$

$$
\begin{equation*}
\text { b) } \frac{\partial}{\partial e} \int_{\Omega}\left(\theta-\theta_{0}\right)^{\top} d P_{\theta_{0}}=I \text { (the unlty matrix), } \tag{3-17}
\end{equation*}
$$

where ( $)^{\text {T }}$ denotes the transpose of ().
2.) The estimate $\hat{e}$ has minlmum mean square error among all locally unblased estimates of $e$ at $e_{0}$, i.e. if $\hat{\sigma}$ is any estimate of $e$ locally unbiased at $e_{0}$, then
$\int\left(\hat{e}-e_{0}\right)\left(\hat{e}-e_{0}\right)^{T} d P_{e_{0}} \leq \int\left(\hat{\sigma}-e_{0}\right)\left(\hat{\sigma}-e_{0}\right)^{T} d P_{e_{0}}$
in the sense of the usual "weak" order relation on a set of non-negative definite square matrices of the same dimension.

Murphy [3] has shown that the locally optimum estimate $\hat{e}$ of $\theta$ at $e=0$ is given by

$$
\begin{equation*}
\hat{e}=\Phi_{0}^{-1} A_{0}(Y) \tag{3-19}
\end{equation*}
$$

where $\Lambda_{0}$ is the vector whose ith component, $\Lambda_{0}$, is given by

$$
A_{O_{i}}(Y)= \begin{cases}\left.\frac{\partial}{\partial \theta_{i}} \ell_{n}\left(\frac{d P}{d Q}\right)\right|_{e=0}, & \text { if } \frac{d P_{e}}{d Q} \neq 0  \tag{3-20}\\ 0 & , \text { otherwise }\end{cases}
$$

and

$$
\begin{equation*}
\Phi_{0}=f_{\Omega} A_{0}\left(Y_{0}\right) \Lambda_{0}^{T}\left(Y_{0}\right) d P_{0} \tag{3-21}
\end{equation*}
$$

in which $Y_{0}$ is given by $(3-10)$ with $e=0$, If $e=0$, the residual mean square error ( $1 . e$. , the error covariance matrix) associated with this estimate is $\bar{\Phi}_{0}^{-1}$, that is

$$
\begin{equation*}
\int_{\Omega} \hat{e}^{-T} \mathrm{~T}_{\mathrm{d}}^{0} 0=\Phi_{0}^{-1} \tag{3-22}
\end{equation*}
$$

Radon-Nikodym derivatives associated with this estimate are the following:

$$
\begin{equation*}
\frac{d Y_{s}}{d Q}=\exp \left(\frac{1}{N_{0}} \int_{0}^{T} s(t, m+e, a) d Y(t)-\frac{1}{2 N_{0}} \int_{0}^{T} s^{2}(t, n+e, a) d t\right) \tag{3-23}
\end{equation*}
$$

(the first integral above being a Wiener integral) and

$$
\begin{equation*}
\frac{d P_{e}}{d Q}=\int \frac{d Y_{s}}{d Q} S(d s), \tag{3-24}
\end{equation*}
$$

this latter integral being taken over the space of realizations of $\{s(t, m+e, a), \dagger \varepsilon[0, T]\}$. When $s(t, m+e, a)=s(\dagger, m+e), i . e . i s$ deterministic, then $S$ is degenerate, i.e., there exists a realization of the process $\{s(t, m+e), t \varepsilon[0, T]\}$ such that $S(\{s\})=1$, in which case $\frac{d P_{e}}{d Q}=\frac{d Y_{s}}{d Q}$. Another special case of interest is when $s(t, m+e, a)$ is a deterministic function of random parameter $a$, which ranges on the reals, $R_{1}$, with induced probability measure $P$. Then the collection of realizations of the process $\{5(t, m+e, a), t \in[0, T]\}$ is indexed by a $\varepsilon R_{1}$, and we may write

$$
\begin{equation*}
\frac{d P_{e}}{d Q}=\int_{R_{1}} \frac{d Y_{s}(t, m+e, a)}{d Q} P(d a) \tag{3-25}
\end{equation*}
$$

where $d Y_{s}(t, m+e, a) / d Q$ denotes the form in (3-23) with the functional form of $s(\dagger, m+e, a)$ substituted as required. If $\mu$ is Lebesque measure on $R_{1}$ and $P$ is absolutely continuous with respect to $\mu$, then we may also write

$$
\begin{equation*}
\frac{d P_{e}}{d Q}=\int_{R_{f}} \frac{d Y_{s}(\dagger, m+e, a)}{d Q} \frac{d P}{d \mu}(a) d \mu(a) \tag{3-26}
\end{equation*}
$$

where $\frac{d P}{d \mu}$ (a) is the probability density of the random variable a.
Application of locally optimum estimation to the MLS receiver design probiem, specialized to the direct-path-signal-plus-noise case, is made by letting $s=y_{D}$, i.e., referring to (2-27),

$$
\begin{equation*}
s(\dagger, m+e, a)=\alpha p\left[\theta_{A}(\dagger)-\theta\right] \cos \left[\omega_{0} t+\beta\right],+\varepsilon[0, T] \tag{3-27}
\end{equation*}
$$

and making the associations

$$
m+e=\left(\begin{array}{c}
\theta  \tag{3-28}\\
\alpha \\
\omega_{0}
\end{array}\right)
$$

$$
\begin{equation*}
a=\beta \tag{3-29}
\end{equation*}
$$

It is illuminating to regard $\beta$ as a known parameter temporarily and regard $s()$ as a "sure" function of $t$ and the parameters $\theta, \alpha, \omega_{0}$. In this case, as explained following ( $3-24$ ), the measure $S$ is degenerate and

$$
\begin{equation*}
\frac{d P_{e}}{d Q}=\frac{d Y_{s}()}{d Q}=\exp \left[\frac{\alpha}{N_{0}} \int_{0}^{T} p\left[\theta_{A}(t)-\theta\right] \cos \left[\omega_{0} t+\beta\right] d Y(t)-\frac{\alpha^{2}}{4 N_{0}} q_{1}(\theta)\right] \tag{3-30}
\end{equation*}
$$

where

$$
\begin{align*}
q_{1}(\theta) & \triangleq 2 \int_{0}^{T} p^{2}\left[\theta_{A}(t)-\theta\right] \cos ^{2}\left[\omega_{0} \dagger+\beta\right] d t  \tag{3-31}\\
& =\int_{0}^{T} p^{2}\left[\theta_{A}(t)-\theta\right] d t \tag{3-32}
\end{align*}
$$

Then, from (3-20)

$$
A_{0}(Y)=\frac{1}{N_{0}}\left(\begin{array}{c}
-\hat{\alpha} \int_{0}^{T} \dot{p}\left[\theta_{A}(t)-\hat{\theta}\right] \cos \left[\hat{\omega}_{0} t+\beta\right] d Y(t)-\frac{\hat{\alpha}^{2}}{4} \dot{q}_{1}(\hat{\theta})  \tag{3-33}\\
\int_{0}^{T} p\left[\theta_{A}(t)-\hat{\theta}\right] \cos \left[\hat{\omega}_{0} t+\beta\right] d Y(t)-\frac{\hat{\alpha}}{2} q_{1}(\hat{\theta}) \\
-\hat{\alpha} \int_{0}^{T} p\left[\theta_{A}(t)-\hat{\theta}\right]+\sin \left[\hat{\omega}_{0} \dagger+\beta\right] d Y(t)
\end{array}\right)
$$

where

$$
\begin{align*}
& \hat{\theta}, \hat{\alpha}, \hat{\omega}_{0} \text { are last estimates, corresponding to } m  \tag{3-34}\\
& \left.\dot{p}\left(\theta_{A}(t)-\hat{\theta}\right) \triangleq \frac{\Delta p\left(\theta_{e}\right)}{d \theta_{e}}\right|_{\theta_{e}}=\left(\theta_{A}(t)-\hat{\theta}\right)  \tag{3-35}\\
& \dot{q}_{1}(\hat{\theta}) \triangleq \frac{d q_{1}(\hat{\theta})}{d \hat{\theta}}=-2 \int_{0}^{T} \mathrm{P}\left[\theta_{A}(t)-\hat{\theta}\right] \dot{p}\left[\theta_{A}(t)-\hat{\theta}\right] d t . \tag{3-36}
\end{align*}
$$

The last quantity, $\dot{q}_{1}(\hat{\theta})$, is very nearly zero chence $q_{1}(\hat{\theta})$ is very nearly a constant), except when $\hat{\theta}$ is near the ends of the interval of coverage, e.g. $\left(-60^{\circ},+60^{\circ}\right)$ for azimuth. To an extent the bivariate nature of p( ), i.e. the channel-cross-coupling effect discussed following (2-2),
also affects the functions $q_{1}()$ and $\dot{q}_{1}()$. Under the assumption these combined effects are negligible, the approximations

$$
\begin{align*}
& \mathrm{q}_{1}(\hat{\theta})=\text { constant }\left(=\int_{0}^{T} \mathrm{p}^{2}\left[\theta_{A}(t)\right] \mathrm{d} t\right)  \tag{3-37}\\
& \dot{q}_{1}(\hat{\theta})=0 \tag{3-38}
\end{align*}
$$

are made with the result that

$$
\begin{equation*}
\Lambda_{0}\left(Y_{0}\right)=\frac{1}{N_{0}} \int_{0}^{T} \operatorname{MdN}(t) \tag{3-39}
\end{equation*}
$$

where

$$
M=\left(\begin{array}{c}
-\hat{\alpha} p\left[\theta_{A}(t)-\hat{\theta}\right] \cos \left[\hat{\omega}_{0}++\beta\right]  \tag{3-40}\\
p\left[\theta_{A}(t)-\hat{\theta}\right] \cos \left[\hat{\omega}_{0} \dagger+\beta\right] \\
-\hat{\alpha} \dagger p\left[\theta_{A}(t)-\hat{\theta}\right] \sin \left[\hat{\omega}_{0} \dagger+\beta\right]
\end{array}\right)
$$

The result (3-39) follows from setting $e=0$ in $Y(t)$ in (3-10) and then substituting the resulting $Y_{0}$ into (3-33) and simplifying.

The above implles, then, by (3-21) that

$$
\begin{align*}
\Phi_{0} & =\frac{1}{N_{0}^{2}} \int_{\Omega}\left[\int_{0}^{T} M d N(t)\right]\left[\int_{0}^{T} M^{\top} d N(t)\right] d P_{0}  \tag{3-4!}\\
& =\frac{1}{N_{0}^{2}}\left\langle\left[\int_{0}^{T} M d N(t)\right]\left[\int_{0}^{T} M^{T} d N(t)\right]\right\rangle \tag{3-42}
\end{align*}
$$

which, since $\{N(+),+\varepsilon[0, T]\}$ is a wiener process with $\langle N(+N(\tau)\rangle$ as given in (3-11), can be written

$$
\begin{equation*}
\Phi_{0}=\frac{1}{N_{0}} \int_{0}^{T} M M^{\top} d t \tag{3-43}
\end{equation*}
$$

Substitution from (3-40) and use of some trigonometric identities (and some nearly obvious notational shorthand) gives

$$
\Phi_{0}=\frac{1}{N_{0}} \int_{0}^{7}\left[\begin{array}{lll}
\hat{\alpha}^{2} \dot{p}^{2}\left(\frac{1+\cos 2[]}{2}\right) & -\hat{\alpha} \dot{p}\left(\frac{1+\cos 2[]}{2}\right) & \hat{\dot{\alpha}^{2}+p \dot{p}\left(\frac{\sin 2[]}{2}\right)} \\
-\hat{\alpha} \dot{p}\left(\frac{1+\cos 2[]}{2}\right) & p^{2}\left(\frac{1+\cos 2[]}{2}\right) & -\hat{\alpha}+p^{2}\left(\frac{\sin 2[]}{2}\right) \\
\hat{\alpha}^{2}+p \dot{p}\left(\frac{\sin 2[]}{2}\right) & -\hat{\alpha}+p^{2}\left(\frac{\sin 2[]}{2}\right) & \hat{\alpha}^{2} t^{2} p^{2}\left(\frac{1-\cos 2[]}{2}\right)
\end{array}\right\} d t
$$

And asymptotically as $\hat{\omega}_{0}+\infty$ (or gets very large),

$$
\Phi_{0}=\frac{1}{N_{0}}\left(\begin{array}{ccc}
\frac{\hat{\alpha}^{2} q_{2}}{2} & 0 & 0  \tag{3-45}\\
0 & \frac{q}{2} & 0 \\
0 & 0 & \frac{\hat{\alpha}^{2} q_{3}(\hat{\theta})}{2}
\end{array}\right)
$$

where, under the approximating assumptions made,

$$
\begin{align*}
& q_{1}(\hat{\theta})=\left.\int_{0}^{T} p^{2}\left[\theta_{A}(t)-\hat{\theta}\right] d t\right|_{\dot{q}_{1}}=0=\int_{0}^{T} p^{2}\left[\theta_{A}(t)\right] d t=q_{1}, \text { a constant } \\
& q_{2}(\hat{\theta})=\left.\int_{0}^{T} p^{2}\left[\theta_{A}(t)-\hat{\theta}\right] d t\right|_{\dot{q}_{1}}=0=\int_{0}^{T} \dot{p}^{2}\left[\theta_{A}(t)\right] d t=q_{2}, \text { a constant } \\
& (3-47)  \tag{3-48}\\
& q_{3}(\hat{\theta})=\left.\int p^{2}\left[\theta_{A}(t)-\hat{\theta}\right] t^{2} d t\right|_{\dot{q}_{1}}=0=q_{1}+t_{R M S}^{2}(\hat{\theta}) .
\end{align*}
$$

Consequently, the covariance matrix, $\Phi_{0}^{-1}$, for the estimation erro, $\hat{e}-e_{0}, i s$ approximately

$$
\dot{\sigma}_{0}^{-1}=N_{0}\left(\begin{array}{ccc}
2 /\left(\hat{\alpha}^{2} q_{2}\right) & 0 & 0  \tag{3-49}\\
0 & 2 / q_{1} & 0 \\
0 & 0 & 2 /\left(\hat{\alpha}^{2} q_{3}(\hat{\theta})\right)
\end{array}\right)
$$

and the locally optimum estimate $\hat{e}$ is obtalned by combining (3-46) through (3-49) and (3-33) with (3-19):

$$
\hat{e}=\left(\begin{array}{c}
-\frac{2}{\hat{\alpha} q_{2}} \int_{0}^{T} \dot{p}\left[\theta_{A}(t)-\hat{\theta}\right] \cos \left[\hat{\omega}_{0} t+\beta\right] d Y(t)  \tag{3-50}\\
\frac{2}{q_{1}} \int_{0}^{T}\left[\left[\theta_{A}(t)-\hat{\theta}\right] \cos \left[\hat{\omega}_{0} t+\beta\right] d Y(t)-\hat{\alpha}\right. \\
\frac{2}{\hat{\alpha} q_{3}(\theta)} \int_{0}^{T} p\left[\theta_{A}(t)-\hat{\theta}\right] \operatorname{ts} \ln \left[\hat{\omega}_{0} t+\beta\right] d Y(t)
\end{array}\right)=\left(\begin{array}{c}
\hat{e}_{\theta} \\
\hat{\theta}_{\alpha} \\
\hat{e}_{\omega 0}
\end{array}\right)
$$

or, alternatively (recall $\beta$ was assumed known)

For instrumentation purposes, $d Y(t)$ would be replaced with $y(t) d t$ and physical dumping integrators employed to integrate the six outputs of three weighted or gated quadrature detectors, corresponding to the above. Other signiflcant observations that can be made include:

1. The estimator is dependent upon the prior estimate values but not the noise varlance parameter, $\mathrm{N}_{0}$.
2. The error covariance is proportlonal to nolse parameter $N_{0}$, and of particular interest is the varlance associated with $\hat{e}_{\theta}$, the estimated $A / C$ angular parameter:

$$
\begin{equation*}
\left\langle\hat{e_{\theta}^{2}}\right\rangle=\left(\frac{2 N o}{a^{2}}\right)\left(\frac{1}{q_{2}}\right) \tag{3-52}
\end{equation*}
$$

where the first factor is a measure of the ratio of noise-to-
estimated signal power, and the second factor is the reciprocal of the integral of $\dot{p}^{2}$, given $\ln (3-47)$.
3. The estimate $\hat{e}_{\alpha}$ in (3-50) involves an integral less the prior estimate of $\alpha, \hat{\alpha}$. Hence the integral is a global estiinate of $\alpha$, given $y(t)$ and prior estimates of $\theta$ and $\omega_{0}$. The error covarlance of the estimate is proportional to $N_{0}$, but independent of other parameters (under the stated assumption $\dot{q}_{1}(\theta)=0$ for all $\theta$ of interest.

Returning now to the more realistic situation in which $\beta$ is regarded as a random variable uniformly distributed on the interval $[-\pi, \pi]$, as described following (2-29), we have from (3-30)

$$
\begin{aligned}
& \frac{d Y}{s}(t, m+e, a) \\
& d Q=\exp \left(-\frac{\alpha^{2} q_{1}(e)}{4 N_{0}}\right) \exp \left(\frac{\alpha}{N_{0}} \int_{0}^{T} p\left[\theta_{A}(t)-\beta\right] \cos \left[\omega_{0} t+\beta\right] d Y(t)\right) \\
&=C_{e} \exp \left[\frac{\alpha}{N_{0}}\left(I_{c_{e}}(Y) \cos \beta-I_{s_{e}}(Y) \sin \beta\right]\right. \\
&=c_{e} \exp \left[\frac{\alpha}{N_{0}} \sqrt{1 Z_{e}(Y)+1 s_{e}^{2}(Y)} \cos \left(\beta-\operatorname{arc} \tan \left(\frac{s_{e}(Y)}{1_{c_{e}}(Y)}\right)\right)\right]
\end{aligned}
$$

where

$$
\begin{align*}
& c_{e} \triangleq \exp \left(-\frac{\alpha^{2} q_{1}(\theta)}{4 N_{0}}\right)  \tag{3-56}\\
& I_{c_{e}}(Y) \triangleq \int_{0}^{T} \mathrm{p}\left[\theta_{A}(t)-\theta\right] \cos \omega_{0} t d Y(t)  \tag{3-57}\\
& I_{s e}(Y) \triangleq \int_{0}^{T} \mathrm{p}\left[\theta_{A}(t)-\theta\right] \sin \omega_{0} t d Y(t) \tag{3-58}
\end{align*}
$$

and $q_{1}(\theta)$ is as given in $(3-32)$. Hence from (3-26)

$$
\begin{align*}
\frac{d P_{e}}{d Q} & =\frac{C_{e}}{2 \pi} \int_{-\pi}^{\pi} \exp \left[\frac{\alpha}{N_{0}} \sqrt{12_{C_{e}}^{2}+12}{\underset{s}{e}}^{e} \cos \left(\beta-\operatorname{arc} \tan \left(\frac{L_{e}}{I_{c_{e}}}\right)\right)\right] d \beta  \tag{3-59}\\
& =C_{e} l_{0}\left(\frac{\alpha}{N_{0}} \sqrt{\left[\frac{2}{C_{e}}(Y)+1 \frac{2}{s_{e}}(Y)\right.}\right)=C_{e} l_{0}\left(\frac{\alpha}{N_{0}} E_{e}(Y)\right) \tag{3-60}
\end{align*}
$$

$$
\begin{equation*}
=\exp \left\{\varepsilon_{n}\left[I_{0}\left(\frac{\alpha}{N_{0}} E_{e}(Y)\right)\right]-\frac{\alpha^{2} q_{1}(\theta)}{4 N_{0}}\right\} \tag{3-61}
\end{equation*}
$$

where

$$
\begin{equation*}
E_{\theta}(Y) \triangleq \sqrt{l_{C_{\theta}}^{2}(Y)+1_{S_{\theta}}^{2}(Y)} \tag{3-62}
\end{equation*}
$$

and $I_{0}$ is the modifted Bessel function of the first kind, zeroth order. The above relates to other results, which are well known [5], [6].

