THE SIGNIFICANCE OF THE SKYLAB ALTIMETER EXPERIMENT
RESULTS AND POTENTIAL APPLICATIONS

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ABSTRACT

The Skylab Altimeter Experiment has proven the capability of the altimeter for measurement of sea surface topography. The geometric determination of the geoid/mean sea level from satellite altimetry is a new approach having significant applications in many disciplines including geodesy and oceanography. A Generalized Least Squares Collocation Technique was developed for determination of the geoid from altimetry data. The technique solves for the altimetry geoid and determines one bias term for the combined effect of sea state, orbit, tides, geoid and instrument error using sparse ground truth data. The influence of errors in orbit and a priori geoid values are discussed. Although the Skylab altimeter instrument accuracy is about ± 1 m, significant results have been obtained in identification of large geoidal features such as over the Puerto Rico trench. Comparison of the results of several passes shows that good agreement exists between the general slopes of the altimeter geoid and the ground truth, and that the altimeter appears to be capable of providing more details than are now available with best known geoids. The altimetry geoidal profiles show excellent correlations with bathymetry and gravity. Potential applications of altimetry results to geodesy, oceanography, and geophysics are discussed.

INTRODUCTION

The Skylab S-193 altimeter experiment was the first of a series of altimeter experiments recommended by the "Williamstown Study"[1], and the NASA "Earth and Ocean Physics Applications Program" (EOPAP)[2]. The primary objective of the Skylab altimeter experiment was to determine the engineering feasibility of the altimeter instrument and demonstrate its capability for measurement of sea surface topography.

Three manned Skylab missions--SL/2, SL/3, and SL/4--provided data from the S-193 system. Geodetic analysis of Skylab S-193 altimeter data from mission SL/2 EREP pass #4, 6, 7, and 9 is the subject of this paper. The overall objective of the Battelle investigation is to demonstrate the feasibility of and necessary conditions in using the altimeter data for the determination of the marine geoid (i.e., the geoid in ocean areas). The geoid is the equipotential surface that would coincide with "undisturbed" mean sea level of the earth's gravity field. "Undisturbed" is the condition that would exist if the oceans were acted on by the earth's force of gravity only and by no other forces such as those due to ocean currents, winds, tides, etc. Thus, determination of the geoid/mean sea level is basic to the understanding of the oceans and associated dynamic phenomena such as currents, tides, circulation patterns and, hence, air-sea interactions. There exist many geoids which have been computed by various methods. Most of these lack the accuracy and quality required for many scientific and practical applications.[3]

Present methods of determining the geoid depend on the knowledge or measurement of the detailed gravity field all over the earth and the use of satellite perturbations to describe the general field. Measurement of the gravity field all over the earth requires the use of land-based, shipborne, airborne and ocean-bottom gravity instruments, a process which will take decades to achieve an accuracy of the order of 1-3 meters. Present knowledge of the geoid on a world-wide basis is probably not better than 5-30 m.

The use of altimetry for determining the geoid is, perhaps, most significant because of its fundamental applications to geodesy, oceanography, geology, geophysics, navigation, national defense, environment, resource development and several other applications. Satellite altimetry offers the most expedient and accurate method for determining the geoid independent of gravity measurements. An accuracy of $\pm$ 10 cm for determining the marine geoid from satellite altimetry is the goal of the NASA's EOPAP.[1] The results obtained from the Skylab S-193 altimeter experiment proved that the concept is feasible. The extent of applications of altimetry data to various disciplines and uses will depend on the degree of accuracy achieved and correlation that can be made between these data and the parameters involved in the application areas.

CONCEPT OF GEOID DETERMINATION FROM SATELLITE ALTIMETRY AND REQUIREMENTS

Figure 1 shows schematic geocentric relations of the various surfaces associated with satellite altimetry. TM is the raw altimeter range which has to be corrected for laboratory instrumental calibration, electromagnetic effects, sea state, and periodic sea surface influences to give TS. S represents the non-periodic "sea level". CT and CE, the geocentric radii of the altimeter and E, its subsatellite point on the reference ellipsoid, are computed from satellite tracking information. EG is the absolute geoidal undulation to be computed from this investigation, while SG is the quasi-stationary departure of the mean instantaneous sea surface from the geoid - the "undisturbed" mean sea level. It can be seen from Figure 1 that the required geoidal undulations are given by

$$EG = ET - TM - MS - SG \tag{1}$$

where, MS represents the sum of the calibration constants and the orbit uncertainties, if any. SG represents the deviation of the surface to which the measurement is made from the geoid. Since we do not have any information on SG which is not considered to vary significantly over the length of profiles corresponding to different submodes of observations, the sum (MS + SG) is considered as the calibration constant.

The basic requirements for determining the geoid from satellite altimetry are:

1. accurate orbit determination;
2. precise altimeter instrumentation;
3. ground truth verification data;
4. methods of interpolating and extrapolating altimetry into unsurveyed areas; and
5. separation of the geoid from sea surface topographic effects.

The geoid to be determined must be in absolute position or geocentric (i.e., centered at the earth's center of mass) and have correct scale, shape and orientation in order to meet the goals of geodesy and also make contributions to the solution of problems in earth gravity modeling, geophysics, oceanography, etc. Correctness of shape depends on the precision of the altimeter and, in theory, absolute centering and orientation are dependent on the satellite orbit ephemeris. The correctness of geoid scale requires that the orbit ephemeris and the altimeter either have no biases or systematic errors, or that such biases and systematic errors must be known to an accuracy better than the error tolerance of the geoid to be computed. Currently and for some time to come, these two scalar conditions cannot be met because of unknown systematic errors or biases in tracking station geocentric coordinates, the earth's gravity model, the tracking systems and the altimeter itself. There is, therefore, a need for other sources of scale and orientation control, which can be satisfied by the use of good ground truth geoid data. Poor ground truth data will only result in a

coarse scale. Fine scale, if necessary, has to be introduced through some form of marine geodetic control. Since there is no established marine geodetic control available at present, the geoid computed in this paper will only have a coarse scale provided by the a priori geoid height input.

APPROACH AND MATHEMATICAL MODEL

The approach to this analysis consisted of two basic steps. The first step was to filter the noisy altimetry data using the Generalized Least Squares Collocation technique. In the second step, the filtered altimetry data were compared with the a priori ground truth geoid in order to determine the calibration constants and the altimetry geoid profile. The basic purpose of this comparison was to have the altimetry geoid data on the same scale as that of the a priori geoid. The details of the procedure used in this analysis is discussed in Mourad, et al.[4] The general mathematical model used in this analysis consists of the linear relationship between the residual altitude from the altimetry data, the a priori geoid, and the calibration constants. For the purpose of this investigation, only one constant term representing the cumulative effect of all the possible systematic errors associated with the residual altitude, is used as a calibration constant.

The basic condition equation is

$$D_1 - R_1 = N_1 + \Delta C + n_i \tag{2}$$

where, R_1 is the measured altimeter range, which is intrinsically related to

1. the geocentric coordinates, X_{si}, Y_{si}, Z_{si}, of the satellite at the instant of measurement;
2. the geoid undulation, N_1, referred to a given reference ellipsoid at the subsatellite point; and
3. the algebraic sum of the biases in all the measurement systems involved.

Except for the noise term (n), equation (2) is the result of rearrangement of equation (1) with ΔC, which is the total bias term involved in the measurement, representing the sum (MS + SG) and with

$$D_1 = F(X_{si}, Y_{si}, Z_{si}, a, f) \tag{3}$$

representing the height of the satellite above the reference ellipsoid given as a function of the geocentric coordinates of the satellite at the instant of observation and with a and f being the parameters (semi-major axis and flattening) defining the size and shape, respectively, of the assumed reference ellipsoid. It is important, however, that both D_1 and N_1 [Equation (2)] refer to the same reference ellipsoid. If they do not, they must be made compatible by effecting the appropriate corrections as and when they are necessary.

The general theory of Least Squares Collocation is presented in Moritz,[5] and the details of its application to filtering the altimetry data are described in Mourad, et al[4]. The filtered altimetry observations, s, are given by:

$$s = N + \Delta C = C_{ss} \overline{C}^{-1} (D - R) \tag{4}$$

where, C_{ss} is the auto-covariance matrix for the geoid undulations corresponding to the altimetry observations,

$$\overline{C} = C_{ss} + C_{nn} \tag{5}$$

with C_{nn} being the error covariance matrix for the altimetry observations. Equation (5) implies that the geoid undulations and the altimetry observations are stochastically independent. The subscript $_i$ is left out in equation (4) to indicate that the equation is in matrix notation and represents a set of several equations corresponding to equation (2).

The elements of the covariance matrix C_{ss} are evaluated from a table of numerical covariance function, from Tscherning and Rapp [6], using linear interpolation method. The covariance between any pair of undulations is given as a function of the spherical distance between these undulations.

