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FLUTTER OF ARTICULATED PIPES AT FINITE AMPLITUDE

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DEPARTMENT OF APPLIED MECHANICS

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STANFORD, CALIFORNIA
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by

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and
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FLUTTER OF ARTICULATED PIPES AT FINITE AMPLITUDE

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SUMMARY

The plane motion of an articulated pipe made of two segments is examined and the flow velocity at which flutter manifests itself is sought. The pressure in the reservoir feeding the pipe is kept constant. In contrast to previous works, the flow velocity is not taken as a prescribed parameter of the system but is left to follow the laws of motion. This approach requires a nonlinear formulation of the problem and the equations of motion are solved using Krylov-Bogoliubov's method. A graph of the amplitude of the limit cycles, as a function of the fluid-system mass ratio, is presented and conclusions are drawn as to the necessity of considering nonlinearities in the analysis.

INTRODUCTION

A review of existing knowledge [1]* on the behavior of pipes conveying fluid, reveals that all earlier analyses assume that the fluid velocity relative to the pipe is a known quantity and is unaffected by the motion of the pipe. In these previous works, the flow velocity is assumed either constant or periodic with a prescribed amplitude and frequency. This approach eliminates the need to find the flow equations of motion, is adequate for infinitesimal transverse amplitudes of motion of the pipe system, but is incapable of predicting what will be the effect of larger amplutudes. This last shortcoming may be of importance when flow velocities are near critical velocities, that is, velocities at which the system begins to flutter.

It is therefore the purpose of the present study to investigate in greater detail the dynamic behavior of pipes in the vicinity of critical velocities. Such an analysis requires a nonlinear formulation of the problem. It will be shown that nonlinear terms are generated by the axial motion of the pipe and by the flow velocity fluctuation. Thus, the relative velocity of the fluid will no longer be assumed a given parameter in the problem,but will have to be determined from its own equations of motion which will be coupled, through the nonlinear terms, to the transverse equation of motion of the pipe. The specific system (see Fig. 1) considered here is a vertically hanging articulated pipe made of two segments. Each articulation is assumed perfect in the sense that it has no damping or restoring force associated with it. The fluid entering the pipe comes from a reservoir maintained at constant pressure and after its passage through the pipe, it is discharged, tangentially to the end of the pipe, to the atmosphere. This articulated pipe system was selected,

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* Numbers in brackets refer to referances at the end of the paper.

rather than a continuous pipe, in order to obtain an analytical solution and therefore get more complete insight into the problem. The related problems of an articulated two-link pipe fed by a reservoir in which the pressure fluctuates with time and the problem of a continuous cantilevered pipe fed by a constant pressure reservoir are also being analyzed for presentation in subsequent papers.

The basic assumptions made are: 1) The fluid is incompressible; this is a reasonable approximation for a liquid since the characteristic time taken by a wave to travel the length of the pipe is very much shorter than the period of the transverse motion of the pipe; 2) The diameter of the pipe is small compared to its length so that the rotatory inertia of an element of the pipe is negligible; 3) The velocity profile of the fluid, at a given cross-section is uniform, modeling fairly well a completely turbulent flow; 4) The motion of the articulated pipes takes place in a plane.

EQUATIONS OF MOTION

The derivation of the relevant equations of motion can readily be obtained by the direct application of Newton's law to our system (see Fig. 1). The first equation is obtained as a moment condition with respect to the upper pivot point on the free-body diagram of the two segments of the pipe. Similarly, the second equation is obtained as a moment condition with respect to the pivot point on the free-body diagram of the lower segment. Finally the equation governing the motion of the fluid in the pipe is obtained by the force condition in the tangential direction on a fluid element and by a subsequent integration over the length of the pipe. The three equations thus obtained are:

$$(m_1 + m_2)g\left(\frac{\ell_1^2}{2} + \ell_1\ell_2\right)\sin\theta_1 + (m_1 + m_2)\Bigg[- \frac{\ell_1\ell_2^2}{2}\dot\theta_2^2\sin(\theta_2 - \theta_1)$$

$$+ \ddot\theta_1\left(\frac{\ell_1^3}{3} + \ell_1^2\ell_2\right) + \ddot\theta_2\frac{\ell_1\ell_2^2}{2}\cos(\theta_2 - \theta_1)\Bigg] + 2m_1\dot v\left[\frac{\ell_1^2}{2}\dot\theta_1 + \ell_1\ell_2\dot\theta_2\cos(\theta_2 - \theta_1)\right]$$

$$+ m_1\ell_1\ell_2\ddot v\sin(\theta_2 - \theta_1) + m_1\dot v^2\ell_1\sin(\theta_2 - \theta_1) = 0 \tag{1}$$

$$(1/2)(m_1 + m_2)g\ell_2\sin\theta_2 + (1/3)(m_1 + m_2)\ell_2^3\ddot\theta_2 + m_1\dot v\dot\theta_2\ell_2^2$$

$$+ (1/2)(m_1 + m_2)\ell_1\ell_2^2\ddot\theta_1\cos(\theta_2 - \theta_1) + (1/2)(m_1 + m_2)\ell_1\ell_2^2\dot\theta_1^2\sin(\theta_2 - \theta_1) = 0 \tag{2}$$

$$P_o - L\dot v^2 + m_1g(\ell_1\cos\theta_1 + \ell_2\cos\theta_2) - m_1\ddot v(\ell_1 + \ell_2)$$

$$+ m_1\dot\theta_1^2[\ell_1^2/2 + \ell_1\ell_2\cos(\theta_2 - \theta_1)] - m_1\ddot\theta_1\ell_1\ell_2\sin(\theta_2 - \theta_1)$$

$$+ (1/2)m_1\dot\theta_2\ell_2^2 = 0 \tag{3}$$

where m_1 and m_2 are the mass, per unit length, of the fluid and the pipe, respectively, g is the acceleration due to gravity, ℓ_1 and ℓ_2 are the lengths of the upper and lower segments, $\dot v$ is the flow velocity, P_o is the force, due to pressure, acting on the fluid at $x = 0$ and $L\dot v^2$ represents the friction force between the fluid and the pipe. Fluid dynamic studies [2] have shown that L depends on the size and roughness of the pipe, on the density of the liquid and on Reynolds number if the latter is not too high. For fully turbulent flow (high Reynolds number), L is independent of Reynolds number and consequently is independent of viscosity.