Substituting the above results in (3-20) we obtain, after some manipulation, the following:
in which new notation appearing is defined as follows:

$$
\begin{align*}
& \hat{\dot{p}} \triangleq p\left[\theta_{A}(t)-\hat{\theta}\right]  \tag{3-64}\\
& \hat{p} \triangleq p\left[\theta_{A}(t)-\hat{\theta}\right]  \tag{3-65}\\
& I_{0}(Y) \triangleq 1_{0}\left(\hat{\alpha} E_{0} E_{0}(Y)\right)  \tag{3-66}\\
& E_{0}(Y) \triangleq \sqrt{1 c_{c_{0}}^{2}(Y)+l_{S}^{2}(Y)} \text { per }(3-62)  \tag{3-67}\\
& I_{c_{0}}(Y) \triangleq \int_{0}^{T}\left[\left[\theta_{A}(t)-\hat{\theta}\right] \cos \hat{\omega}_{0}+\operatorname{tdY}(t) \quad \text { per }(3-57)\right. \tag{3-68}
\end{align*}
$$

$$
\begin{align*}
& I_{s_{0}}(Y) \triangleq \int_{0}^{T}\left[\left[\theta_{A}(t)-\hat{\theta}\right] \sin \hat{\omega}_{0} t d Y(t) \text { per }(3-58)\right.  \tag{3-69}\\
& I_{I_{0}}(Y) \triangleq 1_{1}\left(\frac{\hat{\alpha}}{N_{0}} E_{0}(Y)\right) \tag{3-70}
\end{align*}
$$

Where $I_{1}$ is the modified Bessel function of the first kind, first order. A simplificeition of (3-63) is possible with the definition of four more quantities similar to (3-68) and (3-69) above:

$$
\begin{align*}
& J_{c_{0}}(Y) \triangleq \int_{0}^{T p}\left[\theta_{A}(t)-\hat{\theta}\right] \cos \hat{\omega}_{0} \operatorname{tdY}(t)  \tag{3-7!}\\
& J_{s_{0}}(Y) \triangleq \int_{0}^{T} p\left(\theta_{A}(t)-\hat{\theta}\right] \sin \hat{\omega}_{0}+d Y(t)  \tag{3-72}\\
& K_{C_{0}}(Y) \triangleq \int_{0}^{T} p\left[\theta_{A}(t)-\hat{\theta}\right]+\cos \hat{\omega}_{0} t d Y(t)  \tag{3-73}\\
& K_{s_{0}}(Y) \triangleq \int_{0}^{T} p\left[\theta_{A}(t)-\hat{\theta}\right]+\sin \hat{\omega}_{0}+d Y(t) \tag{3-74}
\end{align*}
$$

Further, the quantity $I_{1_{0}}(Y) / I_{0}(Y)$, essentially a soft-limiter function with initial slope $=1 / 2$ and maximum values of $\pm 1$, will be denoted by $L_{0}(Y)$; it is closely approximated by an arc tangent expression, i.e.,

$$
L_{0}(Y) \triangleq \frac{1_{1}\left(\frac{\hat{\alpha}}{N_{0}} E_{0}(Y)\right)}{i_{\sigma}\left(\frac{Q}{N_{0}} E_{0}(Y)\right)}=\frac{2}{\pi} \operatorname{arc} \tan \left[\frac{\pi}{4} \cdot \frac{\hat{\alpha}}{N_{0}} E_{0}(Y)\right]
$$

With these definitions $\Lambda_{0}(Y)$ can be written

$$
\Lambda_{0}(Y)=\left(\begin{array}{l}
-\frac{\hat{\alpha}^{2} q_{1}(\hat{\theta})}{4 N_{0}}-\frac{\hat{\alpha} L_{0}(Y)}{N_{0} E_{0}(Y)}\left[I_{c_{0}}(Y) d_{c_{0}}(Y)+I_{s_{0}}(Y) J_{s_{0}}(Y)\right]  \tag{3-76}\\
-\frac{\hat{\alpha} q_{1}(\hat{\theta})}{2 N_{0}}+\frac{L_{0}(Y)}{N_{0} E_{0}(Y)}\left[I_{c_{0}}^{2}(Y)+I_{s_{0}}^{2}(Y)\right] \\
-\frac{\hat{\alpha} L_{0}(Y)}{N_{0} E_{0}(Y)}\left[I_{c_{0}}(Y) K_{S_{0}}(Y)-I_{s_{0}}(Y) K_{c_{0}}(Y)\right]
\end{array}\right)
$$

which together with (3-67) through (3-69) and (3-71) through (3-74) indicates the amount of processing of the observation $y(t)$ (or $Y(t)$ ) that is required by the optimal processor which is dented the value of the carrier phase parameter, B. Further, a comparison of (3-33), the known $\beta$ case, with (3-63) adove shows the two expressions for $A$ are analogous if the following interpretations are made:

$$
\begin{align*}
& \cos _{\beta}=\frac{I_{L_{0}}(Y)}{T_{0}(Y)} \cdot \frac{I_{0}(Y)}{E_{0}(Y)}=L_{0}(Y)\left(\frac{I_{c_{0}}(Y)}{E_{0}(Y)}\right)  \tag{3-77}\\
& \sin \beta=\frac{I_{0}(Y)}{I_{0}(Y)} \cdot \frac{I_{s_{0}}(Y)}{E_{0}(Y)}=L_{0}(Y)\left(\frac{I_{S_{0}}(Y)}{E_{0}(Y)}\right) \tag{3-78}
\end{align*}
$$

These may not be optimum or even good estimates of $\cos \beta$ and $\sin \beta$ in the usual sense, but locally optimum estimates of $\theta, \alpha, \omega_{0}$ result from their use. Certainly it can be shown that for very large signal-tom nolse rattos

$$
\begin{align*}
& \cos \beta+\cos \beta  \tag{3-79}\\
& \sin _{\beta}+\sin \beta . \tag{3-80}
\end{align*}
$$

In general we settle for less, however, noting, for example

$$
\begin{equation*}
(\cos \beta)^{2}+\left(\sin ^{\hat{n}} \beta\right)^{2}=L_{0}^{2}(Y) \tag{3-81}
\end{equation*}
$$

which is less than unity for finite signal-to-noise ratios.
This is as far as the stity of the locally optimum estimation technique has progressed. The computation of the error covariance matrix $\Phi_{0}^{-1}$ for the random $\beta$ case has yet to be done (if further effort on this direct-path-signal-plus-noise case is warranted). This matrix is not a function of the observation $Y(t)$; it is needed to complete the solution for the error estlmate $\hat{e}$ via $(3-19)$ as well as providing
the desired reference performance measure. The structure of a representatlve implementation of this receiver algorlthm is shown in Figure lli-l. The quadrature detection represented by the calculations of $I_{c_{0}}, I_{s_{0}}$, $J_{c_{0}}, J_{s_{0}}, K_{c_{0}}, K_{5}$, (3-68), (3-69) and (3-71) through (3-74) Pespectively, would be done with analog circuitry, and the remaining calculations assoclated with the error estimation, (3-19), (3-76), (3-67), (3-75), etc., as well as the A/C angle estimate update, would be done in a digital microprocessor. This algortthm requires a knowledge of the noise power density $N_{0}$. In a baseband simulation of this algorithm the quadrature detection integrations, (3-68), (3-69) and (3-71) through (3-74), would be represented by integrations of the low-frequency (i.e. difference frequency) components of the integrands shown in the equations.

The extenston of the locally optimum estimation algorithm to multipath-corrupted signals will be an early objective in the continuation of the project. The problem will center on modeling as a process the reflection component $y_{R}(t)$ of the recelved signal $y(t)=y_{D}(t)+y_{R}(t)+n(t)$ described in Chapter 11 , and then calculating the required likelihood ratio (Radon-Nikodym derivative). Kailath's work [6], [7], [8] will help in this second aspect. The successful extension to the multipathcorrupted signal is expected to result in an adaptive algorithm, which approach is discussed further in the conclusion to this chapter.

The complexity of the optimal quadrature detector processors that lave resulted motivates a serious consideration of the rectifier-type of envelope detector and the processing optimally of its output to produce the estimate of the $A / C$ angular coordinate. The difficulty In considering this option from the locally optlmum estimation viewpoint stems from the possible inapplicabllity of the latter model (as formulated by Murphy) $\%$ o the rectifled that a suitable likelihood ratio expression cannot be found in the literature and will have to be derived. These problems are related to the fact that, even in the multipath-free case, the corruption of the envelope process (induced by receiver noise) has the following properties:


Figure $111-1 \begin{gathered}\text { Structure of a Locally Optimum Estimator Receiver } \\ \text { for the Direct-Path Signal }+ \text { Noise Case }\end{gathered}$

1) The corruption is not additive.
2) The corruption Is not Gaussian.
3) The corruption does not have mean value zero.

The rectifler-envelope-detector does seem to offer some economic advantages in implementation, however, and the associated analysis and design problems wlll be studied. Concern to date for these types of envelope detectors has been somewhat informal and is discussed in Chapter IV on suboptimal design.

Recursive state estimation (RSE) has some appeal for the MLS application, but algorithms of this class in general have some undestrable characteristics which must be understood and whose effects minimized. In addition, general RSE algorithms are based upon system models in standard state varlable forms, and algorlthms considered for the MLS application must accomodate the special form and uncertainties in the MLS signal model. The adaptation and evaluation of recursive state estimation techniques for the MLS application is presently at an early stage. This is only a brlef description of some relevant propertles of recursive state estimators and a discussion of our initial concerns in the direction of inquiry.

Underlying the RSE approach generally are the following:

1. A valid state-variable model
2. A cholce of some specific criterion of optimality of estimation
3. An objective of optimal estimation based on all observations from some initial time through the present.

The algorithms that result are characterized generally by both an evolving state estimate and an evolving covariance-of-estimation error matrix (assuming model vallidity). The importance of the latter is underscored by the presence of the error covariance as a (matrix-) factor in the processing iKalman) gain for new observations-mas was the case in the localiy-optimum estimation a!gorithm (though the error covariance used there for gain calculations was a static quantity).

The updating of the error covariance in RSE, usually in a "downward ${ }^{\text {¹ }}$ direction as the data base grows, is a distinguishing feature of RSE and one that makes model validity so important. For Ilnear systems with a valid model, for example, the algorithm developed under the least mean squared error criterion produces the exact estimate value (without approximation), and the estimator is finite dimensional and global. y asymptotically convergent. If the model used is not sufficientIy accurate, the expected and characteristic downward drift may be
present in the computed error covariance, but the latter may bear little relation to the covariance of the true error; a form of filter divergence results.

When the system is nonlinear, the exact estimate in general is infinite-dimensional, necessitating for purposes of implementation some form of approximation, which, in turn may affect the modeling validity, Successful extensions of RSE to nonlinear systems in general are finitedimensional algorithmic approximations which are convergent in the locale of the true state valu*; from this standpoint RSE appears as a promising approach to tracking algorithm development for optimal MLS recelvers.

A valid model is clearly a crucial factor in applications of RSE, and requires careful selection of many parameter values. To ease this task somewhat by reducing the number of parameters, one might be tempted In some cases to remove the stochastic forcing function from his model, i.e., to model nis system as either unforced or forced by a known deterministic function. The first implication of this is that the conditional expectation of the state at one point in time, given the state value at some other point in time, is not a random variable; the final consequences may be disastrous. With only observation noise in the model the calculated error covariance matrix tends toward sing!larity (actually may approach the zero matrix) as time and the (implicit) data base grow arbitrarily large. The Kalman gain then becomes very small, limiting severely the influence of later observation. In this situation the estimator is said to be in "data saturation" and is especially prone to the filter divergence syndrome, caused now, in part, by computational errors, such as round-off, etc.

Two ways to avoid data saturation are as follows:

1) Use static noise of sufficiont intensity in the model
2) Use a limited memory approach to restrict the effective data base to the most recent observations.

The first approach is the ustua one taken in RSE applications; the
application of locally optimum estimation to the MLS problem, as described previously, is an example of the second approach. Both approaches effectively place a lower bound on the Kalman gain used in processing new observa†lons.

The digital computation of an evolving error covariance matrix presents certain problems also, arising from the fin!te-wordlength structure of digltal machines. Covariance matrices in general are characterized by the symmetric and non-negative definite properties of matrices. Digital computational errors arising in a RSE algorithm can be categorized into two groups in relation to their effects on filter performance:

1. Those that degrade the performance only slightly in terms of convergence and tracking properties.
2. Those that induce filter divergence.

Computational errors made in the estimate extrapolation and update probably are of the first type and are remedied in subsequent updates. Computational errors made in the covarlance matrix extrapolation and update tend to be of the second type; particularly those associated with the loss of the symmetric and non-negative definite properties Two ways to circumvent this problem are as follows:

1. integrate the covariance matrix equation off-line, if possibie, on a high-precision machine and use the resulting steady-state solution as a matrix constant in the estimator.
2. Reformulate the error covariance propagation problem in terms of propagation of a triangular matrix square root of the error covarlance.

The first approach may result in substantially suboptimal performance during certain periods of time, but provides additional benefits of computational simplicity (on-line) and no possibility of data saturation. The second approach insures the (now impllcit) covariance matrix will have the requisite matrix properties, but at a moderate increase in computational complexity. Optlmality of the estimate is maintained throughour all phases of its evolution, however, a benefit that must be weighed
carefully considering the relative transience of multipath interference in the composite received signal.

Finally, it is noted that stralght-forward appllcation of RSE theory to the MLS receiver problem gives algorithms which assume the carrier phase is known, as with the first appllcation of the locally optimum estimation theory. Extension of the RSE theory is needed to treat the case of interest where the carrier phase is a parameter of no intrinsic interest, modeled as a uniformly distributed random variable on $[-\pi, \pi]$. Extensions would be desirable also to the cases where the signal had been preprocessed with linear or quadratic detectors (rectifier-fypes), and consequently baseband observations were available but which were corrupted effectively by signal dependent noise that was neither Gaussian, additive nor zero mean, as discussed under locally optimum estlmation.

## Conclustons

Both the locally optimum estlmation and recursive state estimation approaches possess both good and bad features in relation to the MLS receiver design problem. It is expected that both algorithms should be consldered for application in the final design. RSE could provide the desired extrapolation between scan periods and into fades, as required, given a valid state model. Locally optimum estimation might then be used to provide needed estimates of the state model parameters, which may themselves vary with time but knowledge is lacking of the assoclated laws of evolution and statistics. Much additional work is needed, of course, to obtaln quantltative design and performance data on such a scheme and to describe the effects of such partitioning of the joint problem of identification-state estimation.

Finally, the maltipath propagation disturbance should be put back into the model and its effect on the estimation algorithm determined. The preferable type of algorithm would be an adaptive one. A nonadaptive approach would appear possible also in which the averaged performance of a class of multipath environments is optimized, but its performence in any one environment could be poor. The adaptive approach
is expected to be one in which the "gates" of the tracking receiver are modulated in some way by estimates of parameters associated with the multipath Interference model.

## CHAPTER IV

## SUBOPTIMAL DESIGN AND SIMULATION EVALUATION

A suboptimal receiver study was undertaken in order to consider some classical receiver designs involving envelope detection and subsequent processing by a form of early-late gating. This provided an opportunity to improve the computer simulation as well as to produce a candidate receiver design which could be used as a performance standard in evaluating future receivers. No attempt was made to optimally process the envelope-detected signal. Simulation results, however, indicate that such a course may well be worthwhile.

Receiver Algort thm Design
Early design involved the development of a centroid receiver which utilized the relation between envelope pulse centroids and the $A / C$ angular coordinate $\theta$. This receiver recomputed centroid positions (and thus $\theta$ ) for each TO-FRO scan, rather than computing error in the estimate of $\theta$ from the previous scan; l.e., it was not a tracking receiver, and since it processed the signal envelope received over the entire scan interval, it had no multipath suppression ability. Several forms of early-late gate tracking algorithms were then considered, with a square-gate version selected as the new receiver design. A development of the square-gate tracking receiver will now be presented.

