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TRANSIENT EFFECT OF LUBRICANT ON ELASTOHYDRODYNAMIC FILM THICKNESS

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16. Abstract

The inlet solution of the elastohydrodynamic lubricated rolling contact problem was obtained by considering lubricants with transient viscosity. The effect of the viscoelastic retardation time of the lubricant on the center film thickness was investigated. The effect of transient viscosity in response to a sudden pressure was insignificant in determining the film thickness in elastohydrodynamic contacts. For the transient effects to become important in film thickness calculations, the retardation time would have to be at least three decades higher than those suggested by other investigators.

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TRANSIENT EFFECT OF LUBRICANT ON ELASTOHYDRODYNAMIC FILM THICKNESS

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SUMMARY

The inlet solution of Elastohydrodynamic lubricated rolling contact problem was obtained considering lubricants with transient viscosity. The effect of the viscoelastic retardation time of lubricant on the center film thickness was investigated.

1. The effect of transient viscosity in response to a sudden pressure was found to be insignificant in determining the film thickness in elastohydrodynamic contacts.

2. For the transient effects to become important in film thickness calculation, the retardation time would have to be at least three decades higher than those suggested by Harrison and Trachman in reference 9.

INTRODUCTION

In lubrication of concentrated contacts such as rolling-element bearings, gears, and cams, it has been found by recent work on elastohydrodynamic (EHD) lubrication that the contacting surfaces are usually separated by a continuous oil film. The level of this film thickness in elastohydrodynamic (EHD) contacts can be predicted by EHD Theories developed by Grubin (ref. 1), Dowson and Higginson (ref. 2), Archard and Cowking (ref. 3), Crook (ref. 4) and Cheng (ref. 5). Similar to the hydrodynamic theories in journal bearings, the minimum film thickness in EHD contacts was found not only to decrease with load and increase with speed and shear viscosity but also to be affected strongly by the pressure-viscosity dependence of the lubricant. In fact, it is because of this drastic increase in viscosity at high pressures, that contacting surfaces are separated by the hydrodynamic action of the lubricant.

With regard to the accuracy of predicting the film thickness, the present EHD theories is only limited to moderately heavy loads and moderately high speeds. Recent work (refs. 6 and 8), have shown that there still exist large discrepancies between the isothermal EHD Theories and X-ray experiments for heavily loaded contacts. The inclusion of heating effects in the inlet of EHD contacts (ref. 7) accounts for some of the discrepancies, but the thermal theory does not predict a load dependence as strong as that measured by X-ray experiments.

In searching for other possible reasons for this discrepancy, Bell and Kannel (ref. 8) suggested that the use of pressure-viscosity coefficients based on static measurements is invalid, because the increase in viscosity due to pressure rise in the high speed and heavily loaded cases may not behave in the same manner as measured in the static experiment. They developed a

Grubin-type inlet EHD theory assuming a short time-delay in the rise of viscosity with pressure. However, in their theory the selection of the time-delay constant is completely arbitrary, and what rheological mechanism governing the time-delay constant for a particular lubricant has not been studied.

More recently, Harrison and Trachman (ref. 9) proposed a Transient pressure-viscosity model which enables one to predict the effective viscosity in the contact as a function of time. Using this theory, they have shown that the calculated effective viscosity as a function of rolling speed correlates very well with that measured by Johnson and Cameron (ref. 10) in the friction experiments.

The object of this work is to incorporate Harrison and Trachman's transient pressure-viscosity model into the isothermal EHD Theory developed by Cheng (ref. 6), and to ascertain whether this transient pressure-viscosity effect will have a strong influence on the film forming capability in heavily loaded EHD contacts.

TRANSIENT VISCOSITY

Doolittle's Empirical Relation

Viscosity is a measure of fluid resistance to deformation and it depends on the state of fluid. Doolittle (ref. 11) adopted the idea that shear viscosity depends on the free volume of the fluid, which is defined as the free volume is the space when the liquid is expanded to a state from the state of absolute zero temperature. If ν_o is the specific volume of liquid at absolute zero temperature and ν is the specific volume at normal state, then the relative free volume is defined as

$$f \equiv \frac{v - v_o}{v_o} \tag{1}$$

By performing a series of experiments, Doolittle found the following empirical relationship between viscosity and relative free volume.

$$\eta_s = A \operatorname{Exp}(B/f) \tag{2}$$

or

$$\ln \eta_s = B/f + \ln A \tag{3}$$

where A and B are material constants differed for each different liquid and B is usually very close to unity. This simple relationship will be used in later analysis to calculate the viscosity for a given state of free volume.

Free Volume Viscosity and its Relation with Shear Viscosity

The liquid structure can be interpreted by assuming that it is composed by a large number of crystal-like group of molecules. These groups of molecules undergo continuous breaking and reforming. Also, the atoms which should be in the neighborhood of some other atoms could be missing and thus produce a hole in that plate. The presence of holes adds an additional structural contribution to the volume response of liquid when pressure or temperature is changed rapidly. If the pressure or temperature is suddenly changed, the liquid volume will undergo contraction or expansion and all molecules will rearrange themselves and producing more holes or filling up some holes. The latter process takes time to reach a new equilibrium state. By means of this structural relaxation process, the state of liquid after changes can be determined only when time scale is given.

In order to describe this time-dependent behavior of liquid volume change, the following two simple models (Fig. 1) are used.

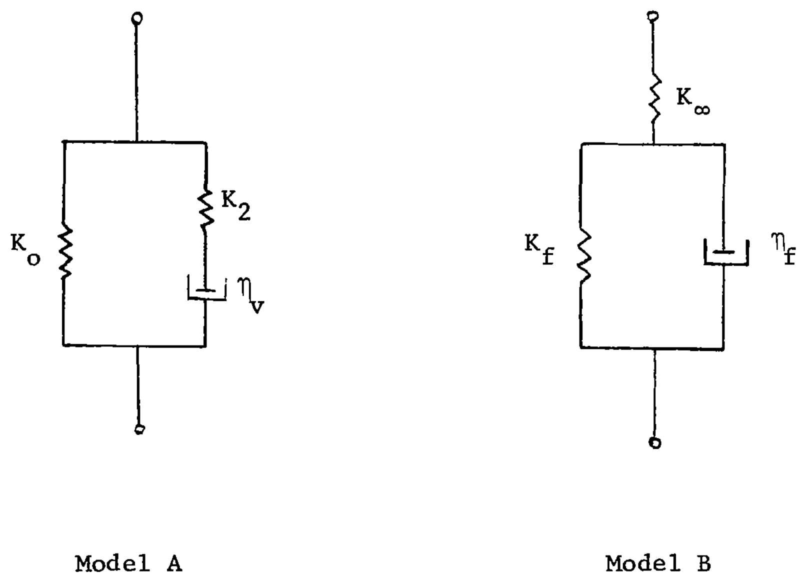


Fig. 1 Models for compressional viscoelasticity

Model A is a generalized Maxwell element with one relaxation time constant and model B is a special Kelvin element. Model A is convenient to correlate with experimental results and model B is good for later mathematical analysis.

In model A, when a constant deformation γ_o is imposed, the stress p(t) follows

$$p(t) = \left[K_o + K_2 \exp(-t/\tau)\right] \gamma_o \tag{4}$$

where $\tau = \frac{\eta_v}{K_2}$ is called relaxation time and in which η_v is the volume viscosity, K_2 is the difference of instantaneous bulk modulus K_∞ and equilibrium bulk modulus K_o.

By setting $t = 0$ in the time dependent modulus in equation (4), one can easily get the instantaneous bulk modulus $K_\infty = K_o + K_2$. When $t = \infty$, this time dependent modulus becomes the steady bulk modulus K_o as can be seen in equation.

In model B, if a pressure p_o is imposed at time $t = 0$, the volume creep $\gamma(t)$ can be written as

$$\gamma(t) = \left\{\frac{1}{K_\infty} + \frac{1}{K_f}\left[1 - \exp(-t/\bar{\tau})\right]\right\} p_o \tag{5}$$

$\bar{\tau}$ is called retardation time, defined by $\bar{\tau} = \frac{\eta_f}{K_f}$ where η_f is the free-volume viscosity and K_f is the free volume bulk modulus.

The instantaneous bulk compressibility $\frac{1}{K_\infty}$ can be obtained by setting $t = 0$ in time dependent bulk compressibility of equation (5). Also, the reciprocal of the steady bulk modulus is equal to $\frac{1}{K_\infty} + \frac{1}{K_f}$ by simply inserting $t = \infty$ in equation (5).

A comparison of the modulus between two models yields

$$\frac{1}{K_o} = \frac{1}{K_\infty} + \frac{1}{K_f} \tag{6}$$

$$K_\infty = K_o + K_2 \tag{7}$$

Apply oscillatory bulk deformation and pressure to both models, one can get the complex bulk modulus as a function of frequency.

For model A

$$K = K_o + K_2(i\omega) = K_o + K_2 \frac{i\omega\tau}{1 + i\omega\tau} \tag{8}$$

For model B

$$\frac{1}{K} = \frac{1}{K_\infty} + \frac{1}{K_f(1 + i\omega\tau)} \tag{9}$$

relate equation (6) and (7)

$$\frac{K_f}{K_o} = \frac{K_\infty}{K_2} \tag{10}$$

relate equation (8) and (9)

$$\eta_f = \eta_v \left(\frac{K_\infty}{K_2}\right)^2 \tag{11}$$

Thus, there are two fixed equations (equation (10) and (11)) governing the relationships between the parameters of these two models.

By measuring the propagation velocity and absorption coefficient of ultrasonic waves propagated through liquid, Litovitz and Davis (ref. 12) obtained a method for calculating volume viscosity η_v. They found that volume viscosity is direct proportional to shear viscosity η_s, and it has the same temperature and pressure dependence as the shear viscosity. Since free volume viscosity η_f is proportional to volume viscosity η_v for a given state of liquid by adapting equation (11) where assuming the ratio $\frac{K_\infty}{K_2}$ is known, it can be concluded that the free volume viscosity η_f is proportional to shear viscosity η_s.

Transient Response of Shear Viscosity to a Single Pressure Step

A method originally derived by Kovac (ref. 13) for solving bulk creep behavior will be used here to calculate the transient shear viscosity of fluid after a finite imposed pressure step. Following his analysis, liquid having initial specific volume v_1 will change to final equilibrium volume v_2 if there is enough time for change. With given value of P, the governing equation by using model B is

$$v_1 - v_2 = v_1 P/K_o \tag{12}$$

If the instantaneous volume change is $v_1 - v_i$ which is equal to $v_1 P/K_\infty$, equation (12) can be written as

$$(v_1 - v_i) + (v_i - v_2) = v_1 P/K_\infty + v_1 P/K_f \tag{13}$$

it follows

$$\frac{v_i - v_2}{v_1} = \frac{P}{K_f} \tag{14}$$

The time dependent part of volume change in model B can be solved from the differential equation considering force balance in parallel spring and dashpot combination

$$P = \frac{\eta_f}{v_1}\frac{dv}{dt} + K_f \frac{v_i - v}{v_1} \tag{15}$$

substitute the value of P in equation (14) into (15)

$$\frac{\eta_f}{K_f}\frac{dv}{dt} = v_2 - v \tag{16}$$

for a finite change of pressure, η_f can't be considered as a constant since η_f is a function of dependent variable v. The governing equation becomes non-linear and it is difficult to solve. However, it was assumed in a previous section that free volume viscosity η_f is proportional to shear viscosity η_s and both depend on free volume in the Doolittle's empirical equation

$$\ln \eta_f = \ln A' + B/f \tag{17}$$

where constant B remains the same and close to unity. Define a parameter s such that

$$s = \ln\left(\frac{\eta_{f_2}}{\eta_f}\right) = B(1/f_2 - 1/f) \tag{18}$$

where f_2 is the final relative free volume for imposed pressure P and η_{f_2} is the final equilibrium free volume viscosity. Equation (17) can be written in terms of parameter s.

$$\frac{\exp(-s)}{s(1 - s\, f_2/B)}\, ds = -\frac{dt}{\tau_2} \tag{19}$$

where τ_2 is a retardation time defined as

$$\tau_2 = \frac{\eta_{f_2}}{K_f} \tag{20}$$

and it will be evaluated at final equilibrium state. The term sf_2/B in equation (19) is much less than unity so that $(1 - sf_2/B)^{-1}$ can be expanded and equation takes the form

$$\frac{\exp(-s)}{s}\, ds + \exp(-s)\, \frac{f_2}{B}\, ds = \frac{-dt}{\tau_2} \tag{21}$$

For a given value of P, this equation can be solved numerically for s and by the relationship

$$\eta_f = \eta_{f_2} \exp(-s)$$

thus

$$\eta_s = \eta_{s_2} \exp(-s) \tag{22}$$

It will give the time-dependent transient shear viscosity for liquid subject to a single pressure step.

