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SOLAR MODULATION OF GALACTIC COSMIC RAYS 4:

LATITUDE DEPENDENT MODULATION

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ABSTRACT

A numerical method is outlined for solving the equation which describes the solar modulation of cosmic rays in models where interplanetary conditions can vary with heliocentric latitude. As an illustration of the use of this method, it is shown how variations in the modulation with latitude could produce the small radial gradients in the intensity that have been observed from the Pioneers 10 and 11 spacecraft.

INTRODUCTION

In Fisk (1971), a numerical method is outlined for solving the equation which governs the solar modulation of galactic cosmic rays in a spherically-symmetric model for the interplanetary medium. In the last few years this method has been used in numerous studies of the modulation problem. It is the purpose of the present paper to extend this method to solve the modulation equation in models where interplanetary conditions vary with heliocentric latitude.

The cosmic-ray flux that is observed in the solar equatorial plane may well depend sensitively on interplanetary conditions at other heliocentric latitudes. For example, observed cosmic rays may have entered the inner solar system over the solar poles, and then diffused across the mean magnetic field into the equatorial plane. The distance that must be travelled by particles following field lines from the interstellar medium to the inner solar system is much shorter over the poles. The magnetic field in this region should lie nearly in the heliocentric radial direction, whereas in the equatorial plane it is wound in a tight spiral. It is also conceivable that cosmic rays may tend to diffuse out of the region near the equatorial plane into the mid-latitude regions, above and below the plane. Solar activity appears to be enhanced in the mid-latitude regions, and thus the cosmic-ray intensity there may be depressed.

As an illustration of the use of the numerical method outlined here, it is shown how latitude variations in the modulation can affect the radial gradient in the intensity seen in the equatorial plane. In particular, it is shown how these variations could produce the small gradients which are measured from Pioneers 10 and 11 (Teegarden et al., 1973; McKibben et al., 1973; Van Allen, 1972 a, b).

THE METHOD

Consider a model in which interplanetary conditions vary with heliocentric radial distance r and polar angle θ; the latter is measured relative to the axis of rotation of the sun. In steady-state conditions, the cosmic-ray omni-directional distribution function f (number of particles per unit volume of phase space, averaged over particle direction) behaves in such a model according to the equation (Parker, 1965; Gleeson and Axford, 1967; Jokipii and Parker, 1970; Fisk et al., 1973):

$$-\frac{p}{3r^2}\frac{\partial}{\partial r}(r^2 V)\frac{\partial f}{\partial p} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \kappa_r \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\, \kappa_\theta \frac{\partial f}{\partial \theta}\right) - V\frac{\partial f}{\partial r} \quad (1)$$

The distribution function f is related to the differential intensity j, per unit interval of kinetic energy T, by $j = f/p^2$ (Forman, 1970).

The term on the left side of (1) and the third term on the right side describe the effects of convection and adiabatic deceleration in the expanding solar wind. Here p is the magnitude of particle momentum, and $V(r,\theta)$ is the solar wind speed. It is assumed that the solar wind flows only in the radial direction. The first and second terms on the right side of (1) describe the effects of diffusion in the radial and polar directions, respectively. The diffusion coefficient κ_r can also be expressed as

$$\kappa_r = \kappa_{\parallel} \cos^2\psi + \kappa_{\perp} \sin^2\psi \quad (2)$$

Here, $\kappa_{\parallel}$ is the diffusion coefficient for propagation parallel to the mean magnetic field, which is assumed to lie on cones of constant θ and to make an angle ψ with the radial direction. The diffusion coefficient $\kappa_{\perp}$ describes the propagation normal to the mean field, along cones of constant θ. In most cases, $\kappa_\perp$ equals κ_θ, the cross-field diffusion coefficient for propagation in the polar direction. Gradient and curvature drifts of the particles in the large-scale interplanetary field are not considered here. However, the effects of such motion could be included in (1) simply by adding a term $\underline{v}_D \cdot \nabla f$, where $\underline{v}_D$ is the drift velocity.

