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A HANDBOOK FOR THE ESTIMATION OF AIRSIDE DELAYS AT MAJOR AIRPORTS (QUICK APPROXIMATION METHOD)

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16. Abstract

The handbook contains a set of curves that allow estimation of the average number of total daily delay minutes at a major airport under a variety of conditions. Demand profiles at each airport are classified with respect to the number of daily peak periods, the percentage of daily flights during peak periods, and the number of peak period operations at the airport. When combined with the saturation capacity of the airport, these descriptors provide sufficient information to allow usage of the handbook.

Examples illustrating the use of the handbook are provided, as well as a brief review and description of the technical approach and of the computer package developed for this purpose.

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Chapter I. Introduction

The estimation of average and total airside delays and delay costs at major airports requires considerable and time-consuming effort, usually centered on an analysis based either on queuing theory or on computer-supported simulation. Alternatively (and preferably, if one can afford it) an extensive data-collection program on delays at the airport of interest can be initiated. Such a program unfortunately must often be carried out over long periods of time and is fraught with statistical pitfalls. Besides, any amount of information is of little value to future planning and forecasting if it is not coupled with an understanding of the underlying relationships between capacity, demand and delays at the airport.

As a means of by-passing such difficulties, the work described here is aimed at providing a simple and practical tool for estimating delay-related statistics quickly and inexpensively. In a way, it is an attempt to provide planners and airport administrators alike with an easy-to-use "handbook" from which airport delays can be obtained using only knowledge of a few basic variables associated with any given airport.

The basic quantity with which the handbook deals is that of average total daily delays (TDDEL), i.e. the total delays suffered in the course of a typical day by aircraft attempting to use the runways of an airport. The delays referred to here are solely those due to normal runway congestion and do not reflect problems that may be due, for instance, to exceptional weather conditions or to other causes. No distinction is made between delays suffered by landing aircraft which have to queue in the air and those suffered by departing aircraft waiting on the ground (the latter being obviously a less severe condition).

It should also be emphasized at the outset that delay estimates provided through this method lay no special claim to extreme accuracy. It is believed however that good approximations (more than adequate for most planning purposes) will most often be obtained. Exceptions do exist, as described in Chapter 2 and in Chapter 3 (which also discuss the question of accuracy in some detail).

Chapter 2 summarizes the technical approach used in arriving at the main product of this work, the TDDEL graphs. The theoretical methodology, the sequence of assumptions used, the computational approach, and a brief discussion of the accuracy and sensitivity of the results are presented in that order.

Chapter 3 is intended as (and written in the form of) a self-sufficient user's guide for the estimation of delay statistics through the TDDEL graphs. It also contains several numerical examples illustrating the use of this tool. The reader who is not interested in the technical details may want to omit Chapter 2 and read Chapter 3 only with no loss of continuity.

Chapter II: Technical Discussion

A. The Computer Programs

The primary tool used for the computation of total daily delays (TDDEL) at airports was the DELAYS set of computer programs which has been developed at the Flight Transportation Laboratory of M.I.T. These programs have been described elsewhere [1] . A summary description of the methodology used by the programs is provided in Appendix I.

Briefly, the programs are used as follows:

i) The input information consists of: the hourly profile of total demand at the airport of interest (total of demanded landings and take-offs); the hourly profile of saturation (or "maximum throughput") capacity at the airport; and the number of runways in use at the airport (for a discussion of the issue of dependent vs. independent runways and the consequent adjustments in airport capacity, the reader is referred to [1]).

ii) The output of the computer programs provides estimates on various delay-related statistics including: the probabilities $p_i(t)$ of having a queue of i aircraft at the airport at time t; the profile of the average queue length during a typical day at the airport; the profile of average delays due to congestion during a typical day; and cumulative statistics for a day such as (average) total delay minutes, (average) total delay costs, average queue length during the day, etc. The quantity of concern in the work under discussion here is the (average) total daily delay (TDDEL) minutes at airports.

iii) In order to compute the various quantities just mentioned, the computer programs obtain upper bound estimates and lower bound estimates for each quantity of interest. A weighted average is then computed from these two limits. The upper bound estimates are computed from a so-called M/M/k queuing model and the lower bound from a M/D/k queuing model (see [1]). Throughout this report the weighting formula used to compute average total daily delay TDDEL is:

$$TDDEL = 1/3\ (TDDEL_{M/M/k}) + 2/3\ (TDDEL_{M/D/k})$$

That is, the upper bound estimate of average total daily delays receives a weight of 1/3 and the lower bound a weight of 2/3. The details and validity of this procedure are discussed in reference [1].

B. Daily Demand Profiles at Major Airports

The daily demand profiles used as inputs for the computation of total daily delays were selected carefully with the aim of rendering the products of this work extensively applicable. For the purpose of identifying the most typical demand profiles at major commercial airports, the two most recent available editions (referring to operations in November 1973 and August 1974) of the publication Profiles of Scheduled Air Carrier Airport Operations: Top 100 U.S. Airports issued by the Aviation Forecast Division of the Federal Aviation Administration were reviewed.

A computer program which (i) "normalized" the demand profiles by dividing the hourly total number of operations at each airport by the total daily number of operations, and (ii) plotted the resulting demand profiles was utilized in order to examine the various types of profiles. (Note that the "normalization" procedure brings all profiles to a common unit namely "hourly demand as a percentage of total daily demand"). On the basis of this procedure, it was decided to use the following two descriptors of demand profiles:

a) <u>The number of daily peaks in demand</u>:

Three classes of demand profiles were identified in this respect:

i) Double peak demand profiles: these profiles exhibit the classical, "textbook" pattern of demand with two quite similar peak demand periods, one associated with the morning peak period and the other with that of the evening. The double peaking pattern seems to be the most common for the airports reviewed. However, few of the largest airports fall into this category.

ii) Single peak demand profiles: these profiles exhibit a distinct, single, more severe, and rather prolonged peak period (usually lasting five or six hours). Such a peaking pattern may be due to special circumstances, most often heavy international traffic, or geographical location, or heavy pleasure traffic at the airport.

iii) No peak (or "uniform") demand profiles: in these cases, the number of operations remains practically constant throughout most of the normal activity hours. The uniformity of demand in these cases is often largely due to capacity problems that force "rationing" of runway slots (a "quota system").

b) <u>Peak hour operations as a percent of total daily operations</u>:

While the number of peak periods (our first descriptive characteristic) is indicative of the general shape of the demand profile, the "peak hour operations as a percent of total daily operations" is a rough indicator of the sharpness of the "peaks and valleys" in the demand profile. Examination of the profiles for the 100 busiest airports led to identification of four categories in this respect, namely (i) 7%, (ii) 8% (iii) 9% and (iv) 10% peaking factors, where:

$$PF = \text{peaking factor} = \frac{\text{no. of operations during peak hour of day}}{\text{total no. of operations during the day}}$$

On the basis of the above a total of 10 basic demand profiles were constructed for the following cases:

1) No peak, 7% - peak-hour profile (NP7)

2) One peak, 7% - peak-hour profile (OP7)

3) Two peak, 7% - peak-hour profile (TP7)

4) No peak, 8% - peak-hour profile (NP8)

5) One peak, 8% - peak-hour profile (OP8)

6) Two peak, 8% - peak-hour profile (TP8)

7) One peak, 9% -peak-hour profile (OP9)

8) Two peak, 9% - peak-hour profile (TP9)

9) One peak, 10% - peak-hour profile (OP10)

10) Two Peak, 10% - peak-hour profile (TP10)

Several remarks are in order at this point:

First, we note that only 10 combinations have been used instead of the possible 12(=3X4). The reason is that no profiles of the "no peak, 9%" and "no peak, 10%" type were observed. This could be expected, since the "no peak" situation is associated with cases in which airports operate at high levels of utilization (and, therefore, operations have to be spread out quite evenly during the course of a day). Consequently, the operations during the peak hour can not constitute a high fraction of all the daily operations, given that many operations take place at times other than the peak hour.

Second, the profiles observed also included many cases in which the peak hour operations constituted 11%, 12% or more of the total for the day. These cases however, invariably involved airports with extremely low operation levels (peak hour operations of 10 or 15 per hour) and, consequently, with obviously insignificant congestion problems. For this reason these cases were ignored.