Transient Response of Shear Viscosity to a Continuous Pressure Change

In EHD problems, the lubricant moving through the gap between two rollers will experience a continuous pressure change from atmospheric pressure up to 4×10^5 psi within a very short time. Since this externally applied pressure is a continuous one instead of a instantaneous pressure jump, the analysis used in the previous section cannot be used here directly. However, by approximating the continuous pressure input as a series of pressure steps as shown in Fig. 2, the previous method for solving the transient shear viscosity can be used repeatedly and successively within each single step. In this case, equation (21) can be written as

$$\frac{\exp(-s_j)}{s_j}\, ds_j + \exp(-s_j)\, \frac{f_{j2}}{B}\, ds_j = -\frac{dt}{\tau_{j2}} \tag{23}$$

and

$$\eta_{s_j} = \eta_{s_{j2}} \exp(-s_j) \tag{24}$$

where variables with subscript j means it belonging to the j th pressure step.

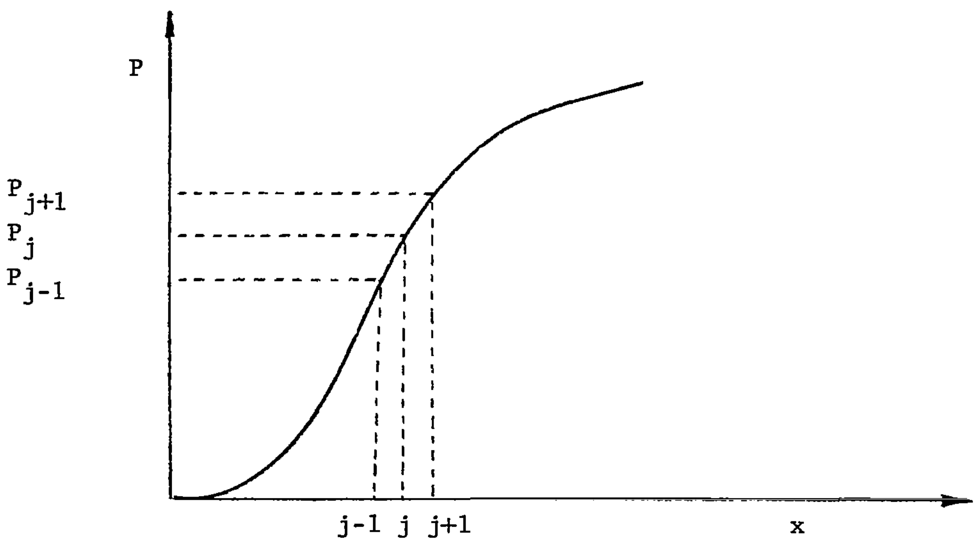


Fig. 2 Approximating the Pressure Distribution by Pressure Steps

s_j for pressure step j varies from initial value s_{j1} to final value s_{jf}. From relationship

$$s_{j1} = \ln\left(\frac{\eta_{s_{j2}}}{\eta_{s_{j1}}}\right) = \ln\left(\frac{\eta_{s_{j2}}}{\eta_{s_{j-1,2}}}\ \frac{\eta_{s_{j-1,2}}}{\eta_{s_{j1}}}\right) \tag{25}$$

since viscosity is continuous between each adjointing steps

$$\eta_{s_{j1}} = \eta_{s_{j-1,f}} \tag{26}$$

Equation (25) becomes

$$s_{j1} = \ln\left(\frac{\eta_{s_{j2}}}{\eta_{s_{j-1,2}}}\right) + \ln\left(\frac{\eta_{s_{j-1,2}}}{\eta_{s_{j-1,f}}}\right) \tag{27}$$

$$= F(P_j,\ P_{j-1}) + s_{j-1,f}$$

thus, initial value of s_j for jth step can be derived from equation (27) once $s_{j-1,f}$ is found in the previous stage. Referring to Harrison and Trachman (ref. 9) retardation time for most oils can be expressed as a function of the equilibrium shear viscosity η_{s_2} and the pressure as follows

$$\tau_2 = \frac{50\ \eta_{s_2}}{3.5 \times 10^5 + 9P} \tag{28}$$

for the jth pressure step, it becomes

$$\tau_{j2} = \frac{50\ \eta_o \exp(\alpha \cdot P_j)}{3.5 \times 10^5 + 9P_j} \tag{29}$$

finally, after substituting equation (27), (29) into equation (23) and approximating ds_j by

$$ds_j \cong s_{jf} - s_{j1} \tag{30}$$

one obtains,

$$0.5 \times \left[\exp(s_{j1})(f_{j2} + 1/s_{j1}) + \exp(s_{jf})(f_{j2} + 1/s_{jf}) \right]$$

$$(s_{jf} - s_{j1}) = - \frac{T_j \; (3.5 \times 10^5 + 9P_j)}{50 \; \eta_o \; \exp(\alpha \, P_j)} \qquad (31)$$

Equation (31) is solved for s_{jf} by using Newton's method.

GOVERNING EQUATIONS FOR FILM THICKNESS

In formulating the elastohydrodynamic equations, the following assumptions are used:

1. The rollers, as shown in Fig. 3, are subject to pure rolling.
2. The deformation is purely elastic.
3. The Hertzian width is much smaller than the width of the disks and the side leakage is neglected. Also, the Hertzian width is small in comparison with the disk radius so that the deformation can be calculated by the half-plane solution.
4. The lubricant is isothermal and the inertia of lubricant is negligible.

Equations governing deformation and pressure are

$$h = h^* + \frac{x^2 - x^{*2}}{R} - \frac{4}{\pi E'} \int_{-\infty}^{x_f} \ln \frac{|\xi - x|}{|\xi - x^*|} \, P(\xi) d\xi \qquad (32)$$

$$\frac{dp}{dx} = 12 \, \eta_s U \left(\frac{h - h^* \rho^*/\rho}{h^3} \right) \qquad (33)$$

In non-dimensional form, above two equations become

$$\frac{dP}{d\bar{x}} = \left(\frac{48}{H^{*2}} \right) \bar{U} \, \bar{\eta}_s \left(\frac{H - \bar{\rho}^*/\bar{\rho}}{H^3} \right) \qquad (34)$$

$$H = 1 + \frac{16 \, \bar{P}_{HZ}^{\,2}}{H^*} \left(\frac{\bar{x}^2}{2} - \frac{1}{\pi} \int_{-\infty}^{\bar{x}_f} P(\xi) \ln \frac{|\bar{\xi} - \bar{x}|}{|\bar{\xi} - \bar{x}^*|} \, d\bar{\xi} \right) \qquad (35)$$

The dependence of the equilibrium viscosity on pressure is assumed to be of the Barus form

$$\eta_{s_2} = \eta_o \exp(\alpha P) \qquad (36)$$

it follows from Equation (24) that transient viscosity η_s becomes

$$\eta_s = \eta_o \exp(\alpha P - s) \tag{37}$$

Density change as a function of pressure is assumed as follows:

$$\rho = \rho_o \left(1 + \frac{cP}{1 + d\,P}\right) \tag{38}$$

where ρ_o is the ambient density, c and d are constants from ASME Report (ref. 14).

Equations (34), (35), (36) and (38) coupled with equation (31) can be solved by numerical method outlined in Appendix B and C.

RESULTS AND DISCUSSION

Typical numerical results were obtained for a run with a load parameter $\bar{P}_{HZ}$ equals to 0.012 and non-dimensional center film thickness H_c equals to 10^{-5}. A typical value of G(G = 3000) is chosen to illustrate the transient effect of the lubricant. The resulting speed parameter $\bar{U}$ for this case is equal to 1.0307×10^{-11} which is very close to the value obtained by Cheng (ref. 6) without considering the transient viscosity. The results of inlet film thickness and pressure distribution for this run are plotted in Fig. 4. The ratio of transient viscosity to equilibrium viscosity as a function of the inlet position is plotted in Fig. 5. As can be seen in this figure, the viscosity ratio remains very close to unity over most of the inlet region. This shows that for typical conditions encountered in an elastohydrodynamic contact the response of lubricant viscosity to pressure is almost immediate in the inlet region. Since the film formation of an elastohydrodynamic contact takes place almost entirely in the inlet region, the transient characteristics of viscosity produce little effect on film thickness. However, in the center region, where the pressure is high, the lubricant viscosity does not respond to the pressure rise immediately. Since the frictional force in an EHD contact is largely governed by the viscosity in the center region, the transient effects become significant in the EHD traction calculation, as shown by Harrison and Trachman (ref. 9).

In order to determine at what level of retardation time τ_2 the lubricant viscosity effects will become significant, a set of arbitrary multiplication factors $M = 10^2$, 10^3, 10^4 and 10^5 is introduced for τ_2. Results for load parameter $\bar{P}_{HZ}$, from 0.003 to 0.012 and normalized center film thickness H_c from 10^{-6} to 10^{-5} are shown in Table 1 and also are plotted in Figs. 5(a) to 6(d) as a function of the rolling speed $\bar{U}$. It is found that for $M = 10^2$

Table 1. OBTAINED NUMERICAL DATA

$H^* = h^*/R$	$\overline{P}_{HZ} = p_{hz}/E'$		Values of Multiplication Factor M 10^2	10^3	10^4	10^5
0.00001	0.003		5.022901×10^{-12}	5.093904×10^{-12}	5.885855×10^{-12}	1.087195×10^{-11}
	0.006		7.212976×10^{-12}	7.455977×10^{-12}	9.659420×10^{-12}	2.136174×10^{-11}
	0.012		1.035142×10^{-11}	1.101366×10^{-11}	1.701148×10^{-11}	4.167230×10^{-11}
0.000005	0.003		1.860423×10^{-12}	1.867464×10^{-12}	2.011805×10^{-12}	3.098025×10^{-12}
	0.006		2.717763×10^{-12}	2.755632×10^{-12}	3.190901×10^{-12}	6.026983×10^{-12}
	0.012	$\overline{U}$ =	3.956260×10^{-12}	4.078289×10^{-12}	5.167988×10^{-12}	1.183219×10^{-11}
0.000002	0.003		4.991350×10^{-13}	4.983464×10^{-13}	5.104673×10^{-13}	6.361610×10^{-13}
	0.006		7.454592×10^{-13}	7.487535×10^{-13}	7.915690×10^{-13}	1.167518×10^{-12}
	0.012		1.097072×10^{-12}	1.105953×10^{-12}	1.230753×10^{-12}	2.195556×10^{-12}
0.000001	0.003		1.835414×10^{-13}	1.830417×10^{-13}	1.844844×10^{-13}	2.055017×10^{-13}
	0.006		2.788145×10^{-13}	2.794191×10^{-13}	2.861793×10^{-13}	3.573406×10^{-13}
	0.012		4.038028×10^{-13}	4.066247×10^{-13}	4.253334×10^{-13}	6.173326×10^{-13}

