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A FINITE ELEMENT FORMULATION FOR EVALUATION OF CRACK BLUNTING EFFECTS IN ELASTO-PLASTIC SOLIDS

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INTRODUCTION

The successful application of linear elastic fracture mechanics (LEFM) to prediction of fracture in metals is founded upon the representation of geometric flaws as infinitely sharp cracks for purposes of analysis. The crack model and linear elastic stress analysis provide stress intensity factors suitable as transfer parameters relating brittle fracture conditions in specimens and structures. Confidence in the scheme derives from successful correlation of failure data, which in practice obtains only under conditions of so-called small scale yielding wherein the flow tip region is subjected to a high degree of elastic constraint.

The sharp crack model allows the use of continuum analysis, linear elasticity, as a basis for predicting a micromechanical process, fracture, by providing a characterization of loading conditions affecting a very small volume of material. In attempting to generalize fracture mechanics for application under conditions of significant yielding the appropriateness of the sharp crack representation must be re-evaluated. In the presence of such yielding not only does the relevance of linear elasticity become questionable, but also the involvement of larger volume of material in the fracture process brings the sharp crack model itself into question.

In lieu of positing microstructural fracture criteria as a basis for engineering correlation of fracture data, one would like to evolve a fracture mechanics for non-brittle conditions which, like LEFM, employs continuum analysis for definition and evaluation of transfer parameters. Both the J-integral [1] and COD [2] approaches to ductile fracture prediction are consequences of this philosophy. It should be noted that, in principle, neither of these approaches compels the use of a sharp crack model.

The model described below is intended to provide a tool for investigation of the behavior of initially sharp flaws which are allowed to blunt as applied loading is increased. The approach is motivated by the observation that, to the extent that actual flaws have tip radii so small that they may be viewed as sharp cracks, blunting must occur under load and will influence internal fields over a finite region near the tip. The hypothesis that the blunting effect is important is reinforced by recent experience with J-integral correlations [3] in which a stretch zone is accounted for in determining a critical loading for fracture.

The case for the utility of this form of analysis is not absolute. Indeed the same argument, "a sharp crack must blunt," could be applied to cases for which LEFM is adequate. The distinction lies in the volume of material involved in the fracture process which must be larger when significant yielding occurs. The formulation presented here will provide a means for assessing the effects and range of influence of crack blunting upon internal fields near flaw tips. Should blunting be judged significant fracture transfer parameters sensitive to its effects must be defined and correlation of fracture data attempted before credible inferences can be drawn.

THE BLUNTING PROBLEM

Consider the problem of a finite length sharp crack in an elasto-plastic solid. We restrict our attention to two dimensional problems. Under initial load application the near tip field will be dominated by the characteristic inverse square root singularity of the Williams eigenvalue expansion [4] for the small strain elastic problem. Were a closed form solution to the large strain elasto-plastic problem available, one would expect to observe immediate yielding near the flaw tip, and immediate blunting. In lieu of such a solution our intent is to provide for numerical prediction of these effects with recognition that resolution will be irrevocably limited by the spatial discretization process inherent in any numerical procedure. The approach taken here is to employ a special finite element in the flaw tip region which is designed to permit anticipated forms of behavior and to account for the effects of geometric changes upon the mechanics of the problem.

Most previous nonlinear analyses of crack problems have incorporated either material or geometric (blunting) nonlinearity, not both. In neither case has a wholly satisfactory solution been obtained. The achievement of any solution at all has required approximations in problem definition sufficient to render the results of debatable utility. Certain aspects of this experience have guided development of the present formulation for the combined nonlinear problem.

1. Asymptotic near field deformation theory plasticity solutions for the small strain problem have been obtained by Hutchinson [5] and Rice and Rosengren [6]. Solution by formulation of a nonlinear eigenvalue problem required neglecting elasticity and limiting attention to power

law plastic materials. These results confirm the existence of singular deformation fields in the presence of (deformation theory) plasticity and suggest a characteristic form analogous to that of the linear elastic problem. The results are of limited use by virtue of the power law material restriction and suspect by their neglect of both elastic effects and history dependence of plastic flow. At best the results are appropriate to high excitation of cracks far removed from exterior boundaries.

2. The limitations inherent in the Hutchinson, Rice results will be removed in the context of numerical solution by the special finite element under development by Swedlow [7,8]. This approach preserves a variable order deformation gradient singularity for arbitrary elasto-plastic solids and employs an incremental theory. These minimum material behavior requirements must be preserved in an analysis of the combined nonlinear problem.