Similarly, there were cases in which more than two traffic peaks could be identified in the course of a day. There was, however, too much variety within this class of profiles to be identifiable as a separate class. Delay estimates in cases where profiles exhibit a three - (or more) peak pattern can probably be obtained approximately from the "no peak" or the "two peak" cases. This point is further discussed later in this chapter.

A fourth remark concerns the construction of the specific profiles from which the delay estimates were computed. Obviously, one needs considerably more than the number of peaks and the percentage of operations during the peak hour of the day for a complete description of airport demand during an average daily cycle. A couple of basic guidelines were therefore drawn for the purpose of constructing the detailed profiles:

i) It was observed from the review of the profiles of the top 100 United States airports, that - almost without exception - the level of operations for at least nine consecutive hours of a day is reduced to a minimum although not necessarily to zero. For the purpose of standardization, it was then assumed that, in all typical profiles, the total operations performed from 22:00 to 7:00 would amount to 10% of the daily total (2% from 22:00 to 23:00 and 1% thereafter). Delays, during this period, are of course negligible but were computed nevertheless. (A recent survey of United States airports conducted by McDonnell-Douglas, concluded that operations from 23:00 to 5:00 constitute approximately 5% of total daily operations at the 59 largest airports Our approximation, therefore, appears to be of the correct order of magnitude).

ii) For most of the 15 remaining hours of the day a significant amount of activity was assumed (≥ 4% of daily operations). From observation, afternoon and evening peak periods seem to last longer than those in the morning and this was incorporated in the profiles used. The specific profiles were finally drawn up with an eye toward approximating to the extent possible, patterns actually observable at a number of locations.

The final resulting ten typical profiles are presented in Figures 1 through 10. Hourly operations (50% landings and 50% take-offs) are plotted by hour of the day as percentage of total daily operations. The precise percentages used for each hour (beginning at midnight) are also listed for each one of the 10 figures.

Figures 11 and 12 compare two of the typical profiles used with a few actual (normalized) demand profiles at major airports. Specifically, Figure 11 compares the TP8 profile with the profiles at Cincinnati (CVG) and Newark

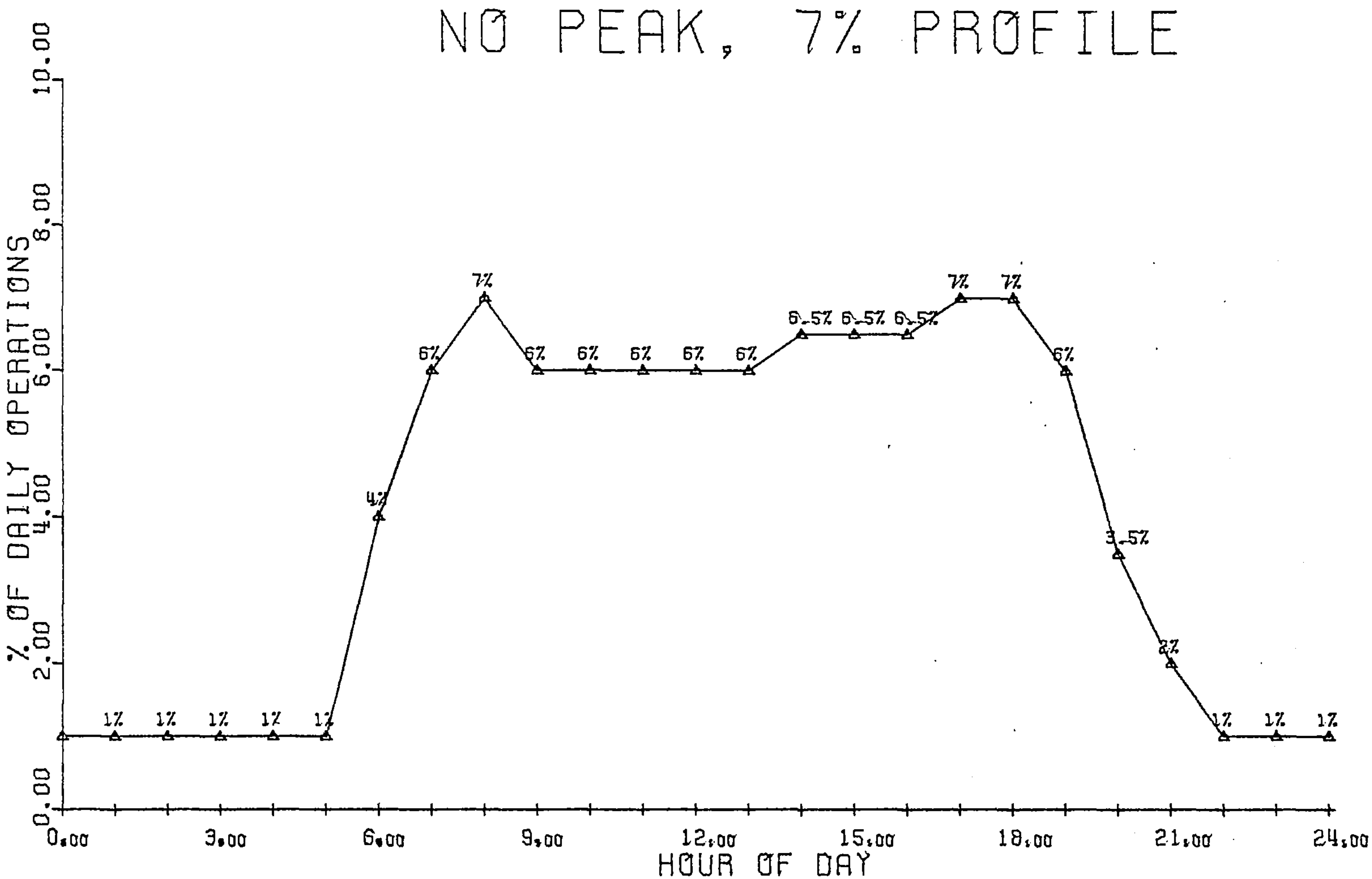

NO PEAK, 7% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0.00
3.00
6.00
9.00
12.00
15.00
18.00
21.00
24.00
1% 1% 1% 1% 1%
4%
6%
7%
6% 6% 6% 6% 6%
6.5% 6.5% 6.5%
7% 7%
6%
3.5%
2%
1% 1% 1%

FIGURE 1.

