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Cosmic-Ray Transport Theory and Out-of-the-Ecliptic Exploration

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Abstract

The reasons for studying cosmic-ray transport theory are summarized and the fundamentally three-dimensional nature of the process is pointed out. Observations in the ecliptic plane cannot unambiguously test transport theories since the solutions to the transport equations depend critically on boundary conditions and variation of parameters such as diffusion tensor out of the ecliptic. Sample calculations are shown which illustrate the problem. It is concluded that out-of-the-ecliptic observations are essential to further test transport theory.

I. Introduction

The study of cosmic-ray transport theory has an old and venerable history, for it is clear that in order to understand anything in cosmic-ray astrophysics we must first understand transport. The observed near-isotropy of cosmic rays was early recognized to imply that the orbits of the particles have been severely distorted in their motion through space. Since the work of Fermi (1949) it has been understood that the motion must be treated statistically, since the plasmas and magnetic fields through which the cosmic rays move are irregular and turbulent. This basic fact leads to the view that the spatial motion of cosmic rays is to a very good first approximation a random walk in three dimensions.

The solar wind provides an excellent local laboratory for testing our ideas of cosmic-ray transport by comparing in situ observations with theory. A comprehensive theory has been developed which has had reasonable success in explaining the various cosmic-ray phenomena in the solar wind (see reviews by Jokipii, 1971 and Völk, 1975). Before going on to discuss this theory in detail, it is useful to emphasize that irrespective of the detailed theory, cosmic-ray transport theory is fundamentally three-dimensional and that this problem is much more severe than for some other aspects of the physics of the interplanetary medium. The general picture is illustrated in figure 1 for particles originating in the galaxy or at the sun. It is evident that, for example, there is no viewing direction in which one sees cosmic rays which have sampled only the ecliptic plane. This conclusion depends only on the turbulent nature of the interplanetary plasma and the consequent stochastic motion of the cosmic-ray particles.

One can, of course, dream up experiments which could be used to study cosmic-ray transport locally. For example, a useful experiment would be to inject a small beam of "tagged" cosmic rays at one point and measure them a distance ~0.1 a.u. away. But such experiments are clearly not possible in the near future. We must be content with what nature provides, and work with the full three-dimensional problem.

The preceding discussion has intentionally been as general as possible in order to emphasize the fundamental nature of the conclusions. In the next section, the current detailed theory of cosmic-ray transport is discussed, then some quantitative calculations are presented which illustrate the effects of uncertainties in parameter outside of the ecliptic.

II. Current Transport Theory

The current theory of cosmic-ray transport has been comprehensively discussed in an earlier review by the author (Jokipii, 1971). The theory works at two levels, which could be termed the "macroscopic" and the "microscopic". This is illustrated schematically in figure 2 which shows the particle being scattered randomly by the magnetic fluctuations. Hence the particle pitch angle θ undergoes a random walk. The details of this scattering process and its relation to the detailed microstructure of the magnetic fluctuations are studied in the microscopic theory, whereas the resulting spatial diffusion is considered in macroscopic theory. Thus, for example, in the simplest form of quasilinear theory, where only planar magnetic fluctuations with wave vector parallel to the average field are considered, the pitch-angle scattering is characterized by the Fokker-Planck coefficient

$$\frac{\langle\Delta\theta^2\rangle}{\Delta t} = \frac{\omega_0^{\ 2}}{|\mu| w B_0^{\ 2}} P_\perp\left(k = \frac{1}{\mu r_c}\right) , \qquad (1)$$

where $\omega_0 = qB_0/mc$ is the particle cyclotron frequency in the average field B_0, $\mu = \cos\Theta$, w = particle speed, $r_c = w/\omega_0$, and $P_\perp(k)$ is the spatial power spectrum of the magnetic fluctuations as a function of wavenumber k (see, e.g., Jokipii 1966, 1971). Other more complex expressions result if other magnetic fluctuation configurations are assumed. Similar expressions result for the other Fokker-Planck coefficients such as $\langle\Delta x^2\rangle/\Delta t$, etc. Hence, if x is a direction normal to B_0, we have

$$\frac{\langle \Delta x^2 \rangle}{\Delta t} = \frac{|\mu| w}{B_0^2} P_\perp (k=0) \quad . \tag{2}$$

The above expressions are examples of microscopic transport theory.

