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IMPROVEMENT IN THE GEOPOTENTIAL DERIVED FROM SATELLITE AND SURFACE DATA (GEM 7 AND 8)

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GODDARD SPACE FLIGHT CENTER
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Abstract. A refinement has been obtained in the earth's gravitational field using satellite and surface data. In addition to a more complete treatment of data previously employed on 27 satellites, the new satellite solution (Goddard Earth Model 7) includes 64,000 laser measurements taken on 7 satellites during the international satellite geodesy experiment (ISAGEX) program. GEM 7, containing 400 harmonic terms, is complete through degree and order 16. The companion solution GEM 8 combines the same satellite data as in GEM 7 with surface gravimetry over 39% of the earth. GEM 8 is complete to degree and order 25. Extensive tests on data independent of the solution shows that the undulations of the geoidal surface computed by GEM 7 has an accuracy (for commission errors only) of about 3 m (rms). The overall accuracy of the geoid estimated by GEM 8 (including short wavelength components not in the solution) is estimated to be about 4-1/4 m (rms), an improvement of almost 1 m over previous solutions. The new combination solution is the first to show signs of "convection rolls" in the upper mantle below the Pacific Ocean.

Introduction

The requirements of the National Aeronautics and Space Administration's Earth and Ocean Physics Applications Program (EOPAP) call for a knowledge of the geoidal surface at the sub meter level [Kaula et al., 1969]. It is well known that current geopotential models are far from this goal. The accuracy in these range from 3 to 5 m (rms) for all undulations down to about 1000 km (half wave length) [e.g., Gaposchkin, 1974, Lerch et al., 1974]. Nevertheless, progress continues to be made with more accurate and comprehensive observations both from satellite tracking and surface gravimetry.

At Goddard Space Flight Center the emphasis has been on using as much of all kinds of precise satellite data as possible. Therefore we use numerical integration to compute precise satellite orbits and variations. The growth of our data base in these solutions is shown in Table 1.

In the models presented here (GEM 7 and 8), the significant new observations are 64,000 laser ranges to 7 satellites in the International Satellite Geodesy Experiment [Brechet, 1970]. However, considerable improvement in the solutions are also attributed to: (1) iteration of the data from an improved earth model, (2) use of new, more accurate one-way Doppler data on the GEOS 1 and 2 satellites, (3) utilization of all the previous Doppler data in overlapping passes, (4) inclusion of shorter wavelength gravitational terms and (5) correction of errors in the computation of diurnal abberation and parallactic refraction for the optical data. In particular, this is the first GEM solution where the Doppler and laser data has nearly full weight corresponding to rms residuals of 4 cm/sec and 3 m. The surface gravimetric "observations", 1656 three hundred nautical mile equal area means [R. H. Rapp, Personal Communication, 1974], were based on 1° x 1° data covering 39% of the earth. GEM 2 used 1° data with 32% coverage [Smith, Lerch and Wagner, 1973].

Table 1

Tracking Data in Goddard Earth Models

Model	Observations: Optical (Satellites)	Observations: Electronic (Satellites)	Observations: Laser (Satellites)	Weekly Orbital Arcs	Highest Complete Degree (Total No. of Coeffs.)
GEM 1 (1971)	120,000 (23)	–	–	300	12 (240)
GEM 5 (1973)	130,000 (24)	274,000 (7)	10,000 (5)	362	12 (241)
GEM 7 (1975)	154,000 (24)	332,000 (9)	76,000 (7)	405	16 (400)

Solutions and Results

The potential coefficients for GEMS 7 and 8 are listed in Table 2. They refer to the definition of the gravitational potential as:

$$V = \frac{\mu}{r}\left\{1 + \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \left(\frac{r_e}{r}\right)^{\ell} \overline{P}_{\ell m}(\sin\phi)\ [C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda]\right\}$$

where μ (398600.8 km^3/sec^2) is the earth's mass including the atmosphere, r_e (6378.145 km) is the earth's mean equatorial radius, $\overline{P}_{\ell m}$ is the fully normalized associated legendre function of degree ℓ and order m [e.g., Kaula, 1966, p. 7] and r, ϕ, λ are the distance to the center of mass, latitude and longitude. Those for GEM 7 have been derived from 562,000 condition equations representing differential corrections to GEM 3 [Lerch et al., 1972] for a maximum of 750 potential coefficients. These equations have been solved by a

Table 2
Normalized Coefficients for GEM 7 and 8
(units of 10^{-6})

