MAGNETOHYDRODYNAMIC AND GASDYNAMIC ASPECTS OF SOLAR-WIND FLOW AROUND TERRESTRIAL PLANETS A CRITICAL REVIEW

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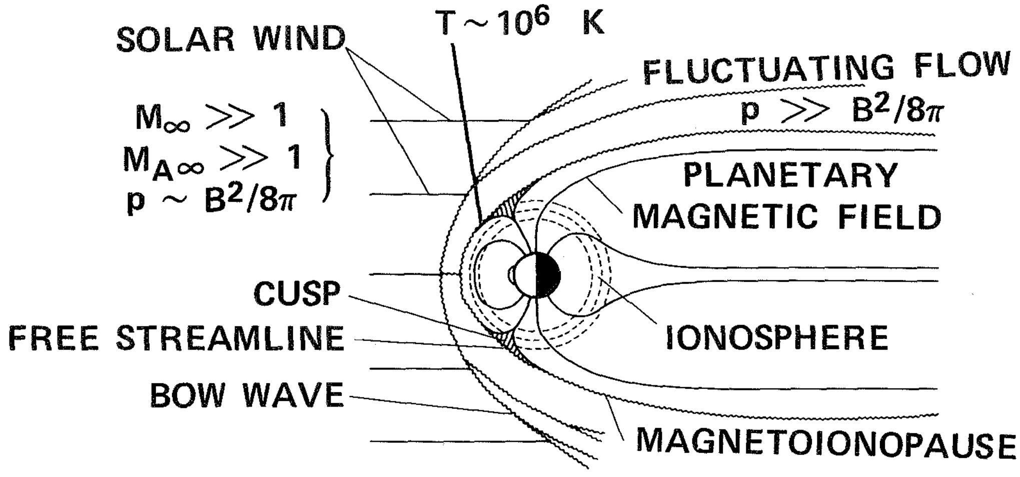
INTRODUCTION

It is now established by numerous observations near Mercury, Venus, Earth, Mars, and also Jupiter and the Moon that many aspects of solar-wind flow around planetary bodies can be understood in terms of continuum fluid models based on familiar equations of magnetohydrodynamics and gasdynamics. That this should be so in spite of the enormous mean-free paths of solar-wind particles was one of the early surprises of the space era, but the conclusion has been confirmed repeatedly with the extension of direct measurements from the Earth to other planets. The considerable variety of atmospheric and magnetic field properties possessed by the planets results, moreover, in a corresponding variety of flow details, and a remarkably rich field of comparative study of solar-wind flow around major objects in the solar system. It is the purpose of this paper to present a review of the fluid aspects of these flows and how they are approximated to obtain tractable mathematical problems, and a commentary on possibilities for further improvements and on some misconceptions that have appeared in applications of the results.

PRINCIPAL FEATURES OF SOLAR-WIND FLOW AROUND TERRESTRIAL PLANETS

Figure 1 provides an outline of the salient features of solar-wind flow around a terrestrial planet as it is presently perceived. The solar wind approaches the planet from approximately the direction of the Sun with supersonic and super-Alfvénic velocity. Its properties vary with time and location in the solar system, but in a manner that is understandable in terms of fluid theories of the solar wind.

Because the solar wind is an ionized medium, and these planets all possess a sufficiently strong magnetic field or a sufficiently dense ionosphere, the solar-wind plasma is unable to flow directly into the planetary surface or atmosphere to any significant degree. It is instead deflected around a surface enclosing the planet which we shall call, for convenience, the magnetoionopause. Its shape and size depend on local conditions in the solar wind and on the properties of the planetary magnetic field and ionosphere. These cannot be deduced from theory, but must be determined by observation.



LEVELS OF THEORETICAL DESCRIPTION

1. COLLISIONLESS NONMAGNETIC PLASMA, CHAPMAN-FERRARO
2. HYBRID GAS DYNAMIC AND CHAPMAN-FERRARO
3. DISSIPATIONLESS MAGNETOHYDRODYNAMIC
4. DISSIPATIVE MAGNETOHYDRODYNAMIC
5. ANISOTROPIC PLASMA
6. ANISOTROPIC MULTI-COMPONENT PLASMA

Figure 1. Principal features of solar-wind flow past terrestrial planets.

The magnetoionopause is a relatively thin region in which a variety of dissipative effects associated with viscosity, heat conduction, and electrical resistance occurs. If its presence is associated primarily with a magnetic field, as is the case for Earth, Mercury (Ness et al., 1974b; 1975a, b; and Ogilvie et al., 1974), and probably Mars (Dolginov et al., 1973; Gringauz et al., 1975), the generally impervious nature of the magnetopause is marred by two cusp-shaped regions through which plasma of solar origin can drain into the magnetosphere. If the boundary is associated more with the ionosphere than a planetary field, as for Venus (see, for example, Bridge et al., 1974 and Ness et al., 1974a), the ionopause separates two significant bodies of plasma, one flowing and the other comparatively stationary. Limited direct evidence combined with general knowledge of other shearing flows suggests that a viscous boundary layer develops along this surface, but its properties remain largely unknown and even controversial at present.