However, in most situations the cosmic-ray angular distribution is driven essentially isotropic by the scattering, so that the transport must be described by the diffusion approximation. If $U(\underset{\sim}{r},t,T)$ is the particle density averaged over pitch angle as a function of position $\underset{\sim}{r}$, time t and energy T, then the diffusion equation reads in the rest frame (Parker 1965, Gleeson and Axford 1967, Jokipii and Parker 1970)

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x_i} \left[\kappa_{ij} \frac{\partial U}{\partial x_j} - V_{w,i} U\right] + \frac{1}{3} \frac{\partial V_{w,i}}{\partial x_i} \frac{\partial}{\partial T} (\alpha \; T \; U) \tag{3a}$$

with an associated flux of particles in the rest frame

$$F_i = -\kappa_{ij} \frac{\partial U}{\partial x_j} + V_{w,i} U - \frac{V_{w,i}}{3} \frac{\partial}{\partial T} (\alpha T U), \tag{3b}$$

where κ_{ij} is the cosmic-ray diffusion tensor, $\underset{\sim}{V}_w$ is the solar wind velocity and $\alpha(T) = (2mc^2 + T)/(mc^2 + T)$. The associated anisotropy is $|\underset{\sim}{\delta}| = 3|\underset{\sim}{F}|/wU$. In a coordinate system with the z-direction oriented along the average magnetic field $\underset{\sim}{B}_0$, κ_{ij} may be written

$$\kappa_{ij} = \begin{bmatrix} \kappa_\perp & \kappa_A & 0 \\ -\kappa_A & \kappa_\perp & 0 \\ 0 & 0 & \kappa_\parallel \end{bmatrix} \tag{4}$$

where the parallel diffusion coefficient $\kappa_\parallel$, the perpendicular diffusion coefficient $\kappa_\perp$ and the antisymmetric diffusion coefficient κ_A can be written

$$\kappa_\parallel = w^2 \int_{-1}^{1} d\mu' \left[\mu' \int^{\mu'} \frac{d\mu}{<\Delta\theta^2>/\Delta t} \right] \tag{5}$$

$$\kappa_\perp = \frac{1}{2} \int_0^1 \frac{<\Delta x^2>}{\Delta t} \, d\mu \tag{6}$$

$$\kappa_A = \frac{1}{3} r_c w \; . \tag{7}$$

The diffusion equation (3) embodies the macroscopic theory which is connected to the microscopic theory by equations (5) - (7). At present, comparison of transport theory with observation must be done through the use of macroscopic theory and then working back to the microscopic theory. We are unable to measure $<\Delta\theta^2>/\Delta t$, etc. directly.

With regard to the problem of out-of-the-ecliptic exploration, it is clear that the solution to equation (3) depends on a knowledge of κ_{ij} and V_w throughout the modulating region and on the value of U on the boundary. There appear to be no purely locally measurable properties of the solution which can be used in checking transport theory.

III. Illustrative Calculations

To illustrate the rather large uncertainties introduced by lack of knowledge of parameters out of the ecliptic, I present in this section a summary of some analytical calculations published elsewhere (Owens and Jokipii, 1971). This problem has also been considered by Sarabhai and Subramanian (1966) and Lietti and Quenby (1968). Consider the 11-year solar cycle modulation of galactic cosmic rays by a solar wind which is not spherically symmetric. The 11-year solar cycle variations are usually regarded as slow enough that the time derivative in equation (3) can be neglected in solving for U. In this calculation the various parameters in the transport equation are allowed to vary with heliographic latitude and the effects of the variation on the density U and the flux $\underset{\sim}{F}$ in the solar equatorial plane are considered.

The following forms for the various parameters were assumed: The various components of the diffusion tensor are independent of energy T and are proportional to heliocentric radius r out to some boundary $r = D$ where $\kappa_{ij} \to \infty$. The cosmic-ray density $U(\underset{\sim}{r},T)$ is assumed to take on a given interstellar value $U_\infty(T) \sim AT^{-2.5}$ at $r = D$.

Finally, we follow the usual practice of circumventing the problem of proper boundary conditions at the Sun by requiring the solution for U to be finite at the origin. This will be shown to lead to negligible error in the present case. See Jokipii (1971) for a more complete discussion of this problem.

