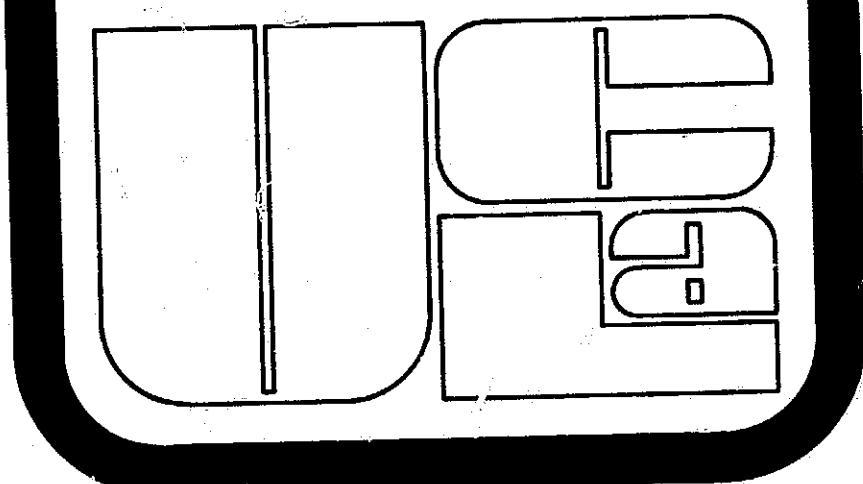


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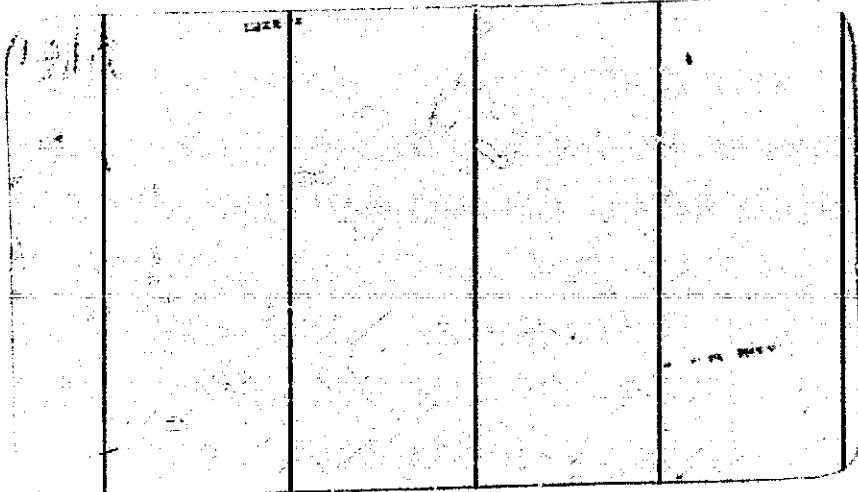
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**LOSSY RADIAL DIFFUSION  
OF  
RELATIVISTIC JOVIAN ELECTRONS**

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# LOSSY RADIAL DIFFUSION OF RELATIVISTIC JOVIAN ELECTRONS

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## ABSTRACT

The radial diffusion equation with synchrotron losses is solved by the Laplace transform method for near-equatorially mirroring relativistic electrons. The evolution of a power law distribution function is found and the characteristics of synchrotron burn-off are stated in terms of explicit parameters for an arbitrary diffusion coefficient of the form  $D_{LL} = D_0 L^\alpha$ . The peaking of the 10.4 cm volume emissivity from Jupiter at  $L=1.8$  provides an estimate  $D_0 \approx 9 \times 10^{-11} \text{ sec}^{-1}$  in the radiation belts; this value is suggested as the appropriate modification for an equatorial field strength of 4.2 Gauss of Birmingham et al.'s (1974) result. Non-synchrotron losses are included phenomenologically and from the phase space densities reported by McIlwain and Fillius (1975) the particle lifetime is estimated as  $\tau \sim 6 \times 10^8 L^{2-\alpha} \text{ sec}$ . Asymptotic forms for the distribution in the strong synchrotron loss regime are provided.

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## I INTRODUCTION

Prior to the Pioneer 10 and 11 missions to Jupiter a considerable effort was made to estimate the fluxes of energetic particles in the radiation belts. The basic starting point was the two-dimensional interferometer maps of non-thermal synchrotron radiation provided by Berge (1966) and Branson (1968) and deconvolved by Beard and Luthey (1973) into equatorial emissivities. The radial diffusion equation with synchrotron losses was then solved (Birmingham et al., 1974; Coroniti, 1974; Stansberry and White, 1974) and parameters varied until a best fit with the equatorial emissivity was found. This procedure gave a determination of the radial diffusion coefficient which was in reasonable agreement with theoretical descriptions of the diffusion process based on atmospheric neutral winds driving a fluctuating dynamo electric field (Brice and McDonough, 1973; Coroniti, 1974).

The data provided by Pioneers 10 and 11 have been consistent with the model of an intense core of electrons with pitch angles near 90° confined to equatorial magnetic latitudes and undergoing inward radial diffusion. The pancaked nature of the pitch angle distribution was demonstrated by Van Allen et al. (1974), who modeled the distribution as  $f \propto \sin^M \theta$  and found that for the 21, 31 MeV detectors  $M = 3.5, 4$  gave good closure for the inbound/outbound Pioneer 10 data between  $4 < L < 12$ . The signature of inward radial diffusion is manifest in all flux diagrams,  $J(L)$ , especially in the vicinity of the Jovian

satellites; if measured phase space densities at constant first invariant decrease towards the planets, as in Fig. 1, an inward diffusive current must result when the particles' third invariant is violated.

The purpose of this paper is to re-examine the radial diffusion problem in light of the measurements of Pioneers 10 and 11. Previous analyses have used models for the electron distribution which differed significantly in form from those measured, and all conclusions heretofore have relied on numerical computations, with underlying physical relationships not manifest (viz., how are Birmingham et al.'s (1974) results modified by an improved measurement of the magnetic field strength?). The mounting evidence that radial diffusion is a fundamental magnetospheric transport process warrants an analytic solution with conclusions of general applicability.

We solve the steady-state radial diffusion equation with synchrotron losses for near-equatorially mirroring relativistic electrons, driven by a diffusion coefficient modeled as  $D_{LL} = D_0 L^\alpha$ , by the Laplace transform method previously utilized by Coroniti (1974). This procedure brings the analysis in close analogy with the problem of heat conduction in solids (Carslaw and Jaeger, 1959). Our solution is similar to that of Birmingham et al. (1974), who used an eigenfunction expansion technique with a delta function source of particles in the outer belt. We, however, concentrate on the evolution of a distribution function of particles (a power law in momenta) whose properties are now fairly well known. The solution is shown to vary as the loss-free

solution up to a point where synchrotron losses become dominant. A precise statement of this synchrotron "burn-off" regime is found, where significant reduction of the phase space density occurs due to rapid synchrotron degradation on a time scale comparable to that of radial diffusion. The Laplace transform technique has the advantage of providing an integral representation for the solution from which asymptotic forms for the distribution in the "burn-off" regime can be obtained. For application to Jupiter, the peak of the 10.4 cm volume emissivity at  $L = 1.8$  (Birmingham et al., 1974) provides a lower bound to the magnitude of the diffusion coefficient (i.e., diffusion is strong enough to transport electrons into at least  $L = 1.8$ ). The decay of the volume emissivity at smaller  $L$  values is presumed to be due to synchrotron degradation of the distribution which provides an upper bound to the diffusion coefficient.