The output resulting from filtering the altimetry data is a set of filtered residual altitudes given by

$$s = N + \Delta C \tag{6}$$

If s and N are accurate, ΔC can be estimated by evaluating equation (6) at any one point. However, in order to minimize the uncertainties, if any, in the ground truth geoid, this equation is evaluated at several points along the profile using Least Squares Adjustment principles. s is considered observable and ΔC and N are parameters to be estimated. In matrix notation, the observation equation corresponding to equation (6) would be

$$V + AV_x + W = 0 \tag{7}$$

where, V is the vector of residuals on -s and V_x is that on ΔC and N. A is the design matrix and W is the misclosure vector obtained by evaluating $(N + \Delta C - s)$ with the 'observed values' for s and the a priori ground truth data for N. The a priori values for ΔC can be assumed to be zero. If the weight matrices of the observables and the parameters are P and P_x respectively, the matrix solution for the residuals for the parameters is given by

$$V_x = -(A^T PA + P_x)^{-1} A^T PW \tag{8}$$

Then, the altimetry geoid undulations, N, are given by

$$N = s - \Delta C \tag{9}$$

If stochastic independence is assumed among observables and among parameters, P and P_x will be diagonal matrices. Those elements of P corresponding to ΔC can be assumed to be zero.

Preliminary examination of the Skylab altimetry data revealed that every time the submode of the altimeter changed, there appeared to be a change in bias in the measured altitude. Consequently, separate bias terms were assumed for data from the segment of the profile observed in a single mode-submode combination. These bias terms can be recovered simultaneously by suitably modifying the design matrix.[4]

Consideration of separate bias terms for different segments may result in a discontinuity from one segment to the other. This can be rectified by constraining the closest ends of the two adjacent segments to have the same value of undulation even though these ends do not correspond to the same point. This is a reasonable assumption considering the accuracy of the Skylab altimetry system combined with the small variation in undulations over short distances of about 8-15 km. In this analysis, these constraints are effected through sequential solution, which means that the effect of the constraints on the parameters is evaluated and added to the solution obtained from equation (8). The reason for this approach is the convenience and efficiency resulting from the special structures of the matrices involved.

RESULTS AND CONCLUSIONS

The major results obtained can be divided broadly into two groups. One group is concerned with the effects of errors inherent in the various input data such as the orbit ephemeris, a priori geoid, etc. The other consists of the results of the actual analysis of the data from the Skylab EREP passes #4, #6, #7, and #9.

Results of Preliminary Data Analysis

The results from the first group have been obtained from the analysis of some preliminary data from EREP pass #9 mode 5. The use of such preliminary data was necessary in the absence of actual data which were not available at the time of the error analysis. Details of the preliminary analysis are described in Fubara and Mourad.[7] Figures 2 and 3 show only two important results. Figure 2 shows the influence of a priori geoid height on the geoid scale. It shows that such a priori geoid input and errors in them affect only the linear scale and not the shape of the geoid. This is due to the fact that any inherent error is modeled by a constant bias. The a priori geoid input was taken from Vincent and Marsh geoid[8] (GG-73 in Figure 3). AA is the resultant altimetry geoidal profile based on GG-73 as a priori input. Two different sets of errors were introduced into GG-73 to produce A-I and B-I. This resulted in altimetry profiles A-O and B-O. It is obvious that AA (control experiment) is shape-wise identical to A-O and B-O. However, the scale of the calibration constant has changed considerably in the same altimetry profiles. Figure 3 shows that the scale and orientation of the computed altimetry geoid is highly dependent on the orbital data used. In this analysis, two sets of orbital data (computed independently using different methods) were used. These resulted in the altimetry geoidal profiles AA and BB. The scale discrepancy was removed through the calibration constant used. However, the difference in orientations remained. The close agreement between AA and GG-73 may be due to the use of basically the same gravity model and coordinate frame in their computations.

To summarize, results demonstrated that:

1. the precision of the altimeter ranges affect the shape of the geoid;
2. the orbit uncertainties affect both the scale and orientation; and
3. available a priori ground truth geoids can give only a coarse scale to the altimetry geoid.

Thus, to get an absolute geoid correct in scale and orientation, marine geodetic controls to offset the systematic errors in the orbit and altimeter data appear to be required.

Results of Final Data Analysis

The analysis of the altimetry data was made only for EREP passes #4, #6, #7, and #9 whose approximate locations in the North Atlantic Ocean are shown in Figure 4. The analysis was accomplished in three basic steps:

1. filtering;
2. estimation of the parameters; and
3. graphical presentation of the results.

The basic inputs for the first step are:

1. the altimeter ranges and the exact correlated time of each measurement;
2. the associated orbit ephemeris;
3. the parameters of the reference ellipsoid; and
4. the covariance function for the geoid undulations.

The satellite altimetry data for the four passes were received on magnetic tapes from NASA/JSC. These data consist of eight altimeter range observations in frames at 1.04 second intervals. However, for the purpose of this investigation, the mean of the eight observations in each frame is considered as one observation. This assumption should not degrade the results for the following collective reasons:

1. A frame of observations covers an effective area of about 6 km by 13 km, since the ground speed of the Skylab was about 7 km/sec and the radius of the radar foot print was about 3 km;[9] and

2. Considering the accuracy of the altimeter system on board the Skylab, the change in geoid over an area of size 6 km by 13 km would be insignificant.

The Skylab Best Estimate Trajectory (SKYBET) data are also available on a tape at intervals of exactly 1/8 of a second. Only the earth fixed geocentric coordinates of the Skylab and the time of observation are input from these data.

The best available estimates for the shape and size of the geodetic reference ellipsoid are given by:

1. flattening = 1/298.255

2. semi major axis diameter = 6,378,142.0 meters.

This is the same as the reference ellipsoid to which the Marsh-Vincent 1973 geoid is referred.[9] The covariance function for geoid undulation is taken from Tscherning and Rapp.[6] This is a numerical covariance function compatible with the reference ellipsoid chosen for the analysis.

The residual altitudes which are filtered in the first step and the ground truth geoid undulations taken from the Marsh-Vincent geoid map[9] form the input for the second step which is the estimation of the calibration constants and the geoidal parameters. In the third step, the estimated geoid profiles are plotted against time along with the ground truth profiles for easy comparison for shape.

The data in each mode of the altimeter were observed in several submodes, each of which consists of several sub-submodes of observations. The altitude measurement of the satellite above the ocean which is the only geodetic data of interest, comes only from Modes 1, 3, and 5 of the Skylab altimeter.[10] The maximum magnitude of the geoid undulations, referred to the best available reference ellipsoid, is of the order of about 125-150 meters. Therefore, any residual altitude of more than a conservative estimate of 300 meters, is an indication of instrument malfunction in the altimeter measuring system. Consequently, data were processed only for those segments of the passes (#4, #6, #7, and #9) corresponding to modes 1, 3, or 5 where the absolute residual altitude is less than 300 meters. It was found that the altimeter observations suitable for geodetic processing come from submodes 0, 1, and 2 in modes 1 and 5 and from submodes 3, 4, and 5 in mode 3.

The results of the data analysis consist of:

1. a set of bias terms (Table I) recovered for various segments of the four passes; and
2. a set of geoidal profiles.

Examples of these geoidal profiles and their corresponding bathymetry and gravity profiles are shown in Figures 5 through 10. The geoidal profiles consist of:

1. ground truth geoid (dashed lines) used in estimating the bias
   terms and establishing the scale;
2. geoid profiles computed from the unfiltered data using the
   computed bias terms (thin lines); and
3. geoid profiles corresponding to the filtered data.

Since the altimetry data were fitted to the ground truth geoid in the determination
of the bias terms, any scale error in the ground truth would also result in error in the
altimetry geoid. Consequently, any deviation of the altimetry geoid profiles from the
ground truth profile will have short periods. In general, the deviation between the two
sets of profiles is within about 2-3 meters with the following exceptions: In pass #7
mode 5 (Figure 9) the deviation ranges from 0 to 12 m. In passes #4 and #6 (Figures 6
and 7), the maximum difference is about 12 m in the Puerto Rico trench area.

A close examination of these differences indicates that these extreme deviations
occur in areas of special features such as trenches, ridges and sea mounts. Passes #4
and #6 cross the Puerto Rico trench on the west side. Pass #7 mode 5 is along the
western edge of the mid-Atlantic ridge while pass #7 mode 3 crosses the Puerto Rico trench
at the Eastern end.

These deviations may be due to several causes:

1. Residual errors due to orbit uncertainties
2. High frequency component of the geoid not reflected in
   the ground truth data
3. Possible (nadir) alignment errors which result in the
   departure of the sensor field of view from the nadir
4. Influence of sea state, tides and ocean circulation effects
5. Possible inaccuracies in the computation of the ground
   truth data
6. Errors introduced as a result of scaling these data off
   small scale world maps.