The flow immediately beyond the magnetoionopause is not the undisturbed incident solar-wind plasma, as envisioned in the pioneering studies for the Earth carried out by Chapman and Ferraro (see Chapman (1963) for a review) decades before the first spacecraft, but a highly-perturbed flow that has passed through a detached bow-shock wave as in figure 1. Such a shock wave has now been identified by direct observations for Mercury, Venus, Earth, Mars, and Jupiter; but not for the Moon where the absence of any significant ionosphere or magnetic field allows the solar wind to flow directly onto the lunar surface.

The existence of the Earth's bow wave was first inferred from data from early spacecraft indicating the presence of a transitional region between the magnetopause and the incident solar wind. To account for the observations, a rather inconsistent model was put forward independently by Axford (1962), Kellogg (1962), and Spreiter and Jones (1963) in which the magnetopause shape is calculated using the collisionless theory of Chapman and Ferraro, and the location of the bow wave was calculated for that shape using gasdynamic theory. While the results were in acceptable agreement with the rudimentary data of the time, the logical foundations of the theory were unsatisfactory.

To remedy the latter, and also to provide more details of the flow, the entire subject was approached anew by Spreiter, Summers, and Alksne (1966) using the equations of dissipationless magnetohydrodynamics as a foundation. That approach removed the logical inconsistencies, and provided a mechanism by which previous results, confirmed by comparison with direct observation, could be recovered by introduction of acceptable approximations. In addition, detailed distribution of flow quantities including the density, velocity, temperature, and magnetic field were calculated for a variety of conditions in the solar wind. These, and similar calculations carried out by others, notably Dryer and his colleagues, have formed a theoretical base with which numerous observations have been compared, and from which further advances have been made. The latter have included extensions to a nonmagnetic planet, for which the studies of Spreiter, Summers, and Rizzi (1970) and Rizzi (1971) are the most quantitative; and to more elaborate descriptions of the plasma properties than provided by the equations of magnetohydrodynamics. The equations of dissipative magnetohydrodynamics, and of anisotropic plasma theory, have been used for the analysis of certain details of the flow, but are so complicated that analysis of the large-scale features of the flow comparable with that carried out with the dissipationless theory does not appear feasible at the present time.

BASIC ASSUMPTIONS, APPROXIMATIONS, AND CONSEQUENCES OF STEADY DISSIPATIONLESS MAGNETOHYDRODYNAMIC DESCRIPTION

Fundamental to the entire application of magnetohydrodynamics to solar-wind flow past planets is a knowledge of the wave pattern associated with steady rectilinear flow with velocity, $\underset{\sim}{v}$, past an infinitesimal point disturbance, as illustrated for a specific set of conditions in the left part of figure 2 from Spreiter, Summers, and Alksne (1966). The ovals represent the propagation speed as a function of angle with the magnetic field vector, $\underset{\sim}{B}$, as viewed in a coordinate system fixed in the undisturbed plasma; the straight lines through the tail of the $\underset{\sim}{v}$ vector represent the standing waves as viewed in a coordinate system fixed with respect to the point disturbance. Note that the latter are not tangent to the ovals, but intersect them at the same points as a circle drawn through the point disturbance with diameter equal to $\underset{\sim}{v}$. When $|\underset{\sim}{v}|$ is sufficiently large compared with the speed of sound, a, and the Alfvén speed, A, there will form, in general, three distinct bow waves–fast, slow, and