Equation (1) is a parabolic partial differential equation. As such, it can be readily solved by using the Crank-Nicholson implicit finite difference technique, which was developed for solving space-time diffusion equations. Momentum in (1) is the analogue of time in the simple diffusion equation. In spherically-symmetric models, where the second term on the right side of (1) is ignored, (1) can be solved by a straight-forward application of the Crank-Nicholson technique, as is done in Fisk (1971). In models where latitude variations are included, the appropriate technique is the alternating - direction modification of the Crank-Nicholson method, which was suggested by Douglas (1962) and elaborated on by Douglas and Gunn (1964). A useful discussion of this method can be found in Carnahan et al. (1969)

In finite-difference schemes a value for f is obtained at a series of grid points f (I, J, K). Here $\theta = I \cdot \Delta\theta$, $r = J \cdot \Delta r + r_o$, and $p = K \cdot \Delta p + p_o$. The spacings between grid points are $\Delta\theta$, Δr, and Δp; $\Delta\theta = \pi/M$, where M is the largest value of I. Boundary conditions must be specified at I = 0 and I = M, corresponding to $\theta = 0$ and π, respectively. Similarly, boundary conditions must be specified at $r = r_o$, the smallest radial distance considered, and at $R = N \cdot \Delta r + r_o$, the largest value of r. An "initial" condition must be given at $p = p_o$, i.e. f (I, J, 0) must be given at all I and J.

In the simple Crank-Nicholson technique, derivatives with respect to θ are replaced with finite difference equations which relate f (I-1, J, K), f (I, J, K) and f (I + 1, J, K) to f (I-1, J, K + 1), f (I, J, K + 1) and f (I + 1, J, K + 1). Similarly, derivatives with respect to r relate f (I, J-1, K), f (I, J, K) and f (I, J + 1, K) to their counterparts at K + 1. Derivatives with respect to p involve f (I, J, K) and f (I, J, K + 1). By starting with the known values of f at K = 0, the value of f at larger K would then have to be determined in this scheme by solving at each step M·N simultaneous linear equations. For most applications this procedure is impractical.

In the alternating-direction modification to the Crank-Nicholson technique, a two-step procedure is followed. In the first step the derivatives with respect to θ are defined in terms of f (I-1, J, K), f (I, J, K) and f (I + 1, J, K), and intermediate values of f* (I-1, J, K + 1), f* (I, J, K + 1), and f* (I + 1, J, K + 1). The derivatives with respect to p also relate f (I, J, K) to f* (I, J, K + 1). The derivatives with respect to r, however, involve only f (I, J-1, K), f (I, J, K) and f (I, J + 1, K). To determine f*, then, requires the solution of M simultaneous, linear equations, N times, which is a more tractable procedure. In the second step, the derivatives with respect to θ remain defined in terms of f at K, and f*, but now the derivatives with respect to r and p are defined in terms of f at K and the required values of f at K + 1. To determine f at K + 1, N simultaneous, linear equations must be solved M times. By starting with f at K = 0, this two-step procedure is repeated at each value of K. A fast algorithm for solving the simultaneous equations can be found in Diaz (1958).

For the boundary conditions at θ = 0 and π it is appropriate to require simply that $\partial f/\partial\theta = 0$. Some caution must be exercised in imposing this condition, however, since (1) contains a term which is proportional to $(1/\tan\theta)\partial f/\partial\theta$. With an application of L'Hospital's rule, $(1/\tan\theta)\ \partial f/\partial\theta = \partial^2 f/\partial\theta^2$ at $\theta = 0$ and π. The maximum radial distance R is chosen so that at $r > R$ the modulation is negligibly small. The boundary condition at $r = R$ is then $f(I, N, K) = f_o(p)$, where $f_o(p)$ is the unmodulated, interstellar distribution function. Cosmic rays are assumed here to impinge on the solar cavity isotropically. For the boundary condition at $r = r_o$, which is assumed to be a value small compared with 1 AU, it is appropriate to require that $\partial f/\partial r = 0$.

It should be noted that in Fisk (1971) the boundary condition at $r = r_o$ was treated with considerable care. The differential number density U was scaled by $r^{1/2}$ or $u = r^{1/2}\ U$, and then u was set equal to zero at $r = 0$. In retrospect, this precaution was unnecessary. The solution near earth, for example, is relatively insensitive to the boundary condition at $r = r_o$. It suffices then simply to choose a condition which is easy to program, and which is not physically unrealistic.