Most of the systematic errors caused by the above would be absorbed in the bias terms
recovered from the data, especially because of the shortness of the segments for which
separate bias terms were considered. The short periodic deviations caused by sea-state,
tides, ocean circulation effects, etc., would be of the order of about 1-2 m. This leads
us to believe that, at least, the larger deviations are due to the short periodic compo-
nents of the geoid not reflected in the ground truth geoid. This is confirmed by the
fact that such deviations occur in the areas of significantly large geoidal features.

Another striking difference noted in these profiles occurs in pass #7 (mode 5) which
passes across the Puerto Rico trench (Figure 9). The trench, as indicated by both profiles,
differs horizontally by about 30 seconds of time; equivalent to about 240 km. Looking at
the gravity anomaly and bottom topography profiles, the altimetry geoid profile appears to
be correct. However, further investigations may be needed in order to determine how a
gross discrepancy such as this could have occurred.

Crossing of passes #4 and #6 (Figures 6 and 7) over the Puerto Rico trench and land
mass area almost at the same place, has provided an ideal opportunity to compare the results
to see the consistency of altimeter system in determining the geoid. The overlapping
segments of the profiles for these passes are shown superimposed in Figure 8. The agree-
ment between the profiles is excellent. The only deviation at the beginning of pass #4
mode 5, is due to the instrument transient response after switching pulse width, beam
width and pointing between modes. This agreement indicates that the altimeter system is
very stable and consistent.

From the results and analysis presented thus far, some general comments/observations and conclusions can be made:

1. The procedure described and applied for filtering the altimetry data and for estimating the calibration constants (bias) and geoid undulations has produced very satisfactory and realistic results;

2. The bias terms recovered for different segments of the same pass are significantly different. There appears to be very little or no correlation among the bias terms associated with the same submodes;

3. The agreement between the general slopes of the two geoid (altimetry and ground truth) profiles shows the viability of the altimetry technique in determining the marine geoid;

4. The magnitude of the deviations of the altimetry geoid from the conventional ground truth leads to the conclusion that these deviations are mostly due to the high frequency components of the geoid rather than due to other causes;

5. The Skylab altimetry data analyzed here have provided ample evidence that the altimetry sensor is very sensitive to local geoidal features such as trenches, ridges, and sea mounts;

6. Excellent agreement between the results obtained for the same place at different times shows that the satellite altimetry is precise and self consistent except for bias terms;

7. The correlation between the altimetry geoid and the gravity anomaly and the ocean bottom topography profiles have shown it to be useful in verifying major discrepancies in the conventional geoid. These correlations may also be useful in other applications such as in geology, geophysics, etc.

CORRELATION OF SKYLAB ALTIMETRY WITH BATHYMETRY AND GRAVITY

There are several obvious correlations of the Skylab altimetry-determined geoid with ocean-bottom topography (bathymetry) and surface free air gravity anomalies. Each of Figures 5, 6, 7, 9, and 10 shows examples of these correlations. In Figure 5, the altimeter responded clearly to the variation in topography and gravity anomalies caused by the Blake escarpment (about 5 m in geoidal heights, correlated with 3600 m depth changes and about 100 mgals in free air gravity anomalies). Note that the ground truth geoidal profile did not show such a change. The free air gravity anomalies show, also, some correlations with bottom topography over the continental shelf and Blake Plateau. Unfortunately, there are no gravity data (indicated by +++ on the profiles) taken at the Blake escarpment. Figure 6 shows very strong correlations, over the Puerto Rico Trench, between the altimetry geoid and both the gravity and bathymetry, (about 15 m geoidal height change corresponding to about 6,000 m change in depth and 400 mgal in free air gravity anomalies). Again, note the difference from the ground truth geoid. The altimeter also responded clearly to the land mass as evidenced by the sharp rise over Puerto Rico.

Passes #4 and #6, over the Puerto Rico trench and Puerto Rico land mass, (Figures 6, 7, and 8) provided an excellent opportunity to show the repeatability of the altimeter. The altimeter data also appear to show some changes due to the shallow water as the Skylab approaches the Lesser Antilles. There is also strong correlation between this altimetry geoid and the gravity, while the ground truth geoid is very smooth. Figure 9 (a segment of pass #7) shows a strong correlation with the Puerto Rico trench (this is the eastern end of the trench which is different from those shown in passes #4 and #6). Also the rise in the altimeter geoid correlates well with the rise in topography going toward the Lesser Antilles. However, the ground truth geoid appears to be shifted laterally by about 200-250 km.

The results of a segment of pass #9 are shown in Figure 10, which indicates a smooth geoid over the whole profile. The geoid variations do not conform to the bottom topography and gravity particularly over the continental slope. It does correlate with the ground truth geoid, however, with one exception where the ground truth profile shows a slight dip corresponding to the continental slope. There is also a relative tilt between the altimetry geoid and the ground truth geoid.

In summary, there are pronounced correlations between the altimeter-determined geoid, the topography and gravity anomaly profiles, particularly those corresponding to special earth structures such as the Puerto Rico trench. In many cases, the altimeter appeared to sense many more details in the geoidal surface than those reflected in the ground truth geoid. There are also some correlations with sea mounts or shallow near-surface topographic features. McGoogan, et al [10,11] showed similar correlations with the Puerto Rico Trench and the Mariana Trench. In addition, he showed some correlations with specific sea mounts and the Flemish cap as well as an overall correlation with the ground truth geoid in around-the-world-profile. Our investigation was limited to the data obtained from the four passes discussed in this paper.

The causes of different gravity anomaly values, hence geoidal features, associated with different but similar bottom topographic features are due to variations in the mass distribution in the earth crust and mantle and to the degree of isostatic compensation. In order to realistically determine and evaluate the various deviations associated with the altimetry geoidal passes, more extensive effort is required. Such an effort could entail constructing sub-bottom profiles, based on various density assumptions and additional geophysical data (magnetic, seismic, geologic) and comparing the results with the geoidal profiles.

FUTURE ALTIMETRY APPLICATIONS

As mentioned earlier, the extent of applications of satellite altimetry will depend, to a large extent, on the degree of accuracy achieved in the determination of the geoid. With the GEOS-3 satellite altimeter (launched April, 1975), a 1-5 m geoid accuracy (absolute) is expected. By 1978/1979 another altimeter is being planned to fly on SEASAT with sub-meter expected accuracy in geoid determination.

Future altimetry applications include the following:

Geodesy/navigation. - Any improvement in the determination of the geoid contributes to geodesy and navigation. With an accuracy of ± 1 m in geoid determination, many geodetic objectives can be achieved. The major contributions are in improving the determination of the size and shape of the earth and a unified datum and coordinate system on a worldwide basis. Determination of the figure of the earth, until the present, has depended largely on continental data. Satellite altimetry is certain to change that trend and include the ocean data covering over 70 percent of the earth's surface. The objectives of the new adjustment of the North American Datum (NAD) is 1 m. The NAD is used for many civilian applications such as surveying, mapping, engineering operations, navigation and resource development. The extension of control points and determining their three-dimensional coordinates to offshore areas as well as the determination of national and international marine boundaries must be established in the same system. Accurate knowledge of absolute deflection of the vertical at sea, if combined with marine geodetic control, could provide the orientation required for all national datums. These absolute deflections are important for improving the accuracy of shipboard inertial navigation systems. Knowledge of the geoid to ± 10 cm from altimetry could provide a worldwide reference for the vertical datum which should contribute to investigations of land subsidence.

Gravity field determination. - The gravity field can be derived from satellite altimetry.[12] A global solution of a ± 1 m altimetry geoid could resolve the gravity field to about 1° x 1° which represents about 180 x 180 geopotential model. This resolution will contribute considerably to improving the geopotential model which is used in determination of satellite orbit and missile trajectories. The altimetry geoid could also provide information on the gravity anomalies which in turn are helpful, when combined with other geophysical data, in exploration geophysics and identification of geological structures.

Mean sea level (MSL). - A ± 10 cm geoid will contribute significantly to MSL determination. Computation of direction and magnitude of MSL slopes and the heights of each ocean relative to the others are important and still unresolved problems. These problems are further complicated because the results of geodetic and oceanographic computations of the parameters involved are different from each other. Determination of heights, directions and magnitude of sea level slopes relative to the continents is a key factor in land and environmental use and in studies of the effect of changes in polar ice caps which affect marine life, meteorology and climate.

Plate tectonics and ocean trenches. - There is a correlation between the geoid/earth's gravity field and geophysical/geological phenomena. Knowledge of these are required in earth and ocean physics studies. Some of these correlations are shown in Skylab data above. To better understand and model this correlation, a knowledge of the fine structures of the ocean geoid is required. These phenomena contribute to a better understanding of continental drift, polar motion and earthquake mechanisms.

Oceanography. - With a ± 10 cm geoid, it is possible to determine the quasistationary departures of the sea surface topography from the geoid.[13] The departures of the sea surface topography from the geoid such as those due to tides, barometric pressure, wind, storm surges have practical applications in the determination of ocean dynamic phenomena such as circulation patterns, mass and nutrient transport, ocean tides, and ocean current influences. Most of these phenomena have important roles in monitoring and preserving the environment, in air-sea interaction and in global numerical weather prediction.