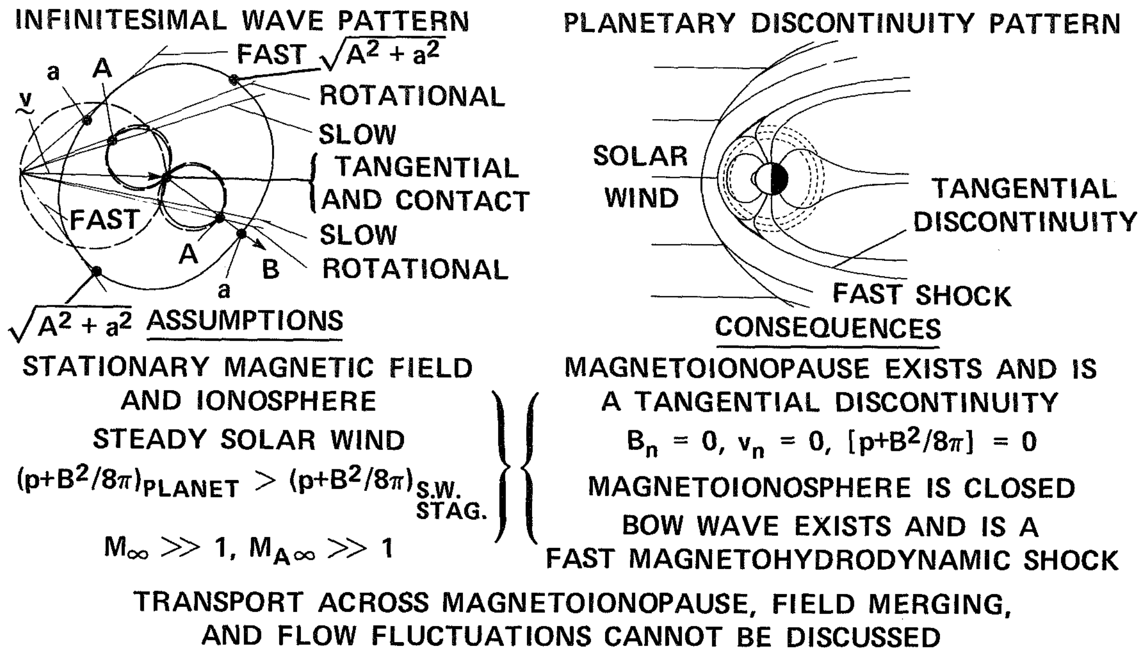


Figure 2. Some basic properties of the steady dissipationless magnetohydrodynamic model.

rotational—each at a different angle with respect to $\underset{\sim}{v}$. In addition, two types of discontinuity surfaces—contact and tangential—extend directly from the point disturbance along the direction of $\underset{\sim}{v}$. The relative importance of the various surfaces depends on conditions that prevail in any specific application. For steady solar-wind flows with $M_\infty = v_\infty/a_\infty \gg 1$ and $M_{A\infty} = v_\infty/A_\infty \gg 1$, the planetary bow wave must be a fast magnetohydrodynamic shock wave, and the magnetopause must be a tangential discontinuity. There is no alternative within the framework of the steady dissipationless magnetohydrodynamic description. The original application was directed toward the Earth, but extension to other planets for which the sum of ionospheric pressure, p, and planetary magnetic pressure, $B^2/8\pi$, exceeds that of the solar wind at a stagnation point leads to the same conclusions, although other details of the flow field may be different quantitatively.

The mathematical problem posed by the magnetohydrodynamic model of solar-wind flow past a planet is very difficult, and a number of approximations are customarily made to obtain a tractable problem. Some of these reflect the status of computational capabilities of a decade ago when many of the calculations still in widespread use were performed, and could be improved upon by introduction of modern techniques and computers. Others of these would probably still have to be employed, either in their present or modified form, to obtain solutions with reasonable effort. Four key approximations in virtually universal use in all present calculations are the following:

1. Because $M_{A_\infty} \gg 1$, the fluid-flow properties approach those of gasdynamics, and the magnetic field can be calculated as a subsequent step from knowledge of the flow.

2. Because $M_\infty \gg 1$, the magnetoionopause shape can be determined independently of the surrounding flow because pressure of the solar wind on the magnetoionopause can be approximated by $p = K\rho_\infty v_\infty^2 \cos^2 \psi$ where $K = 0.881 \; [M_\infty^2/(M_\infty^2 - 1/5)]^{3/2}$ for the ratio of specific heats $\gamma = 5/3$, ρ is the density of the solar wind, ψ is the angle between the normal to the magnetoionopause and $\underset{\sim}{v}_\infty$, and subscript ∞ refers to conditions in the incident solar wind upstream of the bow wave.

3. A simplified representation is introduced to simplify calculation of the magnetoionopause shape; specifically, a dipole planetary magnetic field, a rough approximation for the magnetic contribution of the magnetopause currents, neglect of magnetospheric currents, and a constant scale height ionosphere.

4. The magnetopause shape is approximated by a body of revolution to enable application of existing gasdynamic methods for calculation of flow properties.

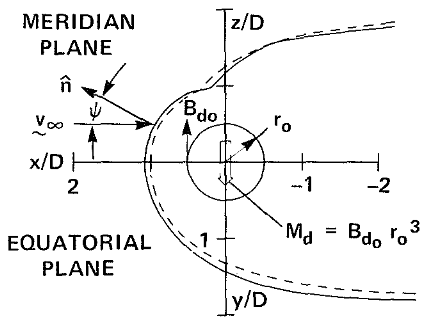
Although certain checks can be performed internally in the theory to determine the accuracy of these approximations, the principal evaluation has been through comparison of the final results with observations. Although the precision of such comparisons is usually not high, the generally good agreement has led to a feeling of confidence in the magnetohydrodynamic model and widespread application of the results.

POSSIBLE IMPROVEMENTS IN THE THEORY

It is evident that improvements can be made in the theory in several places, but there has been no systematic attempt at an overall improvement. Some of these can be accomplished easily; others are very difficult or even beyond present capabilities. In certain instances, improvements have been made in a part of the model, but in a way that is not consistent with the general theory so that the results must be reinterpreted to realize full return for the effort.

