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COMAND – A FORTRAN PROGRAM FOR SIMPLIFIED COMPOSITE ANALYSIS AND DESIGN

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16. Abstract

A FORTRAN program is presented for preliminary analysis and design of multilayered composite panels subjected to inplane loads. All plys are of the same material. The composite is assumed symmetric about the midplane, but need not be balanced. Failure criterion include limit ply strains and lower bounds on composite inplane stiffnesses. Multiple load conditions are considered.

The required input data is defined and examples are provided to aid the user in making the program operational. Average panel design times are two seconds on an IBM 360/67 computer. Results are compared with published literature. A complete FORTRAN listing of program COMAND is provided. In addition, the optimization program NMIN is required for design.

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INTRODUCTION

Early evaluation of composite materials in aerospace structures requires an efficient means of structural sizing for a given application. It is seldom possible to provide simple stress limits as is customary when designing with conventional isotropic materials, since failure of composites is dependent not only on the properties and orientations of the individual plys, but on the nature of the loading as well. Furthermore, by taking advantage of the ply orthotropicity, the designer is free (within certain limits) to actually design the structural material through the proper choice of ply thicknesses and orientations.

COMAND is one of several programs being developed in the Advanced Vehicle Concepts Branch of Ames Research Center to provide a general and consistent approach to structural analysis and design. This program is for the analysis and design of a multilayered composite subject to inplane loads. The principal method of analysis and the failure criterion considered here are those used by Schmit and Farshi (Ref. 1). The optimization algorithm is the method of feasible directions using program CONMIN, which is described in Reference 2. COMAND is intended to provide first level design information for membrane structural behavior. Another program under development includes more general analysis, loading conditions, and failure criterion.*

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*Program COMPOS by J. Mullen, Advanced Vehicle Concepts Branch, Ames Research Center.

The analysis and design capabilities and the basic assumptions of the program are presented in Section I. Section II describes the required input to the program and several examples of the results are presented in Section III. Possible future efforts in composite analysis and design are identified in Section IV. The principle equations used in the analysis are presented in Appendix A. Appendix B is a complete program listing.

SECTION I

ASSUMPTIONS AND RESTRICTIONS

Program COMAND can be used to analyze a given composite panel in which the ply thicknesses are prescribed, or to design the ply thicknesses to satisfy strain and stiffness limitations. Ply orientation angles are prescribed, and are not design variables. Typical loading conditions and ply orientations are shown in Figures 1 and 2, respectively.

The composite analysis and design is based on the following assumptions and restrictions.

1. The panel is subjected to in-plane loads NX, NY and NYX only. Bending and out-of-plane shear loads are not considered. Multiple loading conditions are considered and up to 10 independent loading conditions are allowed.
2. The composite is said to fail when the longitudinal, transverse or shear strain in any single ply exceeds a specified limit in the longitudinal, transverse or shear direction, respectively.
3. The composite is said to fail if the stiffness in the structural X, Y or XY direction is less than a specified lower limit.

4. The individual ply thicknesses are designed to give minimum total panel thickness. Ply thicknesses are treated as continuous variables and several plys may be required to be of equal thickness.
5. All plys are of the same material with the same elastic properties and strain limitations. Ply elastic properties (and therefore, those of the composite) are assumed to be the same in tension and compression.
6. Ply properties are required as program input. Micromechanics analysis is not performed in the program.
7. The composite is assumed to be symmetric about the midplane so that no bending-membrane coupling exists.
8. The composite need not be balanced. That is, a ply with +45 degrees fiber orientation need not be balanced with another ply of -45 degrees orientation. Up to 18 different ply orientations are permitted, allowing for design of composites with ply angles at 10 degree intervals. Ply fiber orientation angles are prescribed and are not design variables.
9. Temperature effects and temperature loading are not considered, except that the material properties and strain limits must be consistent with the design temperature.

SECTION II

PROGRAM INPUT

All program input is listed here. The variables and their definitions are presented first, followed by data organization. No units are provided for the variables. It is required that all units be consistent. That is (for example), if loads are in newtons and thicknesses in meters, moduli must be given in newtons per square meter, strains in meters/meter and stiffness in newtons/meter.

Variables:

TITLE(15)	Anything may be given as a title.
NCALC	Calculation control. If NCALC=0, total composite thickness (weight) is minimized. If NCALC.NE.0. the given composite is analyzed only.
NPLY	Number of plys. Up to 18 plys are allowed.
NDV	Number of design variables. This is the number of ply thicknesses which are allowed to change independently in the optimization process or the number of different thicknesses prescribed for analysis. 1.LE.NDV.LE.NPLY
NLC	Number of loading conditions. Up to 5 loading conditions are allowed.
IPRINT	Print control for the optimization program, CONMIN. IPRINT = 0 gives no print during the optimization. IPRINT = 1 to IPRINT = 4 provide increasing degrees of output during optimization. IPRINT = 2 is usually desirable.
LNK(NPLY)	Design variable linking. LNK(I) gives variable number (ply thickness) associated with the ITH ply. For example, in a four ply problem (NPLY = 4), LNK^T = (1, 2, 2, 3) will impose the requirement that plys 2 and 3 are of the same thickness. In this case NDV = 3.

X(NDV)	Initial thickness of the design variables (IE. X^T = .05, .03, .04). If NCALC.NE.0., the composite is analyzed for ply thicknesses defined in X and linked according to LNK. If J = LNK(I), the thickness of the ITH ply is stored in X(J).
VLB(NDV)	Lower bounds on the design variables. VLB(I).GE.0, I = 1,NDV. It is usually desirable to set at least one VLB(I) = 1.0E-10 if lower bounds of zero are desired, in order to prevent the optimization program from attempting to analyze a panel of zero thickness. If NCALC.NE.0. VLB(I) = 0, I = 1, NDV may be input.
THN(NPLY)	Ply orientations in degrees, referenced to the structural X-axis. THN(I) = Ply orientation of the ITH ply.
EL	Ply longitudinal modulus.
ET	Ply transverse modulus.
GLT	Ply shear modulus.
PRLT	Ply major Poisson's ratio (ply transverse Poisson's ratio, PRTL, is calculated internally).
EPLC	Ply longitudinal compressive strain limit (negative number).
EPLT	Ply longitudinal tensile strain limit (positive number).
EPTT	Ply transverse tensile strain limit (positive number).
GMLT	Ply maximum shear strain limit (positive number).
A11L	Lower bound on composite stiffness in the structural X-DIRECTION.
A22L	Lower bound on composite stiffness in the structural Y-DIRECTION.
A66L	Lower bound on composite shear stiffness.
PN(3, NLC)	Loads, column I corresponds to loading condition I, I = 1, NLC. Row J corresponds to load NX, NY and NXY for J = 1, 2 and 3, respectively of load condition I.

Data Organization:

No. of Cards	Information	Format
1	Title - Anything may be given here	15A4
1	NCALC, NPLY, NDV, NLC, IPRINT	515
1	LNK(I),I=1,NPLY	1515
1-3	X(I),I=1,NDV	8F10.2
1-3	VLB(I),I=1,NDV (Blank card(s) if NCALC.NE.0)	8F10.2
1-3	THN(I),I=1,NPLY	8F10.2
1	EL, ET, GLT, PRLT	4F10.2
1	EPLC, EPLT, EPTC, EPTT, GMLT, A11L, A22L, A66L	8F10.2
NLC	PN(J,I),J=1,3 (One card per loading condition)	3F10.2
	Begin with next set of data. Program terminates if 2 blank cards are read here.	

This information is duplicated in Table 1, along with a data form for convenient reference.

SECTION III

EXAMPLES

Several examples are presented here to aid the user in making the program operational and to provide some insight into design using composite materials. All examples are for a high strength graphite-epoxy composite.

Typical ply unidirectional properties are listed in Table 2 for a fiber volume fraction of 0.6. The table is reproduced directly from Reference 3. Note that the ultimate strain limits are not specified for longitudinal and transverse strain or for shear. However, reasonable values are readily obtained by analyzing a single ply of unit thickness, subject to a set of loads which are equal to the ultimate stresses. For example, given a longitudinal load of 180,000 lb/in. the resulting longitudinal strain will be ultimate strain. Therefore, a single ply composite is analyzed for the following load conditions:

Load Condition	NX	NY	NXY
1	180000.	0.	0.
3	0.	-30000.	0.
2	0.	8000.	0.
4	0.	0.	12000.

Note that a negative NX load is not imposed because the ultimate longitudinal compressive stress is the same in magnitude as the tensile stress. Therefore, the ultimate strains are also equal in magnitude (but opposite in sign).

The program input variables are now:

TITLE:	Determination of strains - G/E composite.
NCALC = 1	Analysis
NPLY = 1	One ply.
NDV = 1	One thickness.
NLC = 4	Four load conditions.
IPRINT = 0	Not used for analysis.
LNK(1) = 1	Ply thickness = X(1).
X(1) = 1.0	Composite thickness.
VLB(1) = 0.	Not used for analysis.
THN(1) = 0.	Zero degree ply orientation.

EL = 21,000,000 Longitudinal modulus.

ET = 1,700,000 Transverse modulus

GLT = 650,000 Shear modulus.

PRLT = 0.21 Major Poisson's ratio.

EPLT = EPLC = EPTT = EPTC = GMLT = 0 - Strain limits set to zero since they are not known.

A11L = A22L = A66L = 0 Not meaningful here

PN(I,J) - Loads, given above.

The input data is listed in Table 3 with the corresponding output in Figure 3. The ultimate strains are now the actual ply strains in the direction of the applied load for the corresponding loading condition. For example, since load condition 1 is the ultimate longitudinal stress, the longitudinal strain, EPL, under this load condition is also ultimate. That is:

EPLT = 0.00857 (table 2 gives 0.00870)

Similarly,

EPLC = -0.00857

EPTC = -0.0176

EPTT = 0.00471 (table 2 gives 0.00475)

GMLT = 0.0185

These are now the limit strains to be used in design.

Example 1 - Quasi-isotropic composite

In order to draw a comparison between graphite epoxy composites and the familiar aluminum materials, a simple case is first considered in which plys are oriented at 15 degree intervals (NPLY = 12) and subject to a single unidirectional load, NX = 20,000 lb/in. (NY=NXY=0). All plys are required to be of the same thickness so that NDV=1 and LNK(I)=1, I,NPLY. The total thickness is minimized. No minimum stiffness limits are imposed, so that A11L=A22L=A66L=0. Lower bounds on the thicknesses are arbitrarily set to 0.00001 in. Initial ply thickness is prescribed as 0.05 in. The input data is listed in Table 4, where the print control for the optimization program, CONMIN, is taken as IPRINT = 2. The program output is listed in Figure 4. The optimum composite thickness is 0.525 inches. The design is constrained by the transverse strain limit in the 90 degree direction (ply number 12). The average stress in the structural X-direction (direction of load) in the composite is 38,000 PSI. Note that this is significantly less than the ultimate stress of 60,000 PSI for a typical aluminum alloy. However, the density of the composite is 0.056 lb/in.**3 as compared to 0.101 lb/in.**3 for aluminum. Therefore, the relative weight of graphite epoxy as compared to aluminum for this example is 0.056*60000/(0.101*38000 = 0.875 giving a 12.5 percent weight savings.

Note that even though the 90 degree ply has failed, some additional load may be carried before all plys fail. Therefore, the failure stress predicted here may be considered analogous to the limit stress, with the ultimate stress being (usually) somewhat higher.

Example 2 - (0. +45, 90) composite design

Due to practical considerations, it is improbable that many different ply orientations will be used in most structures. In this example, the composite is required to be balanced so that the thicknesses of the +45 and -45 degree plys are the same. Then there are three independent design variables (NDV = 3) and the ply thickness linking vector becomes LNK^T = (1, 2, 2, 3). The ply orientation vector is THN^T = (0., 45., -45., 90.). A minimum stiffness of 500,000. lb/in. is required in the structural X-direction. The composite is required to support the following four independent loading conditions:

Load Condition	NX	NY	NXY
1	20000.	0.	0.
2	15000.	-15000.	5000.
3	-15000.	10000.	10000.
4	0.	0.	20000.

The input data is listed in Table 5 and the corresponding output in Fig. 5. The print control for CONMIN is set to IPRINT = 0 in this example and in example 3 for brevity. The optimum composite thickness is 0.578 inches. The active constraints are transverse strain limits and are identified by safety factors of unity in Fig. 5 (3 constraints are active).

Example 3 - (0. ±30, ±60, 90) composite design.

This composite is designed subject to the same constraints and loading conditions as example 2. the only difference is the number of plys and their orientations. The composite is again required to be balanced. In this case, NDV = 4, NPLY = 6, LNK^T = (1, 2, 2, 3, 3, 4), and THN^T = 0., 30., -30., 60., -60., 90.). The input data and output are listed in Table 6 and Fig. 6, respectively. The optimum composite thickness is 0.532 inches and there are six active strain limit constraints as seen from Fig. 6. Note that although the number of plys and their orientations are different from example 2, the total composite thickness is reduced by less than ten percent.

An additional exercise of interest is to eliminate plys which comprise a small percentage of the total thickness, and solve the optimization problem again. For example, a composite made up of ±30 and ±60 degree ply orientations results in an optimum thickness of 0.526 inches. It is instructive to design the 12 ply composite of example 1 subject to this same set of loads, but allowing for different ply thicknesses (require that the composite be balanced for consistency with examples 2 and 3). The resulting thickness is 0.588 inches. Solution of this case is left as an exercise.

Example 4 - Limit stress vs. ply thickness distribution

In order to assess the applicability of this program to preliminary composite design, results obtained using COMAND are compared here with design curves for a (0, ±45, 90) composite subjected to uniaxial tension, compression and shear loading (applied separately). Figures 7-10 are reproduced from Reference 3. A composite with various relative ply thicknesses was analyzed under these separate loading conditions. No stiffness constraints were imposed and the lowest factor of safety was found for all strain failure criterion. The calculated stress was then multiplied by this factor to give the failure (limit) stress. The results are plotted on Figures 7-10 for 25 and 50 percent zero degree plys. Figure 10 compares the extensional modulus, E_x.

The results indicate reasonable comparison for compressive stress, shear stress and extensional modulus. However, considerable discrepancy is found in comparing tensile stress limits. This is because the composite is constrained by transverse strain limits on the 90 degree plys. In Reference 3, one or more plys are allowed to fail without assuming composite failure.

When a single ply fails, this ply is assumed to carry no load. The composite is said to fail only when all plys fail individually. This again demonstrates the difference between the limit stress calculated here and the ultimate stress presented in Reference 3. The difference in results between these two assumptions is usually reduced when multiple sets of combined loadings (practical design situations) are considered.

SECTION IV

DISCUSSION

A short program has been presented by which first estimates are readily obtained for design requirements of composite structures. The program is easily used and requires minimal execution time. Because the failure criterion are extremely load dependent, some judgement is necessary in choosing permissible ply orientations, so that the existence of a given ply orientation does not prevent attainment of an optimum design. This problem is much less prevalent under multiple loading conditions. However, it does suggest that development of an optimization algorithm capable of completely eliminating plys may be fruitful.

For the results to be meaningful, it is important that this program be applied only to structures satisfying (at least approximately) the restrictions imposed in Section I. Of particular importance are the restrictions of inplane loading and composite symmetry about the midplane.

Recognizing the complexities of composite analysis and design as well as the benefits to be gained through the use of these materials, future development work in this area appears warranted.**

**Several of the topics identified here are currently being included in the COMPOS program by J. Mullen at ARC.

These efforts should include more complex loading such as bending, out of plane shear, and temperature loads on nonsymmetric composites. This necessarily requires the inclusion of more sophisiticated analysis techniques and failure criterion. Panel buckling under various force and displacement boundary conditions is also an area of interest because, with increased composite strengths, stiffness requirements become increasingly important, since the probability of failure in this mode is increased with reduced plate thicknesses. Additionally, analysis and design of composites made up of plys of differing elastic properties is a needed and straight forward extension. This will provide the capability of selective reinforcement of conventional isotropic materials as well as use of various combinations of advanced materials. Finally, these capabilities should be incorporated into a general finite element analysis and design program for application to large scale structures of practical interest.