Equation (3a) for $U(r,\Theta,\phi,T)$ becomes, in spherical polar coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(V_w U - \kappa_\parallel \frac{\partial U}{\partial r}\right)\right] - \frac{1}{r\sin\Theta}\frac{\partial}{\partial\Theta}\left[\sin\Theta\left(\frac{\kappa_\perp}{r}\frac{\partial U}{\partial\Theta} - \frac{\kappa_A}{r\sin\Theta}\frac{\partial U}{\partial\phi}\right)\right]$$

$$- \frac{1}{r\sin\Theta}\frac{\partial}{\partial\phi}\left(\frac{\kappa_\perp}{r\sin\Theta}\frac{\partial U}{\partial\phi} + \frac{\kappa_A}{r}\frac{\partial U}{\partial\Theta}\right) - \frac{2V_w}{3r}\frac{\partial}{\partial T}(TU\alpha) = 0. \qquad (8)$$

In what follows, we consider only models which are axisymmetric (independent of ϕ). If $\Theta = 0$ is the axis of solar rotation, this corresponds to a latitude-dependent solar wind. Effects of magnetic sectors could be represented by taking $\Theta = 0$ along a sector. The two terms containing κ_A cancel from the density equation (8) if $\partial\kappa_A/\partial\Theta = \partial\kappa_A/\partial\phi = 0$, as in our axisymmetric model. Thus the terms in κ_A do not contribute, and equation (8) becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(V_w U - \kappa_\parallel \frac{\partial U}{\partial r}\right)\right] - \frac{1}{r^2\sin\Theta}\frac{\partial}{\partial\Theta}\left(\kappa_\perp \sin\Theta\frac{\partial U}{\partial\Theta}\right) - \frac{2V_w}{3r}\frac{\partial}{\partial T}(TU\alpha) = 0. \qquad (9)$$

Since $\kappa_\parallel$ and $\kappa_\perp$ are taken to be independent of energy, equation (9) separates, and one writes

$$U(r,\Theta,T) = S(r,\Theta)\phi(T). \qquad (10)$$

Upon neglecting the small energy dependence of $\alpha(T)$, one finds immediately that S must satisfy

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2(V_w S - \kappa_\parallel \frac{\partial S}{\partial r})\right] - \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left[\kappa_\perp \sin\theta \frac{\partial S}{\partial\theta}\right] = \frac{qV_w S}{r}, \quad (11)$$

with

$$\phi(T) = T^{(3q/2\alpha)-1} . \quad (12)$$

The separation constant q must be chosen to agree with $U_\infty(T) \propto T^{-2.5}$, in which case we choose $q \approx -1$.

A number of different forms for the variation of the parameters V_w, $\kappa_\parallel$ and $\kappa_\perp$ were chosen. Define

$$V_w(\theta,r) = V_o[1+\delta(\theta)], \; \kappa_\parallel(\theta,r) = \kappa_o[1+\varepsilon(\theta)]r$$

$$\text{and} \quad \kappa_\perp = \mu\kappa_\parallel \; . \quad (13)$$

The problem may be solved in terms of a series of Legendre polynomials and the general solution is given by Owens and Jokipii (1971). An illustrative velocity variation is given by $V = V_o[1 + .30P_2(\cos\theta)]$ where V_o is the average velocity and P_2 is the Legendre polynomial of order 2. This velocity variation is illustrated in figure 3. Some typical results of the calculations are illustrated in figures 4, 5, and 6. It is clear that many of the parameters observed in the solar equatorial plane can be substantially changed by varying the solar-wind parameters out of the solar ecliptic plane. Of particular note is the fact that the radial anisotropy observed in the equatorial plane is extremely sensitive to parameters outside the equatorial plane. As shown in figure 6, even the sign of the anisotropy can be changed by relatively small velocity variations.

IV. Conclusions

One may conclude from the above discussion that fundamental ambiguities in testing cosmic-ray transport can be removed by carrying out measurements out of the ecliptic. The out-of-the-ecliptic measurements most necessary to study cosmic-ray propagation are:

a) Measurements of the flux of cosmic rays as a function of solid angle and energy with as much resolution as possible. The anisotropy measurements are easier to carry out on a spinning spacecraft.

b) Simultaneous measurements on board the same spacecraft of the plasma and magnetic field. These measurements should be spaced in time so that good time-series analyses (power spectra, etc.) can be obtained.

c) The spacecraft should go as far out of the ecliptic as possible to insure that any variations with heliographic latitude will be seen.

d) Good base-line measurements at Earth or in the ecliptic plane should be obtained simultaneously.

It is not crucial that these measurements be carried out at constant heliocentric radius, although this might aid interpretation. It appears that measuring protons and possibly electrons will be adequate and it is better to optimize measurements for one species rather than compromise these in order to study composition.

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Figure Captions

Fig. 1. Schematic illustration in a meridian plane of typical trajectories of solar cosmic-rays (dashed line) and galactic cosmic rays (solid line) which reach a detector at 1 a.u. The fundamentally 3-dimensional character of the motion is apparent.