To be more specific, consider the determination of the shape and size of the magnetopause associated with steady solar-wind flow past a dipole field, as is appropriate for Earth, Mercury, and probably Mars. The fluid flow calculations of Spreiter, Summers, and Alksne (1966), developed specifically for the Earth, were performed for axisymmetric flow past the approximate coordinates for the equatorial trace of the magnetopause determined by Spreiter and Briggs (1962a). Here the collisionless model was used with $K\rho_\infty$ equated to the product of the mass and number density of the protons in the solar wind ($m_p n_p$), and with the magnetic field at the magnetopause approximated by assuming it was twice the tangential component of the dipole field, as was done in many earlier analyses of related problems by Chapman and Ferraro. Although $m_p n_p$ represents a good approximation for

$K\rho_\infty$ in the fluid representation, its use is actually erroneous for the collisionless case (Spreiter and Briggs, 1962b). With $K\rho_\infty$ changed to $2m_p n_p$, as appropriate for the collisionless model and revived here to facilitate comparison with the exact solution of Choe et al. (1973), the coordinates of the magnetopause in the equatorial and the noon-midnight meridian planes are as illustrated in figure 3; and the geocentric distance, r_n, of the magnetopause nose is given by $D = (M_d^2/4\pi\, m_p n_p v_\infty^2)^{1/6}$ where $M_d = B_{d_0} r_0^3$ is the magnetic moment, B_{d_0} is the intensity of the dipole field at the equator, and r_0 is the planetary radius.



MAGNETOPAUSE COORDINATES, $K\,\rho_\infty/m_p n_p = 2.$

– – – APPROXIMATE, SPREITER-BRIGGS (1962 a,b)

——— EXACT, CHOE et al., (1973)

$$D = \left(\frac{M_d^2}{4\pi\, m_p n_p v_\infty^2}\right)^{1/6}$$

COORDINATES OF MAGNETOPAUSE NOSE

COLLISIONLESS MODEL, $p = K m_p n_p V_\infty^2 \cos^2\psi$, $K = 2$

$$\frac{r_n}{D} = \begin{pmatrix} 1 \\ 1.069 \end{pmatrix} \begin{matrix} \text{APPROXIMATE} & B_n = 2\,Bd_n \\ \text{EXACT} & B_n = 2.443\,Bd_n \end{matrix}$$

FLUID MODEL, $p = K(\rho_\infty/m_p n_p)\, m_p n_p v_\infty^2 \cos^2\psi$

$K = 0.881$ FOR $\gamma = 5/3$, $M_\infty \gg 1$; $\rho_\infty/m_p n_p = \Sigma\, m_i n_i/m_p n_p = 1.16$ FOR 4% He^+

$$\frac{r_n}{D} = \begin{pmatrix} 1.122 \\ 1.118 \\ 1.195 \end{pmatrix} \begin{matrix} \text{APPROXIMATE, SPREITER et al., (1966)}, K\rho_\infty/m_p n_p = 1,\ B_n = 2B_{d_n} \\ \text{IMPROVED PRESSURE}, K = 0.881,\ \rho_\infty/m_p n_p = 1.16,\ B_n = 2\,B_{d_n} \\ \text{IMPROVED PRESSURE AND FIELD}, K = 0.881,\ \rho_\infty/m_p n_p = 1.16,\ B_n = 2.443\,B_{d_n} \end{matrix}$$

NOTE: PRESSURE IS INADEQUATELY REPRESENTED BY $p \sim \cos^2\psi$ NEAR NEUTRAL POINT WHERE CUSP DEVELOPS

Figure 3. Some improvements in magnetopause coordinate calculation.

