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## NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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## Transformer Design Tradeoffs

Colonel W. T. McLyman

```
(NASA-CR-146554) TRANSFORMEG DESIGN

\title{
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}

April 1, 1976

\section*{PREFACE}

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\section*{LIST OF SYMBOLS}
\begin{tabular}{|c|c|}
\hline \(\mathrm{A}_{\mathrm{c}}\) & effective iron area, \(\mathrm{cm}^{2}\) \\
\hline \(\mathrm{A}_{\mathrm{p}}\) & area product, \(\mathrm{W}_{\mathrm{a}} \times \mathrm{A}_{\mathrm{c}}, \mathrm{cm}^{4}\) \\
\hline \(A_{E}\) & surface area of a transformer, \(\mathrm{crin}^{2}\) \\
\hline \(A_{\text {w }}\) & wire area, \(\mathrm{cm}^{2}\) \\
\hline AWG & American Wire Gauge \\
\hline \(B_{m}\) & flux density, teslas \\
\hline E & voltage \\
\hline \(\eta\) & efficiency \\
\hline f & frequency, Hz \\
\hline I & current, amps \\
\hline \(i_{0}\) & load current, amps \\
\hline \(\mathrm{I}_{\mathrm{p}}\) & primary current, amps \\
\hline \(I_{s}\) & secondary current, amps \\
\hline J & current density, amps/cm \({ }^{2}\) \\
\hline \(J_{p}\) & primary current density, \(\mathrm{amps} / \mathrm{cm}^{2}\) \\
\hline \(J_{s}\) & secondary current density, amps/cm \({ }^{2}\) \\
\hline K & constant \\
\hline \(\mathrm{K}_{\mathrm{j}}\) & current density coefficient \\
\hline \(\mathrm{K}_{\mathrm{s}}\) & surface area coefficient \\
\hline \(\mathrm{K}_{\mathrm{u}}\) & window utili sation factor \\
\hline \(\mathrm{K}_{\mathrm{v}}\) & volume coefficient \\
\hline \(\mathrm{K}_{\mathrm{w}}\) & weight coefficient \\
\hline \(\ell_{\mathrm{m}}\) & magnetic path, cm \\
\hline \(\ell\) & linear dimension, cm \\
\hline MLT & mean length turn, cm \\
\hline N & turns \\
\hline \(P\) & power, watts \\
\hline \(\mathrm{P}_{\text {cu }}\) & copper loss, watts \\
\hline \(P_{\text {fe }}\) & core loss, watts \\
\hline \(P_{\text {in }}\) & inpur power, watts \\
\hline \(P_{0}\) & output power, watts \\
\hline \(\Psi\) & watts/unit area, cm \\
\hline
\end{tabular}

\section*{LIST OF SYMBOLS (cont)}
\begin{tabular}{|c|c|}
\hline \[
P_{P}
\] & primary loss, watts \\
\hline \(\mathrm{P}_{\mathrm{s}}\) & secondary loss, watts \\
\hline \[
P_{\Sigma}
\] & total loss (core and copper), watts \\
\hline \(\mathrm{F}_{\mathrm{t}}\) & apparent power, watts \\
\hline R & resistance, ohms \\
\hline \(\mathrm{R}_{\mathrm{E}}\) & equivalent coremloss (shunt) resistance, ohms \\
\hline \(\mathrm{R}_{\mathrm{cu}}\) & copper resistance, ohms \\
\hline Reg (\%) & transformer regulation in percent \\
\hline \(\mathrm{R}_{0}\) & load resistance, ohms \\
\hline \(\mathrm{R}_{\mathrm{p}}\) & primary resistance, ohms \\
\hline \(\mathrm{R}_{5}\) & secondary resistance, ohms \\
\hline \(\mathrm{R}_{t}\) & total resistance, ohms \\
\hline \(S_{1}\) & conductor area/wire area \\
\hline \(\mathrm{S}_{2}\) & wound area/usable window \\
\hline \(S_{3}\) & usable window area/window area \\
\hline \(\mathrm{S}_{4}\) & usable window area/usable window area + insulation arez \\
\hline T & teslas \\
\hline \(\mathrm{V}_{0}\) & load voltage, volts \\
\hline Vol & volume, \(\mathrm{cm}^{3}\) \\
\hline \(\mathrm{W}_{\mathrm{a}}\) & window area, \(\mathrm{cm}^{2}\) \\
\hline \(W_{t}\) & weight, grams \\
\hline
\end{tabular}

\section*{ABSTRACT}

The adoption by NASA of the metric system for dimensioning to replace the long-used English units imposes a requirement on the U. S, transformer designer to convert from the familiar unit to the less familiar metric equivalents. Material is presented to assist in this transition in the field of transformer design and fabrication.

The conversion process in power electronics requires the use of transformer components which frequently are the heaviest and bulkiest items in the conversion circuit. They also have a significant effect upon the overall performance and efficiency of the system. Accordingly, the design of such transformers has an important effect on overall system weight, powerinversion efficiency, and cost.

For years manufacturers have rated their cores with a number that represents its relative power-handling ability. This method assigns to each core a number which is the product of its window area and core crosssection area, and is called "Area Product Ap"

The author has developed a coordination between the A numbers and current density \(J\) for a given regulation and temperature rise. The area product \(A_{p}\) is a dimension to the fourth power, whereas volume is a dimension to the third power and surface area \(A_{t}\) is a dimension to the second power. The author has developed straight-line relationships for \(A_{p}\) and Volume, \(A_{p}\) and surface area \(A_{t}\) and, \(A_{p}\) and weight. These relationships can now be used as new tools to simplify and standardize the process of transformer design. They also make it possible to design transformers of small bulk and volume or to optimize efficiency.

\section*{INTRODUCTION}

The conversion process in power electronics requires the use of transformers, components which frequently are the heaviest and bulkiest item in the conversion circuits. They also have a significant effect upon the overall performance and efficiency of the system. Accordingly, the design of such transformers has an important influence on overall system weight, power conversion efficiency and cost. Because of the interdependence and interaction of parameters, judicious design tradeoffs are necessary to achieve optimization.

The information presented herein explains the reasons for making such tradeoffs as a guide for making them intelligently.

Manufacturers have for years assigned numeric codes to their colves which represent the relative power handling ability. This mathod assigns to each core a number which is the product of its window area and core cross section area and is called "Area Product", Ap.

Over the last few months, the author became aware of unique relationships between the "Area Product", Ap, characteristic number for transformer cores and several other important parameters which must be considered in transformer design. These numbers were developed by core suppliers to summarize dimensional and electrical properties of C-cores and are listed in their catalogs. Such numbers are available for more than 200 different G-core sizes and configurations.

The author has developed relationships between the \(A_{p}\) numbers and current density J for a given regulation and temperature rise. The area product \(A_{p}\) is a dimension to the fourth power \(\ell^{4}\), whereas volurne is a dimension to the third power \(l^{3}\) and surface area \(A_{t}\) is a dimension to the second power \(\ell^{2}\). Straight-line relationships have been developed for \(A_{p}\) and volume, \(A_{p}\) and surface area \(A_{t}\) and \(A_{p}\) and weight.

These relationships can now be used as new tools to simplify and standardize the process of transformer design. They make it possible to design transformers of smaller buik and volume or to optimize efficiency. While developed specifically for aerospace applications, the information has wider utility and can be used for the derign of non-aerospace transformers as well.

Because of its significance, area product, \(A_{p}\), is treated extensively. Additionally a great deal of information is presented for the convenience of the designer. Much of the material is in graphical or tabular form to assist the designer in making the tradeoffs best suited for his particular application in a minimum amount of time.

One of the basic steps in transformer design is the selection of the proper core material. To aid in the selection of cores a comparison of five common core materials is presented which illustrates their influence on overall transformer efficiency and weight. The designer should also be aware of the cost difference between core materials of the nickel steel families and the silicon steel family. In many instances, the author has found it possible to achieve suitable designs using low cost, silicon steel C-cores when the proper design tradeoffs are made.

\section*{THE DESIGN PROBLEM, GENERALLY}

The designer is faced with a set of constraints which must be observed in the design of any transformer. One of these is the output power, \(P_{o}\), (operating voltage multiplied by maximum current demand) which the secondary winding must be capable of delivering to the load within specified regulation limits. Another relates to minimum efficiency of operation which is dependent upon the maximum power loss which can be allowed in the transforiner. Still another defines the maximum permissible temperature rise for the transformer when used in its intended environment having a defined ambient temperature range.

Other constraints relate to volume occupied by the transformex and particularly in aerospace applications, weight, since weight minimization is an important goal in the design of space flight electronics. Lastly, cost effectiveness is often an important consideration.

Depending upon application, certain of these constraints will dominate. Parameters affecting others may then be traded off as necessary to achieve the most desirable design. It is not possible to optimize all parameters in a single design because of the interaction and interdependence of parameters.

For exarr ple, if volume and weight are of great significance, seductions in both often can be effected by operating the transformer at a higher frequency but at a penalty in efficiency. When the frequency cannot be raised, reduction in weight and volume may still be possible by selecting a more efficient core material, but at a penalty of increased cost. Judicious tradeoffs thus must be effected to achieve the design goals.

A flow chart showing the interrelation and interaction of the various design factors which must be taken into consideration is shown in Figure 1.


Fig. 1. Transformer Design Factors Flow Chart

Various transformer designers have used different approaches in arriving at suitable designs. For example, in many cases a rule of thumb is used for dealing with current detisity. Typically, an assumption is made that a good working level is 1000 circular mils per ampere. This may be practical in many
instances but the wire size needed to meet this requirement may produce a heavier and bulkier transformer than desired or required. The information presented herein makes it possible to avoid the use of this and other rules of thumb and to develop a more economical design with great accuracy.

\section*{THE AREA PRODUCT (Ap)}

The \(A_{p}{ }^{*}\) of a C -type core is the product of the available window a: ea ( \(W_{a}\) ) of the core in square centimeters ( \(\mathrm{cm}^{2}\) ) multiplied by the effective crosssectional area ( \(A_{c}\) ) in square centimeters ( \(\mathrm{cm}^{2}\) ) which may be stated as:
\[
\begin{equation*}
A_{p}=W_{a} A_{c} \quad\left[\mathrm{~cm}^{4}\right] \tag{I}
\end{equation*}
\]

Figure 2 shows in outline form a C-core type transformer typical of those shown in the catalogs of suppliers and uses the letter designations accepted by the industry to indicate certain sit aificant dimensions from which the \(A_{p}\) area product is calculated. From this it can be seen that \(W_{a}\) is the FG product and \(A_{c}\) is the DE product.


Fig. 2. C-Core Transformer

\footnotetext{
*Reference 1.
}

\section*{RELATIONSHIP OF Ap TO TRANSFORMER POWER HANDLING CAPABILITY}

According to the newly developed approach, the power handling capability of a core is related to its area product by an equation which may be stated as:


From the above it can be seen that factors such as flux density, frequency of operation, window utilization factor \(K_{u}\) which defines the maximurn space which may be occupied by the copper in the window and the constant \(K_{j}\) which is related to temperature rise. All have an influence on the transformer area product. The constant \(K_{j}\) is a new parameter that gives the designer control of the copper loss. Derivation is set forth in detail in Appendix D (page 36).

\section*{OUTPUT POWER VS INPUT POWER VS APPARENT POWER CAPABILITY}

Output power ( \(P_{0}\) ) is of greatest interest to the user. To the transformer designer it is the apparent power ( \(P_{t}\) ) which is associated with the geometry of the transformer that is of greater importance. Assume, for the sake of simplicity, the core of an isolation transformer has but two windings in the window area ( \(W_{a}\) ), a primary and a secondary. Also assume that the window area ( \(W_{a}\) ) is divided up in proportion to the power handling capability of the windings using equal current density. The primary winding handles \(P_{i n}\) and the secondary handles \(P_{o}\) to the load, Since the power transformer has to be designed to accommodate the primary \(P_{\text {in }}\) and secondary \(P_{o}\), then:
\[
\begin{align*}
& P_{t}=P_{i n}+P_{0} \\
& P_{t}=\frac{P_{0}}{\eta}+P_{0} \tag{3}
\end{align*}
\]

The designer must be concerned with the apparent power handling capability, \(P_{t}\), of the transformer core and windings. \(P_{t}\) may vary by a factor ranging from 2 to 2.828 times the input power, \(P_{\text {in }}\), depending upon the configuration of the circuit in which the transformer is used becarse of the different RMS current levels in the windings during operation. If the current wave shape in the rectifier transformer becomes interrupted its effective RMS value changes. Transformer size, thus, is not only affected by the load demand but, also, by the different copper (winding) losses incurred in the various circuit arrangements.

For example, for a load of one watt, compare the power handling capabilities required (neglecting transformer and diode losses so that ( \(P_{\text {in }}=P_{0}\) ) for the full-wave bridge circuit of Figure 3, the full-wave center-tapped secondary circuit of Figure 4, and the push-pull center-tapped full-wave circuit in Figure 5.

For the circuit shown in Figure 3,


Fig. 3. Euil Wave Bridge Circuit
the total apparent power \(P_{t}\) is 2 watts, as may be seen from:
\[
\begin{align*}
& P_{t}=(\overbrace{I_{N 1} E_{N 1}}^{P_{\text {in }}})+(\overbrace{I_{N 2}}^{P_{0}})  \tag{4}\\
& P_{t}=2 P_{\text {in }}
\end{align*}
\]
in which \(I_{N 1}\) and \(I_{N 2}\) are the currents associated with the primary and secondary windings, respectively, and \(\mathrm{E}_{\mathrm{N} 1}\) and \(\mathrm{E}_{\mathrm{N} 2}\) are the voltages across the primary and secondary windings, respectively.