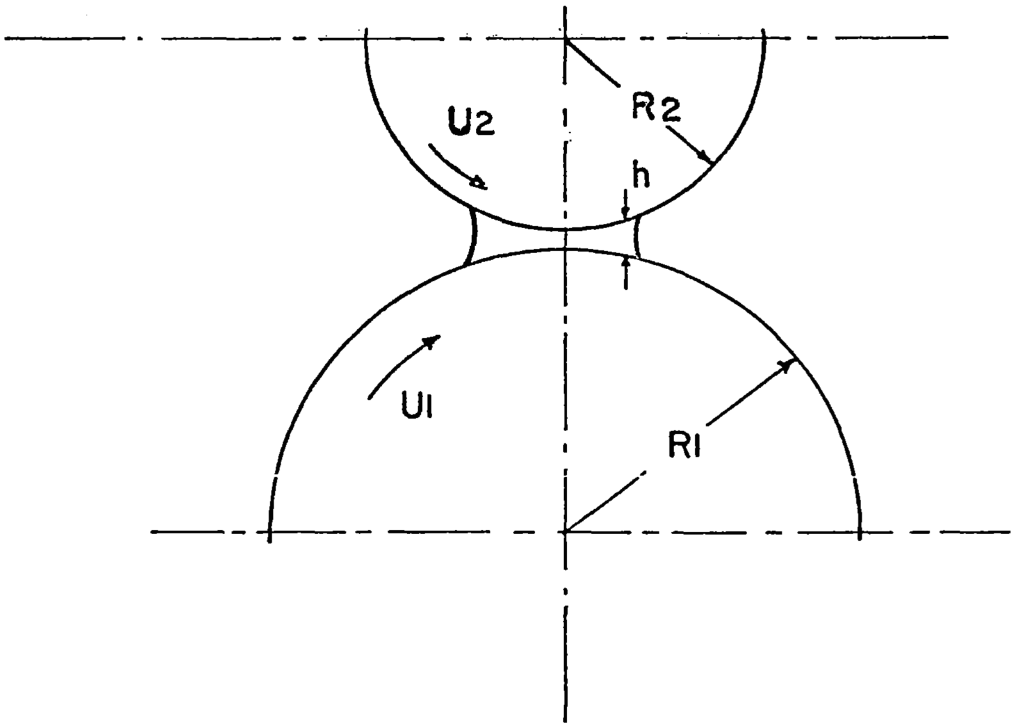


Figure 3 - Geometry of lubricated rollers.