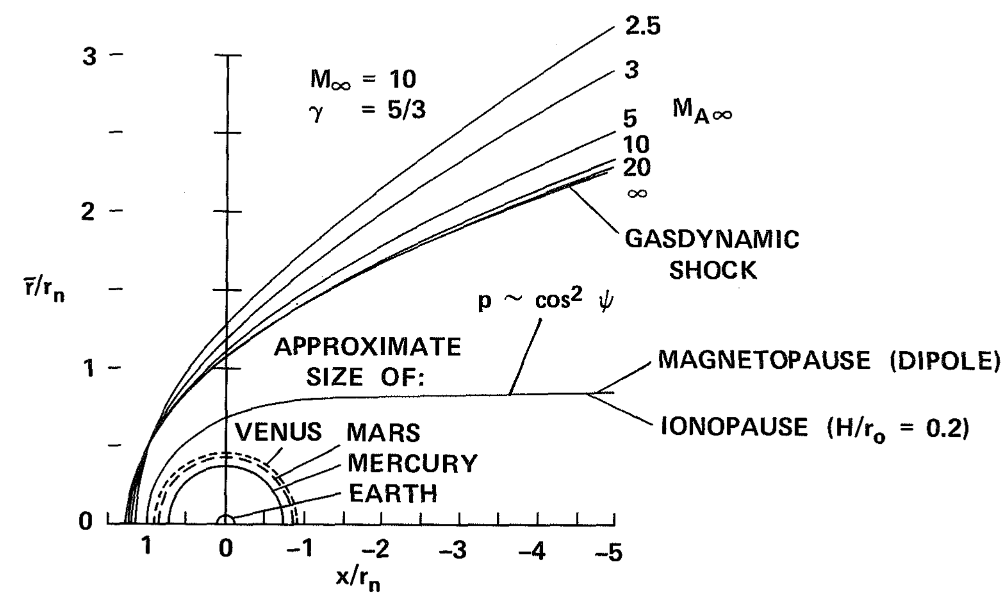
With $K\rho_\infty/m_p n_p$ equated to unity, all coordinates of the magnetopause including r_n increase by a factor of $2^{1/6} = 1.122$. Further improvement can be sought by noting that K is given more accurately in the fluid theory by 0.881 for large M_∞ and $\gamma = 5/3$, in which case $r_n = 1.146$ D. Moreover, the fluid density, ρ_∞, is enhanced above $m_p n_p$ by the presence of minor constituents in the solar wind. If, for example, the number density of ionized helium is taken to be four percent of the protons, as is most often the case (Hirshberg, 1973), the density would be enhanced by 16 percent. Combination of these two effects leads to $r_n = 1.118$ D, virtually the same as the value of 1.122 D used in the original calculations of Spreiter, Summers, and Alksne (1966).

Since those calculations were made, improvements have been achieved in the solution of the Chapman-Ferraro boundary problem so that the magnetic effects of the magnetopause currents need no longer be approximated by a simple doubling of the dipole field. The most complete and accurate solution appears to be that of Choe et al. (1973), but they, like Olson (1969) and Beard and colleagues in a number of intervening studies, continued to equate $K\rho_\infty/m_p n_p$ to 2. Independently of that point, however, their work established, as illustrated in figure 3, that the exact collisionless magnetopause coordinates differ only slightly, and principally in scale, from those of the earlier approximate calculations. Compared with the simple doubling of the dipole field at the magnetopause nose used in the approximate calculations, the exact solution indicates the magnetic field there is 2.443 times the dipole field when the dipole axis is normal to the flow direction; and only slightly different for other orientations of geophysical interest. With this correction, $r_n = 1.069$ D, an increase by a factor of 1.069 above that of the approximate determination with K = 2. Improved values would be 1.199 if $K\rho_\infty/m_p n_p$ were equated to unity; or 1.195 if, for example, 0.881 were used for K together with 1.16 for $\rho_\infty/m_p n_p$ to allow for a four-percent helium concentration.

These differences may seem small, but they assume considerable significance in the determination of the magnetic field of Mercury from the data of Mercury-I and -III spacecraft. Ness (1974b; 1975a, b) has determined the magnetic dipole field of that planet in three ways; once by comparison of observed and calculated bow wave and magnetosphere crossings using the formula of Choe et al. (1973) with $r_n/D = 1.07$ to determine the scale, and twice by fitting the magnetospheric field by either the first few terms of a harmonic expansion or by an eccentric dipole. The resulting values from Mercury-I are 5.6×10^{22}, 5.1×10^{22}, and 3.3×10^{22} G cm^3, respectively. The first two values are considered to be in good agreement in view of both observational and theoretical uncertainties, and also with the value of 4.8×10^{22} G cm^3 determined from the Mercury-III data using the second procedure. The third is a preliminary value superseded by the results of a more complete analysis of the later papers. A change in r_n/D from 1.07 to 1.2 as described above, leads, however, to a dipole moment of 4.0×10^{22} G cm^3 for the first method, although somewhat larger values could be deduced from other acceptable fits to the bow wave and magnetopause crossings. All of these values support the conclusion that the solar wind is held away from Mercury by the planetary magnetic field; but the revised expression for r_n essentially doubles the value for the critical momentum flux described by Ness (1975b) at which the solar-wind particles could begin to impinge directly on the planet surface.

A further area for improvement involves conditions near the neutral points or cusps at high latitudes near the noon meridian. The $\cos^2 \psi$ pressure law is grossly inadequate for regions of the magnetopause that are nearly parallel, or shielded, from the solar-wind flow. The effort of making more accurate calculations of the shape of the magnetopause in these regions using the $\cos^2 \psi$ relation is thus not rewarded by an increase in accuracy of the prediction. Details of the cusp regions have not been worked out quantitatively, but must be qualitatively as described by Spreiter and Summers (1967). In particular, the supersonic flow cannot negotiate the concave region indicated by the $\cos^2 \psi$ solutions, but must separate and subsequently reattach, leaving an enclosed cusp-shaped pocket of hot plasma between the magnetosphere and the flowing solar wind. Since such a configuration is known to be leaky near the tip of the cusp, plasma of solar origin penetrates into the magnetosphere from these regions, a theoretical prediction well supported by numerous observations.