The circuit shown in Figure 4


Fig. 4. Full Wave Center Tapped Circuit
requires an increase of \(20.7 \%\) in \(P_{t}\) due to the increased RMS rating because of the interrupted current flowing in that winding.
\[
\begin{align*}
& P_{t}=\left(I_{N 1} E_{N 1}\right)+\left[\left(0.707 I_{N 2} E_{N 2}\right)+\left(0.707 \mathrm{I}_{\mathrm{N} 3} \mathrm{E}_{\mathrm{N} 3}\right)\right]  \tag{5}\\
& P_{t}=P_{i n}+0.707 P_{i n}+0.707 P_{\text {in }}=2.414 P_{i n}
\end{align*}
\]
and for the circuit shown in Figure 5
\[
N_{1}=N_{2}=N_{3}=N_{4}
\]


Fig。 5. Pushpull Full Wave Center Tapped Circuit
which is typical of a dc to dc converter, requires a \(P_{t}\) increase to 2.828 because of the interrupted current flowing in the primary and secondary windings
\[
\begin{gather*}
\text { since } N_{1}=N_{2}=N_{3}=N_{4} \\
P_{t}=\left[\left(0.707 I_{N 1} E_{N 1}\right)+\left(0.707 I_{N 2} E_{N 2}\right)\right]+\left[\left(0.707 I_{N 3} E_{N 3}\right)+\left(0.707 I_{N 4} E_{N 4}\right)\right]  \tag{6}\\
P_{t}=0.707 P_{i n}+0.707 P_{i n}+0.707 P_{i n}+0.707 P_{i n}=2.828 P_{i n}
\end{gather*}
\]

Thus the circuit configuration in which the transformer is to be used must be considered by the designer when sizing the transformer.

Rather than discuss the various methods previously used by designers, the author believes it will be more useful to consider typical desigri problems and to work out solutions using the approach based upon the newly formulated relationships.

\section*{A SPECIFIC DESIGN PROBLEM AS AN EXAMPLE}

Assume a specification for a transformer design as shown in Figure 4 (page 7) requiring:
\[
\begin{aligned}
& \mathrm{E}_{o} \text { (output voltage) }=10 \text { volts } \\
& I_{o} \text { (output current) }=2.0 \text { amps } \\
& \mathrm{E}_{\mathrm{in}} \text { (input voltage) }=50 \text { volts } \\
& * \text { Operating frequency }(f)=2500 \mathrm{~Hz} \text { (square wave) } \\
& \text { Maximum temperature rise }=25^{\circ} \mathrm{C} \\
& \text { 宗*Transformer efficiency }=95 \% .
\end{aligned}
\]

Assuming the bridge rectifier of Figure 3, and using the efficiency constraint of \(95 \%\), the apparent power handled by the transformer is calculated (from equation (3)) to be: (1.0 volt diode drop ( \(V_{d}\) ) assumed).

Insert values
\[
\begin{gather*}
P_{t}=\frac{P_{0}}{\eta}+P_{o}  \tag{3}\\
P_{t}=I_{o}\left(E_{o}+V_{d}\right) \times\left(\frac{1}{\eta}+1\right) \\
P_{t}=\frac{24}{0.95}+24=49.3 \text { watts }
\end{gather*}
\]

This value determines the apparent power handing capability of the core needed for the transformer. A suitable core selection is made by using the area product listings in the catalogs describing the many \(C\)-core configurations (sizes and shapes) available from the various suppliers.

\footnotetext{
*For high frequency skin effect, see Appendix J (page 57). ** For transformer regulation as a function of efficiency, see Appendix \(E\) (page 39).
}

\section*{Core Selection}

Applying the data from the example to equation (2):
\[
A_{p}=\left(\frac{49.3 \times 10^{4}}{(4)(0.3) *(2500)(0.4)(323)}\right)^{1.16}=1.32 \mathrm{~cm}^{4}
\]

After the Ap has been determined, the geometry of the transformer can be evaluated as described in Appendix \(G\) for weight, Appendix \(C\) for surface area and Appendix \(H\) for volume, and appropriate changes made, if required. Having established the configuration, it is then necessary to determine the core material to complete core selection. Material selection requires consideration of efficiency constraint which is 0.95 in the example. The total transformer losses are
\[
\begin{equation*}
P_{\Sigma}=\frac{P_{0}}{\eta}-P_{0} \tag{7}
\end{equation*}
\]

Inserting values:
\[
P_{\Sigma}=\frac{24}{0.95}-24=1.26 \mathrm{watts}
\]

Maximum efficiency is realized when the copper (winding) losses are equal to the iron (core) losses (see Appendix B, page 27) which is expressed as
\[
\begin{aligned}
& P_{\mathrm{cu}}=P_{\mathrm{fe}}, \text { and the refore } \\
& P_{\mathrm{cu}}=\frac{P_{\Sigma}}{2} \text { and thus } \\
& P_{\mathrm{cu}}=0.63=P_{\mathrm{fe}}
\end{aligned}
\]

Referring to Table 1, column 3 (pages 11 and 12), the AL- 124 core with a \(A_{p}\) of \(1.44 \mathrm{~cm}^{4}\) is closest to the \(1.32 \mathrm{~cm}^{4} A_{p}\) calculated above.

\footnotetext{
This is an arbitrary figure developed through years of experience. It can be scaled upwardly for comparison of materials with higher flux density.
}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
\hline & Car＊ & \(A_{t} \mathrm{~cm}^{2}\) & \(A_{p} \mathrm{~cm}{ }^{+}\) & ML＇T cm &  & 11．4 \(30 \cdot \mathrm{C}\) & \({ }^{\text {P }}\) & \(1 \cdot \sqrt{\frac{M}{\underline{\mu}} 5}\) & \begin{tabular}{l}
\(\Delta \div 25{ }^{+} \mathrm{C}\) \\
\(\mathrm{J}=\frac{\text { ampat }}{\mathrm{cm}^{2}}\)
\end{tabular} &  & \(\mathrm{P}_{\mathbf{Y}}\) & \(1 \cdot \sqrt{\frac{1}{4}}\) & \(\Delta T \mathbf{5 0 *} \mathbf{C}\)
J. 恝品品 & Total Walsht & Valume
\[
\mathrm{cm}^{3}
\] & \(\mathrm{Accm}^{2}\) \\
\hline \(t\) & AL－2 & 20.4 & 1． \(2 \cdot\) & 3.75 & 66.230 & 4． 93 & 0.127 & 0.147 & 370 & 9． HI & 1.46 & 0.273 & 326 & 23． 33 & 7.14 & 0． 265 \\
\hline 2 & AL． 3 & 23.7 & 0.910 & ＋．14 & \(44^{4} \mathbf{3} 30\) & 10． 3 & 0．717 & 0．18＊ & 365 & 11， 3 & 1.67 & 0.269 & 522 & 31.38 & H． 42 & 0.410 \\
\hline 3 & AL－\({ }^{\text {i }}\) & 11．is & 0.7 .7 & 4． 79 & 916 & 16．\({ }^{\text {i }}\) & 1.01 & 0.174 & 345 & \(1 \mathrm{s}\). & 2.35 & 0． 253 & 143 & 52.8 & 14，06 & U． 534 \\
\hline 4 & AL－＊ & 17．\({ }^{\text {a }}\) & 5．0tt & 5.23 & 91430 & 14．H & 1.13 & 0.112 & 341 & 20.6 & 2.63 & 0.253 & \(48 \%\) & 65.1 & 16．68 & 0.716 \\
\hline 5 & AL． 124 & 15． 1 & 1．4 & 5， 30 & 131730 & 27.5 & 1． 36 & 0.157 & 310 & 30.2 & 3.17 & 0.224 & 443 & 80.8 & 22． 50 & 0． 716 \\
\hline 6 & AL－H & －． 3.4 & 2.31 & i． 34 & 22120 & 0.442 & 1．90 & 1.404 & 271 & 0.329 & 4.44 & 2，05 & 395 & 127.85 & 35．46 & 0.106 \\
\hline 7 & AL－4 & 1．4．0 & 3．w & 4．3\％ & 22120 & 0．i3； & 2.07 & 1.39 & 26．1 & 0． 3 HT & 4． 83 & 2.03 & 391 & 155.8 & 41． 4.2 & 1.077 \\
\hline \(\cdots\) & AL－3 \({ }^{\text {a }}\) & \％．\({ }^{\text {\％}}\) & 3.45 & 7．01 & 22120 & 0．јヶ4 & 2.24 & 1.38 & 2 id & 0．6．45： & 5.22 & 2． 31 & 347 & 183.2 & 47． 35 & 1．342 \\
\hline ＇ & AL． 12 & \(\mathrm{NF}_{6} \mathrm{H}\) & 4． 17 & \(\therefore 9\) & 278 & 0． i \(^{\text {a }}\) & 2.61 & 1． 32 & 253 & 0.821 & 6.09 & 1.93 & 371 & 204.2 & 61.38 & 1.26 \\
\hline 11 & AL－15： & 93.7 & 5．1． & 7．3． & \({ }^{325} 20\) & 0.908 & 2． 81 & 1.24 & 240 & 0．997 & 6.36 & 1.81 & 345 & 227.0 & 69.63 & 1．26 \\
\hline 11 & AL． \(\mathrm{FH}^{\text {H }}\) & \(4 \mathrm{H}\). & t．ai & 7.01 & 31220 & 0.831 & 2.94 & 1.33 & 256 & 0． 912 & 6.87 & 1.94 & 374 & 258．0 & 62， 83 & 1， 34 \\
\hline 12 & At． 1 \％ & 11 H & \(\bigcirc{ }^{-18}\) & 7．i．1 & 31020 & 1． 47 & 2． 55 & 1.10 & 211 & 1.61 & 8.26 & 1.60 & 308 & 321.0 & －8． 79 & 1.45 \\
\hline 13 & AL－I； & 120 & \(\because .47\) & 8．0； & \({ }^{3} 4620\) & 1.14 & 3． 58 & 1.23 & 237 & 1.30 & 8.40 & 1． 79 & 946 & 352.0 & 94.43 & 1．80 \\
\hline 14 & AL－16 & 123 & 10．0 & 8． 89 &  & 1． 30 & 3．80 & 1.20 & 233 & 1.13 & 8.89 & 1.76 & 340 & 397.0 & 104.95 & 2.15 \\
\hline 15 & AL－1； & 142 & 14．4 & 10.3 & \({ }^{386} 20\) & 1.51 & 4.25 & 1.185 & 228 & 1，64 & 9.94 & 1.73 & 333 & \＄02．0 & 124．94 & 2． 87 \\
\hline 16 & AL． 14 & 154 & it． & 10．8 & 51120 & 3.10 & 4． 77 & 1.065 & 205 & 2.31 & 11.1 & 1.55 & 299 & 589．0 & 153， 14 & 2．87 \\
\hline 17 & AL－20 & 1Hz & 24，\({ }^{4}\) & 11.5 & 51120 & 2.23 & 5.16 & 1．104 & 213 & 2． 45 & 12.7 & 1.61 & 310 & 715，0 & 187， 015 & 3． 54 \\
\hline th & AL－22 & 202 & ［H．0 & 11，5 & 8.3780 & 2.78 & 6． 05 & 1.043 & 201 & 3.05 & 14．1 & 1． 52 & 293 & 835，0 & 212．04 & 3． 58 \\
\hline 19 & AL．\({ }^{\text {a }}\) S & 220 & 34.9 & 12.7 & \({ }^{637} 20\) & 3.07 & 4.60 & 1.036 & 200 & 3.37 & 15.4 & I． 51 & 291 & 994.0 & 244.67 & 4.48 \\
\hline 20 & AL－24 & 245 & ＊0． 0 & 12， 0 & \({ }^{948} 20\) & 4.32 & 7.35 & 0.922 & 178 & 4.74 & 17.1 & 1.35 & 239 & 1090.0 & 280.91 & 3， 58 \\
\hline
\end{tabular}

Table 1．C－Core Characteristics

\section*{Definitions for Table 1}

Information given is listed by column as:
1. Manufacturer part number
2. Surface area calculated from Figure C 3
3. Area product effective iron area times window area
4. Mean length turn on one bobbin
5. Total number of turns and wire size for two bobbins using a window utilization factor \(K_{u}=0.40\)
6. Resistance of the wire at \(50^{\circ} \mathrm{C}\)
7. Watts loss is based on Figure Cl for a \(\Delta \mathrm{T}\) of \(25^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, total loss is equal to \(2 \mathrm{P}_{\mathrm{cu}}\)
8. Current calculated from column 6 and 7
9. Current density calculated from column 5 and 8
10. Resistance of the wire at \(75^{\circ} \mathrm{C}\)
11. Watts loss is based on Figure Cl for a \(\Delta \mathrm{T}\) of \(50^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, total loss is equal to \(2 \mathrm{P}_{\text {cu }}\)
12. Current calculated from column 10 and 11
13. Current density calculated from column 5 anc 12
14. Effective core weight plus copper weight
15. Transformer volume calculated from Figure H1
16. Core effective cross-section


Fig. 6. Magnetic Material Comparison at a Constant Frequency

Referring to column 14, the weight of the core is 46.6 grams. The core loss in milliwatts per gram is obtained from
\(\frac{0.63 \text { watts }}{46.6 \text { grams }}=0.0135\) which converts to 13.5 milliwatts/gram.

The efficiency of various silicon and nickel steels for various high frequencies and flux density is shown in the graphs of Figure 6*. Reading from the 2.5 KHz frequency curve for a flux density of 0.3 tesla, the loss per gram is about 12 milliwatts per gram, which for 46.6 grams is a total core loss of 560 milliwatts which permits use of a silicon steel core material.

\section*{Winding Parameters}

The power loss in the winding can now be accurately determined. First it is necessary to calculate the number of turns in the primary and secondary. The number of primary turns is calculated from the Faraday law which states:
\[
\begin{equation*}
N=\frac{E \times 10^{4}}{4 B_{m} A_{c} \mathrm{f}} \tag{8}
\end{equation*}
\]

Inserting values from the data:
\[
\mathrm{N}=\frac{50 \times 10^{4}}{(4)(0.3)(0.716)(2500)}=233 \operatorname{turn}(\text { primary })
\]

\footnotetext{
\({ }^{*}\) These curves are for sine waves but are substantially the same for square waves.