We need now to relate the pressure p^* in the reservoir to the force "p_o". From Bernoulli's equation, applied between the surface of the water in the reservoir and its outlet, we obtain:

$$\frac{\dot{v}_o^2}{2} + \frac{p_o}{\rho} = gh^* + \frac{p^*}{\rho} \tag{4}$$

where the subscript "o" refers to quantities measured at the entrance of the pipe, h^* is the height of the fluid in the reservoir, ρ is the density of the fluid and p^* is the pressure in the reservoir. Eq. (4) is now combined with Eq. (3) and after the introduction of the following non-dimentional variables;

$$\alpha = \frac{\ell_1}{\ell_2} \quad , \quad \gamma = \frac{m_1}{m_1 + m_2} \quad , \quad t^* = \sqrt{\frac{g}{\ell_2}}\, t$$

$$\dot{U} = \frac{\dot{v}}{\sqrt{g\ell_2}} \quad , \quad L_o = \frac{L}{m_1} \quad , \quad P^* = \frac{p^*}{\rho \ell_2 g} \quad , \quad H^* = \frac{h^*}{\ell_2} \quad ,$$

Eqs. (1,2,3) become:

$$(\alpha^2 + \alpha^3/3)\ddot{\theta}_1 + (\alpha/2)\ddot{\theta}_2\cos(\theta_2 - \theta_1) + \gamma\alpha^2\dot{U}\dot{\theta}_1 + 2\alpha\dot{U}\dot{\theta}_2\cos(\theta_2 - \theta_1)$$

$$+ (\alpha + \alpha^2/2)\sin\theta_1 + \gamma\alpha\dot{U}^2\sin(\theta_2 - \theta_1)$$

$$+ \gamma\alpha\ddot{U}\sin(\theta_2 - \theta_1) - (1/2)\dot{\theta}_2^{\,2}\sin(\theta_2 - \theta_1) = 0 \tag{5}$$

$$(1/2)\sin\theta_2 + (1/3)\ddot{\theta}_2 + \gamma\dot{U}\dot{\theta}_2 + (1/2)\alpha[\ddot{\theta}_1\cos(\theta_2 - \theta_1) + \dot{\theta}_1{}^2\sin(\theta_2 - \theta_1)] = 0 \quad (6)$$

$$P^* - \dot{U}^2[(1/2) + L_o] + H^* + \alpha\cos\theta_1 + \cos\theta_2 - (\alpha + 1)\ddot{U}$$

$$= -[(1/2)\alpha^2\dot{\theta}_1{}^2 + \alpha\dot{\theta}_1{}^2\cos(\theta_2 - \theta_1) - \alpha\ddot{\theta}_1\sin(\theta_2 - \theta_1) + (1/2)\theta_2{}^2] \quad (7)$$

Since we do not want here to solve the complete nonlinear set of equations, but only to determine the effect of the leading nonlinearities on the system, we will omit nonlinear terms of order 4 and higher. Also, to facilitate manipulations by making certain terms linear in $\dot{U}$, we will assume that:

$$\dot{U} = \dot{U}_o + \Delta\dot{U} \quad \text{where} \quad \dot{U}_o{}^2 = \frac{P^* + H^* + \alpha + 1}{1/2 + L_o} .$$

$\dot{U}_o$ is the steady-state flow velocity under pressure P^* when the pipe is vertically at rest, and $\Delta\dot{U}$ is a small fluctuation of the flow velocity such that $\dot{U}^2 = \dot{U}_o{}^2 + 2\dot{U}_o\Delta\dot{U}$.

Under these modifications, Eqs. (5,6,7) become:

$$\begin{bmatrix} \alpha^2 + \alpha^3/3 & \alpha/2 \\ \alpha/2 & 1/3 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} \gamma\alpha^2\dot{U}_o & 2\gamma\alpha\dot{U}_o \\ 0 & \gamma\dot{U}_o \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$+ \begin{bmatrix} (\alpha + \alpha^2/2) - \gamma\alpha\dot{U}_o{}^2 & \gamma\alpha\dot{U}_o{}^2 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (8)$$

$$\text{where} \quad F_1 = \ddot{\theta}_2\alpha(\theta_2 - \theta_1)^2/4 + \dot{\theta}_2{}^2\alpha(\theta_2 - \theta_1)/2 - \dot{\theta}_1\gamma\alpha^2\Delta\dot{U} - \dot{\theta}_2 2\gamma\alpha\Delta\dot{U}$$

$$+ \dot{\theta}_2\gamma\alpha\dot{U}_o(\theta_2 - \theta_1)^2 + (\alpha + \alpha^2/2)\theta_1{}^3/6 - \gamma\alpha\Delta\ddot{U}(\theta_2 - \theta_1)$$

$$+ \gamma\alpha\dot{U}_o{}^2(\theta_2 - \theta_1)^3/6 - 2\gamma\alpha\dot{U}_o\Delta\dot{U}(\theta_2 - \theta_1)$$

$$F_2 = \quad {}_2{}^3/12 - \gamma\Delta\dot{U}\dot{\theta}_2 + \alpha\ddot{\theta}_1(\theta_2 - \theta_1)^2/4 - \alpha\dot{\theta}_1{}^2(\theta_2 - \theta_1)/2$$

$$(\alpha + 1)\Delta\ddot{U} + (1 + 2L_o)\dot{U}_o\Delta\dot{U} = -\alpha\theta_1{}^2/2 - \theta_2{}^2/2 + \dot{\theta}_1{}^2(\alpha + \alpha^2/2) + \dot{\theta}_2{}^2/2$$

$$- \alpha\ddot{\theta}_1(\theta_2 - \theta_1) \qquad (9)$$

The linear part of Eq. (8) consists of two coupled equations with constant coefficients The stiffness and damping matrix are nonsymmetric, indicating a nonconservative system. Eq. (9) is uncoupled from Eq. (8) in its linear part but not in its nonlinear terms.