The initial momentum p_o is chosen so that at $p > p_o$ the modulation is small. The initial condition is then $f(I, J, 0) = f_o(p_o)$. The solution at values of $p < p_o$ is determined by stepping down in momentum, i.e. Δp is taken to be negative. In this regard it is convenient to change variables in (1) from p to ln p. The spacing between grid points is now $\Delta \ln p$, which is taken to be a constant. At large values of p, then, where there is little modulation, the steps in momentum ($\sim p\Delta\ell n p$) are large. At small p, where f varies rapidly, the step size in p is automatically reduced.

AN ILLUSTRATION EXAMPLE

Measurements from the Pioneer 10 and 11 spacecraft have shown that the radial gradient of the cosmic-ray intensity is small. Teegarden et al. (1973) and McKibben et al. (1973) find that the integral proton gradient (the gradient of protons with energies above ∿ 60 MeV) is only about 4%/AU between 1 and 3 AU. Van Allen (1972 a, b) reports a gradient which is consistent with zero over this distance. McKibben et al. (1975) find that the integral gradient remains at ∿ 4%/AU out to at least 5 AU. McDonald et al. (1975) have argued recently that the corrections for Jovian electrons in their previous analysis, as well as in the analyses of the other experiments, were inadequate. With these corrections taken into proper account, McDonald et al. (1975) suggest that all previous gradient measurements could be reduced.

At the radial distances sampled by Pioneer the principle contribution to the integral proton intensity (protons above ∿ 60 MeV in energy) comes from particles with energies ∿ 1 GeV (e.g., McKibben et al. 1973). Equivalently, the observed integral gradient is a measure of the gradient of these high energy particles. In spherically-symmetric models for the interplanetary medium the high energy gradient can, in turn, be related to the radial diffusion coefficient by the relationship

$$\frac{1}{j}\frac{\partial j}{\partial r} \simeq \frac{CV}{\kappa_r} \qquad (3)$$

where $C = -\partial \ln f/\partial \ln p$ is the Compton-Getting coefficient (Gleeson and Axford, 1968 a; Fisk and Axford, 1969, 1970; Fisk et al., 1973).

The radial diffusion coefficient can, in principle, also be determined from the observed properties of the interplanetary magnetic field. In the last few years there has been contraversy as to exactly how this calculation should be performed (cf. Fisk et al., 1974). Most of the disagreement, however, is concerned with the behavior of particles at relatively low energies. For high energy particles, where the particle gyro-radius r_g is greater than, or on the order of the correlation length λ of the field fluctuations, most theories appear to be in reasonable agreement (e.g. Jokipii, 1966; Klimas and Sandri, 1971).

In the review article by Jokipii (1971), for example, the parallel diffusion coefficient is given as

$$\kappa_{\parallel} \simeq \frac{v\, r_g^{\,2}\, B_o^{\,2}}{P_{xx}(k = 0)} \quad , \quad \text{for } r_g \gtrsim \lambda \tag{4}$$

Here, P_{xx} $(k = 0)$ is the power density in fluctuations normal to the mean field direction, evaluated at zero wavenumber; v is particle speed; and B_o is the mean field strength. The perpendicular diffusion coefficient in this limit is

$$\kappa_{\perp} \simeq \frac{v\, P_{xx}\ (k = 0)}{2\, B_o^{\,2}} \quad , \quad \text{for } r_g \gtrsim \lambda \tag{5}$$