3. The only available analytical solution for the blunting problem was provided by Knowles and Sternberg [9]. They consider a limited class of hyperelastic materials whose large strain behavior is similar to that of power law deformation theory plastic materials. Their near field, high excitation asymptotic solution confirms persistence of deformation gradient singularities in the presence of blunting. Similar to the Hutchinson, Rice results a relatively simple characteristic singularity is identified.

The present combined nonlinear formulation is founded upon the presumption that a relatively simple singular deformation gradient field prevails in the vicinity of a blunting crack, a presumption consistent with the foregoing experience. Furthermore, in view of the fact that a formal near field expansion solution is not within reach, there is little motivation for restrictive simplification of the material model beyond that undertaken to permit economic computation. The analysis is based upon a generalization of conventional J_2 flow theory which is appropriate to the large strain elasto-plastic problem for an arbitrary isotropic hardening material.

A final important feature of the present formulation is suggested by the experience of this author [10,11] and others [12] in attempting solution of the combined nonlinear problem without special attention to modeling the near field. At issue is resolution of the effects of blunting upon internal fields. In small strain finite element crack analysis, a special tip element may be sized in terms of crack length. Once an element size is determined appropriate to a particular formulation that size may be used in a variety of different problems. In crack blunting analysis another size scale becomes important, the "size" of the blunted crack tip. If tip size is thought of in terms of a tip inscribed radius, as suggested by Srawley [13], the size scale of interest may range with load level from 10^{-5} to 10^{-1} times the crack length in the course of a single analysis. The change of size scale with load will depend primarily upon stress-strain curve and to a lesser extent upon loading type: tension, bending, etc. It is clear that no one special element size can give results of equivalent accuracy over any substantial range of excitation in a single analysis, much less from one problem to another. The problem may be treated in either of two ways. The range of influence of the blunting field singularity may be taken as a variable determined by the analysis, or a single value may be used which is appropriate to the load level at which results are desired. The utility of the latter, "simpler," approach cannot be evaluated a priori as an unknown degree of history insensitivity is presumed. No unique solution to this dilemma has presented itself. The formulation outlined below allows the spatial range of influence of the blunting field singularity to be either an unknown, or a fixed value, without influencing the compatability characteristics of the finite element model. Evaluation of the influence of this quantity upon analysis results must be of high priority once an operational computer program is available.

SPECIAL ELEMENT FORMULATION

The elasto-plastic crack blunting finite element formulated below is designed for implementation in a host finite element program capable of solving small strain elasto-plastic problems employing a J_2 flow theory. In principle the host program may employ elements of any order. The present discussion presumes use of an 8 or 9 node isoparametric quadrilateral such as that being developed by Marino [14]. Coupling between host program and special element is in terms of nodal variables (displacement, force) only. The special element and its associated displacement interpolation functions are defined in Lagrangian terms, i.e., on the undeformed geometry. Finite deformation effects are considered within the special element, and may be neglected elsewhere, since the special element employs a large strain elasto-plastic representation which reduces smoothly to familiar J_2 plasticity theory in the limit of infinitesimal deformation.

The following discussion is limited to development of the essential features of the special element. Straightforward operational detail prerequisite to writing a computer program is not included.

Definition and implementation of the special element requires establishment of three distinct, consistent formulations.

1. A field problem with an associated variational principle.
2. An elasto-plastic constitutive model for large strains.
3. An interpolation model for the special element.

Definition of a special element interpolation model incorporating key features of the near crack tip region necessitates use of a Lagrangian coordinate frame. It also requires introduction of algebraic quantities whose evaluation necessitates availability of a variational principle. Consequently, the large strain formulation developed by Osias [15] and Osias and Swedlow [16] is inappropriate to items 1 and 2 above. An alternative, and essentially equivalent, Lagrangian formulation has been developed by Hutchinson [17] and will be employed here. The formulations are discussed and compared by Key, Osias, Belytschko and Hutchinson [18].