If the above parameters can be referenced to the geoid, satellite altimetry should contribute to their solution on a global basis. For example, sea slopes due to ocean currents could cause local rise of water across the current on the order of 1 meter. Current slopes are proportional to their speed. Therefore, mass transport can be determined. The periodic effect of tides in the open ocean is perhaps on the order of about 1 meter. Its determination is important particularly for the separation of the influence of earth tides. Also barometric pressure could cause variations in the slope of the sea surface up to several meters. Other applications that could be possible are in the prediction of tsunamis (seismic sea wave) and storm surges. These, however, may be detected only if they occur during the pass of the altimetry satellites. Tsunamis amplitudes in the open ocean range, perhaps, from a few centimeters to about 1 meter. They have large wavelengths of the order of several thousand km with about a 1-hour period. Storm surges, which is the local build-up of water due to distant violent storms (such as hurricanes and typhoons), could cause damage and reach wave heights of the order of several meters when they hit coastal areas. Their prediction and direction of movement could be of importance not only for coastal areas but also for maritime ship operations.

The various problems affecting the accuracy of achieving a ± 10 cm geoid must be solved in order to arrive at many of the above applications. The Skylab altimetry experiment, however, has demonstrated proof of concept of geoid determination from altimetry data and that satellite altimetry is a potentially valuable tool having many useful applications. If GEOS-3 and SEASAT achieve their objectives, the impact of altimetry on earth and ocean dynamics studies would be significant.

REFERENCES

1. Kaula, W. M. (Chairman), "The Terrestrial Environment: Solid Earth and Ocean Physics", Report of a Study at Williamstown, Massachusetts, to NASA, Report NASA CR-1579, April, 1970.

2. Williams, F. L., "Opening Remarks", Proceedings of the International Symposium on Applications of Marine Geodesy, pp xiii - xvii, Marine Technology Society, Washington, D. C., 1974.

3. Fubara, D.M.J., and Mourad, A. G., "Requirements for a Marine Geoid Compatible with Geoid Deducible from Satellite Altimetry", Proceedings of NOAA/NASA/NAVY Conference on Sea Surface Topography from Space, Edited by J. Apel, NOAA TR ERL 228-AOML7, 1972.

4. Mourad, A. G., Gopalapillai, S., Kuhner, M., and Fubara, D. M., "The Applications of Skylab Altimetry to Marine Geoid Determination:, Final Report prepared by Battelle's Columbus Laboratories to NASA/JSC, under contract NAS9-13276, EPN 440, May 1975.

5. Moritz, Helmut, "Advanced Least Squares Methods", Reports of the Dept. of Geodetic Science, No. 175, The Ohio State University, Columbus, Ohio, 1972.

6. Tscherning, C. C., and Rapp, R. H., "Closed Covariance Expressions for Gravity Anomalies, Geoid Undulations and Deflections of the Vertical Implied by Anomaly Degree Variance Models", Reports of the Dept. of Geodetic Science, No. 208, The Ohio State University, Columbus, Ohio, 1974.

7. Fubara, D. M. and Mourad, A. G. "Results of Geodetic Processing and Analysis of Skylab Altimetry Data", Proceedings of the International Symposium on the Application of Marine Geodesy, pp. 301-314, Marine Technology Society, Washington, D. C. 1974.

8. Vincent, S., and Marsh, J. G., "Global Detailed Gravimetric Geoid", Computer Sciences Corporation and NASA/GSFC, Greenbelt, Md., 1973.

9. Marsh, James G., and Vincent, Samir, "Global Detailed Geoid Computation and Model Analysis", GSFC Report No. X-921-74-131, Greenbelt, Maryland, 1973.

10. McGoogan, J. T., Wells, W. T., and Miller, L. S., "Skylab Altimeter Applications and Scientific Results". Paper Presented at the AIAA/AGU Conference on Scientific Experiments of Skylab, Huntsville, Alabama, October 30 - November 1, 1974.

11. McGoogan, J. T., Leito, C. D., Miller, L. S., and Wells, W. T., "Skylab S-193 Altimeter Experiment Performance, Results and Applications", Proceeding of the International Symposium on Applications of Marine Geodesy, pp. 291-300, Marine Technology Society, Washington, D. C., 1974.

12. Gopalapillai, S., "Non-Global Recovery of Gravity Anomalies from a Combination of Terrestrial and Satellite Altimetry Data", Reports of the Department of Geodetic Science, #210, The Ohio State University, Columbus, Ohio, July, 1974.

13. Apel, J. R., and Byrne, H. M., "Oceanography and the Marine Geoid", Proceedings of the International Symposium on Applications of Marine Geodesy, pp. 59-66, Marine Technology Society, Washington, D. C., 1974.

TABLE I. - RESULTS OF THE BIAS RECOVERY FROM ALTIMETRY DATA

Pass	Mode	Submode	Submode*	Constrained Bias(m) Filtered
4	1	0	1	-23.26
		1	2	-10.25
		2	3	-12.60
	5	0	7	-28.12
		1	8	- 9.02
6	5	0	1	-24.03
		1	2	-11.63
		2	3	- 3.31
7	1	0	1	-52.43
		1	2	-40.28
		2	3	-79.28
	5	0	7	-63.55
		1	8	-52.38
		2	9	-46.56
	3	3	16	-
		4	17	-88.87
		5	18	-87.39
9	5	0	1	-17.07
		1	2	- 6.07
		2	3	0.62
	3	3	10	-19.99
		4	11	-20.53
		5	12	-18.95
		3	18	-36.95
		4	19	-36.26
		5	20	-34.80