ØNE PEAK, 7% PRØFILE

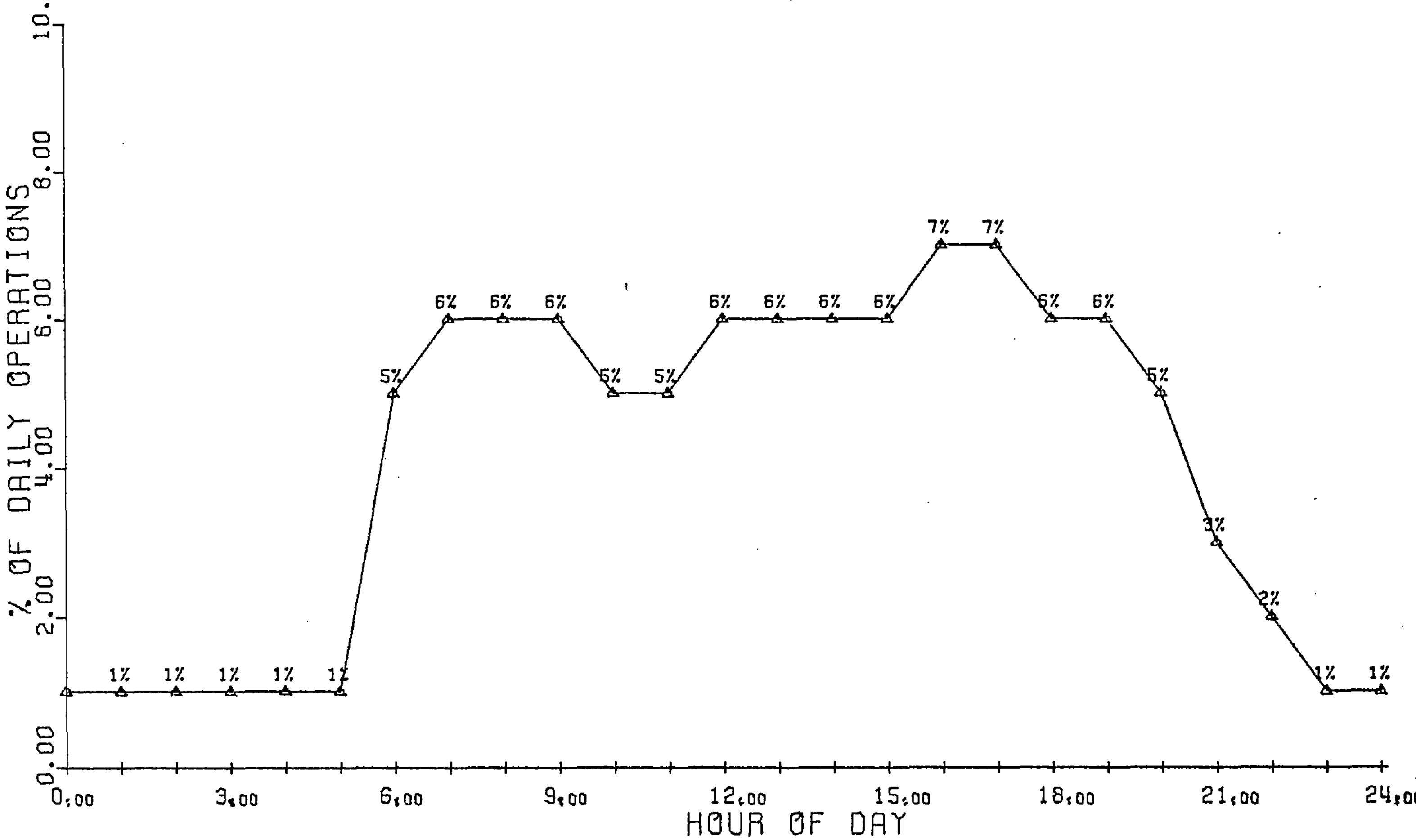


FIGURE 2.

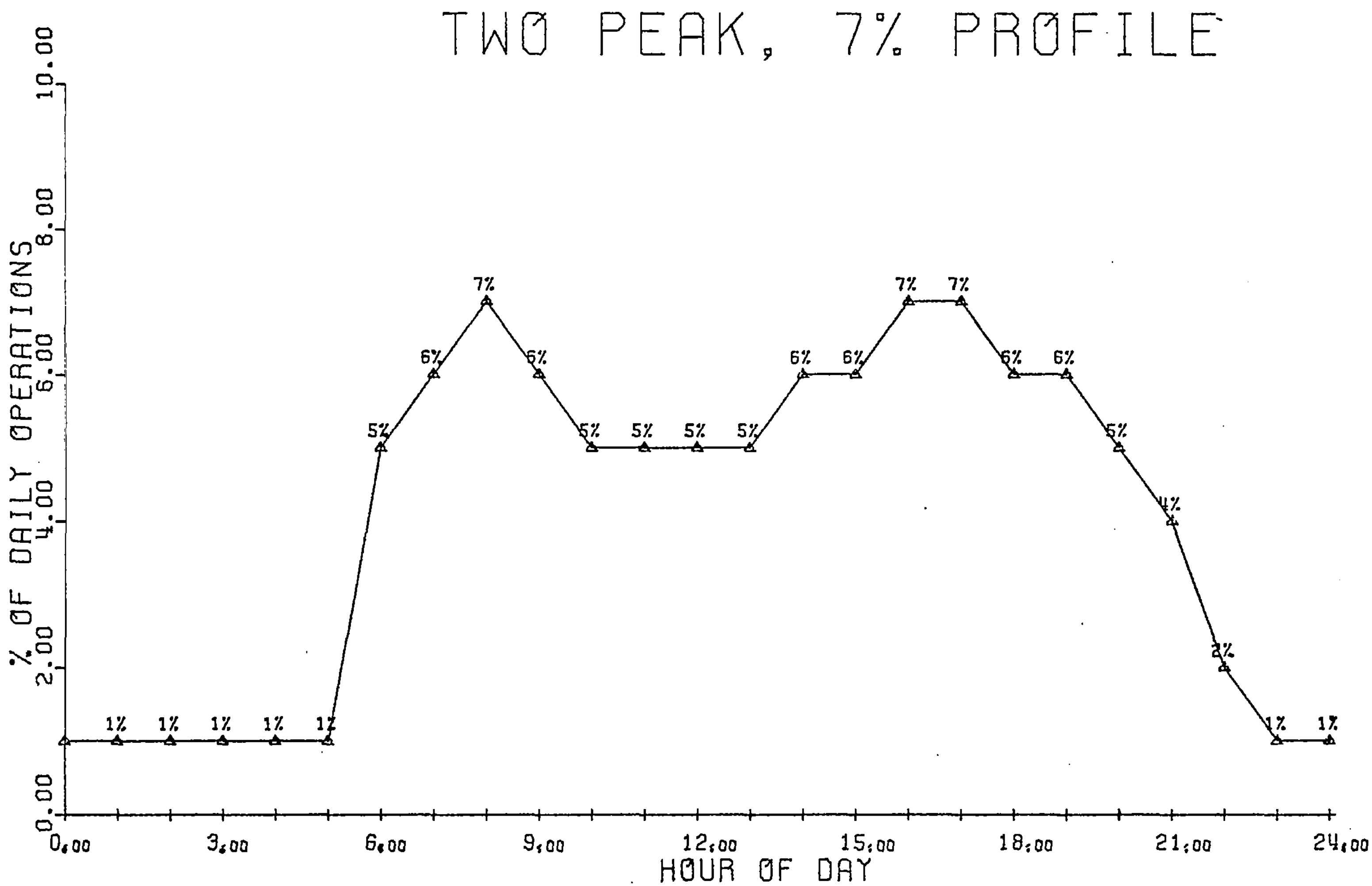
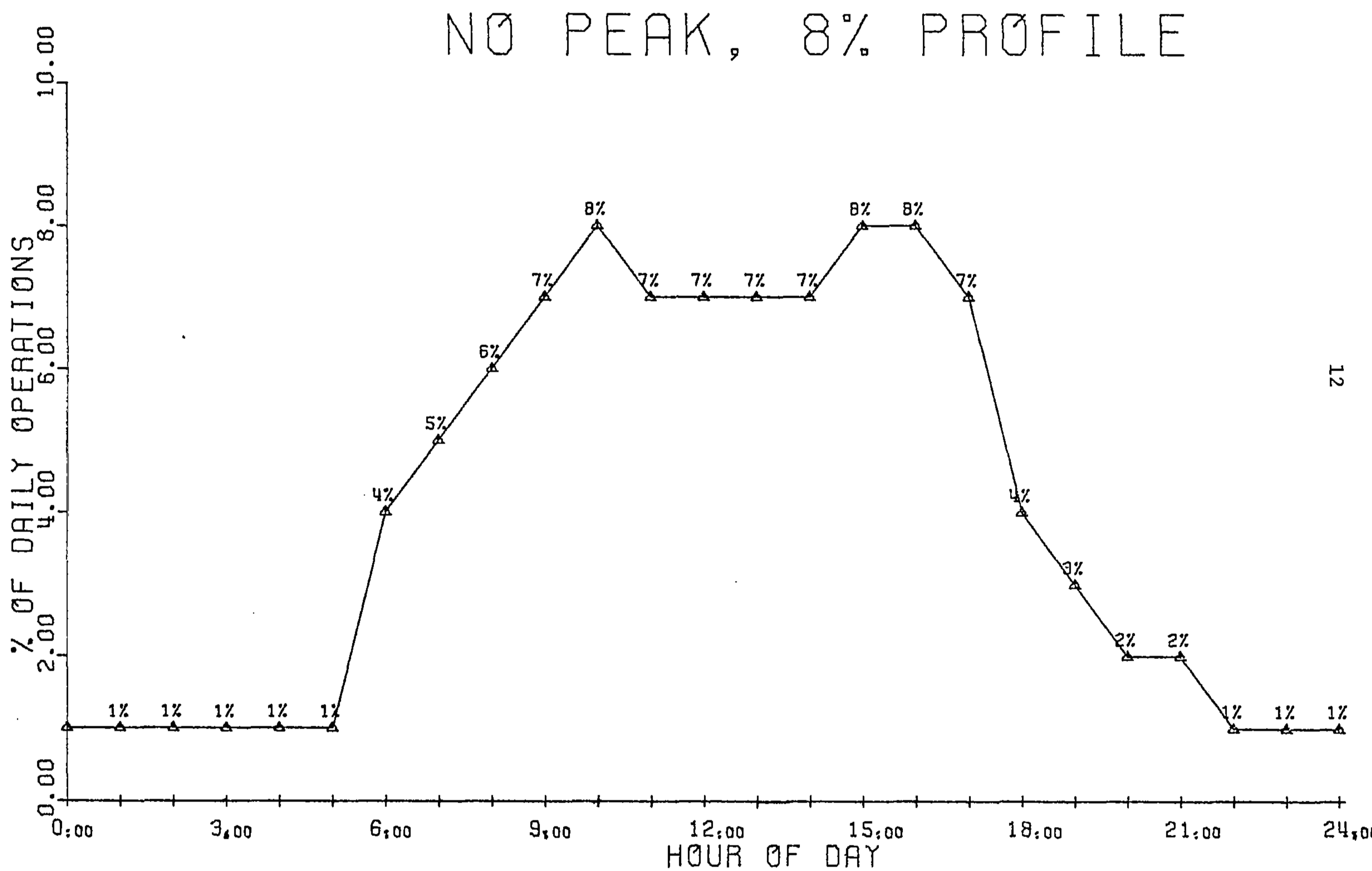