Field problem: We choose a convected coordinate model as defined by Green and Zerna [19] and Nemat-Nasser [20]. The initial geometry, interior B_0 and boundary ∂B_0, is described by material coordinates* X^I in an orthogonal frame with metric G_{IJ} and base vector $\vec{G}_I$. The deformed configuration, B bounded by ∂B, is described by coordinates x^i in a convected frame

*Conventional indicial notation is employed with all indices varying over 1,2,3. The summation convention is employed. Partial differentiation is indicated by a comma and covariant differentiation by a semicolon.

with metric g_{ij} and base vectors $\vec{g}_i$. For a material particle the displacement field is

$$\vec{U} = x^i\vec{g}_i - X^I\vec{G}_I = U^I\vec{G}_I \tag{1}$$

In (1) we choose to identify the material frame tensor components as will be the case throughout this development. Total strain is characterized by Green's strain (2).

$$E_{IJ} = \frac{1}{2}(U_{I;J} + U_{J;I} + U^K_{;I}U_{K;J}) \tag{2}$$

Of the several possibilities it is convenient for our purposes to employ the so-called Kirchoff stresses S^{IJ}

$$S^{IJ} \equiv \sqrt{g/G}\,\sigma^{IJ} \tag{3}$$

In (3), $g = |g_{ij}|$ and $G = |G_{IJ}|$. The σ^{IJ} are material frame components of Cauchy stress. Thus for a surface whose undeformed state outer normal is $\vec{N} = N^I\vec{G}_I$ the deformed state traction is found as:

$$\vec{T} = T^I\vec{G}_I = (S^{IJ} + S^{KJ}U^I_{;K})N_J\vec{G}_I \tag{4}$$

Restricting attention to the case of static problems without body forces equilibrium equations may be established in the form (5).

$$(S^{KI} + S^{KL}U^I_{;L})_{;K} = 0 \tag{5}$$

Large strain elasto-plastic quasi-static behavior is governed by rate equations which are first order homogeneous in time. These equations, of the form (6), motivate formulation of an analysis as an initial- and boundary-value problem for rate quantities. This problem, also first order homogeneous in time, is quasi-linear and, as for the small strain problem [21,22], provides a basis for efficient incremental solution.

$$\dot{S}^{IJ} = P^{IJKL}\dot{E}_{KL} \tag{6}$$

In (6) $\dot{S}^{IJ}$ are the components of the convected rate of Kirohoff stress and $\dot{E}_{IJ}$ are components of the material derivative of Green's strain. As will be shown shortly, the constitutive tensor P^{IJKL} in (6) possesses the symmetry:

$$P^{IJKL} = P^{KLIJ} \tag{7}$$

The rate problem is formed employing rate equilibrium equations (8) and boundary conditions (9) for the quasi-static problem.

$$\dot{S}^{KI}_{;K} + (\dot{S}^{JK}U^{I}_{;J})_{;K} + (S^{JK}V^{I}_{;J})_{;K} = 0 \tag{8}$$

$$\dot{T}^{I} = \dot{S}^{KI} + \dot{S}^{KL}U^{I}_{;L} + S^{KL}V^{I}_{;L})N_{K} \tag{9}$$

Equilibrium equations (8) apply to the undeformed interior configuration B_o and boundary conditions (9) to the undeformed boundary ∂B_o. V^I are the velocity-field components.

A basis for finite element solution may be obtained by establishing a variational statement of the problem defined by equations (6-9). Straightforward manipulation yields the stationary principle (10) for the case where $\dot{T}^I$ is prescribed on $\partial B_T \varepsilon \partial B_o$.

$$\delta I = 0 \tag{10}$$

$$I = \int_{B_o} \{\tfrac{1}{2}P^{IJKL}\dot{E}_{IJ}\dot{E}_{KL} + \tfrac{1}{2}S^{KL}V^{I}_{;L}V_{I;K}\}dv - \int_{\partial B_T} \dot{T}^{I}V_{I}dS$$

In (10) the variation is taken with respect to the velocity field, noting that Green's strain rate is given by:

$$\dot{E}_{IJ} = \frac{1}{2}(V_{I;J} + V_{J;I} + U^{K}_{;I}V_{K;J} + U^{K}_{;J}V_{K;I}) \tag{11}$$

Finite element analysis employing this variational formulation requires definition of a large strain consitutive model providing the symmetry (7) and an interpolation model for the material frame components of velocity within the element. The analysis will perforce take the form of an incremental procedure involving evaluation of the velocity field at a sequence of times during the course of the deformation process.