Fig. 2. Illustration of the "scattering" of a cosmic-ray particle by a magnetic irregularity. If the region over which $\Delta\theta$ is correlated is small, a Fokker-Planck equation results.

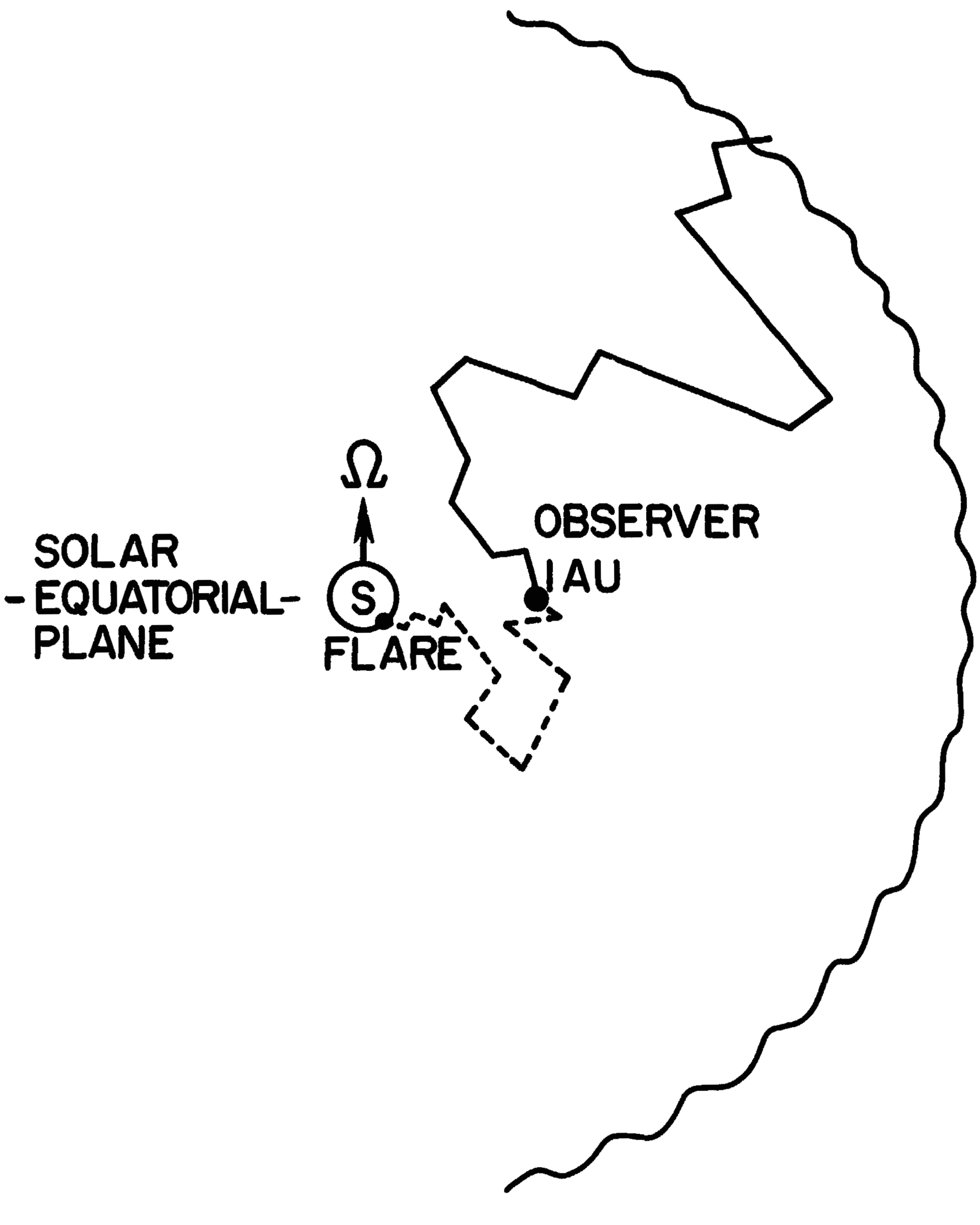
Fig. 3. Illustration of the velocity profile used in some of the calculations. θ is the angle relative to the z-axis.

Fig. 4. Comparison of linearized and exact solutions for the case $V = V_0[1 + 0.30\ P_2(\cos\theta)]$, $\kappa_{\parallel} = \kappa_0 r$, $\mu = \frac{1}{2}$. The density $U(r,\theta)$ is given for both solutions as a function of angle θ $r = 0.2\ D$ in terms of $U_\infty = U(r = D,\theta)$. Straight line shows the corresponding result for an isotropic wind $V = V_0$. The linearized calculation keeps terms only to first order in δ as defined in equation (4). From Owens and Jokipii (1971).