APPENDIX A

COMPOSITE ANALYSIS AND DESIGN EQUATIONS

Analysis Equations

The equations used for analysis and design are presented here. These equations are consistant with the assumptions listed in Section I. Equation numbers beginning with the letter A are consistant with Reference 1.

The analysis is based on the ply materials properties E_L, E_T, G_{LT}, ν_{LT} and ν_{TL}, ply thicknesses, t_i, and orientations, θ_i.

The force deformation equations for the kth load condition are;

$$\{N\}_k = [A]\ \{\varepsilon\}_k \qquad [A1]$$

where

$$\{N\}_k = \begin{Bmatrix} N_{xk} \\ N_{yk} \\ N_{xyk} \end{Bmatrix} \qquad \{\varepsilon\}_k = \begin{Bmatrix} \varepsilon_{xk} \\ \varepsilon_{yk} \\ \gamma_{xyk} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_k}{\partial x} \\ \dfrac{\partial v_k}{\partial y} \\ \dfrac{\partial v_k}{\partial x} + \dfrac{\partial u_k}{\partial y} \end{Bmatrix}$$

$\{N\}_k$ is the vector of applied in-plane loads referenced to the structural x-axis and $\{\varepsilon\}_k$ is the corresponding strain state. u and v are the displacements in the coordinate x and y directions, respectively.

$$A_{rs} = \sum_{i=1}^{NPLY} (C'_{rs})t_i \qquad r,s = 1,2,6 \qquad [A2]$$

where t_i is the thickness of the plys oriented at angle θ_i with respect to the structural x-axis. Coefficients $(C'_{rs})_i$ are defined in terms of θ_i and

and the ply elastic constants as

$$(c'_{11})_i = (c_{11})_i \ \ell_i^4 + 2(c_{12})_i \ \ell_i^2 \ m_i^2 + (c_{22})_i \ m_i^4 + 4(c_{66})_i \ m_i^2 \ \ell_i^2 \qquad [A3]$$

$$(c'_{12})_i = (c_{11})_i \ \ell_i^2 \ m_i^2 + (c_{12})_i \ (\ell_i^4 + m_i^4) + (c_{22})_i \ \ell_i^2 \ m_i^2 - 4(c_{66})_i \ \ell_i^2 \ m_i^2 \qquad [A4]$$

$$(c'_{16})_i = (c_{11})_i \ \ell_i^3 \ m_i + (c_{12})_i \ (m_i^3 \ \ell_i - \ell_i^3 \ m_i) - (c_{22})_i \ m_i^3 \ \ell_i + 2(c_{66})_i \ (m_i^3 \ \ell_i - m_i \ \ell_i^3) \qquad [A5]$$

$$(c'_{22})_i = (c_{11})_i \ m_i^4 + 2(c_{12})_i \ m_i^2 \ \ell_i^2 + (c_{22})_i \ \ell_i^4 + 4(c_{66})_i \ m_i^2 \ \ell_i^2 \qquad [A6]$$

$$(c'_{26})_i = (c_{11})_i \ m_i^3 \ \ell_i + (c_{12})_i \ (\ell_i^3 \ m_i - m_i^3 \ \ell_i) - (C_{22})_i \ m_i \ \ell_i^3 + 2(c_{66})_i \ (m_i \ \ell_i^3 - m_i^3 \ \ell_i) \qquad [A7]$$

$$(c'_{66})_i = (c_{11})_i \ m_i^2 \ \ell_i^2 - 2(c_{12})_i \ m_i^2 \ \ell_i^2 + (c_{22})_i \ m_i^2 \ \ell_i^2 + (c_{66})_i \ (\ell_i^2 - m_i^2)^2 \qquad [A8]$$

where

$$\ell_i = \cos\theta_i \qquad m_i = \sin\theta_i \qquad [A9]$$

$$(c_{11})_i = \frac{E_{Li}}{(1-\nu_{LTi}\nu_{TLi})} \qquad [A10]$$

$$(c_{12})_i = \frac{\nu_{TLi}\, E_{Li}}{(1-\nu_{LTi}\nu_{TLi})} = \frac{\nu_{LTi}\, E_{Ti}}{(1-\nu_{LTi}\nu_{TLi})} \qquad [A11]$$

$$(c_{22})_i = \frac{E_{Ti}}{(1-\nu_{LTi}\nu_{TLi})} \qquad [A12]$$

$$(c_{66})_i = G_{LTi} \qquad [A13]$$

Note that the subscript i is not required on equations [A10]-[A13] since the elastic properties are assumed the same for all plys. The subscript is retained here for consistency.

Given the loads $\{N\}_k$, the membrane strains are obtained from equation [A1] as

$$\{\varepsilon\}_k = [A]^{-1}\,\{N\}_k$$

Finally the strains in the ith ply (kth load condition) are determined from

$$\begin{aligned}
\varepsilon_{1ik} &= \ell_i^2\,\varepsilon_{xk} + m_i^2\,\varepsilon_{yk} + m_i\ell_i\,\gamma_{xyk} \\
\varepsilon_{2ik} &= m_i^2\,\varepsilon_{xk} + \ell_i^2\,\varepsilon_{yk} - m_i\ell_i\,\gamma_{xyk} \qquad [A14] \\
\gamma_{12ik} &= -2m_i\ell_i\varepsilon_{xk} + 2m_i\ell_i\varepsilon_{yk} + (\ell_i^2 - m_i^2)\,\gamma_{xyk}
\end{aligned}$$

If the stresses in the ith ply are required, these may be obtained from the orthotropic elastic stress-strain relationships to be

$$\begin{aligned} \sigma_{1ik} &= (c_{11})_i\, \varepsilon_{1ik} + (c_{12})_i\, \varepsilon_{2ik} \\ \sigma_{2ik} &= (c_{12})_i\, \varepsilon_{1ik} + (c_{22})_i\, \varepsilon_{2ik} \\ \tau_{12ik} &= (c_{66})_i\, \gamma_{12ik} \end{aligned} \qquad [15]$$

Design Equations

The design objective is to minimize the total composite thickness (and therefore weight);

$$\text{Minimize} \qquad W = \sum_{i=1}^{NPLY} t_i$$

Constraints on the design include limit ply strains and lower bounds on stiffness.

The limit strains imposed on the individual plys are expressed as constraint functions as follows:

$$G_{1ik} = \frac{\varepsilon_{1ik}}{EPLC} - 1. \leq 0 \qquad i = 1, NPLY, k = 1, NLC$$

$$G_{2ik} = \frac{\varepsilon_{1ik}}{EPLT} - 1. \leq 0 \qquad i = 1, NPLY, k = 1, NLC$$

$$G_{3ik} = \frac{\varepsilon_{2ik}}{EPTC} - 1. \leq 0 \qquad i = 1, NPLY, k = 1, NLC$$

$$G_{4ik} = \frac{\varepsilon_{2ik}}{EPTT} - 1. \leq 0 \qquad i = 1, NPLY, k = 1, NLC$$

$$G_{5ik} = \frac{|\gamma_{12ik}|}{GMLT} - 1. \leq 0 \qquad i = 1, NPLY, k = 1, NLC$$

where subscript i denotes ply number and subscript k denotes load condition.

Lower bounds on stiffness are expressed as constraint functions;

$$\bar{G}_1 = 1. - A(1,1)/A11L \leq 0.$$

$$\bar{G}_2 = 1. - A(2,2)/A22L \leq 0.$$

$$\bar{G}_3 = 1. - A(3,3)/A66L \leq 0.$$

Constraints on strains are nonlinear functions of the design variables, t_i. The values of these constraints are stored in vector G, (five values per ply, one ply after another) for each load condition in sequence. Constraints $\bar{G}_1$, $\bar{G}_2$ and $\bar{G}_3$ on stiffness are linear functions of the design variables. The values of these constraints are stored after constraints on strains in vector G.

There are 5*NPLY*NLC nonlinear constraints and three linear constraints on the optimization problem. Program "CONMIN" defines a nonlinear constraint as "active" if its value is greater than or equal to a specified value CT (a small negative number). Linear constraints are "active" if their value equals or exceeds a value of CTL. If a given constraint is active the analytic gradient of this constraint with respect to the independent design variables, t_i, must be supplied. This information is obtained by direct differentiation of the constraint functions and is readily calculated using the equations of analysis.