TWO PEAK, 7% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0.00
3.00
6.00
9.00
12.00
15.00
18.00
21.00
24.00
1%
1%
1%
1%
1%
5%
6%
7%
6%
5%
5%
5%
5%
6%
6%
7%
7%
6%
6%
5%
4%
2%
1%
1%

FIGURE 3.



NO PEAK, 8% PROFILE
% OF DAILY OPERATIONS
HOUR OF DAY
0.00
2.00
4.00
6.00
8.00
10.00
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
24:00
1%
1%
1%
1%
1%
4%
5%
6%
7%
8%
7%
7%
7%
7%
8%
8%
7%
4%
3%
2%
2%
1%
1%
1%

FIGURE 4.

ØNE PEAK, 8% PRØFILE

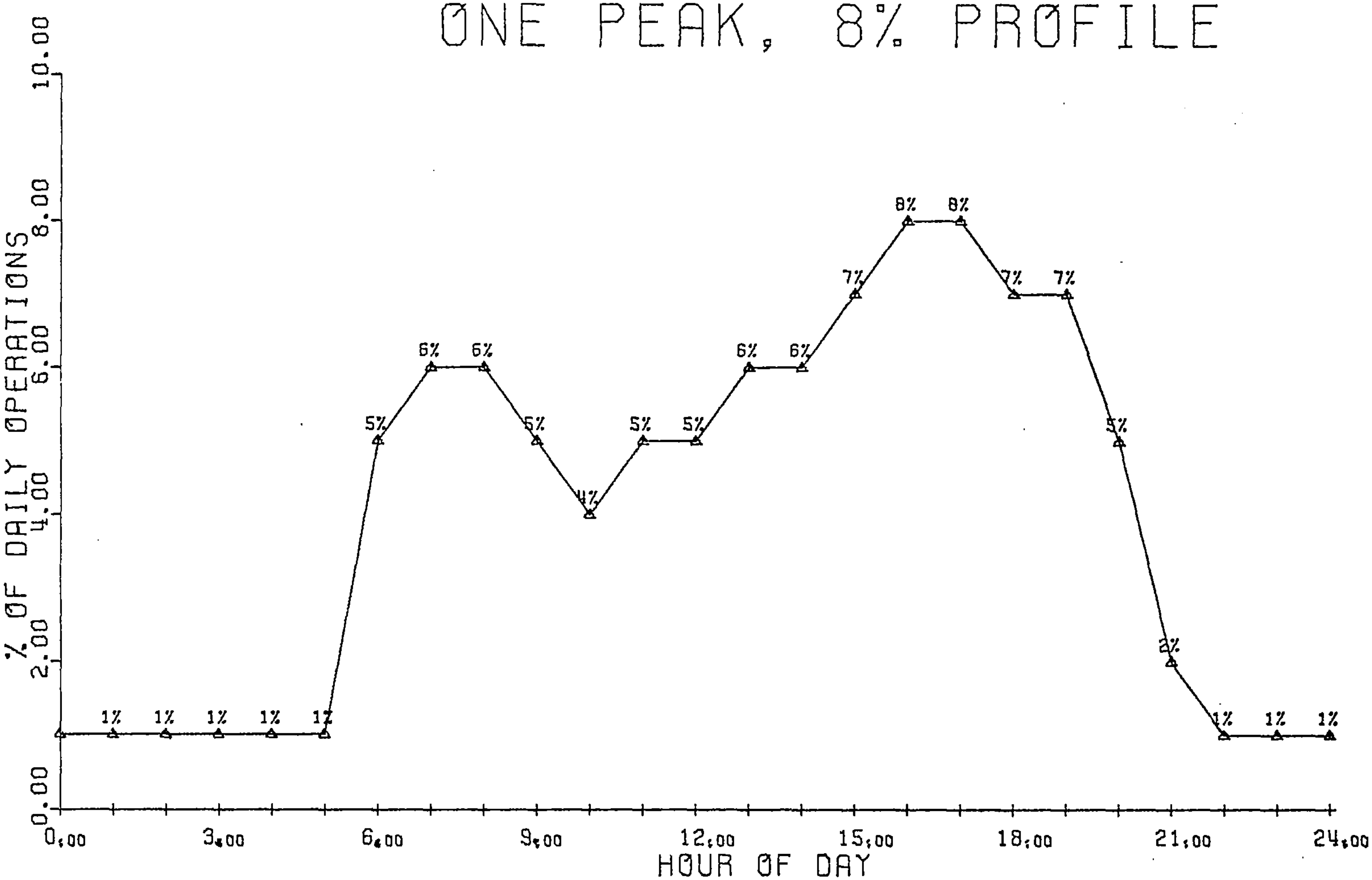


FIGURE 5.