* These numbers are assigned sequentially in each for
  computational convenience.
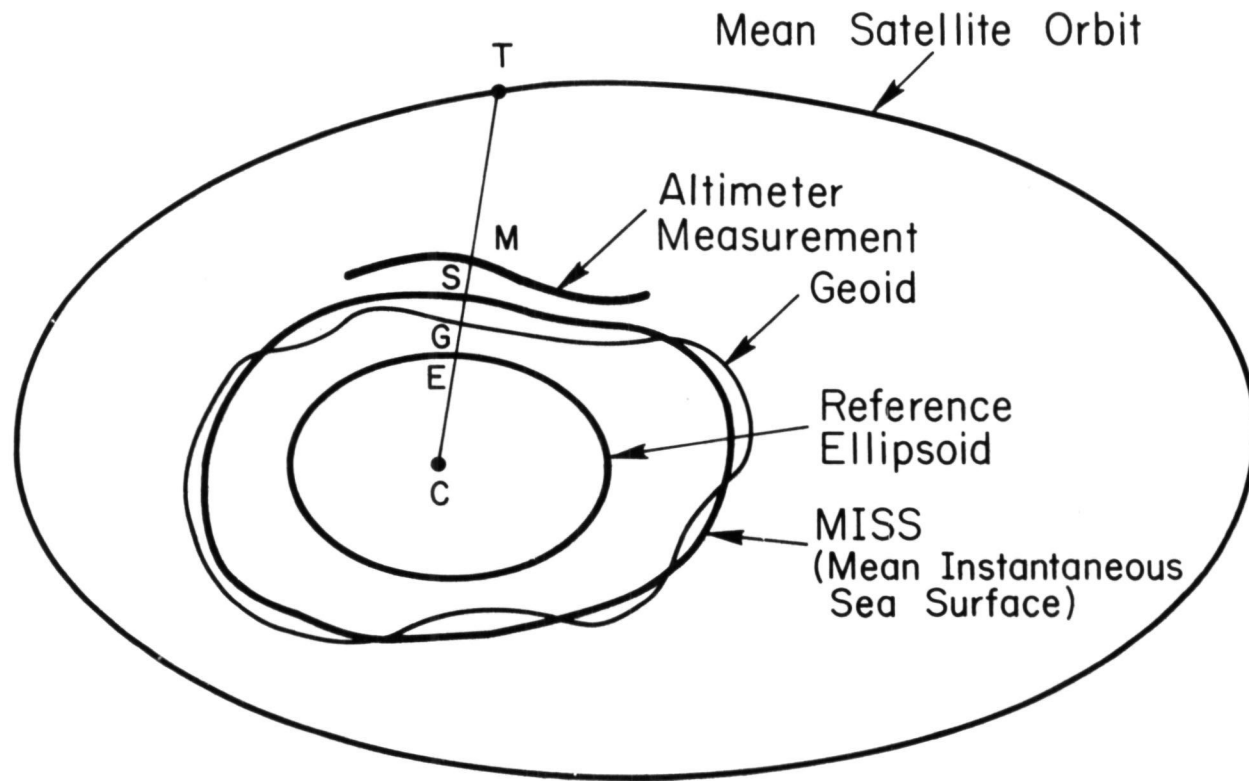
Figure 1. - Surfaces associated with satellite altimetry.
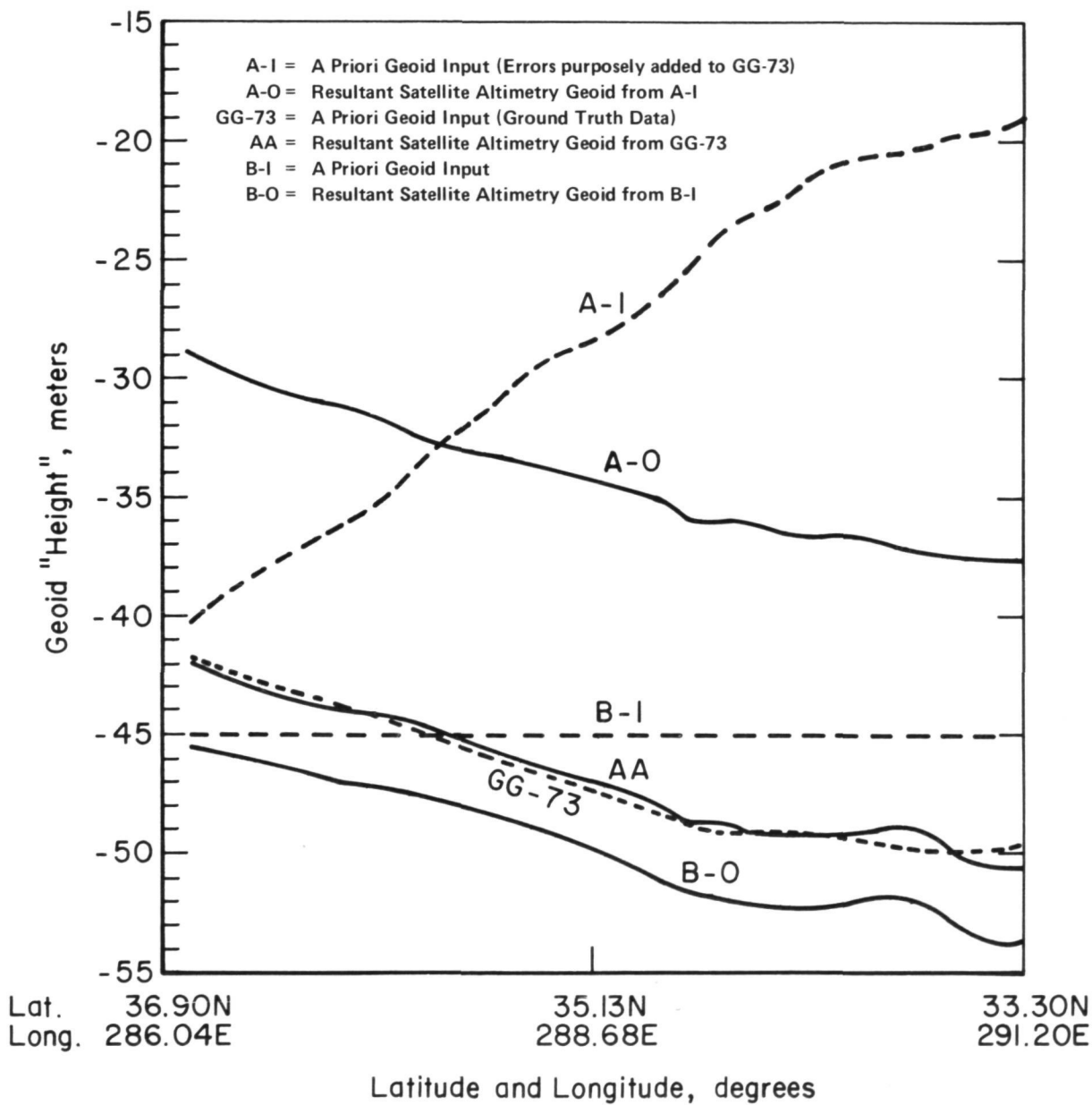
Figure 2. - Effect of errors in a priori geoid height inputs and
scale dependency of calibration constant and geoidal
height on geodetic control (ground truth).
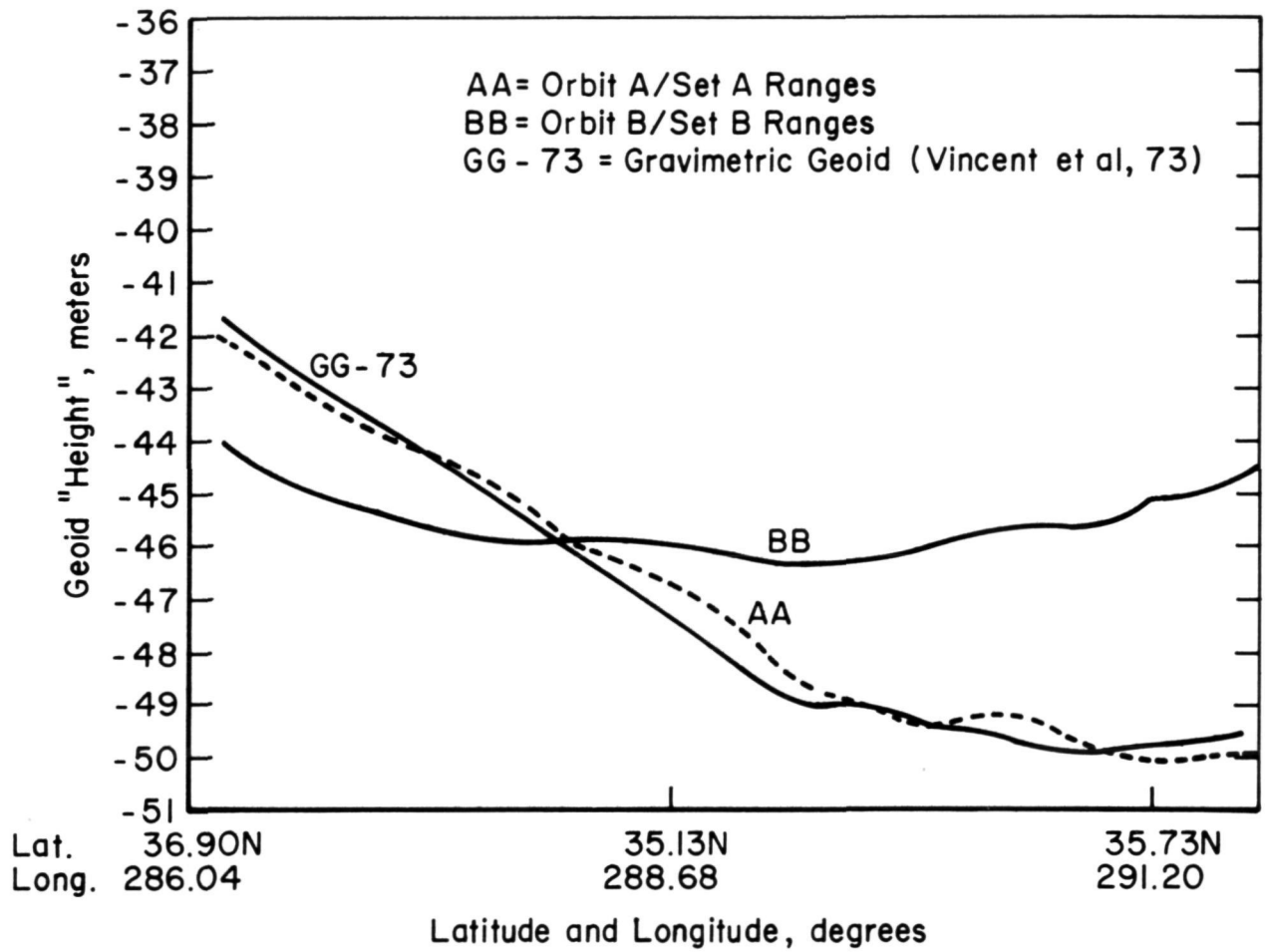
Figure 3. - Conventional Geoid and Satellite Altimetry Geoid Segments
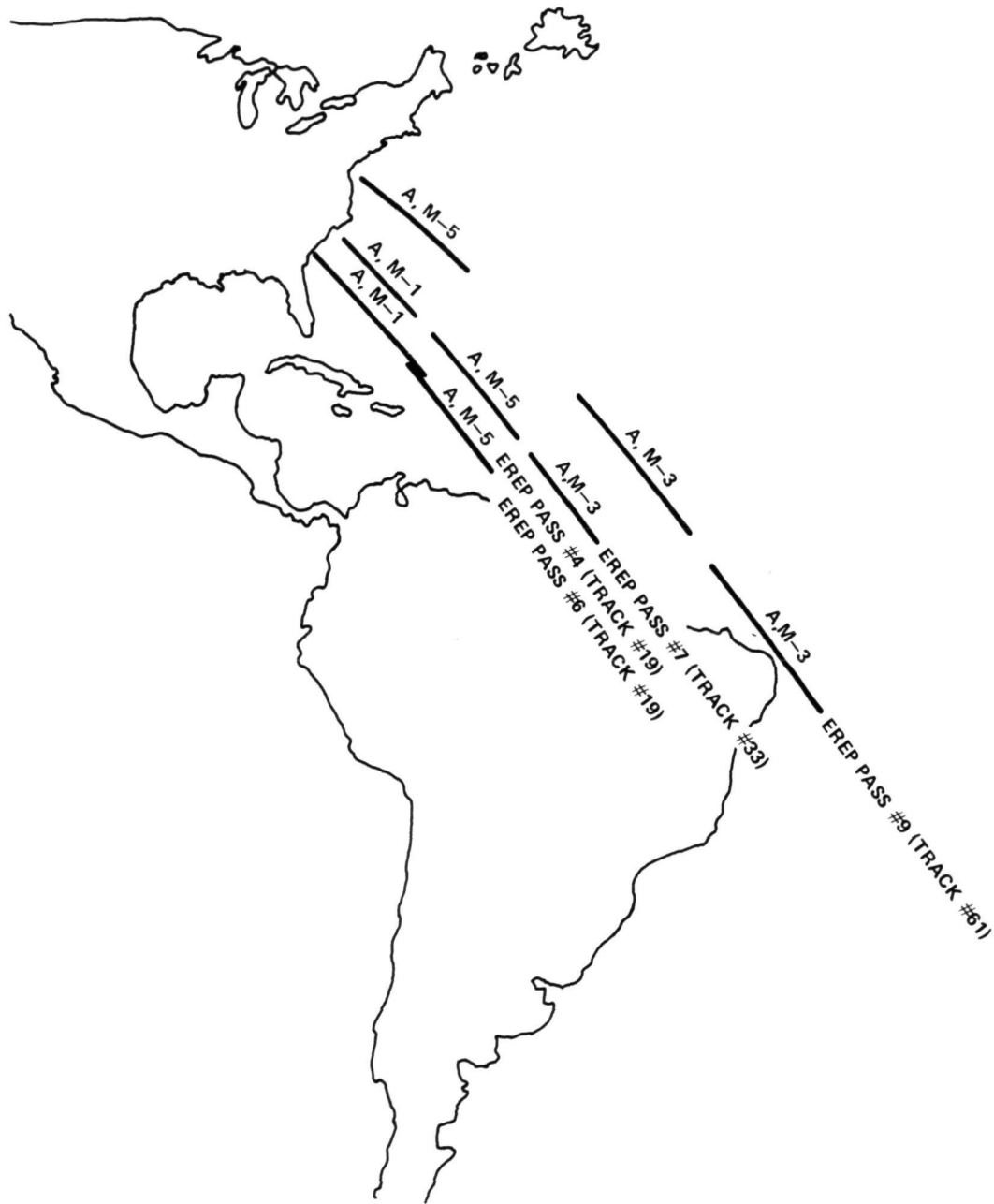(Skylab SL-2 EREP Pass 9 Data)