Since McIlwain and Fillius (1975) have found significant decreases in the phase space density inside of Io which are non-synchrotron associated, we have included a phenomenological loss term and resolved the radial diffusion equation excluding synchrotron losses. It is argued that for a particle lifetime which varies as  $\tau = \tau_0 L^{2-\alpha}$ , the solution is a power law in the loss region, consistent with observation. From the slope of measured phase space densities, the magnitude of particle lifetime,  $\tau_0$ , is estimated.

Finally, if the particle lifetime persists as  $\tau = \tau_0 L^{2-\alpha}$  into the synchrotron emission region ( $L \sim 2$ ), a complete solution which includes both phenomenological and synchrotron losses is provided.

## II SOLUTION OF THE RADIAL DIFFUSION EQUATION WITH SYNCHROTRON LOSSES

The basic equation governing the steady-state radial diffusion with synchrotron losses is (Schulz and Lanzerotti, 1974)

$$\frac{\partial}{\partial t} f(\vec{x}, \vec{p}, t) = L^2 \frac{\partial}{\partial L} \left[ \frac{1}{L^2} D_{LL} \frac{\partial f}{\partial L} \right] - \frac{\partial}{\partial M} [\langle \dot{M} \rangle f] - \frac{\partial}{\partial J} [\langle \dot{J} \rangle f] = 0 \quad (1)$$

$f(\vec{x}, \vec{p}, t)$  is the phase space distribution function averaged over phase angles conjugate to  $M$ ,  $J$ ,  $\phi$  and  $n(\vec{x}, t) = \int d\vec{p} f$  is the local density.  $M = \frac{p^2 \sin^2 \theta}{2mB(L)}$  is the first adiabatic invariant,  $\theta$  is the equatorial pitch-angle,  $J = \oint p_{||} ds$  is the second invariant, and  $L = r/R_J$  is the magnetic shell parameter for a dipole field  $B(L) = B_0 L^{-3}$ . The quantities  $\langle \dot{M} \rangle$  and  $\langle \dot{J} \rangle$  are the bounce-averaged rates of change of  $M$  and  $J$  due to synchrotron energy loss.

Throughout this paper we make the assumptions that most of the electrons mirror near the equator ( $\theta \approx \pi/2$ ) and have ultrarelativistic energies  $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}} \gg 1$ . For this case, the synchrotron degradation of the second invariant is small since  $\langle \dot{J} \rangle \propto \cos^4 \theta$ , and  $\partial/\partial J \langle \dot{J} \rangle \propto \cos^2 \theta$  (Coroniti, 1974); hence the last term in (1) will be dropped compared to  $\langle \dot{M} \rangle$ , which is finite at  $\theta = \pi/2$ . We approximate  $M$  as  $\frac{mc^2}{2} \frac{v^2}{B(L)}$ .

The rate at which  $M$  changes due to synchrotron emission is, to lowest order,

$$\langle \dot{M} \rangle = -\frac{4}{3} \frac{e^4 B_0^2 (L)}{m^3 c^5} \gamma M = -\frac{4\sqrt{2}}{3} \frac{e^4 B_0^{5/2}}{m^{7/2} c^6} \frac{1}{L^{15/2}} M^{3/2} \quad (2)$$

If  $D_{LL} = D_0 L^\alpha$ , equation (1) is of the form

$$\frac{df}{dt} = L^2 \left[ \frac{\partial}{\partial L} D_0 L^{\alpha-2} \frac{\partial f}{\partial L} \right] + \frac{\partial}{\partial M} \left[ \frac{D_s}{L^{15/2}} M^{3/2} f \right] \quad (3)$$

a) Initial Value Problem

The consequences of the synchrotron term can be understood if we drop the diffusion term in equation (3) and solve an initial value problem with  $f(t = 0) = f_0(M) = A M^{-\frac{1}{2}(N+2)} \theta(M_0 - M)$ .  $\theta(x)$  is the Heaviside step function which introduces an arbitrary cutoff,  $M_0$ , at large values of the first invariant.  $N$  is the spectral index of the distribution and  $A$  is a normalization. Two cases are of interest for comparison with the diffusion problem developed in the next section: if  $N = 2$  the solution is

$$f(M, N = 2, t) = \frac{A}{M^2} \left[ 1 - \frac{\sqrt{MD_s}}{2L^{15/2}} t \right] \theta(\bar{M}_0 - M) \quad (4)$$

where

$$\bar{M}_0 = \frac{M_0}{(1 + \frac{\sqrt{MD_s}}{2L^{15/2}} t)^2} \quad (5)$$

If  $t \gg t_s = \frac{2L^{15/2}}{\sqrt{MD_s}}$  (the synchrotron energy half-life of the cutoff) a significant burn-off of particles occurs at  $\bar{M}_0$  even though  $M \ll M_0$ . The second term in equation (4) is a synchrotron correction to the  $N = 2$  spectrum. For  $N = 1$  the solution is

$$f(M, N = 1, t) = \frac{A}{M^{3/2}} \theta(\bar{M}_o - M) \quad (6)$$

with  $\bar{M}_o$  the same as (5). The synchrotron correction is absent since  $f \propto M^{-3/2}$  is a "stationary" solution.

### b) Diffusive Boundary Problem

The diffusive boundary value problem has much the same effect. If the diffusion process cannot transport particles in faster than their synchrotron degradation, burn-off should occur. If we estimate characteristic times from equation (3)

$$\tau_{\text{SYNC}} \sim \frac{L^{15/2}}{D_s/M}, \quad \tau_{\text{R.D.}} \sim \frac{1}{D_o L^{\alpha-2}}$$

then

$$\frac{\tau_{\text{R.D.}}}{\tau_{\text{SYNC}}} \sim \frac{D_s \sqrt{M}}{D_o L^{\alpha+11/2}} = \frac{\sqrt{M}}{D L^{\alpha+11/2}} = 1 \quad (7)$$

where

$$D \equiv \frac{3}{4\sqrt{2}} \frac{D_o^m c^6}{e^4 B_o^{5/2}} \quad (8)$$

roughly marks the transition from the synchrotron burn-off regime to the pure radial diffusion regime (weak synchrotron losses).

In the steady-state equation (3) becomes

$$DL^{19/2} \frac{\partial}{\partial L} \left[ L^{\alpha-2} \frac{\partial f}{\partial L} \right] + \frac{\partial}{\partial M} \left[ M^{3/2} f \right] = 0 \quad (9)$$

Letting  $G = M^{3/2} f$ ,  $y = \sqrt{M}$ , then if  $G(s) \equiv \int_0^\infty e^{-sy} G(y) dy$ ,

$$L^2 \frac{d^2 G}{dL^2} + (\alpha - 2)L \frac{dG}{dL} - \frac{s}{2DL^{\alpha+11/2}} G = 0 \quad (10)$$

where we require

$$\lim_{y \rightarrow 0} e^{-sy} G(L, y) = \lim_{M \rightarrow \infty} e^{-\sqrt{M}} M^{3/2} f(L, M) = 0 \quad (11)$$

The solution of equation (10) subject to the boundary conditions  $G(L = L_i) = 0$  at the surface,  $L_i = 1$ , and  $G(L = L_o) = G_o(y)$  is

$$f(L, M) = \left( \frac{L}{L_o} \right)^{\frac{1}{2}(3-\alpha)} \frac{1}{M^{3/2}} \frac{1}{2\pi i} \int_{-i\infty+\epsilon}^{i\infty+\epsilon} ds \cdot e^{sy} G_o(y) \begin{bmatrix} I_{+v}(xr) I_{-v}(xb) - I_v(xb) I_{-v}(xr) \\ I_v(xa) I_{-v}(xb) - I_v(xb) I_{-v}(xa) \end{bmatrix} \quad (12)$$

where the contour lies to the right of all poles of the integrand,  $I_{\pm v}(xr)$  is the modified Bessel function and

$$x = \frac{1}{|s + 11/2|} \sqrt{\frac{2s}{D}} \quad v = \frac{3 - \alpha}{\alpha + 11/2}$$

$$r = L^{-\frac{1}{2}(\alpha + 11/2)} \quad b = L_i^{-\frac{1}{2}(\alpha + 11/2)} \quad a = L_o^{-\frac{1}{2}(\alpha + 11/2)} \quad (13)$$