Improvements have also been made in the flow calculations by seeking exact solutions of the magnetohydrodynamic equations instead of the approximating gasdynamic equations. To date, this has only been accomplished for the case in which the solar-wind magnetic field is aligned with the flow velocity. In that case, $\underset{\sim}{B} = \lambda\rho\underset{\sim}{v}$ where λ is a universal constant, holds everywhere in the flow; and the equations and boundary conditions of magnetohydrodynamics can be transformed without approximation to those of gasdynamics of a pseudogas having an unusual equation of state (Spreiter, Summers, and Alksne, 1966; Rizzi, 1971; and Spreiter and Rizzi, 1974). Figure 4 presents a summary of bow-wave locations for various Alfvén Mach numbers M_{A_∞} between 2.5 and 20 for a single magnetoionopause shape calculated using the $\cos^2 \psi$ approximation, and either a dipole magnetic field or a nonmagnetized ionosphere having a scale height $H = 0.2\, r_0$ as deduced from the data of Mariner-5 and Venera-4 and -6 to be appropriate for Venus. With Mars appearing to possess a significant magnetic field, only Venus appears to fit the latter category, but recent Mariner-10 measurements have been interpreted (Ness et al., 1974a and Bridge et al., 1974) as indicating a



NOTE: CANNOT SIMULATE PROPER BEHAVIOR BY REPLACING M_∞ IN GASDYNAMIC SOLUTION BY $M_\infty M_{A\infty}/(M_\infty^2 + M_{A\infty}^2 - 1)^{1/2}$ AS OFTEN SUGGESTED.

Figure 4. Improved representation of bow-wave location indicated by magnetohydrodynamic solution for aligned field flow.

substantially smaller value than 0.2 for H/r_0. To provide a scale, the relative size of each of the terrestrial planets is indicated. The principal point of this figure, however, is to show how the magnetohydrodynamic solutions differ from those of gasdynamics. It may be observed that the gasdynamic solution, which represents the limit of the magnetohydrodynamic solutions for infinite $M_{A\infty}$, provides a good approximation for $M_{A\infty}$ greater than about 10, but differs notably for lower values. Moreover, the flanks of the bow wave move away from, and the nose moves toward, the planet with decreasing $M_{A\infty}$. This shows immediately that the procedure of replacing M_∞ in the gasdynamic solution by $M_\infty M_{A\infty}/(M_\infty^2 + M_{A\infty}^2 - 1)^{1/2}$, as is frequently done in an attempt to improve the accuracy, is actually of no avail, at least for aligned flow. Lowering the value for M_∞ in the gasdynamic solution moves the bow wave farther from the planet everywhere; it cannot move it away from the planet on the flanks and toward it at the nose.

A further improvement can be sought by matching the magnetoionospheric $p + B^2/8\pi$ with that actually calculated from the magnetohydrodynamic or gasdynamic solutions rather than with the approximate values obtained using the $\cos^2 \psi$ relation along the equatorial plane. As noted previously, the need to consider a three-dimensional rather than axisymmetric flow would lead to significant complication for a magnetic planet. This difficulty does not occur, at least ideally, for a nonmagnetic planet with ionospheric-type interaction with the solar wind. Rizzi (1971) has carried out a calculation along these lines for a nonmagnetic planet using solar wind and ionospheric properties suggested by observations of Mariner-5. A sample of his results is shown in figure 5. Qualitatively, the results are quite similar to those of the previous gasdynamic analysis (Spreiter, Summers, and Rizzi, 1970), but the ionosphere tail is indicated to taper inward rather than outward, and specific values for flow properties differ significantly because of the low value of 6.75 for $M_{A\infty}$. Additional improvements could be made by introducing a better representation for the ionospheric pressure than provided by the use of a constant scale height.

A more fundamental difference is presently emerging between predictions of the magnetohydrodynamic model and a growing consensus of space scientists studying, primarily, charged particles associated with the Earth. The dissipationless magnetohydrodynamic model clearly leads to an impervious or closed magnetosphere boundary, except at the cusps and in the distant tail. Many experimenters are increasingly convinced that the Earth's magnetosphere is open, although there is no well-developed theory or precise definition of what that statement means (see, for example, McCormac and Evans (1975) for a recent review). It is often defined as meaning that the magnetic field lines from the planet connect with those of the solar wind, but where and in what manner is not specified. To what extent this difference is real is difficult to say. Advocates of an open magnetosphere point to a variety of observations and correlations that can be explained on the basis of connecting field lines, but disregard a body of direct plasma and field measurements indicating the presence of a surface in approximately the location of the theoretical magnetopause possessing properties in good correspondence with those of a magnetohydrodynamic tangential discontinuity surface. On the other hand, it should be recognized that the statement that there is no connection in the

