X-626-76-115
PREPRINT

NASA TM X- 71126

DEPENDENCE OF THE CHARGE EXCHANGE LIFETIMES ON MIRROR LATITUDE

PAUL H. SMITH
N. K. BEWTRA

(NASA-TM-X-71126) DEPENDENCE OF THE CHARGE EXCHANGE LIFETIMES ON MIRROR LATITUDE (NASA) 21 p HC $3.50 CSCL 03B

N76-26135

UNCLAS
G3/93 21172

JUNE 1976

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

Dependence of the Charge Exchange Lifetimes on Mirror Latitude

Paul H. Smith
Laboratory for Planetary Atmospheres
NASA/Goddard Space Flight Center
Greenbelt, Maryland 20771

N. K. Bewtra
Computer Sciences Corporation
Silver Spring, Maryland 20910

ABSTRACT

The dependence of the charge exchange lifetimes on the mirror latitude for ions mirroring off the geomagnetic equator has been re-computed using the improved hydrogen distribution models which have become available since the earlier calculation by Liemohn. The Chamberlain model, with the input parameters determined by recent satellite observations, has been used to define the spatial distribution of the neutral hydrogen environment through which the ring current ions traverse. The resultant dependence of the charge exchange lifetime, τ, on mirror latitude, λ_m, is best fit by the approximation $\tau_m = \tau_e \cos^{3.5}\lambda_m$, where τ_e is the charge exchange lifetime for the equatorial particles. This is a significant change from the $\cos^6\lambda_m$ approximation of the previous results.

The relative importance of the charge exchange mechanism in the recovery of the geomagnetic storm time ring current process has been considered in numerous work since the early suggestion by Dessler and Parker (1959) that this mechanism could be significant. The lifetimes calculated by Liemohn (1961) for the tens of keV ring current protons had been the primary theoretical source for comparing observational results with the charge exchange mechanism (Frank, 1967; Swisher and Frank, 1968; Russell and Thorne, 1970). As Smith et al. (1976) have pointed out, there now exists both more precise cross sections for charge exchange of protons and atomic hydrogen (McClure, 1966), and a much improved hydrogen distribution model (Öpik and Singer, 1961; Chamberlain, 1963), which has been used by Prölss (1973) in computing the proton lifetimes. The dependence of the charge exchange lifetimes on the mirror latitude for ions mirroring off the geomagnetic equator has not been re-examined in light of the improved hydrogen distribution models, and even the most recent articles (Tinsley, 1976; Lyons and Evans, 1976) in the literature still employ the approximation given by Liemohn (1961) under the assumption of the neutral hydrogen density model of Johnson and Fish (1960).

In this letter we use the Chamberlain (1963) model (Öpik and Singer, 1961) to provide the spatial distribution of the neutral hydrogen environment through which the ions traverse. This model appears to be in good agreement with recent Apollo 16 Lyman alpha imagery of the hydrogen geocorona (Carruthers, 1976) and Lyman alpha airglow measurements aboard several satellites (Meier and Mange, 1970; Bertaux and Blamont, 1973; Meier and Mange, 1973). The model assumes a spherically symmetric hydrogen density and is characterized by three parameters: the temperature and density at the exobase, and the satellite critical altitude. Critical satellite altitude, R_{sc}, is defined to be that altitude below which there exist closed orbit satellite particles, and above which the creation of satellite orbits is negligible so that the only orbits present are those that have perigees between the exobase and the critical satellite altitude. In order to determine the functional relationship of the charge exchange lifetime of ions mirroring off the geomagnetic equator, τ_m, to the lifetime of equatorially mirroring ions, τ_e, we performed a full numerical integration for the mirroring particle motion, and fitted the computed ratio, τ_m/τ_e, to a function of the form $\cos^j\lambda_m$, where λ_m is the mirror latitude. The dipole geomagnetic field is assumed in our calculations.