Elasto-plastic Constitutive Equations: The constitutive model which we will employ preserves all of the features of conventional isotropic J_2 flow theory of plasticity. The model utilizes an arbitrary work-hardening stress-strain curve developed from tensile test data and includes provision for treating elastic unloading. Similar to the work of Hibbitt et al. [23] and Osias [15], objectivity is preserved by introduction of a Jaumann stress rate. The formulation is restricted to small elastic deformations.

The model is developed by Hutchinson in [17] and the derivation will not be repeated in detail here. Elasto-plastic flow equations are established by decomposition of the Green's strain rate into elastic and plastic components. Elastic deformation is described by an isotropic hypoelastic generalization of Hookean elasticity. Yielding and plastic flow are controlled by a loading function taken as the second deviatoric invariant of the Kirchoff stress (3).

A generalized expression for total Green's strain rate is found in the form:

$$\dot{E}_{IJ} = \frac{1}{E}[(1+\nu)g_{IK}g_{JL} + \nu g_{IJ}g_{KL}]\hat{S}^{KL} + \frac{\alpha f}{E} g_{IK}g_{JL}s^{KL}\dot{J}_2 \qquad (12)$$

where $\alpha = 1$ for plastic flow and $\alpha = 0$ for elastic behavior. In (12) (E,ν) are the constants of linear elasticity, g_{IK} is the metric of the convected system x^i, s^{KL} are the deviatoric components of S^{IJ} and f is a work hardening parameter taken as a function solely of J_2.

$$J_2 = (\tfrac{1}{2})g_{IK}g_{JL}s^{IJ}s^{KL} \tag{13}$$

$$s^{IJ} = S^{IJ} - (\tfrac{1}{3})g^{IJ}g_{KL}S^{KL}$$

$$\dot{J}_2 = g_{IK}g_{JL}s^{KL}\hat{S}^{IJ}$$

In (12) and (13) the Jaumann stress rate $\hat{S}^{KL}$ preserves flow rule objectivity and is related to the convected rate $\dot{S}^{KL}$ by (14)

$$\hat{S}^{IJ} = \dot{S}^{IJ} + g^{IK}S^{JL}\dot{E}_{KL} + g^{JK}S^{IL}\dot{E}_{KL} \tag{14}$$

Substitution of (14) in (12) and subsequent inversion obtains constitutive equations of the form (6) and provides the symmetry requisite for the variational form (10) to hold.

$$P^{IJKL} = P^{KLIJ} = \frac{E}{(1+\nu)}[\tfrac{1}{2}(g^{IK}g^{JL} + g^{IL}g^{JK}) + \frac{\nu}{(1-2\nu)}g^{IJ}g^{KL} \tag{15}$$

$$- \frac{\alpha}{G}s^{IJ}s^{KL}]$$

$$- (\tfrac{1}{2})[g^{IK}S^{JL} + g^{JK}S^{IL} + g^{IL}S^{JK} + g^{JL}S^{IK}]$$

Material property information enters (12) and (15) through the elastic constants, which take values ascribing to the undeformed state, and through the work hardening functions f and G. Evaluation of (12,15) for simple tension yields the relationships (16)

$$[1 + \tfrac{4}{3}J_2 f]^{-1} = (G - 2J_2)/[G - \tfrac{2}{3}(1 - 2\nu)J_2)] \tag{16}$$

$$= (\tfrac{g}{G})^{1/2}[\frac{E_t}{E} + \frac{g}{E}(1 - 2\nu)]$$

In (16) E_t is the instantaneous slope of the curve relating true stress σ and logarithmic tensile strain ε. $\bar{\nu} = -d\varepsilon_2/d\varepsilon$ is the instaneous contraction ratio, ε_2 being logarithmic transverse strain.

Special Element Interpolation Model: The model is developed to provide a number of kinematic features judged to be necessary for representation of crack blunting under Mode I loading. It is intended for use with the variational formulation (10) and does not introduce kinematic incompatibilities anywhere in the domain of the analysis.

The near crack tip total displacement field is represented as a combination of functions incorporating nonsingular, and variable order singular, gradients. Material frame components of displacement are approximated as functions of the undeformed coordinates so as to preserve reference to the original crack geometry. The singular gradient portion of the displacement model is further characterized by introduction of an explicit length scale R_s which may either be held at a fixed value or allowed to vary during the analysis. This parameter is intended to allow the range of influence of the singular gradient field to vary as crack blunting occurs.