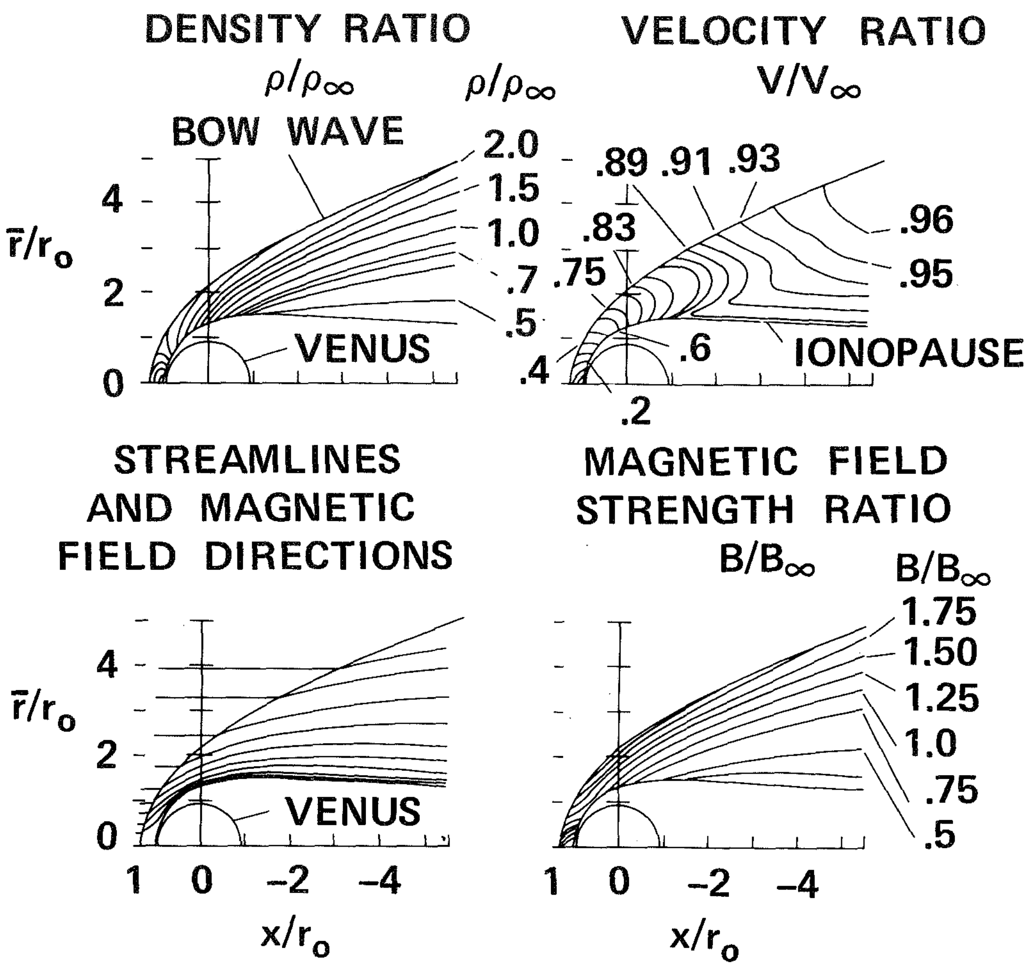


Figure 5. Improved representation of flow-field properties indicated by the magnetohydrodynamic solution for aligned field flow with exact pressure balance at the ionopause, and with conditions selected in accordance with Mariner-5 observations near Venus. $M_\infty = 6.47$, $M_{A\infty} = 6.75$, $\gamma = 5/3$, $H/r_0 = 0.2$.

theoretical model is an obvious idealization that results from the assumption of steady dissipationless flow. It is clear that inclusion of dissipation in the analysis will lead to field merging and connection, but how much occurs and what are the consequences are very difficult questions to answer.

As a further point of difference, Wallis (1972, 1973) has asserted repeatedly that solar-wind interaction with Venus, considered to be a nonmagnetic planet, should be of extended atmospheric interaction type, similar to that of a comet, in which significant ionization processes occur. There does not appear to be widespread support for the idea, (see Cloutier and Daniell (1973) for a commentary) although it is evident that at least some ionizing processes must occur near the ionopause. In any case, analysis of such phenomena is beyond the reach of a single-fluid theory, and must be approached through multi-component theories more typical of plasma studies in which the presence of ions, electrons, and neutrals is considered.

Another point to consider is that observations do not, of course, indicate a zero thickness magnetoionopause or bow wave as indicated by the dissipationless fluid theory. The observed thicknesses are usually small relative to other significant lengths of the overall flow, and are qualitatively understandable in terms of a boundary layer or a viscous shock wave. Conditions associated with these surfaces are frequently fluctuating, however, and not steady as idealized in the usual calculations. Some of the larger-scale fluctuations may be understood in terms of simple extensions of the dissipationless model to include unsteady effects, but successful analysis of the small-scale fluctuations will probably have to await basic advances in turbulence theory. Considering the slow rate of development of ordinary fluid turbulence theory, it will probably be a long time before a satisfactory theory is available for dealing with magnetohydrodynamic turbulence.