The square-gate receiver algorithm produces an estimate of the $A / C$ coordinate by computing the error in the estimate from the previous scan. Let $\theta(t)=\theta\left(t_{k}\right)$ on the $k t h$ scan interval (from 2-i6), and let $\hat{\theta}(k)$ represent the receiver estimate of $\theta\left(t_{k}\right) . \hat{\theta}$ is formed as follows:

$$
\begin{align*}
& \hat{\theta}(k+1) \triangleq \hat{\theta}(k)+\hat{\Delta \theta}(k)  \tag{4-1}\\
& \hat{\Delta \theta}(k) \triangleq \int_{0}^{T} p_{D}(\tau) \theta[\tau, \hat{\theta}(k)] \mathrm{d} \tau \tag{4-2}
\end{align*}
$$

where $\tau$ is the local scan time at the $A / C$ (from 2-23), $P_{D}(\tau)$ is the direct-path signal envelope, on the $(k+l) s+\operatorname{scan}$, and $g[\tau, \hat{\theta}(k)]$ is the gating function (see Figure IV-I). The envelope function is given by:




Figure IV-I Envelope and Gating Functions

$$
\begin{align*}
& P_{D}(\tau) \triangleq \alpha p\left[\theta_{A}(\tau)-\theta\left(\dagger_{K}\right)\right]  \tag{4-3}\\
& \theta_{A}(\tau)= \begin{cases}\theta_{0}+\Omega \tau & 0 \leq \tau \leq T_{S} \\
\theta_{0}+\Omega T S & : \\
T_{S}<\tau<T_{F} \\
\theta_{0}+\Omega T_{S}-\Omega\left(\tau-T_{F}\right): & T_{F} \leq \tau \leq T\end{cases} \tag{4-4}
\end{align*}
$$

where $p(\cdot)$ is the envelope selectivity function defined by ( $\bar{z}-1$ ). In Figure $\mid V-1, t_{p_{1}}$ and $t_{p_{2}}$ are the envelope centroids. The receiver gates
 $\hat{\theta}(k)$. Each gate has helght $G$ and half-width $w$ so that the gating function becomes:

$$
g[\tau, \hat{\theta}(k)]=\left\{\begin{array}{l}
G:{t_{g_{1}}-w \leq \tau \leq t_{g_{1}} \text { or } t_{g_{2}} \leq \tau \leq t_{g_{2}}+w}_{-G: t_{g 1} \leq \tau \leq t_{g_{1}}+w, \text { or } t_{g_{2}}-w \leq \tau \leq t_{g_{2}}} \begin{array}{l}
\text { otherwise }
\end{array} \tag{4-5}
\end{array}\right.
$$

${ }_{g_{f}}$ and ${ }_{g_{s}}$ are the values of $\tau$ when $\theta_{A}(\tau)=\hat{\theta}(k)$ and are given by:

$$
\begin{align*}
& {t_{g_{1}}}=\frac{\hat{\theta}(k)-\theta_{0}}{\Omega}  \tag{4-6}\\
& t_{g_{2}}=T_{s}+T_{F}+\frac{\theta_{0}-\hat{\theta}(k)}{\Omega} \tag{4-7}
\end{align*}
$$

Define the error term $\theta_{e}(k)$ as follows:

$$
\begin{equation*}
\theta_{e}(k) \triangleq \theta\left(t_{k+1}\right)-\hat{\theta}(k) \tag{4-8}
\end{equation*}
$$

It follows from (4-1) that $\hat{\theta}(k)$ is an estimate of $\theta_{\theta}(k)$. A relation between the gate height and width is desired which will drive $\left[\theta\left(t_{k+1}\right)-\hat{\theta}(k+1)\right]$ to zero by equating $\hat{\Delta \theta}(k)$ and $\theta_{e}(k)$. From (4-2) we have:

$$
\begin{equation*}
\hat{\Delta \theta}(k)=\int_{\mathrm{t}_{g_{1}}^{-w}}^{\mathrm{g}_{1}^{+W}} g[\tau, \hat{\theta}(k)] \mathrm{p}_{\mathrm{D}}(\tau) \mathrm{d} \tau+\int_{\mathrm{t}_{\mathrm{g}_{2}}{ }^{+W}}^{g_{2}^{+W}} g[\tau, \hat{\theta}(k)]{p_{D}}(\tau) \mathrm{d} \tau \tag{4-9}
\end{equation*}
$$

$$
\begin{align*}
= & G \int_{-W}^{0} P_{D}\left(\tau+t_{g_{1}}\right) d \tau-G \int_{0}^{W} P_{D}\left(\tau+t_{g_{1}}\right) d \tau-G \int_{-W}^{0} P_{D}\left(\tau+t_{g_{2}}\right) d \tau \\
& +G \int_{0}^{W} P_{D}\left(\tau+t_{g_{2}}\right) d \tau \tag{4-10}
\end{align*}
$$

$P_{D}\left(\tau+\dagger_{g_{1}}\right)$ is found by combining (4-6), (4-7), (4-4), and (4-3). Coupling this result with (4-8), we can rewrite (4-10) as:
$\Delta \hat{\theta}(k)=\alpha G\left[\int_{-w}^{0} p\left(\Omega \tau-\theta_{e}\right) d \tau-\int_{0}^{w} p\left(\Omega \tau-\theta_{e}\right) d \tau-\int_{-w}^{0} p\left(-\Omega \tau-\theta_{e}\right) d \tau+\int_{0}^{w} p\left(-\Omega \tau-\theta_{e}\right) d \tau\right]$ (4-11)
Note that if $\theta_{e}(k)=0, \Delta \hat{\theta}(k)=0$, as expected. Assume $\theta_{e}(k)$ is small. Then:

$$
\begin{gather*}
p\left(\Omega \tau-\theta_{e}\right) \approx p(\Omega \tau)-\theta_{e} \frac{d p(\Omega \tau)}{d(\Omega \tau)}  \tag{4-12}\\
\int_{-w}^{0} p\left( \pm \Omega \tau-\theta_{e}\right) d \tau \approx \int_{-w}^{0} p( \pm \Omega \tau) d \tau \mp \frac{\theta_{e}}{\Omega} \int_{-\Omega w}^{0} d p(\Omega \tau)=p(w) \mp \frac{\theta_{e}}{\Omega}[p(0)-p(\Omega w)]
\end{gather*}
$$

Substituting this result into (4-1i) yields:

$$
\begin{align*}
\hat{\Delta \theta}(k) & =\alpha G\left[\frac{\theta_{e}(k)}{\Omega}(4 p(\Omega w)-4 p(0))\right]  \tag{4-14}\\
& =\frac{-4 \alpha G e_{e}(k)[1-p(\Omega w)]}{\Omega} \tag{4-15}
\end{align*}
$$

Equating $\hat{\theta}(k)$ and $\theta_{e}(k)$, we have:

$$
\begin{equation*}
G=\frac{-\Omega}{4 \alpha[1-p(\Omega W)]} \tag{4-16}
\end{equation*}
$$

It is assumed that the envelope selectivity function $p(\cdot)$ will be known. Thus, given a knowledge of the signal amplitude $\alpha$, the gate height $G$ can be computed for any desired width w such that $\hat{\theta}(k)=\theta_{e}(k)$ (assuming that $\theta_{e}$ is sufficiently small). Although $\alpha$ is not known by the $A / C$, it can be estimated by noting in Flgure IV-I that

$$
\begin{equation*}
\alpha=p_{D}\left(t_{p_{1}}\right), \quad i=1,2 \tag{4-17}
\end{equation*}
$$

An estimate of $\alpha$ can be obtained by averaging signal envelope samples taken at ${ } g_{j}$ and $\dagger_{g_{2}}$, which are estimates of $t_{p_{1}}$ and $t_{p_{2}}$. This estimate can then be substituted for $\alpha \ln (4-16)$ to compute $G$.

The square gate receiver was developed without considering the effects of additive noise or multipath upon the received signal envelope. This approach can be defended in light of the following:

1. The positive and negative areas of the gating function are equal. It is therefore expected that the effect upon the estimate of nolse occurring in a positive gate will be roughly cancelled by that of noise occurring in a negative gate.
2. Any multlpath distortions in the envelope occurring outside the gates wlll be ignored.

The recelver tested in the simulation can be described from (4-1) and (4-2) as:

$$
\begin{equation*}
\hat{\theta}(k+1)=\hat{\theta}(k)+\int_{0}^{T} M\left(t_{k+1}, \tau\right) g[\tau, \hat{\theta}(k)] d \tau \tag{4-18}
\end{equation*}
$$

where $g(\cdot)$ is defined by (4-5) and $p_{D}(\tau)$ has been replaced with the complete signal envelope $M\left(t_{k+1}, \tau\right)$, as presented $i n(2-53)$. On the $k$ th scan $\alpha$ is estimated by averaging $M\left(\dagger_{k}, t_{g_{1}}\right)$ and $M\left(t_{k}, \dagger_{g_{2}}\right)$ (see $4-17$ ), and $G$ is ther updated:

$$
\begin{equation*}
G=\frac{-\Omega}{2\left[M\left(t_{k}, t_{g_{1}}\right)+M\left(t_{k}, t_{g_{2}}\right)\right] \times[1-p(\Omega w)]} \tag{4-19}
\end{equation*}
$$

It was decided to set $w$ equal to the time between centroid and fi, st zero of an envelope pulse of $P_{D}(\tau)$. This selection of gate width seemed intuitively good in that the gate would include most of the direct signa! envelope and yet be narrow enough to exclude most multipath.

The computer simulation Involved a discrete-time version of the local scan time $\tau$, so that the square-gate receiver operated on samples of the envelope $M\left(\dagger_{k}, \tau\right)$. This operation will be described in the next section of thls chapter.

## Simulation Modeling

The simulation objective was to evaluate envelope-detector recelvers, making it necessary to model the l.F. signal envelope $M\left(t_{k}, t\right)$ in (2-53). The signal modeling was basicly that of Chapter II, with the following changes:

1. The "received scan interval" of (2-22) became
$t_{k}+\frac{r\left(t_{k}\right)}{c} \leq t \leq t_{k}+\frac{r\left(t_{k}\right)}{c}+T$
in which fairly insignificant $\frac{\dot{r}(t)}{c}$ terms were dropped, In terms of local scan time $\tau$ this interval became (from 2-24)
$0 \leq \tau \leq T$
$t_{R_{k}}$, defined in (2-25), likewise became
$\dagger_{R_{k}} \approx \dagger_{k}+\frac{r\left(t_{k}\right)}{c}$
2. The signal ampiitudes $\alpha$ and $\alpha_{i}$ in (2-42) and (2-44) were assumed constant. This is valid if the $A / C$ and reflector ranges do not vary appreciably.
3. $t_{k}$ was dropped from the argument of $\theta_{A}(\cdot)$ in (2-42) and (2-44), as $\theta_{A}$ is a periodic function which reinltiates at $t=\dagger_{k}$.
4. The range and angle coordinates of the $A / C$ and reflection points were assumed to be linear functions of time; i.e., $\dot{r}, \dot{\theta}, \dot{\phi}, \dot{r}_{R_{i}}$ $\dot{\theta}_{R_{f}}$, and $\dot{\phi}_{R_{f}}$ were made constants.

The local scan time $\tau$ was discretized to simulate sampling of the received signal envelope $M\left(\dagger_{k}, \tau\right)$. Starting at $\tau=0$, the envelope was sampled across the entire scan interval at a rate equal to the IF bandwidth $\mathrm{B}_{1 F}(160 \mathrm{KHZ})$. From (4-20A) the number of samples taken in one scan interval would be

$$
\begin{equation*}
J_{T}=T \times B_{1 F}+1 \tag{4-23}
\end{equation*}
$$

The local scan time at the j th sample would thus be:

$$
\begin{equation*}
\tau(j)=(j-1) \delta \quad 1 \leq J \leq J_{T} \tag{4-24}
\end{equation*}
$$

where $\delta=1 / B_{1 F}$. The envelope function $M\left(t_{k}, \tau\right)$ can now be expressed in terms of the discrete local scan time by substituting (4-23) into equations (2-47) through (2-50) and incorporating the new changes in modeling enumerafed above:

$$
\begin{align*}
& \left.y_{D_{C}}\left(t_{k}, j\right)=\alpha p\left[\theta_{A}(j-1) \delta\right)-\theta\left(t_{k}\right)\right] \cos \left[\beta\left(t_{k}\right)-\frac{\omega_{c} \dot{r}}{c}(j-1) \delta\right] \\
& \triangleq P_{D}\left(\dagger_{k}, j\right) \cos \psi{ }_{D}\left(\dagger_{k}, j\right) \\
& y_{D_{s}}\left(\dagger_{k}, j\right)=\alpha p\left[\theta_{A}((j-1) \delta)-\theta\left(\dagger_{k}\right)\right] \sin \left[B\left(\dagger_{k}\right)-\frac{\omega_{C} \dot{r}}{c}(j-1) \delta\right] \\
& \triangleq P_{D}\left(\dagger_{k}, j\right) \sin \Psi_{D}\left(\dagger_{k}, j\right) \\
& \gamma_{R_{c}}\left(t_{k}, j\right)=\sum_{i} \alpha_{i} p\left[\theta_{A}\left((j-1) \delta-\frac{\Delta r_{i}\left(t_{k}\right)}{c}\right)-\theta_{R_{i}}\left(t_{k}\right)\right] \cos \left[\beta_{i}\left(\dagger_{k}\right)-\frac{\omega_{c} \dot{r}_{T}\left(\dagger_{k}\right)}{c}(j-1) \delta\right] \\
& \triangleq \sum_{i} P_{R_{i}}\left(t_{k}, j\right) \cos \Psi_{R_{i}}\left(t_{k}, j\right) \\
& y_{R_{s}}\left(\dagger_{k}, j\right)=\sum_{i} \alpha_{i} p\left[\theta_{A}\left((j-1) \delta-\frac{\Delta r_{i}\left(t_{k}\right)}{c}\right)-\theta_{R_{i}}\left(\dagger_{k}\right)\right] \sin \left[\beta_{i}\left(t_{k}\right)-\frac{\omega_{c} \dot{F}_{T_{i}}\left(t_{k}\right)}{c}(j-1) \delta\right] \\
& \text { (4-31) } \\
& \triangleq \sum_{i} P_{R_{i}}\left(\dagger_{k}, j\right) \sin \Psi_{R_{j}}\left(t_{k}, j\right) \tag{4-32}
\end{align*}
$$

Note that the RF phase terms $\Psi_{D}\left(\dagger_{k}, j\right)$ and $\Psi_{R_{i}}\left(\dagger_{k}, j\right)$ are 1 inear functions of time and can be expressed:

$$
\begin{align*}
& \Psi_{D}\left(\dagger_{k}, j\right)=\left\{\begin{array}{l}
\beta\left(\dagger_{k}\right): j=1 \\
\Psi_{D}\left(\dagger_{k}, j-1\right)+\Delta \Psi_{D}: \quad 2 \leq j \leq J_{T}
\end{array}\right.  \tag{4-33}\\
& \text { Where } \Delta \Psi_{D}=-\frac{\omega_{C} \dot{C}}{c} \delta \tag{4-34}
\end{align*}
$$

$$
\Psi_{R_{i}}\left(\dagger_{k}, j\right)=\left\{\begin{array}{l}
\beta_{1}\left(t_{k}\right): J=1 \\
\Psi_{R_{i}}\left(\dagger_{k}, j-1\right)+\Delta \Psi_{R_{i}}\left(\dagger_{k}\right): 2 \leq j \leq J_{T}  \tag{4-36}\\
\text { where } \Delta \Psi_{R_{l}}\left(\dagger_{k}\right)=\frac{-\omega_{c} \dot{r}_{T_{l}}\left(\dagger_{k}\right)}{c} \delta
\end{array}\right.
$$

The envelope $M\left(t_{k}, j\right)$ now foltows directly from (2-54) through (2-56):
$M\left(\dagger_{k}, j\right)=\left[\left(y_{D_{c}}\left(\dagger_{k}, j\right)+y_{R_{c}}\left(\dagger_{k}, j\right)+n_{c}(j)\right)^{2}+\left(y_{D_{s}}\left(\dagger_{k}, j\right)+y_{R_{s}}\left(\dagger_{k}, j\right)+n_{s}(j)\right)^{2}\right]^{1 / 2}$ (4-37)
where $n_{c}(j)$ and $n_{s}(j)$ are the quadrature noise components of (2-51).
The simulation must compute $M\left(t_{k}, j\right), j=1$ to $J_{T}$, for every TO-FRO scan of interest. This also involves computing $A / C$ and reflector ccordinates and velocities at $\dagger_{k}$ (and in some cases $t_{R K}$ ), as these appear as parameters in the equations just developed. A description of the simulation for the $k t h$ scan will now be presented.

Let $t_{k}\left(k=1,2, \ldots k_{\text {max }}\right)$ be evenly spaced, and define:

$$
T_{U D} \triangleq+_{k}-\dagger_{k-1} \quad(=75 \mathrm{milliseconds})
$$

From (4-22) we can update the $A / C$ and reflector coordinates:

$$
\begin{align*}
& r\left(t_{k}\right)=r\left(t_{k-1}\right)+\dot{r} \cdot T_{U D} \\
& \theta\left(t_{k}\right)=\theta\left(t_{k-1}\right)+\dot{\theta} \cdot T_{U D} \\
& \phi\left(t_{k}\right)=\phi\left(t_{k m i}\right)+\dot{\phi} \cdot T_{U D} \\
& r_{R_{i}}\left(t_{k}\right)=r_{R_{i}}\left(t_{k-1}\right)+\dot{r}_{R_{i}} \cdot T_{U D} \\
& \theta_{R_{i}}\left(t_{k}\right)=\theta_{R_{i}}\left(\dagger_{k-1}\right)+\dot{\theta}_{R_{i}} \cdot T_{U D} \\
& \phi_{R_{i}}\left(t_{k}\right)=\phi_{R_{i}}\left(\dagger_{k-1}\right)+\dot{\phi}_{R_{i}} \cdot T_{U D} \tag{4-39}
\end{align*}
$$

For each reflector we compute $r_{T_{i}}\left(\dagger_{k}\right)$ from (2-12) through (2-14):

$$
\begin{align*}
& \zeta_{i}\left(t_{k}\right)=\cos \phi\left(t_{k}\right) \cos \phi_{R_{i}}\left(t_{k}\right) \cos \left[\theta\left(t_{k}\right)-\theta_{R_{l}}\left(t_{k}\right)\right]+\sin \phi\left(t_{k}\right) \sin \phi_{R_{i}}\left(t_{k}\right)  \tag{4-40}\\
& r_{A R_{i}}\left(t_{k}\right)=\left[r^{2}\left(t_{k}\right)-\zeta_{i}\left(t_{k}\right) r\left(t_{k}\right) r_{R_{i}}\left(t_{k}\right)+r_{R_{i}}^{2}\left(t_{k}\right)\right]^{1 / 2}  \tag{4-41}\\
& r_{T_{i}}\left(t_{k}\right)=r_{R_{i}}\left(t_{k}\right)+r_{A R_{i}}\left(t_{k}\right)  \tag{4-42}\\
& \Delta r_{i}\left(t_{k}\right)=r_{T_{i}}\left(t_{k}\right)-r\left(t_{k}\right) \tag{4-43}
\end{align*}
$$

The time derivative of $(4-41)$ can be shown to be:

$$
\begin{align*}
& \dot{r}_{A R_{i}}\left(\dagger_{k}\right)=\frac{r\left(\dagger_{k}\right)}{r_{A R}\left(\dagger_{k}\right)}\left[\dot{r}-\zeta_{i}\left(\dagger_{k}\right) \dot{r}_{R_{i}}\right]+\frac{r_{R_{i}}}{r_{A R_{i}}} \frac{\left.\dagger_{k}\right)}{\left(\dagger_{k}\right)}\left[\dot{r}_{R_{i}}-\zeta_{i}\left(\dagger_{k}\right) \dot{r}\right]  \tag{4-44}\\
& \dot{r}_{T_{i}}\left(\dagger_{k}\right)=\dot{r}_{R_{i}}+\dot{r}_{A R_{i}}\left(\dagger_{k}\right) \tag{4-45}
\end{align*}
$$

From assumptions (2-20) and (4-21) we obtain:

$$
\begin{align*}
& r\left(t_{R K}\right)=r\left(t_{k}\right)+\dot{r} \frac{r\left(t_{k}\right)}{c}  \tag{4-46}\\
& r_{T_{1}}\left(t_{R K}\right) z r_{T_{i}}\left(t_{k}\right)+\dot{r}_{T_{i}}\left(t_{k}\right) \frac{r\left(t_{k}\right)}{c}  \tag{4-47}\\
& \Delta r_{i}\left(t_{R K}\right)=r_{T_{i}}{ }^{\left(t_{R K}\right)-r\left(t_{R K}\right)} \tag{4-48}
\end{align*}
$$

$\Delta \psi_{R_{i}}\left(t_{k}\right)$ is computed from (4-36). Set $\Psi_{D}\left(t_{k}, 1\right)$ to $\beta\left(t_{k}\right)$, a uniform random variable on $[-\pi, \pi]$ supplied by a random number generator. From (2-40) and (4-35) compute:

$$
\Psi_{R_{1}}\left(\dagger_{k}, 1\right)=\beta_{i}\left(\dagger_{k}\right)=\beta\left(t_{k}\right)+\left[\beta_{1}\left(\dagger_{k-1}\right)-\beta\left(t_{k-1}\right)\right]-\frac{\omega_{c}}{c}\left[\Delta r_{i}\left(t_{R K}\right)-\Delta r_{1}\left(t_{R K-1}\right)\right]
$$

Now $M\left(\dagger_{k}, j\right)$ can be computed for each sample on scan $K$.
For $\mathrm{j}=1$ to $\mathrm{J}_{\mathrm{T}}$ :
$P_{D}\left(\dagger_{K}, j\right)=\alpha p\left[\theta_{A}((j-1) \delta)-\theta\left(\dagger_{k}\right)\right]$
$y_{D C}\left(\dagger_{k}, j\right)=P_{D}\left(\dagger_{k}, j\right) \cos \Psi_{D}\left(\dagger_{k}, j\right)$
$\gamma_{D S}\left(\dagger_{K}, j\right)=p_{D}\left(\dagger_{K}, j\right) \sin \Psi_{D}\left(\dagger_{k}, j\right)$
$\Psi_{D}\left(\dagger_{k}, j+1\right)=\Psi_{D}\left(\dagger_{k}, j\right)+\Delta \Psi_{D}$
$P_{R_{i}}\left(\dagger_{k}, j\right)=\alpha_{i} p\left[\theta_{A}\left((j-1) \delta-\frac{\Delta r_{i}\left(t_{k}\right)}{c}\right)-\theta_{R_{i}}\left(t_{k}\right)\right]$
$y_{R_{c}}=\sum_{i} p_{R_{i}}\left(t_{k}, j\right) \cos \psi R_{i}\left(t_{k}, j\right)$
$y_{R_{s}}=\sum_{i} P_{R_{i}}\left(t_{k}, j\right) \sin \psi_{R_{i}}\left(\dagger_{k}, j\right)$
$\Psi_{R_{i}}\left(\dagger_{k}, j+i\right)=\Psi_{R_{i}}\left(\dagger_{k}, j\right)+\Delta \Psi_{R_{i}}\left(\dagger_{k}\right)$
Obtain quadrature noise samples $n_{c}(j)$ and $n_{s}(j)$ from Gaussian random number generator. Compute $M\left(\dagger_{k}, j\right.$ ) using (4-37).