ℓ m	GEM 7	GEM 8	ℓ m	GEM 7	GEM 8	ℓ m	GEM 7	GEM 8
C 2 0	-484.1646	-484.1646	C 16 1	0.0156	0.0136	S 15 2	-0.0304	-0.0479
C 3 0	0.9588	0.9584	S 16 1	-0.0039	0.0057	C 16 2	-0.0025	-0.0085
C 4 0	0.5400	0.5400	C 17 1	-0.0276	-0.0273	S 16 2	0.0231	0.0324
C 5 0	0.0675	0.0681	S 17 1	-0.0196	-0.0095	C 17 2	—	-0.0356
C 6 0	-0.1507	-0.1505	C 18 1	-0.0017	-0.0013	S 17 2	—	0.0353
C 7 0	0.0941	0.0933	S 18 1	-0.0025	0.0024	C 18 2	—	0.0046
C 8 0	0.0503	0.0501	C 19 1	—	-0.0398	S 18 2	—	0.0167
C 9 0	0.0259	0.0262	S 19 1	—	-0.0115	C 19 2	—	0.0469
C 10 0	0.0542	0.0540	C 20 1	—	-0.0158	S 19 2	—	-0.0340
C 11 0	-0.0452	-0.0444	S 20 1	—	-0.0360	C 20 2	—	-0.0230
C 12 0	0.0359	0.0377	C 21 1	—	-0.0052	S 20 2	—	0.0587
C 13 0	0.0386	0.0365	S 21 1	—	0.0218	C 21 2	—	0.0044
C 14 0	-0.0220	-0.0248	C 22 1	—	0.0190	S 21 2	—	0.0153
C 15 0	0.0078	0.0102	S 22 1	—	0.0096	C 22 2	—	0.0136
C 16 0	-0.0055	-0.0031	C 23 1	—	0.0078	S 22 2	—	0.0088
C 17 0	0.0122	0.0108	S 23 1	—	0.0186	C 23 2	—	0.0203
C 18 0	0.0082	0.0061	C 24 1	—	-0.0068	S 23 2	—	-0.0282
C 19 0	0.0021	0.0016	S 24 1	—	-0.0132	C 24 2	—	-0.0350
C 20 0	0.0194	0.0181	C 25 1	—	0.0189	S 24 2	—	0.0149
C 21 0	0.0006	0.0023	S 25 1	—	-0.0032	C 25 2	—	0.0089
C 22 0	-0.0023	0.0008	C 26 1	—	—	S 25 2	—	0.0169
C 23 0	-0.0226	-0.0241	S 26 1	—	—	C 26 2	—	—
C 24 0	-0.0006	-0.0026	C 27 1	—	—	S 26 2	—	—
C 25 0	-0.0026	-0.0026	S 27 1	—	—	C 27 2	—	—
C 26 0	0.0092	0.0078	C 28 1	—	—	S 27 2	—	—
C 27 0	0.0039	0.0045	S 28 1	—	—	C 28 2	—	—
C 28 0	-0.0094	-0.0046	C 29 1	—	—	S 28 2	—	—
C 29 0	0.0021	0.0031	S 29 1	—	—	C 29 2	—	—
C 30 0	—	—	C 30 1	—	—	S 29 2	—	—
C 2 1	-0.0031	-0.0001	S 30 1	—	—	C 30 2	—	—
S 2 1	-0.0099	0.0003	C 2 2	2.4303	2.4345	S 30 2	—	—
C 3 1	2.0296	2.0317	S 2 2	-1.3946	-1.3953	C 3 3	0.7263	0.7162
S 3 1	0.2502	0.2496	C 3 2	0.8972	0.8977	S 3 3	1.4108	1.4169
C 4 1	-0.5326	-0.5374	S 3 2	-0.6193	-0.6233	C 4 3	0.9837	0.9854
S 4 1	-0.4711	-0.4738	C 4 2	0.3465	0.3473	S 4 3	-0.2023	-0.1967
C 5 1	-0.0624	-0.0647	S 4 2	0.6623	0.6657	C 5 3	-0.4588	-0.4617
S 5 1	-0.0840	-0.0835	C 5 2	0.6322	0.6618	S 5 3	-0.2152	-0.2045
C 6 1	-0.0788	-0.0714	S 5 2	-0.3242	-0.3262	C 6 3	0.0556	0.0516
S 6 1	0.0215	0.0300	C 6 2	0.0481	0.0524	S 6 3	-0.0114	-0.0003
C 7 1	0.2692	0.2716	S 6 2	-0.3615	-0.3640	C 7 3	0.2452	0.2339
S 7 1	0.0957	0.0992	C 7 2	0.3263	0.3205	S 7 3	-0.2412	-0.2176
C 8 1	0.0295	0.0196	S 7 2	0.1002	0.0949	C 8 3	-0.0324	-0.0162
S 8 1	0.0520	0.0421	C 8 2	0.0730	0.0736	S 8 3	-0.1104	-0.0963
C 9 1	0.1517	0.1549	S 8 2	0.0594	0.0704	C 9 3	-0.1971	-0.1754
S 9 1	0.0272	0.0170	C 9 2	0.0213	0.0373	S 9 3	-0.0627	-0.0620
C 10 1	0.0795	0.0903	S 9 2	-0.0327	-0.0280	C 10 3	-0.0154	-0.0316
S 10 1	-0.1259	-0.1201	C 10 2	-0.0758	-0.0663	S 10 3	-0.1581	-0.1387
C 11 1	0.0209	0.0182	S 10 2	-0.0366	-0.0512	C 11 3	-0.0193	-0.0546
S 11 1	-0.0201	-0.0016	C 11 2	0.0278	0.0140	S 11 3	-0.1615	-0.1090
C 12 1	-0.0659	-0.0720	S 11 2	-0.1044	-0.1119	C 12 3	0.0486	0.0576
S 12 1	-0.0576	-0.0561	C 12 2	0.0013	-0.0134	S 12 3	0.0232	0.0230
C 13 1	-0.0502	-0.0263	S 12 2	0.0123	0.0293	C 13 3	-0.0201	-0.0187
S 13 1	0.0543	0.0301	C 13 2	0.0355	0.0144	S 13 3	0.1121	0.0636
C 14 1	-0.0199	-0.0016	S 13 2	-0.0430	-0.0355	C 14 3	0.0366	0.0354
S 14 1	0.0552	0.0372	C 14 2	-0.0417	-0.0291	S 14 3	0.0012	-0.0050
C 15 1	0.0137	0.0113	S 14 2	0.0085	-0.0027	C 15 3	-0.0042	0.0251
S 15 1	0.0053	0.0021	C 15 2	-0.0053	0.0010	S 15 3	-0.0067	0.0302

Table 2 (Continued)