The mean lifetime, τ_e, for protons or any other species of ions confined to the equatorial plane for charge exchange decay with atomic hydrogen is given by

$$\tau_e = \frac{1}{n(r_o)\sigma v} \tag{1}$$

where $n(r_o)$ is the hydrogen density in the equatorial plane, v is the ion velocity and σ is the charge exchange cross section for that ion species. However, an ion oscillating between its mirror points, M and M', Figure 1, encounters a variable instantaneous density, n(r), of neutral hydrogen atoms (Chamberlain, 1963) and, also, the time it needs to traverse an element of length along its guiding field line varies with the changing magnetic latitude. Since the ions make a large number of oscillations before charge exchanging with the neutral hydrogen, we can effectively average the instantaneous hydrogen density over a mirror path to get the time averaged density, $\bar{n}$, that the particle encounters. This is expressed as

$$\bar{n} = \frac{\int_{\lambda=0}^{\lambda_m} n(r) \frac{1}{v_\parallel(r)} \, ds}{\int_{\lambda=0}^{\lambda_m} \frac{1}{v_\parallel(r)} \, ds} \tag{2}$$

The above integration is performed along the magnetic field line

$$r = r_o \cos^2\lambda \tag{3}$$

and so,

$$ds = r_o \cos\lambda \sqrt{4-3\cos^2\lambda} \, d\lambda \tag{4}$$

The velocity parallel to the field line, $v_\parallel(r)$, is expressed for the dipole field (Roederer, 1970) as:

$$v_\parallel(r) = v \sqrt{1 - \frac{(4-3\cos^2\lambda)^{\frac{1}{2}}}{\cos^6\lambda} \sin^2\alpha_o} \tag{5}$$

where α_o is the equatorial pitch angle and is related to the mirror latitude as

$$\sin^2\alpha_o = \frac{\cos^6\lambda_m}{\sqrt{4-3\cos^2\lambda_m}} \tag{6}$$

The mean lifetime, τ_m, of the mirroring particles then is given by

$$\tau_m = \frac{1}{\bar{n}\,\sigma v} \qquad (7)$$

We note that the denominator of equation (2) is one quarter of the bounce period and its evaluation provides us with a consistency check on the numerical integration techniques to be discussed later. The denominator integral to be evaluated thus is simply

$$f(\alpha_o) = \int_{\lambda=o}^{\lambda_m} \frac{\cos\lambda \; (4-3\cos^2\lambda)^{\frac{1}{2}} \; d\lambda}{\left[1 - \frac{\sin^2 \alpha_o \; (4-3\cos^2\lambda)^{\frac{1}{2}}}{\cos^6\lambda}\right]^{\frac{1}{2}}} \qquad (8)$$

This integral was first evaluated by Hamlin (1961) and it has a singularity when $\lambda=\lambda_m$. The present calculation includes derivation of this integral only as a check on the evaluation of the numerator in equation (2). If we introduce a new variable proportional to the magnetic field

$$x = \frac{\sqrt{4-3\cos^2\lambda}}{\cos^6\lambda} \; \sin^2 \alpha_o \qquad (9)$$

equation (8) reduces to a form which can be implicitly represented as

$$f(\alpha_o) = \int_{\sin^2 \alpha_o}^{1} \frac{g(x)}{\sqrt{1-x}} \, dx \qquad (10)$$

Krylov (1962) has developed very powerful techniques for numerical integration using Orthogonal Polynomials. It can be shown (Abramowitz and Stegun, 1965) that

$$\int_a^b \frac{g(y)}{\sqrt{b-y}}\, dy = \sqrt{b-a} \sum_{i=1}^{n} w_i\, g(y_i) + R_n \qquad (11)$$

where

$$y_i = a + (b-a)x_i$$

$$x_i = 1 - \xi_i^{\,2}$$

$$\xi_i = i^{th} \text{ positive root of Legendre polynomial } P_{2n}(x)$$

$$w_i = 2\, w_i^{(2n)},\ w_i^{(2n)} \text{ are the Gaussian weights of order } 2n$$

$$R_n = \frac{2^{4n+1}}{4n+1} \quad \frac{[(2n)!]^3}{[(4n)!]^2} \quad g^{(2n)}(\eta),\ a < \eta < b\ .$$

Thus, the integral is evaluated exactly whenever $g(y)$ is a polynomial of degree $(2n-1)$ or less. In our integration we use $n=32$. In Table 1 we show the excellent agreement between the function $f(\alpha_0)$ calculated using the above method and the exact numerical calculation of Hamlin (1961).