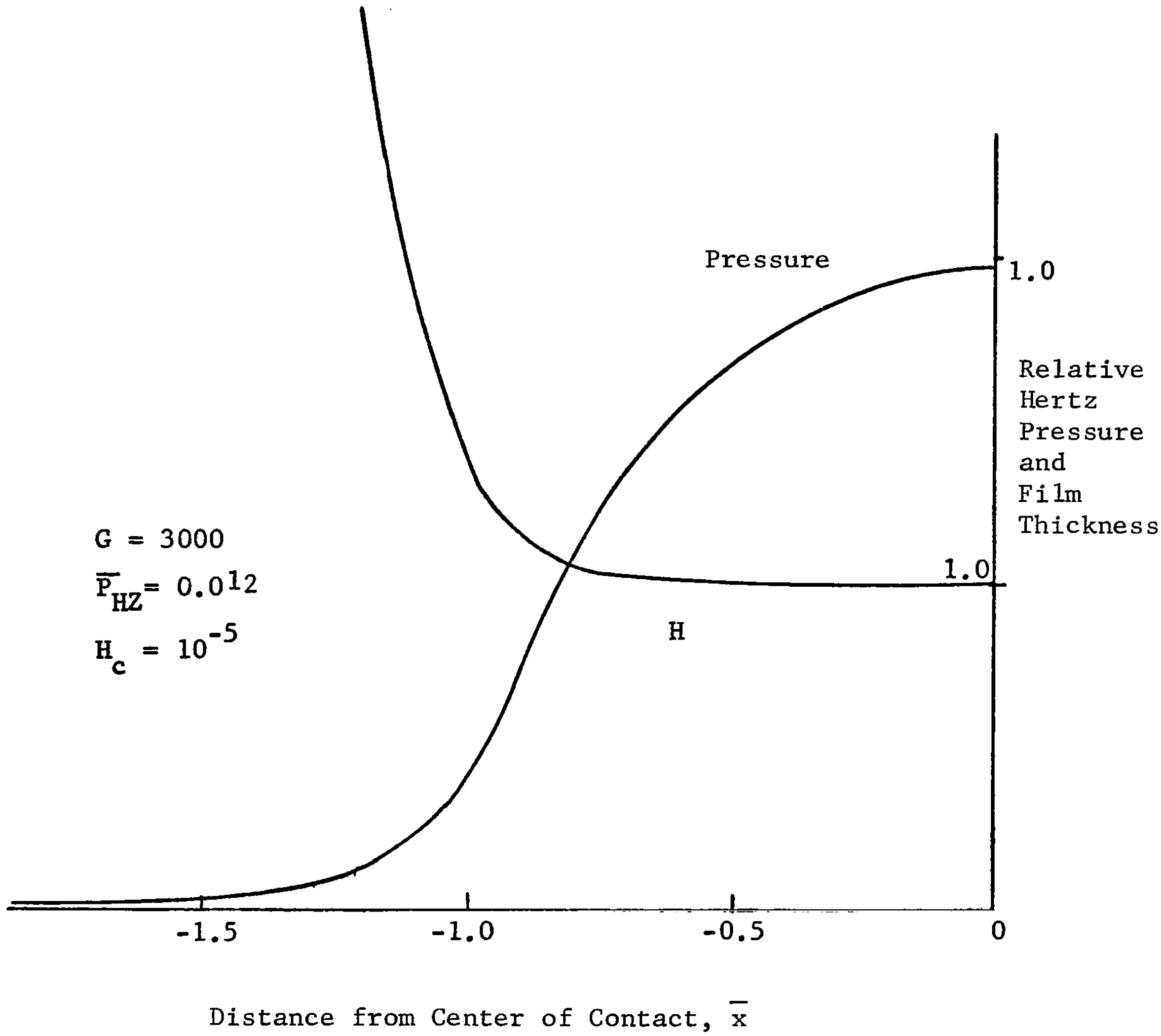


Figure 4 TYPICAL INLET FILM THICKNESS AND PRESSURE DISTRIBUTION

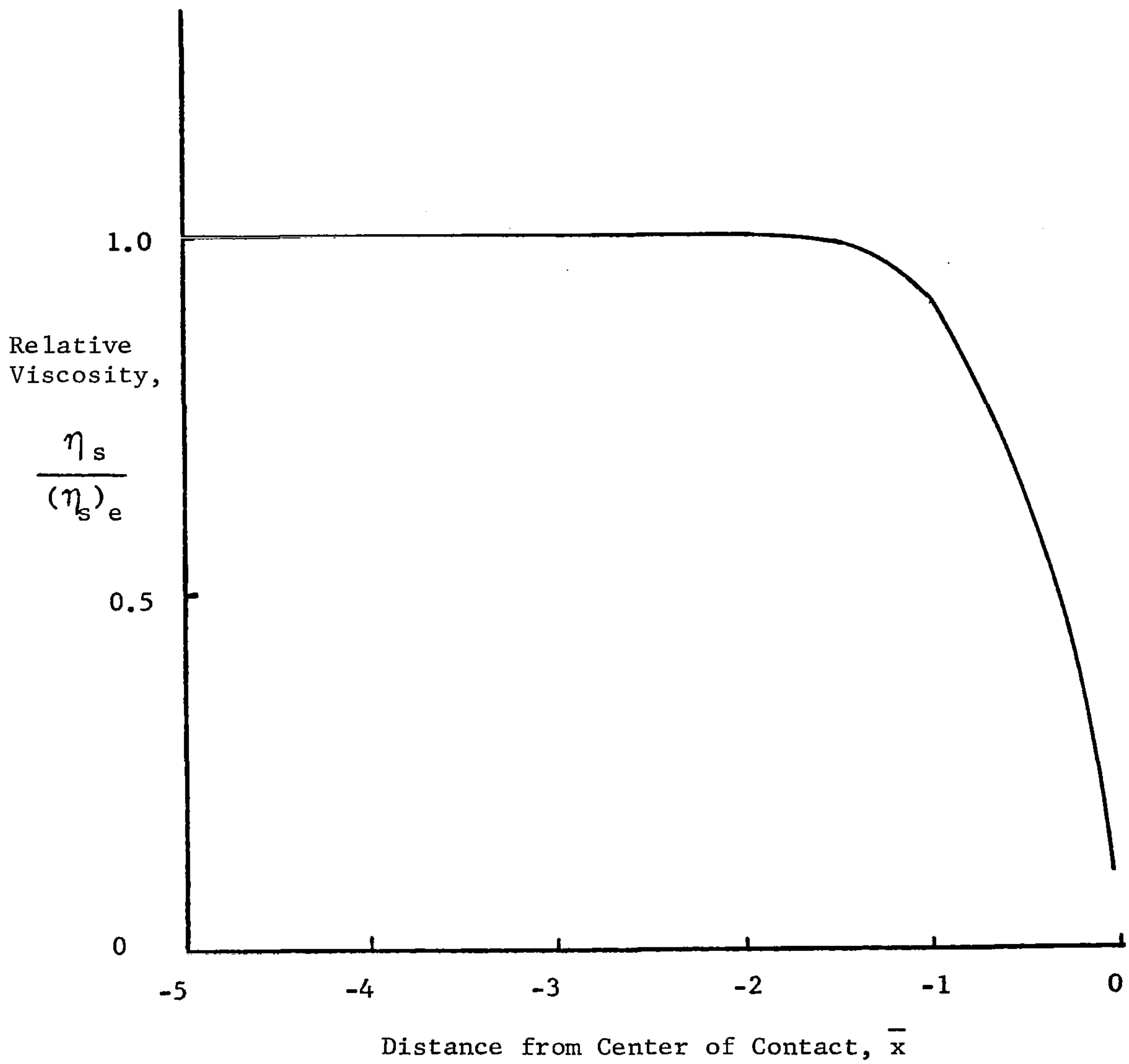
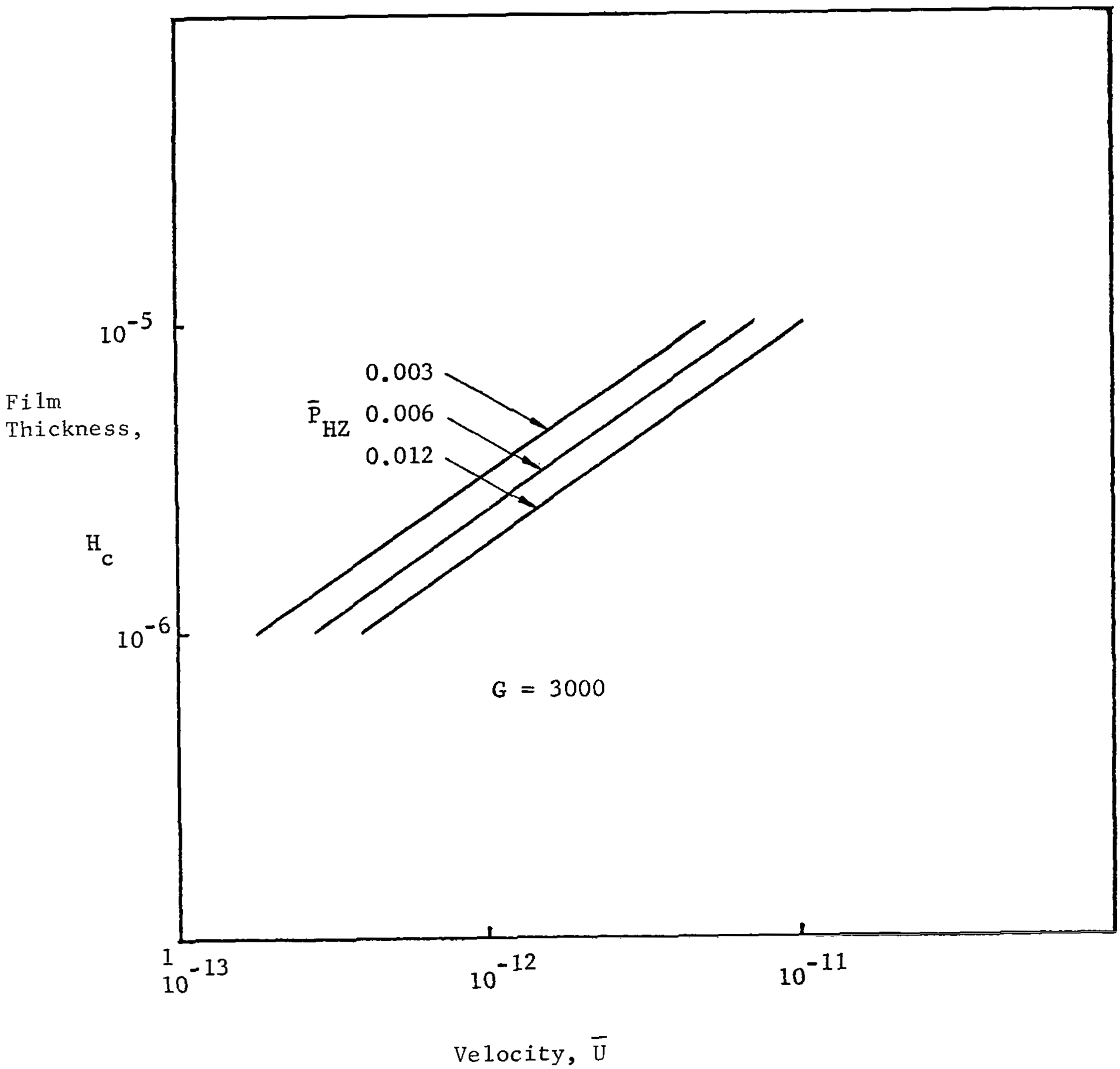
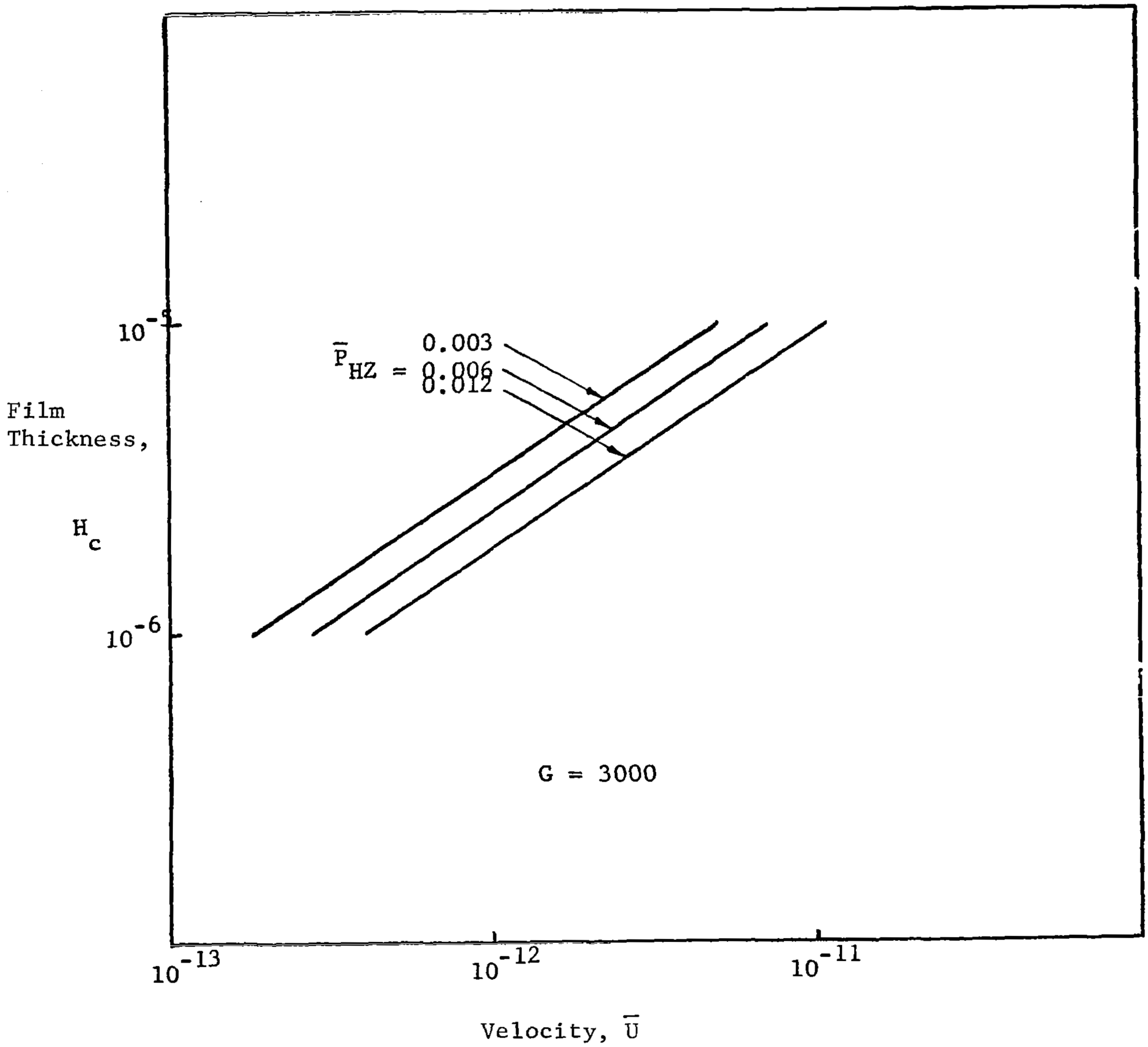