In the FORTRAN computer simulation, the antenna scanning function $\theta_{A}(\cdot)$ is computed by the function subprogram $\operatorname{THA}(T)$, where the argument $T$ is computed in the main program. $\theta_{A}(\cdot)$ is defined by ( $4-4$ ) and illustrated in Figure $\mathrm{IV}-\mathrm{I}$. The angular selectivity function $p(\theta)$ is computed by the funct:on subprogram P(THETA), where THE iA is computed in the main program. $F(\theta)$ is defined as follows, from reference [1, pp. 2-213, 2-214]:
$i p(\theta)= \begin{cases}\frac{\cos \left(\frac{\pi}{2} a\right)}{1-a^{2}} & a \neq 1 \\ \frac{\pi}{4} & a=1 \\ \text { where } a=2.4 \theta\end{cases}$

The simulation program computes $M\left(\dagger_{k}, j\right)$ for $J=1$ to $J_{T}$ and stores these samples in array $M$. $M$ is then passed to subroutine RCVR, which contains the square gate tracking receiver algortthm. The algorthm updates the estimate by computing the error in the estimate of the previous scan. It differs from the recelver developed earlier in this chapter as follows:

1. The signal envelope now exists in discrete form so that the integration of (4-18) is performed using a trapezoldal approximation rule.
2. Discontinuities in the gating function (see Figure IV-I) have been forced to occur at the sample times, so that the gates are well defined. Otherwise, the receiver would produce the same value of $\Delta \hat{\theta}(k)$ in ( $4-18$ ) regardless of where between two samples the gate center occurred.

RCVR operates on the samples of array $M$ one by one, with the result stored in array EST. EST thus contains an evolving estimate, where EST(J ${ }_{T}$ ) represents the final estimate $\theta(k)$ for the $k$ th scan.

After computing $\hat{\theta}(k)$, the simulation program advances to the next scan and recomputes the envelope and estimate. This is continued for as many scans as desired. For efficiency in computing the simulation has been carried out in two FORTRAN programs. The first, MLSRCVR, perforins all simulation calculations and stores results on file. The second program, MLSPLOT, plots these results using a CDC COMPLOT DP-7 digital plotter. Emphasis is placed here on MLSRCVR. The following plots are made:
I. A long-term piot, which plots for every scan $K$ in the simulation: A. the composite signal envelope at the time of direct-signal centroids
B. the error in estimate $\hat{\theta}(k)$
2. A plot of signal envelope $M\left(t_{k}, j\right)$ as a function of local scan time for six selected scans.
3. A short-term plot, which for one selected scan plots the received envelope $M\left(t_{k}, J\right)$ as well as the evolving estinate as functions of local scan time. Also plotted is the evolving estlmate which would resuit from a recelved signal containing no nolse or multipath.

A flow chart for program MLSRCVR is presented in Figures IV-2 through IV-3, while a basic flowchart for MLSPLOT appears in Figure IV-4. Program listings are contained in Appendix A. it should be noted that variable names used in the flowcharts and programs often differ from those used here.

## Performance Evaluation

The square-gate tracking receiver was tested under the following conditions: The $A / C$ with coordinates $r=10 \mathrm{~N} . \mathrm{mi} ., \theta=30^{\circ}, \phi=3{ }^{\circ}$ was approaching the runway with airspeed 300 knots $(\dot{\theta}=\dot{\phi}=0)$. A single specular reflector with coordinates $r_{R_{1}}=1.0 \mathrm{~N} . \mathrm{mi} ., \theta_{R_{1}}=33^{\circ}$, $\phi_{R_{1}}=1.85^{\circ}$, was following the circular path $\dot{r}_{R_{1}}=0, \dot{\theta}_{R_{1}}=-3 \%$ second, $\phi_{R_{1}}=0$. One second later (scan 14), the reflector was at $\theta_{R_{1}}=30^{\circ}$, so that the signal envelope at the $A / C$ dua to the reflector was inside the receiver gates and colncident with the envelope due to the direct signal. After another second ( $k=27$ ), the reflector was at $\theta_{R_{I}}=27^{\circ}$ and the signal envelope due to reflector was outside the recelver gate again. This was considered to be a reallstic fest for the square-gate receiver in that in-gate multipath interference intutively represented a worse case. The simulation was run for 27 scans with the above initial conditions, with signal amplitude $\alpha=1.0$, and multipath amplitude $\alpha_{1}=0.8$ and for several values of initial RF phase difference $\left.E \beta_{1}(1)-\beta(l)\right]$. Also made were two trials with no reflectors and differing signal-tonolse ratios.

All simulation trials were made on the CDC 6400 computer. Plots of receiver performance may be found in Appendix B. Results are fabulated in Figure $\operatorname{lV}-5$. Note that with a 20 db signal-to-noise ratio and no multipath the receiver was able to estimate $\theta$ with an R.M.S. error of $0.015^{\circ}$. R.M.S. errors are shown for the trials with multipath, yet


Figure IV-2 Flowchart for Simulation Program MLSRCVR
(Note: Details of blocks : dentified with encircled letters are given of subsequent pages.)

BLOCK A


Figure IV-2a Initialization


Figure IV-2b A/C and Reflector Initial Conditions


Figure $\operatorname{V}-2 \mathrm{C}$ Updating Signal to kth Scan


Figure IV -2d Computing Envelope Samples

## BLOCKE

## Enter Subroutine RCVR (Recelver Algorithm)

Input: ${ }^{\theta}$ EST ${ }^{\text {(Final Estimate from Previous Scan) }}$
M (Array of Signal Envelope Samples)

Output: EST(Evolving Estimate Array)

$$
\left(\operatorname{EST}\left(J_{T}\right)=\text { Final Estimate of } \theta\right. \text { ) }
$$

Fill Long-Term Plotting Arrays

$$
\begin{gathered}
\operatorname{TMK}(k)=t_{k} \\
\operatorname{CS}(K)=\operatorname{MIN}(\operatorname{CS} I, \operatorname{CS} 2) \\
\operatorname{THER}(K)=\theta-\operatorname{EST}\left(U_{T}\right) \\
E_{\text {RMS }}=E_{\text {RMS }}+[\operatorname{THER}(K)]^{2}
\end{gathered}
$$

Figure IV-2e Estimate Calculation


Figure IV-2f Short-Term Plotting Decisions

Advance A/C Geometry to Start of Next Scan Interval

$$
\begin{aligned}
r_{t k} & =r_{t k}+v T_{U D} \\
\theta & =\theta+\dot{\theta} T_{U D} \\
\phi & =\phi+\dot{\phi} T_{U D}
\end{aligned}
$$

Advance Geometry of ith Specular Reflector to Start of Next Scan Interval

$$
\begin{aligned}
r_{R t_{k}}(i) & =r_{R t_{k}}(i)+v_{R}(i) T_{U D} \\
\theta_{R}(i) & =\theta_{R}(i)+\dot{\theta}_{R}(i) T_{U D} \\
\phi_{R}(i) & =\phi_{R}(i)+\dot{\phi}_{R}(i) T_{U D}
\end{aligned}
$$



Figure IV-2g Advancing Geometry to Next Scan

Advance A/C Geometry to Start of Next Scan Interval
$r_{t k}=r_{t k}+v T_{U D}$ $\theta=\theta+\dot{\theta} \mathrm{T}_{\mathrm{UD}}$ $\phi=\phi+\dot{\phi} T_{U D}$


Advance Geometry of ith Specular Reflector to Start of Next Scan Interval

$$
\begin{aligned}
r_{R t_{k}}(i) & =r_{R t_{k}}(i)+v_{R}(i) T_{U D} \\
\theta_{R}(i) & =\theta_{R}(i)+\dot{\theta}_{R}(i) T_{U D} \\
\dot{\phi}_{R}(i) & =\phi_{R}(i)+\dot{\phi}_{R}(i) T_{U D}
\end{aligned}
$$



Figure IV-2g Advancing Geometry to Next Scan


Figure IV-3 Square-Gate Receiver Flowchart


Figure $1 V-4$ Flowchart for Plotting Program MLSPLOT

| Figure | Jobname | SNR(db.) | No. <br> Reflectors | RF Phase <br> Difference at <br> Coincidence | R.M.S. <br> Error | Comments |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| B-1 | MLSRCMP | 20 | 0 | - | $0.015^{\circ}$ |  |
| B-2 | MLSRCAZ | 8 | 0 | - | $0.059^{\circ}$ |  |
| B-3 | MLSRCJO | 8 | 1 | $180^{\circ}$ | $0.247^{\circ}$ |  |
| B-4 | MLSRCFG | 20 | 1 | $180^{\circ}$ | $0.203^{\circ}$ | Fade; Greatest Error <br> B-5 |
| MLSRC5C | 20 | 1 | $0^{\circ}$ | $0.162^{\circ}$ | Phase Enhancement |  |
| B-6 | MLSRCBE | 20 | 1 | $90^{\circ}$ | $0.173^{\circ}$ |  |
| B-7 | MLSRC7Q | 20 | 1 | $270^{\circ}$ | $0.201^{\circ}$ |  |

Figure IV-5 Simulation Results
these quantitles may not be very useful comparison measures in general. This is especially true for differing multipath environments, since R.M.S. is a time-averaging process and the multipath interference may be non-stationary. A more useful measure may be peak error, which is avallable on the plots.

Note in Figure B-4.a how the signal fades when the direct and reflected slgnals are beam coincident but $180^{\circ}$ out of phase (scan 14). Also note that the greatest error in the estimate $\hat{\theta}(k)$ occurs not at the fade, but as the signal comes out of the fade. In Figure B-4,b one can see how the reflected signal envelope moves across the direct signal envelope as the simulation progresses, and how af scan 13 (1 scan before colncidence) the two signals have nearly cancelled each other. This is contrasted with Figure B-5.b, where phase enhancement causes the signal envelope to increase in amplitude on scan 13. Also note that in general the simulation trials with multipath have very low error near the beginning and end of the trials. This is expected, since a reflector removed $3^{\circ}$ in azimuth from the A/C would produce an out-of-gate signal envelope pulse.

## Conclusion

This study has produced and demonstrated a receiver algorithm of basically intuitive design which has some commendable performance features. The value of the simulation developed for evaluation of candidate algorithms was also demonstrated.

The study of this algorithm suggested two options for further consideration:

1. An algorithm which optimally uses the detected envelope to estimate the $A / C$ angular coordinate.
2. An algorlthm not closely coupled to the exact form of the selectivity function $p(\cdot)$, and hence, inherently suboptimal, but perhaps less sensitive to variations in the shape of p(.) arising from cross-coupling of the angle channels and site-to-site design and installation variations, etc.

The slmulation appears to be validated in the results presented. The principal use in the near future of this tool will be in the study of optimal tracking algorithms. Some refinements will have to be made as we consider both quadrature detection and envelope detection estimation algorithms. Also, a linear evolution of $A / C$ and/or reflector coordinates seems more plausible in a Cartesian frame than in a sperical system. This modification will be made to test the validity of assumption, equations (2-15) through (2-20), made in signal modeling.

## CHAPTER V

## CONCLUSIONS AND RECOMMENDATIONS

A summary has been presented of both theoretical and simulation study results, including

1. The development of detalled signal models cast in the time frame of the $A / C$ receiver;
2. Proliminary optimal estimation studies, involving first an application of the locally optimum estimating algorithm to the direct-slgnal and noise specialization, then a critical overview of some relevant characteristlcs of recursive state estimation approaches;
3. Design of a suboptimal square-gate tracking algorithm which accepts the baseband output signal of a standard IInear AM demodulator, and then evaluation of this algorithm in simulation.

The greater signiflcance of the work done to date is in identifying and bringing into focus concrete problems and problem areas that should be - onsidered, and in suggesting methods of attack.

First among these is the modeling of the trotal reflection component of the signal; the modeling results described gives support to the belief a random field defined with respect to a low-dimension parameter space may be a suitable model. This would motivate a multipath-adaptive approach.

Another problem stems from our ignorance, of the law of evolution of the $A / C$ angular coordinate and the general and unknown variability with time of the law. One approach which should be considered involves slmultaneous model identification and state estimation, possibly in a "layered" algorithm with different criteria of estimation optimality for parameters on different levels.

The study of processors using the output of AM detection should be broadened to include both linear and quadratic AM detection, and also an objective to process optimally the detector outputs in calculating
an estimate of the $A / C$ angular coordinate. The most attractive estimation algorithm for the MLS receiver may well come from such an approach.

Finally, in support of the prototype construction effort to be done In the project continuation a number of decisions must be considered and made, dealing with the cholce of algorithm, cholce of microprocessor and choice of interface equipment. Synchronization and acquisition detalls must be clarifled and a system plan worked out which will allow the microprocessor to perform all its jobs in the requisite real time frame. These tasks are under consideration presently.

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APPENDIX A
COMPUTER PROGRAMS

```
MICROWAVE LANDING SYSTEM RECEIVER S,F. IRWINIJR. S/1/75
```

REVISED $10 / 3 / 75$
MLS RECEIVER SIMULATION PROGRAM, IN THIS PROGRAM THE RECEIVED SIGNAL IG
GFNERATED AND RECEIVER PERFORMANCE EVALUATED. OUTPUT DATA IS STORED ON A
PERMANENT FILE, WHICH IS THEN ACCESSED BY A PLOTTING PROGRAM-.
THE FOLLOWING MUST BE ON DATA CARES AT THE ENO OF THE PROGRAM:
IOPT II
NAME (I),I=1:3 3A10
iNRCVR(I), I=1.3) (NRCVR2(I).I=1,3) 6A10
CHAN $3 A 10$
ERSPEC E20.1
FC.TUO,THO, IHS,TS,TR, 日EAMITH 7E10.2
BIF.ALPHA 2F10.3
ALPHAM F10.3
SNR F10.3
THES E20.1
(RA(I) IF =1.5) 5E15.1
IMAX,KMAX 2 I10
RTK.V.THETA.DTH:PHI.DPH $6(E 10.1)$
RRTK(I),VR(I),THETAR(I),DTHR(I),PHIR(I),DPHR(I),ALPHAR(I),DIFO(I) BEID,I
(FOR I=I TO IMAX)
$(P L(I), I=1,5) \quad 5 E 15,1$
LOGICAL PLTSCAN FLAE
REAL M,MPNM,MC,MS,NC,NS

DIMENSION RRTK(10) THETAR(10), DTHR(10),PHIR(10), DPHR(10)
DIMENSION ALPHAR(10), DIFO(10), TDR(10), PHASER(10), DPHASER(10)
DIMENSION VR(10), NRCVR(3):NAME (3):RTO (10)
OIMENSION CS(30), TMK (30) THER (30) TTA (2000)
DIMENSION RA(5):RA1 (5),RA2(5) \&NRCVR2(3);PL(5)
COMMON TR,TS,TF, THO THS FOMG:TAU
COMMON/PLT/KMAX,ERR*PL
COMMON/SEL/BEAMUTH
COMMON/TRACK/THES,RA
$C$
$C$
$C$
$C$
$C$
$C$
C. INPUT PLOTTING OPTION IOPT: WHERE:
IOPT=1: GENERATE ALL OLOTS
IOPT=2: GENERATE LONG TERM PLOT ONLY
IOPT=3: GENERATE SHORT-TERM PLOT ONLY
READ 11:IOPT
11 FORMAT(II)
${ }^{C}$
INPUT TITLE INFORMATION FOR PLOTTING PROGRAM
C



## 350 CONTINUE

## $C$

C
C INPUT NUMBER OF SCANS AND REFLECTORS
c
C. KMAX $=$ NUMBER OF SCANS

C IMAX =NUMBER OF REFLECTORS
C
READ 400:IMAX,KMAX
400 FOHMAT (2T10)
WRITE(20) KM $\cap$ IIMAX. ALPHA, ALPHAM,SNR,SIGHA,TI I UT
$C$
$C$
$C$

INPUT AIRCRAFT INITIAL. CONDITIONS
C
RTK=RANGE IN NAUTICAL MİLES
V=AIRSPEED IN KNOTS
C THETA=A/C AZIMUTH ANGLE
C DTH $=$ TIME CHANGE IN THETA
PHI=A/C ELEVATION ANGLE
C UPH $=$ TIME CHANGE IN PHI
READ 500 ,RTK, V,THETA \& DTH:PHI \& DPH
500 FORMAT(GE10.1)
WRITE(20) RTK,V,THETA, DTH,PHI, DPH

1501 FORMAT $\left(1 \mathrm{H}_{1}, 10 \mathrm{X}, 3 \mathrm{~A}_{1} \mathrm{D}, 5 \mathrm{X}, 3 \mathrm{~A}_{1} 0\right)$
PRINT 1502, SUR + THETA,THES
1502 FORMAT $11 H 0,10 X, * S N R=*, F 7.2,5 X, * T H E T A=*, F B, 3,5 X, *$ QRIGINAL ESTIMATE $=$ 1**F8.3)
PRINT 1503:KMAX,IMAX