At  $L_0$  we assume that the distribution can be represented by a power law  $f_0 = \frac{A}{M^{\frac{1}{2}}(N+2)}$ . Equation (12) then becomes

$$f(L, M) = A \left( \frac{L}{L_0} \right)^{\frac{1}{2}(3-\alpha)} \frac{1}{M^{\frac{1}{2}}(N+2)} \frac{r(N)}{2\pi i} \int_{-i\infty + \epsilon}^{i\infty + \epsilon} \frac{dt e^{tL}}{t^N} \cdot \frac{\left[ I_v(\lambda r) I_{-v}(\lambda b) - I_v(\lambda b) I_{-v}(\lambda r) \right]}{\left[ I_v(\lambda a) I_{-v}(\lambda b) - I_v(\lambda b) I_{-v}(\lambda a) \right]} \quad (14)$$

with

$$\lambda^2 = \frac{2Mt}{(\alpha + 11/2)^2 D} \quad v = \frac{3-\alpha}{\alpha + 11/2} \quad (15)$$

If  $v$  is an integer the bracket is replaced by its limiting value.

While  $I_{\pm v}(\lambda r)$  has a branch point at the origin, the combination in the denominator of the bracket is a single-valued even function of  $\lambda$ ; as a function of  $t$ , all of the poles of the bracket lie on the negative real axis (Gray and Mathews, 1922). If  $N$  is an integer the contour can be closed by a semi-circle to the left and the integral is evaluated by the theory of residues in Appendix A.

The case of  $N = 1$  is relevant for synchrotron-emitting particles at Jupiter since the time-averaged power spectrum of non-thermal emissions is flat between 200 MHz  $< f <$  3000 MHz (Carr and Gulkis, 1969); if the E-M spectrum varies as  $f^{-\delta}$ ,  $\delta$  is related to the spectral index by  $\delta = \frac{1}{2}(N-1)$  (see equation (36)). Figure 1 is a plot of phase-space densities reported by McIlwain and Fillius (1975) from Pioneer 10 measurements of energetic electrons. For  $L > 10$  the particles have a spectrum

which has  $N \sim 3.5$ . Inside Europa and Io, the spectrum of low energy particles ( $\gamma \sim 2 - 16$ ) hardens considerably and at  $L = 3$  has settled into a profile of  $N \approx 1$ ; the corresponding range of  $M$  is 6.4 - 360 MeV/G.

From Appendix A we have for  $N = 2$

$$f(L, M, N = 2) = A \left( \frac{L}{L_0} \right)^{\frac{1}{2}(3-\alpha)} \frac{1}{M^2} \left[ A_0 + \frac{1}{4} \lambda^2 A_1 \right]$$

$$- \lambda^2 \sum_{n=1}^{\infty} \frac{1}{Y_{v,n}^2} e^{-\frac{Y_{v,n}^2}{\lambda^2}}$$

$$\cdot \frac{\pi J_v(Y_{v,n}^a) J_v(Y_{v,n}^b)}{J_v^2(Y_{v,n}^b) - J_v^2(Y_{v,n}^a)}$$

$$\cdot [J_v(Y_{v,n}^b) Y_v(Y_{v,n}r) - (r \rightarrow b)] \quad (16)$$

and for  $N = 1$ , neglecting edge effects at  $M_0$  ( $M \ll M_0$ )

$$f(L, M, N = 1) = A \left( \frac{L}{L_0} \right)^{\frac{1}{2}(3-\alpha)} \frac{1}{M^{3/2}} \left[ A_0 + \sum_{n=1}^{\infty} e^{-\frac{Y_{v,n}^2}{\lambda^2}} \right] \cdot \frac{\pi J_v(Y_{v,n}^a) J_v(Y_{v,n}^b)}{J_v^2(Y_{v,n}^b) - J_v^2(Y_{v,n}^a)} \cdot [J_v(Y_{v,n}^b) Y_v(Y_{v,n}r) - (r \rightarrow b)] \quad (17)$$

where  $\lambda^2 = \frac{2/M}{(\alpha + 1/2)^2 D}$  and  $A_0, A_1$  are functions of  $L$  defined in Appendix A;  $Y_{v,n}$  are defined such that

$J_v(\gamma_{v,n} b) Y_v(\gamma_{v,n} a) \cdot J_v(\gamma_{v,n} a) Y_v(\gamma_{v,n} b) = 0$ . These solutions can be compared with Birmingham's (1974) equation (11) for the invariant-space distribution function. The similarity with the initial value problem is clear. When  $\bar{\lambda}^2 \rightarrow 0$  only the  $A_0$  term survives to reproduce the loss-free solution to equation (9). The term  $\frac{1}{4}\bar{\lambda}^2 A_1$  is the synchrotron correction to the  $N = 2$  spectrum. The series of Bessel functions contributes when  $\bar{\lambda}^2 \rightarrow \infty$  and strong synchrotron degradation occurs. In fact, if  $\frac{1}{\bar{\lambda}^2} \equiv 0$ , the series in equation (17) is a representation of  $-A_0$  of the type described by Erdélyi et al. (1953). Weak synchrotron losses can be expected when the argument of the exponential is not small and  $\bar{\lambda}^2 \sim \gamma_{v,1}^2$ . A few of the values of  $\gamma_{v,n}$  have been tabulated by Jahnke and Emde (1945). In particular, if  $v = 1/2$  and  $b = 1$ ,  $\gamma_{1/2,n} = \frac{n\pi}{1-a} \approx n\pi$  for  $a = L_0^{-\frac{1}{2}(a+11/2)} \ll 1$ . For other values of  $v$ ,  $\gamma_{v,1}$  is close to  $\pi$ . We conclude that provided

$$\bar{\lambda}^2 = \frac{2\sqrt{M}}{(\alpha + 11/2)^2 D} \lesssim \frac{1}{2}\pi^2 \quad (18)$$

weak synchrotron losses prevail throughout the diffusion to the surface.

The more interesting case involves particles which experience weak losses at  $L_0$  but undergo strong degradation at  $L$ . Inspection of the bracket of equation (14) reveals that the criterion for weak (strong) losses at  $L$  depends on whether  $I_{\pm v}(\lambda r)$  is in the small argument (asymptotic form) for most of that part of the integration up to where the exponential cuts off at  $t = -1$ .

In this case an asymptotic expression for the distribution function

in the burn-off regime can be obtained by expanding the relevant Bessel functions in equation (14) and then inverting the transform. This procedure is carried out in Appendix B. Thus, the appropriate modification of (18) is  $\bar{\lambda}^2 r^2 \lesssim \frac{1}{2}\pi^2$  and provided that

$$\frac{2\sqrt{M}}{(\alpha + 11/2)^2 D} \frac{1}{L^{\alpha+11/2} - 2} \leq \frac{1}{2} \pi^2 \quad (19)$$

weak synchrotron losses will prevail from L to  $L_0$ . This result is similar to the estimate given by equation (7) but differs significantly by the factor of  $(\alpha + 11/2)^{-2}$ . We have arbitrarily chosen the transition criterion at  $\bar{\lambda}^2 r^2 \leq \frac{1}{2} \pi^2$ , but inspection of the argument of the exponential in equations (B.6) or (B.8) suggests an alternative  $\bar{\lambda}^2 r^2 \leq 4$  which modifies the result only slightly. Although the solution (A.14) and in particular equations (16) and (17) are valid for integer values of N, the burn-off criterion (19) and the asymptotic forms are valid generally.