}
(The core cross-section value \(A_{c}\) is obtained from Table I (pages 11 and 12).) The secondary turns are calculated from:
\[
\frac{\text { Primary turns }}{\text { Voltage }}=\frac{233}{50}=4.7 \text { turns per volt }
\]

Since the specified load voltage is 10 volts plus two diode drops, \(4.7 \times 12=57\) turns (secondary).

\section*{Current Density and Wire Size}

The relationship between the area product \(A_{p}\) and current density is:
\[
\begin{equation*}
J=K_{j} A_{p}^{-0.14} \tag{9}
\end{equation*}
\]
in which \(\mathrm{K}_{j}\) is a constant which has a value of 323 for a \(25^{\circ} \mathrm{C}\) rise and a value of 68 for a \(50^{\circ} \mathrm{C}\) rise. Derivation is shown in Appendix D (page 36).

Inserting values:
\[
J=(323)(1.44)^{-0.14}=307 \mathrm{amp} / \mathrm{cm}^{2}
\]

The primary winding current will be:
\[
\frac{\text { input power }}{\text { input voltage }}=\frac{25.2}{50.0}=0.50 \mathrm{amp}
\]

The wire size for the primary is:
\[
\frac{0.50}{307}=0.00162 \mathrm{~cm}^{2}
\]

From the wire table, page 45, No. 25 wire has a diameter of \(0.001623 \mathrm{~cm}^{2}\) and is therefore suitable.

The wire size for the secondary is:
\[
\frac{2.0}{307}=.00651 \mathrm{~cm}^{2}
\]

From the wire table, page 45, No. 19 wire has a diameter of \(0.00653 \mathrm{~cm}^{2}\) ard is therefore suitable.

The power loss in the windings then can be calculated. The resistance of a winding is the mean length turn in cm multiplied by the resistance in microhms per cm and the total number of turns, or:
\[
\mathrm{R}=\operatorname{MLT} \times \mathbb{N} \times(\text { Column } C) \times 10^{-6}
\]

For the primary winding:
\[
\mathrm{R}=5.5 \times 0.00106 \times 233=1.36 \Omega
\]

For the secondary winding:
\[
R=5.5 \times 0.000264 \times 57=0.0827 \Omega
\]

Since power loss is: \(P=I^{2} R\)

Copper loss in the primary is \((0.50)^{2} \times 1.36\) or 0.340 watt. In the secondary, the loss is \((2.0)^{2} \times 0.0827\) or 0.331 . The total loss in the windings is 0.671 watt. Since the power loss in the core is 0.560 watt, the total power loss in the transformer will be 1.23 watts, which will meet the required efficiency parameter.

\section*{Another Design Problem As An Example}

Assume a specification for a transformer design as shown in Figure 4 in which:
\[
\begin{aligned}
& E_{o}=56.0 \text { volts after a diode drop } 1.0 \text { volt } \\
& P_{o}=100 \text { watts to the load } \\
& \mathrm{E}_{\mathrm{IN}}=200 \text { volts } \\
& * \text { Operating frequency }=10 \mathrm{KHz} \text { (square wave) } \\
& \text { Maximum temperature rise }=25^{\circ} \mathrm{C} \\
& \text { N} T \text { ransformer efficiency }=98 \%
\end{aligned}
\]

Because of the diode drop, the actual output power of the transformer is 101. 8 watts. Since Figure 4 shows a center tapped secondary, \(P_{t}\) is \(20.7 \%\) greater than in the first example because of the increased RMS rating as explained in equation (5). Thus
\[
P_{t}=\left(\frac{P_{0}}{\eta}+P_{o}\right) \times 1,207
\]

\section*{Inserting vaiues:}
\[
P_{t}=\left(\frac{201.8}{0.98}+101.8\right) \times 1.207=248 \text { watts }
\]

The proper core is obtained from the area product using equation (2).

Inserting values:
\[
A_{p}=\left(\frac{248 \times 10^{4}}{(4.0)(0.3)\left(10^{4}\right)(0.4)(323)}\right)^{1.14}=1.71 \mathrm{~cm}^{4}
\]

\footnotetext{
For high frequency skin effect, see Appendix J (page 57).
\({ }^{*}\) For transformer regulation as a function of efficiency, see Appendix \(E\) (page 39).
}

After the \(A_{p}\) has been determined, the geometry of the transformer can be evaluated as described in the first example, (page 10), and appropriate changes made, if desired. Having established the configuration, it is then necessary to determine the core material to complete core selection. Material selection requires consideration of efficiency constraint which is 0.98 in the example.

The transformer Iosses are, from equation (7)
\[
P_{\Sigma}=\frac{P_{o}}{\eta}-P_{o}
\]

Inserting values:
\[
P_{\Sigma}=\frac{101.8}{0.98}-101.8=2.08 \text { watts }
\]

Again maximum efficiency is realized when the copper (winding) losses are equal to the iron (core) losses which is expressed as:
\[
\begin{aligned}
& P_{c u}=P_{f e}, \text { and therefore } \\
& P_{c u}=\frac{P_{\Sigma}}{?} \quad \text { and thus } \\
& P_{c u}=1.04=P_{f e}
\end{aligned}
\]

Referring to Table 1 , column 3 (pages 11 and 12), the \(A L-8\) core with an \(A_{p}\) of 2.31 is closest to the \(1.71 \mathrm{~cm}^{4} \mathrm{~A}_{\mathrm{p}}\) caluclated above. Referring to column 14, the weight of the core is 66.6 grams. The core loss in militiwatts per grem is obtained from
\[
\frac{1.04 \text { watts }}{66.6 \text { grams }}=0.0156 \text { which converts to }
\]
\[
15.6 \text { milliwatts/gram }
\]

Knowing the core loss in milliwatts/grams, the designer refers to the graphs of Figure 6 (page 13). Reading from the curve for the 10 KHz frequency of operation which is specified, it appears that for a flux density of 0.3 tesla, the material that comes closest to 15.6 milliwatts per gram is Permalloy 80 which is approximately 12 milliwatts per gram. When nickel steel is used, Table I2 (page 55) in Appendix I provides a weight correction factor. The weight of 66.6 is increased to 76.5 to give a total core loss of 918 milliwatts.

\section*{Winding Parameters}

The power loss in the winding can then be determined. First it is necessary to calculate the number of turns in the primary and secondary. The number of primary turns is calculated from the Faraday law equation (8) which states:
\[
N=\frac{E \times I 0^{4}}{4 B_{m} A_{c} f}
\]

Inserting values from the data:
\[
\mathrm{N}=\frac{200 \times 10^{4}}{(4)(0.3)(0.806)\left(10^{4}\right)}=207 \text { turns (primary) }
\]
(The core crosswsection value \(A_{c}\) is obtained from Table 1, pages 11 and 12).

The secondary turns are calculated from:
\[
\frac{\text { primary turns }}{\text { voltage }}=\frac{207}{200}=1.035 \text { turns per volt }
\]

Since the specified secondary voltage is \(57,1.035 \times 57=59\) turns each side of center tap.

\section*{Current Density and Wire Size}

The relationship between the area product \(A_{P}\) and current density from equation (9) is:
\[
J=K_{j} A_{p}^{-0.14}
\]
in which \(K_{j}\) is a constant which has a value of 323 for a \(25^{\circ} \mathrm{C}\) rise and a value of 468 for a \(50^{\circ} \mathrm{C}\) rise. Derivation is shown in Appendix \(D\) (page 36 ).

Inserting vaiues:
\[
J=(323)(2.31)^{-0.14}=287 \mathrm{amp} / \mathrm{cm}^{2}
\]
the primary winding current will be:
\[
\frac{\text { input power }}{\text { input voltage }}=\frac{104}{200}=0.52 \mathrm{amp} .
\]

The wire size for the primary is:
\[
\frac{0.52}{285}=0.00181 \mathrm{~cm}^{2}
\]

From the wire table, (page 45), No. 25 wire has a diameter of \(0.001623 \mathrm{~cm}^{2}\). The rule is that when the calculated wire size does not fall close to those listed in the table, the next smallest size should be selected.

The wire size for the secondary is:
\[
\frac{\text { output current }(0.707)}{287}=\frac{1.79 \times(0.707)}{287}=0.0044 \mathrm{~cm}^{2}
\]

From the wire table, No. 21 wire has a diameter of \(0.00411 \mathrm{~cm}^{2}\) and is therefore suitable.

The power loss in the winding then can be calculated. From equa.. tion (10), (page 16):
\[
\mathrm{R}=\mathrm{MLT} \times \mathbb{N} \times(\text { Column } C) \times 10^{-6}
\]
for the primary winding:
\[
R=5.74 \times 0.001062 \times 207=1.26 \Omega
\]
for the secondary winding:
\[
R=5.74 \times 0.000419 \times 59=0.142 \Omega
\]
since power loss is:
\[
P=I^{2} R
\]

Copper loss in the primary is \((0.52)^{2} \times 1.26\) or 0.341 watts. In the secondary, the loss is \((1.79 \times 0.707)^{2} \times 0.142 \times 2=0.455\) watts. The total loss in the winding is 0.796 watts. Since the power loss in the core is 0.918 watts, the total power loss in the transformer will be 1.714 watts, which will meet the required efficiency parameter.

The author has put in Appendix \(K\) the area product \(A_{p}\) relationships between volume, surface area, current density, and weight for pot core, tape wound cores (toroids), power cores, laminations, and C cores. Much of the material is in graphical or tabular form ro assist the designer in making the tradeoffs best suited for his particular application in a minimum amount of time.
1. McLyman, C., "Design Parameters of Toroidal and Bobbin Magnetics. Technical Memorandum 33-651, Pages 12-15 Jet Propulsion Laboratory, Pasadena, Ca.
2. Blume, L. F., Transformer Engineering, John Wiley \& Sons's, Inc., New York, N. Y. 1938. Pages 272-282
3. Terman, F.E., Radio Engineers Handbook, McGraw-Hill Book Co., Inc., New York 1943. Pages 28-37

\section*{APPENDIX A}

\section*{TRANSFORMER POWER HANDLING CAPABILITY}

The power handling capability of a transformer can be related to its \(A_{p}\) quantity (which is actually its \(W_{a} A_{c}\) product where \(W_{a}\) is the available core window axea in \(\mathrm{cm}^{2}\) and \(A_{c}\) is the effective cross-sectional area of the core in \(\operatorname{cm}^{2}\) ), as follows.

A form of the Faraday law of electromagnetic induction much used by transformer designers states;
\[
\begin{equation*}
E=K B_{m} A_{c} N f \times 10^{-4} \tag{A1}
\end{equation*}
\]
(The constant \(K\) is taken at 4 for square wave and at 4.44 for sine wave operation.)

It is convenient to restate this expression as:
\[
\begin{equation*}
\mathrm{NA} \mathrm{~A}_{\mathrm{c}}=\frac{\mathrm{E} \times 10^{4}}{4 \mathrm{~B}_{\mathrm{m}} \mathrm{f}} \tag{A2}
\end{equation*}
\]
for the following manipulation.
By definition the window utilization factor is:
\[
\begin{equation*}
K_{u}=\frac{N A_{w}}{W_{a}} \tag{A3}
\end{equation*}
\]
and this may be restated as:
\[
\begin{equation*}
\mathrm{N}=\frac{\mathrm{K}_{\mathrm{u}}{ }^{W} \mathrm{a}}{\mathrm{~A}_{\mathrm{w}}} \tag{A4}
\end{equation*}
\]

If both sides of the equation are multiplied by \(A_{c}\), then:
\[
\begin{equation*}
N A_{c}=\frac{\mathrm{K}_{\mathrm{u}} \mathrm{~W}_{\mathrm{a}} \mathrm{~A}_{\mathrm{c}}}{\mathrm{~A}_{\mathrm{w}}} \tag{A.5}
\end{equation*}
\]

From equation (A2):
\[
\begin{equation*}
\frac{K_{u} W_{a} A_{c}}{A_{w}}=\frac{E \times 10^{4}}{4 B_{m}} \tag{A6}
\end{equation*}
\]

Solving for \(W_{a} A_{c}\) :
\[
\begin{equation*}
W_{a} A_{c}=\frac{E A_{w} \times 10^{4}}{4 B_{m} f K_{u}} \tag{A7}
\end{equation*}
\]

By definition, current density \(J=\mathrm{amp} / \mathrm{cm}^{2}\) which may also be stated:
\[
\begin{equation*}
J=\frac{I}{A_{w}} \tag{A8}
\end{equation*}
\]
which may also be stated as:
\[
\begin{equation*}
A_{w}=\frac{I}{J} \tag{A9}
\end{equation*}
\]

It will be remembered that transformer efficiency is defined as:
\[
\begin{equation*}
\eta=\frac{P_{o}}{P_{i n}} \quad \text { and } \quad P_{i n}=E I \tag{A10}
\end{equation*}
\]

Rewriting equation (A7) as:
\[
\begin{equation*}
E A_{W}=4 B_{m f} K_{u} W_{a} A_{c} 10^{-4}=\frac{E I}{J} \tag{All}
\end{equation*}
\]
and since:
\[
\begin{equation*}
\frac{E I}{J}=\frac{P_{i n}}{I}=\frac{P_{0}}{J \eta} \tag{A12}
\end{equation*}
\]
then:
\[
\begin{align*}
& \left.W_{a} A_{c}\right|_{\text {total }}=\left.W_{a} A^{c}\right|_{\text {Primary }}+\left.W_{a} A_{c}\right|_{\text {Secondary }} \\
& \left.W_{a} A_{c}\right|_{\text {total }}=\frac{P_{o} \times 10^{4}}{J B_{m} B_{u}}+\frac{P_{o} \times 10^{4}}{4 B_{m} f K_{u}^{J}}=\frac{P_{o} \times 10^{4}}{4 B_{m} f K_{u}^{J}}(1 / \eta+1) \tag{AI3}
\end{align*}
\]
and since
\[
\begin{equation*}
P_{t}=\frac{P_{o}}{\eta}+P_{0} \tag{A14.}
\end{equation*}
\]
then
\[
\begin{equation*}
W_{a} A_{c}=\frac{P_{t} \times 10^{4}}{{ }^{4 B} m^{f K_{u}}{ }^{J}} \tag{A15}
\end{equation*}
\]
which may also be stated as applied in Appendix D (page 36) Transformer Current Density as:
\[
\begin{equation*}
A_{p}=\frac{P_{t} \times 10^{4}}{4 B_{m}{ }^{f J K}} \tag{A16}
\end{equation*}
\]

\section*{APPENDIX B}

\section*{TRANSFORMER EFFICIENCY}

The efficiency rating of a transformer is a measure of the effectiveness of the deaign. Efficiency is defined as the ratio of the output power \(P_{0}\) to the input power \(P_{\text {in }}\). The difference between the \(P_{0}\) and the \(P_{\text {in }}\) is due to losses. The total power loss in a transformer is made up of fixed losses in the core and quadratic losses in the windings or copper. Thus
\[
\begin{equation*}
P_{\Sigma}=P_{f e}+P_{c u} \tag{B1}
\end{equation*}
\]
where \(P_{f e}\) represents the core loss and \(P_{c u}\) represents the copper loss.