METHOD OF SOLUTION

To solve the above system of equations we will proceed as follows: a) Solve the linear part of Eq. (8) and determine $\theta_1(t)$, $\theta_2(t)$ for $\dot{U}_o$ critical; b) Use the solution found in a) to calculate the nonlinear terms of Eq. (9); c) Solve Eq. (9) treating the nonlinear terms as forcing functions and obtain $\Delta\dot{U}(t)$; d) Use $\Delta\dot{U}(t)$ in the right-hand side of Eq. (8) and apply Krylov-Bogoliubov's averaging technique.

To find the solution to the linear part of Eq. (8) we assume a solution of the form:

$$\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \mathrm{Re}\left[\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} e^{\lambda t}\right]$$

and substitute into Eq. (8) which yields

$$\begin{bmatrix} (\alpha/2)\lambda^2 & \lambda^2/3 + \lambda\gamma\dot{U}_o + 1/2 \\ (\alpha^2 + \alpha^3/3)\lambda^2 + \gamma\alpha^2\dot{U}_o\lambda + (\alpha + \alpha^2/2) - \gamma\alpha\dot{U}_o^{\,2} & (\alpha/2)\lambda^2 + 2\gamma\alpha\dot{U}_o\lambda + \gamma\alpha\dot{U}_o^{\,2} \end{bmatrix} \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = 0 \qquad (10)$$

For a nontrivial solution to exist, the determinant of the above matrix has to be zero. This leads to a 4th order polynomial in λ. Since we are only interested in critical values of λ, we know that it has to be a pure imaginary number and since the coefficients of the polynomial are all real, we can separate the polynomial into its real and imaginary parts in the following manner:

$$\lambda[\lambda^2\dot{U}_o\alpha\gamma(1 + \alpha)/3 + \dot{U}_o\gamma(1 + \alpha) - \dot{U}_o^{\,3}\gamma^2] = 0 \qquad (11)$$

$$\lambda^4\alpha(\alpha + 3/4)/9 + \lambda^2[(\alpha + 1)^2/3 - \alpha^2/6 - \dot{U}_o^{\,2}\gamma(\alpha/2 + 1/3 - \gamma\alpha)]$$

$$+ (1 + \alpha/2)/2 - \gamma\alpha\dot{U}_o^{\,2}/2 = 0 \qquad (12)$$

The elimination of λ^2 between these two equations yields:

$$A\dot{U}_c^{\,4} = B\dot{U}_c^{\,2} + C = 0$$

where

$$\dot{U}_c \equiv \dot{U}_o \text{ critical} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad (13)$$

and

$$A = \left[\frac{\gamma^2(\alpha + 3/4)}{\alpha(1 + \alpha)^2} - \frac{3\gamma^2(\alpha/2 + 1/3 - \gamma\alpha)}{\alpha(1 + \alpha)}\right]$$

$$B = -\left[\frac{2\gamma(\alpha + 3/4)}{\alpha(1 + \alpha)} - \frac{3\gamma(\alpha/2 + 1/3 - \gamma\alpha)}{\alpha} - \frac{\gamma(\alpha + 1)}{\alpha} + \frac{\gamma}{2} + \frac{\alpha\gamma}{2(1 = \alpha)}\right]$$

$$C = \left[\frac{\alpha + 3/4}{\alpha} - \frac{(\alpha + 1)^2}{\alpha} + \frac{(1 + \alpha/2)}{2} + \frac{\alpha}{2}\right]$$

The substitution of $\dot{U}_c$ into Eq. (11) permits us to find the two complex conjugate, critical, eigenvalues of our system. Eq. (13) gives us two values for $\dot{U}_c$. The value of interest is the higher one since the other value corresponds to the point where the system regains stability (see Fig. 2). To obtain the two corresponding eigenvectors, we substitute $\dot{U}_c$ and the eigenvalue into either one of Eqs. (10), say the first one:

$$R_j \quad \frac{\Theta_{1j}}{\Theta_{2j}} = -\frac{\lambda_j^2/3 + \lambda_j\gamma\dot{U}_c + 1/2}{\lambda_j^2\alpha/2} \qquad j = 1,2$$

Since λ_1 and λ_2 are complex conjugates, R_1 and R_2 are also complex conjugates. We will not investigate the other two eigenvalues since a previous analysis by Bohn and Herrmann [3] shows that they possess a negative real part and consequently they contribute but little to the solution. A typical root locus (as a function of $\dot{U}_o$) for the system is shown in Fig. 2. The motion of the pipe in its flutter mode is therefore:

$$\theta_1 = \text{Re}[R_1\Theta_{21}(\cos\omega t + i\sin\omega t) + \bar{R}_1\Theta_{22}(\cos\omega t - i\sin\omega t)] \tag{14}$$

$$\theta_2 = \text{Re}[\Theta_{21}(\cos\omega t + i\sin\omega t) + \Theta_{22}(\cos\omega t - i\sin\omega t)] \tag{15}$$

where $i\omega = \lambda$. These equations can be expressed more conveniently as:

$$\theta_1 = \Theta\sqrt{a^2 + b^2}\sin(\omega t + \phi_1 + \phi_o) \text{ where } (a + ib) = R_1 \tag{16}$$

$$\theta_2 = \Theta\sin(\omega t + \phi_1)$$

$$\text{where } \Theta = \sqrt{(\text{Re}\Theta_{21} + \text{Re}\Theta_{22})^2 + (-\text{Im}\Theta_{21} + \text{Im}\Theta_{22})^2} \ ,$$

$$\text{tg}\phi_o = \frac{b}{a} \ , \quad \text{tg}\phi_1 = \frac{(\text{Re}\Theta_{21} + \text{Re}\Theta_{22})}{(-\text{Im}\Theta_{21} + \text{Im}\Theta_{22})} \tag{17}$$

Θ and ϕ_1 are determined by the initial conditions. We note, in the above, that Θ_{21} and Θ_{22} are not completely determined since we have two equations and four unknown. If we want, but there is no obligation to do so, we can require the imaginary part of Eqs. (14) and (15) to be zero, which leads to: $\bar{\Theta}_{21} = \Theta_{22}$.