### c) The Diffusion Coefficient

The synchrotron emissions (which peak at  $L \approx 1.8$  for 10.4 cm radiation) span the frequencies  $\Delta f = 200 - 3000$  MHz. If the electrons emit predominantly at the characteristic frequency  $f_c = \frac{3}{2} f_{0\gamma} \gamma^2 = \frac{3}{4\pi} \frac{eB(L)}{mc} \gamma^2$ , then for a surface magnetic field strength of 4.2 Gauss,  $\Delta\gamma = 8 - 32$  at  $L = 1.8$ , corresponding to  $\Delta M = 22 - 352$  MeV/G. Equation (19) can be expressed as

$$D_0 \geq \frac{(3.1 \times 10^{-9}) B_0^{5/2} \sqrt{M(\text{MeV/G})}}{(a + 11/2)^2 L^a + 11/2}$$

$$= \frac{7.7 \times 10^{-10} B_0^{3/2} / f_c(\text{MHz})}{(a + 11/2)^2 L^{a+5/2}} (\text{sec}^{-1}) \quad (20)$$

The Pioneer 10 data (McIlwain and Fillius, 1975) indicate that for  $\Delta M = 6.4 - 360 \text{ MeV/G}$  the spectral index  $N \approx 1$  at  $L = 3$ . The flat synchrotron spectrum is consistent with  $N = 1$  throughout the strong synchrotron radiation region, which extends to at least  $L = 1.8$ . The flat synchrotron spectrum suggests that the radiating electrons diffuse to  $L = 1.8$  without a significant synchrotron degradation of the  $N = 1$  spectrum, we use (20) with the upper value of  $M = 352 \text{ MeV/G}$  ( $f_c = 3000 \text{ MHz}$ ) to arrive at an estimate of the minimum diffusion coefficient  $D_0$ , which is consistent with the preservation of the  $N = 1$  spectrum. We find that the minimum value  $D_0 \text{ min} \times 10^{10} = 0.88, 2.0, 4.6, 11 \text{ sec}^{-1}$  for  $a = 4, 3, 2, 1$ . Note that for the 3000 MHz emitting particles to diffuse to the surface with weak losses requires  $D_0 \times 10^{10} \geq 40, 50, 64, 86$ .

### III NON SYNCHROTRON LOSSES

#### a) General Scaling Laws

For weak synchrotron losses we need retain only the  $A_0$  term in the distribution function, and since  $A_0(L, v) = A_0(L, -v)$

$$f(L) \sim \begin{cases} L^{\frac{1}{2}(3-\alpha)+\frac{1}{2}|3-\alpha|} & : \alpha \neq 3 \\ \ln L & : \alpha = 3 \end{cases} \quad (21)$$

McIlwain and Fillius (1975) have pointed out that Fig. 1 shows  $f(L) \sim L^q$  where  $q=4$ , which is possible in the absence of any losses only if  $\alpha$  is negative. Since serious synchrotron losses yield exponential dependences on  $M$  and  $L$ , and since the weak synchrotron loss criterion is satisfied by these particles within the limits of experimental measurements of the diffusion coefficient, another loss mechanism must be active. If we drop the synchrotron term in equation (1) and replace it by a phenomenological loss term, then

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} \left[ \frac{1}{L^2} D_{LL} \frac{\partial f}{\partial L} \right] - \frac{1}{\tau(L, M)} f = 0 \quad (22)$$

where  $\tau(L, M)$  is the particle lifetime.

The inclusion of energy dependences is straightforward here; letting  $D_{LL} = D_0 L^{\alpha M/2}$  and  $\tau(L) = \tau_0 L^{B \bar{M}/2}$  where  $\bar{M}$  is the first invariant arbitrarily normalized,

$$L^2 \frac{\partial^2 f}{\partial L^2} + (\alpha - 2)L \frac{\partial f}{\partial L} - \frac{M^{-\frac{1}{2}(m+\delta)}}{D_o T_o} L^{2-\alpha-\beta} f(L; M) = 0 \quad (23)$$

Using the boundary conditions  $f(L = L_i) = 0$  and  $f(L = L_o) = f_o(M)$   
the solution of (23) is

$$f(L, M) = \left(\frac{L}{L_o}\right)^{\frac{1}{2}(3-\alpha)} f_o \frac{\left[ I_v(xr) I_{-v}(xb) - I_{-v}(xr) I_v(xb) \right]}{\left[ I_v(xa) I_{-v}(xb) - I_{-v}(xa) I_v(xb) \right]} \quad (24)$$

and

$$v = \frac{3-\alpha}{\alpha+\beta-2} \quad x = \frac{2}{1\alpha+\beta-2} \sqrt{\frac{1}{M} M^{-\frac{1}{2}(m+\delta)}}$$

$$r = L_i^{-\frac{1}{2}(\alpha+\beta-2)} \quad b = L_i^{-\frac{1}{2}(\alpha+\beta-2)} \quad a = L_o^{-\frac{1}{2}(\alpha+\beta-2)} \quad (25)$$

For weak losses, the solution (24) behaves like equation (21).

For strong losses, the solution behaves asymptotically as

$$f(L, M) \sim \left(\frac{L}{L_o}\right)^{\frac{1}{2}(3-\alpha)} f_o \frac{\sqrt{a}}{r} \frac{\sinh[x(b-r)]}{\sinh[x(b-a)]} \quad (26)$$

Arbitrary values of  $\beta$  yield exponential dependences on  $L$ . But,  
for the special case  $\beta = 2 - \alpha$  it can be shown that (24) reduces  
to

$$f(L, M) \xrightarrow{\beta=2-\alpha} \left(\frac{L}{L_o}\right)^{\frac{1}{2}(3-\alpha)} f_o \frac{\left( \frac{L}{L_o} \right)^{\frac{1}{2}(3-\alpha)C}}{\left[ \frac{1 - (L_i/L)^{(3-\alpha)C}}{1 - (L_i/L_o)^{(3-\alpha)C}} \right]} \quad (27)$$

where

$$c = \left[ 1 + \frac{4}{(3-\alpha)^2 D_0 \tau_0} \bar{M}^{-\frac{1}{2}(m+\delta)} \right]^{-\frac{1}{2}} \quad (28)$$

then outside of the surface absorption region  $\left[ (L_i/L)^{1/(3-\alpha)} c \ll 1 \right]$ , the solution scales like

$$f(L, M) \approx \left( \frac{L}{L_0} \right)^q f_0(M) \quad (29)$$

where

$$q = \frac{1}{2}(3-\alpha) + \frac{1}{2}|3-\alpha|c \quad (30)$$

Solving for the magnitude of the particle lifetime

$$\frac{1}{D_0 \tau_0} \bar{M}^{-\frac{1}{2}(m+\delta)} = q(q + \alpha - 3) \quad (31)$$

which is, of course, what will result if we assume a power law for  $f$  a priori and insert it into (22). If  $m + \delta = 0$  and if we choose  $\alpha = 4, 3, 2$ , then estimating the slope from Fig. 1 as  $q = 4$ , we have as an estimate for  $D_0 \tau_0 = \frac{1}{20}, \frac{1}{16}, \frac{1}{12}$ .