FIGURE 5 VARIATION OF RELATIVE VISCOSITY ALONG THE ROLLER CONTACT



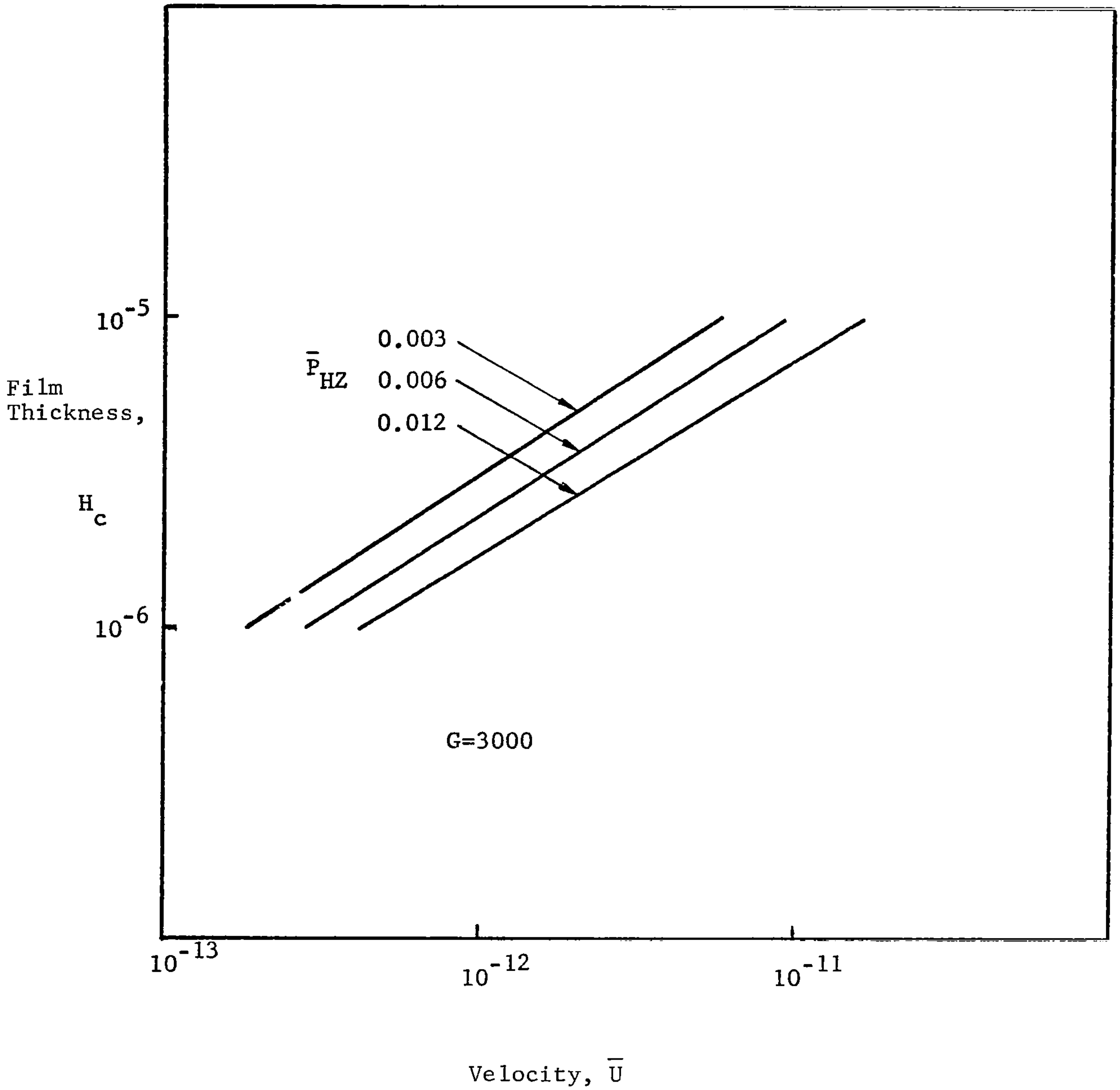
(a) Multiplication Factor $M = 10^2$

FIGURE 6 FILM THICKNESS AS A FUNCTION OF VELOCITY FOR VARYING VALUES OF CONTACT STRESS AND MULTIPLICATION FACTOR M



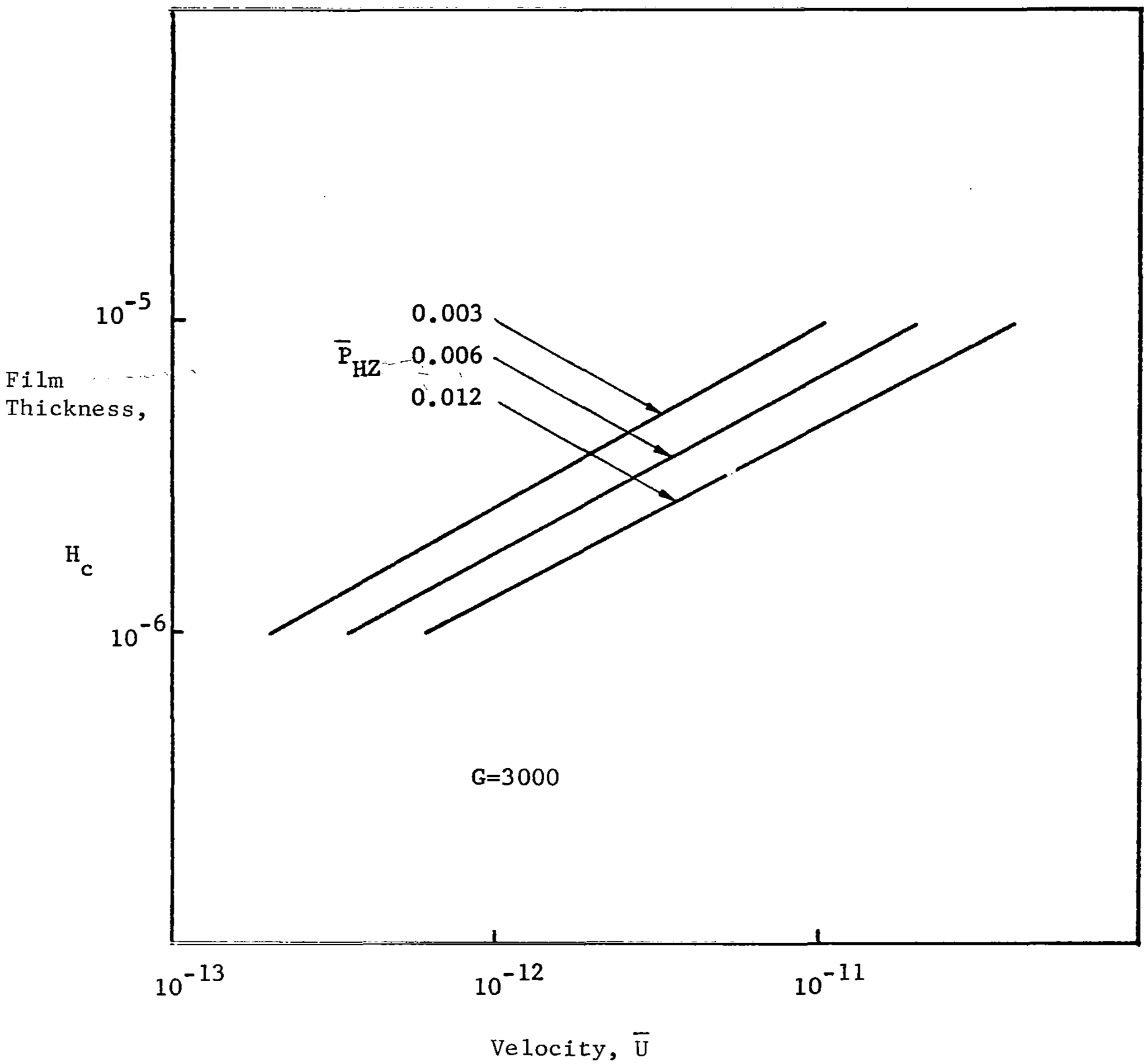
(b) Multiplication Factor M= 10^3

Figure 6 (cont'd)



(c) Multiplication Factor $M = 10^4$

Figure 6 (cont'd)

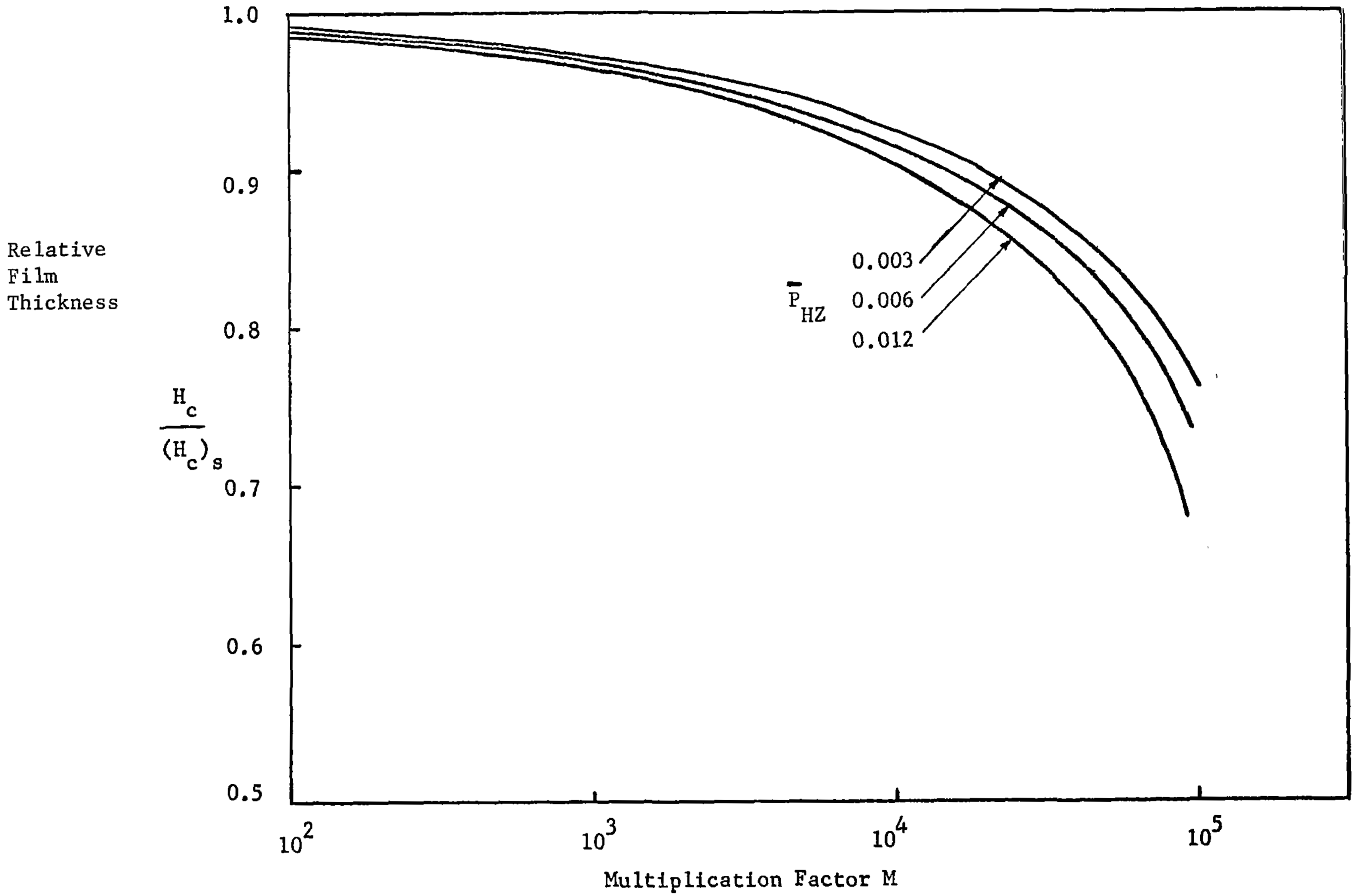


(d) Multiplication Facter $M=10^5$

Figure 6 (cont'd)

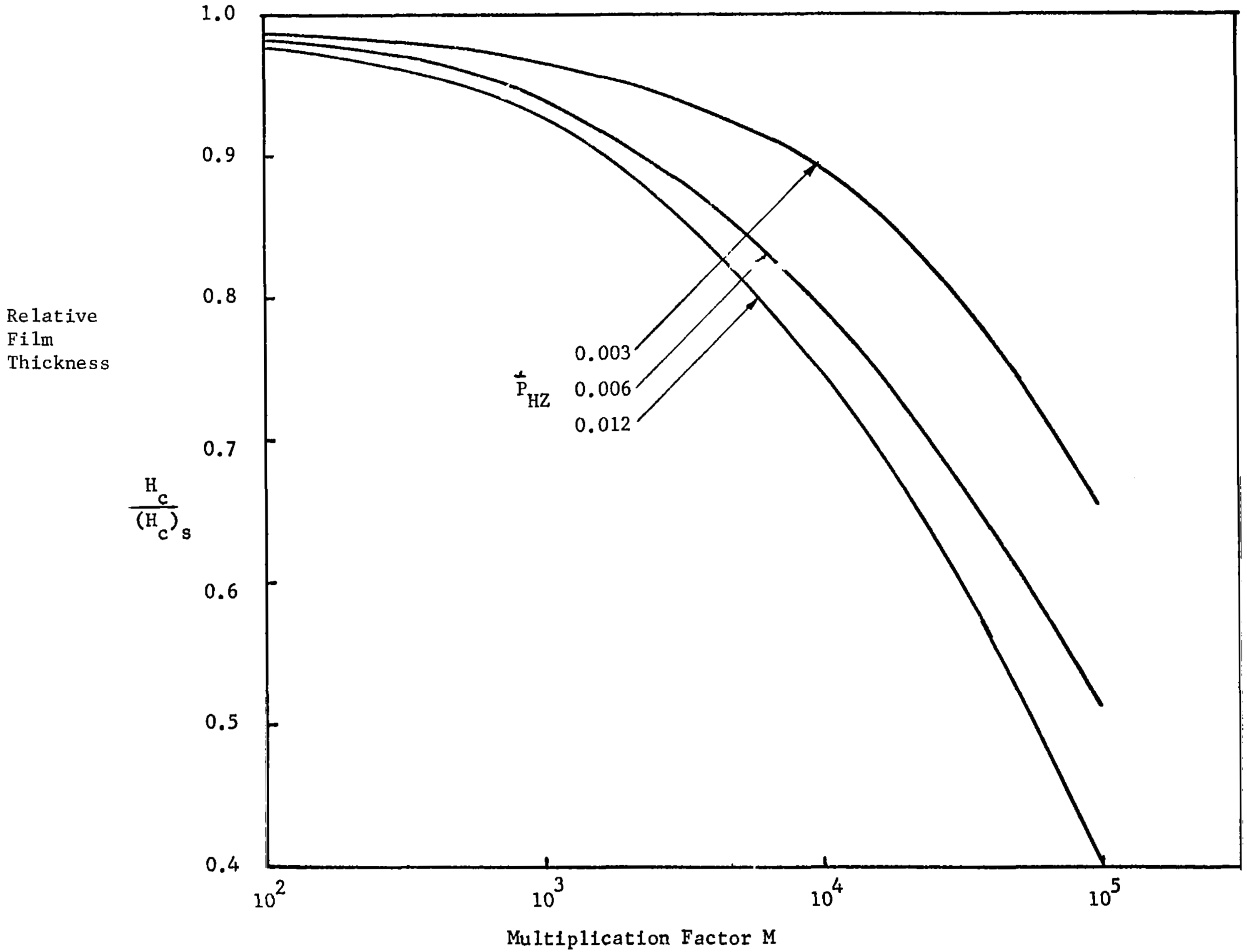
the film thickness H_c is not reduced significantly comparing to that without considering the effect of transient viscosity. Significant reductions occur as M increases beyond two decades.

The ratio of center film thickness calculated with the transient effect to that without this effect are plotted as a function of multiplication factor M in Fig. 7. It can be seen that significant reduction of film thickness begin to occur when the multiplication factor M approaches 10^3. It is somewhat unlikely that the level of retardation time of the lubricants under typical EHD condition can reach values several decades higher than those predicted by Harrison and Trachman (ref. 9).



a) $\overline{U} = 10^{-2}$, G = 3000

FIGURE 7 RELATIVE FILM THICKNESS AS A FUNCTION OF MULTIPLICATION FACTOR FOR VARYING VALUES OF CONTACT STRESS AND $\overline{U}$



b) $\bar{U} = 10^{-11}$, G = 3000

Figure 7 (cont'd)

SUMMARY OF RESULTS

The inlet solution of Elastohydrodynamic lubricated rolling contact problem was obtained considering lubricants with transient viscosity. The effect of the viscoelastic retardation time of lubricant on the center film thickness was investigated.