Maximum efficiency is achieved when the fixed loss is equal to the quadratic loss as shown by the equations on page 28. Transformer loss versus output load current is shown in Figure Bl, below.


Fig. B1. Transformer Loss Versus Ottput Load Current

The copper loss increases as the square of the output power multiplied by a constant K which is thus:
\[
\begin{equation*}
P_{c u}=K P_{o}^{2} \tag{B2}
\end{equation*}
\]
which may be rewritten as
\[
\begin{equation*}
P_{\Sigma}=P_{f e}+K P_{o}^{2} \tag{B3}
\end{equation*}
\]

\section*{Since}
\[
P_{i n}=P_{o}+P_{\Sigma}
\]
and the efficiency is
\[
\eta=\frac{P_{o}}{P_{o}+P_{\Sigma}}
\]
then:
\[
\eta=\frac{P_{0}}{P_{0}+P_{f e}+K P_{o}^{2}}=\frac{P_{0}}{P_{f e}+P_{0}+K P_{0}^{2}}
\]
and, differentiating with respect to \(P_{0}\) :
\[
\begin{gather*}
\frac{d \eta}{d P_{o}}=-P_{o}\left[P_{f e}+P_{o}+K P_{o}^{2}\right]^{-2}\left(1+2 K P_{o}\right) \\
+\left[P_{f e}+P_{o}+K P_{o}^{2}\right]=0 \text { for max } \eta \\
-P_{o}\left(1+2 K P_{o}\right)+\left(P_{f e}+P_{o}+K P_{o}^{2}\right)=0 \\
-P_{o}-2 K P_{o}^{2}+P_{f e}+P_{o}+K P_{o}^{2}=0 \\
\therefore P_{f e}=K P_{o}^{2}=P_{c u} \tag{B4}
\end{gather*}
\]

\section*{APPINDIX C}

\section*{RELATIONSHIP OF Ap TO CONTROL OF TEMPERATURE RISE}

\section*{Temperature Rise}

Not all of the \(P_{\text {in }}\) input power to the transformer is delivered to the load as the \(P_{o}\). Some of the input power is converted to heat by hysteresis and eddy currents induced in the core material, and by the resistance of the windings. The first is a fixed loss arising from core excitation and is termed "core loss." The second is a variable loss in the windings which is related to the current demand of the load and thus varies as \(I^{2} R\). This is termed the quadratic or copper loss.

The generated heat produces a temperature rise which must be controlled to prevent damage to or failure of the windings by breakdown of the wire insulation at elevated temperatures. Such heat is dissipated only from the exposed surfaces of the transformer by a combination of radiation and convection, and thus is dependent upon the total exposed surface area of the core and windings.

Ideally, maximum efficiency is achieved when the fixed and quadratic losses are equal. Thus:
\[
\begin{equation*}
P_{\Sigma}=P_{f e}+P_{c u} \tag{Ci}
\end{equation*}
\]
and
\[
\begin{equation*}
P_{c u}=\frac{P_{\Sigma}}{2} \tag{C2}
\end{equation*}
\]

When the copper loss in the primary winding is equal to the copper loss in the secondary, the current density in the primary is the same as the current density in the secondary:
\[
\begin{equation*}
\frac{P_{p}}{R_{p}}=\frac{P_{s}}{R_{s}} \tag{C3}
\end{equation*}
\]
and:
\[
\begin{equation*}
\frac{P_{\Sigma}}{R_{t}}=\frac{2 P_{P}}{R_{p}} / 2=\frac{4 P_{p}}{R_{p}}=\left(2 I_{p}\right)^{2} \tag{C4}
\end{equation*}
\]
then:
\[
\begin{equation*}
J_{p}=\frac{I_{p}}{W_{a} / 2}=\frac{2 I_{p}}{W_{a}}=J_{s}=J \tag{C5}
\end{equation*}
\]

\section*{Calculation of Temperature Rise}

Temperatare rise in a transformer winding cannot be predicted with complete precision, despite the fact that many different techniques are described in the literaturt for its calculation. One reasonably accurate method for open core and winding construction is based upon the assumption that core and winding losses may be ?nmped together as:
\[
\begin{equation*}
P_{\Sigma}=P_{f e}+P_{c u} \tag{C6}
\end{equation*}
\]
and the assumption that thermal energy is dissipated throughout the surface area of the core and winding assembly.

Transfer of heat by radiation occurs because any body raised to a 'emperature above its surroundings sixits heat energy in the form of waves. In accordance with the Stefan-Boitzmann law,* this may be expressed as:
\[
\begin{equation*}
W_{r}=K \in\left(T 2^{4}-T 1^{4}\right) \tag{C7}
\end{equation*}
\]
in which
\[
\begin{aligned}
W_{r} & =\text { watts per square inch of surface } \\
K & =3.68 \times 10^{-11}
\end{aligned}
\]

Weference No. 2
\(\epsilon=\) emissivity factor
\(\mathrm{T} 2=\) hot body temperature in absolute degrees
\(T I=\) ambient or surrounding temperature in absolute degrees.
Transfer of heat by convection occurs when a body is hotter than the surrounding medium, which usually is air. A thin layer of air in intimate contact with the hot body is heated by conduction and expands, rising to take the absorbed heat with it. The next layer being colder, replaces the risen layer, and in turn on being heated also rises. This continues until all of the medium surrounding the body is at the body temperature. Transfer of heat by convection* \({ }^{*}\) is stated as:
\[
\begin{equation*}
W_{c}=K F \theta^{\eta} \sqrt{p} \tag{C8}
\end{equation*}
\]
in which:
\[
\begin{aligned}
W_{c}= & \text { watts loss per square inch } \\
K= & I .4 \times 10^{-3} \\
F= & \text { air friction factor (unity for a vertical surface) } \\
\theta= & \text { temperature rise, degrees } C \\
p= & \text { relative barometric pressure (unity at sea level) } \\
\eta= & \text { exponential value ranging from } 1.0 \text { to } 1.25, \text { depending on the shape } \\
& \text { and position of the surface being cooled. }
\end{aligned}
\]

The total loss dissipated from a plane vertical surface is expressed by the sum of equations (C7) and (C8),
\[
\begin{equation*}
W=3.68 \times 10^{-11} \in\left(\mathrm{~T} 2^{4}-\mathrm{T} 1^{4}\right)+1.4 \times 10^{-3} \mathrm{~F} \theta^{1.25} \sqrt{\mathrm{p}} \tag{C9}
\end{equation*}
\]

\section*{Temperature Rise Versus Surface Area Dissipation}

The temperature rise which may be expected for various levels of power Ioss is shown in the nomograph of Figure \(C l\) below. It is based on equation (C9)

WReference Na. 2


Fig. C1. Temperature Rise Versus Surface Dissipation
relying on data obtained from Reference 2 for heat transfer effected by a combination of \(55 \%\) radiation and \(45 \%\) convection, from surfaces having an emissivity of 0.95 , in an ambient of \(25^{\circ} \mathrm{C}\), at sea level. Power loss (heat dissipation) is expressed in watts/ \(\mathrm{cm}^{2}\) of total surface area. Heat dissipation by convection from the upper side of a horizontal flat surface is on the order of 15 to \(20 \%\) more than from vertical surfaces. Heat dissipation from the underside of a horizontal flat surface depends upon surface area and conductivity. Surface Area Required for Heat Dissipation

The effective surface area \(A_{t}\) required to dissipate heat (expressed as watts loss per unit area) is:
\[
\begin{equation*}
A_{\Sigma}=\frac{P_{\Sigma}}{\underline{Y}} \tag{ClO}
\end{equation*}
\]
in which \(\Psi\) is the power density or the average power'lost per unit area of the heat dissipating surface of the transformer and \(P_{\Sigma}\) is the total power lost or dissipated.

Surface area \(A_{t}\) of a transformer can be related to the area product \(A_{p}\) of a C-core transformer. The straightline logarithmic relationship shown in Figure C2 below, has been Flotted from the data shown in Table 1 (pages 11 and 12).


Fig. C2. Surface Area Versus Area Product Ap

The relationship is obtained from the conventional slope relationship:
\[
\text { Slope }=\frac{\log A_{t 2} / A_{t 1}}{\log A_{p 2} / A_{p l}}
\]
according to:

in which the subscripts denote the extremes of the values in each column.
From this it appears that:
\[
\begin{equation*}
A_{t}=K_{s}\left(A_{p}\right)^{0.5}=\frac{P_{\Sigma}}{\Psi} \tag{CII}
\end{equation*}
\]
and that (from Fig. C1)
\[
\begin{aligned}
\Psi & =0.03 \mathrm{~W} / \mathrm{cm}^{2} @ 25^{\circ} \mathrm{C} \text { rise } \\
\Psi & =0.07 \mathrm{~W} / \mathrm{cm}^{2} @ 50^{\circ} \mathrm{C} \text { rise }
\end{aligned}
\]
in which the constant \(K_{s}\) has been derived empirically by averaging the data presented in Table 1 (pages 11 and 12) columns 2 and 3. Golumn 3 wasincreased to account for the gross area of the iron and \(K_{s}\) therefore is 39.2 .

\section*{Calculation of Surface Area of C.Cores}

Table 1 (pages 11 and 12) is a tabulation of data relating to selected \(C\)-cores of standard manufacture. The surface areas \(A_{t}\) of those cores were calculated in accordance with the dimensional relations shown in Figures C3 and \(C 4\) below, which derive from the geometry of the core and windings of C-type core transformers as fabricated to industry standards.

\(A_{t}=4 E(2 E+F)+(E D) 4+2(D+F)(G)+2(2 F+2 E)(G)+2(D+F)(2 F+2 E)\)

Fig. C3. Surface Area Calculation


Fig. C4. Industrial Description

\section*{APPENDIX D}

\section*{TRANSFORMER GURRENT DENSITY}

Current density \(J\) of a transformer can be related to the surface area \(A_{t}\) of a C-core transformer for a given temperature rise. The straightline logarithmic relationship shown in Figure Dl below, has been plotted from the data shown in Table 1 (pages 11 and 12).


Fig. DI. Current Density Versus Surface Area for a \(25^{\circ} \mathrm{C}\) and \(50^{\circ} \mathrm{C}\) Rise

The relationship is obtained from the conventional slope relationship:
\[
\text { Slope }=\frac{\log J_{1} / J_{2}}{\log A_{t}^{\prime} / A_{t}^{Z}}
\]
according to:


The relationship is:
\[
\begin{equation*}
J=K_{1} A_{t}^{-0.28} \tag{D1}
\end{equation*}
\]
in which \(\mathrm{K}_{1}\) is a constant which is calculated to be 776 for a \(25^{\circ} \mathrm{C}\) temperature rise and 1120 for a \(50^{\circ}\) temperature fise.

The relationship of current density \(J\) to the area product \(A_{p}\) for a given temperature rise can be derived as follows.

The surface area \(A_{t}\) relation to the area product \(A_{p}\) derived in equation CII of Appendic C, states:
\[
\begin{equation*}
A_{t}=K_{s}\left(A_{p}\right)^{0.5} \tag{D2}
\end{equation*}
\]

Combining the equations D1 and D2
\[
\begin{gather*}
A_{t}^{-0.28}=\frac{J}{K_{1}}=\left(K_{s} A_{p}^{0.5}\right)^{-0.28} \\
J=K_{1}\left(K_{s} A_{p}^{0.5}\right)^{-0.28} \\
J=K_{1} K_{s}^{-0.28} A_{p}^{-0.14} \\
K_{j}=K_{1}\left(K_{s}\right)^{(-0.28)} \\
J=K_{j} A_{p}^{-0.14} \tag{D3}
\end{gather*}
\]
where:
\(\mathrm{K}_{\mathrm{j}}\) for \(25^{\circ} \mathrm{C}\) rise is 323 and \(\mathrm{K}_{\mathrm{j}}\) for \(50^{\circ}\) rise is 468 from the data of Table 1 (pages 11 and 12 ) in columns 3 and 6 and 3 and 10. This expression may now be inserted in equation (A16) from Appendix \(A\) which is:
\[
A_{p}=\frac{P_{t} \times 10^{4}}{4 B_{m}{ }^{\text {fJK }}}
\]
yielding:
\[
\begin{gather*}
A_{p}=\frac{P_{t} \times 10^{4}}{4 B_{m} f K_{u}\left(K_{j} A_{P}^{-0.14}\right)} \\
A_{p}^{0.86}=\frac{P_{t} \times 10^{4}}{4 B_{m} E K_{u} K_{j}} \\
A_{p}=\left(\frac{P_{t} \times 10^{4}}{4 B_{m} K_{u} K_{j}}\right)^{1,16} \tag{D4}
\end{gather*}
\]

Figure D2 utilizes the efficiency rating in watts loss in terms of two different, but coramonly used allowable temperature rises for the transformer over ambient temperature. The data presented are used as bases for indicating the needed transformer surface area \(A_{t}\) (in \(\mathrm{cm}^{2}\) ).