Table 1

Values of $f(\alpha_o)$ from Hamlin (1961) and Present Calculation

Mirror Latitude λ_m (degrees)	Hamlin Values	Present Values
0	0.7405	0.740
4	0.7449	0.742
8	0.7601	0.757
12	0.7842	0.781
16	0.8159	0.812
20	0.8535	0.849
25	0.9063	0.902
30	0.9626	0.956
35	1.020	1.014
40	1.076	1.068
45	1.129	1.119
50	1.179	1.169
55	1.224	1.213
60	1.264	1.252

The ratio τ_m/τ_e can be computed for given mirror latitudes, λ_m, by calculating the two integrals in equation (2) to determine $\bar{n}$, the time averaged neutral hydrogen density, and using equations (1) and (7). The calculated ratios are given in Table 2 for λ_m between 0^o and 60^o, and the corresponding equatorial pitch angle, α_e, is also presented. The temperature at the exobase was taken to be $T_c = 1000^oK$, the critical satellite particle altitude was taken to be $R_{sc} = 2.5R_c$, and the L-value of the mirroring particles was L = 4. R_c is the geocentric distance of the exobase taken at an altitude of 500 km. A least squares fit of the calculated ratio, τ_m/τ_e, to a function of the form $\cos^j\lambda_m$ determined a value of j = 3.4. This is in contrast to the previously used (Liemohn, 1961) function $\cos^6\lambda_m$.

The primary cause of the difference in the value of the exponent of the cosine function is the difference in the neutral hydrogen density distribution between the Johnson and Fish (1960) model which Liemohn (1961) used and the Chamberlain (1963) model which we have employed. In the Johnson and Fish (1960) model the hydrogen density falls off more rapidly as a function of radial distance than it does in the Chamberlain (1963) model (Figure 2).

In Figure 3 we show the calculated ratio of lifetimes compared to the fitted function $\cos^{3.4}\lambda_m$, and compared to the previously used (Liemohn, 1961) function $\cos^6\lambda_m$. As can be seen from Figure 3, the function $\cos^{3.4}\lambda_m$ is a good approximation to the calculated values. The difference between the two functional approximations is not as great for latitudes near 0^o, corresponding to near 90^o equatorial pitch angle, as it is at the larger mirror latitudes. The difference at 40^o mirror latitude (21.6^o equatorial pitch angle), however, represents nearly a factor of 2 increase in the

Table 2

Ratios of Charge Exchange Lifetimes for Mirroring Particles to Equatorial Particles for Various Mirror Latitudes

Mirror Latitude λ_m (degrees)	Ratio τ_m/τ_e	Equatorial Pitch-angle α_e (degrees)
0.	1.	90.
5.	0.988	79.44
10.	0.954	69.17
15.	0.898	59.42
20.	0.823	50.32
25.	0.731	41.97
30.	0.628	34.38
35.	0.519	27.58
40.	0.411	21.56
45.	0.309	16.33
50.	0.218	11.89
55.	0.142	8.24
60.	0.084	5.34

charge exchange lifetimes for ions mirroring at that latitude.

As we stated earlier the hydrogen density in the Chamberlain model depends on three parameters: density, N_c, at the exobase, temperature T_c, at the exobase and the critical satellite particle altitude, R_{sc}. These parameters in turn vary with the geophysical conditions (solar activity, magnetic activity, season, etc.), and the MSIS model of Hedin (private communication, 1976) uses the measured geophysical conditions to determine N_c and T_c. We have considered the effect on our functional approximation, $\tau_m/\tau_c = \cos^j\lambda_m$, of varying these three parameters.

1) The ratio τ_m/τ_e is independent of N_c but the actual lifetime, τ, does depend on the density at the exobase.
2) The ratio of lifetimes does depend weakly on T_c. The variation in τ_m/τ_e for a temperature variation of 900^0K to 1200^0K is shown in Figure 3 at point (b) for $\lambda_m = 30^0$. It should be noted that, for the major geomagnetic storm periods during the early life of the Explorer 45 (S^3-A) satellite (Smith and Hoffman, 1974), Hedin's MSIS model (private communication, 1976) determines T_c to fall in the range 970^0K to 1050^0K. In computing the ratios presented in Table 2, $T_c = 1000^0$K was used.
3) The recent observations of the Apollo 16 Lyman alpha measurements are adequately described by a value of $R_{sc} = 2.5R_c$ particularly in the region near $3R_E$ (Carruthers, 1976). However, for the purposes of examining the effect on τ_m/τ_e we have considered a range of R_{sc} values. The variation in τ_m/τ_e for R_{sc} between

$2.25R_c$ and $2.75R_c$ is shown in Figure 3 at point (c) for $\lambda_m = 40^0$.