The observed power density at small wavenumbers yields P_{xx} $(k = 0)$ $\simeq 1 - 2\cdot10^2$ gauss^{2}cm (See, for example, the power spectra given in Jokipii and Coleman (1968), or in Fisk and Sari (1973)). The mean field strength near $r = 1$ AU is $B_o \simeq 4.5\cdot10^{-5}$ gauss. Thus, (4) and (5) yield respectively

$$\kappa_{\parallel} \simeq 2\cdot10^{21}\ \beta R^2 \tag{6}$$

and

$$\kappa_{\perp} \simeq 1.2\cdot10^{21}\ \beta \tag{7}$$

where R is particle rigidity in units of GV, and $\beta = v/c$, with c the speed of light. These values for $\kappa_{\parallel}$ and $\kappa_{\perp}$ agree closely with the results given in Jokipii (1971). With a typical correlation length of $\lambda \sim 1.5 \cdot 10^{11}$ cm (Jokipii and Coleman 1968; Fisk and Sari, 1973), (6) and (7) should be valid for particles with rigidities $R \gtrsim 2$ GV, or equivalently, for protons with energies $T \gtrsim 1$ GeV. The solar wind speed is $V \simeq 400$ km/sec. Thus, by use of (3), (6) and (7) predict a gradient for 1-GeV protons, or equivalently an integral gradient, of $\sim$ 15%/AU.

Conceivably, (6) and (7) could be small by a factor of 4. These expressions are evaluated here at T = 1 GeV, which is at the lower limit of their range of validity. An observed integral gradient of $\sim$ 4%/AU, is then not necessarily inconsistent with theory. However, if the integral gradient is in fact small compared with 4%/AU, as is indicated in the observations of Van Allen (1972 a, b) or is implied in the suggestion of McDonald et al. (1975), then there appears to be a conflict between the theory which is applied in spherically-symmetric models, and observation.

One possible solution to this problem is to disgard the assumption of a spherically-symmetric interplanetary medium. If there are more particles above and below the solar equatorial plane, and these particles tend to diffuse into the region of the plane preferentially at small radial distances, such a process reduces the radial gradient. The spiral pattern of the interplanetary field is, in fact, less tightly wound above and below the equatorial plane. Among other results, this effect increases the term $\cos^2\psi$ in (2) at higher latitudes, and thus it may reduce the modulation here, and increase the particle density. The particles above and below the plane will diffuse onto the plane preferentially at small r, provided that κ_θ increases less rapidly than proportional to r^2, as can be seen from (1).

There is another criterium which must be satisfied for a latitude-dependent modulation model to be acceptable. The observed proton spectrum has a slope near unity at energies $\lesssim$ 80 MeV, i.e. $\partial \ln j/\partial \ln T \approx 1$ or $\partial \ln f/\partial \ln p \approx 0$ (e.g. Rygg and Earl, 1971). In this energy range, then, the terms on the right of (1) must all sum to nearly zero, or they must all be individually small. For the case considered here, a balance of the terms does not seem to be possible. Particles are diffusing into the region near the equatorial plane, and thus the second term on the right of (1) is positive. If, after entering the equatorial plane, these particles tend to diffuse outward in radial distance, the first term in (1) can be negative. However, since particles also diffuse inward along the equatorial plane from the interstellar medium, this first term can never cancel the second. Circumstances in which the first and third terms can be made individually small are discussed in detail in Fisk et al. (1973). The second term can be made adequately small by requiring that κ_θ becomes small at low energies.

Consider then the following model. For particles with $r_g \geq \lambda$ $\kappa_\parallel$ and $\kappa_\perp$, as are given in (4) and (5), are used. For particles with $r_g \leq \lambda$, $\kappa_\parallel$ is taken to be the form given in Jokipii (1971), which is expressed here as

$$\kappa_\parallel \simeq \frac{v\, r_g^{\,2}\, B_o^{\,2}}{P_{xx}(k = 0)}\left(\frac{\lambda}{r_g}\right)^{3/2}, \text{ for } r_g \leq \lambda \qquad (8)$$

It is assumed in (8) that the power spectrum of field fluctuations falls off as $k^{-3/2}$ for large k. It is also recognized that (8) yields a value for $\kappa_\parallel$ which is probably too small at low energies, and thus it will tend to overestimate the radial gradient. For example, with $P_{xx}(k = 0) = 1.5\cdot 10^{2}\,\text{Gauss}^2\text{cm}$, $B_o = 4.5\cdot 10^{-5}$ Gauss, and $\lambda = 1.5\cdot 10^{11}$cm, (8) predicts that the mean free path for a 10 MeV proton is $\lambda_{mfp} \sim 0.015$ AU. Solar flare observations, however, appear to indicate that $\lambda_{mfp} \sim 0.1$ AU at these energies (Ma Sung et al., 1975).