ℓ m	GEM 7	GEM 8	ℓ m	GEM 7	GEM 8	ℓ m	GEM 7	GEM 8
C 16 3	-0.0643	-0.0173	S 18 4	—	0.0091	C 22 5	—	0.0090
S 16 3	-0.0599	-0.0330	C 19 4	—	-0.0034	S 22 5	—	-0.0037
C 17 3	—	-0.0003	S 19 4	—	-0.0604	C 23 5	—	0.0059
S 17 3	—	0.0033	C 20 4	—	-0.0235	S 23 5	—	-0.0030
C 18 3	—	0.0226	S 20 4	—	-0.0457	C 24 5	—	-0.0343
S 18 3	—	-0.0035	C 21 4	—	0.0203	S 24 5	—	-0.0086
C 19 3	—	0.0059	S 21 4	—	0.0393	C 25 5	—	0.0011
S 19 3	—	-0.0193	C 22 4	—	-0.0017	S 25 5	—	-0.0035
C 20 3	—	-0.0305	S 22 4	—	0.0059	C 26 5	—	—
S 20 3	—	-0.0003	C 23 4	—	-0.0131	S 26 5	—	—
C 21 3	—	0.0333	S 23 4	—	-0.0179	C 27 5	—	—
S 21 3	—	0.0076	C 24 4	—	0.0061	S 27 5	—	—
C 22 3	—	0.0223	S 24 4	—	0.0256	C 28 5	—	—
S 22 3	—	-0.0054	C 25 4	—	0.0250	S 28 5	—	—
C 23 3	—	-0.0339	S 25 4	—	-0.0051	C 29 5	—	—
S 23 3	—	-0.0072	C 26 4	—	—	S 29 5	—	—
C 24 3	—	-0.0026	S 26 4	—	—	C 30 5	—	—
S 24 3	—	-0.0016	C 27 4	—	—	S 30 5	—	—
C 25 3	—	-0.0005	S 27 4	—	—	C 6 6	0.0035	0.0109
S 25 3	—	-0.0037	C 28 4	—	—	S 6 6	-0.2500	-0.2593
C 26 3	—	—	S 28 4	—	—	C 7 6	-0.3569	-0.3492
S 26 3	—	—	C 29 4	—	—	S 7 6	0.1276	0.1266
C 27 3	—	—	S 29 4	—	—	C 8 6	-0.0743	-0.0628
S 27 3	—	—	C 30 4	—	—	S 8 6	0.3053	0.2953
C 28 3	—	—	S 30 4	—	—	C 9 6	0.0357	0.0514
S 28 3	—	—	C 5 5	0.1685	0.1525	S 9 6	0.1933	0.1930
C 29 3	—	—	S 5 5	-0.6751	-0.6808	C 10 6	-0.0179	-0.0106
S 29 3	—	—	C 6 5	-0.2532	-0.2590	S 10 6	-0.1022	-0.0956
C 30 3	—	—	S 6 5	-0.5254	-0.5291	C 11 6	-0.0393	-0.0229
S 30 3	—	—	C 7 5	0.0057	-0.0084	S 11 6	0.0089	0.0151
C 4 4	-0.1966	-0.1954	S 7 5	0.0479	0.0371	C 12 6	-0.0034	0.0038
S 4 4	0.3063	0.3053	C 8 5	-0.0188	-0.0210	S 12 6	0.0353	0.0178
C 5 4	-0.2859	-0.2823	S 8 5	0.0868	0.0807	C 13 6	-0.0785	-0.0668
S 5 4	0.0497	0.0518	C 9 5	0.0297	0.0063	S 13 6	-0.0128	0.0115
C 6 4	-0.1051	-0.1020	S 9 5	-0.0560	-0.0532	C 14 6	-0.0360	-0.0252
S 6 4	-0.4583	-0.4562	C 10 5	-0.0282	-0.0610	S 14 6	-0.0175	0.0059
C 7 4	-0.2816	-0.2673	S 10 5	-0.0571	-0.0451	C 15 6	0.0231	0.0444
S 7 4	-0.1102	-0.1243	C 11 5	0.0749	0.0358	S 15 6	-0.0594	-0.0589
C 8 4	-0.2467	-0.2341	S 11 5	0.0962	0.1010	C 16 6	-0.0413	-0.0015
S 8 4	0.0645	0.0680	C 12 5	0.0706	0.0428	S 16 6	0.0027	-0.0384
C 9 4	-0.0097	-0.0090	S 12 5	0.0183	-0.0025	C 17 6	—	-0.0635
S 9 4	0.0057	0.0125	C 13 5	0.1350	0.0705	S 17 6	—	-0.0248
C 10 4	-0.1140	-0.1113	S 13 5	0.0729	0.0540	C 18 6	—	-0.0126
S 10 4	-0.0642	-0.0889	C 14 5	0.0920	0.0350	S 18 6	—	-0.0220
C 11 4	-0.0504	-0.0448	S 14 5	-0.0452	-0.0454	C 19 6	—	0.0389
S 11 4	-0.0407	-0.0904	C 15 5	0.0622	0.0100	S 19 6	—	0.0625
C 12 4	-0.0514	-0.0628	S 15 5	0.0399	0.0528	C 20 6	—	0.0440
S 12 4	-0.0167	-0.0254	C 16 5	-0.0386	-0.0326	S 20 6	—	-0.0140
C 13 4	0.0046	-0.0040	S 16 5	-0.0114	0.0003	C 21 6	—	0.0023
S 13 4	-0.0186	-0.0126	C 17 5	—	-0.0209	S 21 6	—	-0.0145
C 14 4	-0.0310	-0.0078	S 17 5	—	0.0086	C 22 6	—	-0.0044
S 14 4	0.0192	-0.0089	C 18 5	—	0.0127	S 22 6	—	0.0047
C 15 4	-0.0668	-0.0358	S 18 5	—	-0.0118	C 23 6	—	-0.0365
S 15 4	0.0063	-0.0137	C 19 5	—	-0.0228	S 23 6	—	0.0124
C 16 4	0.0395	0.0373	S 19 5	—	-0.0214	C 24 6	—	-0.0290
S 16 4	0.0005	0.0477	C 20 5	—	-0.0195	S 24 6	—	0.0261
C 17 4	—	-0.0179	S 20 5	—	0.0224	C 25 6	—	0.0237
S 17 4	—	0.0629	C 21 5	—	0.0027	S 25 6	—	-0.0106
C 18 4	—	0.0098	S 21 5	—	-0.0079	C 26 6	—	—

Table 2 (Continued)

ℓ m	GEM 7	GEM 8
S 26 6	—	—
C 27 6	—	—
S 27 6	—	—
C 28 6	—	—
S 28 6	—	—
C 29 6	—	—
S 29 6	—	—
C 30 6	—	—
S 30 6	—	—
C 7 7	-0.0225	0.0133
S 7 7	0.0228	-0.0006
C 8 7	0.0669	0.0763
S 8 7	0.0754	0.0786
C 9 7	-0.1133	-0.0765
S 9 7	-0.0645	-0.0640
C 10 7	-0.0004	0.0175
S 10 7	-0.0035	-0.0049
C 11 7	-0.0081	0.0129
S 11 7	-0.0681	-0.0774
C 12 7	-0.0330	-0.0333
S 12 7	0.0442	0.0403
C 13 7	-0.0180	0.0023
S 13 7	0.0233	0.0039
C 14 7	0.0146	0.0271
S 14 7	-0.0225	0.0056
C 15 7	0.0704	0.0604
S 15 7	-0.0114	0.0302
C 16 7	-0.0200	-0.0017
S 16 7	0.0075	0.0015
C 17 7	—	0.0325
S 17 7	—	-0.0251
C 18 7	—	0.0143
S 18 7	—	-0.0136
C 19 7	—	0.0071
S 19 7	—	-0.0018
C 20 7	—	-0.0166
S 20 7	—	-0.0142
C 21 7	—	-0.0209
S 21 7	—	0.0217
C 22 7	—	0.0083
S 22 7	—	0.0202
C 23 7	—	-0.0057
S 23 7	—	-0.0151
C 24 7	—	0.0143
S 24 7	—	0.0294
C 25 7	—	-0.0159
S 25 7	—	-0.0053
C 26 7	—	—
S 26 7	—	—
C 27 7	—	—
S 27 7	—	—
C 28 7	—	—
S 28 7	—	—
C 29 7	—	—
S 29 7	—	—
C 30 7	—	—
S 30 7	—	—
C 8 8	-0.1341	-0.1089
S 8 8	0.1065	0.1106
C 9 8	0.2182	0.2432
S 9 8	-0.0124	-0.0054
C 10 8	0.0256	0.0461
S 10 8	-0.1090	-0.0935
C 11 8	0.0011	0.0116
S 11 8	0.0329	0.0679
C 12 8	-0.0223	-0.0154
S 12 8	-0.0010	0.0123
C 13 8	-0.0072	-0.0027
S 13 8	0.0101	0.0175
C 14 8	-0.0341	-0.0172
S 14 8	-0.0067	-0.0267
C 15 8	-0.0274	-0.0173
S 15 8	0.0049	0.0337
C 16 8	-0.0300	-0.0254
S 16 8	-0.0194	0.0064
C 17 8	—	0.0261
S 17 8	—	-0.0348
C 18 8	—	0.0157
S 18 8	—	0.0252
C 19 8	—	0.0238
S 19 8	—	-0.0186
C 20 8	—	-0.0168
S 20 8	—	0.0573
C 21 8	—	0.0097
S 21 8	—	-0.0268
C 22 8	—	0.0001
S 22 8	—	-0.0142
C 23 8	—	-0.0077
S 23 8	—	0.0182
C 24 8	—	0.0056
S 24 8	—	-0.0216
C 25 8	—	0.0444
S 25 8	—	-0.0067
C 26 8	—	—
S 26 8	—	—
C 27 8	—	—
S 27 8	—	—
C 28 8	—	—
S 28 8	—	—
C 29 8	—	—
S 29 8	—	—
C 30 8	—	—
S 30 8	—	—
C 9 9	-0.0547	-0.0482
S 9 9	0.0901	0.0856
C 10 9	0.1226	0.1237
S 10 9	-0.0559	-0.0438
C 11 9	-0.0280	-0.0315
S 11 9	0.0471	0.0659
C 12 9	0.0542	0.0276
S 12 9	-0.0010	0.0058
C 13 9	0.0347	0.0129
S 13 9	0.0702	0.0700
C 14 9	0.0710	0.0164
S 14 9	-0.0049	0.0064
C 15 9	0.0227	-0.0032
S 15 9	0.0586	0.0297
C 16 9	0.0167	-0.0086
S 16 9	-0.0708	-0.0515
C 17 9	—	-0.0435
S 17 9	—	-0.0519
C 18 9	—	0.0089
S 18 9	—	0.0356
C 19 9	—	-0.0164
S 19 9	—	-0.0104
C 20 9	—	0.0558
S 20 9	—	-0.0123
C 21 9	—	-0.0087
S 21 9	—	0.0275
C 22 9	—	0.0096
S 22 9	—	0.0177
C 23 9	—	0.0027
S 23 9	—	-0.0118
C 24 9	—	-0.0214
S 24 9	—	-0.0082
C 25 9	—	-0.0103
S 25 9	—	-0.0054
C 26 9	—	—
S 26 9	—	—
C 27 9	—	—
S 27 9	—	—
C 28 9	—	—
S 28 9	—	—
C 29 9	—	—
S 29 9	—	—
C 30 9	—	—
S 30 9	—	—
C 10 10	0.1007	0.1013
S 10 10	-0.0163	-0.0233
C 11 10	-0.0449	-0.0461
S 11 10	-0.0078	-0.0046
C 12 10	-0.0021	-0.0033
S 12 10	0.0499	0.0471
C 13 10	0.0446	0.0353
S 13 10	-0.0277	-0.0337
C 14 10	0.0505	0.0361
S 14 10	0.0054	-0.0069
C 15 10	0.0141	-0.0033
S 15 10	0.0127	0.0034
C 16 10	-0.0041	0.0109
S 16 10	0.0279	-0.0047
C 17 10	—	0.0055
S 17 10	—	0.0259
C 18 10	—	0.0320
S 18 10	—	-0.0041
C 19 10	—	-0.0284
S 19 10	—	-0.0169
C 20 10	—	-0.0031
S 20 10	—	-0.0165
C 21 10	—	-0.0172
S 21 10	—	-0.0221
C 22 10	—	-0.0024
S 22 10	—	0.0431
C 23 10	—	-0.0011
S 23 10	—	0.0113
C 24 10	—	0.0129
S 24 10	—	-0.0004
C 25 10	—	0.0088
S 25 10	—	-0.0169
C 26 10	—	—
S 26 10	—	—
C 27 10	—	—
S 27 10	—	—