A velocity field approximation appropriate to formulation of a rate, or incremental problem is obtained by differentiation of the displacement model with respect to time. Whereas some of the algebraic variables in the displacement model appear in a nonlinear fashion, their time rates appear linearly as unknown coefficients in the velocity representation. Thus a basis is provided for formulation of an incremental finite element analysis requiring solution of linear equations.

The special element domain is defined on the undeformed state and is the region $0 \leq \theta \leq \pi$, $0 \leq R/R_e \leq 1$ of Fig. 1. (R,θ) are undeformed material, or Lagrangian, coordinates and R_e is fixed. The value of R_e must be large enough to insure that the region within which singular behavior is affected by blunting is contained within the special element. In application R_e will be of order $10^{-2}a$, where a is crack length. The domain B_e is partitioned into N pie-shaped subregions, or base elements. Figure 1 shows the case N = 3; actual application will require N to be of order 10.

The total displacement field is approximated as:

$$U_I(X^1,X^2,t) = U_{oI}(X^1,X^2,t) + U_{sI}(R,\theta,t) \tag{18}$$

where* U_{oI} are defined independently within each base element and have nonsingular gradients. The U_{sI} are defined over the full special element domain and contain terms leading to singular gradients.

For null U_{sI} the special element would consist of an array of conventional higher order, nonsingular finite elements surrounding the crack tip. The displacement interpolation functions U_{oI} within each such element are constrained only by the requirements of geometric and displacement compatibility with each other and with host program elements across R/R_e = 1.0. Figure 1 suggests the use of 6 node isoparametric triangles; the base elements could also be degenerate, nonsingular, 8 or 9 node isoparametric quadrilaterals. The U_{oI} must account for rigid motions and uniform deformation states. Without prejudice as to specific form we will denote these functions for the m^{th} base element as*

$$U^m_{oI}(X^1,X^2,t) = \sum_{\alpha=1}^{n} A^m_{I\alpha}(t)\phi_{I\alpha}(X^1,X^2) \text{ (no sum on I)} \tag{19}$$

*Henceforth Latin indices o,s,m are reserved for use as defined here and do not denote tensor components.

where time dependence appears through unknown coefficients $A^m_{I\alpha}(t)$, and n is the number of nodes associated with the base element.

The special element displacement interpolation model is completed by specification of U_{sI} which is defined over the domain $0 \leq R/R_s \leq 1.0$, $0 \leq \theta \leq \pi$, where $R_s \leq R_e$ is to be determined as part of the analysis. For $R \leq R_s$, and within base element m, U_{sI} is written as:

$$U_{sI}(R,\theta) = R_I(\frac{R}{R_s},\theta)\Theta^m_I(\theta_m) \tag{20}$$

for

$$R_s/R_e \leq 1.0, \quad R/R_s \leq 1.0$$

$$(m - 1)\Delta\theta \leq \theta \leq m\Delta\theta, \quad 0 \leq \theta_m \leq \Delta\theta \quad \Delta\theta = \pi/N$$

The precise functional form of the functions R_I and Θ^m_I is somewhat arbitrary. If an exact asymptotic solution for the blunting problem were available it would be employed here. In lieu of such a solution we employ (20) with the functional forms (21-23).

$$R_I = (\frac{R}{R_s})^{n_I(\theta)} [1 - \frac{R}{R_s} + \ln\frac{R}{R_s} + \frac{1}{2}(\frac{R}{R_s})^2 - (\frac{R}{R_s})^3 + \frac{1}{2}(\frac{R}{R_s})^4] \tag{21}$$

where

$$n_I(\theta) = n_{oI} + n_{1I}\sin\theta + n_{2I}\sin 2\theta + \ldots + n_{pI}\sin p\theta \tag{22}$$

$$\Theta^m_I(\theta_m) = \sum_{\gamma=1}^{\ell} B^m_{I\gamma}\sin\gamma\theta_m + \sum_{\gamma=1}^{\ell} C^m_{I\gamma}\cos\gamma\theta_m \tag{23}$$

where the $B^m_{I\gamma}$ and $C^m_{I\gamma}$ are time dependent.