The fact that the observed distribution behaves like a power law in the loss region,  $L < 10$ , supports the argument that  $\tau(L) \sim L^{2-\alpha}$ . This situation, as stated above, is only the requirement that anomalous losses just balance the diffusion source and that  $f$  behave as a power law in  $L$ .

If the corotation drift dominates over the curvature-gradient drift at Jupiter for these particles, we expect  $m = 0$  (Coroniti, 1974). Figure 1 demonstrates a weak dependence on  $\bar{M}$  inside

the loss region (i.e.,  $q \neq q(\bar{M})$ ) which supports the approximation  $\delta = 0$ .

Although the loss mechanism has not been confirmed, loss cone precipitation is the leading candidate. For a dipole field the minimum precipitation lifetime on strong pitch-angle diffusion is (Coroniti, 1974)  $\tau_{\min} = \frac{2L R_J \gamma}{c(\gamma^2 - 1)^{\frac{1}{2}}}$  which is independent of energy for relativistic electrons. For the size of the diffusion coefficient suggested in section II ( $D_0 \sim 10^{-10} \text{ sec}^{-1}$ ), we find that for  $D_0 \tau_c = \frac{1}{20}$ ,  $\tau \gg \tau_{\min}$  and that if pitch-angle diffusion is the loss mechanism, the weak pitch-angle diffusion regime prevails.

We note that if the particle lifetime persists as  $\tau = \tau_0 L^{2-\alpha}$  into the synchrotron emission region and that no other, different mechanism takes over at small  $L < 2$  values, then if  $m = \delta = 0$ , synchrotron losses can be included and all of the formulas of section II still hold with the only substitution  $v \rightarrow \frac{3+\alpha}{\alpha+11/2} \left[ 1 + \frac{4}{(3-\alpha)^2 D_0 \tau} \right]^{\frac{1}{2}}$ . The conclusions reached from equation (19) are, of course, still valid.

### b) Volume Emissivity

On the assumption that weak synchrotron losses persist up to the surface of Jupiter (i.e.,  $D_0 > 40 \times 10^{-10} \text{ sec}^{-1}$ ) and that the lifetime  $\tau = \tau_0 L^{2-\alpha}$  still applies for  $L < 3$ , we ask how does the equatorial emissivity vary with  $L$ . If the decay in the emissivity for  $L < 1.8$  is due not to 1) synchrotron degradation or 2) a different, very intense loss mechanism

which commences just inside of  $L < 2$ , then the decay is due to surface absorption. If  $m + \delta = 0$  and  $C = \left[ 1 + \frac{4}{(3 - \alpha)^2 D_0 \tau_0} \right]^{\frac{1}{2}}$ , equation (27) becomes

$$f(L, \vec{p}) = L^{\frac{1}{2}(3-\alpha)+\frac{1}{2}13-\alpha1C-\frac{3}{2}(N+2)} \left[ 1 - (1/L)^{13-\alpha1C} \right] \frac{1}{p^{N+2}} \\ = g(L) \frac{1}{p^{N+2}} \quad (32)$$

The monochromatic emissivity is defined as

$$\eta_f(\vec{n}) = \int d\vec{p} P_f(\vec{p}, \vec{n}) f(\vec{x}, \vec{p}) \quad (33)$$

where  $P_f(\vec{p}, \vec{n})$  is the power radiated at the frequency  $f$  in the direction  $\vec{n}$  by an electron with momentum  $\vec{p}$ . For simplicity assume the electrons radiate at only the characteristic frequency

$$P_f(\vec{p}, \vec{n}) \propto \gamma^2 B^2(L) \delta(f - f_c) \quad (34)$$

For relativistic particles where  $\gamma \approx \frac{p}{mc}$ , equation (33) scales as

$$\eta_f(\vec{n}) \sim g(L) \frac{1}{L} \int dp \frac{\delta(f - f_c)}{p^{N-2}} \sim g(L) L^{-\frac{3}{2}(N+1)} \int du \frac{\delta(f - u)}{u^{\frac{1}{2}(N-1)}} \quad (35)$$

and we have

$$\eta_f(\vec{n}) \sim L^{\frac{1}{2}(3-\alpha)+\frac{1}{2}13-\alpha1C-3(N+3/2)} \left[ 1 - (1/L)^{13-\alpha1C} \right] f^{-\frac{1}{2}(N-1)} \quad (36)$$

The emissivity maximizes due to surface absorption (no synchrotron losses) at  $L = L'$  where

$$\begin{aligned}
 L' &= \left[ 1 + \frac{13 - \alpha IC}{3(N + 3/2) - \frac{1}{2}(3 - \alpha) - \frac{1}{2}13 - \alpha IC} \right]^{\frac{1}{13 - \alpha IC}} \\
 &= \left[ 1 + \frac{2q + \alpha - 3}{2(N + 3/2) - q} \right]^{\frac{2q + \alpha - 3}{13 - \alpha IC}}
 \end{aligned} \tag{37}$$

#### IV. DISCUSSION

The electrons measured by Pioneers 10 and 11 were in the weak synchrotron loss regime and equation (32) is relevant for them. The uni-directional flux  $\bar{j}(\vec{x}, \vec{p}) = p^2 f(\vec{x}, \vec{p})$  and the omni-directional integral flux is then approximately

$$J(\gamma > \Gamma) \approx L^{\frac{1}{2}(3-\alpha)+\frac{1}{2}3-\alpha} C^{-\frac{3}{2}(N+2)} \quad (38)$$

For  $L > 8$ , where losses are not apparent at high energies, if  $\alpha \geq 4$ ,  $J(\gamma > \Gamma) \approx L^{-3/2(N+2)}$ . In this region  $N \approx 3.5$  for electrons of energies  $E > 35$  MeV and  $J(\gamma > 70) \approx L^{-8.2}$ , consistent with observation.

If we suppose that the diffusion coefficient is large enough to transport the significant synchrotron emitting electrons to the surface, and also that the characteristics of the anomalous loss mechanism do not suddenly change for  $L < 2$ , then the electron flux should drop because of absorption at the surface and the emissivity should peak at  $L'$  given by (37). For reasonable values of parameters  $L'$  is insensitive to  $q$  and  $\alpha$ : if we choose  $\alpha = 4$  and  $N = 1$ ,  $L' = 1.13 - 1.17$  for  $q = 0.5$ . If the resolution of the volume emissivity is high enough to preclude these values, the particles must undergo serious synchrotron losses before reaching the surface. This conclusion puts an upper bound on the magnitude of the diffusion coefficient. With the assumption that the 3000 MHz emitting particles diffuse to  $L = 1.8$  without serious synchrotron degradation, we obtained an estimate for the minimum diffusion coefficient of

0.88, 2.0, 4.6,  $11 < D_o \text{ min} \times 10^{10}$  for  $\alpha = 4, 3, 2, 1$ . Since the 10.4 cm emissivity decreases significantly by  $L = 1.4$ , we can estimate an upper limit on  $D_o$  by assuming that the synchrotron loss criterion (20) is not satisfied at  $L = 1.4$ . Substituting  $f_c = 3000$  MHz and  $L = 1.4$  in (20), the range of  $D_o$  values is bounded by

$$0.88, 2.0, 4.6, 11 < D_o \times 10^{10} < 4.5, 7.9, 14, 26 \text{ (sec}^{-1}\text{)} \quad (39)$$

for  $\alpha = 4, 3, 2, 1$ . The criteria of minimum synchrotron degradation at  $L = 1.8$  and serious losses at  $L = 1.4$  limits the range of  $D_o$  to less than a factor of 4.