TWO PEAK, 8% PROFILE

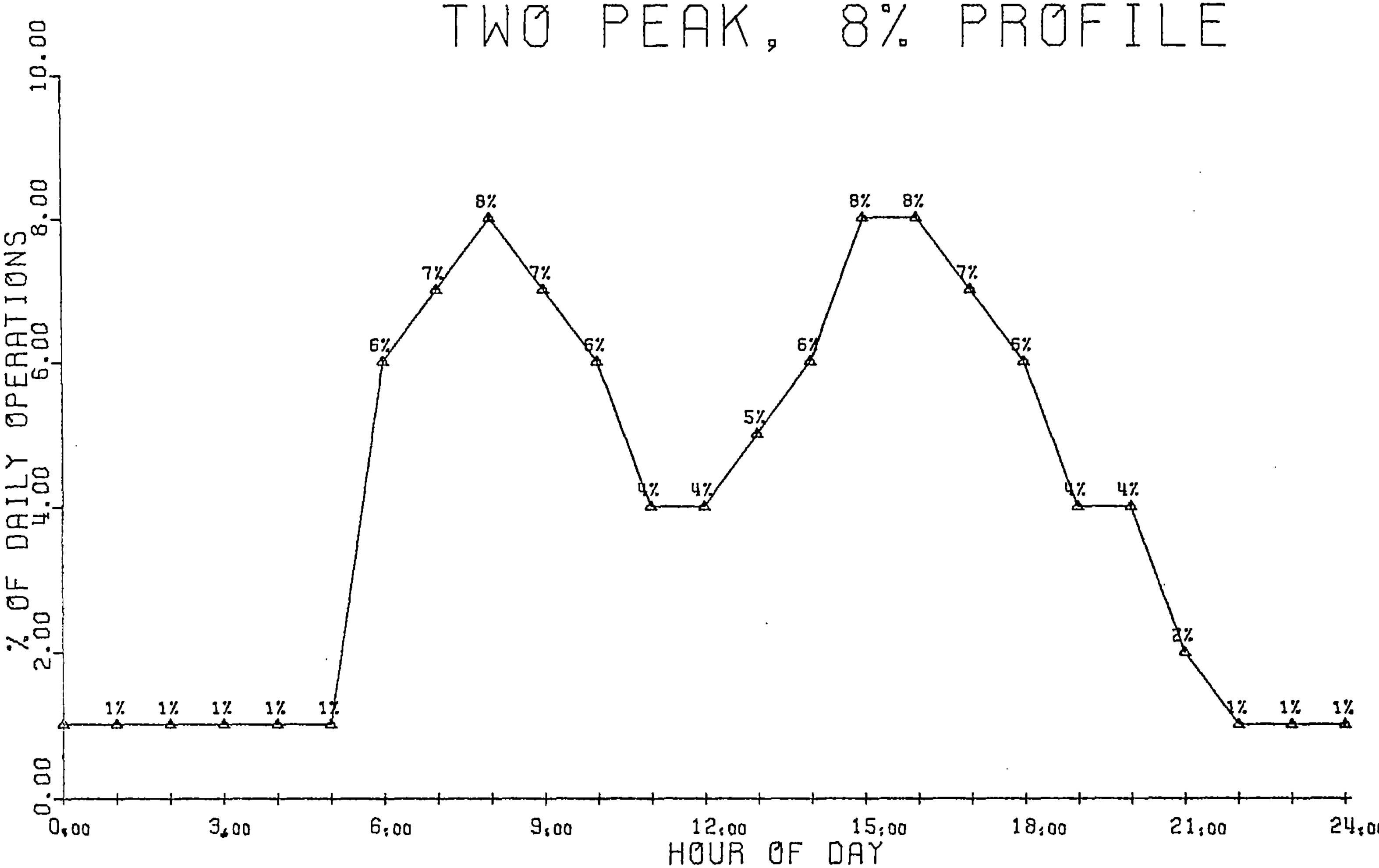


FIGURE 6.

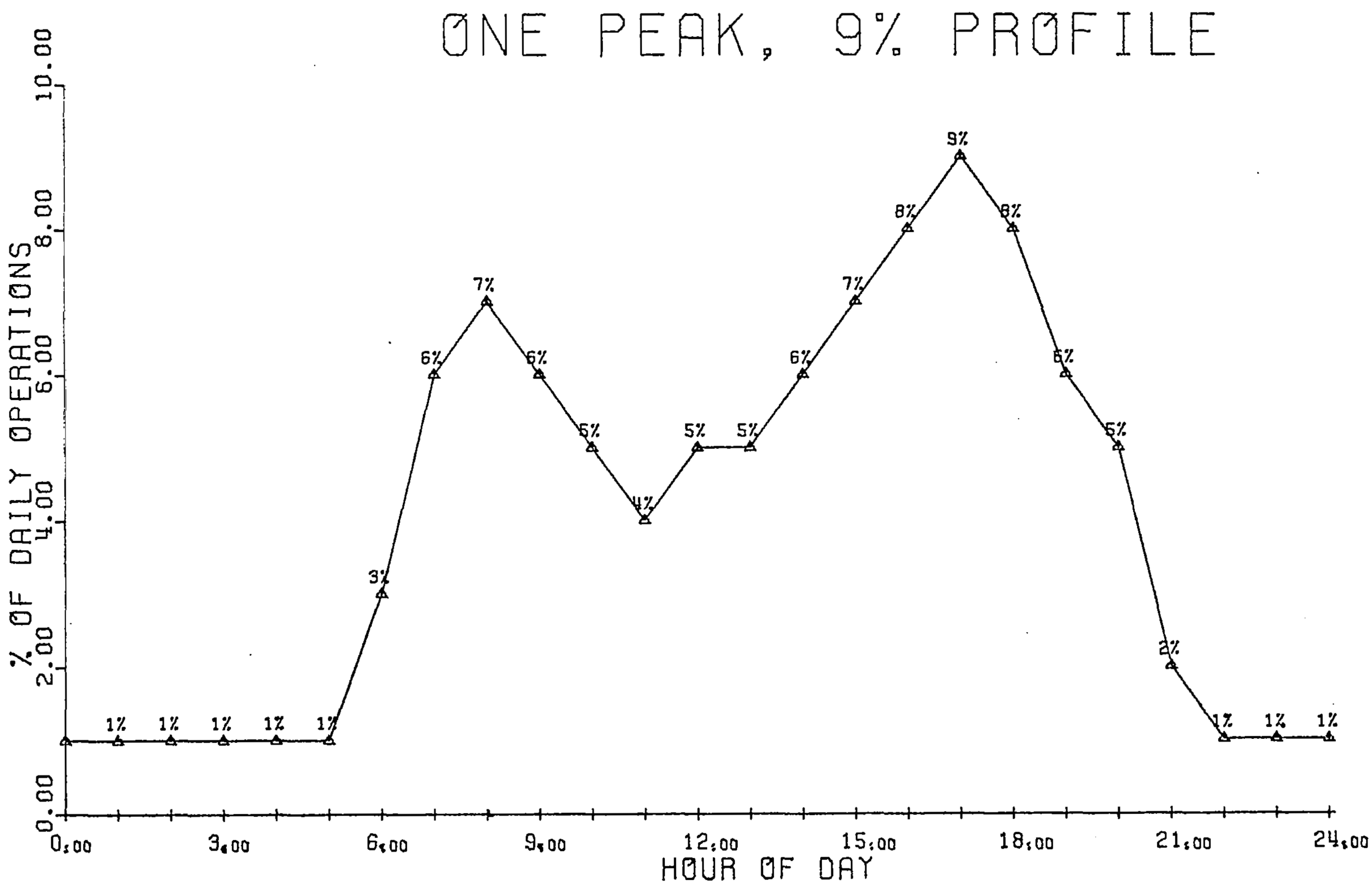

ONE PEAK, 9% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
24:00
1%
1%
1%
1%
1%
3%
6%
7%
6%
5%
4%
5%
5%
6%
7%
8%
9%
8%
6%
5%
2%
1%
1%
1%

FIGURE 7.