A graphical representation of the mode shape through a cycle is presented in Fig. 3. The *angular* velocities of the two segments are indicated by curved arrows and each of the four sketches is taken at the instant when one segment has no angular velocity. The sketches are separated by almost exactly 90°.

Now that we have determined the mode shape of interest, we will eliminate the damped mode, thus reducing our fourth-order system to second order. To achieve this elimination we follow Foss' work [4] on the uncoupling of the equations of motion of a damped linear system. His approach must, however, be modified since he assumes that all the matrices of the system are symmetric. To take care of this difficulty we must have recourse to the concept of the adjoint problem, that is, the problem obtained by transposing all the matrices of our system. It can be shown that the eigenvalues of this new problem are the same as those of the original problem but that the eigenvectors are different. Yet, these eigenvectors have the crucial property of being orthogonal to the eigenvectors of the original system, thus permitting the separation of the modes of our system. The details of how to proceed will not be given here because of limitations in space; they can be found in reference [5].

Thus, following the method described above, Eq. (8) can be rewritten as:

$$\ddot{x} + (\omega^2 + \beta^2)x = 2\beta\dot{x} + \frac{r\dot{f} + s\dot{g}}{r^2 + s^2} + \frac{rf + sg}{r^2 + s^2}\left(-\frac{r}{s}\,\omega - \beta\right) + f\,\frac{\omega}{s} \tag{18}$$

where

$$x = \Theta/2 \sin\psi \quad , \quad \psi = \omega t + \phi_1 \tag{19}$$

$$(\beta + i\omega) = \lambda$$

$$(r + is) = 2\omega i \begin{Bmatrix} 1 \\ R_a \end{Bmatrix}^T \begin{bmatrix} \alpha^2 + \alpha^3/3 & \alpha/2 \\ \alpha/2 & 1/3 \end{bmatrix} \begin{Bmatrix} R \\ 1 \end{Bmatrix} + \gamma\dot{U}_o \begin{Bmatrix} 1 \\ R_a \end{Bmatrix}^T \begin{bmatrix} \alpha^2 & 2\alpha \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} R \\ 1 \end{Bmatrix} \tag{20}$$

$f = F_1 + cF_2$ where F_1 and F_2 are defined in Eq. (8)

$g = dF_2$

$$(a + ib) = R = -\frac{(1/3)\lambda^2 + \lambda\gamma\dot{U}_o + 1/2}{\lambda^2\alpha/2} \tag{21}$$

$$(c + id) = R_a = -\frac{\lambda^2(\alpha^2 + \alpha^3/3) + \lambda(\gamma\alpha^2\dot{U}_o) + 3\alpha/2 - \gamma\alpha\dot{U}_o^{\,2}}{\lambda^2\alpha/2} \tag{22}$$

The damping coefficient β is a small quantity, since we are interested only in flow velicities close to critical and therefore it will be eliminated in Eq. (18) except in the linear damping term. Also, because of the smallness of that term, it has been transferred to the right-hand side of the equation, which contains the small nonlinear terms. The nonlinear terms F_1 and F_2 are not completely known yet because they depend on $\Delta\dot{U}$ and $\Delta\ddot{U}$. To determine these fluctuations of the flow velocity we combine Eqs. (9) and (16) to obtain

$$\Delta\ddot{U}(\alpha + 1) + \Delta\dot{U}(1 + 2L_o) = (\theta^2/2)[(A + B) - (A - B)\cos 2\psi + C \sin 2\psi] \tag{23}$$

where $A = -a^2\alpha/2 - 1/2 + (\alpha + \alpha/2)\omega^2b^2 + \alpha\omega^2(a - b^2)$

$B = -\alpha b^2/2 + (\alpha + \alpha^2/2)\omega^2a^2 + \omega^2/2 - \alpha\omega^2b^2$

$C = -\alpha ab - (2\alpha + \alpha^2)\omega^2ab + \alpha\omega^2(b - 2ab)$

The steady-state homogeneous solution is $\Delta\dot{U} = 0$. The particular solution is assumed to be of the form

$$\Delta\dot{U} = \theta^2(K + U_1 \sin 2\psi + U_2 \cos 2\psi) \tag{24}$$

and is substituted into Eq. (23). After a comparison of the coefficients of similar terms we obtain:

$$K = \frac{A + B}{2\dot{U}_o(1 + 2L_o)} \tag{25}$$

$$U_1 = \frac{\dot{U}_o(1 + 2L_o)C/2 - (A - B)\omega(\alpha + 1)}{\dot{U}_o^{\,2}(1 + 2L_o)^2 + 4\omega^2(\alpha + 1)^2} \tag{26}$$

$$U_2 = \frac{-\dot{U}_o(1 + 2L_o)(A - B)/2 - C\omega(\alpha + 1)}{\dot{U}_o^{\,2}(1 + 2L_o)^2 + 4\omega^2(\alpha + 1)^2} \tag{27}$$

Now that the right-hand side of Eq. (18) is known, we will solve this equation using the Krylov-Bogoliubov (K.B) method [6], keeping only the first term in the asymptotic expression. This method is also known as the method of "averaging".

The K.B. method essentially assumes that in the region close to the critical point, the motion will be oscillatory, as it is at the critical point, except that the amplitude and phase of the motion will change slowly due to the effect of the small damping and of the small nonlinearities.