The effects on τ_m/τ_e of either a T_c variation (900^0K to 1200^0K) or an R_{sc} variation ($2.25R_c$ to $2.75R_c$) are quite small. We have computed τ_m/τ_e for different combinations of T_c and R_{sc} in the stated ranges and the maximum variations in the ratio are shown in Figure 3 at points (a) for $\lambda_m = 20^0$ and $\lambda_m = 50^0$. The ratio τ_m/τ_e was again fitted to the $\cos^j\lambda_m$ function, with the points weighted inversely by the spread in τ_m/τ_e at each λ_m, and $j = 3.5$ was determined. In fitting the ratio using various extreme combinations we found, for T_c between 900^0K and 1200^0K, for R_{sc} between $2.25R_c$ and $2.75R_c$ and for L-values between 2.5 and 10, that j falls in the range $j = 3.5 \pm 0.2$. Recall, that for $T_c = 1000^0$K, $R_{sc} = 2.5R_c$ and $L = 4$, the functional approximation was $\tau_m = \tau_e \cos^{3.4}\lambda_m$.

This new determination of the dependence of the charge exchange lifetimes on the mirror latitude for ions mirroring off the geomagnetic equator requires that previous agreements or disagreements of the charge exchange mechanism with satellite observations need to be re-examined using at least the general relation

$$\tau_m = \tau_e \cos^{3.5 \pm 0.2} \lambda_m .$$

Acknowledgement

We wish to thank R. E. Hartle, R. A. Hoffman, and D. P. Stern for their very helpful discussions on this letter.

REFERENCES

Abramowitz, M., and J. A. Stegun, Handbook of Mathematical Functions, National Bureau of Standards Applied Mathematics Series, U.S. Government Printing Office, Washington, D. C., 1965.

Bertaux, J. L., and J. E. Blamont, Interpretation of OGO-5 Lyman alpha measurements in the upper geocorona, J. Geophys. Res., 78, 80-91, 1973.

Carruthers, G. R., T. Page, and R. R. Meier, Apollo 16 Lyman alpha imagery of the hydrogen geocorona, J. Geophys. Res., 81, 1664-1672, 1976.

Chamberlain, J. W., Planetary coronal and atmospheric evaporation, Planet. Space Sci., 11, 901, 1963.

Dessler, A. J., and E. N. Parker, Hydromagnetic theory of geometric storms, J. Geophys. Res., 64, 2239, 1959.

Frank, L. A., On the extraterrestrial ring current during geomagnetic storms, J. Geophys. Res., 72, 3753-3767, 1967.

Hamlin, D. A., R. Karplus, R. C. Vik, and K. M. Watson, Mirror and ozimuthal drift frequencies for geomagnetically typed particles, J. Geophys. Res., 66, 1-4, 1961.

Johnson, F. S., and R. A. Fish, The telluric hydrogen corona, Astrophys. J., 131, 502-515, 1960.

Krylov, V. I., Approximate Calculations of Integrals, MacMillan Company, New York, N. Y., 1962.

Liemohn, H., The lifetimes of radiation belt protons with energies between 1 KeV and 1 MeV, J. Geophys. Res., 66, 3593-3595, 1961.

Lyons, L. R., and D. S. Evans, The inconsistency between proton charge exchange and the observed ring current decay, SEL preprint #244, 1976.

McClure, G. W., Electron transfer in proton-hydrogen-atom collisions: 2-117 KeV, Phys. Rev., 148, 47-54, 1966.

Meier, R. R., and P. Mange, Geocoronal hydrogen: An analysis of the Lyman alpha airglow observed from OGO-4, Planet. Space Sci., 18, 803-821, 1970.

Meier, R. R., and P. Mange, Spatial and temporal variations of the Lyman alpha airglow and related atomic hydrogen distributions, Planet. Space Sci., 21, 304-327, 1973.

Öpik, E. J., and S. F. Singer, Distribution of density in a planetary exosphere II, Phys. Fluids, 4, 221, 1961.

Prölss, G. W., Decay of the magnetic storm ring current by the charge-exchange mechanism, Planet. Space Sci., 21, 983-992, 1973.

Roederer, J. G., Dynamics of Geomagnetically trapped radiation, Physics and Chemistry in Space, Vol. 2, New York, Springer-Verlag, 1970.