Table 2 (Continued)

ℓ m	GEM 7	GEM 8	ℓ m	GEM 7	GEM 8	ℓ m	GEM 7	GEM 8
C 28 10	—	—	S 18 12	-0.0200	-0.0230	C 30 13	—	—
S 28 10	—	—	C 19 12	-0.0174	-0.0201	S 30 13	—	—
C 29 10	—	—	S 19 12	-0.0031	-0.0064	C 14 14	-0.0518	-0.0519
S 29 10	—	—	C 20 12	-0.0105	-0.0077	S 14 14	-0.0072	-0.0047
C 30 10	—	—	S 20 12	0.0287	0.0096	C 15 14	0.0021	0.0052
S 30 10	—	—	C 21 12	0.0023	-0.0065	S 15 14	-0.0248	-0.0232
C 11 11	0.0605	0.0629	S 21 12	0.0369	0.0286	C 16 14	-0.0161	-0.0158
S 11 11	-0.0692	-0.0462	C 22 12	-0.0189	-0.0142	S 16 14	-0.0421	-0.0390
C 12 11	0.0190	0.0142	S 22 12	-0.0027	-0.0146	C 17 14	-0.0097	-0.0169
S 12 11	-0.0062	-0.0046	C 23 12	—	0.0024	S 17 14	0.0135	0.0117
C 13 11	-0.0292	-0.0300	S 23 12	—	-0.0099	C 18 14	-0.0101	-0.0124
S 13 11	-0.0055	0.0047	C 24 12	—	0.0129	S 18 14	-0.0049	-0.0102
C 14 11	0.0312	0.0292	S 24 12	—	-0.0301	C 19 14	-0.0023	-0.0005
S 14 11	-0.0310	-0.0380	C 25 12	—	-0.0333	S 19 14	-0.0144	-0.0159
C 15 11	0.0162	0.0089	S 25 12	—	0.0112	C 20 14	0.0181	0.0219
S 15 11	-0.0054	-0.0145	C 26 12	—	—	S 20 14	-0.0065	-0.0030
C 16 11	0.0225	0.0030	S 26 12	—	—	C 21 14	0.0074	0.0132
S 16 11	0.0050	-0.0096	C 27 12	—	—	S 21 14	0.0068	0.0113
C 17 11	0.0081	0.0066	S 27 12	—	—	C 22 14	0.0027	0.0099
S 17 11	-0.0413	-0.0172	C 28 12	—	—	S 22 14	0.0033	0.0127
C 18 11	-0.0258	-0.0269	S 28 12	—	—	C 23 14	-0.0005	-0.0008
S 18 11	-0.0061	0.0074	C 29 12	—	—	S 23 14	-0.0076	-0.0047
C 19 11	0.0345	0.0305	S 29 12	—	—	C 24 14	-0.0181	-0.0273
S 19 11	0.0071	0.0183	C 30 12	—	—	S 24 14	0.0045	-0.0028
C 20 11	—	0.0226	S 30 12	—	—	C 25 14	-0.0003	-0.0273
S 20 11	—	-0.0005	C 13 13	-0.0609	-0.0581	S 25 14	0.0195	0.0067
C 21 11	—	0.0022	S 13 13	0.0676	0.0681	C 26 14	0.0143	0.0092
S 21 11	—	-0.0309	C 14 13	0.0256	0.0325	S 26 14	0.0310	0.0142
C 22 11	—	-0.0013	S 14 13	0.0439	0.0403	C 27 14	0.0137	0.0263
S 22 11	—	-0.0084	C 15 13	-0.0253	-0.0232	S 27 14	0.0010	-0.0002
C 23 11	—	-0.0069	S 15 13	-0.0031	-0.0039	C 28 14	-0.0409	-0.0300
S 23 11	—	0.0100	C 16 13	0.0084	0.0143	S 28 14	-0.0015	-0.0050
C 24 11	—	0.0168	S 16 13	0.0016	-0.0023	C 29 14	-0.0312	-0.0406
S 24 11	—	-0.0012	C 17 13	0.0168	0.0175	S 29 14	-0.0096	-0.0091
C 25 11	—	-0.0042	S 17 13	0.0249	0.0274	C 30 14	—	—
S 25 11	—	0.0189	C 18 13	-0.0192	-0.0106	S 30 14	—	—
C 26 11	—	—	S 18 13	-0.0358	-0.0402	C 15 15	-0.0191	-0.0240
S 26 11	—	—	C 19 13	-0.0094	-0.0086	S 15 15	-0.0103	-0.0065
C 27 11	—	—	S 19 13	-0.0202	-0.0163	C 16 15	0.0078	0.0087
S 27 11	—	—	C 20 13	0.0029	0.0160	S 16 15	-0.0267	-0.0246
C 28 11	—	—	S 20 13	0.0082	0.0021	C 17 15	0.0069	0.0024
S 28 11	—	—	C 21 13	-0.0219	-0.0188	S 17 15	0.0082	0.0183
C 29 11	—	—	S 21 13	0.0162	0.0159	C 18 15	—	-0.0614
S 29 11	—	—	C 22 13	-0.0243	-0.0114	S 18 15	—	-0.0194
C 30 11	—	—	S 22 13	0.0193	0.0120	C 19 15	-0.0329	-0.0088
S 30 11	—	—	C 23 13	-0.0070	-0.0062	S 19 15	-0.0217	-0.0305
C 12 12	-0.0071	-0.0050	S 23 13	0.0049	0.0022	C 20 15	-0.0331	-0.0250
S 12 12	-0.0115	-0.0121	C 24 13	-0.0149	-0.0098	S 20 15	-0.0329	-0.0117
C 13 12	-0.0322	-0.0309	S 24 13	0.0038	-0.0012	C 21 15	0.0170	0.0272
S 13 12	0.0889	0.0893	C 25 13	0.0251	0.0166	S 21 15	0.0182	0.0138
C 14 12	0.0095	0.0125	S 25 13	0.0111	0.0157	C 22 15	—	-0.0070
S 14 12	-0.0316	-0.0313	C 26 13	0.0051	0.0039	S 22 15	—	0.0219
C 15 12	-0.0347	-0.0330	S 26 13	0.0040	0.0026	C 23 15	—	0.0175
S 15 12	0.0149	0.0131	C 27 13	0.0164	0.0020	S 23 15	—	-0.0045
C 16 12	0.0164	0.0164	S 27 13	0.0212	0.0317	C 24 15	—	-0.0208
S 16 12	0.0102	0.0044	C 28 13	-0.0559	-0.0641	S 24 15	—	0.0013
C 17 12	0.0273	0.0277	S 28 13	0.0092	0.0104	C 25 15	—	-0.0047
S 17 12	0.0229	0.0194	C 29 13	-0.0063	-0.0158	S 25 15	—	-0.0076
C 18 12	-0.0353	-0.0263	S 29 13	-0.0074	0.0023	C 26 15	—	—