Figure 4. - S193 - Skylab SL-2 data tracks altimeter
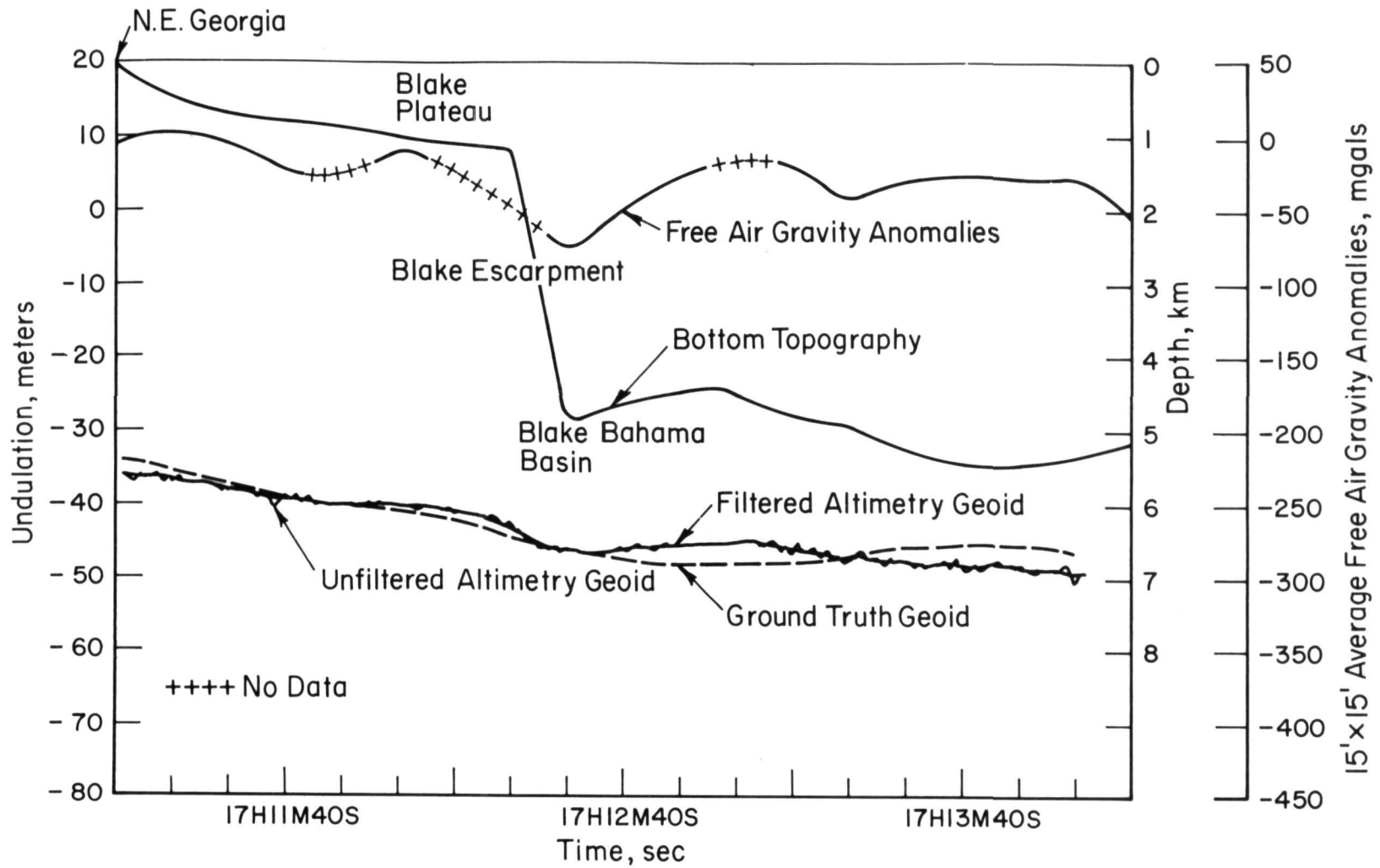modes 1, 3, and 5.

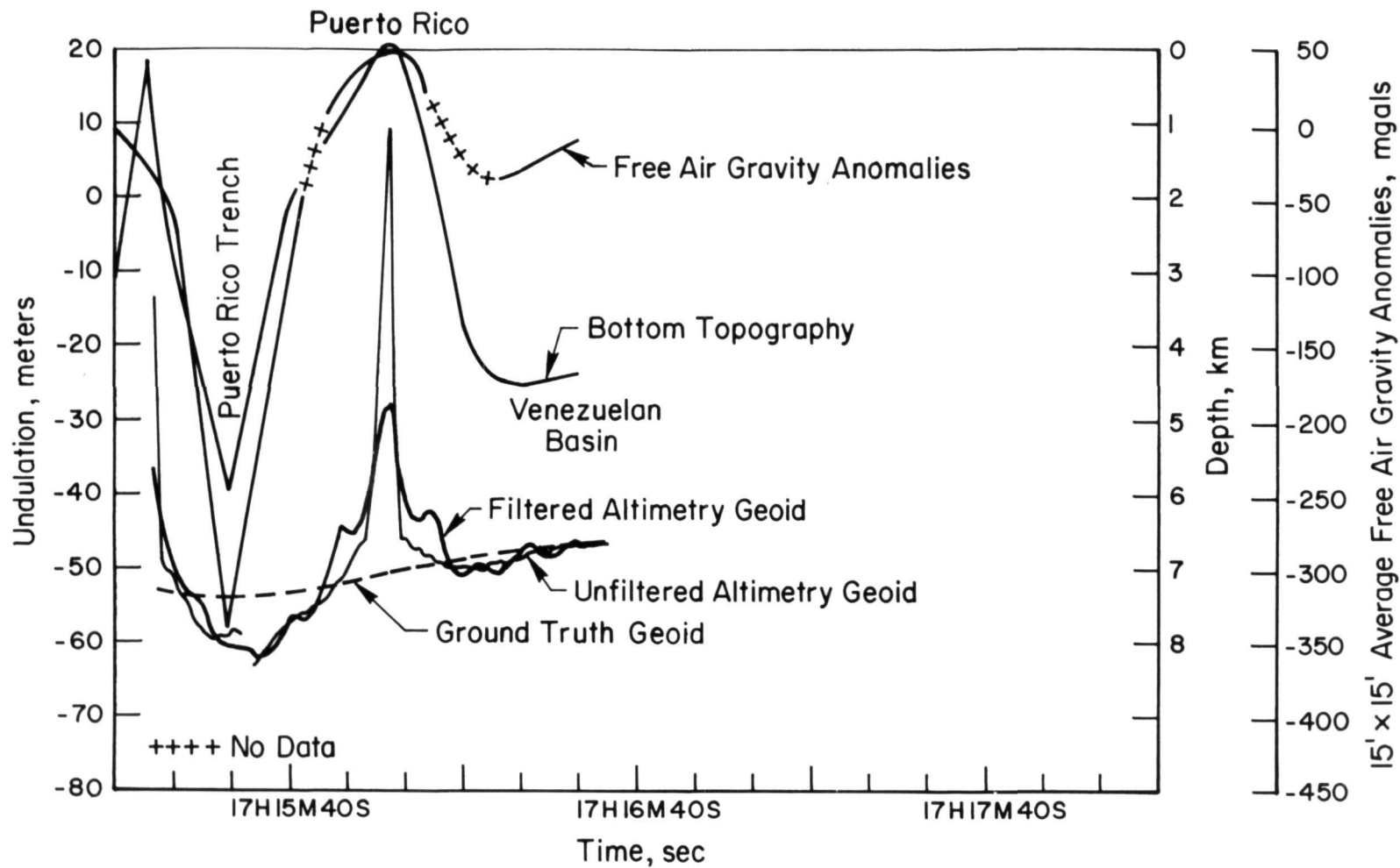Figure 5. - Geoid undulations computed from Skylab pass-4 altimetry data-mode 1.