In our particular case, the oscillatory motion of interest is:

$$x = (\theta/2)\sin\psi = (\theta/2)\sin(\omega t + \phi) \tag{28}$$

$$\dot{x} = (\dot{\theta}/2)\omega\cos\psi = (\dot{\theta}/2)\omega\cos(\omega t + \phi) \tag{29}$$

which satisfies the left-hand side of Eq. (18). We now assume that the small terms of the right-hand side of Eq. (18) will cause θ and ϕ to vary slowly with time. For this to be true, Eq. (29) requires that:

$$(\dot{\theta}/2)\sin\psi + (\theta/2)\dot{\phi}\cos\psi = 0 \tag{30}$$

We also obtain, by a differentiation of Eq.(29):

$$\ddot{x} = (\dot{\theta}/2)\omega\cos\psi - (\theta/2)\omega(\omega + \dot{\phi})\sin\psi \tag{31}$$

Equations (28) and (31) transform equation (18) into:

$$(\dot{\theta}/2)\omega\cos\psi - (\theta/2)\dot{\phi}\omega\sin\psi = 2\beta\dot{x} + \left(\frac{r\dot{f} + s\dot{g}}{r^2 + s^2}\right) - \frac{r\omega}{s}\left(\frac{rf + sg}{r^2 + s^2}\right) + f\frac{\omega}{s} \tag{32}$$

which can be combined with Eq. (30) to obtain:

$$\dot{\theta} = \left[\frac{2}{\omega}\, 2\beta\dot{x} + \left(\frac{r\dot{f} + s\dot{g}}{r^2 + s^2}\right) - \frac{r\omega}{s}\left(\frac{rf + sg}{r^2 + s^2}\right) + f\frac{\omega}{s}\right]\cos\psi \tag{33}$$

$$\theta\dot{\phi} = \left[\frac{2}{\omega}\, 2\beta\dot{x} + \left(\frac{r\dot{f} + s\dot{g}}{r^2 + s^2}\right) - \frac{r\omega}{s}\left(\frac{rf + sg}{r^2 + s^2}\right) + f\frac{\omega}{s}\right]\sin\psi \tag{34}$$

Since θ and ϕ change very slowly, we assume them to be constant for the duration of a cycle and integrate Eqs. (33) and (34) to find the average of $\dot{\theta}$ and $\dot{\phi}$:

$$\dot{\theta}_{av} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{2}{\omega}\left[2\beta\dot{x} + \left(\frac{r\dot{f} + s\dot{g}}{r^2 + s^2}\right) - \frac{r\omega}{s}\left(\frac{rf + sg}{r^2 + s^2}\right) + \frac{f\omega}{s}\right]\cos\psi\, dt \tag{35}$$

$$\theta\dot{\phi}_{av} = \frac{\omega}{2\pi}\int_{o}^{2\pi/\omega} -\frac{2}{\omega}\left[2\beta\dot{x} + \left(\frac{r\dot{f} + s\dot{g}}{r^2 + s^2}\right) - \frac{r\Omega}{s}\left(\frac{rf + sg}{r^2 + s^2}\right) + \frac{f\Omega}{2}\right]\sin\psi \, dt \qquad (36)$$

The damping term, $2\beta\dot{x}$, in the argument of the above integrals depends on θ to the first power, but terms in f and g depend on θ to the third power, which will permit us to find equilibrium points for certain amplitudes. If we are exactly at the critical point, $\beta = 0$ and, therefore, the nonlinear terms will decide whether or not the critical point is stable.

The evaluation of the right-hand side of Eqs. (35) and (36) turns out to be extremely tedious since r , s , f and g are complicated expressions. The details of this evaluation, which are given in [5], will only be summarized here. The terms in brackets in Eqs. (35) and (36) can be shown to generate, after expansion, terms in $\sin\psi$, $\cos\psi$, $\sin 3\psi$ and $\cos 3\psi$ but the averaging will retain only the terms in $\sin\psi$ in Eq. (36) and only the terms in $\cos\psi$ in Eq. (35), these terms being affected by a coefficient of one-half. Thus Eqs. (35) and (36) become:

$$\dot{\theta}_{av} = (\omega\beta\theta + K_1\theta^3)/\omega \qquad (37)$$

$$\theta\dot{\phi}_{av} = -K_2\theta^3/\omega \qquad (38)$$

where

$$K_1 = \frac{r(f_1\omega + cg_1\omega)}{r^2 + s^2} + \frac{sdg_1\omega}{r^2 + s^2} - \left(\frac{r\omega}{s}\right)\frac{r(f_2 + cg_2) + sdg_2}{r^2 + s^2} + \frac{\omega}{s}(f_2 + cg_2)$$

$$K_2 = \frac{r(-f_2\omega - cg_2\omega) - sdg_2\omega}{r^2 + s^2} - \left(\frac{r\omega}{s}\right)\frac{r(f_1 + cg_1) + sdg_1}{r^2 + s^2} + \frac{\omega}{s}(f_1 + cg_1)$$

where, from Eqs. (38, 18, 8, 24)

$$f_1 = -(1/16)\omega^2[3(1 - a)^2 + b^2] + \alpha\omega^2(1 - a)/8$$

$$-\gamma\alpha^2\omega(U_1 a/2 - Kb + U_2 b/2) - \gamma\alpha\omega U_1$$

$$-\gamma\alpha\dot{U}_o\omega(1 - a)b/2 + (\alpha + \alpha^2/2)(a^3 + ab^2)/8$$

$$+\gamma\alpha[(1 - a)U_1\omega - b\omega U_2] + (1/8)\gamma\alpha\dot{U}_o{}^2[(1 - a)^3 + (1 - a)b^2]$$

$$-2\gamma\alpha\dot{U}_o[(1 - a)(K - U_2/2) - bU_1/2]$$

$$f_2 = \alpha\omega^2(1 - a)b/8 - 3\alpha\omega^2 b/8 - \gamma\alpha^2\omega[a(U_2/2 + K) - bU_1/2]$$

$$-\gamma\alpha\omega(2K + U_2) + \gamma\alpha\dot{U}_o\omega[(1 - a)^2/4 + 3b^2/4]$$

$$+(1/8)(\alpha + \alpha^2/2)(a^2 b + b^3) + \gamma\alpha[(1 - a)U_2\omega + bU_1\omega]$$

$$-(1/8)\gamma\alpha\dot{U}_o{}^2[(1 - a)^2 b + b^3] - 2\gamma\alpha\dot{U}_o[(1 - a)U_1/2 - b(K + U_2/2]$$

$$g_1 = 1/16 - \gamma\omega U_1/2 - (1/4)\alpha\omega^2\{a[3(1 - a)^2/4 + b^2/4] - (1 - a)\ b^2/2\}$$

$$-(1/2)\alpha\omega^2[(1 - a)(a^2 + 3b^2)/4 + ab^2/2]$$

$$g_2 = -\gamma\omega(K + U_2/2) + (1/4)\alpha\omega^2\{ab(1 - a)/2 - b[(1 - a)^2/4 + 3b^2/4]\}$$

$$(1/2)\alpha\omega^2[ab(1 - a)/2 + b(3a^2 + b^2)/4]$$

Only β in Eqs. (37) and (38) needs to be determined; we can either solve numerically for the roots of the fourth-order polynomial given by equating the determinant of Eq. (10) to zero or, since β will be a small quantity, we can solve analytically the polynomial neglecting terms of order two and higher in β. This second method is presented in [5].