Fig. 5. Total cosmic-ray flux as a function of r,θ, and with F_ϕ suppressed. The pattern is azimuthally symmetric and even about the equator. (a) The parameters $V(\theta) = V_0[1 + 0.30\ P_2(\cos\theta)]$, $\kappa_{\parallel}(r,\theta) = \kappa_0 r$ and $\mu = \frac{1}{2}$ were used. The position of Earth for D = 5 a.u. is indicated. Of particular interest is the

virtual source of particles in the equatorial plane at $r \approx 0.5$ D. (b) Same as (a) except that $V(\Theta) = V_0[1 + 0.46\ P_2(\cos\Theta)]$. Note that the flux is much larger and virtual source has moved out to $r \sim 0.9$ D. From Owens and Jokipii (1971).

Fig. 6. Anisotropy in the equatorial plane at $r = 0.2$ D for $V(\Theta) = V_o + V_2 P_2(\cos\Theta)$, as a function of V_2/V_o. Note that in our model the anisotropy is radial. From Owens and Jokipii (1971).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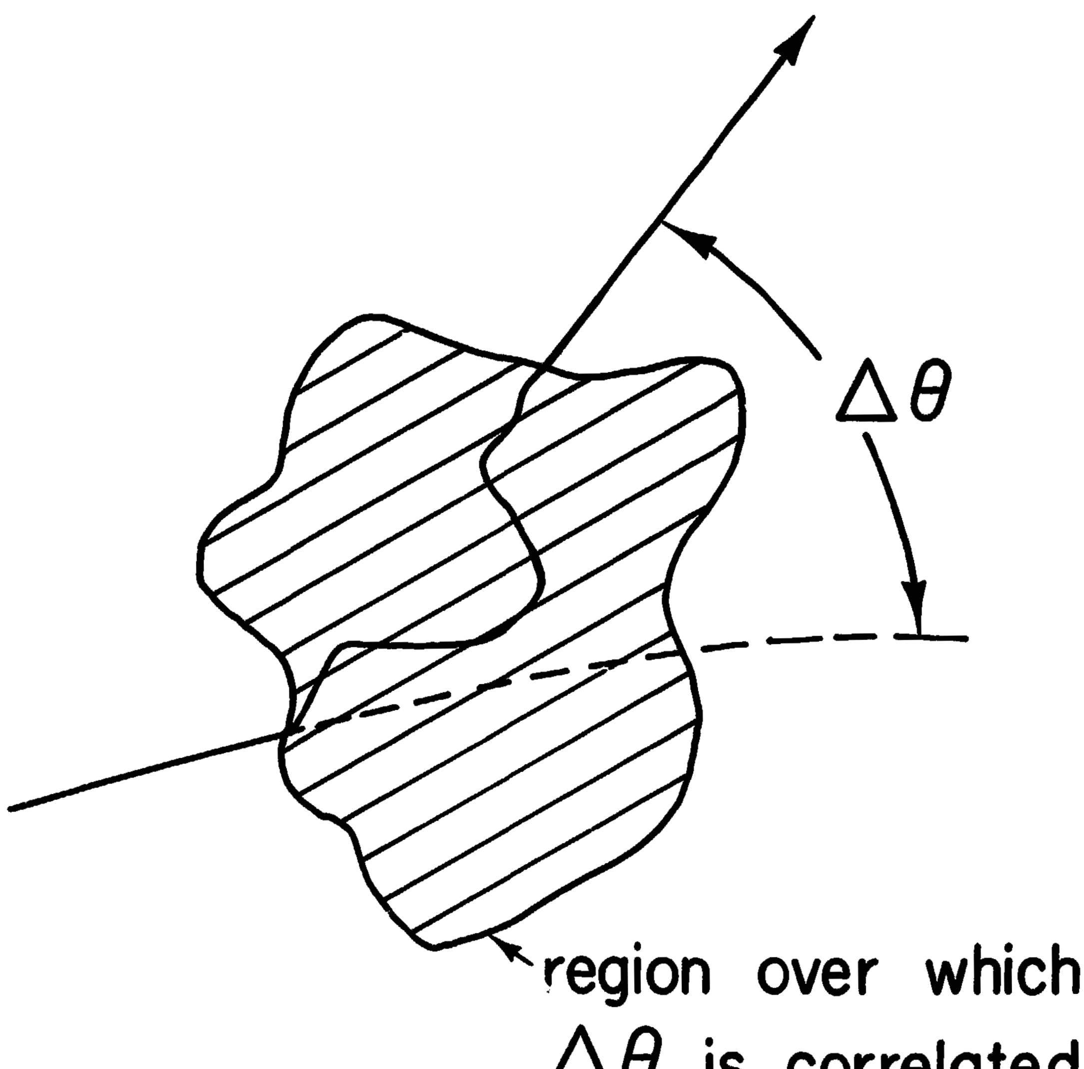
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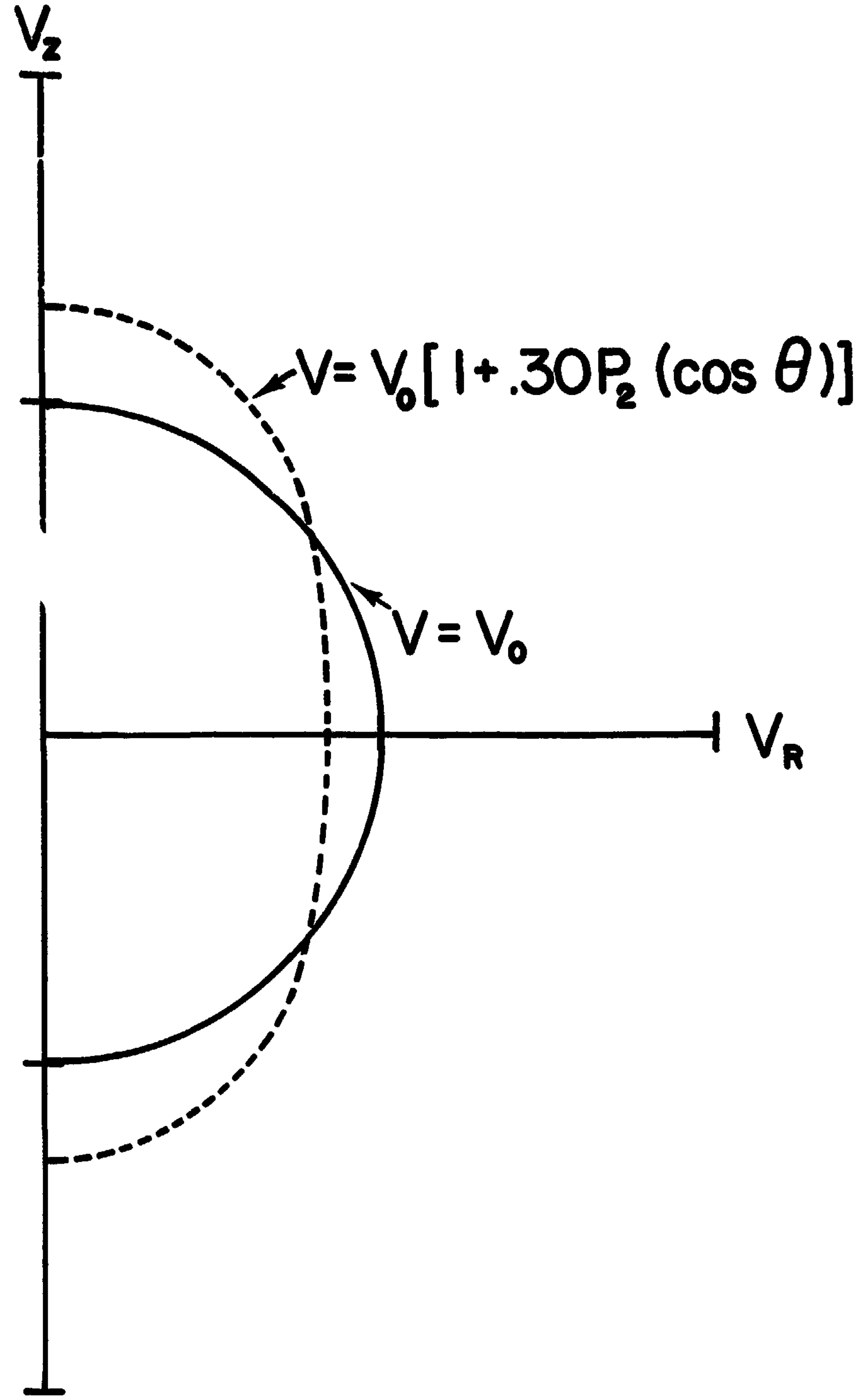