Table 2 (Continued)

ℓ m	GEM 7	GEM 8	ℓ m	GEM 7	GEM 8	ℓ m	GEM 7	GEM 8
S 26 15	—	—	C 27 17	—	—	S 20 20	—	-0.0142
C 27 15	—	—	S 27 17	—	—	C 21 20	—	-0.0036
S 27 15	—	—	C 28 17	—	—	S 21 20	—	0.0158
C 28 15	—	—	S 28 17	—	—	C 22 20	—	0.0056
S 28 15	—	—	C 29 17	—	—	S 22 20	—	0.0081
C 29 15	—	—	S 29 17	—	—	C 23 20	—	-0.0285
S 29 15	—	—	C 30 17	—	—	S 23 20	—	-0.0054
C 30 15	—	—	S 30 17	—	—	C 24 20	—	-0.0126
S 30 15	—	—	C 18 18	—	-0.0151	S 24 20	—	0.0029
C 16 16	-0.0367	-0.0294	S 18 18	—	-0.0416	C 25 20	—	-0.0032
S 16 16	0.0164	0.0009	C 19 18	—	0.0660	S 25 20	—	-0.0142
C 17 16	-0.0313	-0.0258	S 19 18	—	0.0180	C 26 20	—	—
S 17 16	0.0045	-0.0024	C 20 18	—	-0.0038	S 26 20	—	—
C 18 16	—	0.0449	S 20 18	—	-0.0221	C 27 20	—	—
S 18 16	—	0.0161	C 21 18	—	0.0355	S 27 20	—	—
C 19 16	—	-0.0081	S 21 18	—	0.0023	C 28 20	—	—
S 19 16	—	-0.0022	C 22 18	—	0.0160	S 28 20	—	—
C 20 16	—	-0.0029	S 22 18	—	0.0107	C 29 20	—	—
S 20 16	—	0.0054	C 23 18	—	0.0087	S 29 20	—	—
C 21 16	—	0.0049	S 23 18	—	-0.0069	C 30 20	—	—
S 21 16	—	-0.0125	C 24 18	—	0.0159	S 30 20	—	—
C 22 16	—	-0.0392	S 24 18	—	0.0130	C 21 21	—	0.0005
S 22 16	—	-0.0099	C 25 18	—	-0.0051	S 21 21	—	-0.0240
C 23 16	—	0.0543	S 25 18	—	0.0007	C 22 21	—	-0.0115
S 23 16	—	-0.0057	C 26 18	—	—	S 22 21	—	0.0390
C 24 16	—	0.0120	S 26 18	—	—	C 23 21	—	0.0274
S 24 16	—	0.0161	C 27 18	—	—	S 23 21	—	-0.0362
C 25 16	—	0.0068	S 27 18	—	—	C 24 21	—	-0.0271
S 25 16	—	-0.0250	C 28 18	—	—	S 24 21	—	0.0501
C 26 16	—	—	S 28 18	—	—	C 25 21	—	0.0111
S 26 16	—	—	C 29 18	—	—	S 25 21	—	-0.0367
C 27 16	—	—	S 29 18	—	—	C 26 21	—	—
S 27 16	—	—	C 30 18	—	—	S 26 21	—	—
C 28 16	—	—	S 30 18	—	—	C 27 21	—	—
S 28 16	—	—	C 19 19	—	-0.0314	S 27 21	—	—
C 29 16	—	—	S 19 19	—	-0.0199	C 28 21	—	—
S 29 16	—	—	C 20 19	—	0.0036	S 28 21	—	—
C 30 16	—	—	S 20 19	—	0.0070	C 29 21	—	—
S 30 16	—	—	C 21 19	—	-0.0403	S 29 21	—	—
C 17 17	-0.0221	-0.0458	S 21 19	—	0.0174	C 30 21	—	—
S 17 17	-0.0022	-0.0206	C 22 19	—	0.0067	S 30 21	—	—
C 18 17	-0.0439	0.0068	S 22 19	—	-0.0001	C 22 22	—	-0.0039
S 18 17	-0.0192	0.0030	C 23 19	—	0.0200	S 22 22	—	-0.0055
C 19 17	—	0.0424	S 23 19	—	0.0080	C 23 22	—	0.0056
S 19 17	—	-0.0346	C 24 19	—	-0.0279	S 23 22	—	0.0296
C 20 17	—	-0.0246	S 24 19	—	-0.0166	C 24 22	—	-0.0217
S 20 17	—	0.0310	C 25 19	—	-0.0048	S 24 22	—	-0.0203
C 21 17	—	0.0109	S 25 19	—	0.0202	C 25 22	—	0.0044
S 21 17	—	-0.0009	C 26 19	—	—	S 25 22	—	-0.0009
C 22 17	—	0.0408	S 26 19	—	—	C 26 22	—	—
S 22 17	—	-0.0511	C 27 19	—	—	S 26 22	—	—
C 23 17	—	-0.0181	S 27 19	—	—	C 27 22	—	—
S 23 17	—	0.0068	C 28 19	—	—	S 27 22	—	—
C 24 17	—	-0.0290	S 28 19	—	—	C 28 22	—	—
S 24 17	—	-0.0031	C 29 19	—	—	S 28 22	—	—
C 25 17	—	-0.0070	S 29 19	—	—	C 29 22	—	—
S 25 17	—	0.0053	C 30 19	—	—	S 29 22	—	—
C 26 17	—	—	S 30 19	—	—	C 30 22	—	—
S 26 17	—	—	C 20 20	—	0.0020	S 30 22	—	—