1. The effect of transient viscosity in response to a sudden pressure was found to be insignificant in determining the film thickness in elastohydrodynamic contacts.

2. For the transient effects to become important in film thickness calculation, the retardation time would have to be at least three decades higher than those suggested by Harrison and Trachman in reference 9.

APPENDIX A

NOMENCLATURE

a	semi-major axis of an elliptical contact
A	constant in Doolittle's relation
A'	constant used in relation for free volume viscosity
b	semi-minor axis of an elliptical contact
B	constant in Doolittle's relation
c	coefficient in density function
d	coefficient in density function
c_1	$16\ \bar{P}_{HZ}^2/H^*$
c_3	$48\ \bar{U}/H^{*2}$
$1/E'$	$\frac{1}{2}(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2})$
E_1, E_2	Young's Modulus for rollers 1 and 2
f	fractional free volume
f_2	equilibrium state fractional free volume
f_{j2}	equilibrium state fractional free volume for pressure step j
G	$\alpha E'$
h	film thickness
h_o	inlet film thickness at $x = -b$
h^*	reference film thickness at $\frac{dp}{dx} = 0$, $h^* = h_c$
h_c	center film thickness at $x = 0$
$(h_c)_s$	center film thickness for the case without transient viscosity effects
h_{min}	minimum film thickness
H	h/h^*
H^*	h^*/R
H_c	h_c/R
$(H_c)_s$	$(h_c)_{s/R}$

k, j	grid point numbers for the x coordinate
k_a	grid point numbers at $x = x_a$
n	iteration number
K	complex bulk modulus
K_1, K_2, K_3	used in Eq. (h), (i) and (j)
K_o	low frequency bulk modulus
K_∞	high frequency modulus
K_f	bulk modulus associated with molecular rearrangement of free volume
K_{f_j}	K_f for jth pressure step
K_r	complex relaxational modulus
K_2	high frequency value of K_r
p	pressure
$\overline{P}_{HZ}$	p_{HZ}/E'
P	p/p_{HZ}
q	$1 - \frac{1}{\overline{\eta}_s}$
Q	see Eq. (g)
R	$R_1R_2/(R_1 + R_2)$
R_1, R_2	radius of roller 1 and 2
s	$\ln\left(\frac{\eta_{f_2}}{\eta_f}\right)$
s_j	$\ln\left(\frac{\eta_{f_{j2}}}{\eta_{f_j}}\right) = \ln\left(\frac{\eta_{s_{j2}}}{\eta_{sj}}\right)$
s_{j1}	initial value of s_j in pressure step j
s_{jf}	final value of s_j in pressure step j

T_j	time required for lubricant pass through jth divided region
$\bar{U}$	$\dfrac{\eta_o(u_1 + u_2)}{2E'R}$
u_1, u_2	velocity of rollers 1 and 2
v	specific volume
v_o	specific volume at zero absolute temperature
v_1	initial specific volume
v_2	final equilibrium specific volume
v_i	instantaneous volume response
x	coordinate along the film
x^*	reference coordinate at $\frac{dp}{dx} = 0$
x_a	coordinate separating the inlet region into two subregions
x_b	coordinate separating the outlet region into two subregions
$\bar{x}$	x/b
x_f	coordinate at the termination of the film
$\bar{\alpha}$	αP_{HZ}
α	pressure-viscosity coefficient
η_s	shear viscosity of the lubricant
η_f	free volume viscosity
η_{f_2}	equilibrium state free volume viscosity
$\eta_{f_{j2}}$	equilibrium state free volume viscosity for jth pressure step
η_{s_j}	shear viscosity for jth pressure step

η_{s_2} equilibrium state shear viscosity

$\eta_{s_{j2}}$ equilibrium state shear viscosity for jth pressure step

η_o inlet viscosity

η_v volume viscosity

ρ density of the lubricant

ρ_o ambient density

ρ^* density at $x = x^*$

$\bar{\rho}$ ρ/ρ_o

τ relaxation time $\frac{\eta_v}{K_2}$

$\bar{\tau}$ retardation time $\frac{\eta_f}{K_f}$

τ_2 $= \frac{\eta_{f2}}{K_f}$

τ_{j2} $= \frac{\eta_{f_{j2}}}{K_{f_j}}$

ν_1, ν_2 Poisson's ratio of rollers 1 and 2

$\bar{\xi}$ dummy variable for $\bar{x}$

Ψ see Eq. (f)

APPENDIX B

NUMERICAL ANALYSIS

The region interested is the inlet half of the contact zone, which can be further divided into two sub-regions as shown in Fig. 8. In the first sub-region, pressure distribution is obtained by direct integration of the Reynold's Equation with introduced dimensionless function q where

$$q = 1 - \frac{1}{\bar{\eta}_s} \tag{a}$$

equation (29) can be written as

$$\frac{dq}{d\bar{x}} = \frac{48\bar{U}}{H^{*2}} \frac{d(\ln \bar{\eta}_s)}{dp} \left(\frac{H - \bar{\rho}^*/\bar{\rho}}{H^3}\right) \tag{b}$$

it can be integrated

$$q(x) = \frac{48\bar{U}}{H^{*2}} \int_{-\infty}^{\bar{x}} \frac{d(\ln \bar{\eta}_s)}{dP} \left(\frac{H - \bar{\rho}^*/\bar{\rho}}{H^3}\right) d\bar{\xi} \tag{c}$$

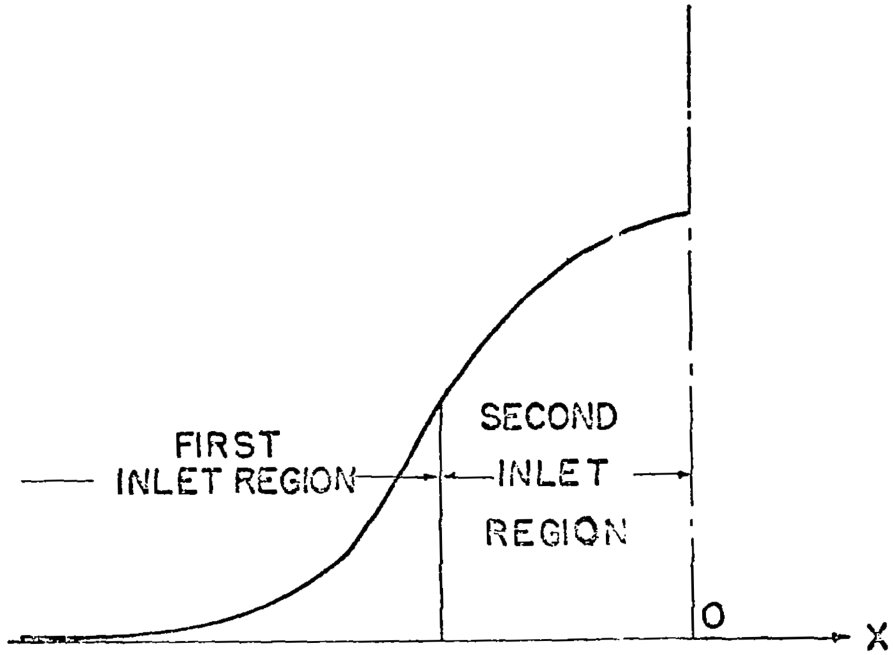
For a given viscosity as a function of pressure, the pressure distribution can be obtained by solving the equation

$$\bar{\eta}_s(P) = \frac{1}{1 - q(\bar{\mathbf{x}})} \tag{d}$$

In the second subregion, the pressure distribution can be obtained by solving the combined equations (29) and (30).

$$\frac{H^3}{\bar{\eta}_s} \frac{dP}{d\bar{x}} - c_3 \left[1 + c_1 \left(\frac{\bar{x}^2}{2} - \frac{1}{\pi} \int_{-\infty}^{\bar{x}_f} P(\bar{\xi}) \ln \frac{|\bar{\xi} - \bar{x}|}{|\bar{\xi}|} d\bar{\xi}\right) - \frac{\bar{\rho}^*}{\bar{\rho}}\right] = 0 \tag{e}$$

where $C_1 = 16\ \bar{P}_{HZ}^2 / H^*$ and $C_5 = 48\bar{U}/H^{*2}$. In the discretized form, it becomes $\Psi_k = 0$.



Distance from Center of Contact

FIGURE 8 DIVISION OF PRESSURE IN THE INLET REGION

$$\Psi_k = \frac{(H_{k-\frac{1}{2}})^3}{\overline{\eta}_{s_{k-\frac{1}{2}}}} \frac{(P_k - P_{k-1})}{(\overline{x}_k - \overline{x}_{k-1})} - C_3 \left[1 + C_1 \frac{(\overline{x}_{k-\frac{1}{2}})}{2} - \right.$$

$$\left. \frac{1}{\pi} \sum_{j=1,3,5\ldots}^{k_f - 2} P_j Q(k-\tfrac{1}{2}, j) - \frac{\overline{\rho}^*}{\overline{\rho}_{k-\frac{1}{2}}} \right] \tag{f}$$

These are a set of n equations to be solved by Newton-Raphson Method for P_K. Where $Q(K-\frac{1}{2},j)$ are the quadrature formulae for the singular logarithmic Kernel (ref. 6).

$$Q(k-\tfrac{1}{2},j) = \frac{1}{2} \sum_{m=1}^{3} \left[K_m(k,j) + K_m(k-1,j) - K_m(K_o,j) - K_m(K_o - 1,j) \right] \tag{g}$$

where

$$K_1(k,j_j) = \frac{1}{2\delta_j}(-3V_j - V_{j+2}) - \frac{\overline{V}_j}{3\delta_j^2} - u_j(\ln|u_j| - 1) \tag{h}$$

$$K_2(k,j_j) = \frac{2}{\delta_j}(V_j + V_{j+2}) + \frac{2\overline{V}_j}{3\delta_j^2} \tag{i}$$

$$K_3(k,j_j) = \frac{1}{2\delta_j}(-V_j - 3V_{j+2}) - \frac{\overline{V}_j}{3\delta_j^2} + u_{j+2}(\ln|u_{j+2}| - 1) \tag{j}$$

also

$$\delta_j = \overline{\xi}_{j+1} - \overline{\xi}_j$$

$$u_j = \overline{\xi}_j - \overline{x}_k$$

$$V_j = \frac{u_j^2}{2}(\ln|u_j| - \tfrac{3}{2})$$

$$\bar{V}_j = u_j(V_j - \frac{u_j^2}{6}) - u_{j+2}\ (V_{j+2} - \frac{u_j^2 + 2}{6}) \tag{k}$$

along with above equations, a set of n equations based on n grid points between $-\infty \langle \bar{x} \langle 0$ can be rewritten again

$$\left[\exp(-s_j)/s_j\right] ds_j + \exp(-s_j)\ \frac{f_{j2}}{B}\ ds_j = -\ \frac{dt}{\tau_{j2}} \tag{l}$$

and

$$\eta_{s_j} = \eta_o\ \exp(\alpha\ P_j - s_j) \tag{m}$$

The following are the outlines of numerical procedures for solving the governing equations:

1. Given a set of H, $\bar{P}_{HZ}$, G values
2. Assume a pressure profile for $-\infty \langle \bar{x} \langle 0$
3. Calculate H(x) for $-\infty \langle \bar{x} \langle 0$
4. Calculate density, viscosity for $-\infty \langle \bar{x} \langle 0$
5. Integrate the following integral in the first inlet region

$$I(x) = \int_{-\infty}^{\bar{x}} \frac{d(\ln\ s)}{dP}\ (\frac{H - \bar{\rho}^*/\bar{\rho}}{H^3})\ d\bar{\xi}$$

for $-\infty \langle \bar{x} \langle x_a$

6. Calculate $\bar{U}$

$$\bar{U} = \frac{H^{*2}}{48} \cdot \frac{q(x_a)}{I(x_a)}$$

7. Solve equations (f), (l) by Newton-Raphson method.
8. Check the convergence for pressure. If not, repeat calculating procedure from step number 3.
9. Final solutions are in the forms of $\bar{U}$, P, and H.