APPENDIX B

PROGRAM LISTING

A complete FORTRAN listing of program "COMAND" is given here. In addition, program "CONMIN" is required and this program is described in reference 2. The general program organization is shown in block diagram form in figure 11.

```
C     PROGRAM COMAND - A FORTRAN PROGRAM FOR COMPOSITE ANALYSIS
C     AND DESIGN.
      COMMON X(20),DF(20),G(500),ISC(500),IC(20),A(20,20),S(20),G1(500),
     *G2(500),C(20),MS1(40),B(20,20),VLB(20),VUB(20),SCAL(20)
      COMMON /CNMN1/ IPRINT,NDV,ITMAX,NCON,NSIDE,ICNDIR,NSCAL,NFDG,FDCH,
     1FDCHM,CT,CTMIN,CTL,CTLMIN,THETA,PHI,NAC,DELFUN,DABFUN,LINOB
     2J,ITRM,ITER,INFOG
      COMMON /COMPOS/ NPLY,EL,ET,GLT,PRLT,PRTL,EPLC,EPLT,EPTC,EPTT,GMLT,
     1NLC,A11L,A22L,A66L,TN(18),THN(18),CP(3,3,18),PN(3,5),AA(3,3),BB(3
     2,3),EP(3),DEP(3),LNK(18)
      DIMENSION TITLE(15)
      EXTERNAL COMP3
C     PROGRAM FOR MULTILAYERED COMPOSITE PANEL OPTIMIZATION.
C     BY G. N. VANDERPLAATS                          SEPT., 1973.
C     NASA-AMES RESEARCH CENTER, MOFFETT FIELD, CALIF.
C     REQUIRED DIMENSIONS:  TN(NPLY),THN(NPLY),CP(3,3,NPLY),PN(3,NLC),
C                           LNK(NPLY).  OTHERS REMAIN AS NOW DIMENSIONED
C     STORAGE REQUIREMENTS (DECIMAL WORDS, CDC):
C     PROGRAM -
C         COMAND = 2000
C         CONMIN = 6000
C     ARRAYS = 3000 FOR PRGRAM AS DIMENSIONED IN REF. 1.
C     REF. 1 - VANDERPLAATS, G. N., SCOMAND - A FORTRAN PROGRAM FOR
C         SIMPLIFIED COMPOSITE ANALYSIS AND DESIGNS, NASA TM X-**,***,
C         AUG. 1975.
C     REF. 2 - SCHMIT, L. A., AND FARSHI, B.:  OPTIMUM LAMINATE DESIGN
C         FOR STRENGTH AND STIFFNESS.  INT. J. FOR NUMERICAL METHODS
C         IN ENGINEERING, VOL. 7, NO. 4, PP. 519-536, 1973.
C     EQUATION NUMBERS LISTED IN THIS PROGRAM ARE FROM THE ABOVE
C     REFERENCE.
C     REF. 3 - CONMIN - A FORTRAN PROGRAM FOR CONSTRAINED FUNCTION
C         MINIMIZATION:  USERS MANUAL, BY G. N. VANDERPLAATS,
C         NASA TM X-62,282, AUGUST, 1973.
C     THIS PROGRAM USES CONMIN VERSION II, DATED JULY, 1975.
C     ASSUMPTIONS:
C       BOUNDARY CONDITIONS ARE PRESCRIBED LOADS NX, NY AND NXY.
C       ALL PLYS HAVE SAME MATERIAL PROPERTIES AND FAILURE STRAINS.
C       FAILURE CRITERION ARE MAX PLY LONGITUDINAL, TRANSVERSE AND SHEAR
C         STRAINS, AND STIFFNESS LIMITS ON A11, A22 AND A66.
C       WHEN ANY ONE PLY FAILS, THIS IS DEFINED AS COMPOSITE FAILURE.
C       MEMBRANE LOADS ONLY - MULTIPLE LOADING CONDITIONS ARE
C       CONSIDERED.
C       SYMMETRY ABOUT MIDPLANE IS ASSUMED.
C       COMPOSITE NEED NOT BE BALANCED.
C
C     NCALC = CALCULATION CONTROL
C         0: DO OPTIMIZATION
C      NE.0: DO ANALYSIS ONLY
C     NPLY = NUMBER OF PLYS.  SYMMETRY ASSUMED.
C           PROGRAM TERMINATED IF NPLY=0.
```

```
C     NLC = NUMBER OF LOAD CONDITIONS.
C     NDV = NUMBER OF DESIGN VARIABLES OR INDEPENDENT PLY THICKNESSES
C           FOR ANALYSIS.
C     IPRINT = PRINT CONTROL FOR CONMIN.
10    READ (5,140) TITLE
      READ (5,200) NCALC,NPLY,NDV,NLC,IPRINT
      IF (NPLY.EQ.0) STOP
C     DESIGN VARIABLE LINKING.  LNK(I) = DESIGN VARIABLE
C     ASSOCIATED WITH ITH PLY.
      READ (5,200) (LNK(I),I=1,NPLY)
C     INITIAL THICKNESS.
      READ (5,210) (X(I),I=1,NDV)
C     TOTAL INITIAL COMPOSITE THICKNESS.
      OBJ=0.
      DO 20 I=1,NPLY
      J=LNK(I)
      OBJ=OBJ+X(J)
20    TN(I)=X(J)
C     LOWER BOUNDS ON DESIGN VARIABLES.
      READ (5,210) (VLB(I),I=1,NDV)
C     PLY ORIENTATION IN DEGREES.
      READ (5,210) (THN(I),I=1,NPLY)
C     PLY MATERIAL PROPERTIES.
      READ (5,210) EL,ET,GLT,PRLT
      PRTL=PRLT*ET/EL
C     STRAIN AND STIFFNESS LIMITS.
      READ (5,210) EPLC,EPLT,EPTC,EPTT,GMLT,A11L,A22L,A66L
C     LOADS FOR EACH LOAD CONDITION.
      DO 30 I=1,NLC
C     LOADS - NX, NY AND NXY FOR THIS LOAD CONDITION.
30    READ (5,210) (PN(J,I),J=1,3)
C     NCON = NUMBER OF CONSTRAINTS.
      NCON=5*NPLY*NLC
      DO 40 I=1,NCON
40    ISC(I)=0
      N1=NCON+1
      ISC(N1)=1
      ISC(N1+1)=1
      ISC(N1+2)=1
      IF (A11L.GT.1.0E-10) NCON=NCON+1
      IF (A22L.GT.1.0E-10) NCON=NCON+1
      IF (A66L.GT.1.0E-10) NCON=NCON+1
C     PRINT INPUT INFORMATION.
      IF(NCALC.EQ.0) WRITE(6,460)
      IF(NCALC.NE.0) WRITE(6,470)
      WRITE (6,220)
      WRITE (6,150) TITLE
      WRITE (6,230) NPLY,NLC
      WRITE (6,240) EL,ET,GLT,PRLT,PRTL
      WRITE (6,250)
```

```
      DO 50 I=1,NPLY
50    WRITE (6,270) I,TN(I),THN(I),LNK(I)
      GMLT1=-GMLT
      WRITE (6,280) EPLC,EPLT,EPTC,EPTT,GMLT1,GMLT
      WRITE (6,290) A11L,A22L,A66L
      WRITE (6,300)
      DO 60 I=1,NLC
60    WRITE (6,310) I,(PN(J,I),J=1,3)
C     INITIALIZE CONMIN PARAMETERS TO DEFAULT VALUES.
      ITMAX=30
      NSIDE=1
      ICNDIR=0
      NSCAL=0
      NFDG=0
      FDCH=0.
      FDCHM=0.
      CT=0.
      CTMIN=0.
      CTL=0.
      CTLMIN=0.
      THETA=0.
      PHI=0.
      DELFUN=0.
      DABFUN=0.
      LINOBJ=1
      ITRM=0
C     CONVERT PLY ANGLES TO RADIANS.
      DO 70 I=1,NPLY
      THN(I)=THN(I)/57.295776
C     UPPER BOUNDS ON DESIGN VARIABLES ARBITRARILY SET = 100.
70    VUB(I)=100.
C     PLY STIFFNESS COEFICIENTS.
      CALL COMP1 (NPLY,THN,EL,ET,GLT,PRLT,PRTL,CP)
      NN1=20
      NN2=500
      NN3=20
      NN4=20
      NN5=40
      IF (NCALC.EQ.0) CALL CONMIN (COMP3,OBJ,X,DF,G,ISC,IC,A,S,G1,G2,C,M
     *S1,B,VLB,VUB,SCAL,NN1,NN2,NN3,NN4,NN5)
C     PRINT ANALYSIS RESULTS.
      WRITE (6,320)
      WRITE (6,150) TITLE
      WRITE (6,330)
      PP=100./OBJ
C     PLY THICKNESSES AND PERCENT OF TOTAL THICKNESS.
      DO 80 I=1,NPLY
      J=LNK(I)
      TN(I)=X(J)
      PCT=PP*TN(I)
```

```
80    WRITE (6,260) I,TN(I),PCT
C     TOTAL THICKNESS.
      WRITE (6,340) OBJ
C     STIFFNESSES AND FLEXABILITIES.
      CALL COMP2 (NPLY,TN,CP,AA,BB)
C    PLY STRAINS AND COMPOSITE STRESSES FOR ALL LOAD CONDITIONS.
      WRITE (6,350)
      DO 100 II=1,NLC
      WRITE (6,360) II
C     COMPOSITE STRAINS.
      EP(1)=BB(1,1)*PN(1,II)+BB(1,2)*PN(2,II)+BB(1,3)*PN(3,II)
      EP(2)=BB(2,1)*PN(1,II)+BB(2,2)*PN(2,II)+BB(2,3)*PN(3,II)
      EP(3)=BB(3,1)*PN(1,II)+BB(3,2)*PN(2,II)+BB(3,3)*PN(3,II)
C     PLY STRAINS AND SEFETY FACTORS.
      DO 90 J=1,NPLY
      THETA=THN(J)
      AL=COS(THETA)
      AM=SIN(THETA)
      AL2=AL*AL
      AM2=AM*AM
C     STRAINS.
      EP1=AL2*EP(1)+AM2*EP(2)+AL*AM*EP(3)
      EP2=AM2*EP(1)+AL2*EP(2)-AL*AM*EP(3)
      EP3=2.*AL*AM*(EP(2)-EP(1))+(AL2-AM2)*EP(3)
C     SET STRAINS TO MINIMUM ABSOLUTE VALUE OF 1.0E-20 TO PREVENT
C     DIVIDE BY ZERO.
      IF (ABS(EP1).LT.1.0E-20) EP1=1.0E-20
      IF (ABS(EP2).LT.1.0E-20) EP2=1.0E-20
      IF (ABS(EP3).LT.1.0E-20) EP3=1.0E-20
C     SAFETY FACTOR.
      SF1=EPLC/EP1
      SF2=EPTC/EP2
      SF3=-GMLT/EP3
      IF (EP1.GT.0.) SF1=EPLT/EP1
      IF (EP2.GT.0.) SF2=EPTT/EP2
      IF (EP3.GT.0.) SF3=-SF3
      IF (SF1.GT.100.) SF1=100.
      IF (SF2.GT.100.) SF2=100.
      IF (SF3.GT.100.) SF3=100.
      WRITE (6,370) J,EP1,SF1,EP2,SF2,EP3,SF3
90    CONTINUE
C     COMPOSITE STRAINS.
      WRITE (6,380) (EP(K),K=1,3)
100   CONTINUE
C     COMPOSITE STRESSES.
      WRITE (6,390)
      DO 120 I=1,NLC
      DO 110 J=1,3
110   G(J)=PN(J,I)/OBJ
      WRITE (6,400) I,(G(J),J=1,3)
```

```
120   CONTINUE
C     COMPOSITE STIFFNESSES.
      WRITE (6,160)
      SF=100.
      IF (A11L.GT.1.0E-10) SF=AA(1,1)/A11L
      IF (SF.GT.100.) SF=100.
      WRITE (6,170) AA(1,1),A11L,SF
      SF=100.
      IF (A22L.GT.1.0E-10) SF=AA(2,2)/A22L
      IF (SF.GT.100.) SF=100.
      WRITE (6,180) AA(2,2),A22L,SF
      SF=100.
      IF (A66L.GT.1.0E-10) SF=AA(3,3)/A66L
      IF (SF.GT.100.) SF=100.
      WRITE (6,190) AA(3,3),A66L,SF
      WRITE (6,410)
      DO 130 I=1,3
      DO 130 J=1,3
      BB(I,J)=OBJ*BB(I,J)
130   AA(I,J)=AA(I,J)/OBJ
C     COMPOSITE STRESS-STRAIN RELATIONSHIPS.
      WRITE (6,420) ((AA(I,J),J=1,3),I=1,3)
      WRITE (6,430)
C     COMPOSITE STRAIN-STRESS RELATIONSHIPS.
      WRITE (6,440) ((BB(I,J),J=1,3),I=1,3)
C     COMPOSITE ELASTIC CONSTANTS.
      EX=1./BB(1,1)
      EY=1./BB(2,2)
      GXY=1./BB(3,3)
      PRXY=-BB(1,2)/BB(1,1)
      PRYX=-BB(1,2)/BB(2,2)
      WRITE (6,450) EX,EY,GXY,PRXY,PRYX
      GO TO 10
C
140   FORMAT (15A4)
150   FORMAT (/14X,5HTITLE/14X,15A4)
160   FORMAT (//25X,30HCOMPOSITE MEMBRANE STIFFNESSES/27X,6HACTUAL,7X,8H
     1REQUIRED/27X,5HVALUE,9X,5HVALUE,8X,4HS.F.)
170   FORMAT (19X,3HA11,E13.5,1X,E13.5,3X,F7.2)
180   FORMAT (19X,3HA22,E13.5,1X,E13.5,3X,F7.2)
190   FORMAT (19X,3HA66,E13.5,1X,E13.5,3X,F7.2)
200   FORMAT (16I5)
210   FORMAT (5F10.2)
220   FORMAT(//41X,2HOF//30X,25HSYMMETRIC COMPOSITE PANEL)
230   FORMAT (//27X,25HNO. OF PLYS              =,I5,/,27X,25HNO. OF LOAD
     1 CONDITIONS    =,I5)
240   FORMAT (//25X,35HPLY PROPERTIES - ALL PLYS IDENTICAL/26X,22HLONGIT
     1UDINAL MODULUS =,E12.5/26X,22HTRANSVERSE MODULUS   =,E12.5/26X,22H
     2SHEAR MODULUS        =,E12.5/26X,22HPOISSONS RATIO, L-T =,E12.5/2
     36X,22HPOISSONS RATIO, T-L =,E12.5)
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250   FORMAT (//14X,31HPLY THICKNESSES, ORIENTATIONS,,27HAND DESIGN VARI
     1ABLE NUMBERS/20X,47HPLY NO.   THICKNESS       THETA    DES. VAR. NO
     *.)
260   FORMAT (21X,I9,3X,E13.5,F10.2)
270   FORMAT (15X,I9,3X,E13.5,3X,F7.2,5X,I5)
280   FORMAT (//35X,17HPLY STRAIN LIMITS/14X,23HLONGITUDINAL STRAIN.GE.,
     1E13.5,9H .AND.LE.,E13.5/14X,23H  TRANSVERSE STRAIN.GE.,E13.5,9H .A
     2ND.LE.,E13.5/14X,23H       SHEAR STRAIN.GE.,E13.5,9H .AND.LE.,E13.
     35)
290   FORMAT (//35X,16HSTIFFNESS LIMITS/33X,7HA11.GE.,E12.5/33X,7HA22.GE
     1.,E12.5/33X,7HA66.GE.,E12.5)
300   FORMAT (//40X,5HLOADS/15X,10HLOAD COND.,6X,2HNX,12X,2HNY,12X,3HNXY
     1)
310   FORMAT (15X,I6,6X,E12.5,2X,E12.5,2X,E12.5)
320   FORMAT (1H1,26X,30HDESIGN AND/OR ANALYSIS RESULTS)
330   FORMAT (///35X,15HPLY INFORMATION/25X,32HPLY NO.     THICKNESS
     1PERCENT)
340   FORMAT (35X,10H----------,3X,10H----------/23X,11HTHICKNESS =,E12.
     15,4X,6H100.00)
350   FORMAT (////37X,11HPLY STRAINS/33X,23HS.F. = SAFETY FACTOR/33X,26H
     1EPL  = LONGITUDINAL STRAIN/33X,24HEPT  = TRANSVERSE STRAIN/33X,19H
     2EPLT = SHEAR STRAIN)
360   FORMAT (//36X,10HLOAD COND.,I5/5X,7HPLY NO.,6X,3HEPL,8X,4HS.F.,8X,
     13HEPT,8X,4HS.F.,8X,4HEPLT,7X,4HS.F.)
370   FORMAT (5X,I5,3X,E12.5,2X,F7.3,2X,E12.5,2X,F7.3,2X,E12.5,2X,F7.3)
380   FORMAT (/5X,47HCOMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS/5X,
     16HEPX = ,E12.5,3X,6HEPY = ,E12.5,3X,7HEPXY = ,E12.5)
390   FORMAT (//25X,30HMEMBRANE STRESSES IN COMPOSITE/15X,10HLOAD COND.,
     15X,7HSIGMA-X,6X,7HSIGMA-Y,7X,6HTAU-XY)
400   FORMAT (15X,I5,7X,3E13.5)
410   FORMAT (//20X,42HCOEFFICIENTS OF STRESS-STRAIN RELATIONSHIPS/28X,26
     1HRELATED TO STRUCTURAL AXES)
420   FORMAT (10X,6HC11 = ,E12.5,4X,6HC12 = ,E12.5,4X,6HC16 = ,E12.5,4X,
     1/32X,6HC22 = ,E12.5,4X,6HC26 = ,E12.5/20X,9HSYMMETRIC,25X,6HC66 =
     2,E12.5)
430   FORMAT (//20X,42HCOEFFICIENTS OF STRAIN-STRESS RELATIONSHIPS/28X,26
     1HRELATED TO STRUCTURAL AXES)
440   FORMAT (10X,6HQ11 = ,E12.5,4X,6HQ12 = ,E12.5,4X,6HQ16 = ,E12.5,4X,
     1/32X,6HQ22 = ,E12.5,4X,6HQ26 = ,E12.5/20X,9HSYMMETRIC,25X,6HQ66 =
     2,E12.5)
450   FORMAT (//26X,27HCOMPOSITE ELASTIC CONSTANTS/11X,5HEX = ,E12.5,5X,
     15HEY = ,E12.5,4X,6HGXY = ,E12.5/9X,7HNUXY = ,E12.5,3X,7HNUYX = ,E1
     *2.5)
460   FORMAT(1H1,38X,6HDESIGN)
470   FORMAT(1H1,37X,8HANALYSIS)
      END
```

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      COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND - COMP1          JULY, 1974

      SUBROUTINE COMP1 (NPLY,THN,EL,ET,GLT,PRLT,PRTL,CP)
      DIMENSION CP(3,3,18), THN(18)
C     ROUTINE TO CALCULATE PLY STIFFNESS COEFICIENTS - ALL PLYS THE SAME
C     ELASTIC PROPERTIES.
C     BY G. N. VANDERPLAATS                           SEPT., 1973.
C     NASA-AMES RESEARCH CENTER, MOFFETT FIELD, CALIF.
C     MATERIAL ELASTIC PROPERITES.
      PR1=1./(1.-PRLT*PRTL)
C     EQUATION A10
      C11=EL*PR1
C     EQUATION A11
      C12=PRTL*EL*PR1
C     EQUATION A12
      C22=ET*PR1
C     EQUATION A13
      C66=GLT
C     GO FOR ALL PLYS.
      DO 10 I=1,NPLY
      THETA=THN(I)
C     EQUATION A9
      AL=COS(THETA)
      AM=SIN(THETA)
      AL2=AL*AL
      AL3=AL*AL2
      AL4=AL2*AL2
      AM2=AM*AM
      AM3=AM*AM2
      AM4=AM2*AM2
C     EQUATION A3
      CP(1,1,I)=C11*AL4+2.*C12*AL2*AM2+C22*AM4+4.*C66*AL2*AM2
C     EQUATION A4
      CP(1,2,I)=C11*AL2*AM2+C12*(AL4+AM4)+C22*AL2*AM2-4.*C66*AL2*AM2
C     EQUATION A5
      CP(1,3,I)=C11*AL3*AM+C12*(AL*AM3-AL3*AM)-C22*AL*AM3+2.*C66*(AL*AM3
     1-AM*AL3)
C     EQUATION A6
      CP(2,2,I)=C11*AM4+2.*C12*AL2*AM2+C22*AL4+4.*C66*AL2*AM2
C     EQUATION A7
      CP(2,3,I)=C11*AL*AM3+C12*(AL3*AM-AL*AM3)-C22*AL3*AM+2.*C66*(AL3*AM
     1-AL*AM3)
C     EQUATION A8
      CP(3,3,I)=C11*AL2*AM2-2.*C12*AL2*AM2+C22*AL2*AM2+C66*(AL4-2.*AL2*A
     1M2+AM4)
C     IMPOSE SYMMETRY ON CP.
      CP(2,1,I)=CP(1,2,I)
      CP(3,1,I)=CP(1,3,I)
      CP(3,2,I)=CP(2,3,I)
   10 CONTINUE
      RETURN
      END
```

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      COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND - COMP2          JULY, 1974

      SUBROUTINE COMP2 (NPLY,TN,CP,A,B)
      DIMENSION A(3,3), B(3,3), TN(18), CP(3,3,18)
C     ROUTINE TO CALCULATE MEMBRANE STIFFNESSES AND FLEXIBILITIES OF
C     COMPOSITE MADE UP OF NPLY PLYS, EACH WITH THE SAME MATERIAL
C     PROPERTIES.
C     BY G. N. VANDERPLAATS                           SEPT., 1973.
C     NASA-AMES RESEARCH CENTER, MOFFETT FIELD, CALIF.
C     STIFFNESS COEFICIENTS.
C     ZERO A.
      DO 10 I=1,3
      DO 10 J=1,3
   10 A(I,J)=0.
C     BUILD A BY SUPERPOSITION.
C     EQUATION A2
      DO 30 I=1,NPLY
      T=TN(I)
      DO 20 J=1,3
      DO 20 K=J,3
   20 A(J,K)=A(J,K)+T*CP(J,K,I)
   30 CONTINUE
C     IMPOSE SYMMETRY ON A.
      A(2,1)=A(1,2)
      A(3,1)=A(1,3)
      A(3,2)=A(2,3)
C     FLEXABILITY COEFICIENTS - INVERSE OF STIFFNESS.
C     BUILD B=A-INVERSE.
      DET=A(1,1)*A(2,2)*A(3,3)+2.*A(1,2)*A(1,3)*A(2,3)-A(1,1)*A(2,3)*A(2
     1,3)-A(2,2)*A(1,3)*A(1,3)- A(3,3)*A(1,2)*A(1,2)
      DET=1./DET
      B(1,1)=DET*(A(2,2)*A(3,3)-A(2,3)*A(2,3))
      B(1,2)=DET*(A(1,3)*A(2,3)-A(1,2)*A(3,3))
      B(1,3)=DET*(A(1,2)*A(2,3)-A(1,3)*A(2,2))
      B(2,2)=DET*(A(1,1)*A(3,3)-A(1,3)*A(1,3))
      B(2,3)=DET*(A(1,2)*A(1,3)-A(1,1)*A(2,3))
      B(3,3)=DET*(A(1,1)*A(2,2)-A(1,2)*A(1,2))
C     IMPOSE SYMMETRY ON B.
      B(2,1)=B(1,2)
      B(3,1)=B(1,3)
      B(3,2)=B(2,3)
      RETURN
      END
```