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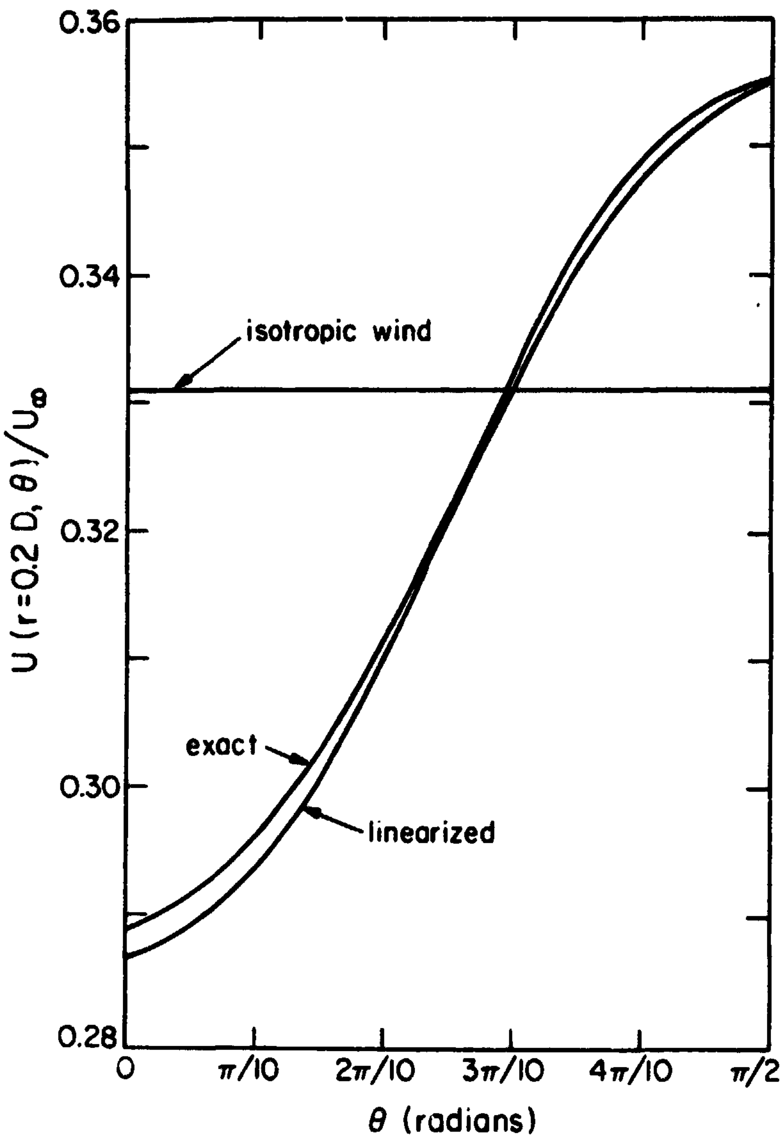
$V = V_0$
V_R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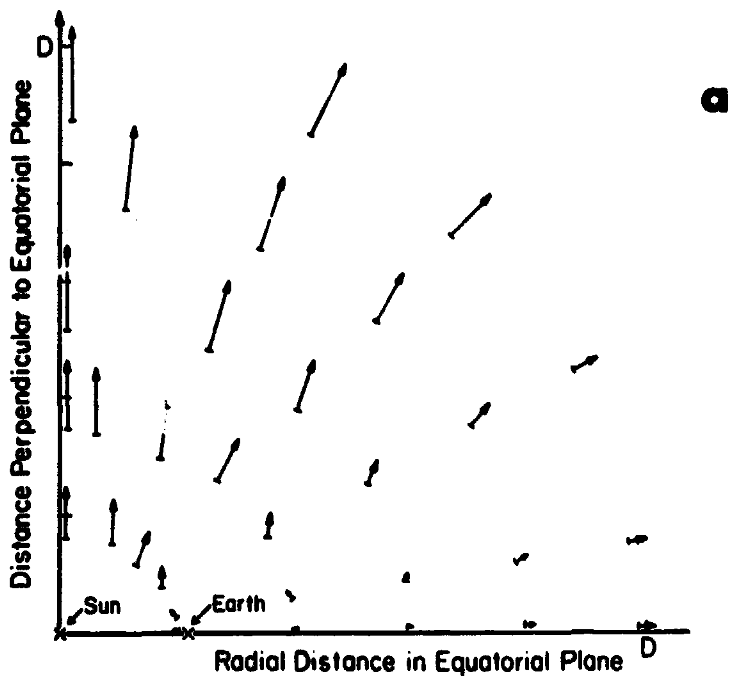
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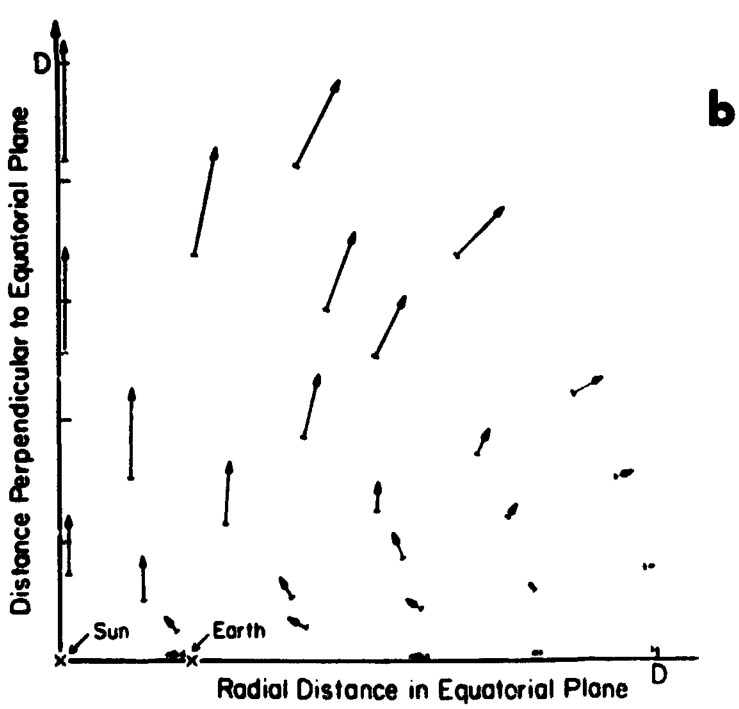