APPENDIX C

NUMERICAL PROGRAM

The complete computer program coded in FORTRAN IV is listed in this Appendix for solving the Transient Viscosity EHD problem.

```
      PROGRAM WANG  (INPUT,OUTPUT,PUNCH,TAPE5=INPUT, TAPE6=OUTPUT)
C     EHD01
C     NASA-EHD  INLET FILM FOR LINE CONTACT FOR LOAD UP TO 400,000 PSI
      DIMENSION DP(60),A(30,30),C(30),SUMA(60),KBAR(20),FAC(10)
      COMMON P(60),H(60),X(60),PHZBA(20),Q(60,60),UBA(20),HSA(20)
      COMMON VISD(60),DEN(60),DEND(60),PLUB(20),SA(60),SMA(60),DX(60)
      COMMON VIS1(35),VIS2(35),VIS3(35),VIS(60)
      COMMON AFA,PHZB,HSB,UB,ED,EN,NR,NW,KF,KO,KR
      COMMON P1,P2,DPV,BTA,IT,UBG
C     READ BASIC INPUT DATA
      NR=5
      NW=6
      READ(NR,1)
      WRITE(NW,1)
      READ (NR,2)NRUN
      DO 1000 NRR=1,NRUN
      READ(NR,2) KG, KA, KO, KF, KR, NKER,NAVIS
      READ(NR,2) NS1, NS2, NS3, NS4, NS5, NS6, NS7, NS8, NS9, NS10
      READ(NR,2) ITH, ITP, ITE
      READ(NR,3) EPSH, EPSP, EPSE
      KKF=KF-1
      READ(NR,3) (DX(K),K=1,KKF)
      X(1)=-5.0
      DO 99 K=2,KF
   99 X(K)=X(K-1)+DX(K-1)
      READ (NR,3)(P(K),K=1,KA)
      WRITE(NW,4) (K, P(K), K=1, KA)
      PKA=P(KA)
      DO 106 K=KA,KF
  106 P(K)=SQRT(1.0-X(K)**2)
      KKA=KA-1
      TEMP=P(KA)/PKA
      DO 250 K=1,KKA
  250 P(K)=TEMP*P(K)
      PI=3.141593
C     READ LOAD, SPEED AND LUB. PARAMETERS
      READ(NR,3) (PLUB(N), N=1, 8)
      READ(NR,3) (FAC(N),N=1,8)
      READ(NR,2) NHM, NPHZM
      READ(NR,3) (HSA(N), N=1, NHM)
      READ(NR,3) (PHZBA(N), N=1, NPHZM)
      IF(NAVIS .EQ. 0) GO TO 101
      READ (NR,3)(VIS1(K),K=1,31)
      WRITE(NW,24)
      WRITE(NW,20)(VIS1(K),K=1,31)
      DO 102 K=1,31
  102 VIS2(K)=ALOG(VIS1(K))
  101 CONTINUE
      WRITE(NW,7)
      WRITE(NW,2) KG, KA, KO, KF, KR, NKER
      WRITE(NW,8)
      WRITE(NW,2) ITH, ITP, ITE
      WRITE(NW,9)
      WRITE(NW,20) EPSH,EPSP, EPSE
      WRITE(NW,10)
      WRITE(NW,20) (X(K),K=1,KF)
      IF(NKER .EQ. 0 ) GO TO 91
```

```
      CALL KERCAL
C                                                                   NS1
      IF(NS1 .EQ. 0) GO TO 92
      WRITE(NW,11)
      WRITE(NW,20) ((Q(K,J), J=1, KF), K=1, KO)
      GO TO 92
   91 READ(NR,21) ((Q(K,J),J=1,KF),K=1,KO)
   92 WRITE(NW,12)
      WRITE(NW,20)(PLUB(N),N=1,8)
      WRITE(NW,13)
      WRITE(NW,20)  (HSA(N),N=1,NHM)
      WRITE(NW,14)
      WRITE(NW,20)  (PHZBA(N),N=1,NPHZM)
      DO 1000 NPHZ=1,NPHZM
      READ(NR,2)  (KAAR(N),N=1,NHM)
      PHZB=PHZBA(NPHZ)
      WRITE(NW,15)PHZB
      AFA=PLUB(1)*PHZB
      ED=PLUB(2)*PHZB*2.0/PI
       EN=PLUB(3)*PHZB*2.0/PI
      IF(NAVIS .EQ. 0) GO TO 93
      BTA=PLUB(4)*PHZB
       P1=PLUB(5)/PHZB
      P2=PLUB(6)/PHZB
      DPV=(P2-P1)/30.
      DPV2=DPV*2.0
      VIS3(1)=(VIS2(2)-VIS2(1)+AFA*DPV)/DPV2
      VIS3(31)=(VIS2(31)+BTA*DPV -VIS2(30))/DPV2
      DO 103 K=2,30
  103 VIS3(K)=(VIS2(K+1)-VIS2(K-1))/DPV2
C                                                                   NS1
      IF(NS1 .EQ. 0 ) GO TO 93
      WRITE(NW,25)
      WRITE(NW,20)(VIS3(K),K=1,31)
      WRITE(NW,3) P1, P2, DPV, DPV2
   93 DO 1000 NH=1, NHM
      DO 999  M=1,4
      FACT=FAC(M)
      HSB=HSA(NH)
      KA=KAAR(NH)
      WRITE(NW,18) HSB
      G=PLUB(1)
      C1=16.0*PHZB**2/HSB
      C3=48.0/HSB**2
      C4=C1/PI
      UBG=(HSB**0.75/(1.26*G**0.6*PHZB**(-0.27)))**(10./7.)
      WRITE(NW,23)UBG
      IT=1
  107 CALL HCAL(KO)
C                                                                        NS1
      IF(NS1 .EQ. 0) GO TO 109
      WRITE(NW,5)
      WRITE(NW,4)  (K, H(K), K=1, KO)
  109 CALL DVD  (1,KO,2,0)
      CALL VDTD  (FACT,KA)
      CALL DVD  (1, KO,1, 0)
      PS=DEN(KO)
      KKA=KA+1
```

```
      STOP
    1 FORMAT(72H
     1                                    )
    2 FORMAT(16I5)
    3 FORMAT(8E10.3)
    4 FORMAT(7(1X,I2,1X,E13.6))
    5 FORMAT(/6H H(K) /)
    6 FORMAT(/6H P(K) /)
    7 FORMAT(5H   KG, 5H   KA, 5H   KO, 5H   KF, 5H   KR, 5H NKER )
    8 FORMAT(5H  ITH, 5H  ITP, 5H  ITE)
    9 FORMAT(10H    EPSH   ,10H    EPSP   , 10H    EPSE    )
   10 FORMAT(6H XH(K) )
   11 FORMAT(7H Q(K,J))
   12 FORMAT(8H PLUB(N))
   13 FORMAT(7H HSA(N))
   14 FORMAT(9H PHZBA(N))
   15 FORMAT(6H PHZB=,E13.6)
   16 FORMAT(8H SUMA(K))
   17 FORMAT(4H SQA)
   18 FORMAT(5H HSB=,E13.6)
   20 FORMAT(1X,9E13.6)
   21 FORMAT(5E15.7)
   22 FORMAT(/ 5H  IT=, I5, 5H  UB=,E13.6)
   23 FORMAT(/10H UBGRUBIN=,E13.6/)
   24 FORMAT( / 8H VIS1(K)/)
   25 FORMAT(/8H VIS3(K) /)
   26 FORMAT (33H MULTIPLICATION FACTOR FOR TAU 2=,E13.6)
      END
```

```
      CALL OVD(KKA,KO,2, 1)
      TP=H(KA)-DS/DEN(KA)
      IF(TP .GT.0.0) GO TO 116
      DO 115 K=1,KA
  115 H(K)=H(K)-TP
  116 SUMA(1)=0.0
      DO 135 K=1,KA
      IF(K-1) 117,117,118
  117 Z1=(H(K)-DS/DEN(K))/H(K)**3
      GO TO 119
  118 Z1=(H(K)-DS/DEN(K))/H(K)**3
  119 IF(K-1) 133,133,132
  132 SUMA(K)=SUMA(K-1)+0.5*(X(K)-X(K-1))*(Z1+Z2)*VISD(K)
  133 Z2=Z1
  135 CONTINUE
      SQA=1.0-1.0/VIS(KA)
      DO 125 K=1,KA
      SQ=SQA*SUMA(K)/SUMA(KA)
      SSS=SA(K)
      IF(SQ-1.0) 125,141,141
  141 WRITE(NW,4)  (N,SUMA(N),N=1,KA)
      WRITE  (NW,4)  (N,VISD(N),N=1,KA)
      WRITE  (NW,4)  (N,DEN(N),N=1,KA)
      WRITE(NW,4)  (N,H(N),N=1,KA)
      WRITE  (NW,20) DS
  125 P(K)=PMU(SQ,SSS)
      CALL HCAL  (KO)
C                                                                  NS3
      IF(NS3 .EQ. 0) GO TO 126
      KK4=KA-4
      WRITE(NW,5)
      WRITE(NW,4)(K,H(K),K=KK4,KA)
      WRITE(NW,6)
      WRITE(NW,4)  (K, H(K), K=1, KF)
      WRITE(NW,16)
      WRITE(NW,4)(K,SUMA(K),K=1,KA)
  126 CONTINUE
      UB=SQA/(C3*SUMA(KA))
      IF( IT .GT. 1) UB=(UB+UBP)*0.5
      N=KO-KA
C                                                                  NS6
      IF(NS6 .EQ. 0) GO TO 136
      WRITE  (NW,3) C1, C3, C4, UB, DS, SQA
      WRITE(NW,4)  (K,VIS(K),K=1,KO)
      WRITE  (NW,4)  (K, VISD(K),K=1,KO)
      WRITE  (NW,4)  (K,DEN(K),K=1, KO)
      WRITE  (NW,4)  (K,DEND(K),K=1, KO)
  136 CONTINUE
      KKA=KA+1
      KKO=KO-1
      DO 170 K=KKA,KO
      HH=(H(K)+H(K-1))*0.5
      KK=K-KA
      DO 160 J=KA, KKO
      JJ=J-KA+1
      IF(J.EQ.KA) GO TO 158
      A(KK,JJ)= C3*UB*0.5* C4*(Q(K,J)+Q(K-1,J))
      GO TO 158
```

```
  158 SPQ=0.
      DO 137 L=1,KA
  137 SPQ=SPQ+(Q(K,L)+Q(K-1,L))*P(L)
      A(KK,JJ)= C3*UB*0.5*C4*SPQ/P(KA)
  138 IF(J.EQ.K) GO TO 156
      IF(J.EQ.K-1) GO TO 157
      GO TO 160
  156 SIGN=1.0
      GO TO 159
  157 SIGN=-1.0
  159 A(KK,JJ)=A(KK,JJ)+HH**3/(X(K)-X(K-1))/VIS(K)
     1*(-(P(K)-P(K-1))        *VISD(K)*0.5+SIGN) -C3*UB*0.5*DS*DEND(K)/
     2 DEN(K)**2
  160 CONTINUE
      C(KK)=-HH**3/(X(K)-X(K-1))*(P(K)-P(K-1))/VIS(K)+C3*UB*(HH-DS /
     1DEN(K))
  170 CONTINUE
C                                                                 NS4
      IF(NS4 .EQ. 0) GO TO 174
      DO 171 KK=1,N
      WRITE(NW,4)KK,C(KK)
  171 WRITE(NW,4)(JJ,A(KK,JJ),JJ=1,N)
  174 CALL MATINV (A, N, C, 1, DET)