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COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND - COMP3          JULY, 1974

      SUBROUTINE COMP3 (INFO,OBJ,X,DF,G,IC,A,N1,N2,N3)
      COMMON /CNMN1/ IPRINT,NDV,ITMAX,NCON,NSIDE,ICNDIR,NSCAL,NFDG,FDCH,
     1FDCHM,CT,CTMIN,CTL,CTLMIN,THETA1,PHI,NAC,DELFUN,DABFUN,LINO
     2BJ,ITRM,ITER,INFOG
      DIMENSION X(N1),DF(N1),G(N2),IC(N3),A(N3,N1)
C     EXTERNAL ROUTINE FOR CONMIN FOR COMPOSITE PANEL DESIGN.
C     BY G. N. VANDERPLAATS                             SEPT., 1973.
C     NASA-AMES RESEARCH CENTER, MOFFETT FIELD, CALIF.
C     THIS IS A BUFFER BETWEEN CONMIN AND COMP4.
      CALL COMP4 (INFO,OBJ,NDV,CT,CTL,NAC,X,DF,G,A,IC,N1,N2,N3)
      RETURN
      END
```

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COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND - COMP4          JULY, 1974

      SUBROUTINE COMP4 (INFO,OBJ,NDV,CT,CTL,NAC,X,DF,G,A,IC,NN1,N
     *N2,NN3)
      COMMON /COMPOS/ NPLY,EL,ET,GLT,PRLT,PRTL,EPLC,EPLT,EPTC,EPTT,GMLT,
     1NLC,A11L,A22L,A66L,TN(18),THN(18),CP(3,3,18),PN(3,5),AA(3,3),BB(3
     2,3),EP(3),DEP(3),LNK(18)
      DIMENSION IMP(3), X(NN1),DF(NN1),G(NN2),IC(NN3),A(NN3,NN1)
C     ROUTINE TO CALCULATE FUNCTION VALUE, CONSTRAINT VALUES AND
C     GRADIENT OF FUNCTION AND ACTIVE CONSTRAINTS FOR COMPOSITE
C     ANALYSIS AND DESIGN PROGRAM - COMAND.
C     BY G. N. VANDERPLAATS                             SEPT., 1973.
C     NASA-AMES RESEARCH CENTER, MOFFETT FIELD, CALIF.
      IF (INFO.GT.2) GO TO 20
C     OBJECTIVE
      OBJ=0.
      DO 10 I=1,NPLY
      J=LNK(I)
      TN(I)=X(J)
10    OBJ=OBJ+TN(I)
      IF (INFO.EQ.1) RETURN
      IF (INFO.LE.2) GO TO 50
20    CONTINUE
C     GRADIENT OF OBJECTIVE
      DO 30 I=1,NDV
30    DF(I)=0.
      DO 40 I=1,NPLY
      J=LNK(I)
40    DF(J)=DF(J)+1.
      IF (INFO.EQ.3) RETURN
50    CONTINUE
C     CONSTRAINTS AND GRADIENT OF ACTIVE CONSTRAINTS.
      NCTOT=0
      IF (INFO.EQ.4) NAC=0
C     STIFFNESS AND FLEXIBILITIES.
      CALL COMP2 (NPLY,TN,CP,AA,BB)
      DO 170 I=1,NLC
C     INVERSE OF EQUATION A1.
      EP(1)=BB(1,1)*PN(1,I)+BB(1,2)*PN(2,I)+BB(1,3)*PN(3,I)
      EP(2)=BB(2,1)*PN(1,I)+BB(2,2)*PN(2,I)+BB(2,3)*PN(3,I)
      EP(3)=BB(3,1)*PN(1,I)+BB(3,2)*PN(2,I)+BB(3,3)*PN(3,I)
      DO 170 J=1,NPLY
      THETA=THN(J)
      AL=COS(THETA)
      AM=SIN(THETA)
      AL2=AL*AL
      AM2=AM*AM
C     EQUATION A14
      EP1=AL2*EP(1)+AM2*EP(2)+AL*AM*EP(3)
      EP2=AM2*EP(1)+AL2*EP(2)-AL*AM*EP(3)
      EP3=2.*AL*AM*(EP(2)-EP(1))+(AL2-AM2)*EP(3)
      NCTOT=NCTOT+1
```

```
      N1=NCTOT
C     LONGITUDINAL STRAIN CONSTRAINT - COMPRESSION.
      G(NCTOT)=EP1/EPLC-1.
      NCTOT=NCTOT+1
C     LONGITUDINAL STRAIN CONSTRAINT - TENSION.
      G(NCTOT)=EP1/EPLT-1.
      NCTOT=NCTOT+1
C     TRANSVERSE STRAIN CONSTRAINT - COMPRESSION.
      G(NCTOT)=EP2/EPTC-1.
      NCTOT=NCTOT+1
C     TRANSVERSE STRAIN CONSTRAINT - TENSION.
      G(NCTOT)=EP2/EPTT-1.
      NCTOT=NCTOT+1
C     SHEAR STRAIN CONSTRAINT.
      G(NCTOT)=ABS(EP3)/GMLT-1.
      IF (INFO.LT.4) GO TO 160
      MAC=NAC
      DO 60 K=N1,NCTOT
      IF (G(K).GE.CT) MAC=MAC+1
60    CONTINUE
      IF (MAC.EQ.NAC) GO TO 160
      N2=NAC+1
      DO 70 II=N2,MAC
      DO 70 JJ=1,NDV
70    A(II,JJ)=0.
      MAC=NAC
      N2=N1
      DO 150 KK=1,NPLY
      K=LNK(KK)
C     GRADIENT OF STRAINS - EQUATION 37.
      DO 80 K1=1,3
      TMP(K1)=0.
      DO 80 K2=1,3
80    TMP(K1)=TMP(K1)+CP(K1,K2,KK)*EP(K2)
      DO 90 K1=1,3
      DEP(K1)=0.
      DO 90 K2=1,3
90    DEP(K1)=DEP(K1)-B8(K1,K2)*TMP(K2)
      DEP1=AL2*DEP(1)+AM2*DEP(2)+AL*AM*DEP(3)
      DEP2=AM2*DEP(1)+AL2*DEP(2)-AL*AM*DEP(3)
      DEP3=2.*AL*AM*(DEP(2)-DEP(1))+(AL2-AM2)*DEP(3)
      NAC=MAC
      N1=N2
      IF (G(N1).LT.CT) GO TO 100
C     GRADIENT OF ACTIVE LONGITUDINAL COMPRESSIVE STRAIN CONSTRAINT.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      A(NAC,K)=A(NAC,K)+DEP1/EPLC
      IC(NAC)=N1
100   N1=N1+1
```

```
      IF (G(N1).LT.CT) GO TO 110
C     GRADIENT OF ACTIVE LONGITUDINAL TENSILE STRAIN CONSTRAINT.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      A(NAC,K)=A(NAC,K)+DEP1/EPLT
      IC(NAC)=N1
110   N1=N1+1
      IF (G(N1).LT.CT) GO TO 120
C     GRADIENT OF ACTIVE TRANSVERSE COMPRESSIVE STRAIN CONSTRAINT.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      A(NAC,K)=A(NAC,K)+DEP2/EPTC
      IC(NAC)=N1
120   N1=N1+1
      IF (G(N1).LT.CT) GO TO 130
C     GRADIENT OF ACTIVE TRANSVERSE TENSILE STRAIN CONSTRAINT.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      A(NAC,K)=A(NAC,K)+DEP2/EPTT
      IC(NAC)=N1
130   N1=N1+1
      IF (G(N1).LT.CT) GO TO 140
C     GRADIENT OF ACTIVE SHEAR STRAIN CONSTRAINT.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      SIGN=1.
      IF (EP3.LT.0.) SIGN=-1.
      A(NAC,K)=A(NAC,K)+SIGN*DEP3/GMLT
      IC(NAC)=N1
140   CONTINUE
150   CONTINUE
160   CONTINUE
170   CONTINUE
C     CONSTRAINTS ON STIFFNESS.
      N1=NCTOT
      IF (A11L.LT.1.0E-10) GO TO 180
C     CONSTRAINT ON A11.
      NCTOT=NCTOT+1
      G(NCTOT)=1.-AA(1,1)/A11L
180   IF (A22L.LT.1.0E-10) GO TO 190
C     CONSTRAINT ON A22.
      NCTOT=NCTOT+1
      G(NCTOT)=1.-AA(2,2)/A22L
190   IF (A66L.LT.1.0E-10) GO TO 200
C     CONSTRAINT ON A66.
      NCTOT=NCTOT+1
      G(NCTOT)=1.-AA(3,3)/A66L
200   IF (INFO.LT.4.OR.N1.EQ.NCTOT) RETURN
      IF (A11L.LT.1.0E-10) GO TO 230
      N1=N1+1
```

```
      N1=NCTOT
C     LONGITUDINAL STRAIN CONSTRAINT - COMPRESSION.
      G(NCTOT)=EP1/EPLC-1.
      NCTOT=NCTOT+1
C     LONGITUDINAL STRAIN CONSTRAINT - TENSION.
      G(NCTOT)=EP1/EPLT-1.
      NCTOT=NCTOT+1
C     TRANSVERSE STRAIN CONSTRAINT - COMPRESSION.
      G(NCTOT)=EP2/EPTC-1.
      NCTOT=NCTOT+1
C     TRANSVERSE STRAIN CONSTRAINT - TENSION.
      G(NCTOT)=EP2/EPTT-1.
      NCTOT=NCTOT+1
C     SHEAR STRAIN CONSTRAINT.
      G(NCTOT)=ABS(EP3)/GMLT-1.
      IF (INFO.LT.4) GO TO 160
      MAC=NAC
      DO 60 K=N1,NCTOT
      IF (G(K).GE.CT) MAC=MAC+1
   60 CONTINUE
      IF (MAC.EQ.NAC) GO TO 160
      N2=NAC+1
      DO 70 II=N2,MAC
      DO 70 JJ=1,NGV
   70 A(II,JJ)=0.
      MAC=NAC
      N2=N1
      DO 150 KK=1,NPLY
      K=LNK(KK)
C     GRADIENT OF STRAINS - EQUATION 37.
      DO 80 K1=1,3
      TMP(K1)=0.
      DO 80 K2=1,3
   80 TMP(K1)=TMP(K1)+CP(K1,K2,KK)*EP(K2)
      DO 90 K1=1,3
      DEP(K1)=0.
      DO 90 K2=1,3
   90 DEP(K1)=DEP(K1)-BB(K1,K2)*TMP(K2)
      DEP1=AL2*DEP(1)+AM2*DEP(2)+AL*AM*DEP(3)
      DEP2=AM2*DEP(1)+AL2*DEP(2)-AL*AM*DEP(3)
      DEP3=2.*AL*AM*(DEP(2)-DEP(1))+(AL2-AM2)*DEP(3)
      NAC=MAC
      N1=N2
      IF (G(N1).LT.CT) GO TO 100
C     GRADIENT OF ACTIVE LONGITUDINAL COMPRESSIVE STRAIN CONSTRAINT.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      A(NAC,K)=A(NAC,K)+DEP1/EPLC
      IC(NAC)=N1
  100 N1=N1+1
```

```
      IF (G(N1).LT.CT) GO TO 110
C     GRADIENT OF ACTIVE LONGITUDINAL TENSILE STRAIN CONSTRAINT.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      A(NAC,K)=A(NAC,K)+DEP1/EPLT
      IC(NAC)=N1
  110 N1=N1+1
      IF (G(N1).LT.CT) GO TO 120
C     GRADIENT OF ACTIVE TRANSVERSE COMPRESSIVE STRAIN CONSTRAINT.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      A(NAC,K)=A(NAC,K)+DEP2/EPTC
      IC(NAC)=N1
  120 N1=N1+1
      IF (G(N1).LT.CT) GO TO 130
C     GRADIENT OF ACTIVE TRANSVERSE TENSILE STRAIN CONSTRAINT.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      A(NAC,K)=A(NAC,K)+DEP2/EPTT
      IC(NAC)=N1
  130 N1=N1+1
      IF (G(N1).LT.CT) GO TO 140
C     GRADIENT OF ACTIVE SHEAR STRAIN CONSTRAINT.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      SIGN=1.
      IF (EP3.LT.0.) SIGN=-1.
      A(NAC,K)=A(NAC,K)+SIGN*DEP3/GMLT
      IC(NAC)=N1
  140 CONTINUE
  150 CONTINUE
  160 CONTINUE
  170 CONTINUE
C     CONSTRAINTS ON STIFFNESS.
      N1=NCTOT
      IF (A11L.LT.1.0E-10) GO TO 180
C     CONSTRAINT ON A11.
      NCTOT=NCTOT+1
      G(NCTOT)=1.-AA(1,1)/A11L
  180 IF (A22L.LT.1.0E-10) GO TO 190
C     CONSTRAINT ON A22.
      NCTOT=NCTOT+1
      G(NCTOT)=1.-AA(2,2)/A22L
  190 IF (A66L.LT.1.0E-10) GO TO 200
C     CONSTRAINT ON A66.
      NCTOT=NCTOT+1
      G(NCTOT)=1.-AA(3,3)/A66L
  200 IF (INFO.LT.4.OR.N1.EQ.NCTOT) RETURN
      IF (A11L.LT.1.0E-10) GO TO 230
      N1=N1+1
```

```
      IF (G(N1).LT.CTL) GO TO 230
C     GRADIENT OF ACTIVE CONSTRAINT ON A11.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      IC(NAC)=N1
      DO 210 K=1,NDV
210   A(NAC,K)=0.
      DO 220 KK=1,NPLY .
      K=LNK(KK)
220   A(NAC,K)=A(NAC,K)-CP(1,1,KK)/A11L
230   IF (A22L.LT.1.0E-10) GO TO 260
      N1=N1+1
      IF (G(N1).LT.CTL) GO TO 260
C     GRADIENT OF ACTIVE CONSTRAINT ON A22.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      IC(NAC)=N1
      DO 240 K=1,NDV
240   A(NAC,K)=0.
      DO 250 KK=1,NPLY
      K=LNK(KK)
250   A(NAC,K)=A(NAC,K)-CP(2,2,KK)/A22L
260   IF (A66L.LT.1.0E-10) RETURN
      N1=N1+1
      IF (G(N1).LT.CTL) GO TO 290
C     GRADIENT OF ACTIVE CONSTRAINT ON A66.
      NAC=NAC+1
      IF (NAC.EQ.NN3) RETURN
      IC(NAC)=N1
      DO 270 K=1,NDV
270   A(NAC,K)=0.
      DO 280 KK=1,NPLY
      K=LNK(KK)
280   A(NAC,K)=A(NAC,K)-CP(3,3,KK)/A66L
290   CONTINUE
      RETURN
      END
```

References

1. Schmit, L.A., Jr., and Farshi, B.: Optimum Laminate Design for Strength and stiffness. Int. J. For Numerical Methods in Engineering, Vol. 7, No. 4, pp. 519-536, 1973.