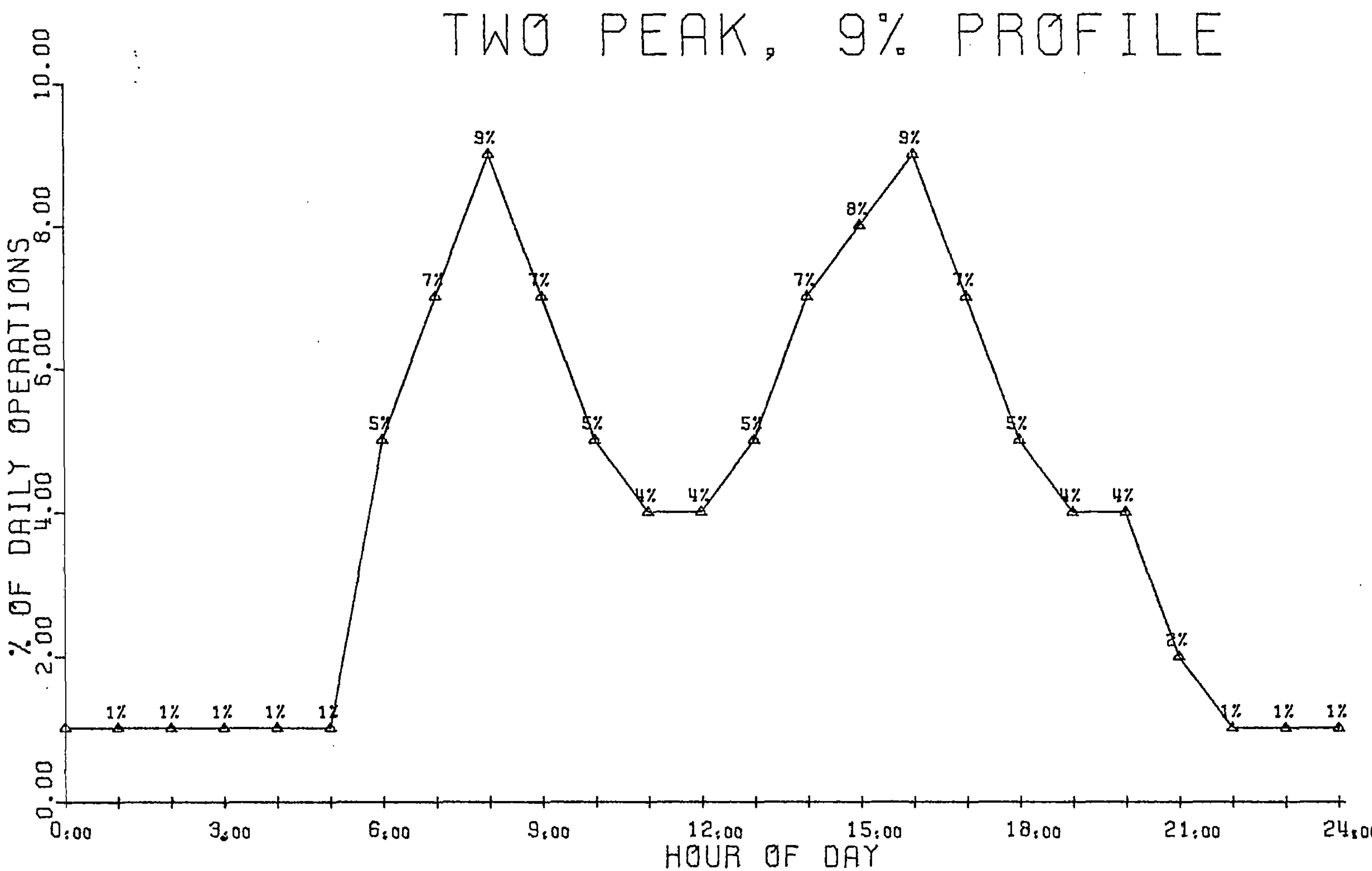
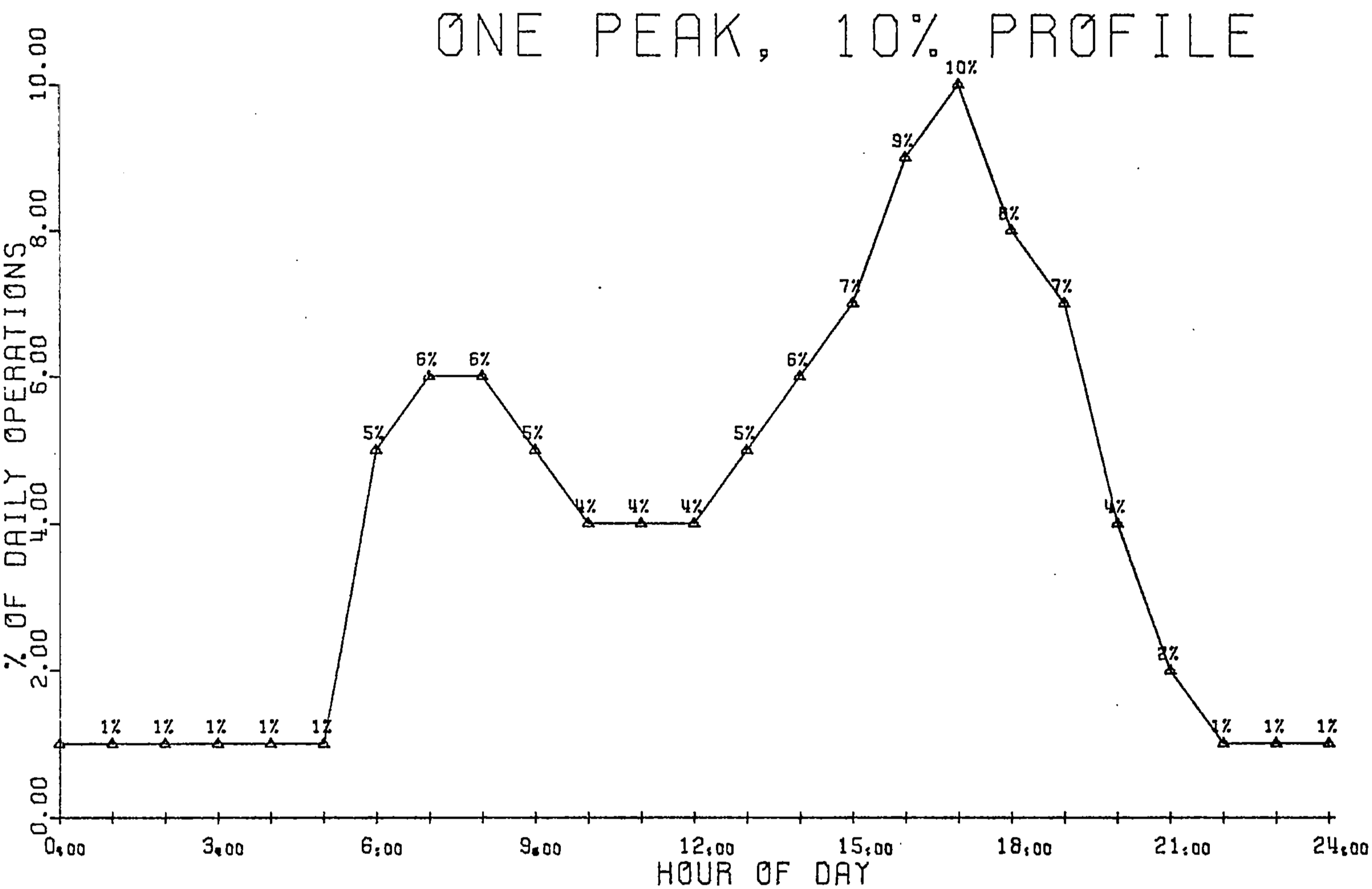

TWO PEAK, 9% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
24:00
1%
1%
1%
1%
1%
5%
7%
9%
7%
5%
4%
4%
5%
7%
8%
9%
7%
5%
4%
4%
2%
1%
1%
1%

FIGURE 8.


ONE PEAK, 10% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0.00
3.00
6.00
9.00
12.00
15.00
18.00
21.00
24.00
1%
1%
1%
1%
1%
5%
6%
6%
5%
4%
4%
4%
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6%
7%
9%
10%
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7%
4%
2%
1%
1%
1%

FIGURE 9.