Fig. D2. Surface Area Versus Total Watt Loss for a \(25^{\circ} \mathrm{C}\) and \(50^{\circ} \mathrm{C}\) Rise

\section*{APPENDIX E}

\section*{REGULATION AS A FUNCTION OF EFFICIENCY}

The size of a transformer usually is determined either by a temperature rise limit, or by allowable voltage regulation, assuming that size and weight are to be minimized.

Figure El below shows circuit diagram of a transformer with one secondary.


Fig. El. Transformer Circuit Diagram
The analytical equivalent is shown in Figure E2.


Fig. E2. Transformer Analytical Equivalent
This assumes that distributed capacitance in the secondary can be neglected because the secondary voltage is not excessive. Ais? the winding
geometry is designed to limit the leakage inductance to a level low enough to be neglected under most operating conditions.

Transformer voltage regulation can be expressed as:
\[
\begin{equation*}
\operatorname{Reg}(\%)=\frac{V_{0}\left(N_{0} L_{0}\right)-V_{0}\left(F . L_{0}\right)}{V_{0}\left(N . I_{0}\right)} \times 100 \tag{El}
\end{equation*}
\]
in which \(V_{o}\) ( \(N\). L. ) is the no load voltage and \(V_{o}\) (F. L.) is the full load voltage.

The output voltage computed using Figure E1 is:
\[
\begin{equation*}
V_{o}=\frac{R_{o}}{R_{0}+R_{s}} \frac{\left(\mathbb{N}^{2} R_{p}\right)\left\|\left(N^{2} R_{E}\right)\right\|\left(R_{o}+R_{s}\right)}{N^{2} R_{p}} N E \tag{E2}
\end{equation*}
\]

For the usual condition of
\[
N^{2} R_{E} \gg N^{2} R_{p} \|\left(R_{o}+R_{s}\right)
\]
\(V_{0}\) simplifies to
\[
\begin{equation*}
V_{o}=V_{o}\left(F \cdot L_{0}\right)=\frac{R_{o}}{R_{o}+\left(N^{2} R_{p}+R_{s}\right)} N E \tag{E3}
\end{equation*}
\]

For equal window areas allocated for the primary and secondary windings, it can be shown that \(N^{2} R_{p}=R_{s}\).

For simplicity

Let
\[
R_{c u} \equiv N^{2} R_{p}+R_{s}=2 R_{s}
\]

At no load (N. L. ) \(R_{o}\) approaches inficity, therefore:
\[
\begin{equation*}
V_{0}\left(\mathbb{N}, L_{0}\right)=\mathbb{N E} \tag{E4}
\end{equation*}
\]
\[
\begin{align*}
& \operatorname{Reg}(\%)=\frac{N E-\frac{R_{o}}{R_{0}+R_{c u}} N E}{N E}=100  \tag{E5}\\
& =\left(1-\frac{R_{o}}{R_{o}+R_{c u}}\right) \times 100  \tag{E6}\\
& =\frac{R_{c u}}{R_{0}+R_{c u}} \times 100 \tag{E7}
\end{align*}
\]

Thus it appears that regulation is independent of the transformer turns ratio.

Regulation as a function of copper loss, multiply the equation \(E 7\) by \(I_{0}^{2}\)
then
\[
\begin{align*}
\operatorname{Reg}(\%) & =\frac{P_{\mathbf{c u}}}{P_{o}+P_{c u}} \times 100  \tag{E9}\\
P_{i n} & =P_{\mathbf{c u}}+P_{f e}+P_{o} \tag{E10}
\end{align*}
\]

Regulation as a function of efficiency
\[
\begin{equation*}
\frac{P_{o}}{P_{i n}}=\frac{P_{o}}{P_{c u}+P_{f e}+P_{o}}=\eta \tag{Ell}
\end{equation*}
\]

By definition
\[
P_{c u}=P_{f e}
\]

Solving for \(P_{c u}+P_{f e}\)
\[
\begin{gather*}
\frac{P_{\mathrm{o}}(1-\eta)}{\eta}=P_{\mathrm{o}}\left(\frac{1}{\eta}-1\right)=P_{\mathrm{cu}}+P_{\mathrm{fe}}=2 P_{\mathrm{cu}}  \tag{El2}\\
\frac{\operatorname{Reg}(\%)}{100}=\frac{1}{1+\frac{P_{\mathrm{o}}}{P_{\mathrm{cu}}}}=\frac{1}{1+\frac{2}{1 / \eta-1}}=\frac{1-\eta}{1+\eta}  \tag{E13}\\
\operatorname{Reg}(\%)=\frac{1-\eta}{1+\eta} \times 100 \tag{E14}
\end{gather*}
\]

Efficiency as a function of regulation, multiply both sides of the equation by \((1+\eta):\)
\[
\begin{equation*}
\operatorname{Reg}(\%)+\eta \operatorname{Reg}(\%)=100-\eta 100 \tag{E15}
\end{equation*}
\]
solve for
\[
\begin{gather*}
\eta 100+\eta \operatorname{Reg}(\%)=100-\operatorname{Reg}(\%)  \tag{E16}\\
\eta(100+\operatorname{Reg}(\%)=100-\operatorname{Reg}(\%)  \tag{E17}\\
\eta=\frac{100-\operatorname{Reg}(\%)}{100+\operatorname{Reg}(\%)} \tag{E18}
\end{gather*}
\]

\section*{APPENDIX F \\ WINDOW UTILIZATION FACTOR}

The fraction \(K_{u}\) of the available core window space which will be occupied by the winding (copper) is calculated from areas \(S_{1}, S_{2}, S_{3}\), and \(S_{4}\) :
\[
\begin{equation*}
K_{u}=S_{1} \times S_{2} \times s_{3} \times s_{4} \tag{F1}
\end{equation*}
\]
where
\[
\begin{aligned}
& S_{1}=\frac{\text { conductor area }}{\text { wire area }} \\
& S_{2}=\frac{\text { wound area }}{\text { usable window area }} \text { and, } \\
& S_{3}=\frac{\text { usable window area }}{\text { window area }} \\
& S_{4}=\frac{\text { usable window area }}{\text { usable window area insulation area }}
\end{aligned}
\]
in which
conductor area = copper area
wire area \(=\) copper area + insulation axea
wound area \(=\) number of turns \(x\) wire area of one turn
usable window are = available window area minus residual area which results from the particular winding technique used
window area \(=\) available window area
insulation area = area usable for winding insulation
\(S_{1}\) is dependent upon wire size. Columns \(A\) and \(D\) of Table F1, page 44 may be used for calculating some typical values such as for AWG 10 , AWG 20, AWG 30 and AWG 40.

Table F1. Wire Table




\section*{TEMPERATURE CORRECTION FACTORS}

The values shown in Fig. I are based upon a correction factor of 1.0 at \(20^{\circ} \mathrm{C}\). For other temperatures the effect upon wire resistance can be calculated by multiplying the resistance value for the wire size shown in column \(C\) of Table 2 by the appropriate correction factor shown on the graph. Thus, Corrected Resistance \(=\mu \Omega / \mathrm{cm}\left(\right.\) at \(\left.20^{\circ} \mathrm{C}\right) \times \zeta\)


Fig. Fl. Resistance Correction Fac'or ( \(h\), Zeta) for wire temperature between \(-50^{\circ}\) and \(100^{\circ} \mathrm{C}\)

CONVERSION DATA FOR WIRE SIZES FROM \#10 to \(\# 44\)
Columns \(A\) and \(B\) in Table \(F I\) give the baxe area in the commonly used circular mils notation and in the metric equivalent for each wire size. Column C gives the equivalent resistance in microhms/centimeter ( \(\mu \Omega / \mathrm{cm}\) or \(10^{-6} \Omega /\) cm .). Golumns \(D\) to \(L\) relate to coated wires showing the effect of insulation on size and the number of turns and the total weight in grams/centimeter.

The total resistance for a given winding may be calculated by multiplying the MLT (mean length/turn) of the winding in centimeters, by the microhms cm for the appropriate wire size (Column C), and the total number of turns. Thus
\[
R=(M L T) \times(N) \times(\text { Column } C) \times 10^{-6}
\]
[ohms]

The weight of the copper in a given winding may be calculated by multiplying the \(\mathrm{MI} T\) by the grams/cm (Column \(I\) ) and by the total number of turns. Thus
\[
\left.W_{t}=(M L T) \times(N) \times(\text { Column } L) \quad \text { [grams }\right]
\]

Turns per square inch and turns per square cm are based on \(60 \%\) wire fill factor.

Thus:
\[
\begin{aligned}
& \text { AWG } 10=\frac{52.61 \mathrm{~cm}}{55.90 \mathrm{~cm}}=0.941 ; \\
& \text { AWG } 20=\frac{5.188 \mathrm{~cm}}{6.065 \mathrm{~cm}}=0.855 ; \\
& \text { AWG } 30=\frac{0.5067 \mathrm{~cm}}{0.6785 \mathrm{~cm}}=0.747 ; \text { and } \\
& \text { AWG } 40=\frac{0.04869 \mathrm{~cm}}{0.0723 \mathrm{~cm}}=0.673
\end{aligned}
\]
\(S_{2}\) is the fill factor for the usable window area. It can be shown that for circular cross-section wire wound on a flat form the ratio of wire \(\mathrm{cm}^{2}\) to the area required for the turns can never be greater than 0.91. In practice, the actual maximum value is dependent upon the tightness of winding, variations in insulation thickness, and wire lay. Consequently, the fill factor is always less than the theoretical maximum.

As a typical working value for copper wire with a heavy synthetic filn insulation, a ratio of 0.60 may be safely used.

The term \(\mathrm{S}_{3}\) defines how much of the available window space may actually be used for the winding. The winding area available to the designer depends on the bobbin configuration. A single bobbin design offers an effective \(W_{a}\) between 0.835 to 0.929 while a two bobbin configuration offers an effective \(W_{a}\) between 0.687 to 0.872 . A good value to use for both configurations is 0.75 .

The term \(\mathrm{S}_{4}\) can vary from 1.0 to 0.80 and defines how much of the usable window space is actually being used for insulation. If the transformer has multiple secondaries having significant amounts of insulation \(S_{4}\) could be as low as 0.8.

A typical value for the copper fraction in the window area is about 0.40. For example, for AWG 20 wire, \(S_{1} \times S_{2} \times S_{3} \times S_{4}=0.855 \times 0.060 \times 0.75 \times\) \(1.0=0.385\), which is very close to 0.4 .

This may be stated somewhat differently as:
\[
0.4=\frac{A_{w} \text { Bare }}{\left.A_{w}\right|_{\left(S_{1}\right)} ^{\text {Total }}} \times \text { Fill Factor } \times \frac{W_{a(e f f)}}{W_{2}} \times \text { Insulation Factor }
\]

\section*{APPENDIX G}

\section*{TRANSFORMER WEIGHT}

The total weight \(W_{t}\) of a transformer can be related to the area product \(A_{p}\). The straightine logarithmic relationship shown in Figure Gl below, has been calculated from the data shown in Table 1 (pages 11 and 12).


Fig. Gi. Transformer Total \(W_{t}\) Versus Area Product Ap

This relationship is obtained from the conventional slope relationship:
\[
\text { Slope }=\frac{\log \left(W_{t 2} / W_{t 1}\right)}{\log \left(A_{\mathrm{p} 2} / A_{\mathrm{p} 1}\right)}
\]
in which the \(W_{t}\) and \(A_{p}\) values are the extremes of the data shown in columns 14 and 15 for weight, and column 3 for area product.

The relationship is:
\[
\begin{equation*}
W_{t}=K_{w} A_{p}^{0.75} \tag{G1}
\end{equation*}
\]
in which the constant \(K_{w}\) has been derived empirically by averaging the data presented in columns 3, 14 and 15 of Table 1 (pages 11 and 12) and is 66.6.

Table 12 (page 55) shows how weight varies as a function of selected different magnetic materials used for transiormer C-cores. Magnetic materials for C-cores are discussed in Appendix I (page 54).

Derivation of the relationship is according io the following: Weight \(W_{t}\) varies in accordance with the cube of any linear dimension \(\ell\) (designated \(\ell^{3}\) below), whereas, area product \(A_{p}\) varies as the fourth power:
\[
\begin{align*}
W_{t} & =K_{1} \ell^{3}  \tag{G2}\\
A_{p} & =K_{2} \ell^{4}  \tag{G3}\\
Q^{4} & =\frac{A_{p}}{K_{2}}  \tag{G4}\\
\ell & =\left(\frac{A_{p}}{K_{2}}\right)^{0.25} \tag{G5}
\end{align*}
\]
\[
\begin{align*}
& Q^{3}=\left[\left(\frac{A_{p}}{K_{2}}\right)^{0.25}\right]^{3}=\left(\frac{A_{p}}{\bar{K}_{2}}\right)^{0.75}  \tag{G6}\\
& W_{t}=K_{1}\left(\frac{A_{p}}{K_{2}}\right)^{0.75}  \tag{C7}\\
& K_{w}=\frac{K_{1}}{K_{2}^{0.75}}  \tag{G8}\\
& W_{t}=K_{w} A_{p}^{0.75} \tag{C9}
\end{align*}
\]
in which \(K_{1}\) is a constant depending upon the core material, and \(K_{2}\) is related to core and window dimensions.