Thus complete definition of the model requires in addition to specification of N and R_e, the choice of lengths for the series (22) and (23). The number of unknown time dependent coefficients in (23) is reduced by requiring that (24,25) hold at the radial boundaries between base elements.

$$\Theta^m_I = \Theta^{m+1}_I \tag{24}$$

$$\frac{\partial}{\partial \theta_m} \Theta_I^m = \frac{\partial}{\partial \theta_{m+1}} \Theta^{m+1} \tag{25}$$

The form of the displacement field model U_{sI} of (20) as detailed in (21-25) follows from provision for a variable order displacement gradient singularity in the vicinity of the crack tip. The singularity is introduced such that the special element displacement field is single-valued and further such that at $R/R_s = 1.0$ for $0 \leq R_s/R_e \leq 1.0$:

$$\frac{\partial U_{sI}}{\partial R} = \frac{\partial U_{sI}}{\partial \theta} = U_{sI} = 0 \tag{26}$$

i.e., the singular gradient field vanishes smoothly as $R \rightarrow R_s$, which may vary during the analysis.

The velocity field interpolation model for use in (10) is established by differentiating (18) and provides for each component at time t:

$$V_I(X^I,t) = \dot{U}_{oI} + \dot{U}_{sI} \tag{27}$$

As a consequence of the form (20) of the displacement model, the velocity field will be single valued and will couple smoothly to the host program velocity model across $R/R_e = 1.0$. Furthermore, the singular gradients terms, $\dot{U}_{sI}$ and its first derivatives, vanish at $R/R_s = 1.0$. The velocity field interpolation model may be written within the m^{th} base element as

$$V_I = \dot{U}_{oI}^m + \dot{R}_I \Theta_I^m + R_I \dot{\Theta}_I^m \quad \text{(no sum on I)} \tag{28}$$

$$\dot{R}_I = \frac{\partial R_I}{\partial n} \dot{n}_{oI} + \frac{\partial R_I}{\partial n} \sum_{\beta=1}^{p} \frac{\partial n_I(\theta)}{\partial n_\beta} \dot{n}_{\beta I} + \frac{\partial R_I}{\partial R_s} \dot{R}_s \quad \text{(no sum on I)} \tag{29}$$

$$\dot{\Theta}_I^m = \sum_{\gamma=1}^{\ell} \dot{B}_{I\gamma}^m \sin \gamma\theta_m + \sum_{\gamma=1}^{\ell} \dot{C}_{I\gamma}^m \cos \gamma\theta_m \tag{30}$$

$$\dot{U}_{oI}^m = \sum_{\alpha=1}^{n} A_{I\alpha}^m \phi_{I\alpha}(X^1,X^2) \quad \text{(no sum on I)} \tag{31}$$

Thus at any time during the deformation process the deformed geometry is established through the displacement field (20) by coefficients $A^m_{I\alpha}$ of (19), $n_{\beta I}$ of (22) and $B^m_{I\gamma}$, $C^m_{I\gamma}$ of (23) as well as the value of R_s appearing in (21). Presuming these quantities to have been established by preceeding analysis, the velocity field is found by substitution of (27) into the functional (10) which will be quadratic in the time rate quantities appearing as coefficients in (28-31). The variational processs will provide linear equations for ($\dot{A}^m_{I\alpha}$, $\dot{B}^m_{I\gamma}$, $\dot{C}^m_{I\gamma}$, $\dot{R}_s$, $\dot{n}_{oI}$, n_{BI}), the unknown coefficients.

Solution for the time rates and integration over an increment of time will establish new values for the displacement field parameters, $A^m_{I\alpha}$ etc., allowing a new rate problem to be defined at the later time. In this manner a complete deformation history may be established by solution of a sequence of linear algebraic problems.

BLUNTING MODEL APPLICATION

The above formulation will permit solution of crack blunting problems within the framework of conventional incremental finite element analysis. Computation cost will be directly dependent upon the number of base elements employed within the special element as well as the number of terms carried in the series portions of the interpolation model. The incremental stiffness equations will be linear and well-conditioned. Solution may employ conventional reduction techniques.

Problem definition will require specification of loading history, geometry, and material properties as well as non-zero initial values of the coefficients in $n_I(\theta)$ (22) and R_s in (21). Typically one would employ the small strain elastic values for the coefficients, i.e., $n_{oI} = 1/2$,

$n_{\beta I} = 0$ ($\beta = 1,\ldots, p$). An initial value for R_s is less obvious. A value of the order of the osculating circle for the deformed elastic tip radius seems appropriate. The particular value giving the most accurate elastic stress intensity factor would be a viable starting point. Solution sensitivity to initial values of R_s will have to be investigated by numerical experimentation. Should variability of R_s prove to be an undesirable feature the model permits its value to be fixed.