Mogro-Campero has reported (see Fillius et al., 1975) a measurement of the diffusion coefficient at  $l_0$  of  $D(L) = D_{l_0} \left(\frac{L}{5.9}\right)^{n_d}$  where  $1.5 \times 10^{-8} < D_{l_0} < 3.1 \times 10^{-7} \text{ sec}^{-1}$  and  $3.6 < n_d < 4.0$ . Using the extreme values of  $n_d$  gives  $0.12, 0.25 < D_o \times 10^{10} < 2.6, 5.2$  for  $n_d = 4, 3.6$ . We note that in (39) the actual value of  $D_o$  is likely to be closer to  $D_o \text{ min}$ , since the upper value of  $M$  (352 MeV/G) which we have used corresponds to the 3000 MHz ( $\sim 10.4$  cm) emitting particles, and the 10.4 cm emissivity peaks near  $L = 1.8$ . Thus, if we estimate  $D_o \text{ min}$  as a probable value for  $D_o$ , this value agrees well with Mogro-Camperos' measurement. Simpson et al. (1974) have reported a measurement of the diffusion coefficient  $2 \times 10^{-7} < D_{l_0} < 10^{-6} \text{ sec}^{-1}$ ; assuming  $\alpha = 4$ ,  $1.5 < D_o \times 10^{10} < 7.7$ , which is also consistent with equation (39).

Birmingham et al. (1974), using a pre-Pioneer 10 value of 10 Gauss for the magnetic field, concluded that a diffusion

coefficient of  $1.7 \times 10^{-9} L^{1.95} \text{ sec}^{-1}$  gave the best fit to the 10.4 cm emissivity. We note that equation (20) gives for  $B_0 = 10 \text{ Gauss}$  and  $\alpha = 2$  the value  $D_0 \text{ min} = 1.7 \times 10^{-9} \text{ sec}^{-1}$  when  $f_c = 3000 \text{ MHz}$  and  $L = 1.8$ .

Our conclusions can be ordered in decreasing L value:

1) The fact that the phase space density obeys a power law reasonably well in the loss region  $3 < L < 10$  of Fig. 1 suggests the electron lifetime varies as  $\tau = \tau_0 L^{2-\alpha}$  and that the relation (31) holds in this region.

2) The inclusion of synchrotron losses which are characterized by exponential dependences in both the particle energy spectrum and particle flux,  $J(L)$ , in the strong synchrotron loss regime demonstrates that strong synchrotron losses do not occur for 3000 MHz emitting electrons for  $L \leq 1.8$ . This provides a lower bound for the magnitude of the diffusion coefficient given by (39).

3) If the characteristics of the loss mechanism do not alter and intensify at  $L \leq 1.8$  then the decay of the 10.4 cm volume emissivity is concomitant with strong synchrotron losses and the diffusion coefficient is bracketed by equation (39).

4) The scaling of the particle lifetime as  $L^{2-\alpha}$  and the magnitude, which is estimated as  $6 \times 10^8 \text{ sec}$  from equations (39) and (31) for  $\alpha = 4$ , indicate that if pitch-angle diffusion of the electrons is the culprit, the weak pitch-angle diffusion regime prevails.

5) In the strong synchrotron loss limit the power law

dependence in energy of the distribution should collapse and behave exponentially as equation (B.8). An in situ measurement of the distribution ( $> 352$  Mev/G) inside  $L \leq 1.8$  would test the hypothesis that serious synchrotron losses are present. We note also that for  $\alpha = 4$  and  $D_0 = 8.8 \times 10^{-11} \text{ sec}^{-1}$ , equation (20) requires the 200 MHz (150 cm) volume emissivity to peak at  $L \approx 1.5$ .

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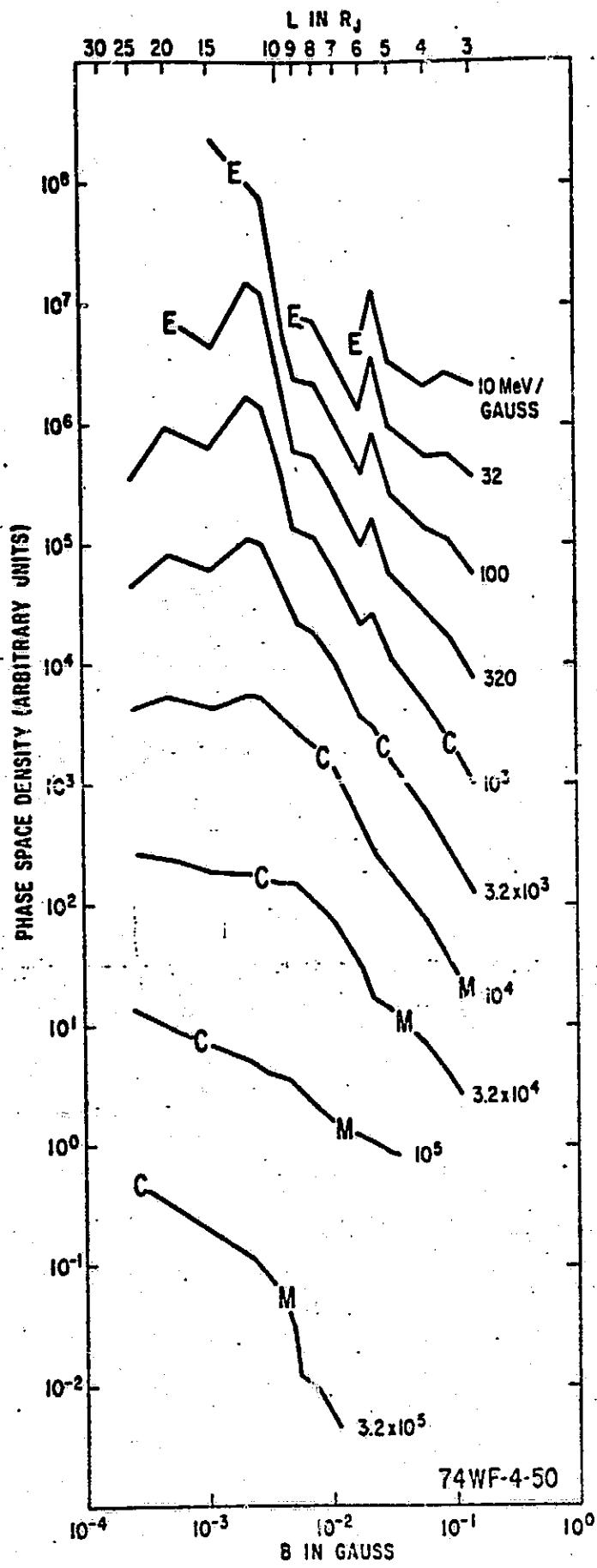
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### Figure Caption

**Figure 1. Phase space densities reported by McIlwain and Fillius (1974) from Pioneer 10 data, evaluated at constant first invariant M.**

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## APPENDIX A

We must evaluate

$$I = \frac{\Gamma(N)}{2\pi i} \int_{-i\infty+\epsilon}^{i\infty+\epsilon} \frac{dt}{t^N} e^t \varphi(t) \quad (A.1)$$

where

$$\varphi(t) = \frac{I_v(\lambda r) I_{-v}(\lambda b) - I_v(\lambda b) I_{-v}(\lambda r)}{I_v(\lambda a) I_{-v}(\lambda b) - I_v(\lambda b) I_{-v}(\lambda a)} \quad (A.2)$$

If  $N$  is an integer, the integrand is single-valued and analytic except for simple poles of  $\varphi(t)$  on the negative real axis and the pole at the origin. Also, if  $N = 1$  we must introduce an upper limit cutoff for  $f_0(M)$  to satisfy equation (11).