STABILITY ANALYSIS

The effect of nonlinearities in the critical region can now be investigated from Eqs. (37) and (38). Equation (37) indicates that a positive β is destabilizing since it leads to $\dot{\theta}_{av} > 0$. Also, if K_1, which represents the effect of nonlinearities, is positive, we have also a destabilizing effect.

The only way the nonlinarities can change a stable behavior into an unstable one, and conversely, is for β and K_1 to be of opposite sign. When this is the case there is an amplitude $\theta_{L.C.}$ for which $\dot{\theta}_{av} = 0$ and which is known as the amplitude of the limit cycle (L.C.). The L.C. is said to be stable if for θ greater or smaller than $\theta_{L.C.}$ the amplitude of the motion approaches $\theta_{L.C.}$. The L.C. is said to be unstable if for θ greater or smaller than $\theta_{L.C.}$ the amplitude moves away from $\theta_{L.C.}$

Hence, from the previous discussion, we can identify three different possibilities, as far as stability is concerned: a) $K_1 < 0$ and $\beta > 0$ which leads to a stable limit cycle; b) $K_1 > 0$ and $\beta < 0$ which leads to an unstable limit cycle; c) K_1 and β do not meet any of the two previous requirements. In this last case, β and K_1 have the same sign and reinforce one another, leading to the same conclusion regarding stability as a linear analysis. In a) and b) above, the amplitude of the limit cycle is given by:

$$\theta_{L.C.} = \sqrt{-\frac{\omega\beta}{K_1}} \tag{39}$$

Figure 4 shows a plot of $\theta_{L.C.}$ as a function of γ, the ratio of the mass of the fluid to the mass of the pipe plus fluid. Also indicated on Fig. 4 is the type of limit cycle. There are two regions of interest. The first one extends from $\gamma = 0$ to $\gamma = .1$ and is characterized by destabilizing contribution of the nonlinearities which thus permit only limit cycles for flow velocities lower than critical. The second region, from $\gamma = .1$ to $\gamma = .17$, is characterized by stabilizing nonlinearities and consequently limit cycles exist only for velocities higher than critical. In the first region, if $\theta < \theta_{L.C.}$, the oscillation will die out but if $\theta > \theta_{L.C.}$, θ will grow continuously. In the second region, if $\theta < \theta_{L.C.}$, the motion will increase until it reaches the limit cycle and if $\theta > \theta_{L.C.}$, motion will decay until it reaches the limit cycle. Naturally, the most desirable region is the second one since it permits to operate with complete confidence up to th critical velocity and even if one overshoots the critical velocity a little, the amplitude of motion will remain bounded.

For γ greater than approximately .17 flutter cannot exist, since λ can no longer be pure imaginary. We also observe that for γ close to .1, the amplitude of the limit cycle goes to infinity. This indicates that the nonlinearities contribute nothing to the motion for such a value of γ.

It must be pointed out that in all the above discussions Eq. (38) was never mentioned. The reason for this omission lies in the fact that $\dot{\phi}_{av}$ is the change in the frequency of the motion, which is of no interest as far as stability is concerned.

EXPERIMENT

The experimental apparatus (with copper tubes) used by Bohn and Herrmann [3] proved very useful in carrying out the following qualitative experiment: With the pipe vertically at rest, the fluid velocity is progressively increased until the slightest disturbance causes the system to flutter. The flow is then reduced by a very small amount and the system is subjected to small disturbances. It is observed that as long as the disturbances are small, the motion damps out but that if the initial conditions are sufficiently large, the amplitude of motion grows until it reaches a very large value. This behavior shows the existence of an unstable limit cycle which is also what the theory predicts (Fig. 4) since the effective γ for the system is about .072.

CONCLUSIONS

This study has shown that the effect of finite amplitudes can significantly change the qualitative behavior of the articulated pipe system. Nevertheless, the results permit us to increase our confidence in the predictions of the linear analysis since Fig. 4 clearly illustrates that an amplitude of motion of at least ten degrees is required to change the flow velocity by only a few percent. It thus seems that in most engineering applications the effect of the nonlinearities can be safely neglected knowing that designers are not likely to allow a system to operate within a few percent of the critical velocity.