\section*{APPENDIX H}

\section*{TRANS FORMER VOLUME}

The volume of a transformer can be related to the area product \(A_{p}\) of a C-core transformer, treating the volume as shown in Figure HI below as a solid cube quantity withou subtraction of anything for the core window.


Fig. H1. CwCore Volume

The straight-line logarithmic relationship plotted in Figure H2 below, has been calculated from data in Table I, using the data shown in Figure HI above.

The relationship is obtained from the conventional slope relationship:
\[
\text { Slope }=\frac{\log (\text { Vol. } 2 / \text { Vol. 1) }}{\log \left(A_{\mathrm{p}} 2 / A_{\mathrm{p}} 1\right)}
\]
in which the Vol. and \(A_{p}\) values are the extremes of the data shown in column 15 for volume, and column 3 for area product.

The volume/area product relationship is:
\[
\begin{equation*}
\text { Vol. }=K_{v} A_{p}^{0.75} \tag{H1}
\end{equation*}
\]
in which \(\mathrm{K}_{\mathrm{v}}\) is a constant related to core configuration. It is 17.9 for a C -core, which has been derived by averaging the values in Table 1.


Fig. H2. Transformer Volume Versus Area Product \(A_{p}\)

\section*{APPENDIX I}

MAGNETIC CORE MATERIAL TRADEOFF

The relationships between area product \(A_{p}\) and certain parameters are associated only with such geometric properties as surface area and volume, weight, and the factors affecting temperature rise such as current density. A has no relevance to the magnetic core materials used, but since the designer often must make tradeoffs between such goals as efficiercy and size which are influenced by core material selection, some useful data is presented below.

In the many articles written about inverter and converter transformer design, recommendations with respect to choice of core material usuelly are a compromise selection of material characteristics such as those tabulated in Table I1, and graphically displayed in Figure I1. The selected data are typical of commercially available core materials suitable for the mentioned applications.


Fig. II. The Typical d.c. B-H Loops of Magnetic Material

Table Il. Magnetic core material characteristics
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Trade names & Composition & Saturated flus density, \(T^{1}\) & DC coercive force, amp-turn/ cm & Squareness ratio & Material density, \(\mathrm{g} / \mathrm{cm}^{3}\) & Luss factor at 3 kHz and \(0.5 \mathrm{~T}, \mathrm{~W} / \mathrm{kg}\) \\
\hline \begin{tabular}{l}
Magnesil \\
Silectron \\
Microsil \\
Supersil
\end{tabular} & \[
\begin{array}{r}
3 \% \mathrm{Si} \\
97 \% \mathrm{Fe}
\end{array}
\] & 1.5-1.8 & 0.5-0. 75 & \(0.85-1.0\) & 7.63 & 33.1 \\
\hline Deltarmax Orthonol \(49 \mathrm{Sq}\). & \(50 \% \mathrm{Ni}\) \(50 \% \mathrm{Fe}\) & 1.4-1.6 & 0.125-0.25 & \(0.94-1.0\) & 8.24 & 17.66 \\
\hline Allegheny 4750 48 Alloy Carpenter 49 & \[
\begin{aligned}
& 48 \% \mathrm{Ni} \\
& 52 \% \mathrm{Fe}
\end{aligned}
\] & 1.15-1.4 & 0.062-0.187 & \(0.80-0.92\) & 8.19 & 11.03 \\
\hline 4-79 Permalloy Sq. Permalloy \(80 \mathrm{Sq} . \mathrm{Mu} 79\) & \[
\begin{gathered}
79 \% \mathrm{Ni} \\
17 \% \mathrm{Fe} \\
4 \% \mathrm{Mo}
\end{gathered}
\] & 0.66-0.82 & 0.025-0.05 & 0.80-1.0 & 8.73 & 5.51 \\
\hline Supermalloy & \(7 \mathrm{~B} \% \mathrm{Ni}\) \(17 \% \mathrm{Fe}\) 5\% Mo & 0.65-0.82 & 0.0037-0.01 & \(0.40-0.70\) & 8.76 & 3. 75 \\
\hline \multicolumn{7}{|l|}{\({ }^{1} 1 \mathrm{~T}=10^{4} \mathrm{G}\)} \\
\hline \multicolumn{7}{|l|}{\({ }^{2} 1 \mathrm{~g} / \mathrm{cm}^{3}=0.036 \mathrm{lb} / \mathrm{in} .^{3}\)} \\
\hline
\end{tabular}

Table 12. Core material characteristics
\begin{tabular}{|c|c|c|}
\hline Material & Density & Factor* \\
\hline Magnesil & 7.63 & 1.000 \\
Supermender & 8.15 & 1.066 \\
48 Alloy & 8.19 & 1.073 \\
Orthonol & 8.24 & 1.079 \\
Sq Permalloy & 8.73 & 1.144 \\
Supermailoy & 8.77 & 1.148 \\
\hline\({ }^{*}\) Weight factor. & \\
\hline
\end{tabular}

As can be seen，the material which provides the highest \(⿴ 囗 十 ⺝ 丶\) silicon，produces the smallest component size．If size is the most important consideration，this would determine the choice of materials．On the other hand，the type 78 SUPERMALLOY material（see the \(5 / 78\) curve in Figure Il）， has the lowest flux density and this material would result in the largest size transformer．However，this material has the lowest coercive force and lowest core loss of any of the available materials．These factors might well be decisive in other applications．

Inverter transformer design usually is aimed at achieving the smailest size with the highest efficiency，and with adequate performance for the widest range of environmental conditions．Unfortunately，the material which produces the smallest size has the lowest efficiency，and conversely，the highest effi－ ciency materials result in the largest size．Thus tradeoffs must be made between the allowable transformer size and the minimum tolerable efficiency． Choice of core material is thas based upon achieving the best characteristic for the most critical or important design parameter，with acceptable compro－ mises on all other parameters．

Fortunately，there is such a wide choice of core sizes available（Table 1 ， pages 11 and 12 ，lists only 20 out of more than 200 commercially available）， that relative proportions of iron and copper can be varied without changing the \(A_{p}\) area product．＊

\footnotetext{
Whowever，at frequencies above about 20 kHz ，eddy current losses are so much greater then hysteresis losses that it is necessamy to use very thin（ 1 and 2 mil）strip cores．
}

\section*{APPENDIX J}

SKIN EFFECT

\section*{Skin Effect}

It is now common practice to operate dc-to-dc converters at frequencies up to 50 kHz . At higher frequencies, skin effect alters the predicted efficiency since the current carried by a conductor is distributed uniformly across the conductor cross-section only at dc and at low frequencies. The concentration of current near the wire surface at higher frequencies is termed the skin effect. This is the result of magnetic flux lines which circle only part of the conductor. Those portions of the cross section which are circled by the largest number of flux lines exhibit greater reactance.

Skin effect accounts for the fact that the effective alternating current resistance to direct current ratio is greater than unity \({ }^{*}\). The magnitudes of the effects due to increased frequency on conductivity, magnetic permeability and inductax, e are sufficient to require further consideration of the size of the con. ductor. The depth of the skin effect is expressed by:
\[
\begin{equation*}
\left.\operatorname{depth}(\mathrm{cm})=6.61 / \mathrm{E}^{1 / 2}\right) \mathrm{K} \tag{J1}
\end{equation*}
\]
in which \(K\) is a constant according to the relationship:
\[
\begin{equation*}
\mathrm{K}=[(1 / \mathrm{fr}) \rho / \rho \mathrm{c}]^{1 / 2} \tag{J2}
\end{equation*}
\]
in which:
\[
\begin{aligned}
\mu r= & \text { relative permeability of conductor material ( } \mu \mathrm{r}=1 \text { for copper and } \\
& \text { other nonmagnetic materials) } \\
\rho= & \text { resistivity of conductor material at any temperature } \\
\rho c= & \text { resistivity of copper at } 20^{\circ} \mathrm{C}=1.724 \text { microhmmentimeter } \\
\mathrm{K}= & \text { unity for copper }
\end{aligned}
\]

\footnotetext{
*Reference 3 .
}

Figures JI and J 2 below show respectively, skin depth as a function of frequency according to equation (J2) above, and as related to the AWG radius, or as \(R_{a c} / R_{d c}=1\) versus frequency. *


Fig. J1. Skin Depth Versus Frequency


Fig. J2. Skin Depth Equal to AWG Radius Versus Frequency

\footnotetext{
\({ }^{*}\) The data presented is for sine wave excitation. The author could not find any data for square wave excitation.
}

\section*{APPENDIX K}

\section*{AREA PRODUCT Ap RELATIONSHIP}

There is a unique relationship between the "Area Product", Ap characteristic number for transformer cores and sevexal other important parameters which must be considered in transformer design.

The power handling capability of a transformer can be related to its \(A_{p}\) quantity (which is actually its \(W_{a} A_{c}\) product where \(W_{a}\) is the available core window area in \(\mathrm{cm}^{2}\) and \(\mathrm{A}_{\mathrm{c}}\) is the effective cross-sectional area of the core in \(\mathrm{cm}^{2}\) ).

These relationships can now be used as new tools to simplify and standardize the process of transformer design. They make it possible to design transformers of smaller bulk and volume or to optimize efficiency.

Table K1 was developed using the least-squares curve fit from the data obtained in Tables K2 through K6. The asaa product Ap relationships with volume, surface area, current density, and weight for tape wound cores, C type core, powder cores, laminations and pot core are found in Figures Kl through K20.

Table Kl. Transformer Configuration Constants
\begin{tabular}{|l|c|c|c|c|c|c|}
\hline & \(\mathrm{K}_{\mathrm{j}} 25^{\circ} \mathrm{C}\) & \(\mathrm{K}_{\mathrm{j}} 50^{\circ} \mathrm{C}\) & \(\eta\) & \(\mathrm{K}_{\mathrm{s}}\) & \(\mathrm{K}_{\mathrm{w}}\) & \(\mathrm{K}_{\mathrm{v}}\) \\
\hline Pot cores & 433 & 632 & -0.17 & 33.8 & 48.0 & 14.5 \\
\hline Powder cores & 290 & 423 & -0.12 & 32.5 & 58.8 & 13.1 \\
\hline Lamination & 366 & 534 & -0.12 & 41.3 & 68.2 & 19.7 \\
\hline C type cores & 323 & 468 & -0.14 & 39.2 & 66.6 & 17.9 \\
\hline Tape wound cores & 250 & 365 & -0.13 & 50.9 & 82.3 & 25.0 \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
\hline & Core & \(A_{t} \mathrm{~cm}^{2}\) & \(A_{p} \mathrm{~cm}^{4}\) & MLT cm & * AwG & S \({ }^{-1} 50{ }^{\circ} \mathrm{C}\) & \(\mathrm{Pr}_{\mathbf{E}}\) & \(t=\sqrt{\frac{W}{7}}\) & \[
\begin{gathered}
\Delta T 25{ }^{\circ} \mathrm{C} \\
\mathrm{~J}=1 / \mathrm{cm}^{2}
\end{gathered}
\] & ת. \({ }^{\text {¢ }} 75^{\circ} \mathrm{C}\) & \(\mathrm{Pr}_{2}\) & \(1 \times \sqrt{\frac{W}{7}}\) & \[