If we close the contour by a semicircle to the left, which gives no contribution, since at best  $\varphi(t) \sim e^{\sqrt{t}}$  compared to  $e^t$  (a rigorous treatment can be found in Carslaw and Jaeger, 1959), then by residue theory

$$I = \left( \frac{d}{dt} \right)^{N-1} \left[ e^t \varphi(t) \right]_{t=0} + r(N) \sum_{\text{res}} \frac{e^t}{t^N} \text{Res}[\varphi(t)] \quad (A.3)$$

Note that the anti-symmetric combination  $\varphi(t)$  can be written

$$\varphi(t) = \frac{I_v(\lambda r) K_v(\lambda b) - I_v(\lambda b) K_v(\lambda r)}{I_v(\lambda a) K_v(\lambda b) - I_v(\lambda b) K_v(\lambda a)} \quad (A.4)$$

$$\text{If } \lambda = e^{\frac{i\pi}{2}} z \text{ and } Y_v(z) = \frac{J_v(z) \cos v\pi - J_{-v}(z)}{\sin v\pi}$$

then

$$\varphi(t) = \frac{J_v(zr) Y_v(zb) - J_v(zb) Y_v(zr)}{J_v(za) Y_v(zb) - J_v(zb) Y_v(za)} \quad (\text{A.5})$$

If the roots of  $J_v(x) Y_v(x\frac{b}{a}) - J_v(x\frac{b}{a}) Y_v(x) = 0$  are denoted by  $\gamma_{v,n}$ , the poles lie in the t plane at  $t = -t_{v,n} = -\frac{1}{\lambda^2} \gamma_{v,n}^2$ .

The second term in (A.3) can be expressed as

$$I_2 = \Gamma(N) \sum_{n=1}^{\infty} \frac{e^{-t}}{t^{N-1}} \left. \frac{I_v(\lambda r) I_{-v}(\lambda b) - I_v(\lambda b) I_{-v}(\lambda r)}{t \frac{d}{dt} [I_v(\lambda a) I_{-v}(\lambda b) - I_v(\lambda b) I_{-v}(\lambda a)]} \right|_{t=-t_{v,n}} \quad (\text{A.6})$$

Using the property that the Wronskian  $[I_v, I_{-v}] = -\frac{2}{\pi} \sin v\pi$ ,

it is straightforward to show that

$$\begin{aligned} & \left. -t \frac{d}{dt} [I_v(\lambda a) I_{-v}(\lambda b) - I_v(\lambda b) I_{-v}(\lambda a)] \right|_{t=-t_{v,n}} \\ &= \frac{\sin v\pi}{\pi} \frac{J_v^2(\gamma_{v,n} b) - J_v^2(\gamma_{v,n} a)}{J_v(\gamma_{v,n} a) J_v(\gamma_{v,n} b)} \end{aligned} \quad (\text{A.7})$$

and the second term of (A.3) is evaluated as

$$\begin{aligned} I_2 &= \pi \Gamma(N) \left( -\frac{1}{\lambda^2} \right)^{N-1} \sum_{n=1}^{\infty} \frac{e^{-\frac{1}{\lambda^2} \gamma_{v,n}^2}}{\gamma_{v,n}^{2(N-1)}} \\ &\quad \cdot \frac{J_v(\gamma_{v,n} a) J_v(\gamma_{v,n} b)}{J_v^2(\gamma_{v,n} b) - J_v^2(\gamma_{v,n} a)} [J_v(\gamma_{v,n} b) Y_v(\gamma_{v,n} r) - (r \leftrightarrow b)] \end{aligned} \quad (\text{A.8})$$

The first term in (A.3) is due to the pole at the origin and is

$$I_1 = \sum_{\ell=0}^{N-1} \binom{N-1}{\ell} \phi^{(\ell)}(t=0) \quad (A.9)$$

Now  $\phi(t) = \frac{D(r, b, t)}{D(a, b, t)}$ , and utilizing the series expansion of

we have

$$D(r, b, t) = \sum_{n=0}^{\infty} \sum_{k=0}^n \left(\frac{1}{2}\lambda\right)^{2n} t^n$$

$$\cdot \left[ \frac{(r/b)^{2k+v} b^{2n}}{k! (n-k)!} \frac{1}{\Gamma(v+k+1)} \frac{1}{\Gamma(-v+n-k+1)} \right] \quad (A.10)$$

from which we must evaluate  $\phi^{(\ell)}(t=0)$ . The general result is cumbersome, so we shall give only the leading and first order terms. Define

$$A_0(L) = (a/r)^v \left[ \frac{1 - (r/b)^{2v}}{1 - (a/b)^{2v}} \right] \quad (A.11)$$

$$A_1(L) = \frac{b^2 (a/r)^v}{[1 - (a/b)^{2v}]^2} \left\{ \left[ \frac{(r/b)^2}{(-v+1)} \left[ 1 - (r/b)^{2(v-1)} \right] \right. \right.$$

$$+ \frac{1}{(v+1)} \left[ 1 - (r/b)^{2(v+1)} \right] \left. \right\} \left\{ 1 - (a/b)^{2v} \right.$$

$$- (r \rightarrow a) \} \quad (A.12)$$

Then

$$I = A_0 + (N-1) \frac{1}{4} \lambda^2 A_1 + (N-2) 0(\bar{\lambda}^4) + \pi \Gamma(N) (-\bar{\lambda}^2)^{N-1}$$

$$\cdot \sum_{n=1}^{\infty} \frac{e - \frac{\gamma_{v,n}}{\bar{\lambda}^2}}{\gamma_{v,n}^{2(N-1)}} \frac{J_v(\gamma_{v,n}a)}{J_v^2(\gamma_{v,n}b) - J_v^2(\gamma_{v,n}a)} \\ \cdot [J_v(\gamma_{v,n}b) Y_v(\gamma_{v,n}r) - (r \leftrightarrow b)] \quad (A.13)$$

and

$$f(L, M) = A \left( \frac{L}{L_0} \right)^{\frac{1}{2}(3-\alpha)} \frac{1}{M^{\frac{1}{2}(N+2)}} \quad (A.14)$$

Special cases are

$$A_0(L) \xrightarrow[v \rightarrow 0]{} \frac{\ln r/b}{\ln a/b} \quad (A.15)$$

$$A_1(L) \xrightarrow[v \rightarrow 0]{} \frac{\ln b/r \{ 1 - (a/b)^2 [1 + \ln b/a] \} - \ln b/a \{ 1 - (r/b)^2 [1 + \ln b/r] \}}{[\ln b/a]^2} \quad (A.16)$$

$$A_1(L) \xrightarrow[v \rightarrow \pm \sqrt{1 - a^2/b^2}]{} \frac{\frac{1}{2}(a/r)^2}{[1 - a^2/b^2]^2} \left\{ \left[ 1 - r^4/b^4 - 4r^2/b^2 \ln b/r \right] \left[ 1 - a^2/b^2 \right] \right. \\ \left. - (r \leftrightarrow a) \right\} \quad (A.17)$$

As mentioned previously if  $N = 1$  we should introduce a cutoff at high momenta  $f_0(M) = \frac{A}{M^{3/2}} \theta(M_0 - M)$ . The modification alters the previous calculations only slightly and the result is, for  $N = 1$

$$f(L, M, N = 1) = A \left( \frac{L}{L_0} \right)^{\frac{1}{2}(3-\alpha)} \frac{1}{M^{3/2}} \theta(M_0 - M)$$

$$\left[ A_0 + \sum_{n=1}^{\infty} e^{-\frac{r_v n}{\lambda^2} (1 - \sqrt{\frac{M}{M_0}})} \frac{\pi J_v(\gamma_{v,n} b) J_v(\gamma_{v,n} r)}{J_v^2(\gamma_{v,n} b) - J_v^2(\gamma_{v,n} r)} \right] \cdot [J_v(\gamma_{v,n} b) Y_v(\gamma_{v,n} r) - (r \rightarrow b)] \quad (A.18)$$