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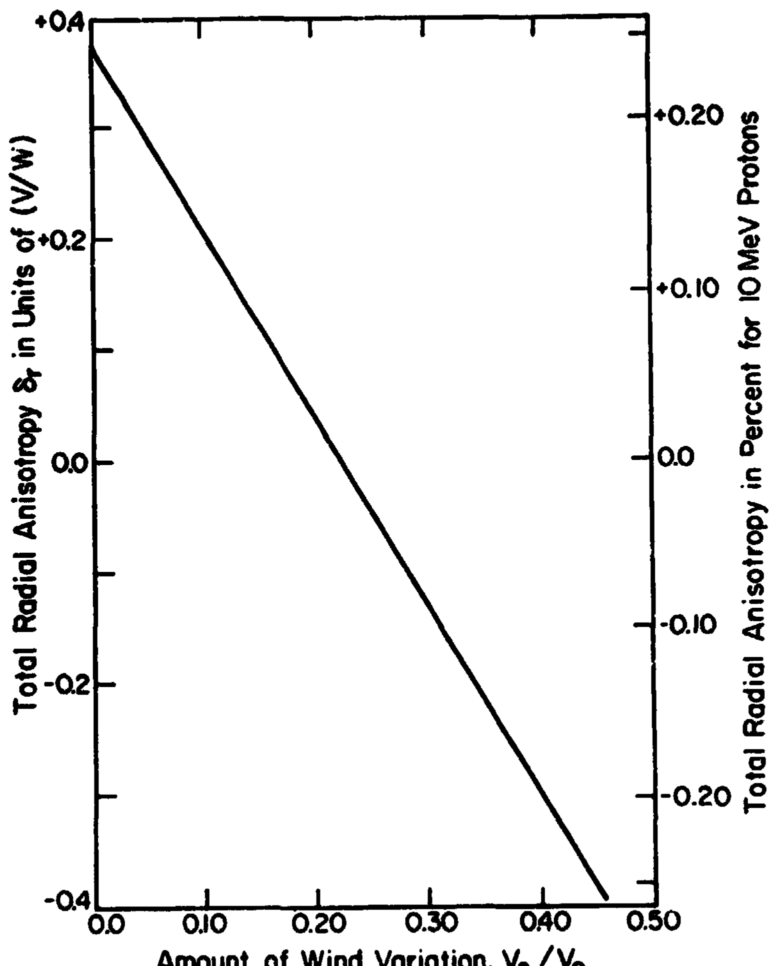
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Total Radial Anisotropy δr in Units of (V/W)
Total Radial Anisotropy in Percent for 10 MeV Protons
Amount of Wind Variation, V2/V0