PRILYT 1504,IOPT, CHAN, ERSPEC

PRINT 1505, (NAME (I) +IF1,3) , JB:DAY
1505 FDRMAT (1HO +10X.*RUN BY *.3A10.EX.*.JOENAME: *,A10.5X; *DATE: *,A10)
PRINT 1506, AI_PHA, ALPHAM, BEF

$1 H=*, F 7,0, * H Z \# 1$
PRINT 1507
1507 FORMAT (1HO:/1,20X**MLS FUNCTION PROPERTIES:*)
PRINT 1508,FC.TUD.THO.THS:TR;TS:BEAMWTH


PRINT 1509
1509 FORMAT(1HO*//,20X.*TRACKING RECEIVER CONSTANTS:*)
PRINT 1510 (RA(I) 1 I $=1: 5)$
1510 FOKMAT (1H0.10X.*RA= (**5(2X.E11.3),*)**)
PRINT 1511
1511 FORMAT(1HO.//.20Xन\#A/C INITIAL CONDITIUNS:*)
PRINT 1512,RTK,V,THETA,DTH,PHI, DPH
 $16.2,5 \mathrm{X}, * \mathrm{PHI}=*, \mathrm{~F} 6.2,5 \mathrm{X}, * \mathrm{DPH}=*, F 6,2$ )
RTK=RTK*MPNM
$\mathrm{V}=\mathrm{V} * \mathrm{MPI} \mathrm{M} / \mathrm{SPH}$

DPHASED=WCC*V*TAU

```
        IF(IMAX,EQ,0) GO TO 1030
```

        D0 \(1000 \quad \mathrm{I}=1\) I IMAX
        REAU BOD, RRTK(I), VR(I), THETAR(I), DTHR(I), PHIR(I), DPHR(I), ALPHAR(I)
        1. DIFO(I)
        800 FORMAT (BE10.1)
            PKINT 1513,I
    1513 FORMAT (1HO, / / 20X:*INITIAL CONDITIONS FOR REFLECTOR*II2)
        PRINT 1512, MRTK(I):VR(I), THETAR (I), DTHR(I) \&PHIR(I);DPHR(I)
        PRINT 1514, ALPHAR (I), DIFO(I)
    1514 FOHVAT (1H0,10X,*ALPHA=*,F7.2.10X**PHASE DTF. FROA DIRECT SIGNALGRA
    

1I). ALPHAR(I)
RRTK(I)=RRTK(I)*MPNM
VR(I) $=$ VR(I) *MPNM/SPH
1000 CONTINUE
IF (IMAX.GE 3) GOTO 1050
1030 IU=3-INAX
D0 $1040 \quad I=1, I J$
WRITE(20) C4C,CIC.CIC.C
2040 CONTINUE
C
C
c
C INPUT SHORT-TERM PLDTTING CONSTANTS:
c
ARRAR PL CONTAINS CONSTANTS FOR ACCESS BY SUBROUTINE SHORT, WHICH DETERMIAES
C WHAT SCAN WILL HAME A DETAILED SHORT-TERM PLOT OF RECEIVER PERFORMANCE,
C IF ARRAY PL IS NOT NEEDED. IT MUST BE FILLED WITH ZEROS.
c
3050 READ 340 ( $\mathrm{PL}(I)$-I $=1.5$ )

1515 FORMAT $(1 \mathrm{HO}, 10 \mathrm{X} * * \mathrm{PL}=(*: 5(2 \mathrm{X}, \mathrm{E} 16.8), *) * *)$
$I N C=K$ MA $\overline{x / 6}$
INC2=1
ERHS $=0.0$


```
TM(J)=(J-1)*TAU
```

1525 CONTINUE
WIRITE（20）TM
DO $50000 \mathrm{~K}=1$ ，KMAX
$T K=(K-1) * T U D$
IDU二RTKくC
$\qquad$
T1K＝TK＋TOD
$R=R T K+V * T D D$
PHASED $=2.0 * P I *(\operatorname{RANF}(0.0)=.5)$
PRINT 1517，K，R，THETA，PHI

$1=*$ EI $6,8,5 X, *$ PHI（DEG）$=*$ E16．8）
IF（IMAX．EQ．0）GO TO 2050
$00 \quad 2000 \quad 1=1$ I ImAX
ZETA＝CGS（PHI）＊COS（PHIR（I））＊COS（THETA－THETAR（I））＋SIN（PHI）＊SIN（PHIR（
151）

VAR＝（RTK＊（V－ZETA＊VR（I））＋RRTK（I）＊（VR（I）－ZETA＊V））／RAR
TDR（I）$=($ RRTK（I）$)+$ RARI／C
RR＝RRTK（I）$+V R(I) * T D R(I)$
RAR＝RAR＋VAR＊TDR（I）
RT $=H R+R A K$
OPHASER（I）$=$ WCC＊（VR（I）＋VAR）$* T A U$
IF（K．NE． 1 ）GO TO 1900
OIF＝DIFO（I）
$R O=R$
RTO（I）＝RT
GO TO 1950
1900 OIF $=0$ IFO（I）WCC＊
$001800 \quad \mathrm{JK}=1 \cdot 10$
IF（ABS（DIF）．LT．2．0＊PI）GO TO 1950
TF（DIF．GT．0．0）DIF＝DIF－2．0＊PI
IF（OIF，LT．0．0）DIF＝DIF＋2．0＊PI
1800 CONTINUE
1950 PHASER（I）$=$ PHASEDTDIF
DIFD＝DIF＊ $150.0 / \mathrm{PI}$
PREAT 1518．I，RR，THETAR（I）\＆PHIR（I

1PHI二＊，E16．8）
PRINT 1519．0IFD
 1GREES＊）
2000 COINTINUE
－$C$
C GET SAMPLE NUMBERS CORRESPONDING TO DIRECT PATH CENTROIDS
C
$2050 \mathrm{JP} 1=1+\mathrm{IF} \mathrm{IX}((\mathrm{THETA}-\mathrm{TH} 0) /(\mathrm{TAU} * O M G)+0.5)$
$\triangle P 2=1+1 F I X((T H S-T H E T A) / O M G+T F) / T A U+0.5)$
C


CS(K)=AMIN1 CSS1CS2
ERR=THETA-EST (UT)
THER(K)=ERR
ERMS=ERMS + (THER (K)) **2
PRINT 1520.EST(JT) , THETA, THER (K)

10R=*, E16.8)
$\frac{c}{c}$
$\stackrel{C}{C}$
C DECIDE IF SHORT-TERM PLOT IS OESIKED
C IF PLTSCAN IF TRUE,S.T. PLOT HAS ALREADY BEEN MADE

| C |
| :--- |
| C |

    IF(PLTSCAN)40000:25000
    25000 CSK=CS \((K)\)
    CALL SHONT(CSK, K, PLTSCAN)
    C
    C IF SHORT RETURNS PLTSCANETRUE, A SHORT-TERM PLOT IS TO BE MADE FOR THIS SCAN
            IF(PLTSCAN) \(26000: 40000\)
    C
    C STORE ON FILE THE ARRAYS NEEDED FOR THE SHORT-TERM PLOT
    c
    26000 FLAG \(=. T\).
        PRINT 26050
    26050 FORMAT \(2 \mathrm{HO} \cdot /, * A\) SHORT-TERM PERFORMANCE PLOT WILL BE MADE FOR THIS
        1 SCAN*)
        EST \((J T+1)=T H E T A\)
        50 \(27000 \mathrm{I}=1,5\)
        \(\mathrm{RAC}(I)=R A(I)\)
        \(R A(I)=R A 1\) ( 1 )
    27000 CONTINUE
    THES2=THES
    THES \(=\) THES 1
    CALL RCVR(DM,JT.ESI)
    DO 28000 . \(I=1.5\)
    RA(I)=RA己 (I)
    28000 CONTINUE
    THES = THES2
    PRINT 2B100.ESI(JT)
    28100 FO\&゙TAT(1HO.10X.*ESTIMATE FOR OIRECT SIGNAL= *FEIG.8)
    WRITE(20) FLAG
    WAITE(20) EST
    URITE(20) M
    WKITE(20) RM
    WRITE(20) ESI
    waITEizo) TA
    YPLIE(20) DM
    \(65 \quad 1043000\)
    40000 FLAG: \(=. F\).
    WaIte(20) FLAG
    \(\frac{C}{C}\)
    C CHECK TU SEE IF THIS IS ONE OF THE 6 SCANS FOR WHICH A SHORT-TERM PLOT OF THE
    C RECEIVED SIGNAL IS DESIRED
    ```
C
43000 IF(K.NE, INC?) GO TO 45000
    IF(K.GT.1+5*INC) GO TO 45000
C STORE ON FILE THE SAMBLES OF THE RECEIVEO SIGNAL TUNGESS THIS IS THE SAME
C SCAN FOR WHICH THE DETAILED SHOKT-TERM PLOI IS TO BE MADEI.
    IFT.NOT: FLAGI WRITE(2O) M
    IMC2=IHC2+INC
C
45000 RTK=RTK+V*TUD
    PRINT 1521,FLAG
    1521 FORMAT(1Fi0,10X**FLAG=*,L2)
            THETA=THETA+DTH*TUD
            PrII=PHI+DPH*TUD
            IFIImAX,EO.0) GO TO 50000
            0044000 I=1.IWAX
            RZTK(I)=RRTK(I)+VR(I)*TUD
            THETAR(I)=THETAR(I)+DTHR(I)*TUD
            P+IE(I)=PHIE(I)+DPHK(I)*TUD
41000 coidTMMUL
50000 CDidTrfuE
    ERNS=SSNT (EHMS/KMAX)
C
C
    STORE ON FTLE THE ARRAYS NEEUEUTOR THE LONG-TERM PLOT
C
            WRITE(20)ERMS:TMKICS,THER
            FRINT 15E7
    1527 FORMAT(1H1,9X,*SCAN K*:12X,*TMK(K) (MILLSEC)*,13X**ES(K)*1,7X:*THE
        1R(K) (UEG)*,/1
            D0.60000 K=1.KMAX
            PRINT 1523,K,TMK(K),CS(K),THER(K)
    1528 FORMAT(11X,13,3110X.E16.81)
    60000 COHTINUE
    PRINT 1529,ERMS
    1529 FORMAT(1H0.%111X.*RMS ERROR= **TG.B)
        STOP
    END
```


## FUNCTION GAUSS (X)

$C$
$C$
$C$
$C$
C GAUSS UTILIZES THE FORTRAN RANOUM NO. FUNCTION RANFI TO GENERATE RANOMA C NUABEKS WITH STANDABU GAUSSIAN UISTRIBUTION(ZERO MEAN. UNITY VARIANCEI. C
C THE 1 ST CALL TO GAUSS IN THE MAIN PROGRAM SHOULD BE WITH X AS AN IRRATIONAL $C$ NUMBEF(EG. $X=S Q R T(2$.$) TO INITIALIZE RANF. ALL OTHER CALLS TO GAÚSS ARE MADE$ C WITH $X=0$.
C
LOGICAL HAS
IF (X.EO.O.) GO TO 1000
$A=R A N F(X)$
HAS $=, ~ F A L S E$.
1000 IF(HAS) $2000 \cdot 3000$
2000 GAUSS $=$ SAVED
HAS=. FALSE.
RETUKN
$3000 \mathrm{~A}=\operatorname{SQRT}(-2.0 * \operatorname{ALOG}(\operatorname{RANF}(0.1))$
$\mathrm{P}=3.141592705$
$T=2 * P * R A N F(0,1$
SAVED=A*SIN(i.
GAUSS $=A * C O S(T)$
HAS = TRUE.
RETURN
END

FUNGIION THA(T)
C THETA-A.
C
$C$
$C$
$C$
$C$
$C$

## FUNCIION P(THETA)

$C$
$C$
$C$
$C$
$C$
$C$
$C$
$C$
$C$ END

SUAROUTINE SHORT(SaKyPS)
LOGICAL FS
DIMENSION PL(5)
COMMON/PLT/KMAXIERRIPL

RETURN
End

SUBROUTINE RCVRTM,UT,EST:


EST(J) $\operatorname{EST}(\sqrt{1})$ 6000 CONT I.JUE RETURN
END
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

$\qquad$
$\qquad$
$\qquad$
UPIGMA至 PACE IS OF POOR QUAUTH螢

## TAPE12＝PLOT FILE TAPE1る＝PERMAMENT FILE

    THIS PROOIAF PLOTS THE RESULTS OF THE PROGRAM MLSRCVR: WHTCH ARE
    C STORED FOK A SUGRT TIME ON PERMANENT FILE ALL INFORMATION NEEDED BY
    _IHIS PROCRAF TS JO BE READ FROM THE FILE IM UNFORMATTED FORM. THE
    OROLR IS AS FOLLOWG; A SLASH DENOTING END-DF-RECORD:
    I OPT \(+N A M E, N R C V R, C H / N, E R S P E C \cdot U B, D A Y /\)
    KMAXIIMAX, ALPHA, ALPHAM,SNR, SIGMA,T1, JT/
    RTK, VITHETA, JTH:PHI, UPH/
    RRTK(1),VR(1), THET/R(1), DTHR(1), PHIR(1), DPHR(1), ALPHAR(1)/
    RITTK(2), VI: (2), THET UR(2), DTHR(2), PHIR(2), DPHR (2), ALPHAR(2)/
    RRTK(3), VF (3), THE;AR (3), DTHR (3), PHLF (3), DPHR(3), ALPHAR(3)/
    TM/ (SCAN TIME ARRAY)
    OATA FOR DETAILED AND SFLECTED SHORT-TERM PLOTS/ IEACH SCAN IS
    HFPRESCITII RY A FLAG. IF FLAG= TRUE - A A DETAILED SHORT TERM PLOT
    IS TO bE MADE. PLOTS OF THE COMPOSITE SIGNAL WILL BE MADE FOR SIX
    UNIFORMLY SPACED SCANS, BEGINNING WITH THE FIRST. 1
    ESTERR,THIK,CS,THER/
        LOGICAL FLAG
        REAL NOISE M1,M2
        UIMENSIOH CS (30). THER(30), TMK (30) -NOISE (30).TM(2000):Y(2000)
    
DIMENSION ACRATE(5), SRIRAT(4), SR2RAT(4), SR3RAT (4), DPAMPA (2)
OIMENSION KIAMFA(2), F2AMPA(2), RJAMPA(2) SMA(?), SNRD日A (2), KTKA(2)
DIMENSIOH ESTLRA(4), NRCVR(3), KMAXA(3), STLEL (3), TRANGL (3), NAME (3)

1) NRCVR2 (3)
CALL CALCOMP(I2)
$C$
$C$
$C$
$C$
READ INITIALIZATION DATA FROM FILE
READ(13) IOPT, NAME, NRCVR, NRCVR2, CHAN, ERSPEC, JB DAY
READ (13) KM
READ(13) RTK,V,THETA,DTH,PHI QPP
$0010 \quad 1=1,3$
READ(13) RPTK(I),VR(I), THETAR(I), DTHR(I) \&PHIR(I), DPHR(I), ALPHAR(I)
10 CONTINUE
$C$
$C$
$C$
$C$
EHCODE PLOT LABELS
ENCODF (27:5.KMAXA)KMAX
ENCODF ( 16.10 .0 PAMPA$)$ ALPHA
EVCODE 126,10 , RIAMPA) ALPHAR(I)
ENCODE $(10,10 \cdot F 2 A M P A) A L P H A R(2)$
ENCODE $(1,10,13 A M P A) A L P H A R(3)$
ORIGINAL PAGE IS
OF POOR QUA
ENCODE (1月:10.SMA)ALFHAM
ENCODE (45, 15:ACPOS)RTK,THETA,PHI
ENCODE(42.20, ACRATEIVIDTH,DPH
ENCODE (41.25.SR1POS)RATK(1), THETAR(1),PHIR(1)
ENCODE (37,30:SR1RATIVR(1), DTHi (1), DPHR(1)
ENCODE (41:25,SLZPOS)RRTK(2),THLTAR(2),PHIR(2)

```
-- ENCODE(37,30,SR2RAT)VR(2),DTHR(2),DPHR(2)
        FNCOOE(41,25,SR3POS)RRTK(3),THETAR(5).PHIR(3)
            ENCOUE(37,30,SR3RAT)VR(3),OTHR(3),DPHR(3)
            SN2=SIGMA*SQRT(2.0)
            ENCODE(6:35:SNA)SN2
                    ENCOUE(16,40.SNROBA)SNK
                    ENCODE(40,50,ERSPCA)ERSPEC
            5 FORMAT (*LONG-TERM PLOT FOK *,I2,* SCANS*)
            10 FORMAT (*AMPLITUDE E*,FG.2)
            15 FORMAT(*INITIAL.A/C POSITION =**FG.2.** *,FG.2,****FG.2)
            20 FORMAT (*INITIAL A/C RATE = *,F7.2,*; *,FG.2.** *,FG.2)
            25. FORMATI*INITIAL POSITION = *,FG.2.*:*,FG.2.**_*,F6.21
            30 FORMAT(*INITIAL RATE ='*,F6.2,*, *,F6.2,*, *,FG.2)
            35 FORMAT (FE.2)
            4 0 ~ F O R M A T ( * ) ( S N R ~ = ~ * , F 5 . 2 , * ~ D B ) * ) ~
            SO FORMATI*RMS ERROR SPECIFICATION=**FG.3** DEGREES*)
    C
    C
    READ SCAN TIME AXIS AND SCALE FOR PLOTTING
    G_READ(13) TM
            REALL SCALE(TM,25,0,UT,1)
            VMIN=TM(UT+1)*1000
            DELTA=TM(UT+2)*1000
    c
    C INITIALIZE FOR SELECTEU SHORT-TERM PLOTS
    C
        IHC=KMAX/G
        KPLT=1
        00 6000 K=1.GMAX
        READ(I3) FLAG
        IF(.NOT.FLAG) GO-TO 2000
        KTRUE=K
        IF(K.NE.KPLT) GO TO-4050
        KPLT=KPLLT+INC
        4050 IF(IOPT:NE,2) G0 TO 5000
        00 4500 KS=1:6
        REAO(13) Y
        4500 COHTINUE
        G0 T0-6000
        2000 IF(K,EQ,KPLT) GO TO 3000
        GO 10 6000
    3000 IF(K.GT.5$INC+1) GO_T0 6000
        KPL.T =KPLTT+INC
        READ(13) Y
    GO T0 6000
C
```