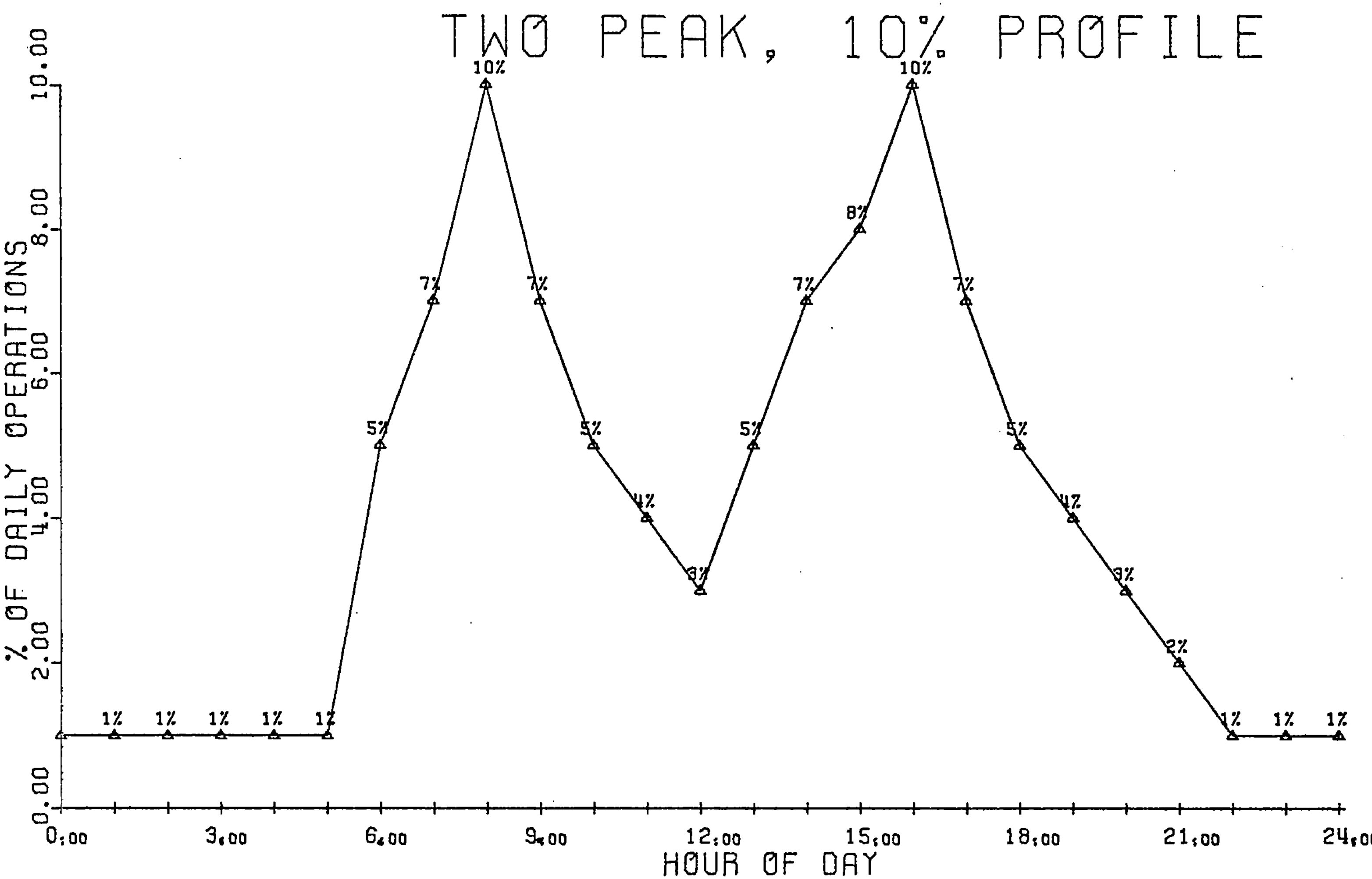

TWO PEAK, 10% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
24:00
1%
5%
7%
10%
4%
3%
8%
2%

FIGURE 10.

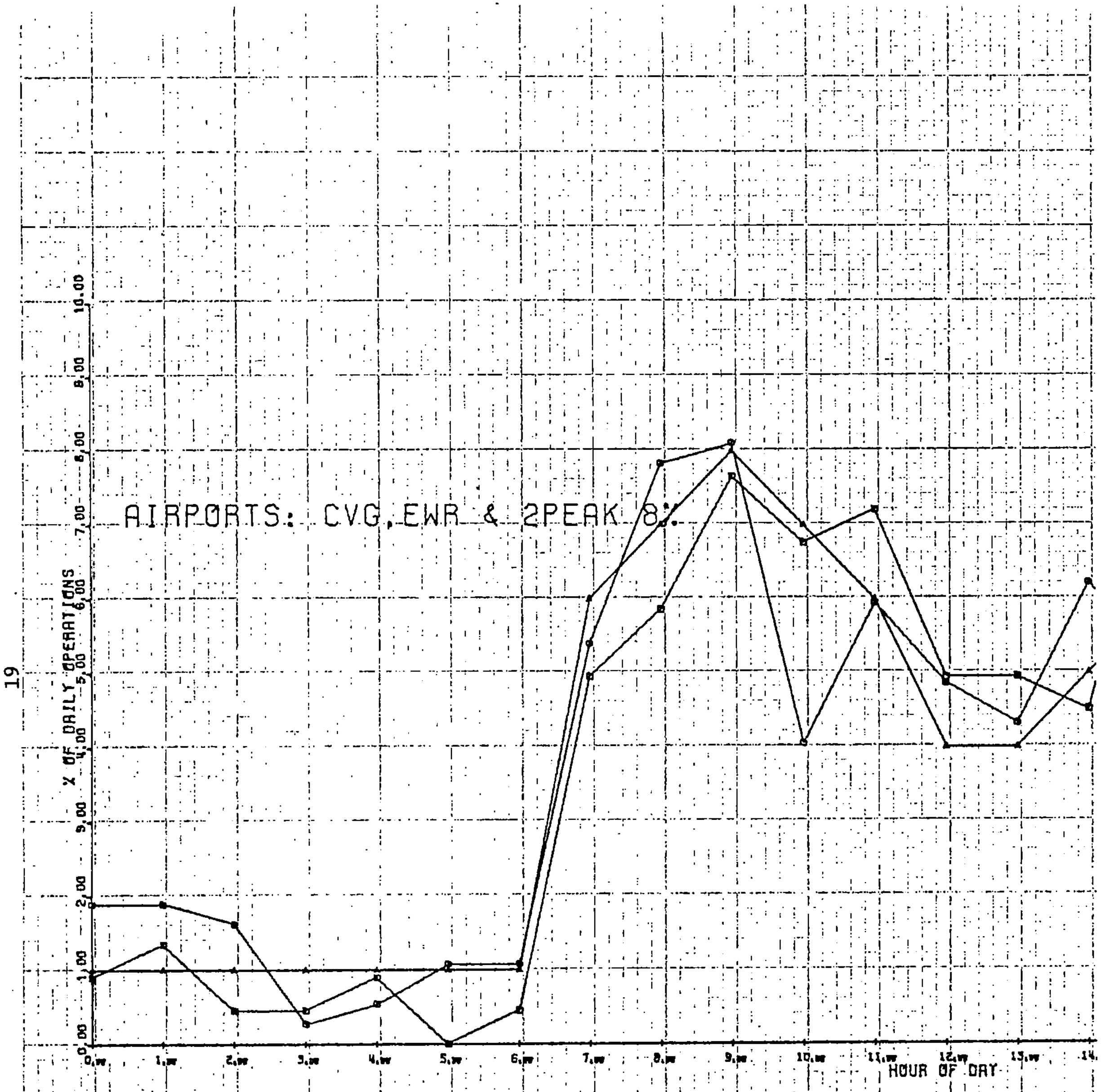


FIGURE 11.