2. Vanderplaats, Garret, N.: CONMIN - A FORTRAN Program for Constrained Function Minimization - User's Manual, NASA TM X-62,282, Aug. 1973.

3. Advanced Composites Design Guide, Volume I - Design, Wright-Patterson Air Force Base, Ohio, January 1973.

COMAND DATA ORGANIZATION:

Block	Number of Cards	INFORMATION	FORMAT
A	1	Title - Anything may be given here	15A4
B	1	NCALC, NPLY, NDV, NLC, IPRINT	515
C	1	LNK(I),I=1,NPLY	1215
D	1-3	X(I),I=1,NDV	8F10.2
E	1-3	VLB(I),I=1,NDV (Blank card(s) if NCALC.NE.0)	8F10.2
F	1-3	THN(I),I=1,NPLY	8F10.2
G	1	EL, ET, GLT, PRLT	4F10.2
H	1	EPLC, EPLT, EPTC, EPTT, GMLT, A11L, A22L, A66L	8F10.2
I	NLC	PN(J,I),J=1,3 (One card per loading condition)	3F10.2
		Begin with next set of data - Program terminates if 2 blank cards are read here.	

TABLE 1 - DATA ORGANIZATION

DATA SHEET

COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND

Col Block	1	5	11	21	31	41	51	61	71
A									
B									
C									
D									
E									
F									
G									
H									
I									

TABLE 1 - DATA ORGANIZATION - CONCLUDED

TABLE 1.2.1-III. KEY UNIDIRECTIONAL PROPERTIES

HIGH-STRENGTH GRAPHITE/EPOXY -[0]

$V_f = 0.60$

				RT	350°F
Design strengths*	Longitudinal tensile ultimate	Ksi	F_L^{tu}	180.0	180.0
	Transverse tensile ultimate	Ksi	F_T^{tu}	8.0	4.0
	Longitudinal compression ultimate	Ksi	F_L^{cu}	180.0	70.0
	Transverse compression ultimate	Ksi	F_T^{cu}	30.0	12.0
	In-plane shear ultimate	Ksi	F_{LT}^{su}	12.0	6.8
	Interlaminar shear ultimate	Ksi	F^{isu}	13.0	8.0
	Ultimate longitudinal strain	μin./in.	ϵ_L^{tu}	8,700.0	9,650.0
	Ultimate transverse strain	μin./in.	ϵ_T^{tu}	4,750.0	4,100.0
Elastic properties [typical]	Longitudinal tension modulus	Msi	E_L^t	21.0	18.7
	Transverse tension modulus	Msi	E_T^t	1.7	0.87
	Longitudinal compression modulus	Msi	E_L^c	21.0	18.7
	Transverse compression modulus	Msi	E_T^c	1.7	0.87
	In-plane shear modulus	Msi	G_{LT}	0.65	0.32
	Longitudinal Poisson's ratio		ν_{LT}	0.21	0.21
	Transverse Poisson's ratio		ν_{TL}	0.017	0.010
Physical constants [typical]	Density	lb/in.^{3}	ρ	0.056	0.056
	Longitudinal coefficient of thermal expansion	μin./in./°F	α_L	-0.21	-0.005
	Transverse coefficient of thermal expansion	μin./in./°F	α_T	16.0	21.8

References: 1.2-15, -19, -21

*Typical Design Allowable, reference section 1.2.0

Table 2.- Material properties.

DATA SHEET

COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND

Col Block	1	5	11		21		31	41	51	61	71
A	DETERMINATION OF LIMIT STRAINS - G/E COMPOSITE										
B	1	1	1	4	0						
C	1										
D	1.0										
E	0.										
F	0.										
G	21000000.		17000000.		650000.		.21				
H	0.		0.		0.		0.	0.	0.	0.	0.
I	180000.		0.		0.						
	0.		-30000.		0.						
	0.		8000.		0.						
	0.		0.		12000.						

TABLE 3 - DETERMINATION OF LIMIT STRAINS - G/E COMPOSITE

DATA SHEET

COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND

Col / Block	1	5	11		21		31		41		51		61		71	
A	QUASI-ISOTROPIC COMPOSITE UNDER UNIAXIAL LOAD															
B	0	12	1	1	2											
C	1	1	1	1	1	1	1	1	1	1	1	1				
D	.05															
E	.00001															
F	0.		15.		-15.		30.		-30.		45.		-45.		60.	
	-60.		75.		-75.		90.									
G	21000000.		17000000.		650000.		.21									
H	-.00857		.00857		-.0176		.00471		.0184		0.		0.		0.	
J	20000.															

TABLE 4 - QUASI-ISOTROPIC COMPOSITE UNDER UNIAXIAL LOAD - EXAMPLE 1

DATA SHEET

COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND

Col Block	1	5	11		21	31	41	51	61	71
A	(0, 45, -45, 90) GRAPHITE EPOXY COMPOSITE - EXAMPLE 2									
B	0	4	3	4	0					
C	1	2	2	3						
D	.1		.1		.1					
E	.00001		.00001		.00001					
F	0.		45.		-45.	90.				
G	21000000.		17000000.		650000.	.21				
H	-.00857		.00857		-.0176	.00471	.0184	500000.	0.	0.
I	20000.		0.		0.					
	15000.		-15000.		5000.					
	-15000.		10000.		10000.					
	0.		0.		20000.					

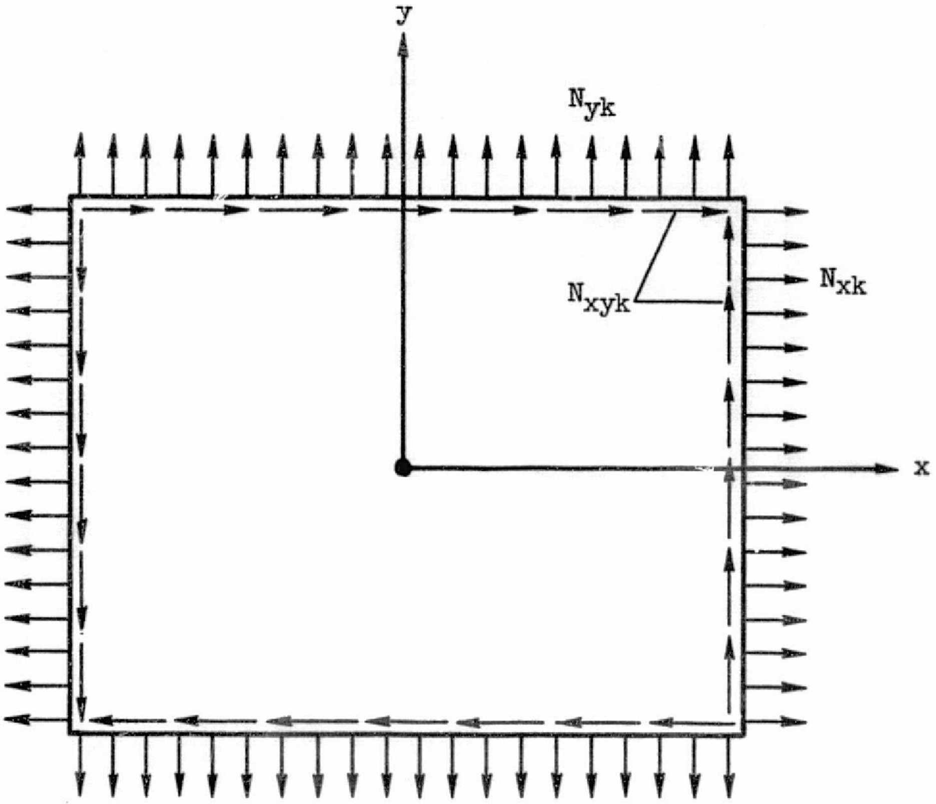
TABLE 5 - (0, ±45, 90) GRAPHITE EPOXY COMPOSITE - EXAMPLE 2

DATA SHEET

COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND

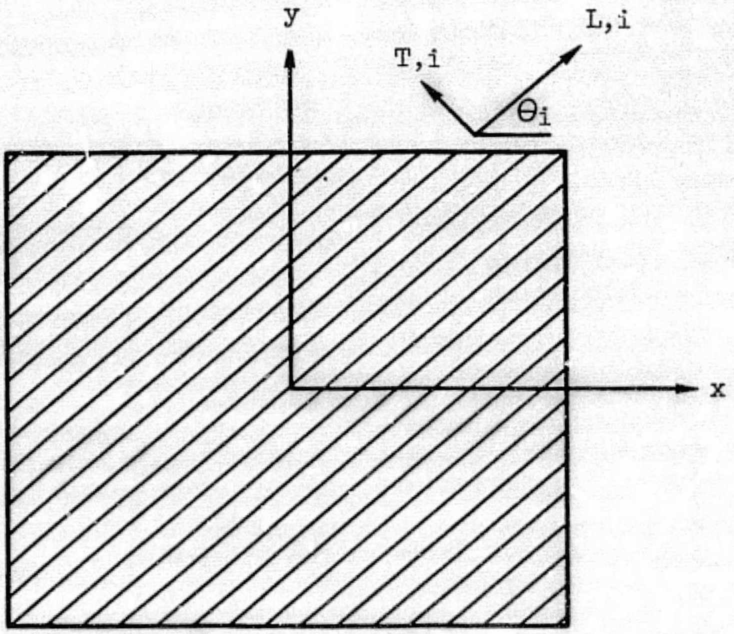
Col / Block	1	5	11		21		31	41	51	61	71
A	(0, 30, -30, 60, -60, 90) GRAPHITE EPOXY COMPOSITE - EXAMPLE 3										
B	0	6	4	4	0						
C	1	2	2	3	3	4					
D	.1		.1		.1		.1				
E	.00001		.00001		.00001		.00001				
F	0.		30.		-30.		60.	-60.	90.		
G	21000000.		17000000.		650000.		.21				
H	-.00857		.00857		-.0176		.00471	.0184	500000.	0.	0.
I	20000.		0.		0.						
	15000.		-15000.		5000.						
	-15000.		10000.		10000.						
	0.		0.		20000.						

TABLE 6 - (6, ±30, ±60, 90) GRAPHITE EPOXY COMPOSITE - EXAMPLE 3

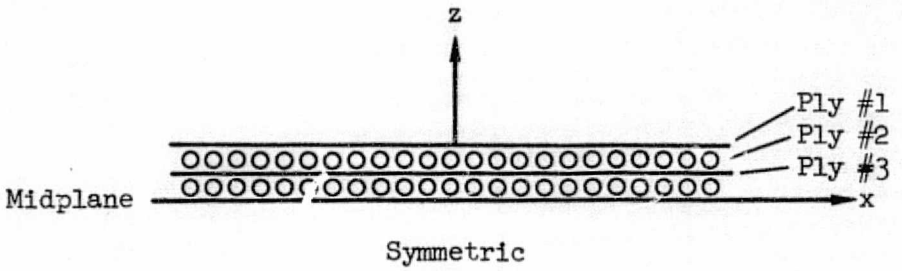


Inplane loads N_x, N_y, N_{xy}
Load condition k.

Figure 1.- Typical composite loading.



Ply orientation, θ_i



Symmetric composite layup

Figure 2.- Typical ply orientation.

```
                     ANALYSIS

                        OF

            SYMMETRIC COMPOSITE PANEL

TITLE
 DETERMINATION OF LIMIT STRAINS - G/E COMPOSITE

           NO. OF PLYS                 =     1
           NO. OF LOAD CONDITIONS      =     4

         PLY PROPERTIES - ALL PLYS IDENTICAL
          LONGITUDINAL MODULUS =  .21000E+08
          TRANSVERSE MODULUS   =  .17000E+07
          SHEAR MODULUS        =  .65000E+06
          POISSON'S RATIO, L-T =  .21000E+00
          POISSON'S RATIO, T-L =  .17000E-01

PLY THICKNESSES, ORIENTATIONS,,AND DESIGN VARIABLE NUMBERS
      PLY NO.   THICKNESS        THETA    DES. VAR. NO.
         1      .10000E+01       0.00          1

                  PLY STRAIN LIMITS
LONGITUDINAL STRAIN.GE.  0.              .AND.LE.  0.
  TRANSVERSE STRAIN.GE. -0.              .AND.LE. -0.
       SHEAR STRAIN.GE.  0.              .AND.LE. -0.

                  STIFFNESS LIMITS
                 A11.GE.-0.
                 A22.GE.-0.
                 A66.GE.-0.

                        LOADS
 LOAD COND.       NX                NY              NXY
     1          .18000E+05   0.                 0.
     2       0.             -.30000E+05         0.
     3       0.              .80000E+04         0.
     4       0.             0.                  .12000E+05
```

```
                   DESIGN AND/OR ANALYSIS RESULTS

       TITLE
        DETERMINATION OF LIMIT STRAINS - G/E COMPOSITE

                        PLY INFORMATION
               PLY NO.   THICKNESS    PERCENT
                  1      .10000E+01   100,00
                         ----------   ----------
            THICKNESS =  .10000E+01   100,00

                          PLY STRAINS
                     S.F.  = SAFETY FACTOR
                     EPL   = LONGITUDINAL STRAIN
                     EPT   = TRANSVERSE STRAIN
                     EPLT  = SHEAR STRAIN

                          LOAD COND.    1
PLY NO.       EPL         S.F.        EPT        S.F.         EPLT        S.F.
   1      .85714E-03      0,000   -.18000E-03    0,000     .10000E-19     0,000

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =   .85714E-03   EPY =  -.18000E-03   EPXY =  0.

                          LOAD COND.    2
PLY NO.       EPL         S.F.        EPT        S.F.         EPLT        S.F.
   1      .30000E-03      0,000   -.17647E-01    0,000     .10000E-19     0,000

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =   .30000E-03   EPY =  -.17647E-01   EPXY =  0.

                          LOAD COND.    3
PLY NO.       EPL         S.F.        EPT        S.F.         EPLT        S.F.
   1     -.80000E-04      0,000    .47059E-02    0,000     .10000E-19     0,000

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =  -.80000E-04   EPY =   .47059E-02   EPXY =  0.

                          LOAD COND.    4
PLY NO.       EPL         S.F.        EPT        S.F.         EPLT        S.F.
   1      .10000E-19      0,000    .10000E-19    0,000     .18462E-01     0,000

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =  0.               EPY =  0.              EPXY =   .18462E-01

                  MEMBRANE STRESSES IN COMPOSITE
           LOAD COND.    SIGMA-X       SIGMA-Y         TAU-XY
               1         .18000E+05  0.               0.
               2        0.           -.30000E+05      0.
               3        0.            .80000E+04      0.
               4        0.           0.               .12000E+05
```

Figure 3.- Determination of limit strains - G/E composite.

```
          COMPOSITE MEMBRANE STIFFNESSES
            ACTUAL        REQUIRED
            VALUE          VALUE          S.F.
     A11   .21075E+08   -0.                100.00
     A22   .17061E+07   -0.                100.00
     A66   .65000E+06   -0.                100.00


       COEFFICIENTS OF STRESS-STRAIN RELATIONSHIPS
              RELATED TO STRUCTURAL AXES
C11 =   .21075E+08    C12 =   .35828E+06    C16 =  0.
                      C22 =   .17061E+07    C26 =  0.
    SYMMETRIC                               C66 =   .65000E+06


       COEFFICIENTS OF STRAIN-STRESS RELATIONSHIPS
              RELATED TO STRUCTURAL AXES
Q11 =   .47619E-07    Q12 =  -.10000E-07    Q16 =  0.
                      Q22 =   .58824E-06    Q26 =  0.
    SYMMETRIC                               Q66 =   .15385E-05


              COMPOSITE ELASTIC CONSTANTS
EX =    .21000E+08     EY =   .17000E+07    GXY =    .65000E+06
NUXY =  .21000E+00   NUYX =   .17000E-01
```