```
C
    C
    5000 L.BLE%
        GO TO 62000
    62100 CONTIINUE
    READ(13) Y
    THETALEY(UT+1)
    CALL SCALE (Y,2.0.JT,1)
    ESTI=Y(UT+1)
    ESTE=Y(UT+2)
    READ(13) Y
    CALL SCALE(Y,2.0.UT*1)
    S2LEY(NT -2)
    READ(13) f
    CALL SCALE(Y,2.0.\T,1)
    S22EY(JT+2)
    READ(13) Y
    CALL SCALE(Y.2.0.JT,I)
    ESI1=Y(UT+1)
    ESI2=Y(JT+2)
    REAO(13)Y
    READ(13)Y
    CALL, SCALE (Y.2.0.UT.1)
    S23=Y(JT+2)
    M1]=0.0
    M2=AMAX1(S21.S22.S23)*2.0
    BACKSPACE 13
    BACKSPACE 13
    BACKSPACE 13
    LÁCKSPACE 13
    BACKSPACE 13
    BACKSPACE 13
    READ(13) Y
    IF(ESI2.GF.ESTZS GO TO 5110
    ESII=EST1
    ESI2=EST2
    GO TO 5130
    5110 ESTI=ESII
    EST2=ESI2
    EST2=ESI2
    5130 Y(UT+1)=EST1.
    Y(UT+2)=EST2
C
    C O DRAW THE RECEIVER OUTPUT FROM THE COMPOSITE SIGNAL
            CALL PLOT(0.0,0.2.-3)
            S=-EST1/EST2
            IF(ESTI.GT.0,0) S=0.0
            XMAX=T1/TM(JT+2)
            XMI=X婇AX +0.1
            XM2=XMAX +0.2
            CALL PLOT (0,0,S,3)
            CALL PLOT(XMAXIS*2)
            CALL AXIS(0.0.0.0:1H,1.12.0190.0.EST1,EST21
            S1=(Y(1)-EST{)/EST 2+0.1
            CALL SYMBOL{O,1,S1+0.14,14H( DEG.1,0,0%14)
```


## IF（Y（1）．GE．1．0）GO TO 5134

CALL SYMBOL $(0,5,51,0,14,1 H 0.0 .0,1)$
5134 CALL NUMBER（0．2．S1，0．14．Y（1）：0．044HF7．2）
C EST
CALL LINE（TM．Y，JTII－O．O）
S1二（Y（JT）－EST1）／EST2
CALL SYMBOL（X191，S1．0．14：14H（ DEG．）．0．0．14）
IF（Y（UT）．GE．1．0）GO TO 5135
$X M 3=X M 2+0.3$
CALL SYMBOL（XM3，S1，0．14，1H0，0．0．11）
5135 cALL NUMBER（ X 讧2，S1，0．14，Y（JJ），0．0，4HF7．2）
CALL SYNBOL（－2．5．1．56．0．14．9HCANDIDATE：0．0．9）
CALL SYMBOL（－2．5，1．28，0．14，11HKCVR OUTPUT，0．0．11）
CALL SYMBOL（ $-2,5,1,00,0,14,9 H(D E G R E E S), 0.0,9)$
CALL SYMBOL（ $-2.5,0.58,0.14,16 H C O M P O S I T E S I G N A L, 0.0116)$
CALL SYMBOL $(-2 \cdot 5 \cdot 0 \cdot 30 \cdot 0.14 \cdot 10 \mathrm{HPLUS}$ NOISE．0．0．10）
C
DRAW TGE COMPOSITE SIGNAL AMPLITUDE VS TIME PLOT
READ（13）Y
$Y(J T+1)=M 1$
$Y(J T+2)=M 2$
CALL PLOT（0．0．2．5．－3）
CALL PLOT（XMAX，0，0，2）
CALL AXIS $10,0,0,0,1 H, 1,1,0,90.0 . Y(J T+1), Y(J T+2))$
C Pl
CALL LINE（TM．Y，JT•I：OPD）
CALL SYMEOĹ（－2．5．0．7．0．14－12HAMPLITUEE DF，0．0．12）
CALL＿SYMBOL $(-2.5,0.42,0.14,16 H C O M P O S I T E$ SIGNAL，0．0．16）
CALL SYMEOLT－2．5．0．14，0．14．10HPLUS NOISE，0．0．10．
$c$
C DRAW THE MULTIPATH AMPLITUDE VS TIME PLOT
C
READ（士3）Y
Y（JT＋1）＝信1
$Y(J T+2)=\mathrm{N}_{2}$
CALL PLOT（0．0．1．5．－3）
CALL PLOT（XMAX，0．0．2）

C RM
CALL LINE（TM，Y，JTं1．0．0）
CALL SYMBOL $(-2,5,0,7,0.14,12$ HAMPLITUDE OF， $0.0,12)$
CALL SYMBOL（－2．5，0．42．0．14．9HMULTIPATH，0．0．9）
CALL SYMBOL（－2：5：0：14：0．14．12HINTERFERENCE：0．0．12）
c
c
C DRAN THE RECEIVER OUTPUT VS TIME PLOT（ONLY DIRECT SIGNAL PRESENT）
FALL PLOT（0．0． $1.5 \div-3)$
READ（13）Y
$Y(J Y+1)=E \subset S$
$Y(J T+2)=E S I 2$
S＝－ESII／ESI2
IF ESSI1．GT．0．0）$S=0.0$
CALL PLOT（0．0． Si 3 ）
CALL PLOT（XMAX：S．2）

## CALL AXIS(0.0.0.0.1H $1,2.0,90.0, E S I 1, E S I 2)$

S1~1Y11) $E S I 11 / E S I 2 \div 0.1$
CALL SYMBOL (0.1.SI.0.14.14 14 (
IF(Y(1).GE.1.0) GO TO. 5138
CALL SYMBOL(0.5,SI,0.14,2H0,0.0.1)
5138 CALL NUMBERIO.2.SI, R.14,Y(1):0,0.4HF7,2L
c ESI
CALL LINEITM,Y•UT:1.010)
Si=(Y)JT) EESII)/ESI2
CALL SYMBOL (XM1,SI:0.14,14H1
$0 E 6.2,0.0 .141$
IF(Y(UT).GE.1.0)GO TO 5136
$X M 3=X M_{2}+0.3$
CALL SYMBOL (XM3, $51+0.14,1 \mathrm{HO} 0.0 .1)$
5136 CALL NUMBER(XM2, S1,0.24.Y(UT), 0.0. 4 HF7. 21
CALL SYMBOL(-2.5.1.56.0.14:9HCANDIDATEO0.049)
CALL SYMBOL $-2.5,1.28 .0 .14 \cdot 11$ HRCVR OUTPUT.0.0.11)
CALL SYMBOL(-2.5.1.00.0.14.9H(DEGREES) (0.0.9)
CALL SYMBOL (-2.5, 0. 58. $0.14,11$ HEIRECT. PATHE0.0.11)
CALL SYMBOL $(-2,5,0.30,0,14,14 \mathrm{HCOMPONENT}$ ONLY,0.0.14)
$C$
$\frac{C}{C}$ DRAW THE SCAN ANGLE AND DIRECT SIGNAL AMPLITUDE VS TIME PLOTS
CALL PLOT $(0.0,2,54-3)$
REAO(13) Y
CALL SCALE (Y-2.0.UTT11)
$S=-Y(U T+\overline{1}) / Y(J T+2)$
CALL PLOT $0,0, S, 3)$
CALL PLOT (XMAX -2 )
CALL AXIS (0.0, O. O. IH $1,1,2,0,90.0, Y(U T+1), Y(U T+2))$
C TA
CALL DASHLN (TM+Y+UT+1+0.0)
CALL SYMBOL $-2.5,1.3,0.14,10 \mathrm{HS}, \overline{A N}$ ANGLE. 0.0 .101
CALL SYMBOL(-2.5.1.02.0.14.9H(DEEREES),0.0.9)
CALL SYMBGL(-2,5,0.6,0.14, BHITAASHEOI, 0.0.8)
CALL PLOT $(0,0, S:-3)$
READ(13) Y
$Y(U T+1)=M 1$
$Y(U T+2)=M 2$
C OM
CALL LINE(TM.Y:UT11,0.0)
CALL AXIS (XMAX.0.0.1H:-1.1.0,90,0:Y(JT+1LYY(JT+2))
CALL SYMBOL (25.25:.65:0.14,12HAMPL ITUDE OF.0.0.12)
CALL SYMBOL (25.255 $3.310 .14,11$ HDIRECT PATH10.0.11)
CALL SYMBOL (25.25,0.06.0.14.9HCOMFDNENT,0.0.9)
CALL SYMBOL $25.25, * 25,14,12 H(S O L I D$ LINE), 0.0,12)
SET origin an inch past last folu and at bottom of page
ADJUST $=8 \cdot 9+S$
CALL PLOT(2ठ.5:-ADJUST:-3)
$\stackrel{c}{c}$
c
c LABEL TI TLE BLOCK
ENCODE127:5100.STLBL)K
5100 FORMAT (*SHORT - TERM PLOT FOR SCAN *, I2)

```
    KTK=(K-1)*75
    ENCODE(15.5150.STLAL2)KTK
    5150 FORMAT(*TK = **I4.*MSEC.*)
    ENCODE(29,5200,TRANGL)THETA1
    5200 FOHMAT (*A/C AT *,FG,2,* DLGREES ÄZIMUUTI*)
    CALL SYMBOL(1,0,10,22,0,14,5TLEL,0,0,27).
    G0 T0 64000
    64100 COHTINUE
    CALL SYMBOL1.75,.865,0.14,TRANGL,0.0.291
    CALL SYMBDL(0.75,0.586,0.14:STLBL2,0.0:15)
    6000 CONTINUE
    IF(IOPT.EQ.3) GO TO. 17000
    READ(13) ESTERR:TMKICS:THER
    IFIIOPIAEQ.2) GO TO 10000
    CALL NEWIEOK
    BACKSPACE 13
    KPLT=5*INC+1
    C
    C
    c
LBLE?
            N1=1
            G0 T0 62000
    62200 COINTINUE
    CALL PLOT(0.0,0.2:-3)
    OO 15000 KCOM=I,KMAX
    K=kMAX+1-KCOM
    FLAGE,FALSE.
    IF(K,EQ.KTRUE) GO TO 8000
    IF(K.NE.KPLT) GO TO 14000
    KPLT=KPLT-INC
    BACKSPACE I3
    12000 READ(13) Y
    IF(K.EQ.5*INC+:) GO TO 12500
    CALL PLOT (0.0.1.75,-3)
    12500 CALL PLOT(25.0:0.0.2)
    IF(N1.NE.1) GO TO 12510
    CALL SCALE(Y,1-0,JT+1)
    AM1=0.0
    AH2=Y(JT+2)
    N1=2
    12510 Y(JT+1)=AM1
    Y(JT+2)=AM2
    CALT AXIS(0.0.0.0.1H 1.1.,0790.0.Y(JT+1).Y(JT+2)!
    CALL LINEITM.Y.JT:1,0,1)
    BACKSPACE 13
    IF(.NOT.FLAG) GO TO 14000
    BACKSPACE 13
    14000 BACKSPACE 13
```

$-$

## 15000 CONTINUE

${ }^{C}$ C LABEL_AXES
00.65050 NK=1,6.

NSCAN=1+(NK-1)*INC
ENCODE(7.65010.RSCANA)NSCAN
65010 FORMAT (*SCAN * I I $)$
$K T K=75 *($ IVSCAN-1)
ENCODE (12,65020,KTKA)KTK
65020 FORMAT (*TK = *, I4.**MS*)
CALL SYMBOL (-2.5,0.57,0.14, NSCANA $0.0,0,7)$
CALL SYMBOL $(-2,5,0,29,0,14$ EKTKA, O, 0,112)
IF (NK.EG.6) GO TO 65050
CALL PLOT(0.0,-1.75,-3)
65050 CONTINUE
CALL PLOT $(26.5,=0.9,-3)$
CALL SYMBOL $0,0,10.22,0.14$-4OHAMPLITUDE OF COMPOSITE SIGNAL PLUS N
10ISE,0,0.40)
GO TO 64000
64200 CONTINUE
CALL NEWBLOK
GO TO 1.6000
8000 IF (K.NE,KPLT) GO TO 9000
FLAG $=$, TRUE:
$K P L T=K P L T=I N C$
DO $11000 \mathrm{~KB}=1,5$
BACKSFACE 13
11000 COMTINUE
G0 TO 12000
$9000 \quad 0010000 \mathrm{~KB}=1.6$
BACKSPACE 13
10000 CONT INUE
G0 TO 14000
$C$
$C$
$C$
$C$
$C$
$C$
$C$
$C$
$C$
$C$
$C$
$C$
$C$
16000 LBL=3
${ }_{C}^{C}$ FILL NOISE ARRAY FOR PLOT
DO 68000 KL二i +KMAX
NOISETKL) $=\operatorname{SIGMA} * \operatorname{SQRT}(2.0)$
68000 CONTINUE

DRAW THE EDGES AND FOLOING MARKS