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LIST OF ILLUSTRATIONS

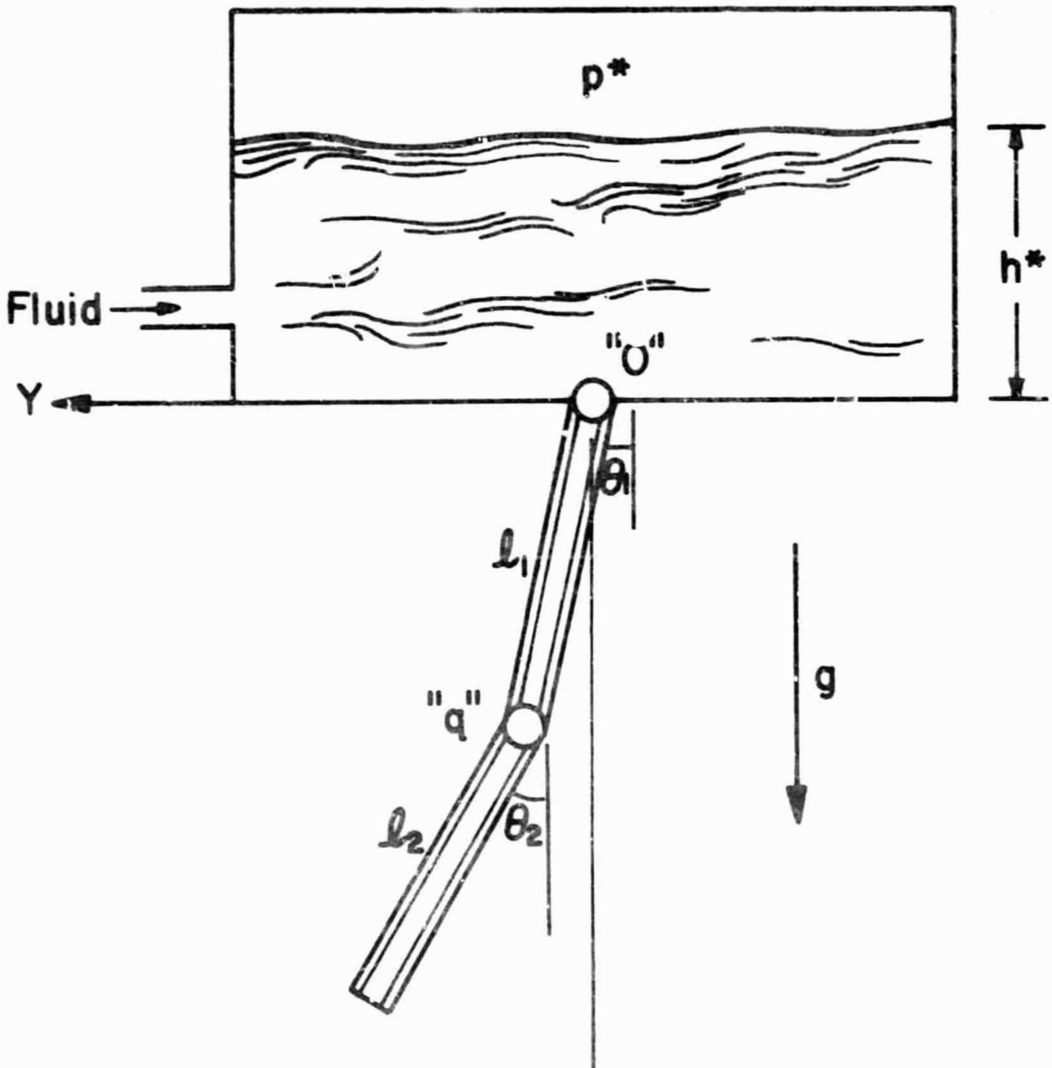

Figure 1 Reservoir - articulated pipe system.