The special element will permit blunting problem solution within the constraints implied by the form of the interpolation model. The ad hoc form of this model makes it most appropriate for evaluation of integral parameters, e.g., crack tip displacements or J-integral values. Results for internal field quantities will be directly controlled by the form of the particular interpolation functions employed. The utility of the analysis must ultimately be evaluated on the basis of its ability to support correlation of fracture data for ductile materials.

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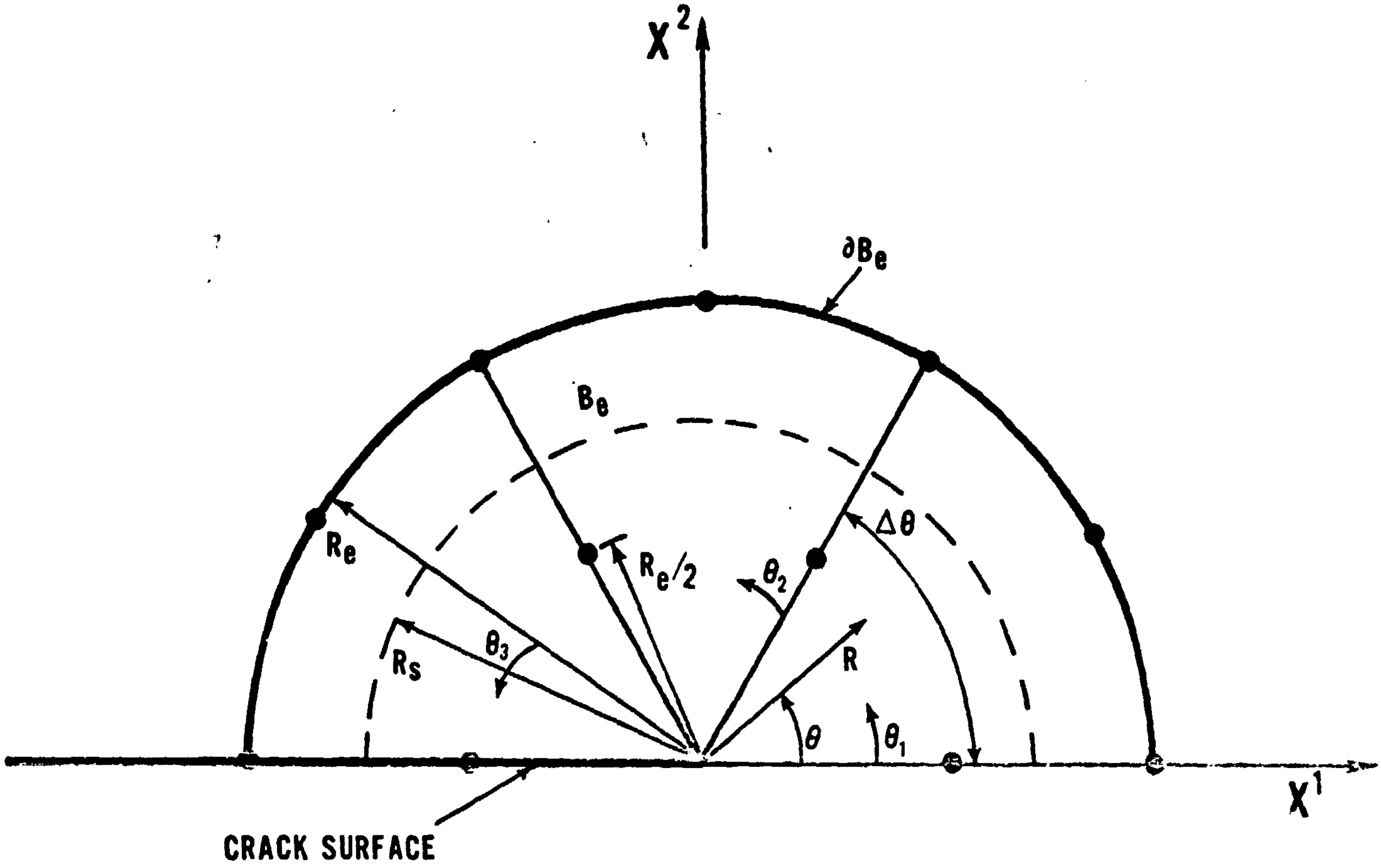

FIGURE 1: SUBPARAMETRIC ELEMENT CONFIGURATION
FOR CRACK BLUNTING ANALYSIS

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