## APPENDIX B

a) If the arguments of all the Bessel functions in equations (14) and (A.1) are large, the asymptotic expansion of  $I_{\pm\nu}$  produces

$$\phi(t) \sim \sqrt{a/r} e^{\lambda(a-r)}$$

$$\cdot \left[ \frac{1 + \frac{4\nu^2 - 1}{8\lambda} \left( \frac{1}{r} - \frac{1}{b} \right) + \frac{(4\nu^2 - 1)(4\nu^2 - 9)}{128\lambda^2} \left( \frac{1}{b^2} + \frac{1}{r^2} - \frac{4\nu^2 - 1}{4\nu^2 - 9} \frac{2}{rb} \right) + \dots}{[r \rightarrow a]} \right] \quad (B.1)$$

In which we have retained the dominant exponentials of both numerator and denominator. Inverting the denominator yields

$$\phi(t) \sim \sqrt{a/r} e^{\lambda(r-a)}$$

$$\cdot \left[ 1 + \frac{4\nu^2 - 1}{8\lambda} \left( \frac{1}{r} - \frac{1}{a} \right) + \frac{(4\nu^2 - 1)(4\nu^2 - 9)}{128\lambda^2} \left( \frac{1}{r^2} - \frac{1}{a^2} \right) + \frac{(4\nu^2 - 1)^2}{64\lambda^2 a} \left( \frac{1}{a} - \frac{1}{r} \right) \right] \quad (B.2)$$

Using the inverse transform

$$s^{-1} [s^{u-\frac{1}{2}} e^{-\sqrt{ks}}] = \frac{e^{-k/8y}}{2^u \sqrt{\pi} y^{u+\frac{1}{2}}} D_{2u} \left( \frac{\sqrt{k}}{2y} \right) \quad (B.3)$$

where  $D_{2u}(z)$  is the parabolic cylinder function (see Abramowitz and Stegun (1970) for definitions of all functions in this paper)

we have for the distribution function

$$\begin{aligned}
 f(L, M) &\sim A \left( \frac{L}{L_0} \right)^{\frac{1}{2}(3-\alpha)} \frac{\Gamma(N)}{M^{\frac{1}{2}(N+2)} \sqrt{r}} \sqrt{\frac{a}{r}} \\
 &\cdot e^{-\frac{1}{8\lambda^2 r^2} \frac{(1-a/r)^2}{2^{\frac{1}{2}-N} \sqrt{\pi}} D_{1-2N}(w)} \\
 &\cdot \left[ 1 - \frac{4v^2 - 1}{8\lambda a} (1-a/r) \frac{\sqrt{2D_{-2N}(w)}}{D_{1-2N}(w)} + \frac{(4v^2 - 1)^2}{32\lambda^2 a^2} \right. \\
 &\cdot \left. (1-a/r) \left[ 1 - \frac{1}{2} \frac{4v^2 - 9}{4v^2 - 1} (1+a/r) \frac{D_{-1-2N}(w)}{D_{1-2N}(w)} + \dots \right] \right] \quad (B.4)
 \end{aligned}$$

with

$$w = \frac{\bar{\lambda}r(1-a/r)}{\sqrt{2}}$$

If N is a positive integer (B.4) reduces to

$$\begin{aligned}
 f(L, M) &\sim A \left( \frac{L}{L_0} \right)^{\frac{1}{2}(3-\alpha)} \frac{\Gamma(N)}{M^{\frac{1}{2}(N+2)} \sqrt{r}} \sqrt{\frac{a}{r}} i^{2N-1} i^{2N-2} \text{ERFC}(w/\sqrt{2}) \\
 &\cdot \left[ 1 - \frac{4v^2 - 1}{4\lambda a} (1-a/r) \frac{i^{2N-1} \text{ERFC}(w/\sqrt{2})}{i^{2N-2} \text{ERFC}(w/\sqrt{2})} \right. \\
 &+ \frac{(4v^2 - 1)^2}{16\lambda^2 a^2} (1-a/r) \left[ 1 - \frac{1}{2} \frac{4v^2 - 9}{4v^2 - 1} (1+a/r) \right] \\
 &\cdot \left. \frac{i^{2N} \text{ERFC}(w/\sqrt{2})}{i^{2N-2} \text{ERFC}(w/\sqrt{2})} \right] \quad (B.5)
 \end{aligned}$$

For large  $w \gg 1$

$$i^{2N-2} \text{ERFC}(w/\sqrt{2}) \sim \frac{2}{\sqrt{\pi}} \frac{e^{-\frac{1}{4}\lambda^2 r^2(1-a/r)^2}}{\left[\frac{\lambda r(1-a/r)}{2}\right]^{2N-1}} \quad (\text{B.6})$$

which demonstrates the exponential dependence on  $M$  and  $r^2 = L^{-(\alpha+1)/2}$  for strong synchrotron losses. Equation (B.4) is useful when  $\lambda a \gtrsim 1$ , for which values the series in equation (A.18) converges very slowly.

b) The intermediate case of weak losses at  $L = L_0$  is obtained by treating  $\lambda a \ll 1$ ,  $\lambda b \gg 1$  and  $\lambda r \gg 1$ . Expand  $I_{\pm v}(\lambda a)$  in small argument form and keep only the leading term. If  $v \neq 0$ ,

$K_v(\lambda a) \sim \frac{1}{2} r(v)(\lambda a)^{-v}$  dominates. We have then

$$\phi(t) \sim \sqrt{\frac{2}{\lambda r}} \frac{1}{\Gamma(v)} (\frac{1}{2} \lambda a)^v e^{-\lambda r} \left[ 1 + \frac{4v^2 - 1}{8\lambda r} + \frac{(4v^2 - 1)(4v^2 - 9)}{128\lambda^2 r^2} + \dots \right] \quad (\text{B.7})$$

which gives

$$f(L, M) = A \left( \frac{L}{L_0} \right)^{\frac{1}{2}(3-\alpha)} \frac{\Gamma(N)}{M^{\frac{1}{2}(N+2)}} \frac{1}{\Gamma(v)} (\frac{1}{2} \lambda a)^v \sqrt{\frac{2}{\lambda r}} 2^{-\mu} e^{-\frac{1}{8}\lambda^2 r^2} D_{2\mu}(w)$$

$$+ \left[ 1 + \frac{4v^2 - 1}{8\lambda r} \frac{\sqrt{2} D_{2\mu-1}(w)}{D_{2\mu}(w)} + \frac{(4v^2 - 1)(4v^2 - 9)}{64\lambda^2 r^2} \frac{D_{2\mu-2}(w)}{D_{2\mu}(w)} \right] \quad (\text{B.8})$$

where

$$\mu = \frac{1}{2}(v + \frac{1}{2}) - N \quad w = \frac{\lambda r}{\sqrt{2}}$$

for large values of w

$$D_{2\mu}(w) \sim w^{2\mu} e^{-\frac{1-\lambda^2}{8\lambda^2} r^2} \left[ 1 + \frac{4r(\frac{1}{2} - \mu)}{\lambda r} + O\left(\frac{1}{\lambda^2 r^2}\right) \right] \quad (B.9)$$

and  $f(L)$  exhibits the same exponential dependence as equation (B.4) when  $\lambda^2 r^2 \geq 4$ .

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