Russell, C. T., and R. M. Thorne, On the structure of the inner magnetosphere, Cosmic Electrodynamics, 1, 67-89, 1970.

Smith, P. H., and R. A. Hoffman, Direct observations in the dusk hours of the characteristics of the storm time ring current particles during the beginning of magnetic storms, J. Geophys. Res., 79, 966-971, 1974.

Smith, P. H., R. A. Hoffman, and T. A. Fritz, Ring current proton decay by charge exchange, J. Geophys. Res., 81, (in press), 1976.

Swisher, R. L., and L. A. Frank, Lifetimes for low-energy protons in the outer radiation zone, J. Geophys. Res., 73, 5665-5672, 1968.

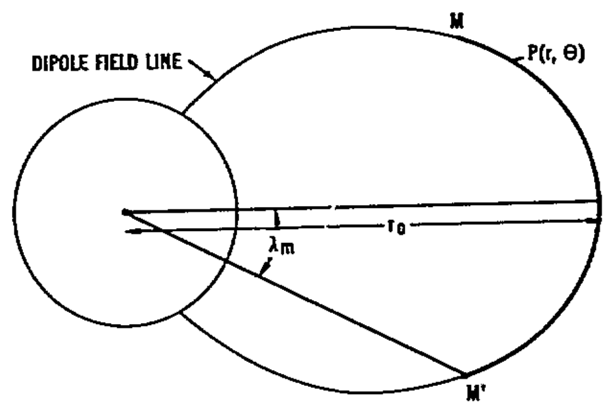
Tinsley, B. A., Evidence that the recovery phase ring current consists of helium ions, University of Texas at Dallas preprint, 1976.

FIGURE CAPTIONS

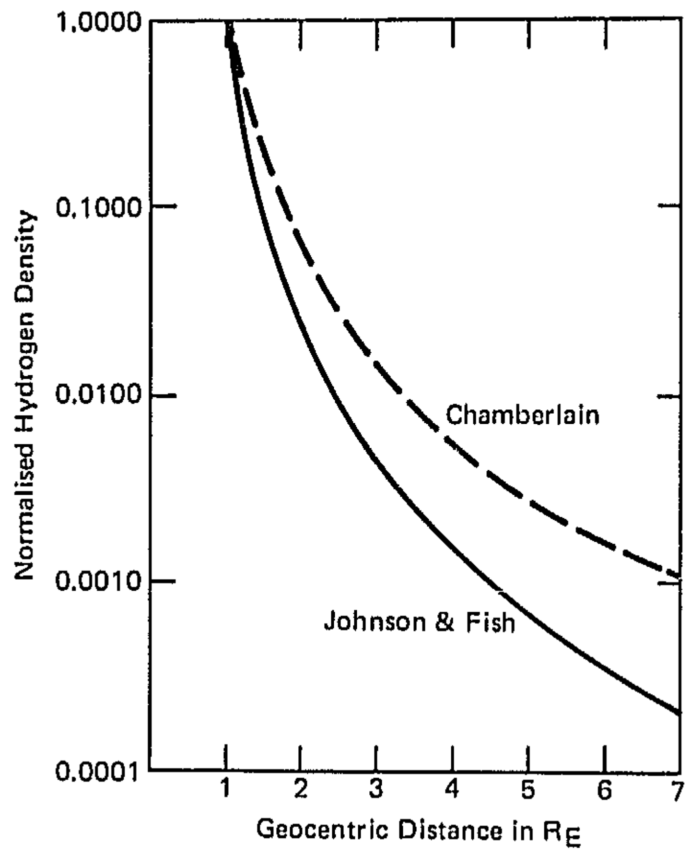
Figure 1. Cross section of earth dipole field in a magnetic meridian plane.

Figure 2. Functional dependence of atomic hydrogen density on the radial distance from center of earth as given by the Johnson and Fish model (shown as solid line) and the Chamberlain model (shown as dashed line). Values of $T_c = 1200^oK$ and $R_{sc} = 2.5R_c$ are used. A unit density is assumed for each model at the exobase (500km).