Figure 3.- Concluded.

```
                             DESIGN

                               OF

                    SYMMETRIC COMPOSITE PANEL

TITLE
 QUASI-ISOTROPIC COMPOSITE UNDER UNIAXIAL LOAD - EXAMPLE 1


            NO. OF PLYS                  =   12
            NO. OF LOAD CONDITIONS       =    1


         PLY PROPERTIES - ALL PLYS IDENTICAL
          LONGITUDINAL MODULUS  =  .21000E+08
          TRANSVERSE MODULUS    =  .17000E+07
          SHEAR MODULUS         =  .65000E+06
          POISSON'S RATIO, L-T  =  .21000E+00
          POISSON'S RATIO, T-L  =  .17000E-01


PLY THICKNESSES, ORIENTATIONS, AND DESIGN VARIABLE NUMBERS
      PLY NO.    THICKNESS       THETA    DES. VAR. NO.
         1       .50000E-01       0.00         1
         2       .50000E-01      15.00         1
         3       .50000E-01     -15.00         1
         4       .50000E-01      30.00         1
         5       .50000E-01     -30.00         1
         6       .50000E-01      45.00         1
         7       .50000E-01     -45.00         1
         8       .50000E-01      60.00         1
         9       .50000E-01     -60.00         1
        10       .50000E-01      75.00         1
        11       .50000E-01     -75.00         1
        12       .50000E-01      90.00         1


                         PLY STRAIN LIMITS
LONGITUDINAL STRAIN.GE.  -.85700E-02 .AND.LE.    .85700E-02
  TRANSVERSE STRAIN.GE.  -.17600E-01 .AND.LE.    .47100E-02
       SHEAR STRAIN.GE.  -.18400E-01 .AND.LE.    .18400E-01


                       STIFFNESS LIMITS
                    A11.GE. 0.
                    A22.GE. 0.
                    A66.GE. 0.


                             LOADS
LOAD COND.       NX             NY                 NXY
     1        .20000E+05    0.                  0.
```

Figure 4.- Quasi-isotropic G/E composite under uniaxial load — Example 1.

```
* * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                   *
*                 C O N M I N                       *
*                                                   *
*            FORTRAN PROGRAM FOR                    *
*                                                   *
*     CONSTRAINED FUNCTION MINIMIZATION             *
*                                                   *
* NASA/AMES RESEARCH CENTER, MOFFETT FIELD, CALIF.  *
*                                                   *
*          VERSION II      JULY, 1975               *
*                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * *


CONSTRAINED FUNCTION MINIMIZATION

CONTROL PARAMETERS

IPRINT  NDV     ITMAX   NCON     NSIDE   ICNDIR  NSCAL   NFDG
  2       1      30      60        1        2      0       0

LINOBJ  ITRM
  1       3

    CT              CTMIN             CTL              CTLMIN
-.10000E+00      .40000E-02      -.10000E-01        .10000E-02

   THETA             PHI            DELFUN            DABFUN
 .10000E+01      .50000E+01       .10000E-03        .60000E-03

   FDCH             FDCHM
 .10000E-01      .10000E-01

LOWER BOUNDS ON DECISION VARIABLES (VLB)
  1)      .10000E-04

UPPER BOUNDS ON DECISION VARIABLES (VUB)
  1)      .10000E+05

ALL CONSTRAINTS ARE NON-LINEAR


INITIAL FUNCTION INFORMATION

OBJ =     .600000E+00

DECISION VARIABLES (X-VECTOR)
  1)      .50000E-01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.14809E+01  -.51908E+00  -.92703E+00  -.12727E+01  -.10000E+01  -.14387E+01
  7)     -.56134E+00  -.94760E+00  -.11958E+01  -.85310E+00  -.14387E+01  -.56134E+00
 13)     -.94760E+00  -.11958E+01  -.85310E+00  -.13232E+01  -.67678E+00  -.10038E+01
 19)     -.98575E+00  -.74557E+00  -.13232E+01  -.67678E+00  -.10038E+01  -.98575E+00
 25)     -.74557E+00  -.11655E+01  -.83447E+00  -.10806E+01  -.69882E+00  -.70621E+00
 31)     -.11655E+01  -.83447E+00  -.10806E+01  -.69882E+00  -.70621E+00  -.10078E+01
 37)     -.99217E+00  -.11574E+01  -.41188E+00  -.74557E+00  -.10078E+01  -.99217E+00
 43)     -.11574E+01  -.41188E+00  -.74557E+00  -.89239E+00  -.11076E+01  -.12136E+01
 49)     -.20184E+00  -.85310E+00  -.89239E+00  -.11076E+01  -.12136E+01  -.20184E+00
 55)     -.85310E+00  -.85014E+00  -.11499E+01  -.12342E+01  -.12495E+00  -.10000E+01
```

```
ITER =    1     OBJ =    .52504E+00

DECISION VARIABLES (X-VECTOR)
  1)      .43753E-01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.15496E+01  -.45042E+00  -.91661E+00  -.13116E+01  -.10000E+01  -.15013E+01
  7)     -.49870E+00  -.94012E+00  -.12238E+01  -.83213E+00  -.15013E+01  -.49870E+00
 13)     -.94012E+00  -.12238E+01  -.83213E+00  -.13694E+01  -.63063E+00  -.10044E+01
 19)     -.98372E+00  -.70924E+00  -.13694E+01  -.63063E+00  -.10044E+01  -.98372E+00
 25)     -.70924E+00  -.11892E+01  -.81084E+00  -.10921E+01  -.65582E+00  -.66426E+00
 31)     -.11892E+01  -.81084E+00  -.10921E+01  -.65582E+00  -.66426E+00  -.10089E+01
 37)     -.99105E+00  -.11799E+01  -.32791E+00  -.70924E+00  -.10089E+01  -.99105E+00
 43)     -.11799E+01  -.32791E+00  -.70924E+00  -.87703E+00  -.11230E+01  -.12441E+01
 49)     -.87875E-01  -.83213E+00  -.87703E+00  -.11230E+01  -.12441E+01  -.87875E-01
 55)     -.83213E+00  -.82874E+00  -.11713E+01  -.12676E+01  -.14203E-04  -.10000E+01


ITER =    2     OBJ =    .52504E+00      NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .43753E-01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.15496E+01  -.45042E+00  -.91661E+00  -.13116E+01  -.10000E+01  -.15013E+01
  7)     -.49870E+00  -.94012E+00  -.12238E+01  -.83213E+00  -.15013E+01  -.49870E+00
 13)     -.94012E+00  -.12238E+01  -.83213E+00  -.13694E+01  -.63063E+00  -.10044E+01
 19)     -.98372E+00  -.70924E+00  -.13694E+01  -.63063E+00  -.10044E+01  -.98372E+00
 25)     -.70924E+00  -.11892E+01  -.81084E+00  -.10921E+01  -.65582E+00  -.66426E+00
 31)     -.11892E+01  -.81084E+00  -.10921E+01  -.65582E+00  -.66426E+00  -.10089E+01
 37)     -.99105E+00  -.11799E+01  -.32791E+00  -.70924E+00  -.10089E+01  -.99105E+00
 43)     -.11799E+01  -.32791E+00  -.70924E+00  -.87703E+00  -.11230E+01  -.12441E+01
 49)     -.87875E-01  -.83213E+00  -.87703E+00  -.11230E+01  -.12441E+01  -.87875E-01
 55)     -.83213E+00  -.82874E+00  -.11713E+01  -.12676E+01  -.14203E-04  -.10000E+01


ITER =    3     OBJ =    .52504E+00      NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .43753E-01

CONSTRAINT VALUES (G-VECTOR)
  1)     -.15496E+01  -.45042E+00  -.91661E+00  -.13116E+01  -.10000E+01  -.15013E+01
  7)     -.49870E+00  -.94012E+00  -.12238E+01  -.83213E+00  -.15013E+01  -.49870E+00
 13)     -.94012E+00  -.12238E+01  -.83213E+00  -.13694E+01  -.63063E+00  -.10044E+01
 19)     -.98372E+00  -.70924E+00  -.13694E+01  -.63063E+00  -.10044E+01  -.98372E+00
 25)     -.70924E+00  -.11892E+01  -.81084E+00  -.10921E+01  -.65582E+00  -.66426E+00
 31)     -.11892E+01  -.81084E+00  -.10921E+01  -.65582E+00  -.66426E+00  -.10089E+01
 37)     -.99105E+00  -.11799E+01  -.32791E+00  -.70924E+00  -.10089E+01  -.99105E+00
 43)     -.11799E+01  -.32791E+00  -.70924E+00  -.87703E+00  -.11230E+01  -.12441E+01
 49)     -.87875E-01  -.83213E+00  -.87703E+00  -.11230E+01  -.12441E+01  -.87875E-01
 55)     -.83213E+00  -.82874E+00  -.11713E+01  -.12676E+01  -.14203E-04  -.10000E+01
```

Figure 4.- Continued.

```
ITER =    4    OBJ =    .52504E+00    NO CHANGE IN OBJ

DECISION VARIABLES (X-VECTOR)
  1)      .43753E-01

CONSTRAINT VALUES (G-VECTOR)
  1)    -.15496E+01  -.45042E+00  -.91661E+00  -.13116E+01  -.10000E+01  -.15013E+01
  7)    -.49870E+00  -.94012E+00  -.12238E+01  -.83213E+00  -.15013E+01  -.49870E+00
 13)    -.94012E+00  -.12238E+01  -.83213E+00  -.13694E+01  -.63063E+00  -.10044E+01
 19)    -.98372E+00  -.70924E+00  -.13694E+01  -.63063E+00  -.10044E+01  -.98372E+00
 25)    -.70924E+00  -.11892E+01  -.81084E+00  -.10921E+01  -.65582E+00  -.66426E+00
 31)    -.11892E+01  -.81084E+00  -.10921E+01  -.65582E+00  -.66426E+00  -.10089E+01
 37)    -.99105E+00  -.11799E+01  -.32791E+00  -.70924E+00  -.10089E+01  -.99105E+00
 43)    -.11799E+01  -.32791E+00  -.70924E+00  -.87703E+00  -.11230E+01  -.12441E+01
 49)    -.87875E-01  -.83213E+00  -.87703E+00  -.11230E+01  -.12441E+01  -.87875E-01
 55)    -.83213E+00  -.82874E+00  -.11713E+01  -.12676E+01  -.14203E-04  -.10000E+01
```

```
FINAL OPTIMIZATION INFORMATION

 OBJ =     .525036E+00

 DECISION VARIABLES (X-VECTOR)
   1)      .43753E-01

 CONSTRAINT VALUES (G-VECTOR)
   1)    -.15496E+01  -.45042E+00  -.91661E+00  -.13116E+01  -.10000E+01  -.15013E+01
   7)    -.49870E+00  -.94012E+00  -.12238E+01  -.83213E+00  -.15013E+01  -.49870E+00
  13)    -.94012E+00  -.12238E+01  -.83213E+00  -.13694E+01  -.63063E+00  -.10044E+01
  19)    -.98372E+00  -.70924E+00  -.13694E+01  -.63063E+00  -.10044E+01  -.98372E+00
  25)    -.70924E+00  -.11892E+01  -.81084E+00  -.10921E+01  -.65582E+00  -.66426E+00
  31)    -.11892E+01  -.81084E+00  -.10921E+01  -.65582E+00  -.66426E+00  -.10089E+01
  37)    -.99105E+00  -.11799E+01  -.32791E+00  -.70924E+00  -.10089E+01  -.99105E+00
  43)    -.11799E+01  -.32791E+00  -.70924E+00  -.87703E+00  -.11230E+01  -.12441E+01
  49)    -.87875E-01  -.83213E+00  -.87703E+00  -.11230E+01  -.12441E+01  -.87875E-01
  55)    -.83213E+00  -.82874E+00  -.11713E+01  -.12676E+01  -.14203E-04  -.10000E+01

 THERE ARE     1 ACTIVE CONSTRAINTS
 CONSTRAINT NUMBERS ARE
    59

 THERE ARE     0 VIOLATED CONSTRAINTS

 THERE ARE     0 ACTIVE SIDE CONSTRAINTS

 TERMINATION CRITERION
      ABS(1-OBJ(I-1)/OBJ(I)) LESS THAN DELFUN FOR  3 ITERATIONS
      ABS(OBJ(I)-OBJ(I-1))   LESS THAN DABFUN FOR  3 ITERATIONS

 NUMBER OF ITERATIONS =    4

 OBJECTIVE FUNCTION WAS EVALUATED             5  TIMES

 CONSTRAINT FUNCTIONS WERE EVALUATED          5  TIMES

 GRADIENT OF OBJECTIVE WAS CALCULATED         2  TIMES

 GRADIENTS OF CONSTRAINTS WERE CALCULATED     2  TIMES
```

Figure 4.- Continued.

```
                    DESIGN AND/OR ANALYSIS RESULTS

TITLE
 QUASI-ISOTROPIC COMPOSITE UNDER UNIAXIAL LOAD - EXAMPLE 1


                         PLY INFORMATION
              PLY NO.    THICKNESS      PERCENT
                 1       .43753E-01       8.33
                 2       .43753E-01       8.33
                 3       .43753E-01       8.33
                 4       .43753E-01       8.33
                 5       .43753E-01       8.33
                 6       .43753E-01       8.33
                 7       .43753E-01       8.33
                 8       .43753E-01       8.33
                 9       .43753E-01       8.33
                10       .43753E-01       8.33
                11       .43753E-01       8.33
                12       .43753E-01       8.33
                         ----------   ----------
          THICKNESS =    .52500E+00     100.00


                           PLY STRAINS
                      S.F.  = SAFFTY FACTOR
                      EPL   = LONGITUDINAL STRAIN
                      EPT   = TRANSVERSE STRAIN
                      EPLT  = SHEAR STRAIN


                              LOAD COND.     1
PLY NO.     EPL         S.F.       EPT          S.F.       EPLT         S.F.
   1     .47099E-02    1.820   -.14677E-02    11.991   -.74776E-10  100.000
   2     .42961E-02    1.995   -.10539E-02    16.700   -.30888E-02    5.957
   3     .42961E-02    1.995   -.10539E-02    16.700    .30888E-02    5.957
   4     .31655E-02    2.707    .76694E-04    61.412   -.53500E-02    3.439
   5     .31655E-02    2.707    .76694E-04    61.413    .53500E-02    3.439
   6     .16211E-02    5.287    .16211E-02     2.905   -.61777E-02    2.978
   7     .16211E-02    5.287    .16211E-02     2.905    .61777E-02    2.978
   8     .76694E-04  100.000    .31655E-02     1.488   -.53500E-02    3.439
   9     .76694E-04  100.000    .31655E-02     1.488    .53500E-02    3.439
  10    -.10539E-02    8.132    .42961E-02     1.096   -.30888E-02    5.957
  11    -.10539E-02    8.132    .42961E-02     1.096    .30888E-02    5.957
  12    -.14677E-02    5.839    .47099E-02     1.000    .12648E-08  100.000

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =   .47099E-02    EPY =  -.14677E-02    EPXY =  -.74776E-10


                    MEMBRANE STRESSES IN COMPOSITE
          LOAD COND.      SIGMA-X       SIGMA-Y         TAU-XY
              1          .38095E+05  0.                0.


                   COMPOSITE MEMBRANE STIFFNESSES
                     ACTUAL        REQUIRED
                     VALUE           VALUE          S.F.
            A11   .47030E+07    0.                 100.00
            A22   .47031E+07    0.                 100.00
            A66   .16187E+07    0.                 100.00


              COEFICIENTS OF STRESS-STRAIN RELATIONSHIPS
                       RELATED TO STRUCTURAL AXES
     C11 =    .89576E+07     C12 =    .27914E+07     C16 =    .38574E-03
                             C22 =    .89576E+07     C26 =   -.15584E+00
              SYMMETRIC                              C66 =    .30831E+07


              COEFICIENTS OF STRAIN-STRESS RELATIONSHIPS
                       RELATED TO STRUCTURAL AXES
     Q11 =    .12364E-06     Q12 =   -.38530E-07     Q16 =   -.19630E-14
                             Q22 =    .12364E-06     Q26 =    .62547E-14
              SYMMETRIC                              Q66 =    .32435E-06


                       COMPOSITE ELASTIC CONSTANTS
      EX =    .80877E+07     EY =     .80877E+07     GXY =    .30831E+07
    NUXY =    .31162E+00   NUYX =     .31162E+00
```