\begin{aligned}
& \Delta T 50^{\circ} \mathrm{C} \\
& \mathrm{~J}=\mathrm{I} / \mathrm{cm}^{2}
\end{aligned}
\] & \[
\begin{aligned}
& \text { Toter } \\
& \text { Welabt }
\end{aligned}
\] & \[
\begin{aligned}
& \text { Volume } \\
& \mathrm{cm}^{3}
\end{aligned}
\] & \(\mathrm{A}_{c} \mathrm{~cm}^{2}\) \\
\hline 1 & 55051 & 7.19 & 0.0437 & 2.12 & 8685 & 0.215 & 0.216 & 0.706 & 435 & 0.236 & 0.503 & 1.03 & 635 & 5. 82 & 1.39 & 0.113 \\
\hline 2 & 55121 & 12.3 & 0.137 & 2.71 & \(160 \quad 25\) & 0.513 & 0. 369 & 0.599 & 369 & 0.563 & 0.861 & 0, 874 & 538 & 13.3 & 3.11 & 0.196 \\
\hline 3 & 55848 & 17.3 & 0.259 & 4.95 & 25725 & 0. 897 & 0, 519 & 0.537 & 344 & 0.985 & 1.211 & 0.783 & 502 & 21,3 & 5.07 & 0.232 \\
\hline 4 & 55059 & 21.9 & 0.466 & 3.39 & \({ }^{316} 25\) & 1.27 & 0.657 & 0. 508 & 314 & 1.39 & 1.533 & 0.742 & 458 & 32.3 & 7.28 & 0. 327 \\
\hline 5 & 55894 & 30.8 & 1.021 & 4. 51 & 35125 & 1. \(\mathrm{B}^{7}\) & 0. 924 & 0.496 & 306 & 2.06 & 2.16 & 0.724 & 447 & 59.4 & 12.4 & 0.639 \\
\hline 6 & 55586 & 48.6 & 1. 䀢1 & 4.37 & \(002 \quad 25\) & +. 69 & 2.46 & 0. 394 & 244 & 5. 15 & 3. 40 & 0.574 & 355 & 94.9 & 23.3 & 0.458 \\
\hline 7 & 55071 & 44.7 & 1.966 & 4.7 & 65625 & 3, 70 & 1. 34 & 0.425 & 263 & 4.07 & 3. 13 & 0.620 & 383 & 94.4 & 21.0 & 0.666 \\
\hline - & 55076 & 51.6 & 2.46 & 4.88 & \(815 \quad 25\) & 4. 71 & 1. 55 & 0.405 & 250 & 5.17 & 3.61 & 0.590 & 365 & 113.0 & 25.7 & 0.670 \\
\hline 9 & 55083 & 66.8 & 4, 57 & 6, 02 & 45925 & 6, 84 & 2,00 & 0. 382 & 236 & 7.50 & 4.68 & 0.558 & 345 & 178.0 & 39.1 & 1.06 \\
\hline 10 & 55090 & 89.4 & 6.19 & B. 65 & 137285 & 10.8 & 2.68 & 0.352 & 225 & 11.8 & 6.26 & 0.513 & 329 & 271.0 & 59.5 & 3.32 \\
\hline 11 & 55434 & 86. 9 & B. 48 & 7. 48 & 950 & 8. \(4^{19}\) & 2.00 & 0. 391 & 250 & 9.32 & 6. 08 & 0.571 & 965 & 291.0 & 58.1 & 1.95 \\
\hline 12 & 55716 & 100.0 & 9.38 & 6, 54 & 168425 & 13.0 & 3.000 & 0.339 & 217 & 14.3 & 7.00 & 0.494 & 317 & 303, 0 & 69.0 & 1.24 \\
\hline 13 & 55110 & 124.0 & 13,66 & 7. 09 & \(2125 \quad 25\) & 17.8 & 3. 72 & 0. 322 & 206 & 19.6 & 8.68 & 0.470 & 301 & 405.0 & 93.4 & 1.44 \\
\hline
\end{tabular}




\section*{Definitions for Table K2}

Information given is listed by column as:
1. Manufacturer part number
2. Surface area calculated from Figure K21
3. Area product effective iron area times window area
4. Mean length turn
5. Total number of turns and wire size using a window utilization factor \(K_{u}=0.40\)
6. Resistance of the wire at \(50^{\circ} \mathrm{C}\)
7. Watts loss is based on Figure Cl for a \(\Delta T\) of \(25^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, total loss is equal to \(2 P_{c u}\)
8. Current calculated from column 6 and 7
9. Current density calculated from column 5 and 8
10. Resistance of the wire at \(75^{\circ} \mathrm{C}\)
11. Watts loss is based on Figure Cl for a \(\Delta T\) of \(50^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, total loss is equal to \(2 \mathrm{P}_{\mathrm{cu}}\)
12. Current calculated from column 10 and 11
13. Current density calculated from column 5 and 12
14. Effective core weight for silicon plus copper weight
15. Transformer volume calculated from Figure K24
16. Core effective cross-section


Fig. Kl. Current Density Versus Area Product \(A_{p}\)
for a \(25^{\circ} \mathrm{C}\) and \(50^{\circ} \mathrm{C}\) Rise for Powder Cores


Fig. K2. Surface Area Versus Area Product \(A_{p}\) for Powder Cores


Fig. K3. Volume Versus Area Product Ap for Powder Cores


Fig. K4. Total Weight Versus Area Product Ap for Powder Cores

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 21 & 12 & 13 & 14 & 15 & 16 \\
\hline & Core & \(\mathrm{A}_{\mathrm{t}} \mathrm{cm}^{2}\) & \(A_{P} \mathrm{~cm}^{4}\) & MLT cm & v AFG & \(8650{ }^{\circ} \mathrm{C}\) & \(\mathrm{P}_{\mathbf{\Sigma}}\) & \(I=\sqrt{\frac{W}{\Omega}}\) & \(\Delta T 25{ }^{\circ} \mathrm{C}\)
\(J=1 / \mathrm{cm}^{2}\) & n- \(75^{\circ} \mathrm{C}\) & \(\mathrm{P}_{\mathbf{\Sigma}}\) & \(\mathrm{I}=\sqrt{\frac{\mathrm{W}}{\mathrm{n}}}\) & \[
\begin{aligned}
& \Delta T 50^{\circ} \mathrm{C} \\
& \mathrm{~J}=1 / \mathrm{cm}^{2}
\end{aligned}
\] & Tatal Weight & \[
\begin{gathered}
\text { Volume } \\
\mathrm{cm}^{3}
\end{gathered}
\] & \(\mathrm{A}_{\mathrm{c}} \mathrm{cm}^{2}\) \\
\hline 1 & 9x5 & 2.93 & 0.0065 & 1.85 & 2530 & 0.175 & 0.098 & 0.529 & 1044 & 0. 192 & 0. 230 & 0.774 & 1527 & 1.12 & 0.367 & 0.10 \\
\hline 2 & \(11 \times 7\) & 4. 35 & 0.0152 & 2.2 & 3730 & 0.309 & 0.130 & 0.458 & 904 & 0. 339 & D. 304 & 0. 670 & 1322 & 2.08 & 0.662 & 0.16 \\
\hline 3 & 14:8 & 6.96 & 0.0393 & 2.8 & 7430 & 0.787 & 0.208 & 0. 363 & 716 & 0. 864 & 0.487 & 0.531 & 1048 & 4.18 & 1.35 & 0.25 \\
\hline 4 & \(18 \times 11\) & 11.3 & 0.114 & 3, 56 & 14330 & 1.934 & 0.339 & 0.296 & 584 & 2.12 & 0. 791 & 0.432 & 853 & B. 37 & 2.78 & 0.43 \\
\hline 5 & \(22 \times 13\) & 17.0 & 0. 246 & 4.4 & 20730 & 3.46 & 0.510 & 0.271 & 535 & 3.80 & 1.190 & 0.396 & 782 & 17.3 & 5.17 & 0.63 \\
\hline 6 & \(26 \times 16\) & 23.9 & 0.498 & 5.2 & 9625 & 0.592 & 0.717 & 0.778 & 479 & 0.650 & 1.67 & 1.13 & 696 & 28.5 & 8. 65 & 0.94 \\
\hline 7 & \(30 \times 19\) & 32.8 & 1.016 & 6.0 & 14425 & 1.024 & 0.984 & 0.693 & 427 & 1. 12 & 2.30 & 1.01 & 622 & 48.9 & 13.9 & 1.36 \\
\hline 8 & \(36 \times 22\) & 44.0 & 2.01 & 7.3 & 18925 & 1.636 & 1.34 & 0.639 & 394 & 1.79 & 3.14 & 0.937 & 577 & 77.6 & 22.0 & 2.01 \\
\hline 9 & \(47 \times 28\) & 76.0 & 5.62 & 9.3 & 34525 & 3.81 & 2.28 & 0. 547 & 337 & 4.18 & 5. 32 & 0. 790 & 492 & 173.0 & 48.6 & 3.12 \\
\hline 10 & \(59 \times 36\) & 122.0 & 13.4 & 12.0 & 60.825 & 8.65 & 3. 66 & 0.459 & 285 & 9.50 & 8.54 & 0.670 & 413 & 379.0 & 98.3 & 4.85 \\
\hline
\end{tabular}
Table K3. Pot cores characteristics


\section*{Definition for Table K3}

Information given is listed by column as:
1. Manufacturer part number
2. Surface area calculated from Figure KZl
3. Area product effective iron area times window area
4. Mean length turn
5. Total number of turns and wire size using a window utilization factor \(K_{u}=0.40\)
6. Resistance of the wire at \(50^{\circ} \mathrm{C}\)
7. Watts loss is based on Figure Cl for a \(\Delta \mathrm{T}\) of \(25^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the fransformer surface area, total loss is equal to \(2 P_{c u}\)
8. Current calculated from column 6 and 7
9. Current density calculated from column 5 and 8
10. Resistance of the wire at \(75^{\circ} \mathrm{C}\)
11. Watts Loss is based on Figure Cl for a \(\Delta\) T of \(50^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, tc tal loss is equal to \(2 P_{\mathrm{cu}}\)
12. Gurrent calculated from column 10 and 11
13. Current density calculated from column 5 and 12
14. Effective core weight for silicon plus copper weight
15. Transformer volume calculated from Figure K24
16. Core effective cross-section


Fig. K5. Current Density Versus Area Product Ap for a \(25^{\circ} \mathrm{C}\) and \(50^{\circ} \mathrm{C}\) Rise for Pot Cores


Fig. K6. Surface Area Versus Area Product \(A_{p}\) for Pot Cores


Fig. K7. Volume Versus Area Product \(A_{p}\) for Pot Cores


Fig. K8. Total Weight Versus Area Product \(A_{p}\) for Pot Cores

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
\hline & Cors & \(A_{t} \mathrm{~cm}^{2}\) & \(A_{p} \mathrm{~cm}^{4}\) & MLT cm & AwG & n-50*' & \(\mathrm{P}_{\mathbf{2}}\) & \(I=\sqrt{\frac{W}{n}}\) & \[
\begin{array}{r}
\Delta \mathrm{T} 25^{\circ} \mathrm{C} \\
\mathrm{~J}=1 / \mathrm{cm}^{2}
\end{array}
\] & п \({ }^{\text {¢ }} 75{ }^{\circ} \mathrm{C}\) & \(\mathrm{P}_{2}\) & \(1 \pm \sqrt{\frac{W}{8}}\) & \[
\begin{aligned}
& \Delta T 50^{\circ} C_{2} \\
& \mathrm{~J}=1 / \mathrm{cm}^{2}
\end{aligned}
\] & \[
\begin{aligned}
& \text { Tokal } \\
& \text { waithe }
\end{aligned}
\] & \[
\begin{gathered}
\text { Volume } \\
\mathrm{cm}^{3}
\end{gathered}
\] & \(\mathrm{A}_{\mathrm{c}} \mathrm{cm}^{2}\) \\
\hline 1 & EE-3031 & \(4+11\) & 0.0090 & 1.72 & & 0, 58 & 0. 123 & 0.323 & 638 & 0.645 & 0.288 & 0.472 & 932 & 2.04 & 0.651 & 0.056 \\
\hline 2 & EE.2829 & 6.63 & 0.0254 & 2.33 & \(1+7 \quad 30\) & 1.30 & 0.199 & 0.276 & 546 & 1,43 & 0.464 & 0.403 & 795 & 3.78 & 1.35 & 0.101 \\
\hline 3 & E1-187 & 14.4 & 0. 120 & 3.20 & 31.450 & 3.82 & 0.432 & 0.237 & *69 & 4.19 & 1.01 & 0.347 & 685 & 10.2 & 4.34 & 0.225 \\
\hline 4 & EE-2425 & 23. \({ }^{\text {P }}\) & 0. 325 & 5.08 & 49830 & 9.61 & 0.714 & 0.192 & 380 & 10.5 & 1.67 & 0.281 & 555 & 24.6 & 9.22 & 0, 403 \\
\hline 5 & EE-2627 & 40.6 & 1.01 & 5.74 & 24525 & 1.68 & 1.22 & 0.602 & 371 & 1.85 & 2.84 & 0.876 & 540 & 61.3 & 19.1 & 0.907 \\
\hline 6 & E1-375 & 47.7 & 1.36 & 6.30 & 35025 & 2.62 & 1.43 & 0.522 & 322 & 2.87 & 3. 34 & 0.762 & 470 & 74.1 & 25.3 & 0.907 \\
\hline 7 & E1-50 & 57.7 & 1.95 & 7.00 & \(263 \quad 25\) & 2.21 & 1.73 & 0.625 & 385 & 2.43 & 4.04 & 0.912 & 562 & 124.0 & 36.1 & 1.61 \\
\hline 8 & El-21 & 66.0 & 2.62 & 7.97 & 37225 & 3.34 & 1.98 & 0.544 & 335 & 3,60 & 4.62 & - 0.793 & 449 & 140.0 & 39.2 & 1. 61 \\
\hline 9 & El-625 & 90.0 & 4.76 & 8. 8 * & \({ }_{503} 25\) & 5.47 & 2. 70 & 0. 505 & 312 & 5.79 & 6. 30 & 0.737 & 453 & 223.0 & 60.0 & 2, 52 \\
\hline 10 & E1-75 & 130.0 & 9.87 & 20.6 & 21120 & 0.826 & 3.90 & 1.54 & 296 & 0.906 & 9.10 & 2.24 & 432 & 417.0 & 104.0 & 3, 63 \\
\hline 11 & E1-87 & 176.0 & 18.3 & 12.3 & 29630 & 1.34 & 5.28 & 1. 10 & 270 & 1.48 & 12.3 & 2.04 & 393 & 616.0 & 164.0 & 4.94 \\
\hline 12 & E1-100 & 230.0 & 31.2 & 14.5 & 38680 & 2.07 & 6.90 & 1.29 & 249 & 2.27 & 16.1 & 1.86 & 363 & 953.0 & 246.0 & 6.45 \\
\hline 13 & E1-112 & 292.0 & 47.9 & 14.0 & 49280 & 2.91 & 8. 76 & 1.23 & 237 & 3. 19 & 20.7 & 1.79 & 344 & 1370.0 & 350.0 & 8.16 \\
\hline 14 & Et-125 & 361.0 & 76. 3 & 17.7 & \(625 \quad 20\) & 4.09 & 10.8 & 1.15 & 222 & 4.49 & 25.3 & 1.68 & 324 & 1870.0 & 481.0 & 10.08 \\
\hline 15 & E1-138 & +32.0 & 112.0 & 19.5 & 74020 & 5. 33 & 13.0 & 1.10 & 213 & 5.85 & 30.2 & 1.61 & 310 & 2560.0 & 629.0 & 12.19 \\