      WRITE(NW,3)DET
C                                                                 NS 5
      IF(NS5.EQ.0) GO TO 190
      WRITE(NW,4)  (KK, C(KK), KK=1, N)
  190 PKA=P(KA)
      CV=1.
      KK0=K0-1
      DO 180 K=KA,KK0
      KK=K-KA+1
      DP(K)=C(KK)
      IF(ABS(DP(K))-EPSP) 176,176,175
  175 CV=0.0
  176 P(K)=P(K)+DP(K)
  180 CONTINUE
      KKA=KA-1
      DO 181 K=1, KKA
  181 P(K)=P(K)*P(KA)/PKA
      IF(CV.EQ.1.0) GO TO 210
      IF (IT.GT.ITP) GO TO 999
      IT=IT+1
C                                                       NS7
      IF(NS7 .EQ. 0) GO TO 200
  210 WRITE(NW,6)
      WRITE(NW,4)  (K, P(K), K=1, K0)
      WRITE(NW,5)
      WRITE(NW,4)  ( K,H(K), K=1, K0)
      WRITE(NW,16)
      WRITE(NW,4)(K,SUMA(K),K=1,KA)
  200 WRITE(NW,22) IT,UB
      WRITE  (NW,26) FAC(M)
      IF (CV.EQ.1.0) GO TO 999
      UBP=UB
      GO TO 107
  999 CONTINUE
  990 CONTINUE
```

```
      SUBROUTINE HCAL (KK)
      COMMON P(60),H(60),X(60),PHZBA(20),Q(60,60),UBA(20),HSA(20)
      COMMON VISD(60),DEN(60),DEND(60),PLUB(20),SA(60),SMA(60),DX(60)
      COMMON VIS1(35),VIS2(35),VIS3(35),VIS(60)
      COMMON AFA,PHZB,HSB,UB,ED,EN,NR,NW,KF,KO,KR
      COMMON P1,P2,DPV,BTA,IT,UBG
      PI=3.141593
      C1=16.*PHZB**2/HSB
      DO 10 K=1,KK
      H(K)=0.
      DO 1 J=1,KF
    1 H(K)=H(K)+P(J)*Q(K,J)
      H(K)=1.0+C1*(0.5*X(K)**2-H(K)/PI)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE KERCAL
      COMMON P(60),H(60),X(60),PHZBA(20),Q(60,60),UBA(20),HSA(20)
      COMMON VISD(60),DEN(60),DEND(60),PLUB(20),SA(60),SMA(60),DX(60)
      COMMON VIS1(35),VIS2(35),VIS3(35),VIS(60)
      COMMON AFA,PHZ3,FS3,UB,ED,EN,NR,NW,KF,KO,KR
      COMMON P1,P2,DPV,BTA,IT,UBG
      DO 1 I=1, KF
      DO 1 J=1, KF
    1    Q(I,J)=0.0
      KKF=KF-2
      DO 8 K=1, KO
      Q(K,1)=0.0
      F5=X(K)
      DO 8 J=1, KKF, 2
      U=X (J)-F5
      U2=X(J+2)-F5
      AU=ABS(U)
      AU2= ABS(U2)
      IF(AU) 50, 51, 50
   50 AU=ALOG(AU)
   51 IF(AU2) 52, 6, 52
   52 AU2=ALOG(AU2)
    6 DJ=X(J+1)-X(J)
      F2=3.0*DJ
      UQ=U*U
      U2Q=U2*U2
      FK=UQ*(AU-1.5)*0.5
      FK2=U2Q*(AU2-1.5)*0.5
      FKB=U*(FK-UQ/6.0)-U2*(FK2-U2Q/6.0)
         Q(K,J)= ((-3.0*FK-FK2)/2.0-FKB/F2)/DJ-U*(AU-1.0)+    Q(K,J)
         Q(K,J+1)= (2.0*(FK+FK2)+2.0*FKB/F2)/DJ
         Q(K,J+2)=((-FK-3.0*FK2)/2.0-FKB/F2)/DJ+U2*(AU2-1.0)
    8 CONTINUE
      DO 300 K=1, KO
      DO 300 J=1, KF
  300 Q(K,J)=Q(K,J)-Q(KR,J)
      RETURN
      END
```

```
      SUBROUTINE VDTD (FACT,KA)
      DIMENSION  F2(60),TAU(60)
      COMMON P(60),H(60),X(60),PHZBA(20),Q(60,60),UBA(20),HSA(20)
      COMMON VISD(60),DEN(60),DEND(60),PLUB(20),SA(60),SMA(60),DX(60)
      COMMON VIS1(35),VIS2(35),VIS3(35),VIS(60)
      COMMON AFA,PHZB,PSB,UB,ED,EN,NR,NW,KF,KO,KR
      COMMON P1,P2,DPV,BTA,IT,UBG
      IF(IT.EQ.1)   UB=UBG
      IF (IT.GT.1)   GO TO 1
      SIN=0.00001
      S=SIN
      SM=AFA*P(1)
    1 IMAX= 50
      DENM=1.0+EN*0.015/PHZB/(1.0+ED*0.015/PHZB)
      EPS= 0.001
      DO 5 N=1,KO
      F2(N)=  (DENM-DEN(N))/DEN(N)
      TAU(N)= 0.08*DX(N)/UB*PHZB**2*(PLUB(7)/PHZB+9*P(N))/EXP(AFA*P(N))
      TAU(N)= TAU(N)/FACT
    5 CONTINUE
      WRITE (NW,4)  (TAU(N),N=1,KO)
      IF (IT.GT.1)   GO TO 12
      WRITE (NW,180)
      GO TO 13
   12 S=SA(1)
      SM=SMA(1)
   13 DO 100 N=1,KO
      I=1
   10 ES=EXP(-S)
      ESM=EXP(-SM)
      T1=ES*(F2(N)+1.0/S)
      T2=ESM*(F2(N)+1.0/SM)
      PSI=0.5*(T1+T2)*(S-SM)+TAU(N)
      DPSI=0.5*((-T1-ES/S**2)*(S-SM)+(T1+T2))
      IF (PSI.GT. 0.0)   GO TO 20
      DS=-PSI/DPSI
    9 IF (ABS(DS)-EPS)   11,11,15
   11 IF (ABS(PSI)-EPS)    95,95,15
   15 IF (I-IMAX) 16,300,300
   16 I=I+1
      S=S+DS
      GO TO 10
   20 S=S*0.1
      GO TO 10
   95 IF(N-KO)   96,96,100
   96 S=S+DS
      SA(N)=S
      SM=S+AFA*(P(N+1)-P(N))
      SMA(N)=SM
  100 CONTINUE
  388 CONTINUE
      VIS(1)=1.0
      VISD(1)=AFA
      DO 115 K=2,KO
      IF  (K-KA)  310,310,311
  310 PK=P(K)
      SK=SA(K)
```

```
      GO TO 312
  311 PK=(P(K)+P(K-1))*0.5
      SK=(SA(K)+SA(K-1))*0.5
  312 VIS(K)=EXP(AFA*PK-SK)
  115 VISD(K)= AFA-(SA(K)-SA(K-1))/(P(K)-P(K-1))
      NS8=1
      IF(NS8.EQ.0)  GO TO 120
    4 FORMAT(7(1X,I2, 1X,E13.6))
  150 FORMAT (8H    SA(K))
    7 FORMAT  (1X,E13.6,8X,E13.6)
  155 FORMAT (26H     F2              TAU          ,I3)
  171 FORMAT (10H DIVERGENT)
  180 FORMAT (3X,1HN,2X,1HI,5X,2HES,11X,3HESM,10X,2HT1,10X,2HT2,10X,
     13HPSI,10X,4HOPSI,10X,2HOS,12X,1HS)
  181 FORMAT (1X,2I3,8(1X,E12.5))
  300 WRITE (NW,171)
  120 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DVD(K1,K2,IS,KHAF)
      COMMON P(60),H(60),X(60),PHZBA(20),Q(60,60),UBA(20),HSA(20)
      COMMON VISD(60),DEN(60),DEND(60),PLUB(20),SA(60),SMA(60),DX(60)
      COMMON VIS1(35),VIS2(35),VIS3(35),VIS(60)
      COMMON AFA,PHZG,HSB,UB,ED,EN,NR,NW,KF,KÖ,KR
      COMMON P1,P2,DPV,BTA,IT,UHG
      IF(IS.EQ.1) 200,205
  200 PK=P(K2)
      IF (KHAF.EQ.1)   PK=(P(K2)+P(K2-1))*0.5
      DEN(K2)=1.0+EN*PK/(1.0+ED*PK)
      DEND(K2)=EN/(1.0+PK*ED)**2
      GO TO 215
  205 DO 210 K=K1,K2
      PK=P(K)
      IF(KHAF .EQ.1)   PK=(P(K)+P(K-1))*0.5
      DEN(K)= 1.0+EN*PK/(1.0+ED*PK)
  210 DEND(K)=EN/(2.0+PK*ED)**2
  215 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATINV (A,N,B,M,DETER)
      DIMENSION A(30,30),B(30),IPIVO(30),PIVOT(30)
C     MATRIX INVERSION WITH ACCOMPANYING SOLUIION OF LINEAR EQUATI
      DETER =1.0
      DO 20 J=1,N
   20 IPIVO(J)=J
      DO 55. I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
      AMAX=0.0
      DO 105 J=1,N
      IF (IPIVC(J)-1) 60,105,60
   60 DO 100 K=1,N
      IF (IPIVC(K) -1) 80, 100, 600
   80 IF (ABS (AMAX)-ABS (A(J,K))) 85,100,100
   85 IROW=J
      ICOLU =K
      AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
      IPIVO(ICCLU)=IPIVO(ICOLU)+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
      IF (IROW-ICOLU)  140, 260, 140
  140 DETER =-DETER
      DO 200 L=1,N
      AMAX=A(IROW,L)
      A(IROW,L)=A(ICOLU,L)
  200 A(ICOLU,L)=AMAX
      AMAX=B(IROW)
      B(IROW)=B(ICOLU)
      B(ICOLU)=AMAX
  260 PIVOT(I)=A(ICOLU,ICOLU)
      DETER =DETER*PIVCT(I)
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
      A(ICOLU,ICOLU)=1.0
      DO 350 L=1,N
  350 A(ICOLU,L)=A(ICOLU,L)/PIVOT(I)
      B(ICOLU)=B(ICOLU)/PIVOT(I)
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
      IF(L1-ICOLU)  400, 550, 400
  400 AMAX=A(L1,ICOLU)
      A(L1,ICOLU) =0.0
      DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLU,L)*AMAX
      B(L1)=B(L1)-B(ICCLU)*AMAX
  550 CONTINUE
  600 RETURN
      END
```

```
FUNCTION PMU(QQ,SSS)
COMMON P(60),H(60),X(60),PHZBA(20),Q(60,60),UBA(20),HSA(20)
COMMON VISD(60),CEN(60),DEND(60),PLUB(20),SA(60),SMA(60),DX(60)
COMMON VIS1(35),VIS2(35),VIS3(35),VIS(60)
COMMON AFA,PHZB,HSB,UB,ED,EN,NR,NW,KF,KO,KR
COMMON P1,P2,DPV,BTA,IT,UBG
PMU=(-ALOG(1.0-QQ)+SSS)/AFA
RETURN
END
```

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