Table 2 (Continued)

ℓ m	GEM 7	GEM 8	ℓ m	GEM 7	GEM 8	ℓ m	GEM 7	GEM 8
C 23 23	—	-0.0105	C 28 24	—	—	C 29 26	0.0155	0.0157
S 23 23	—	-0.0229	S 28 24	—	—	S 29 26	-0.0386	-0.0388
C 24 23	—	0.0096	C 29 24	—	—	C 30 26	—	—
S 24 23	—	0.0017	S 29 24	—	—	S 30 26	—	—
C 25 23	—	0.0223	C 30 24	—	—	C 27 27	0.0027	0.0022
S 25 23	—	-0.0146	S 30 24	—	—	S 27 27	-0.0084	-0.0066
C 26 23	—	—	C 25 25	—	-0.0000	C 28 27	-0.0053	-0.0043
S 26 23	—	—	S 25 25	—	0.0193	S 28 27	0.0015	0.0016
C 27 23	—	—	C 26 25	—	—	C 29 27	—	—
S 27 23	—	—	S 26 25	—	—	S 29 27	—	—
C 28 23	—	—	C 27 25	—	—	C 30 27	—	—
S 28 23	—	—	S 27 25	—	—	S 30 27	—	—
C 29 23	—	—	C 28 25	—	—	C 28 28	0.0109	0.0126
S 29 23	—	—	S 28 25	—	—	S 28 28	0.0043	0.0017
C 30 23	—	—	C 29 25	—	—	C 29 28	—	—
S 30 23	—	—	S 29 25	—	—	S 29 28	—	—
C 24 24	—	-0.0040	C 30 25	—	—	C 30 28	-0.0078	-0.0084
S 24 24	—	0.0013	S 30 25	—	—	S 30 28	0.0015	0.0001
C 25 24	—	0.0258	C 26 26	—	—	C 29 29	—	—
S 25 24	—	0.0074	S 26 26	—	—	S 29 29	—	—
C 26 24	—	—	C 27 26	0.0059	0.0060	C 30 29	—	—
S 26 24	—	—	S 27 26	-0.0254	-0.0261	S 30 29	—	—
C 27 24	—	—	C 28 26	0.0107	0.0106	C 30 30	—	—
S 27 24	—	—	S 28 26	-0.0033	-0.0033	S 30 30	—	—

least squares adjustment of 400 harmonic coefficients, 369 station coordinates, and 5000 electronic pass-biases. The observing stations consisted of 3200 orbit parameters, 24 Baker-Nunn cameras, 31 minitrack optical test site cameras, 6 international cameras, 33 tranet Doppler receivers, 5 Goddard range and range rate antennas, 7 C-band antennas, 14 minitrack arrays, 3 French lasers, 6 Smithsonian lasers, and 6 Goddard lasers.

The new satellite solution is superior to GEM 5 in performance on the 362 data-arcs common to both models. This is due mainly to the reiteration of the data and the finer detail of GEM 7. Sensitive tests on independent data also attest to the superiority of GEM 7. Figure 1 shows the results of orbital reductions of 22 short (6 hour) arcs of one station, 4-pass laser ranges to the BE-C satellite. The "noise" level of these ranges is 50 cm and is closely approached only with the use of GEM 7. All three models shown use considerable amounts of BE-C data, though not this set. Figure 2 shows results of similar reductions of one day arcs of

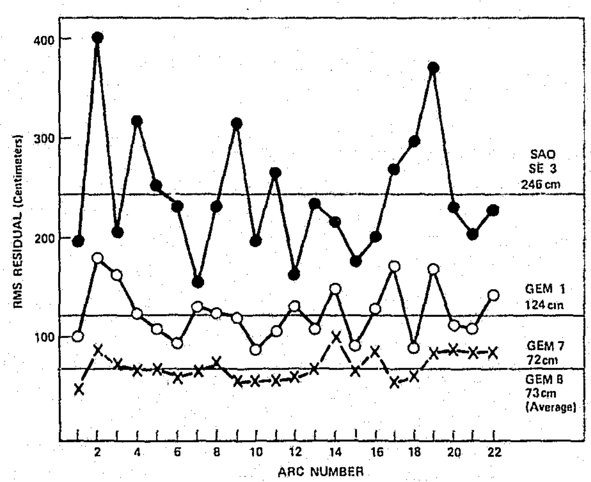

Figure 1. Godlas Range Residuals to BE-C with Recent Geopotentials (Arcs are 6 Hours Long)

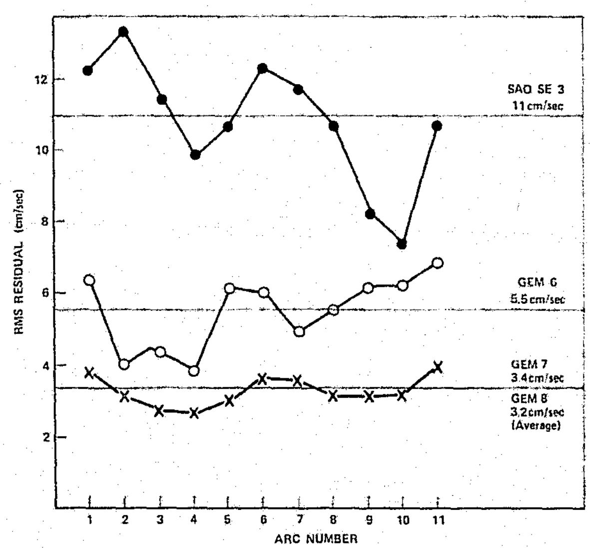

Figure 2. S-Band 2-Way Doppler Residuals (11 Daily ERTS Arcs)

data on the earth resources technology satellite (ERTS) from 10 worldwide S-band stations. Here the orbit also is new to all models (99° inclination, 900 km altitude). Ionospheric refraction uncertainties limit the usefulness of these measurements to 2 cm/sec. Again, this limit is approached with GEM 7.