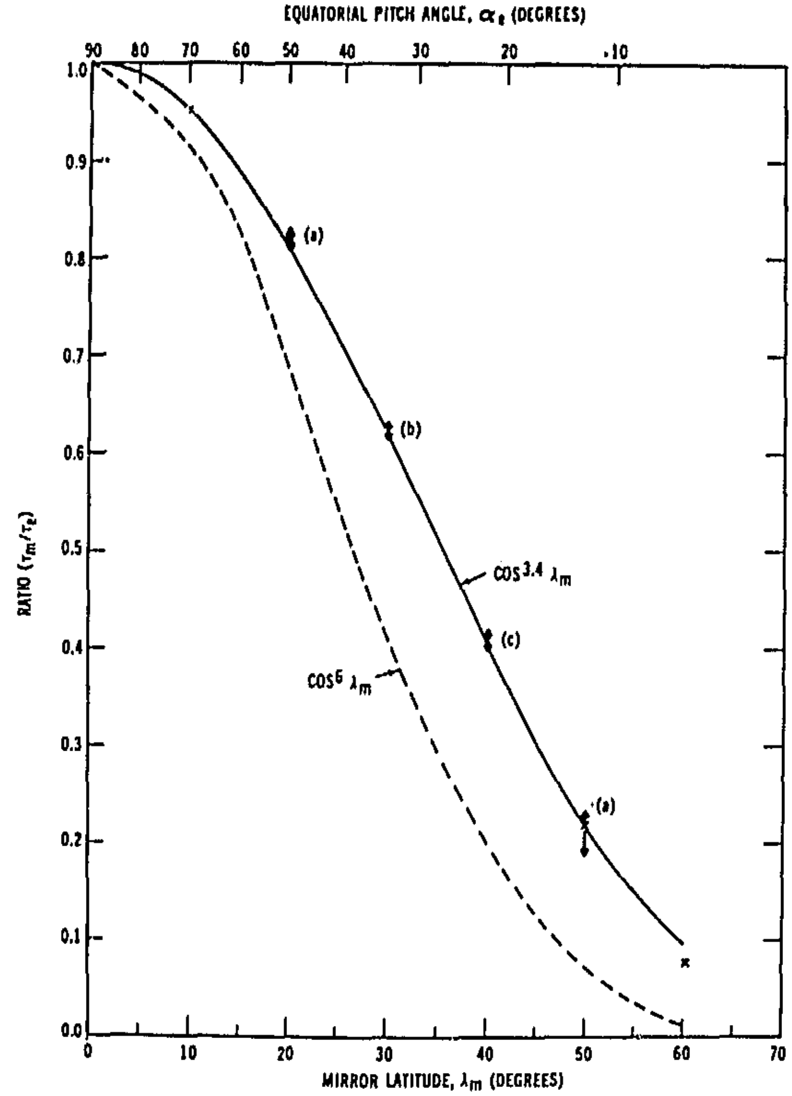
Figure 3. Ratios of lifetimes of mirroring particles to equatorial particles as a function of mirror latitude (and of equatorial pitch angle). Values shown by the x's correspond to input parameters of $T_c = 1000^oK$ and $R_{sc} = 2.5R_c$ in the Chamberlain model. The approximate functional fit, $\cos^{3.4}\lambda_m$, is shown by the solid line. For comparison, the function corresponding to $\cos^6\lambda_m$ (Liemohn, 1961) is shown by the dashed line. Variations in the ratio for ranges of R_{sc} between $2.25R_c$ and $2.75R_c$ and T_c from 900^oK to 1200^oK are shown by points (a) ($\lambda_m = 20^o$ and 50^o). Variations in the ratios are shown for fixed $R_{sc} = 2.5R_c$ and T_c ranging from 900^oK to 1200^oK at point (b), and for fixed $T_c = 1000^oK$ and R_{sc} ranging from $2.25R_c$ to $2.75R_c$ at point (c).



DIPOLE FIELD LINE
M
P(r, Θ)
r0
λm
M'



1.0000
0.1000
0.0100
0.0010
0.0001
Normalised Hydrogen Density
Chamberlain
Johnson & Fish
1
2
3
4
5
6
7
Geocentric Distance in R_E



EQUATORIAL PITCH ANGLE, α_e (DEGREES)
90
80
70
60
50
40
30
20
10
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
RATIO (τ_m/τ_e)
(a)
(b)
(c)
(a)
$\cos^{3.4}\lambda_m$
$\cos^6\lambda_m$
0
10
20
30
40
50
60
70
MIRROR LATITUDE, λ_m (DEGREES)