Figure 6. - Geoid undulations computed from Skylab pass-4 altimetry data-mode 5.
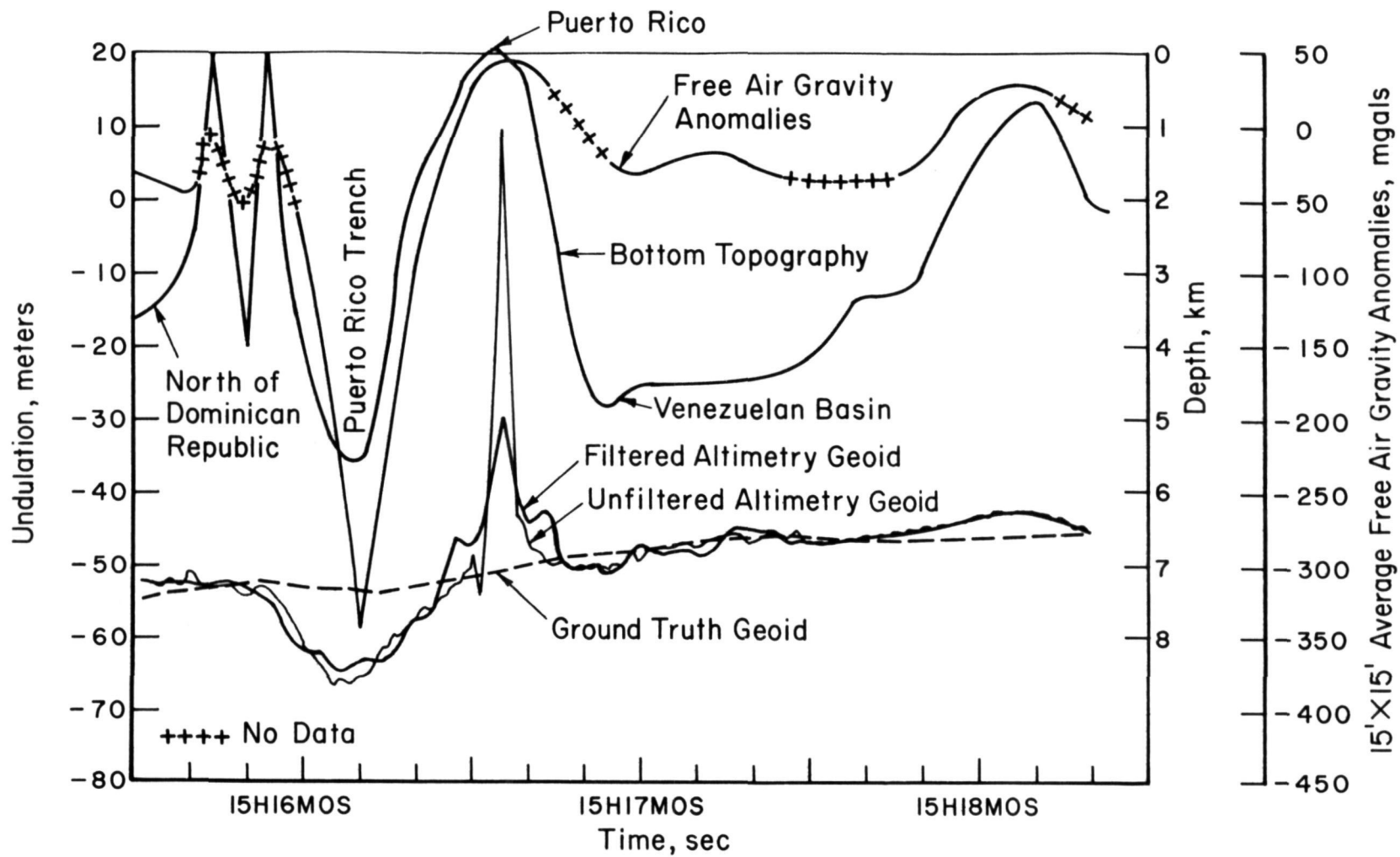
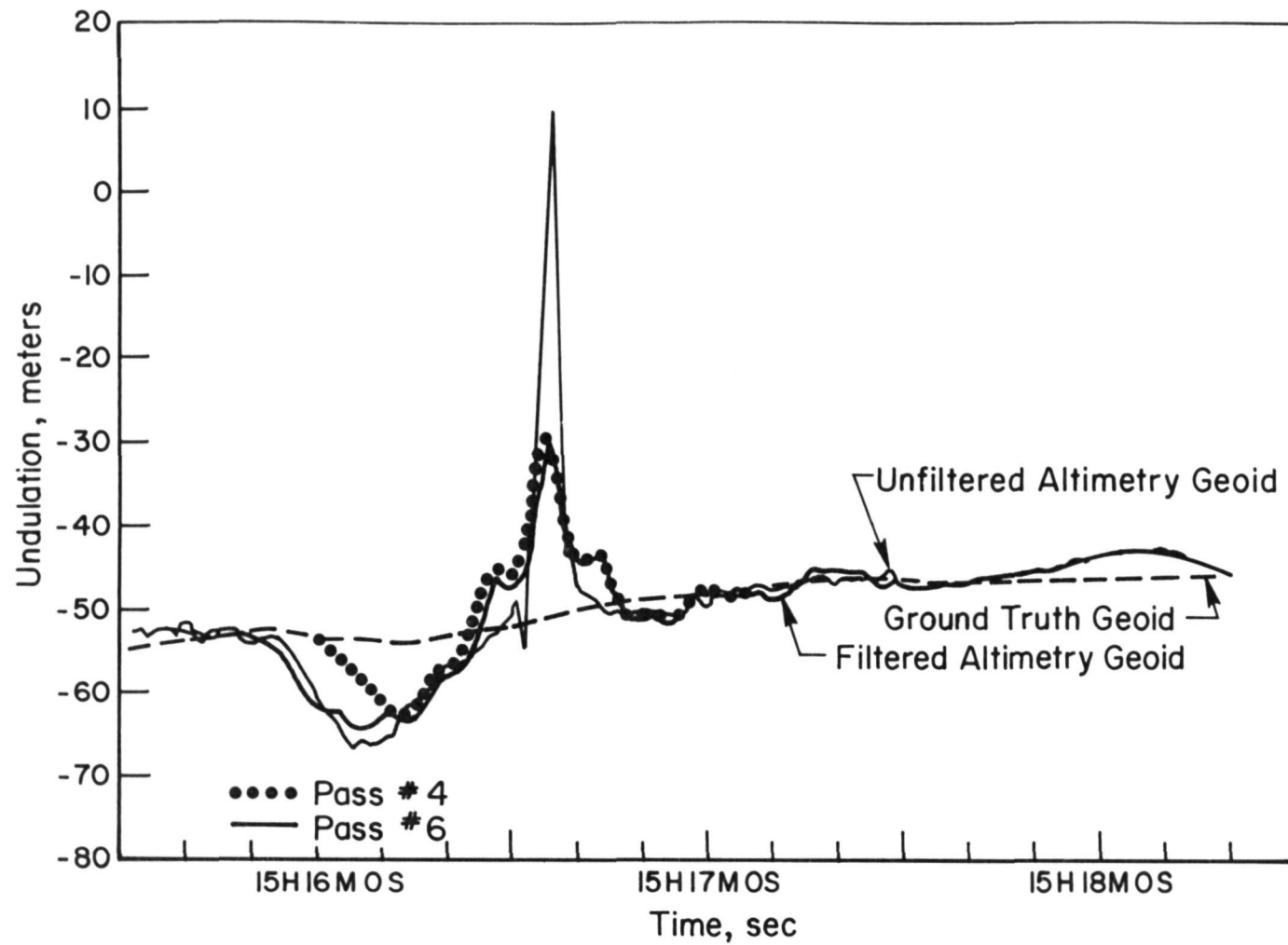Figure 7. - Geoid undulations computed from Skylab pass-6 altimetry data-mode 5.