The improvement of GEM 7 is also seen with respect to the gravimetry used in GEM 8 [Figure 3]. (GEM 8 essentially absorbs all the gravimetric information.) There is striking evidence here that, in spite of the improvement at shorter wavelengths GEM 7 is still relatively weak there. The formal precision of the GEM 7 coefficients also confirms this weakness in all but the zonal and resonant terms.

Indeed the improvement of resonant terms in GEM 7 and 8 is confirmed from comparisons with linear constraints developed from long term tracking of deep resonant orbits external to these models (Table 3). It is encouraging that the gravimeter data improves even the lowest order harmonics. The absolute accuracy of these fields, quoted in the abstract, is based primarily on the comparisons with the resonant and gravimetric data. This accuracy is also confirmed by comparison with independent zonal coefficients obtained from long-term orbit perturbations by Cazenave et. al, 1971 and Wagner, 1973.

The overall features of the gravity field have not changed substantially in 10 years (Figures 4 and 5) but there is new detail in GEM 8 that has not been seen in previous solutions. Figure 6 shows the portion of its anomaly field from 13 through 22nd degree, presumably arising from irregularities in the upper mantle. In two regions of the Pacific, west of Hawaii and west of Peru, run strong lines of alternating anomalies trending westward with the apparent motion of the Pacific and Nazca plates (B. Marsh, Private Communication, 1975; EOS, 56, 1063, Dec. 1975). These may be evidence of convective rolls in the mantle (Richter and Parsons, 1975) below these rapidly moving plates. They do not seem to be ephemeral features of a weak solution, a full (30, 30) field with the same data shows the same features. But gravimetry is sparse in the Pacific. Confirmation of these anomaly trends waits on utilization of altimetry from the GEOS 3 satellite.

Table 3

Residuals Using Independent Resonant Data
(Accelerations and Lumped Harmonics)

FIELD	RESONANT ORDER							
	1 and 2*	9**	10**	11**	12**	13**	14**	TOTAL**
GEM 1	15.3	16.0	57.3	33.0	56.1	8.8	4.9	23.9
GEM 7	3.3	12.0	20.3	11.9	17.6	7.3	5.5	9.1
GEM 8	2.1	14.3	38.4	5.8	13.1	7.7	4.7	7.1
Resonant (Using Data)	1.2	5.0	6.2	1.5	1.4	2.8	0.8	1.3
(No. of Constraints)	(11)	(14)	(4)	(24)	(14)	(18)	(24)	(224)

*Data is RMS Residual in units of 10^{-8} rad/day^2 (drift of 24 hour-satellite)
**Data is RMS Residual in units of 10^{-9} (normalized lumped potential coefficient)

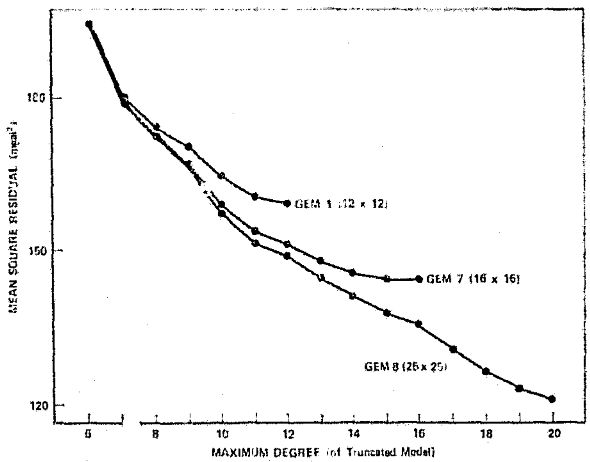

Figure 3. Gravity Anomaly Residuals with Truncated Geopotentials (Observations are Mean Gravity Anomalies for 1352 5° Equal Area Blocks)

Acknowledgments

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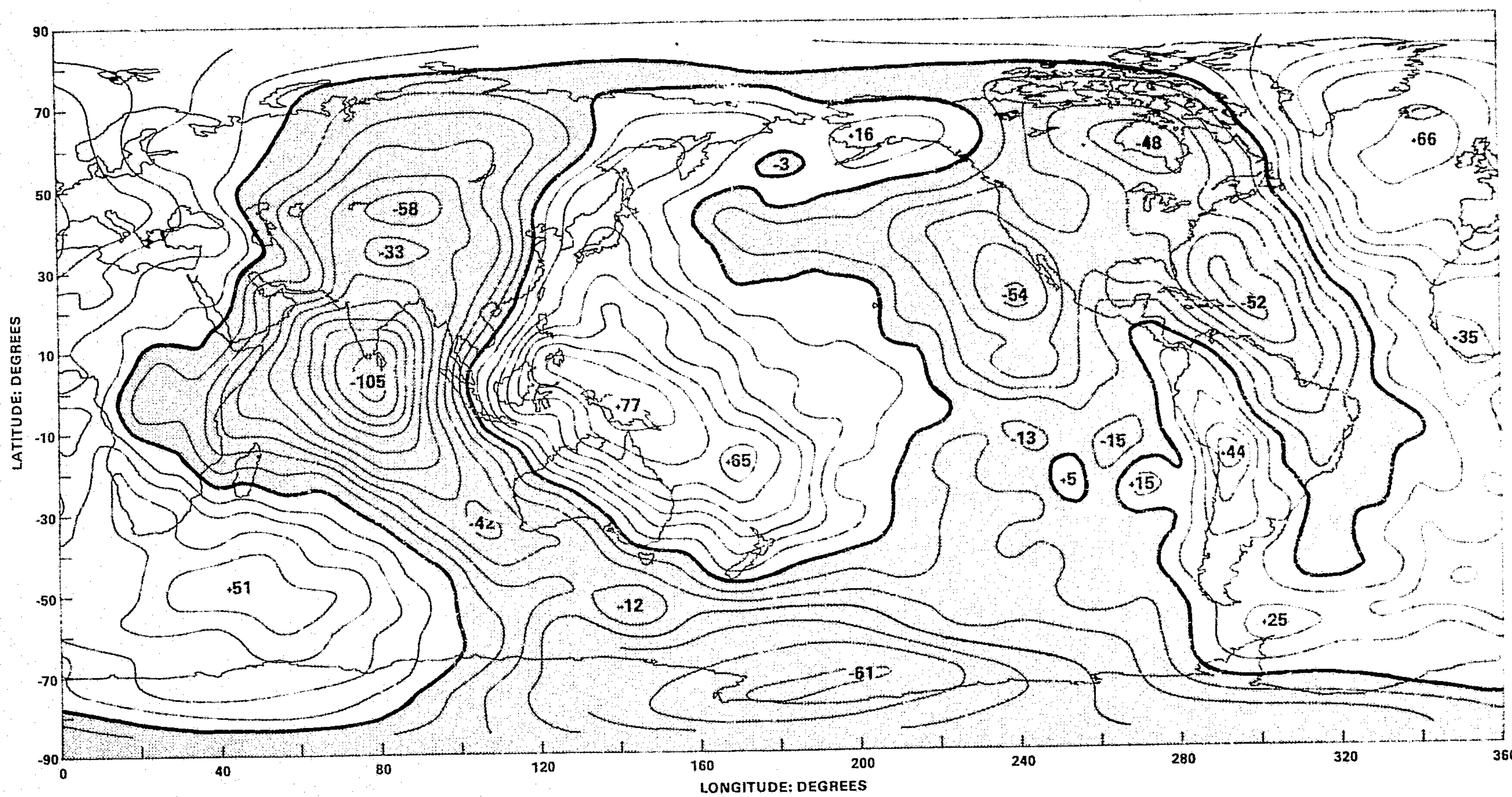