Figure 4.- Concluded.

```
                              DESIGN

                                OF

                     SYMMETRIC COMPOSITE PANEL

TITLE
 0, 45, -45, 90 GRAPHITE EPOXY COMPOSITE - EXAMPLE 2


               NO. OF PLYS                =    4
               NO. OF LOAD CONDITIONS     =    4


            PLY PROPERTIES - ALL PLYS IDENTICAL
             LONGITUDINAL MODULUS =  .21000E+08
             TRANSVERSE MODULUS   =  .17000E+07
             SHEAR MODULUS        =  .65000E+06
             POISSON'S RATIO, L-T =  .21000E+00
             POISSON'S RATIO, T-L =  .17000E-01


PLY THICKNESSES, ORIENTATIONS,,AND DESIGN VARIABLE NUMBERS
      PLY NO.    THICKNESS       THETA    DES. VAR. NO.
        1        .10000E+00       0.00         1
        2        .10000E+00      45.00         2
        3        .10000E+00     -45.00         2
        4        .10000E+00      90.00         3


                        PLY STRAIN LIMITS
LONGITUDINAL STRAIN.GE.  -.85700E-02 .AND.LE.    .85700E-02
  TRANSVERSE STRAIN.GE.  -.17600E-01 .AND.LE.    .47100E-02
       SHEAR STRAIN.GE.  -.18400E-01 .AND.LE.    .18400E-01


                        STIFFNESS LIMITS
                      A11.GE.  .50000E+06
                      A22.GE. 0.
                      A66.GE. 0.


                              LOADS
 LOAD COND.         NX             NY               NXY
     1          .20000E+05   0.               0.
     2          .15000E+05  -.15000E+05       .50000E+04
     3         -.15000E+05   .10000E+05       .10000E+05
     4         0.           0.                .20000E+05
```

Figure 5.- (0, ±45, 90) Graphite epoxy composite — Example 2.

```
                 DESIGN AND/OR ANALYSIS RESULTS

   TITLE
   0, 45, -45, 90 GRAPHITE EPOXY COMPOSITE - EXAMPLE 2


                     PLY INFORMATION
          PLY NO.     THICKNESS       PERCENT
             1        .15956E+00       27.56
             2        .18014E+00       31.16
             3        .18014E+00       31.16
             4        .58507E-01       10.12
                      ----------     ----------
       THICKNESS =    .57815E+00      100.00



                     PLY STRAINS
                S.F.  = SAFETY FACTOR
                EPL   = LONGITUDINAL STRAIN
                EPT   = TRANSVERSE STRAIN
                EPLT  = SHEAR STRAIN

                            LOAD COND.    1
PLY NO.     EPL          S.F.       EPT          S.F.        EPLT         S.F.
   1     .41561E-02     2.062   -.21131E-02     8.329   -.10912E-09   100.000
   2     .10215E-02     8.390    .10215E-02     4.611   -.62692E-02     2.935
   3     .10215E-02     8.390    .10215E-02     4.611    .62692E-02     2.935
   4    -.21131E-02     4.056    .41561E-02     1.133    .15167E-08   100.000

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =   .41561E-02   EPY =  -.21131E-02   EPXY =  -.10912E-09


                            LOAD COND.    2
PLY NO.     EPL          S.F.       EPT          S.F.        EPLT         S.F.
   1     .47019E-02     1.823   -.62811E-02     2.802    .23485E-02     7.835
   2     .38466E-03    22.280   -.19638E-02     6.962   -.10983E-01     1.675
   3    -.19638E-02     4.364    .38466E-03    12.245    .10983E-01     1.675
   4    -.62811E-02     1.364    .47019E-02     1.002   -.23485E-02     7.835

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =   .47019E-02   EPY =  -.62811E-02   EPXY =   .23485E-02


                            LOAD COND.    3
PLY NO.     EPL          S.F.       EPT          S.F.        EPLT         S.F.
   1    -.41736E-02     2.053    .47157E-02      .999    .46970E-02     3.917
   2     .26195E-02     3.272   -.20774E-02     8.472    .88893E-02     2.070
   3    -.20774E-02     4.125    .26195E-02     1.798   -.88893E-02     2.070
   4     .47157E-02     1.917   -.41736E-02     4.217   -.46970E-02     3.917

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =  -.41736E-02   EPY =   .47157E-02   EPXY =   .46970E-02


                            LOAD COND.    4
PLY NO.     EPL          S.F.       EPT          S.F.        EPLT         S.F.
   1    -.10912E-09   100.000    .32206E-09   100.000    .93939E-02     1.959
   2     .46970E-02     1.825   -.46970E-02     5.747   -.47357E-09   100.000
   3    -.46970E-02     1.825    .46970E-02     1.003   -.13359E-08   100.000
   4    -.58270E-09   100.000    .79560E-09   100.000   -.93939E-02     1.959

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =  -.10912E-09   EPY =   .32206E-09   EPXY =   .93939E-02
```

```
                 MEMBRANE STRESSES IN COMPOSITE
       LOAD COND.      SIGMA-X        SIGMA-Y          TAU-XY
           1          .34593E+05   0.                0.
           2          .25945E+05  -.25945E+05      .86482E+04
           3         -.25945E+05   .17296E+05      .17296E+05
           4          0.           0.              .34593E+05

                 COMPOSITE MEMBRANE STIFFNESSES
                   ACTUAL          REQUIRED
                   VALUE            VALUE          S.F.
           A11   .58090E+07      .50000E+06       11.62
           A22   .38556E+07    0.                100.00
           A66   .21290E+07    0.                100.00

              COEFICIENTS OF STRESS-STRAIN RELATIONSHIPS
                      RELATED TO STRUCTURAL AXES
   C11 =   .10048E+08      C12 =   .33908E+07     C16 =   .46600E-03
                           C22 =   .66689E+07     C26 =  -.18925E+00
           SYMMETRIC                              C66 =   .36825E+07

              COEFICIENTS OF STRAIN-STRESS RELATIONSHIPS
                      RELATED TO STRUCTURAL AXES
   Q11 =   .12014E-06      Q12 =  -.61086E-07     Q16 =  -.31545E-14
                           Q22 =   .18101E-06     Q26 =   .93100E-14
           SYMMETRIC                              Q66 =   .27156E-06

                    COMPOSITE ELASTIC CONSTANTS
    EX =   .83235E+07       EY =   .55246E+07     GXY =   .36825E+07
  NUXY =   .50845E+00     NUYX =   .33747E+00
```

Figure 5.- Concluded.

```
                         DESIGN

                           OF

                SYMMETRIC COMPOSITE PANEL

TITLE
 0, 30, -30, 60, -60, 90 GRAPHITE EPOXY COMPOSITE - EX. 3

          NO. OF PLYS                 =     6
          NO. OF LOAD CONDITIONS      =     4

        PLY PROPERTIES - ALL PLYS IDENTICAL
         LONGITUDINAL MODULUS =  .21000E+08
         TRANSVERSE MODULUS   =  .17000E+07
         SHEAR MODULUS        =  .65000E+06
         POISSON'S RATIO, L-T =  .21000E+00
         POISSON'S RATIO, T-L =  .17000E-01

PLY THICKNESSES, ORIENTATIONS,,AND DESIGN VARIABLE NUMBERS
     PLY NO.    THICKNESS       THETA     DES. VAR. NO.
        1       .10000E+00       0.00          1
        2       .10000E+00      30.00          2
        3       .10000E+00     -30.00          2
        4       .10000E+00      60.00          3
        5       .10000E+00     -60.00          3
        6       .10000E+00      90.00          4

                     PLY STRAIN LIMITS
LONGITUDINAL STRAIN.GE.  -.85700E-02 .AND.LE.   .85700E-02
  TRANSVERSE STRAIN.GE.  -.17600E-01 .AND.LE.   .47100E-02
       SHEAR STRAIN.GE.  -.18400E-01 .AND.LE.   .18400E-01

                   STIFFNESS LIMITS
                  A11.GE.  .50000E+06
                  A22.GE. 0.
                  A66.GE. 0.

                           LOADS
LOAD COND.       NX              NY              NXY
    1        .20000E+05  -0.               -0.
    2        .15000E+05   -.15000E+05      .50000E+04
    3       -.15000E+05    .10000E+05      .10000E+05
    4       0.            0.               .20000E+05
```

```
                     DESIGN AND/OR ANALYSIS RESULTS

TITLE
 0, 30, -30, 60, -60, 90 GRAPHITE EPOXY COMPOSITE - EX. 3

                       PLY INFORMATION
            PLY NO.    THICKNESS     PERCENT
               1       .11368E+00     21.35
               2       .91215E-01     17.13
               3       .91215E-01     17.13
               4       .11328E+00     21.28
               5       .11328E+00     21.28
               6       .97205E-02      1.83
                       ----------   ----------
         THICKNESS =   .53239E+00    100.00

                        PLY STRAINS
                   S.F.  = SAFETY FACTOR
                   EPL   = LONGITUDINAL STRAIN
                   EPT   = TRANSVERSE STRAIN
                   EPLT  = SHEAR STRAIN

                           LOAD COND.     1
PLY NO.     EPL        S.F.       EPT        S.F.       EPLT        S.F.
   1     .43339E-02   1.977  -.19309E-02    9.115  -.19195E-10  100.000
   2     .27677E-02   3.096  -.36468E-03   48.262  -.54255E-02    3.391
   3     .27677E-02   3.096  -.36468E-03   48.262   .54255E-02    3.391
   4    -.36468E-03  23.500   .27677E-02    1.702  -.54255E-02    3.391
   5    -.36468E-03  23.500   .27677E-02    1.702   .54255E-02    3.391
   6    -.19309E-02   4.438   .43339E-02    1.087   .12260E-08  100.000

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =   .43339E-02   EPY =  -.19309E-02   EPXY =  -.19195E-10

                           LOAD COND.     2
PLY NO.     EPL        S.F.       EPT        S.F.       EPLT        S.F.
   1     .46986E-02   1.824  -.60633E-02    2.903   .27193E-02    6.766
   2     .31856E-02   2.690  -.45503E-02    3.868  -.79604E-02    2.311
   3     .83061E-03  10.318  -.21953E-02    8.017   .10680E-01    1.723
   4    -.21953E-02   3.904   .83062E-03    5.670  -.10680E-01    1.723
   5    -.45503E-02   1.883   .31856E-02    1.479   .79604E-02    2.311
   6    -.60633E-02   1.413   .46986E-02    1.002  -.27193E-02    6.766

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =   .46986E-02   EPY =  -.60633E-02   EPXY =   .27193E-02

                           LOAD COND.     3
PLY NO.     EPL        S.F.       EPT        S.F.       EPLT        S.F.
   1    -.42159E-02   2.033   .45249E-02    1.041   .54386E-02    3.383
   2     .32433E-03  26.423  -.15298E-04  100.000   .10289E-01    1.788
   3    -.43857E-02   1.950   .46947E-02    1.003  -.48504E-02    3.794
   4     .46947E-02   1.825  -.43857E-02    4.013   .48504E-02    3.794
   5    -.15297E-04 100.000   .32433E-03   14.522  -.10289E-01    1.788
   6     .45249E-02   1.894  -.42159E-02    4.175  -.54387E-02    3.383
```

Figure 6.- (0, ±30, ±60, 90) Graphite epoxy composite — Example 3.

```
COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =  -.42159E-02   EPY =   .45209E-02   EPXY =   .54366E-02


                                LOAD COND,    4
PLY NO,      EPL         S,F,          EPT        S,F,        EPLT          S,F,
   1     -.19195E-10  100,000     .60884E-10  100,000    .10877E-01     1,692
   2      .47100E-02    1,820    -.47100E-02    3,737    .54386E-02     3,383
   3     -.47100E-02    1,820     .47100E-02    1,000    .54386E-02     3,383
   4      .47100E-02    1,820    -.47100E-02    3,737   -.54387E-02     3,383
   5     -.47100E-02    1,820     .47100E-02    1,000   -.54387E-02     3,383
   6     -.98674E-09  100,000     .10284E-08  100,000   -.10877E-01     1,692

COMPOSITE STRAINS REFERENCED TO STRUCTURAL AXIS
EPX =  -.19195E-10   EPY =   .60884E-10   EPXY =   .10877E-01


                  MEMBRANE STRESSES IN COMPOSITE
        LOAD COND,      SIGMA-X         SIGMA-Y         TAU-XY
            1          .37567E+05   0.              0.
            2          .28175E+05   -.28175E+05     .93917E+04
            3         -.28175E+05    .18783E+05     .18783E+05
            4          0.            0.             .37567E+05


                  COMPOSITE MEMBRANE STIFFNESSES
                    ACTUAL         REQUIRED
                    VALUE           VALUE          S,F,
            A11    .53648E+07     .50000E+06      10,73
            A22    .37784E+07    0.              100,00
            A66    .18587E+07    0.              100,00


             COEFFICIENTS OF STRESS-STRAIN RELATIONSHIPS
                    RELATED TO STRUCTURAL AXES
      C11 =    .10077E+08     C12 =    .31620E+07     C16 =    .84078E-04
                              C22 =    .70971E+07     C26 =   -.34145E-01
             SYMMETRIC                                C66 =    .34537E+07


             COEFFICIENTS OF STRAIN-STRESS RELATIONSHIPS
                    RELATED TO STRUCTURAL AXES
      C11 =    .11537E-06     C12 =   -.51398E-07     C16 =   -.51096E-15
                              C22 =    .16380E-06     C26 =    .16207E-14
             SYMMETRIC                                C66 =    .28955E-06


                    COMPOSITE ELASTIC CONSTANTS
       EX =    .86681E+07     EY =    .61049E+07     GXY =    .34537E+07
     NUXY =    .44553E+00   NUYX =    .31378E+00
```

Figure 6.- Concluded.