Figure 8. - Comparison of the altimetry geoid profiles over the Puerto Rico trench and land mass as obtained in passes 4 and 6.

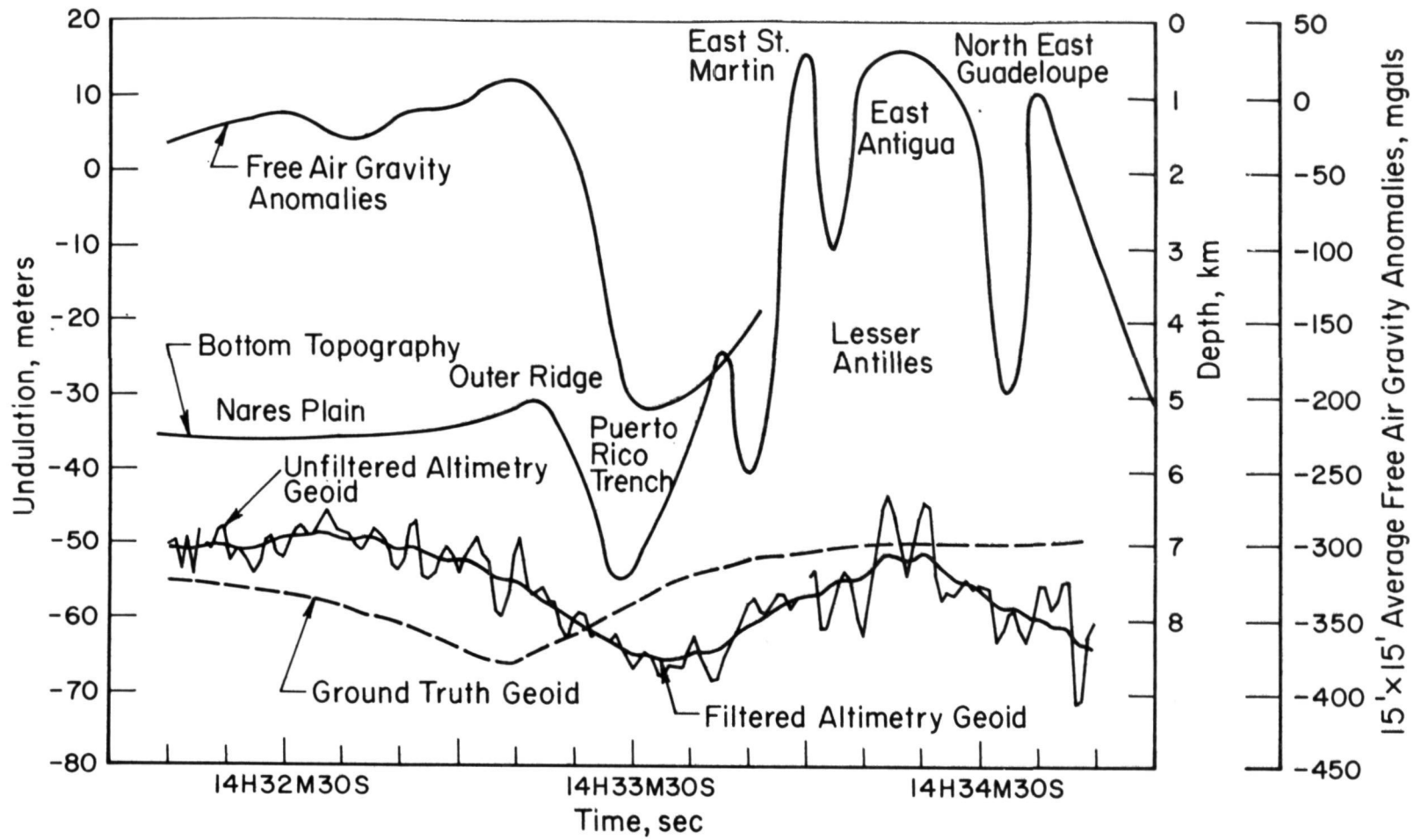Figure 9. - Geoid undulations computed from Skylab pass-7 altimetry data-mode 5.
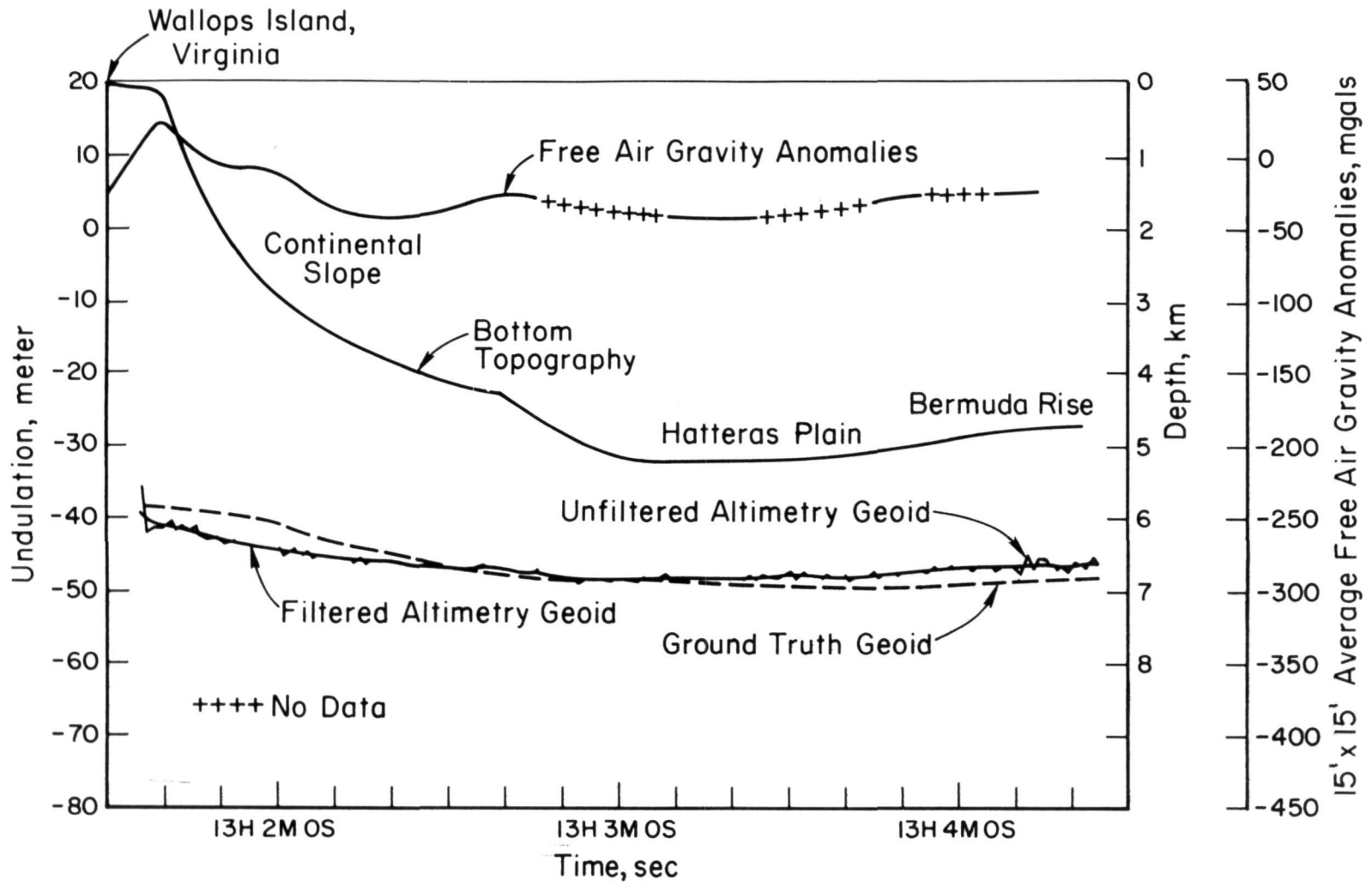
Figure 10. - Geoid undulations computed from Skylab pass-9
altimetry data-mode 5.