Figure 4. Geoid Height Contours – GEM 8 (Referenced to an Ellipsoid of Flattening 1/298.255), Contour Interval = 10 Meters

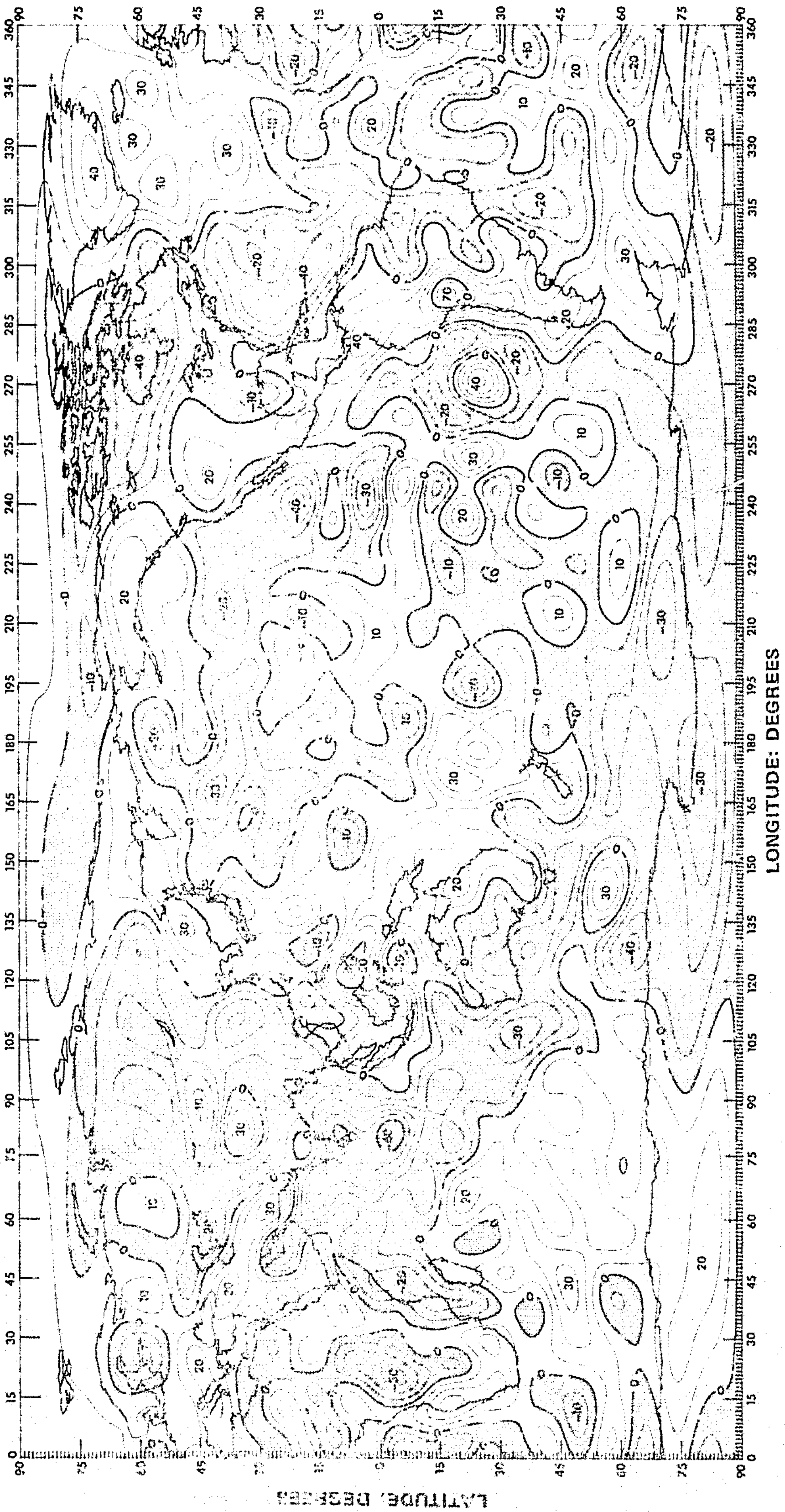

Figure 5. Contour Map of Free Air Gravity Anomalies, GEM 8 Model, Contour Interval 10 mgal

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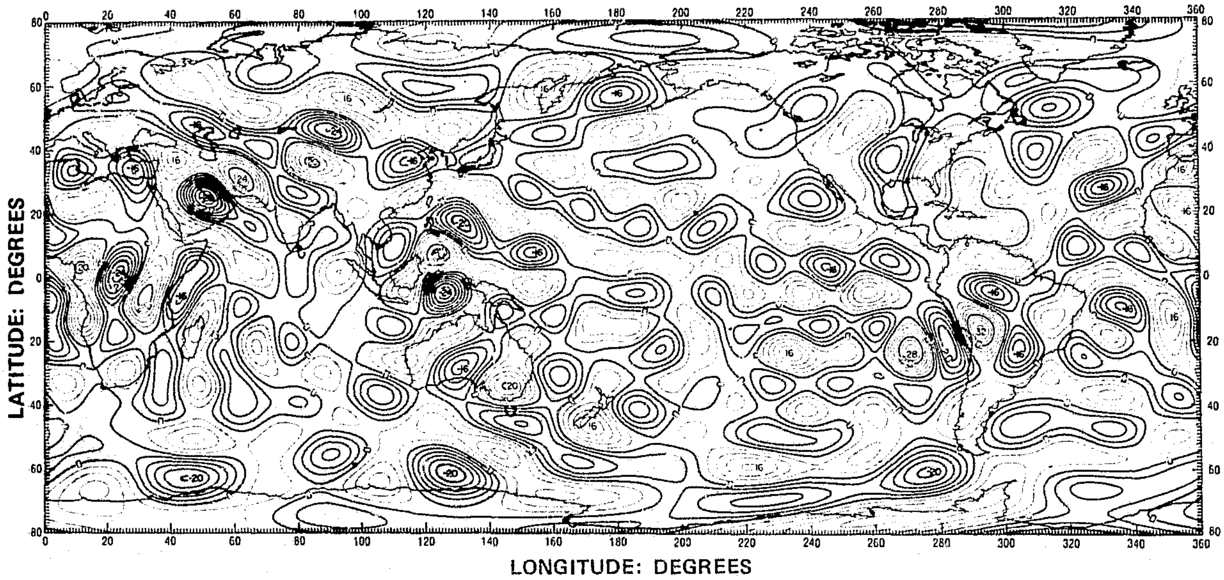

Figure 6. Contour Map of Free Air Gravity Anomalies, GEM 8 Model, Coefficients of Degrees 13 through 22, Contour Interval: 4 mgal