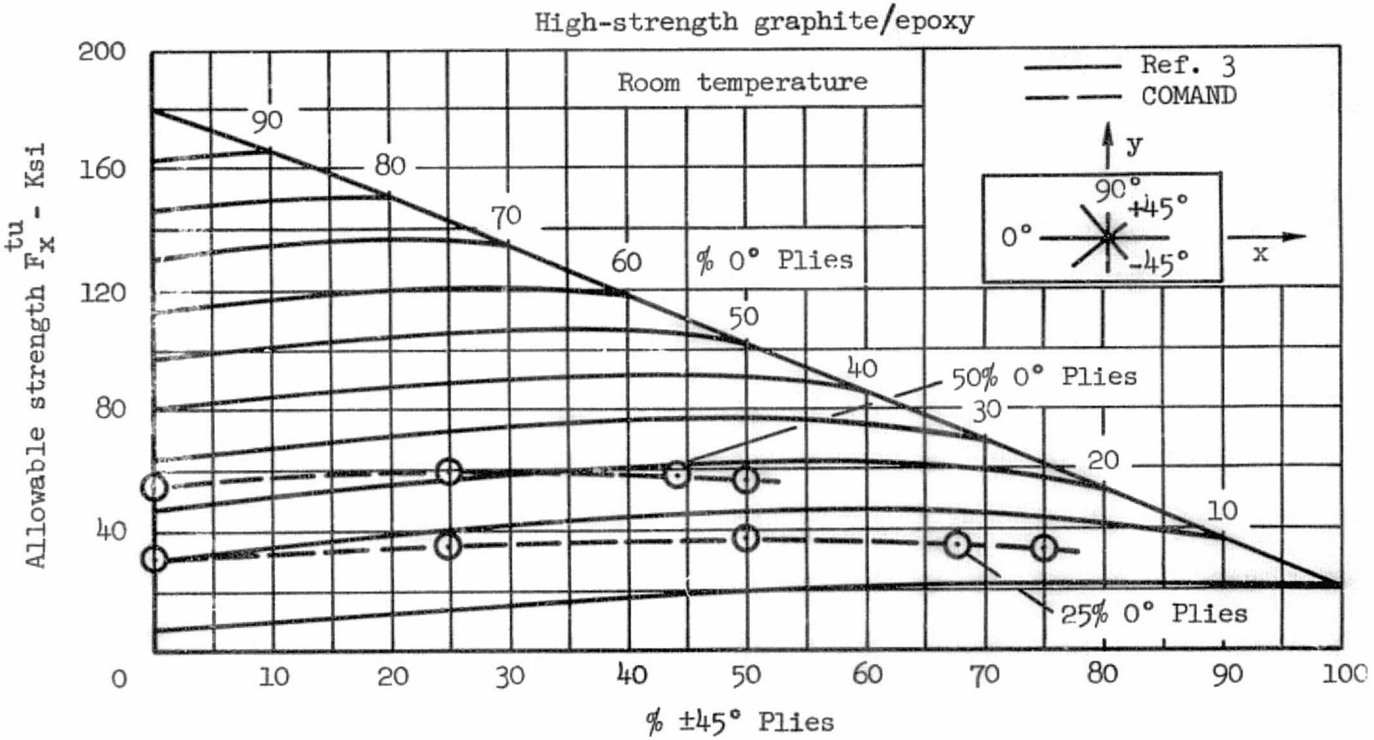


Figure 7.- Ultimate tensile strength F_x^{tu} high-strength graphite/epoxy - $\left[0_i/\pm45_j/90_k\right]^x$ family.

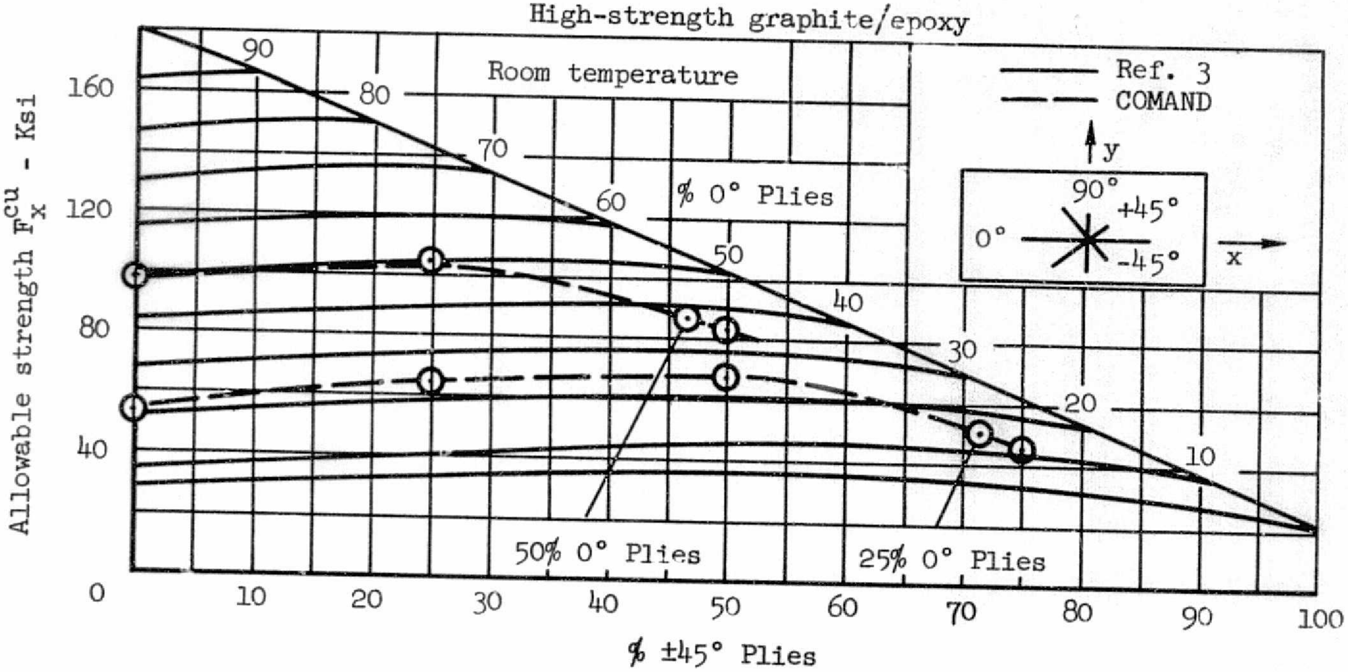


Figure 8.- Ultimate compressive strength F_x^{cu} high-strength graphite/epoxy - $[0_i/\pm45_j/90_k]$ family.

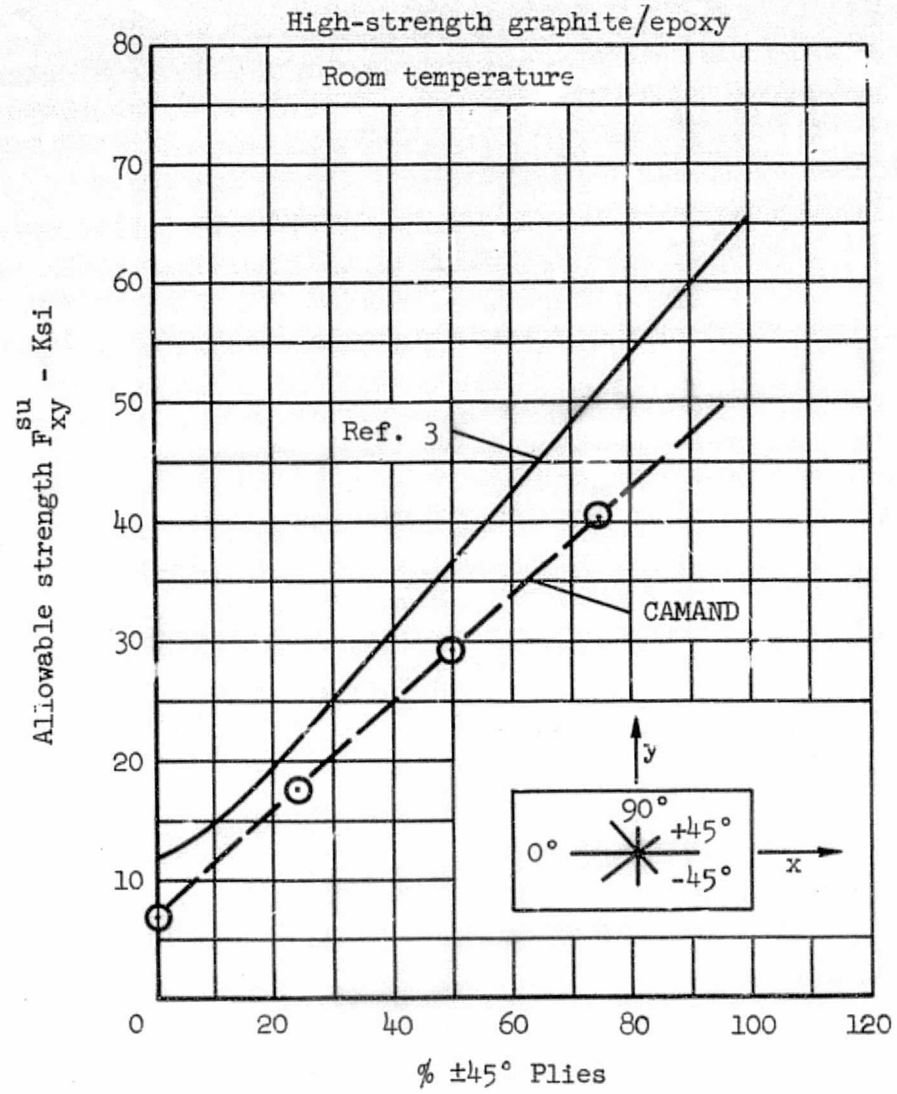


Figure 9.- Ultimate shear strength F_{xy}^{su} high-strength graphite/epoxy - $[0_i/\pm45_j/90_k]$ family.

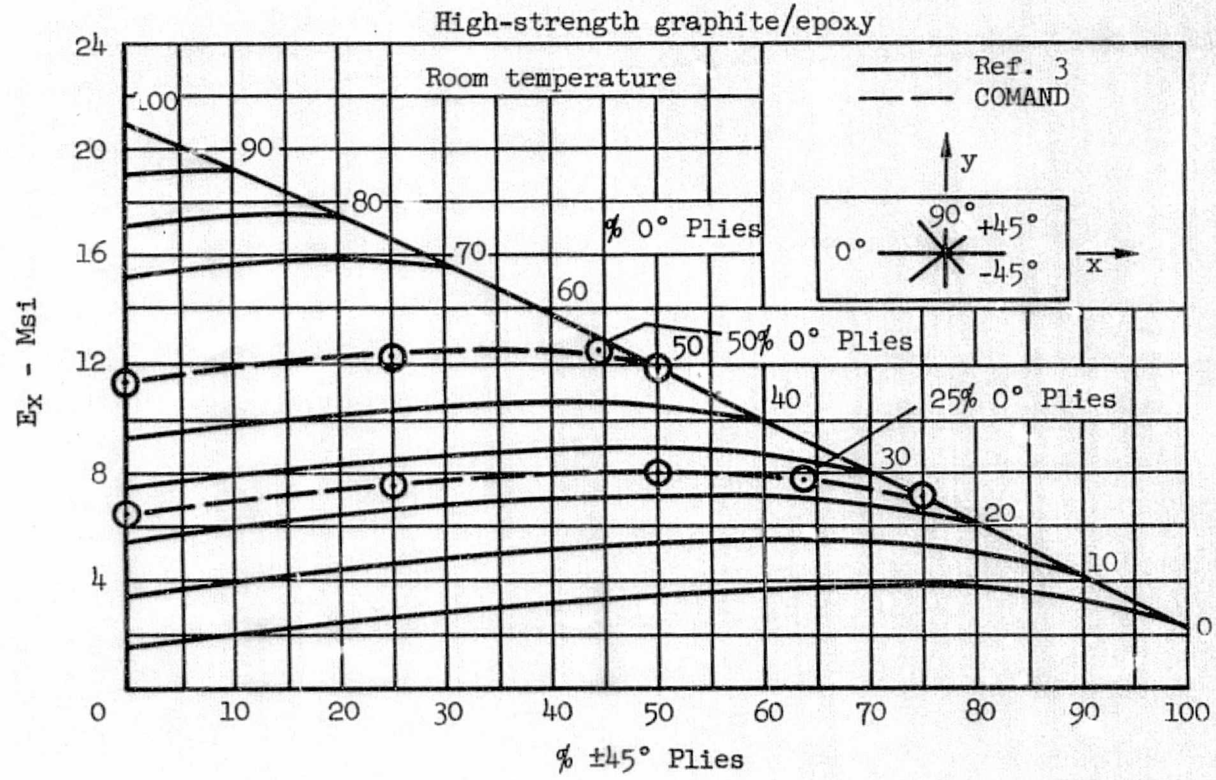


Figure 10.- Extensional modulus E_x high-strength graphite/epoxy - $\left[0_i/\pm45_j/90_k\right]$ family.

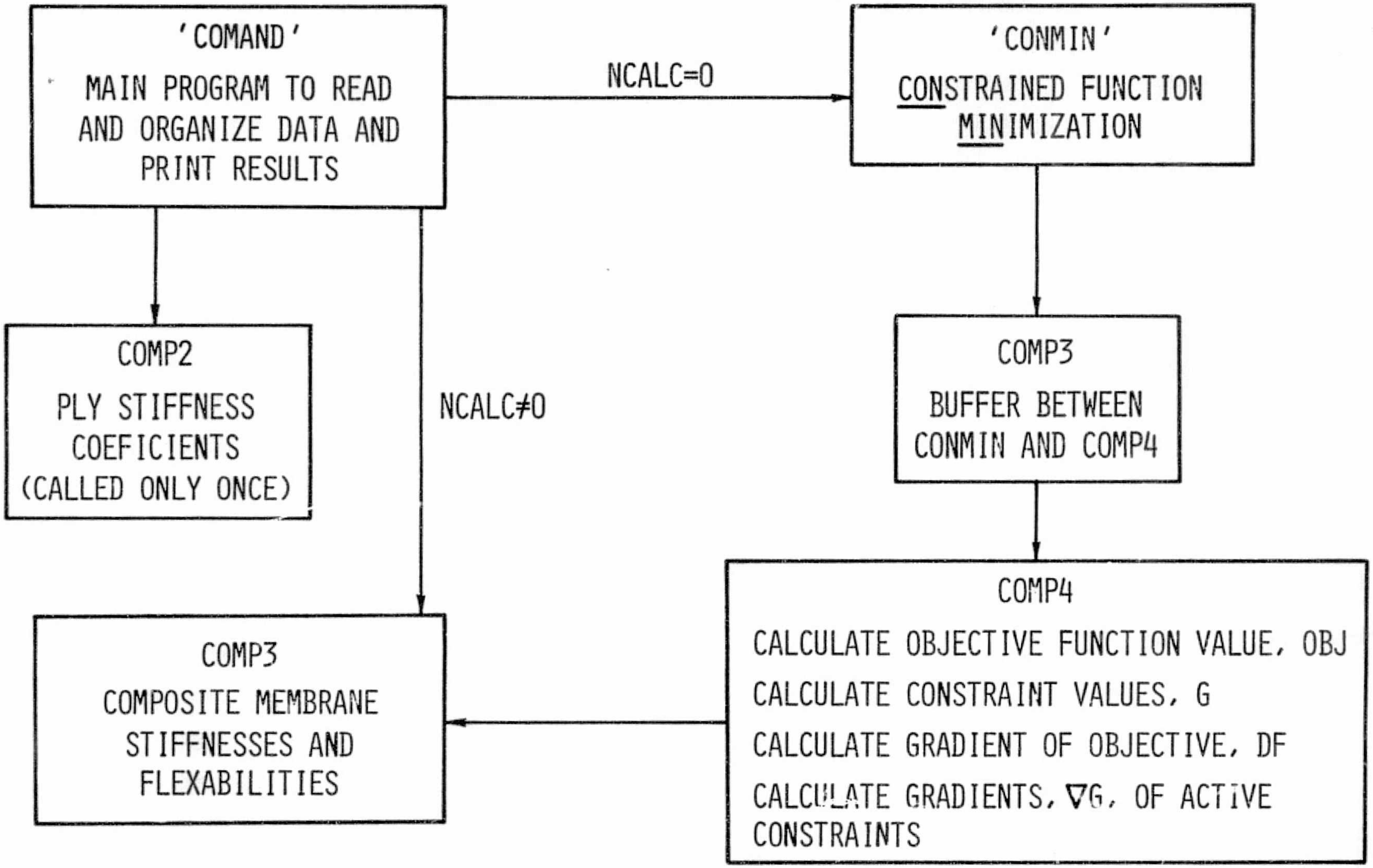


FIGURE 11.- 'COMAND' BLOCK DIAGRAM.