```
    CALL PLOT(0.0.-0.5.3)
    CALL PLOT(0.0,10.5.2)
    CALL SYMOOL(8.51-.4.0.21,13,0.0.-1)
    CALL SYMBOL(0,5,10.4,0.21,13,0.0,-1)
    CALL SYMBOL(14.5,+.4.0.21,13.0.0.m1)
    CALL SYMBOL(14,5,10,4,0,21,13,0.0,-1)
    CALL PLOT(20.5,=0.503)
    CALL FLOT(20.5,10.5+2)
    C
    C MOVE ORIGIN TO 3/4 INCH FROM BUTTOM OF PAPER
    CALL PLOT (0,0,25*-3)
    C
    CALL SYMBOL(1.5:8.35:0.14.16HAV, AMPLITUUEEOF:0.0.16)
    CALL SYMBOL(1.5:0.07.0.14.16HCOMPOSTTE SIGNAL,0.0.16)
    CALL SYMHOL{1,5,7,79,0.14:17HAT TIME OF DIRECT1D.0,171
    CALL SYMBOL(1.5,7.51.0.14,15HSIGNAL CENTROID*0.0.15)
    C:1L SYMBOL(1,5,7.23,0.14,12H(SOLID_INE),0.0.12)
    C:LL SYMBOL{1.5,6.67,0.14,15HRMS NOISE LEVEL,0.0.151
    CALL SYMBOL11.5,6,35,0.14+13H(DASHED LINE),0.0,13)
    C LABEL BOTIOM AXIS
    C
    CALL SYMROL(1.5:2.1,0.14.16HESTIMATION ERROR:0.0.16)
    CALL SYMBOL (1,5,1.82,0.14,16H(A/C ANGLEEEST:).0.0.16)
    CALL SYMBOL(1.B11.54*0.14,9H(DEGREES),0.0.9)
    C
    MOVE ORIGLN AN INCH TO RIGHT OF LETTERING AND UP
    CALL PLOT(4.5,5.5:-3)
    C
    CALL SCALE(TMK:7:O.KMAX:1)
    CS(KMAX+1)=0.0
    CALL SCALE(CS,4,0,KMAX+1:1)
    CS}(KMMXX+1)=CS(KMAXX+2
    CS(KMAX+2)=CS(KMAX+3)
    NOISE(KMAXX+1)=CS (KMAX+1)
    NOISE(KMAX +2)=CS(KMAXX+2)
    C
    CALL AXIS(0.0.0.0.35HTYME(SEC) SINCE START OF FIRST SCAN,-35,7.O:
    10.0.TMK(KMAX +1),TMK(KMAX+2))
    CALL AXIS(0.0.0.0.1H.1.4.0.90.0.CS(KMAX+1),CS(KMAK+2))
    C
    CALL LINE(TMK,CS.KMAX,1,010)
    IF(IOPT.EO.2) GO TO 3005
    IMX =5*INC+1
    DO 3001 I=1,IMXIINC
    SX=(IMK(I)-TMK(KMAX+1))/TMK(KMAXX+2)
```

SI=(CS(I)-CS(KMAX+1))/CS(KMAX+2)
CALL SYMBOL $(S X, S 1 \cdot 0,14,11 \cdot 0,0,=1)$

## 3001 CONT INUE

$S X=(T M K(K T R U E)-T M K(K M A X+1)) /$ TMK (KMAX+2)
$S 1=(C S(K T R U E)-C S(M M A X+1)) / C S(K M A X+2)$
CALL SYMBOL SSX:S1,0.14:1,0.0. $=1 \mathrm{~L}$
3005 CALL DASHLN(TMK, NOISE,KMAK,I:ÖO)
$c$
C MOVE ORIGIN BACK DOWN
CALL PLOT $(0,0,-5.5,-3)$
C SCALE ERROR DATA
$C$
CALL SCALE(THER.4.0.KMAX.1)
C COMPUTE TRUE ORIGIN
$c$
TOK=0.0
IF (THER (KMAX+1).GE 0) GOTO 3111
TOR = - THER $(K M A X+1) / T H E R(K M A X+2)$
3111 CONTINUE
$\stackrel{C}{C}$
C DRAN AXES
C
CALL AXIS 10,0, TOR, $35 H T I M E$ (SEC) SINCE START OF FIRST SCAN. -35 .

CALL AXIS $10.0,0.011 \mathrm{H}, 1+4.0 .90,0$. THER (KMAX +1 ). THER (KMAX +2$)$ )
C DRAW PLOT WITH AN ASTERISK AT SELECTED POINTS
CALL LINE (TMK, THER +KMAX, 1,0:0)
IF (IOPT. EQ.2) GO TO 3006
$003009 \mathrm{I}=1 \mathrm{I}$ IMX, INC

$51=(T H E R(I)-T H E R(K M A X+1)) / T H E R(K M A X+E)$
CALL SYMBOL (SX,S1,0.14.11,0.0.w1)
3009 CONTANUE
$S X=(T M K(K T R U E)-T M K(K M A X+1))$ TMK (KMAX -2$)$
S1=(THER(KTRUE) -THER (KMAX+1))/THER(KMAX +2 )
CALL SYMEOL (SX,S1, $0.14,1,0.00-1)$
$C$
C MOVE ORIGIN AND CREATE TITLE BLOCK
3006 CALL PLOT $19.75 \cdot=0.75,-31$
CALL SYMBOL(1,0.10.22,0.14,KMAXA,0.0,27)
GO T0 64000
64300 CONTINUE
ENCUDE (36.45,ESTERA)ESTERR
45 EORTAT 4 RMS ESTIMATION ERROR $\approx *$ F5.3.* OEGREES*)
CALI SYMBOL $1.75, .865,0.14,164 E S T I M A T I O N$ ERROR. 0.0 .161
CALL SYMBOL (.95,.655:0.105:ESTERA,0.0.36)
CALL SYMBOL $6.95, .445,0.105$, ERSPCA,0.0.401
IF(IOPT.EQ.2) GO TO 17000
CALL SYMBOL $1.75, .165,0.07,11,0,-1)$
CA SYMBOLI.85:165.0.07.45HSHORI-TERM PLOT OF COMPOSITE SIGNAL A
1VA1 ABBLE*0.0.45)
CALL SYMBOL $(0.75: 0.025,0.071 \pm 0.00-1)$
 17 AVAI! ruLEIG0.0.146) GOTO 17000

CALL PLOT $(0,0,10.5,2)$
CALL SyMbut (0,5,-0,4,0.21,13.0.0, -1)
CALL SYMOUL $(3.5,10.4,0.21, \pm 3.0 .0,-1)$
CALL SY: $30 \mathrm{OL}(15.5,10.4,0,21,13,0.0,-1)$
CALL SYM $10 \mathrm{~L}(15.5,40.4,0.21,13,0.0,-1)$
CALL SYMFHL (22.5,-0.4, : . $21,13,0.0,-1)$
EALL SYFEOL(22.5. 10.4.0.21,13,0.0. -1)
CALL SYMBOL(29.5,10.4,0.21,13,0,0,-1)
CALL SYNUOL(29.5:-0.4.0.21,13.0.0.0-1)
CALL PIOT (36.5.10.5.3)
CALL PLOT ( $36.5,40.5 .2$ )
limAW THE TIME AXIS
CALL PLOT(4.0.0.2.-3)
CALL AXIS $10.0,0,0,32 H R E C E I V E R S C A N T I M E I M I L L I S E C O N D S 3,-32,25,0$,
10.0.V:IIJ.JELTA)

GOTO(62100:62200)LBL $\qquad$

CALL SYFBOL(2.11,9.94,0.14,0AY,0.0.10)
CALL $5 Y$ IIOL (1.4.9.6t, 0.14 .16HPKOGRAM: MLSRCVR.0.0.16)
CALL SYMADL (1.4.9.3B,0.14.9HNOMNAME: 0.0 .9 )
CALL SYNEOL (2.43,9.38,0.14,J13,0.0.7)
CALE EYMiTOL (2.10.9.10.0.14:1HAME:0.0.30)
CALL SYNEOL (2.10,9.16, 0.14, NAME, $0.0,10$ )
CALL SYMiOL $1.75,0.54,0.14,8$ HCHANNEL: 0.0.0.8)
CALL SYMAOLI1.93.0.54,0.14,CHAN.0.0,101
CALL SYMFOL (.75.0.12,0.14,14HSIGNAL MAKEUT 90.0 .14 )
CALL SYMIOL!.75,7.98,0.07.47HPOSITION SPECIFIED EY RANGE IN NAUTIC

1AL MILES. 0.0 .471
CALL SYMBOL 1.75 :7.84:0.07,41HDEGREES OF AZIMUTH, DEGREES DF ELEVAT 1 ION. 0,0.411
CALL SYMEOL (.75.7.7.0.07.46HRATES SPECIFIED BY VELOCITY JN KNOTS: 1 DEGHEES 0.0 .461
CALL_SYMBOL 1.75,7.56,0.07,35HAZIMUTH/SECL.DEGREES ELEVATION/SEC."
10.0.35)

C ADUUSI ORIGIN
CALL PLOT $(0,0,0,5,-3)$
CALL SYMBOL (.75,6,78,0.14,11HOIRECT PATH•0,0.1i)
CALL SYMBOL(1.05.6.57.0.105.DPAMPA.0.0.113)
CALL SYMBOL (1,05,6.36,0.105,ACPOS, 0.0.45)
CALL SYMEOL (1.05:6.15:0.105,ACRATE:0.0.41)
CALL SYMBOL 1.95.5.765:0.14.18HSPECULAR YULTIPATH 0.0 .0181
IFIMAX.GE. 1160 TO 64010
CALL SYMBOL (3.2,5.765,0.14,5H-NONE ,0.0.5)
GO TO 64020
64010 CALL SYMBOL (.95.5.485.0.1225.11HREFLECTOR 1.0.0.11)
CALL SYMBOL (1.05,5.275,0.105,R1AMPA.0.0,18)
CALL SYMBOL $11.05,5,065,0.105$, SR1POS.0.0.4. 41
CALL SYMBOL (1.05,4.855,0.105.SR1RAT,0.0.37)
IF (IMAX.EQ.1) GO TO 64020
CALL SYMBOL (.95,4.61,0.1225,11HREFLECTOR 2.0.0.11)
CALL SYMEOL (1.05.4.40.0.105,H2AMPA:0.0.18)
CALL SYMBOL (1.05,4.19.0.105.SR2POS:0.0.141)
CALL SYMHOL $(1,05,3.98,0.105$, SR2RAT, 0.0 .37$)$
IF(IMAX.EQ.2) GOTO 64020
CALL SYMBOL $0.95,3.735,0.1225,11$ HREFLECTOR $3,0.0,11)$
CALL SYMBOL (1.05,3.525:0.105,R3AMPA,0.0.18)
CALL SYMBOL (1.05,3.315,0.105,SK3POS:0.0.E1)
CALL SYMBOL (1.05,3.105,0.105, SKERAT,0.0;37)
64020 CALL SYMBOLI.95.2.72.0.14.19HSCATTERED MULTIPATH.0.0.191
$C \overline{A L L}$ SYMBOL $(1,05,2,51,0,105+S M A \cdot 0.0,1 B)$
CALL SYMBUL $1.95,2.125,0.14,24$ HFRONT-END RECEIVER NOISE.0.0.24)
CALL SYMBOL ( $1.05,1.915,0.105,3$ H0 $=0.0 .35$
CALL PLOT(1.1.2.02.3)
CALL PLOT (1.15,2.0 2,21
CALL SYMBOL (1.14.1.915,0.07.1HN.0.0.1)
CALL SYMBOL (1.4.1.915,0.105.SNA.0.0.6)
CALL SYMBOL(1.05.1.705,0.105,SNRCBA,0.0.16)
CALL SYMBOL (.75.1.285.0.14.2OHCANOTOATE RECETVER:. 0.0.20)
CALL SYMBOL (1.05:1.075.0.105.NFCVR:0.0130)
CALL SYMEOL (1.05.0.87,0.105, NRCUR2,0.0.30)
CALL PLOT (0.0.-0.41,-3)
60 TO164100,64200,643001LEL
$-\frac{C}{C}$
17000 CALL ENDPLT
STOP
END
$\qquad$
$\qquad$
$\qquad$
$\qquad$

AFPENDIX B
COMPUTER FLOTS


FIGURE B-1.a
LONG-TERM PLOT FOR 27 SCANS
DATE: 10/07/7E
PROGRAM: MLSRCVR
JGBNAME: MLSRCMP
S. H. IRWIN, JR.
CHANNEL: FIZIMUTH
SIGNAL MAKELPPGSITIGN SPECTFIED $\mathrm{B}^{V}$ RGNEE IN NGUTICEL MILES,DEGREES GF RZIHUTH, DEGRE'ES OF ELEVATION.RATES SPECIFIED \& VELGCI!Y IN KNDTS, DEGREESAZIHITH/ASEC, DEGREES ELEVATIUN/SEC.
DIRECT PATH
AMPLITUDE = ..... 1.00
INITIAL A/C PGSITION $=10.00$. ..... 30.00, $\quad 3.00$
INITIAL A/C RATE $=-300.00, .00$, .....  0
SPECULAR MULTIPATH -NONE
SCATTERED MULTIPATH
AMPLITUDE = ..... 00
FRONT-END RECEIVER NOISE
$\sigma_{N}=$ ..... 10
(SNR $=20.00 \mathrm{D3})$
CANDIDATE RECEIVER:
SQUARE GATE TRACKING RECEIVER
(G0-BIT WGRO LENGTH)
ESTIMATION ERROR
RMS ESTIMATIGN ERRGR $=.015$ DEGREES
RMS ERROR SPECIFICATION $=.010$ DEGREES

* SHORT-TEFH PLDT OF COHPGSITE SIGNAL BYATLABLE© DEARILED SHOT-TERM PEAFGRMANCE PLOT GVAILABLE


| 1.00 | 4.50 | 5.00 | 5.50 | 6.00 | 6.50 | 7.00 | 7.50 | 8.00 | 8.50 | 9.00 | 9.50 | 10.00 | 10.56 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |




| 10.00 | 10.50 | 11.00 | 11.50 | 12.00 | 12.50 |
| :--- | :--- | :--- | :--- | :--- | :--- |

FIGURE B-I. b

## AMPLITUDE OF COMPOSITE SIGNAL. PLUS NOISE

DATE: 10/07/75
PROGRAM: MLSRCVR
JOBNAME: MLSRCMP
S. H. IRWIN, JR.

CHANNEE: AZIMUTH
SIGNAL MAKEUP





DIRECT PATH
AYPLITHDE $=1.00$
INITIAL A/C POSITION $=10.00,30.00,3.00$ INITIAL A/C RATE $=-300.00 . \quad .00, .0$

SPECULAR MULTIPATH -NONE

SCATTERED MULTIPATH
HRM! ITUDE = .00
FRONT-ENL RETEIVER NOISE
$\sigma_{H}=.10$
(SNR $=20.09$ 08)
CAVDIDATE RECEIVER:
sounae cate tracking receiveh (60-8IT MGRD LENGTH)




AMPLTTUDE GF DIRECT PATH COMPONENT (SOLID LTNE)


( 29.97 DEG.)

FIGURE B-I.c
SHORT-TERM PLOT FOR SCAN 17 DATE: 10/07/75 PRITCRAM: MLSRCVR JOBNAME: MLSRCMP
S. H. IRWIN, JR.

CHRNNEL: AZTMUUTH
SIGNAL MAKEUP

OFCREES OF AZIMUTH, grGREES OF ELEYATIDN.
 AZTKUIH/SEC, DEGEES ELEVGTION/SEC

DIRECT PATH
RMPL.TTUDE = 1,00 INTITRL R/L PASITION $=10.00,30.00,3.00$ INITIAL A/C RATE $=-300.00$, .00. . 0

SPECULAR MULTIPATH -NONE

```
    SCATTERED MULTIPATH
    AMPLITUDE = .00
    FRGNT-ENO RECETVER NOISE
    UN= . }1
    [5NR = 20.00 08)
CRNDIDATE RECEIVER: SQlarre ghie thacking receiver (GD-BII KORD (ENGTH)
```

A/C AT 30.00 DEGREES RZIMUTH
TK $=1200 \mathrm{MSEC}$.

AV. AMPLITUDE AF COMPOSITE SIGNAL AT TIME OF DIRECT SIGNAL CENTROID (SOLLID LINE)

RMS NOISE LEVEL (DASHED LINE)

ESTIMATION ERROR (A/C ANGLE-EST.) (DEGREES)
FIGURE B-2.a
LONG-TERM PLOT FOR 27 SCANS
DATE: 10/08/75
PROGRAM: MLSRCVRJUBNAME: MLSRCAZ5. H. IRWIN, JR.
CHANNEL: AZIMUTH
SIGNAL MAKEUP
PGSIT GN specified by range in nautical miles.degrees gr azimuth, degrees ef elevation.RATES SPE:IFIED BY VELGCITY IN KNGTS, DEGREESRZIMUTH/SEC, DEGREES ELEVHTIEN/SEC.
DIRECT PATH
AMPLITUDE $=1.00$ INITIAL A/C POSITION $=10.00,30.00,3.00$ INITIRL A/C RATE $=-300.00, .00, .0$
SPECLLAR MULTIPATH -NONE
SCATTERED MULTIPATH AMPLITUDE $=.00$
FRONT-END RECEIVER NOISE ..... $\sigma_{\mathrm{N}}=\quad .40$
(SNR $=8.00 \mathrm{DG})$
CFVDIDATE RECEIVER:
SQUARE GATE TRACKING RECEIVER (60-BIT WORO LENGTH)
ESTIMATION ERROR
RMS ESTIMATION ERROR $=.059$ DEGREES RMS ERROR SPECIFICATION $=.010$ DEGREES

|  |  |
| :---: | :---: |
| $\mathrm{TK}_{\text {creas }}=300 \mathrm{Ms}$ |  |
|  | $\left.{ }^{c}\right]$ Whald |
|  | B <br> ? |
| scan 17 TK $=1200$ Ms | B ヘi <br>  |
| $\begin{aligned} & \text { SCAN } 21 \\ & \text { TK }=1500 \mathrm{MS} \end{aligned}$ |  |
|  |  |









Foimoun frame
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## 

## 

## 

FIGURE B-2.b
AMPLITUDE OF COMPOSIIE SIGNFL PLUS NOISE DATE: 10/08/75
PRgGRAM: MLSRCVR
JOBNAME: MLSRCAZ
S. H. IRWIN, JR.

## CHANNEL: AZIMUTH

SIGNAL MAKEUP

DECFEES OF RZDMUTH, DECREES OF EEVATIOM,
PATES SFECIFIED B' vELOCTIY TN RNDTS, DECEEES AZTHUTH/SEC, DEGRELS ELEVAIIOUSEC.

## OIRECT PATH

AHPLITUDE $=1.00$
INITIFL A/C POSITION $=10.00,30.00,3.00$ INITEAL R/C RRTE $=-300.00, .00, .0$

SPECULAR MULTIPAIH -NDNE

```
SCRTTERED MULTIPATH
GMPLTUDE = . 00
FRGNT-END RECEIVER NOISE
\(\sigma_{\mathrm{H}}=.40\)
(SNG \(=8.00 \mathrm{DB})\)
```

CFINDIDATE RECEIVER:
SGLIARE GATE TRACKING RECETVER (60-BIT WURO LENGTH)




HOLDOTE FRAME 2


AMPLITUDE OF DIRECT PATH COMPONENT (SOLID LINE)

30.00 DEG.)

## 

FIGURE B-2.c
SHORT-TERM PLOT FOR SCAN 17
DATE: 10/08/75
PROGRAM: MLSRCVR JOBNAME: MLSRCAZ
S. H. IRWIN, JR.

## CHANNEL: AZIMUTH

SIGNAL MAKEUP
PiSticioh specified br rave in muricate hales.

RATES SPECIFIED B VELOCTY IN KiTS. DEGREES PRIHUTH/SEC, DEGREES EEVMTIEN/SEC.

DIRECT PATH
AMPLITUDE $=1.00$
INITIAL $\operatorname{A} / \mathrm{C}$ POSITION $=10.00$, 30.00 .3 .00 INITIAL 9/C RATE $=-300.00, .00 . \quad .0$

SPECULAR MULTIPATH -NONE