Among the various possibilities, there appears to be a significant range of phenomena involving dissipative effects that invite closer examination than has yet been given, although it is also evident that their analysis raises substantial difficulties that must be overcome. In addition to the expected problems of solving the more complicated equations for dissipative magnetohydrodynamics, there exists a major question regarding what are to be used for the coefficients of viscosity, heat conduction, and electrical resistance. It is clear that the simple Coulomb scattering formulae for a nonmagnetized plasma cited by Parker (1963) in his review of solar-wind theory and used in some subsequent analyses of planetary flows are totally inadequate. The expression

$$\mu = 10^{-16}\, T^{5/2} = 0.469\, v_{t_p}\, \rho\, \ell_d \ \text{g/cm s}$$

for the viscosity of fully-ionized hydrogen having a representative value of 22 for the Coulomb logarithm, can be used as an example.

Here,

- T = the temperature in degrees Kelvin (K),
- v_{t_p} = the thermal velocity of the protons,
- ρ = the density, and
- ℓ_d = the effective mean-free path for cumulative deflection of 90° by Coulomb interactions.

For typical solar-wind conditions of $n_p = 10\ \text{cm}^{-3}$, $v = 5 \times 10^7$ cm/s, $T = 10^5$ K, and reference length, D, taken as the radius of the Earth (6.37×10^8 cm), this expression leads to a Reynolds number $R = \rho v D/\mu = 0.002$. Such a value is not at all typical of aerodynamic-like solar-wind flow past a planet; but is more representative of a small ball sinking through tar! Use of such a value would lead to the prediction of enormously thick boundary layers and shock waves, completely different from those observed. There is no dilemma, however, since the particles were assumed in the derivation to travel in straight lines between collisions which, even in the more conservative sense of cumulative small Coulomb deflections, turn out to be separated by mean distances of the order of half an astronomical unit when the above-stated conditions are applied to the formula. This is obviously inappropriate for planetary applications which involve phenomena of much smaller scale.

Part of the answer to this apparent deficiency is provided by the obvious fact that the presence of a magnetic field in the solar wind prevents the particles from traveling in straight lines between collisions, and causes them to spiral along the moving magnetic field lines. This reduces the transport transverse to the field lines approximately as the square of the ratio of the gyroradius of the protons to the distance ℓ_d. If a representative value of 1.4×10^{-6} is used for this ratio, corresponding to a magnetic field of 5×10^{-5} G, the Reynolds number in the example cited above would increase to 9×10^8. Such a value is typical of that encountered in ordinary aerodynamics, and is consistent with the generally good agreement between observations and the results of dissipationless fluid theories, including the concept of relatively thin shock waves and magnetoionopause surfaces.

However, all is not that simple. The magnetic field does not reduce the transport coefficients equally in all directions; in fact, it does not reduce the values for transport parallel to the field lines at all. The dissipative part of the proposed fluid model is thus highly anisotropic. The direction of anisotropy, moreover, depends on the properties of the flow and cannot be specified in advance. Since there is at present virtually no theoretical development of the behavior of such a fluid for any application, the space scientist desiring to explain these features of the flow in terms of an anisotropic dissipative fluid is faced with the task of achieving major theoretical advances or, as is more often the case, being satisfied with hopefully describing what he thinks will happen in qualitative terms based on analogy with the known behavior of isotropic fluids. In view of the extreme anisotropy of the solar-wind plasma and the fact that the Coulomb deflection times upon which the analysis is based are much longer than the times required for solar-wind particles to traverse the significant part of a planetary flow field, it is evident that considerable caution should be exercised in relying on such descriptions. When one goes further into questions of fluctuations and turbulence, either in the main body of the flow or associated with the bow shock or magnetoionopause boundary layer, the difficulties compound, and there seems little hope for definitive analysis in the near future.

CLOSING REMARKS

In summary, a review has been presented of the fluid aspects of solar-wind flow past terrestrial planets, how they are approximated to obtain tractable mathematical problems, and how improvements can be made in the theoretical models currently in use. Some of the latter can be achieved relatively easily, others appear virtually impossible at the present time. It is important that the more promising avenues be explored vigorously, since proper understanding of planetary properties through space exploration can best be achieved by a combined observational-theoretical approach, of which the effort and cost of the theoretical studies is very small compared to that of the experimental program.

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QUESTIONS

Spreiter/Vaisberg: I should like to add one complication to your comprehensive list. Measurements of the spectra of α particles on the Prognoz satellites showed that the α component of the solar wind behaved differently from the proton component. In the magnetosheath, the velocity of the α component is higher than that of the proton component and their directions may differ. That is probably due to their different decelerations on the electric potential barrier of the bow shock due to the different masses and charges of the components. Thus, the ram pressure on the magnetopause may differ more than by simple addition of a second ion component.

Spreiter: Yes, that would certainly seem to be the case.