Particles are assumed to diffuse across the mean field direction by scattering off of fluctuations which have scale-sizes comparable to r_g. As can be seen from the formula given in Jokipii (1971), $\kappa_\perp$ is then

$$\kappa_\perp = \kappa_\theta \simeq \frac{v\, P_{xx}(k=0)}{2B_o^2}\left(\frac{r_g}{\lambda}\right)^{3/2}, \text{ for } r_g \lesssim \lambda \tag{9}$$

This form has the required behavior that κ_θ becomes small at low energies.

Cross-field diffusion in which particles follow random-walking field lines across the mean field direction is thus ignored here. This latter process, which is discussed by Jokipii and Parker (1969), supposedly yields a diffusion coefficient that is roughtly equal to (7) at all energies. In practice, however, as is indicated by observations of solar particles, $\kappa_\perp$ at low energies is nowhere near this large (Krimigis et al., 1971).

The quantity $P_{xx}(k = 0)/B_o^2$ is taken to be a constant equal to $7.4 \cdot 10^{10}$cm, independent of r and θ. (Note that near earth $P_{xx}(k=0) = 1.5 \cdot 10^2$ Gauss^{2}cm and $B_o = 4.5 \cdot 10^5$ Gauss.) The cross-field diffusion coefficient given in (5) is then constant, or equivalently κ_θ/r^2 varies as $1/r^2$. Higher energy particles will thus enter the region of the equatorial plane preferentially at small r. This behavior for $P_{xx}(k = 0)/B_o^2$ can result provided that the correlation length, and the amplitude of the fluctuations relative to the mean field are both constants. Thus, λ is taken to be $1.5 \cdot 10^{11}$cm, independent of r and θ. The solar wind speed is taken to be similarly constant at V = 400km/sec.

The mean magnetic field is assumed to execute the Archimedes spiral pattern appropriate for each latitude (Parker, 1963). The field is normalized so that at r = 1 AU, $B_o = 4.5 \cdot 10^{-5}$ Gauss, and $\psi = 45°$. In the model considered here the variation in the field magnitude and direction with polar angle θ is solely responsible for the latitude dependence of the modulation.

The outer edge of the modulating region is placed at R = 25 AU. The unmodulated interstellar spectrum is taken to be $f_o \propto (T_o + c^2p^2)^{-1.3}$, p, where T_o is particle rest energy. With this form, the spectrum for the differential number density is given by a power law in total energy, with spectral index -2.6. The unmodulated spectrum is normalized to be equal to the unmodulated spectrum that was used by Urch and Gleeson (1972) in their extensive study of the modulation.

With the above parameters and upon assuming that the particles are protons, (1) has been solved numerically by using the technique outlined in the previous section. Shown in Figure 1 is the calculated differential proton intensity near earth. As can be seen here, this spectrum provides a good fit to the proton spectrum that is observed in 1972, at the beginning of the Pioneer 10 mission. The calculated spectrum also provides a reasonable fit to the spectra observed by McKibben et al. (1975) at later dates, from Pioneers 10 and 11. Also shown in Figure 1 is the calculated differential radial gradient, determined here between the distances 1 and 5 AU, in the equatorial plane. Rather than yielding a gradient ∿ 15%/AU, (6) and (7) in this latitude-dependent model yield a gradient for 1-GeV protons of < 1%/AU. The integral proton gradient (protons above 60 MeV in energy) is in fact ∿ 0.6%/AU. Clearly, variations in the modulation with heliocentric latitude could be responsible for the small gradients seen from Pioneers 10 and 11.

There is some additional evidence which supports the explanation for the small gradients given here. Axford et al. (1975) report that the azimuthal anisotropy seen from Pioneers 10 and 11 for particles with energies $\geq$ 480 MeV/nucleon is smaller than expected. The observed anisotropy is $\xi_\phi = 0.46 \pm .11\%$, whereas the theory, as it is applied in spherically-symmetric models, can yield $\xi_\phi > 1\%$. Axford et al. (1975) suggest that an unusually large ratio of $\kappa_\perp/\kappa_\parallel$ for particles with energies $\gtrsim$ 1 GeV/nucleon could be responsible for the reduced anisotropy. An alternative explanation is that particles diffusing onto the equatorial plane from higher latitudes, as in the model discussed here, cause an outward radial streaming which in turn diminishes ξ_ϕ.