\hline 16 & E1-150 & 518.0 & 158.0 & 21.2 & \({ }^{193} 20\) & 6.99 & 15, 5 & 1.05 & 203 & 7.67 & \(3 \mathrm{E}, 3\) & 1.54 & 296 & 3560,0 & 829.0 & \(14.5 \ddagger\) \\
\hline 17 & E1-175 & 704.0 & 292.0 & 24.7 & 10100 & 0. 85 & 21.1 & 1,034 & 199 & 10.8 & +9.3 & 1. 51 & 291 & 5180.0 & 1312.0 & 19.75 \\
\hline 18 & E1-36 & 778, 0 & 361.0 & 26.5 & 170120 & 16.6 & 23.3 & 0.836 & 261 & 18.3 & 54.5 & 1.22 & 235 & 5930.0 & 1654.0 & 17.03 \\
\hline 19 & E1-19 & 1093.0 & \(6 \mathrm{6s} .0\) & 31.7 & 20106 & 33.8 & 32.8 & 0.696 & 134 & 37.1 & 76, 5 & 1.015 & 19b & 8694.0 & 2873.0 & 19.75 \\
\hline
\end{tabular}



\section*{Definitions for Table K4}

Information given is listed by column as:
1. Manufacturer part number
2. Surface area calculated from Figure K22
3. Area product effective iron area times window area
4. Mean length turn on one bobbin
5. Total number of turns and wire size for one bobbin using a window utilization factor \(K_{u}=0,40\)
6. Resistance of the wire at \(50^{\circ} \mathrm{C}\)
7. Watts loss is based on Figure Cl for a \(\Delta T\) of \(25^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, total loss is equal to \(2 P_{c u}\)
8. Current calculated from column 6 and 7
9. Current density calculated from column 5 and 8
10. Resistance of the wire at \(75^{\circ} \mathrm{C}\)
11. Watts loss is based on Figure Cl for a \(\Delta \mathrm{T}\) of \(50^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, total loss is equal to \(2 \mathrm{P}_{\mathrm{cu}}\)
12. Current calculated from column 10 and 11
13. Current density calculated from column 5 and 12
14. Effective core weight for silicon plus copper weight
15. Transformer volume calculated from Figure K25
16. Core effective cross-section


Fig. K9. Current Density Versus Area Product \(A_{p}\) for \(25^{\circ} \mathrm{C}\) and \(50^{\circ} \mathrm{C}\) Rise for Laminations


Fig. K10. Surface Area Versus Area Product Ap for Laminations


Fig. Kll. Volume Versus Area Product \(A_{p}\) for Laminations


Fig. K12. Total Weight Versus Area Product Ap for Laminations

\section*{Definitions for Table K5}

Information given is listed by column as:
1. Manufacturer part number
2. Surface area calculated from Figure E 23
3. Area product effective iron area times window area
4. Mean length turn on one bobbin
5. Total number of turns and wire size for two bobbins using a window utilization factor \(K_{u}=0.40\)
6. Resistance of the wire at \(50^{\circ} \mathrm{C}\)
7. Watts loss is based on Figure Cl for a \(\Delta T\) of \(25^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, total loss is equal to \(2 \mathrm{P}_{\mathrm{cu}}\)
8. Gurrent calculated from column 6 and 7
9. Current density calculated from column 5 and 8
10. Resistance of the wire at \(75^{\circ} \mathrm{C}\)
11. Watts loss is based on Figure Cl for a \(\Delta T\) of \(50^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, total loss is equal to \(2 \mathrm{P}_{\mathrm{cu}}\)
12. Current calculated from column 10 and 11
13. Current density calculated from column 5 and 12
14. Effective core weight for silicon plus copper weight
15. Transformer volume calculated from Figure K26
16. Core effective cross-section


Fig. Kl3. Surrent Density Versus Area Product Ap for \(25^{\circ} \mathrm{C}\) and \(50^{\circ} \mathrm{C}\) Rise for C -Type Cores


Fig. K14. Surface Area Versus Area Product Ap for'C-Type Cores



Fig. K15. Volume Versus Area Product Ap for C-Type Cores


Fig. Kl6. Total Weight Versus Area Product \(A_{p}\) for C-Type Cores

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
\hline & Care & \(\mathrm{A}_{\mathrm{t}} \mathrm{cm}^{2}\) & \(A_{p} \mathrm{~cm}^{4}\) & MLT cm & N AwG & 8 8650\％ & \(\mathrm{P}_{\mathbf{I}}\) & \(1 \pm \sqrt{\frac{w}{n}}\) & \[
\begin{aligned}
& \Delta T 25^{\circ} \mathrm{C} \\
& \mathrm{~J}=\mathrm{I} \mathrm{~cm}^{2}
\end{aligned}
\] & п¢75＇c & \(\mathrm{P}_{\text {z }}\) & \(I=\sqrt{\frac{W}{51}}\) & \[
\begin{aligned}
& \Delta T 50^{\circ} \mathrm{C} \\
& \mathrm{~J}=\mathrm{L} / \mathrm{cm}^{2}
\end{aligned}
\] & Total Weipht & \[
\underset{\mathrm{cm}^{3}}{\text { Votume }}
\] & \(\mathrm{A}_{\mathrm{c}} \mathrm{cm}^{2}\) \\
\hline 1 & 52．02 & 7.26 & 0，0100 & 2，05 & 30230 & 2，35 & 0． 218 & 0.215 & 425 & 2.58 & B． 508 & 0.313 & 619 & 3.75 & 1.42 & 0.022 \\
\hline 2 & 52153 & 8． 29 & 0.0176 & 2.22 & 30230 & 2.54 & 0． 249 & 0.221 & 436 & 2． 80 & 0.580 & 0．322 & 636 & 4.60 & 1.71 & 0.053 \\
\hline 3 & 52107 & 11.1 & 0． 0201 & 2.21 & 60630 & 5.04 & 0． 333 & 0.180 & 357 & 5． 59 & 0．78i & 0． 263 & 520 & 7.64 & 2.63 & 0.022 \\
\hline 4 & 32403 & 13.5 & 0.0267 & 2.30 & 6.2130 & 5.43 & 0． 405 & 0.193 & 391 & 5.96 & 0.943 & 0． 281 & 556 & 10.4 & 3.48 & 0， 022 \\
\hline 5 & 52057 & 27.4 & 0.0659 & 2.53 & 105730 & 9.78 & 0． 322 & 0.163 & 324 & 10.7 & 1．22 & 0． 238 & 471 & 15.1 & 4.98 & 0， 043 \\
\hline \(\bullet\) & 52000 & 15.2 & 0.0787 & 2.70 & 60630 & D． 22 & 0.456 & 0． 191 & 378 & 6． 8 L 2 & 1． 0 o & 0.278 & 550 & 11.7 & 3.99 & 0．086 \\
\hline 7 & 52063 & 20．7 & 0.132 & 2.85 & 101730 & 11.0 & 0.621 & 0， 267 & 331 & 12.1 & 1．引引 & 0.644 & 483 & 18.9 & 6.20 & 0． 086 \\
\hline 8 & 52002 & 24.8 & 0.144 & 2．88 & 111430 & 12.2 & 0.654 & 0.263 & 323 & 13.4 & 1.53 & 0.239 & 472 & 20.6 & 6． 72 & 0.086 \\
\hline \(\square\) & 52007 & \(2{ }^{2} .6\) & 0，3B0 & 3． 87 & 98830 & 14．4 & 0． 828 & 0.169 & 334 & 15，8 & 2.93 & 0.246 & 487 & 32.2 & 9.84 & 0． 257 \\
\hline 10 & 52167 & 31.5 & 0.516 & 4． 23 & 108030 & 16.1 & 0.945 & 0.171 & 338 & 17.6 & 2.21 & 0.250 & 494 & 39.9 & 11.9 & 0． 343 \\
\hline 11 & 52094 & 30.4 & 0． 392 & 4．47 & 1015 & 17.3 & 0.912 & 0.162 & 32.1 & 19．0 & 2.13 & 0.237 & 468 & 42.8 & 12.2 & 0．386 \\
\hline 12 & 52004 & ＋6． 1 & 0． 725 & 4.02 & 3558 & 0． 160 & 1.38 & 1.20 & 234 & 0.515 & 3.23 & 1.77 & \(3+1\) & 70.2 & 21.3 & 0.171 \\
\hline 13 & 52032 & 56．5 & 1.76 & 4.65 & 31520 & 0． \(3+3\) & 1.69 & 1，25 & 240 & 0.596 & 3.95 &  & 351 & 93.5 & 27， 8 & 0.343 \\
\hline \(1+\) & 52026 & 01.0 & 2．18 & 5． 28 & 31520 & 0，616 & 1．63 & 1.22 & 235 & 0.676 & 4． \(2 \overline{7}\) & 1.77 & 342 & 116.0 & 32.8 & 0.514 \\
\hline 13 & 5203k & 6.5 .4 & 2．\({ }^{1} 1\) & 5.97 & 31520 & 0．0．47 & 1.98 & 1.19 & 230 & 0.765 & 4，61 & 1.74 & 334 & 139.0 & 38.3 & 0.686 \\
\hline is & 52013； & 99． 9 & 4.68 & 6.33 & \({ }^{505} 80\) & 1．14 & 2.67 & 1.06 & 204 & 1.3 & 6． 22 & 1.55 & 298 & 210.0 & 59.0 & 0.686 \\
\hline 17 & 5205\％ & 116.0 & 6.81 & 6.76 & 73720 & 1． 65 & 3． 48 & 0，970 & \(1 \mathrm{BT}^{\text {d }}\) & 2.0 & 8． 12 & 1.42 & 273 & 303.0 & 86.4 & 0.666 \\
\hline \(1{ }^{1+}\) & 52815 & 170.0 & 4.35 & B． 88 & 50520 & 1． 1.0 & 3． 30 & 0． 4.96 & 192 & 1.82 & 7． 70 & 1.45 & 280 & 378． 0 & 17．4 & 1.371 \\
\hline 17 & 52¢1； & 173．0 & 14．5． & －．\({ }^{\text {d }}\) & \({ }_{69818} 17\) & 0．98 & 5.37 & 1.66 & 160 & 1.065 & 12.5 & 2． 33 & 274 & 562.0 & 163.0 & 0.686 \\
\hline 20 & 32031 & 456．0 & 19， 8 & 8． 23 & 1114 & 1． 70 & 7.68 & 1．30 & 145 & 1，86 & 17．9 & 2.19 & 211 & 431.9 & 272．0 & 0.686 \\
\hline \(\therefore 1\) & 54：03 & く＜0．0 & 24.3 & 6．78 & \(688 \quad 17\) & 1． 12 & 6.60 & 1． 72 & 165 & 1.23 & 15．4 & 2． 5 ： & 241 & 741.0 & 214.0 & 1． 371 \\
\hline \(\therefore\) & 72149 & 304， 0 & 39.4 & 9.49 & 110417 & 1． 04 & 0.12 & 1.53 & 147 & 2.13 & 21.3 & 2． 24 & 415 & 1182.0 & 34.0 & 1．371 \\
\hline \(\pm 3\) & 3402. & 23 F .0 & ＋9． 1 & 15.3 & \(6{ }^{188}\) & 1.74 & T． 68 & 1.63 & 157 & 1.58 & 57，9 & 2．38 & 229 & 1106.0 & 201.0 & 4． 742 \\
\hline \(\therefore\) & 3 P 104： & 347.0 & 78.7 & 12.0 & 110417 & 2.45 & 10.4 & 1． 45 & 140 & 2.69 & 24.3 & 2.12 & 204 & 1681．0 & ＋53．0 & 2． 742 \\
\hline 4 & － 2100 & ＋22．0 & 145．0 & 15.4 & 108917 & 3.11 & 12.7 & 1.43 & 138 & 3.41 & 29.5 & 2.08 & 200 & 2459.0 & 633.0 & 5.142 \\
\hline \(\therefore\) & －214 & 478．0 & 510.0 & 20.3 & 2871 & 10.8 & 26.3 & 1.1 & 106 & 13．8 & 61.5 & 1．61 & 155 & 7100．0 & 1891.0 & 6． 855 \\
\hline \(\therefore\) & ： 51 & 1014.0 & 813．0 & 22， 2 & 2856 & 11.7 & 44．4 & 1.02 & 98.1 & 12.9 & 71.0 & 1.66 & 159 & 8891.0 & 2290.0 & 10．968 \\
\hline \multicolumn{17}{|l|}{} \\
\hline
\end{tabular}

\section*{\(\begin{array}{r}1 \\ \hdashline \quad 1 \\ \hline\end{array}\)}

\section*{Definitions for Table K6}

Information given is listed by column as:
1. Manufacturer part number
2. Surface area calculated from Figure K2l
3. Area product effective iron area times window area
4. Mean length turn
5. Total number of turns and wire size using a window utilization factor \(\mathrm{K}_{\mathrm{tu}}=0.40\)
6. Resistance of the wire at \(50^{\circ} \mathrm{C}\)
7. Watts loss is besed on Figure Cl for a \(\Delta \mathrm{T}\) of \(25^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, total loss is equal to 2 F
8. Current calculated from column 6 and 7
9. Current density calculated from column 5 and 8
10. Resistance of the wire at \(75^{\circ} \mathrm{C}\)
11. Watts loss is based on Figure Cl for a \(\Delta \mathrm{T}\) of \(50^{\circ} \mathrm{C}\) with a room ambient of \(25^{\circ} \mathrm{C}\) surface dissipation times the transformer surface area, total loss is equal to \(2 \mathrm{P}_{\mathrm{cu}}\)
12. Current calculated from column 10 and 11
13. Current density calculated from column 5 and 12
14. Effective core weight plus copper weight
15. Transformer volume calculated from Figure K24
16. Core effective cross-section


Fig. K17. Current Density Versus Area Product Ap for \(25^{\circ} \mathrm{C}\) and \(50^{\circ} \mathrm{C}\) Rise for Tape-Wound Toroids \({ }^{\circ}\)


Fig. K18. Surface Area Versus Area Product Ap for Tape-Wound Toroids


Fig. KI9. Volume Versus Area Product \(A_{p}\) for Tape-Wound Toroids


Fig. K20. Total Weight Versus Area Product \(A_{p}\) for Tape-Wound Toroids


\section*{A. Surface: then}


Fig. K21. Tape Wound Core, Power Cores, and Pot Cores Surface Area \(A_{t}\)

Fig. K22. Lamination Surface Area \(A_{t}\)

Fig. K23. C Core Surface Area \(A_{t}\)


Fig. K24. Tape wound core, Powder Core, and Pot Cores

Fig. K25. Laminations

Fig. K26. C Type Cores```