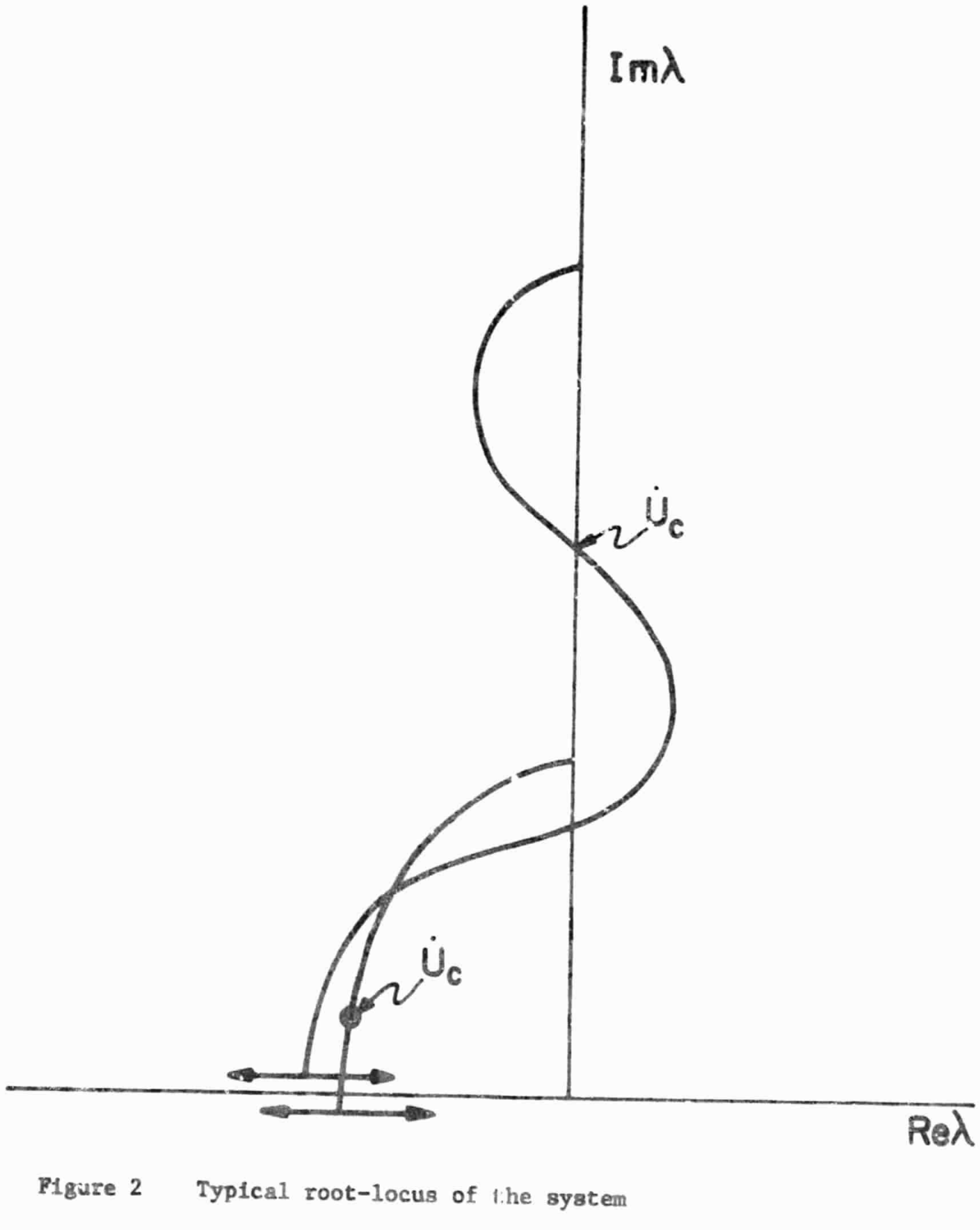

Figure 2 Typical root-locus of the system

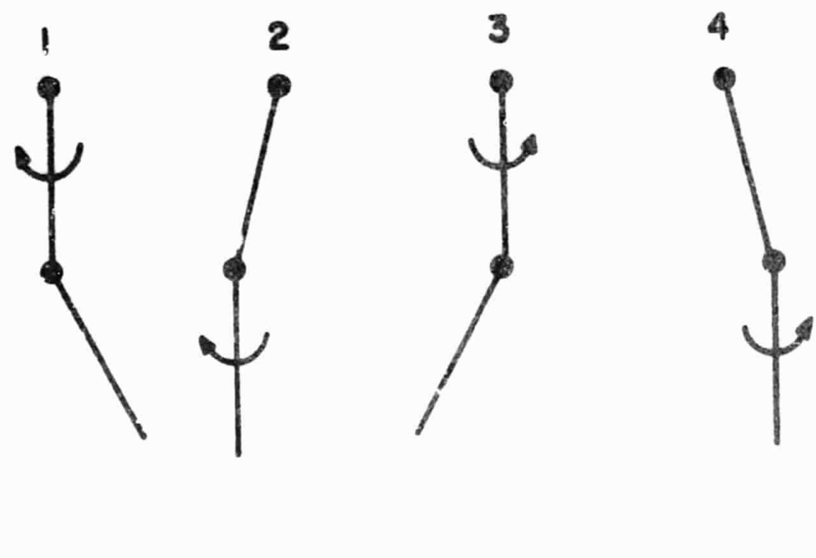

Figure 3 Flutter mode shape of the pipe.

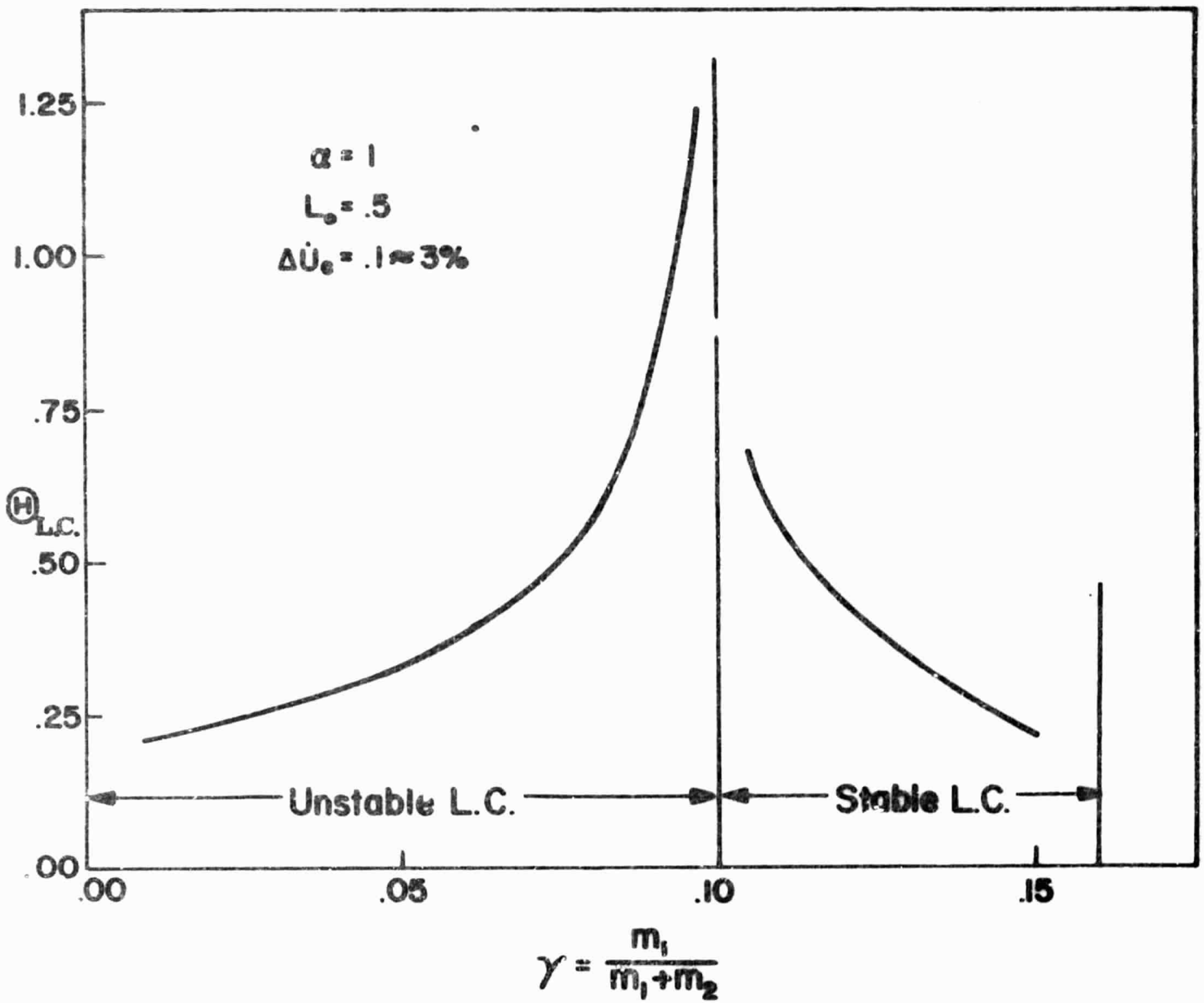

Figure 4 Limit-cycle amplitudes (rad.) vs. mass ratio γ