As can be seen in the discussion in Fisk (1974), in steady-state conditions

$$\xi_\phi \approx \tan \psi \left(\frac{3CV}{v} - \xi_r \right) \qquad (10)$$

where ξ_r is the radial anisotropy. It is assumed in (10) that $\kappa_\perp/\kappa_\parallel << 1$. In the model given here, for example, $\kappa_\perp/\kappa_\parallel < 10^{-2}$ for particles above 1 GeV in energy, at $r \gtrsim 3$ AU. Also the azimuthal and polar gradients are assumed to be small; the latter assumption is appropriate since in the model given here the modulation is symmetric about the equatorial plane.

In spherically-symmetric models, $\xi_r \approx 0$ for particles with energies greater than several hundred MeV/nucleon (Gleeson and Axford, 1968 b; Fisk and Axford, 1969). At r = 3 AU, for example, $\tan \psi \approx 3$. With $C \approx 1$ and $V = 400$ km/sec, (10) then yields in such models $\xi_\phi \approx 1.4\%$ for 1 GeV protons. In contrast, in the latitude dependent model considered here ξ_r is positive, as can be seen in Figure 1. At r = 3 AU in the equatorial plane, ξ_r shown here is $\sim 0.3\%$ for 1 GeV protons. With the same values for ψ, C and V, (10) then yields in this case $\xi_\phi \approx 0.48\%$, in good agreement with the observations of Axford et al. (1975).

The radial anisotropy predicted here would be difficult to detect from the Pioneer spacecraft. The spin axes of these spacecraft point in the radial direction. Also, a detailed examination of the numerical solution obtained here reveals that the radial anisotropy is less important near 1 AU. Thus, this anisotropy is not expected to influence the diurnal anisotropy that is seen by neutron monitors.

CONCLUDING REMARKS

In addition to providing an explanation for the small gradients seen from Pioneers 10 and 11, the example presented here also illustrates how sensitive the cosmic-ray intensity observed near earth may be to latitude-variations in the modulation. Recall that the variation in modulation with latitude in this model is caused solely by the expected latitude variations in the direction and magnitude of the interplanetary magnetic field. This effect alone, however, introduces considerable polar gradients in the cosmic-ray intensity, and as a result, significantly alters the flux seen near earth. Shown in Figure 2 is the calculated intensity at 1 AU, plotted as a function of polar angle θ, for two different energies. The curves marked $\kappa_\theta = 0$ are solutions to (1) with the parameters given in the previous section, but now with polar diffusion ignored. The variation in the modulation conditions with latitude in this case causes the low-energy intensity to vary by nearly three orders of magnitude between the polar end equational regions. The curves marked $\kappa_\theta \neq 0$ are plots of the solution obtained in the previous section with polar diffusion included. Note in particular the 25 MeV curve. The polar diffusion coefficient is extremely small in this case ($\kappa_\theta \sim 7 \cdot 10^{18}\ cm^2 sec^{-1}$), and yet polar diffusion can alter the intensity near earth by a factor $\sim$ 5.

It is reasonable to expect that the cosmic-ray modulation observed near earth will be understood only when information is available on modulating conditions at other heliocentric latitudes.

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FIGURE CAPTIONS

Figure 1 A plot vs. kinetic energy of the differential intensity near earth, the differential radial gradient in the equatorial plane between 1 and 5 AU, and the differential radial anisotropy in the equatorial plane at 3 AU. These curves are obtained from the numerical solution to (1) which is discussed in the text. The data points are the observed proton intensity during quiet-times in mid - 1972. These points, which were obtained from the GSFC experiment on IMP-5, were kindly provided by M. A. Van Hollebeke. Also shown is the unmodulated interstellar spectrum which was used in the numerical example.