```
SCATTERED MLLTIPATH
    AMPLITUDE = .00
FRONT-END RECEIVER NOISE
    \sigmaN= .40
    (9NR = 8.00 Dg]
```

CANDIDATE RECEIVER:
sQuare gate tracking receiver (60-BIT WORD LENGTH)

A/C AT 30.00 DEGREES AZIMUTH PK $=1200 \mathrm{MSEC}$.

FIGURE B-3.aLONG-TERM PLOT FOR 27 SCANSDATE: 10/07/75
PROGRAM: MLSRCVRJOBNAME: MLSRCJO5. H. IRWIN, JR.
CHANNEL: AZIMUTH
SIGNAL MAKEUP PGSITION SPECIFIED E RRNEE IN KAUTICAL HILES, DEGREES GF AZIHUTH, DECREES OF ELEVATIGN. RATES SPECIFIED BY VELGCITY IN KNGTS, DEGREES AZIMUTH/SEC, DEGREES ELEVATION/SEC.
DIRECT PATH
AMPLITUDE $=1.00$
INITIAL A/C POSITION $=10.00,30.00,3.00$
INITIAL A/C RATE $=-300.00, .00, .0$
SPECULAR MULTIPATH
REFLECTOR 1
AMPLITUDE = .....  80
INITIAL POSITION $=1.00,33.00,1.85$
INITIRL RATE $=.00,-3.04, .00$
R.F. PHASE DIFFERENCE $=180^{\circ}$ AT PULSECOINCIDENCE
SCATTERED MULTIPATH
AMPLITUDE $=.00$
FRONT-END RECEIVER NOISE
$\sigma_{\mathrm{N}}=$ ..... 40
$(S N R=8.00 \mathrm{DJ})$
CANDIDATE RECEIVER:
SQUARE GATE TRACKING RECEIVER (60-BIT WURD LENGTH)
ESTIMATION ERRORRMS ESTIMATION ERROR $=.247$ DEGREESRMS ERROR SPECIFICATION = . 010 DEGRFES
\% SHGRT-TERM FLDT GF COMPDSITE SICNML RWFILABLE
O. OETALIED SHGRT-TERH PERFGRHPNCE PLOT RVAHLABLE

| ${ }_{\text {chean }}^{\text {TK }}={ }^{1} 0$ Ms |  |
| :---: | :---: |
|  | $\stackrel{\circ}{\circ}$ <br>  |
|  | $\stackrel{\circ}{\circ}$ <br>  |
|  | - |
| scan 17 TK $=1200$ ms |  |
| $\begin{aligned} & \text { SCAV } 21 \\ & T K=1500 \mathrm{MS} \end{aligned}$ |  |








FoLDOUR MRAME 2
1

## 

## 

FIGURE B-3.b

## amplitude of composite signal pius noise

DATE: 10/07/75
PROGRAM: MLSRCVR JCBNAME: MLSRCJO
S. H. IRWIN, JR.

CHANNEL: AZIMUTH
SIGNAL MAKEUP



DIRECT PATH
AMPLITUDE $=1.00$
INITIAL R/C POSITION $=10,00,30.00,3300$ INITIAL G/C RATE $=-300.00, .00, .0$

SPECULAR MULTIPATH
REFLECTOR 1
AMPLTTME $=.80$
INITIAL POSITION $=1.00,33.00,1.85$ INITIAL RATE $=\quad 00,-3.04, \quad .00$ RF. PHASE DIFFERENCE $=180^{\circ}$ aT PULSE COINCIDENCE

## SCATTERED MULTIPATH AMPLITUDE = . 00

FRONT-END RECEIVER NOISE $U_{n}=.40$ $(5 N R=8.00 \mathrm{DB})$

CANDIDATE RECEIVER: GOUGE GATE TRACKING RECEIVER (GO-BLTT WORD LENGTH)



HOLDOUR $\mathrm{FRAME}=$


H4xthath

FIGURE B-3.c
SHORT-TERM PLOT FOR SCAN 17
DRTE: 10/07/75
PROCRAM: MLSRCVR
JOBNAME: MLSRC.JO
5. H. IRWIN, JR.

CHANNEL: RZIMUTH
SIGNAL MAKEUP

DEGAEES GT RZDRNTH, DEGRES OF ELENATIEN.
FifIES SPECIFIED BY YELOCHY IN KROTS, DEGREES

DIRECT PATH
AMPLITUDE $=1.00$
INITIAL A/C POSITION = 10.00, 30.00, 3.00
INITIAL ACC RRTE $=-300.00 . \quad .00 . \quad .0$
SPECULAR MULTIPATH

## REFLECTOR 1

RMPLITUDE $=$.日O
INITIAL POSITION $=1.00 .33 .00,1.85$
INITIAL RRTE $=\quad .00,-3.04 . \quad .00$
RF PHASE DIFFERENCE $=180^{\circ}$ AT PLLSE COINCIDENCE

> SCATTERED MULTIPATH
> AMPLITUDE $=\quad .00$
> FRONT-END RECEIVER NOISE
> On $^{2}=.40$
> (SNR $=8.00 \mathrm{DB})$

CANDIDATE RECEIVER:
sounre gate tracking receiver (60-BIT WGRD LENGTH)

A/C AT 30.00 DEGREES AZIMUTH
$T K=1200 M S E C$.

AV. AMPLITUDE OF COMPOSITE SIGNAL AT TIME OF DIRECT SIGNAL CENTROID (SOLID LINE)

RMS NOISE LEVEL (DASHED LINE)


FIGURE B-4.a
LONG-TERM PLOT FOR 27 SCANS
DATE: 10/06/75
PROGRAM: MLSRCVR
JUBNAME: MLSRCFG
S. H. IRWIN, JR.
CHANNEL: AZIMUTH
SIGNAL MAKEUP
PGSITIGN SPECIFIED BY RANEE IN NAUTICAL MILES. DEGREES OF RZIMLLIH, DEGREES GF ELEVATIGN. RATES SPECIFIED EY VELUCITY IN KNGTS, DEGREES AZIMUTH/SEC, DEGREES ELEVAIIGN/SEC.
DIRECT PATH
AMPLITUDE $=1.00$
INITIAL A/C POSITION $=10.00,30.00$, ..... 3.00
INITIAL A/C RATE $=-300.00, .00, .0$
SPECULAR MULTIPATH
REFLECTOR 1
AMPLITUDE = ..... 80
INITIAL POSITION $=1.00,33.00,1.85$
INITIAL RATE $=.00,-3.04, \quad .00$
RF PHASE DIFFERENCE $=180^{\circ}$ AT PULSECOINCIDENCE
SCATTERED MULTIPATH
AMPLITUDE = ..... 00
FRONT-END RECEIVER NOISE
$\sigma_{N}=$ ..... 10
$(S N R=20.00 \mathrm{DB})$
CANDIDATE RECEIVER:SQUARE GATE TRACKING RECEIVER(G0-BIT WGRD LENGTH)
ESTIMATION ERRORRMS ESTIMATION ERROR $=.203$ DEGREESRMS ERROR SPECIFICATION $=.010$ DEGREES (1) DETAILED SHGRT-TERM PERFGRHANEE PLOT GVAILAELE

#  




(1)


FOLDOUR FBASMA 2


FIGURE B-4.b
amplitude of camposite signal plus naise DRTE: 10/06/75 PRIGCRAM: MLSRCVR JOBNAME: MLSRCFG
S. H. IRWIN. JR.

CHANNEL: AZIMUTH
SIGNAL MAKEUP

UEGEES EF fZITMTH, DEGREES OT ELEYGTIDH.
FHTES SFECTFIED Br VEAGCTIY IN KHTTS, DEGEETS AZIHUTH/SEC, DECREES ELEYATTOI/SEL.
DIRECT PATH
AMPLITUDE $=1.00$
INIIIAL A/C PGSITION = 10.00, $30.00,3.00$ INITIAL R/C RATE $=-300.00$. .00. . 0

SPECULAR MULTIPATH
REFLECTOR i
AMPLITUDE $=$. . 0
INITIAL POSITION $=1.00,33.00,1.05$
INITIAL RATE $=$.00, -3.04. . 00
RF PHASE DIFFERENCE $=180^{\circ}$ AT PULSE colncidence

SCATTERED MULTIPATH
R9PLITUEE = .00
FRONT-END RECEIVER NOISE
$\sigma_{H}=.10$
(SNR $=20.00 \mathrm{DB}$ )
CANDIDATE RECEIVER: SQUARE GATE TRACKING RECEIVER (GO-BIT WURD LENGTH)

SCRN ANGLE (DEGREES)
(DASHED)

CANDIDATE RCVR EUTPUT (DEGREES)

DIRECT PATH CGIMPONENT ONLY


AMPLITUDE OF MULTIPATH INTERFERENCE

AMPLITUDE OF COMPOSITE SIGNAL PLUS NOISE

CANDIDATE RCVR GUTPUT (DECREES)

COMPOSITE SICNFL PLUS NGISE



| 10 | 10.00 | 10.50 | 11.00 | 11.50 | 12.00 |
| :--- | :--- | :--- | :--- | :--- | :--- |

FIGURE B-4.c SHORT-TERM PLOT FGR SCAN 17 DATE: 10/06/75 PREGRAM: MLSRCVR JGENAME: MLSRCFG S. H. IRWIN, JR.

CHANNEL: AZTMLITH
SIGNAL MAKEUP
PUSITION SPECIFIED BY REHEE TN MAUTICET hILES.
DECREES CF RZIHUTH, DEGAEES OF ELEVATICH.
ARTES SFECIFIED BY VELDCTTY IIH KNOTS, bEtnEES म2IMUTH/EEC. DEGREES ELEVATION/SEC.

## DIRECT PATH

AMPLITUEE $=1.00$
INITIAL R/C POSITION $=10.00,30.00,3.00$
INITIRL A/C RATE $=-300.00, .00, .0$
SPECULAR MULTIPATH
REFLECTOR I
AMPLITUDE $=$. . 0
INITIFL POSITION = $1.00,33.00,1.05$ INITIRL RATE $=.00,-3.04, \quad .00$
RF PHASE DIFFERENCE $=180^{\circ}$ at PULSE COINCIDENCE

SCATTERED MULTIPATH RMPLITUDE $=.00$

FRONT-END RECEIVER NOISE
$\sigma_{N}=\quad+10$
$(5 N R=20.00 \mathrm{DB})$
CANDIDATE RECEIVER:
SQufre cate tracking receiver (GU-EIT WGRO LENGTH)

A/C AT 30.00 DEGREES AZIMUTH
$T K=1200 M S E C$.
AV. AMPLITUDE OF
COMPCSITE SIGNAL
AT TIME OF DIRECT
SIGNAL CENTROID
(SOLID LINE)

ESTIMATION ERROR ( $\mathrm{A} / \mathrm{C}$ ANGLE-EST:) (DEGREES)

## FOLDOUN सRAME

FIGURE B-5.a.
LONG-TERM PLOT FOR 27 SCANS
DRTE: 10/10/75
PROGRAM: MLSRCVR
JOBNAME: MLSRC5C
5. H. IRWIN, JR

CHANNEL: AZIMUTH
SIGNAL MAKEUP
POSTIIGN SPECIFIED BY RANGE IN NRUTICAL MILES DEGREES OF RZIMUTH, DEGREES OF ELEVATHON. RATES SPECIFIED BY VELDCIT' IN KNGTS, DEEREES RZTMUTH/SEC, DEGREES ELEVATIGN/SFC.
DIRECT PATH
AMPLITUGE $=1.00$
INITIAL A/C POSITION $=10.00,30.00,3.00$
INITIAL $\mathrm{A} / \mathrm{C}$ RATE $=-300.00, .00, .0$
SPECULAR MULTIPGTH
REFLECTOR 1
AMPLITUDE $=. B D$
INITIAL POSITION $=1.00 \quad 33.00 \quad 1.85$
INITIAL RATE $=.00-3.04 . \quad .00$
RF PHASE DIFFERENCE $=0^{\circ}$. AT PULSE
COINCIDENLE

## SCATTERED MULTIPATH <br> AMPLITUDE = <br> . 00

FRONT-END RECEIVER NOISE
$\sigma_{\mathrm{N}}=\quad .10$
(SNR $=20.00 \mathrm{DB})$

## CANDIDATE RECEIVER: <br> SQUARE GATE TRACKING RECEIVER (EO-BIT WORD LENGTH)

```
ESTIMATION ERROR
RMS ESTIMATION ERROR = . 162 DEGREES
RMS ERROR SPECIFICATION \(=.010\) DEGREES
```



#  2 

Sequ,


| 5.00 | 5.50 | 6.00 | 6.50 | 7.00 | 7.50 | 8.00 | 8.50 | 9.00 | 9.50 | 10.00 | 10.50 | 11.00 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

MOLDOUK GRAROA

$\begin{array}{llllll}10.00 & 10.50 & 11.00 & 11.50 & 12.00 & 12.50\end{array}$

FIGURE B-5.6
AMPLITUDE OF COMPGSITE SIGNAL FlUS NOISE DATE: 10/10/75
PROGRAM: MLSRCVR
JGBNAME: MLSFIC5C
S. H. IRWIN, JR.

CHANNEL: AZIMUTH
SICNAL MAKEUP
PEISTION Frecrited ET RAMEE In wifltcri Mras.

RAFES SPECIFTEB BY VELDETIY IN KMOTS, DEGRES


DIRECT PRTH
AMPLITUDE $=1.00$
INITIAL A/C POSITION $=10.00,30.00,3.00$ INITIAL F/C RATE = -300 00, .00, . 0

SPECULAR MULTIPATH
REFLECTGR 1
PMPLITUDE $=.80$
INITIAL POSITION $=1,00,33.00, \quad 1.85$ INITIRL RATE $=.00-3.04$.00 RF PHRSE DIFFERENCE $=0^{\circ}$ RT PULSE COINCIDENCE

## SCATTEREO MULTIPATH

 AYPLITUDE = .00```
    FRONT-END RECEIVER NOISE
    \(\sigma_{\mathrm{H}}=\quad .10\)
    (5N日 \(=20.0008\) )
```

CRNDIDATE RECEIVER:
Sourfe ghte tracking receiver (60-BLT WORD LENGTH:

SCAN PNGLE (DEGREES)
(DASHED)

CANDIDATE RCVR OUTPUT (DEGREES)

DIRECT PATH COMPONENT ONLY

AMPLITUDE बF MULTIPATH INTEREERENCE


( 30.00 DEG.)

( 29.86 DEG.)
$\begin{array}{llllll}10.00 & 10.50 & 11.00 & 11.50 & 12.00 & 12.50\end{array}$

FIGURE B-5.c
SHORT-TERM PLOT FOR SCAN 17
DATE: 10/10/75
PROGRAM: MLSFCVA JOBNAME: MLSRC5C
S. H. IRWIN, JR.

CHANNEL: AZIMUTH
SIGNAL MAKEUP
position specified by raver in mauica moles
DECAEES dF qZIMUTH, DECREES OF ELEVATEOS.
 RZJMETHSEC. DEGREES ELEVAT FON/GEC.
direct path
RMPLITUDE $=1.00$ INITIRL R/C POSITION $=10.00,30.00$,
INITIRL $A / C$ RATE $=-300.00$
.00
.000

## SPECULAR MULTIPATH

REFLECTOR 1
PMPLITUDE $=$. BO
INIIIAL PGSITION $=1.0033 .00 \quad 1.85$ INITIRL RATE $=.00,-3.04, \quad .00$ RF PHASE DIFFERENCE $=0^{\circ}$ AT PULSE coincionsee

SCATTERED MULTIPATH
AMPLITUDE $=.00$
FRONi-END RECEIVER NOISE
OH= .10
$15 N R=20.00001$
CANDTDATE RECEIVER: SQLARE GATE TRACKTNG REC, - ${ }^{\text {-VER }}$ (60-B1 1 WORE LENGTH)

A C AT 30.00 DEGREES AZIMUTH
$T K=120 G M S E S$.
AV. AMPLITUDE OF
COMPOSITE SIGNAL
AT TIME OF OIRECT
SIGNAL CENTROID
(SOLID LINE)

FIGURE B-6.a
LONG-TERM PLOT FOR 27 SCANS
DATE: 10/10/75PROGRAM: MLSRCVRJOBNAME: MLSRC8ES. H. IRWIN, JR.
CHANNEL: AZIMUTH
SIGNAL MAKEUP
PGGITIGN SPECIFIED BY RANGE IN NAU'ITCAL MILES, DEGREES OF AZIMUTH, DEGREE OF ELEVATION. RATES SPECIFIED BY VEL.JCITY TN KKㅗㅗTS, DEGREES AZIMUTH/SEC, DEGREES ELEVATIGN/SEC.
DIRECT PATH
AMPLITUDE $=1.00$
INITIAL ATC POSITION $=10.00,30.00$, ..... 3.00
INITIAL A/C RATE $=-300.00$, ..... 00, . C
SPECULAR MULTIPATH
REFLECTOR 1
AMPLITUDE = ..... 80
INITIAL POSITION $=1.00,33.00,1.85$
INITIRL RATE = .00, -3.04. .....  00
RF PHASE DIFFERENCE $=90^{\circ} \mathrm{AT}$ PULSECOINCIDENCE
SCATTERED MULTIPATH ..... AMPLITUDE $=.00$
FRONT-END RECEIVER NOISE$G_{N}=.10$
$(S N R=20.00 \mathrm{DB})$
CANDIDATE RECEIVER:
square gate tracking receiver(60-BIT WORD LENGTH)
ESTIMATION ERROR
RMS ESTIMATION ERROR = . 173 DEGREES RMS ERROR SPECIFICATION = . 010 DEGREES


## 

#  

 $\therefore$. ::




|  | 10.00 | 10.50 | 11.00 | 11.50 | 12.00 | 12.50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

```
GGURE 5-65
```



```
    ##TE: :0N107%5
```



```
    JasNGME: MLSRCSE
        5. H. IRWEN. S.
CHANE: RTEMGY
STOURL MFKEIP
```






```
    ORRECT PATH
        Amplytue = 1.00
        IMTINL AT PESTTEN = 10.00. 30.00. 3.00
        INTIFL F/C ROTE = -300.35, .00. .D
    SPECULAR FULTIPATH
    REFIECTGR:
        RMPLITUDE = .80
        INITIAL FESITION = 1.00, 33.00. 1.85
        INITIRL RATE = .00. -3.04, .000
        RF PHASE DIFFERENCE = 90' AT PULSE
        coinctaence
```

    SCATTERED MULTIPATH
        AMPLITUDE \(=.00\)
        FRONT-END RECEIVER NOISE
        \(\mathrm{OW}_{\mathrm{w}}=.10\)
        \((S N R=20.00 \mathrm{DB})\)
    CANDIDATE RECETVER: SQuare chte thackine receiver (6R-BIT MURD LENCTH)




FIGURE B-6.c SHORT-TERM PLOT FOR SCAN 17

DATE: 10/10/75
PROGRAM: MLSRCVR
JOBNAME: MLSRCBE
S. H. IRWIN, JR.

CHANNEL: AZIMUUTH
STGNAL MAKEUP





DIRECT PATH
AMPEITUDE = 1,00
INIIIAL. A/C POSITION $=10.00, \quad 30.00, \quad 3.00$
INIFIAL R/C RRTE $=-300.00$.
$.00 . \quad .0$
SPECULAR MULTIPATH
REFLECTOR 1
RMPLITUUE $=$. 80
INITIAL PGSITIEN $=1.00,33.00,1.65$ INITIRL RRTE $=.00,-3.04, .00$ RF PHASE DIFFERENCE $=90^{\circ}$ AT PULLSE CDINCIDENCE

SCATTEREQ MULTIPATH AMPLITUEE = .00

FRONT-END RECEIVER NOISE
砛= .10 (SNR $=20.00 \mathrm{DB})$

CANDTDATE RECETVER: sQuaite gate tracking receiver (60-EIT WORD LENETH)

A/C AT 30.00 DEGREES AZIMUTH $T K=1200 \mathrm{MSEC}$.
FIGURE B-7.a
LONG-TERM PLOT FGR 27 SCANS
DRTE: 10/10/75
PRDGRAM: MLSRCVR
JOBNAME: MLSRC7日
S. H. IRWIN, JR.
CHANNEL: AZIMUTH
SIGNAL MAKEUP
PGSITION SPECTFIED BY RANEE IN MRUTICRL MILES, DEGREES GF GZIMLTM, DEEREES OF ELEVGTION. RATES SPECIFIED EY VELOALTH IN RNGTS. DEGREES AZIMLTH/GEC. DEGREES ELEVATION/GEC.
DIRECT PATH
AMPLITUDE $=1.00$
INITIRL $A / E$ POSITION $=10.00,30.00$, ..... 3.00
INITIAL A/C RATE $=-300.00$, ..... $.00, .0$
SPECULAR MLLTIPPATH
REFLECTOR 1
AMPLITUDE = ..... 80
INITIAL POSITION $=$ ..... 1.00, $33.00,1.85$
INITIAL RATE $=.00,-3.04$, ..... 00
RF PHASE DIFFERENCE $=270^{\circ}$ AT PULSECOINCIDENCE
SCATTERED MULITIPATH ..... AMPLITUDE $=.00$
FRONT-END RECEIVER NOISE $\sigma_{\mathrm{N}}=$ ..... 10

        \((S N R=20.00 \mathrm{DB})\)
    CANDIDATE RECEIVER:
ESTIMATION ERRORRMS ESTIMATION ERROR = . 201 DEGREESRMS ERROR SPECIFICATION $=.010$ DEGREES
SQUARE GATE TRACKING RECEIVER
(60-BIT WORD LENGTH)

* SHGT-TERM PLOT bF CGWPGSTTE SIGNRL AVPILRELE

AV. AMPLITUDE OF COMPOSITE SIGNAL AT TIME OF DIRECT SIGNAL CENTROID (SOLID LINE)

RMS NOISE LEVEL (DASHED LINE)




#  

位

[^0]

| 1.00 | 5.50 | 6.00 | 6.50 | 7.00 | 7.50 | 8.00 | 8.50 | 9.00 | 9.50 | $10.0 \%$ | 10.50 | 11.00 | 11.50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

RECEIVER SCRN TIME(MILLISECONDS)

FIGIRE B-7.6
amplitude of composite signal plus noise DRTE: 10/10/75 PROGRAM: MLSRCVR JOBNATHE: MLSRC7O
S. H. IRWIN, JR.

CHANNEL: AZTMUTH
SIGNAL MAKEUP
pasition specified by gace in mautica nices.



DIRECT PATH
AMPLTTUEE $=1.00$
INITIAL A/C PGSITION $=10.00, \quad 30.00, \quad 3.00$ INITIAL A/C RATE $=-300.00, \quad .00, \quad .0$

## SPECULAR MULTIPATH

REFLEETOR 1
AMPLITUDE $=.80$
INITIAL POSTTIEN $=1.00,93.00,1.65$
INITIAL RATE $=.00,-3.04 . \quad .00$ RF PHASE DIFFERENCE $=270^{\circ}$ AT PULSE coincidence

SCATTERED MULTIPATH
AMPLITUDE $=.00$

```
FRUNT-END RECEIVER NOISE \(\sigma_{k}=.10\)
\((5 N R=20.00 \mathrm{DB})\)
```

CANDIDATE RECEIVER: SQUARE GATE TRACKING RECEIVER (60-BIT WORD LENGTH)



FIGURE B-7.c

Shert-term plot for ican 17

DATE: 10/10/75

PROGRAM: MLSRCVR

JOBNAME: MLSRC7C

S. H. IRWIN, JR.

CHANNEL: AZIMUTH



DIRECT PATH
AIPLITUDE $=1.00$
INITIAL A/C POSITION $=10.00,30.00,3.00$
INITIRL A/C RRTE $=-300.00, .00, .0$

## SPECULAR MULTIPATH

## REFLECTBR

AMPLITUDE $=.60$
INITIAL POSITION = $1.00,33.00,1.85$ INITIRL RRTE $=.00,-3.04 . \quad .00$ RF PHASE DIFFERENCE $=270^{\circ}$ AT PULSE coincinence

## 

SCRTTERED N:ULTIPATH
AMPLITUDE $=.00$
FRONT-END RECEIVER NOISE OH= $\quad .10$
$(S N R=20.00 \mathrm{DB})$
CANEIDATE RECEIVER: SQufRe gate tracking receiver ( $60-$ BIT HORD LENGTH)

月/C AT 30.00 DEGREES AZIMUTH
TK = 1200MSEC.


[^0]:    