Figure 2. A plot vs. polar angle θ of the differential intensity at 1 AU, for two energies: T = 1 GeV and 25 MeV. The curves marked $\kappa_\theta = 0$ are solutions to (1), obtained by using the parameters described in the text, but with polar diffusion ignored. The curves marked $\kappa_\theta \neq 0$ are the corresponding solutions with polar diffusion included.

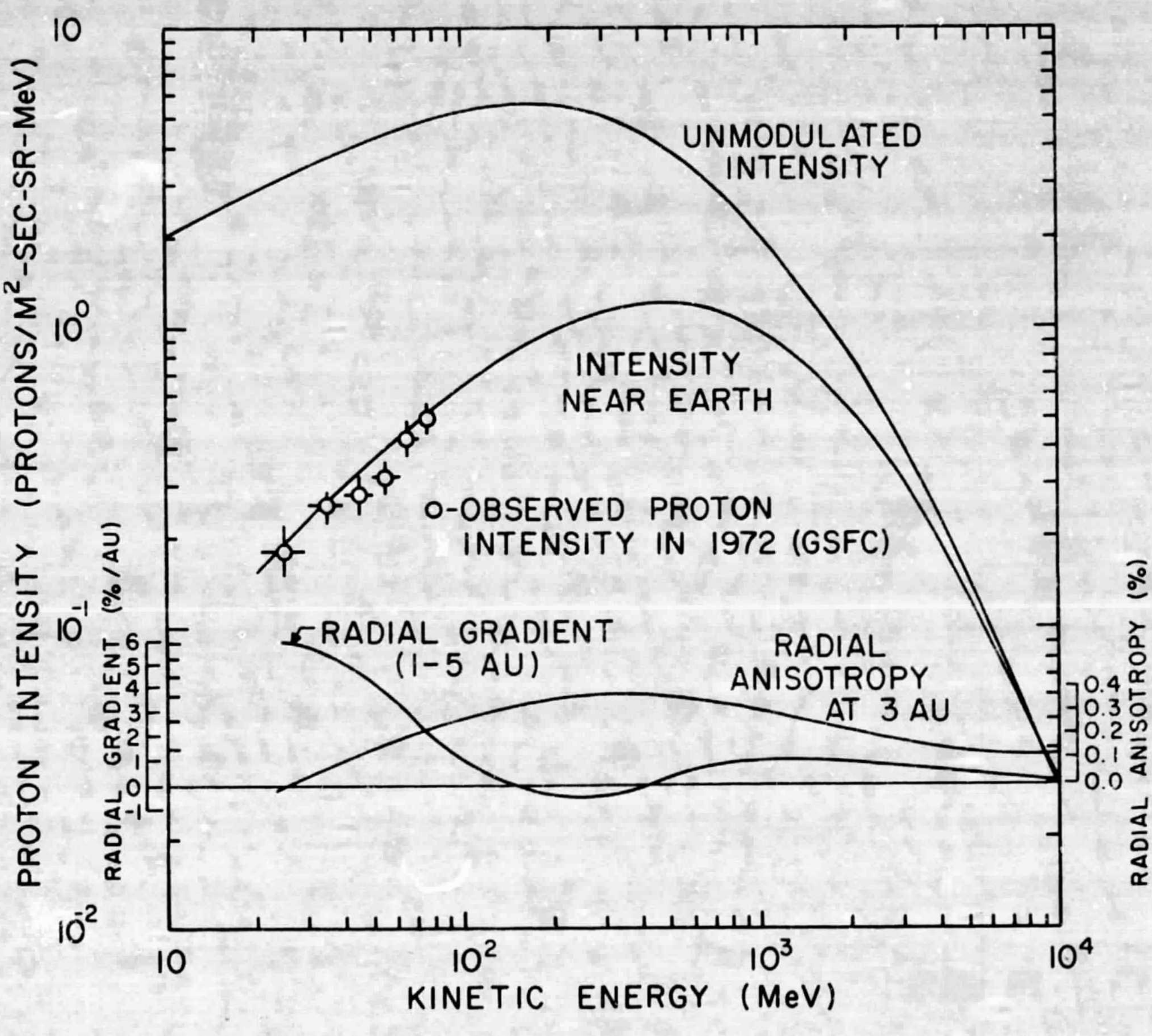

FIGURE 1

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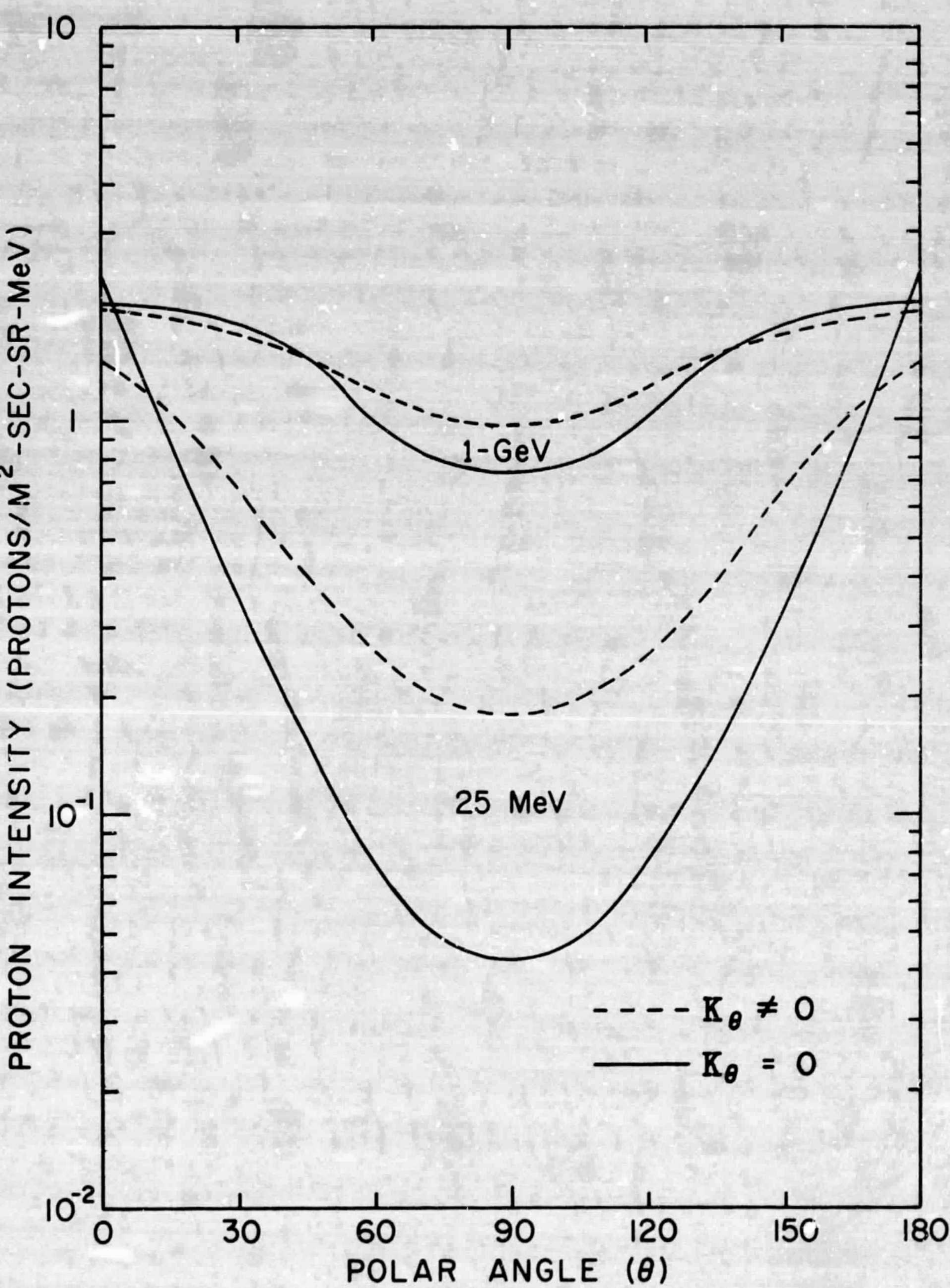

FIGURE 2

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