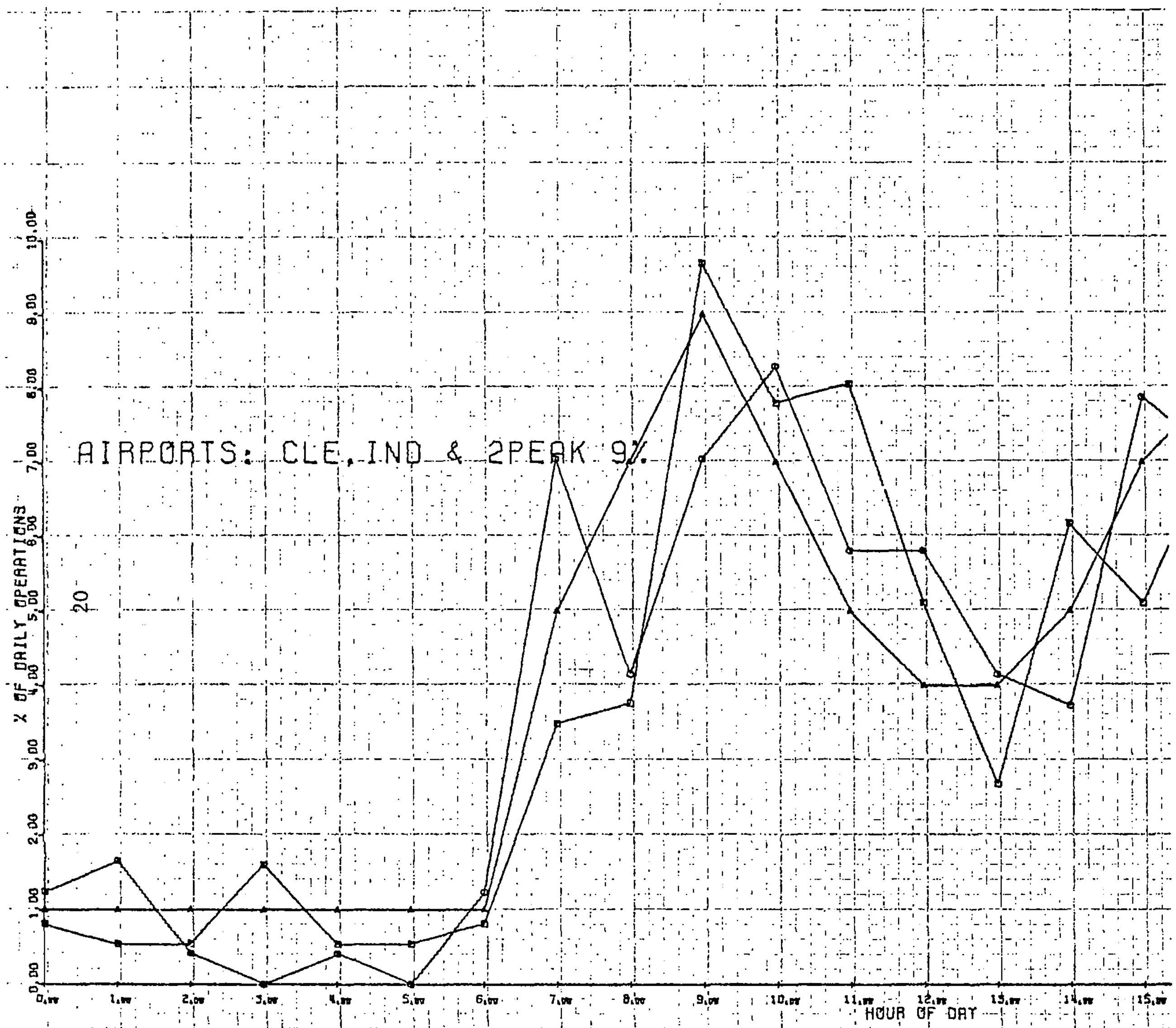


FIGURE 12.

(EWR), while Figure 12 compares the TP9 profile to those at Cleveland (CLE) and Indianapolis (IND). It can be seen that the "fit" is very good in these four cases (which, however, is not always the case with other airport patterns). The question of "fit" will be further discussed in the section on Sensitivity Analysis in this chapter.

C. Estimating Total Daily Delays

Different airports, naturally, have different runway capacities. The measure of capacity which was used here was saturation capacity (or "maximum throughput"), i.e. the maximum number of operations that can be conducted at the airport for a given set of weather conditions and traffic mix and without violating ATC separation rules. The better known - but less precisely defined - practical hourly capacity (PHCAP), i.e. the level of operations at which the average delay at the airport is 4 minutes, is equal to about 80% of the saturation capacity.

The saturation capacities used as inputs in the computation of total daily delays were (i) 48; (ii) 66; (iii) 86; (iv) 96; (v) 107; (vi) 114; (vii) 123; and (viii) 160 operations per hour, assuming 50% landings and 50% take-offs. These, correspond, approximately to practical hourly capacities of 39, 53, 70, 80, 90, 97, 105, and 138 operations per hour. Obviously, these capacities cover the complete spectrum of known capacities at major airports in the United States, beginning with the single runway airport (saturation capacity of about 48) and going all the way to the largest capacity airports.

The level of demand was then varied for each case under consideration, as follows:

The peak hour demand was set successively at 70%, 80%, 90%, 100% and 110% of the saturation capacity of the airport. For example, consider the case of an airport with a (saturation) capacity of 96 operations per hour and with a "no peak, 7%" (NP7) type of demand profile. For such an airport five computer runs were performed using the profile of Figure 1 and assuming that for each of the peak hours (i.e., between 8:00 and 9:00, between 17:00 and 18:00, and between 18:00 and 19:00 - see Figure 1) operations amount, first, to 67 per hour (70% of 96), then to 77 (80% of 96), then to 86 (90% of 96), then to 96 (100% of 96) and finally to 106 per hour (110% of 96). (Demand during the remaining hours of the day was, of course, adjusted accordingly so as to maintain, in all five cases, the same profile as that of Figure 1). Note that the 70%, 80%, 90% 100%, 110% cases represent a spectrum of situations ranging from a practical lack of congestion (70% case) to over-saturation (110% case). Situations with less than 70% peak-hour demand are of limited interest, since delays under such circumstances are quite small and due only to the randomness of demand at the airport. At the other end, no airport could be expected to regularly absorb demand exceeding its saturation capacity by more than 10%. Should that be done the level of delays would be unacceptable, as has been shown previously by numerous studies and as illustrated by the present results. Although under sharply deteriorating weather conditions it is possible to exceed the 110% level temporarily, we are here only interested in long-term average conditions. In any case, some extrapolation (for demands below 70% or above 110%) can be performed on the prepared graphs, as discussed briefly in Chapter 3.

In summary, the total number of computer runs performed were as follows: For each of the 10 typical profiles, 8 different airport saturation capacities were examined, each at five different relative levels of peak hour demand (70%, 80%, 90%, 100% and 110%). Thus a total of 10 X 8 X 5 = 400 cases were run in the computer using the DELAYS package described in Section A of this chapter. From each run, a single number, the average total number of daily delay minutes (TDDEL) was obtained. A curve on the TDDEL graphs was generated by plotting and connecting the 5 delay figures (corresponding to 70%, 80%, 90%, 100%, and 110% of saturation capacity) calculated for every combination of one of the 10 profiles with one of the 8 saturation capacities. The resultant TDDEL graphs as shown in Chapter 3 were produced on semilog paper by the Calcomp plotter.

A final note to complete this description is in order. The number of airport runways assumed (this is necessitated by the nature of the DELAYS program, see the Appendix) were: 1 runway in the 48 saturation capacity case; 2 runways in the 66, 86, 96 and 107 saturation capacity cases; and 3 runways in the case of capacities of 114, 123 and 160 operations. These choices appeared to be logical ones for each of the capacities under consideration. In any case, the delay estimates, particularly when it comes to total daily numbers and to utilization levels close to the saturation point are not sensitive to the exact number of runways (but very sensitive to the total capacity of the airport). Therefore, the exact number of runways used for the computations is not expected to affect greatly the accuracy of the results.

D. Sensitivity Analysis

Much effort was expended in exploring the sensitivity of the results of this work (the TDDEL graphs) to the variation of the input parameters, especially to changes in the typical profiles used to compute total daily delays.

The primary test of sensitivity consisted of using as inputs for the DELAYS program the demand profiles of several commercial airports - as well as some imaginary demand profiles - and comparing the actual delay obtained through the DELAYS program with the figures predicted by the TDDEL graphs of Chapter 3. These tests were also used as aids in adjusting some of the ten typical profiles (Figures 1 through 10) to achieve better performance in delay estimation.

The main conclusions of this effort were:

a) The total daily delay estimates are, to a large extent, dominated by delays taking place during the peak traffic periods of the day. Thus, the estimates are very insensitive to the exact shape of the demand profiles at times other than the peak traffic periods. This confirmed the emphasis placed here on the number of peak periods and the number of operations during the peak hour of the day. The user of the TDDEL graphs should concentrate primarily on classifying his/her demand profile with respect to these two items and not be overly concerned about the precise patterns in the "valleys" of the demand profile.

b) It follows from a) that the delay estimates can change appreciably with changes in the details of the demand profile during peak periods. This is especially true when the peak period demand is at 90% or more of the saturation capacity. Therefore, in cases where the peak period pattern with which a user of the TDDEL curves is dealing happens to be appreciably different from any of those used for the typical profiles of Figures 1-10, the TDDEL estimates should be viewed only as first-order approximations.

c) After the adjustment of the ten typical profiles, almost all cases tested, for demand profiles reasonably close to the ten typical demand profiles and with peak hour demands of 90% or less of saturation capacity, were within a ± 20% zone from the level of total daily delays predicted by the TDDEL curves. High accuracy was also achieved for cases with peak hour demand at or above the saturation capacity level for the 9% peak and the 10%-peak profiles.

d) Success with the 7% and 8% profiles was mixed for demand profiles which during peak hours reach or exceed the saturation level of the airport. In cases where the demand profiles are relatively smooth (such as at Chicago's O'Hare Airport, ORD, or at LaGuardia Airport in New York, LGA) the estimates from the TDDEL graphs were in good agreement with the actual delay figures obtained through the DELAYS program. However, in cases where a demand profile exhibits a "jagged" pattern with several peaks (such as the demand profile of Atlanta, ATL) the discrepancy between the two total delay figures could be high for high demand levels. In one case, with a TP8 profile, the observed difference (2,966 minutes from the DELAYS program vs. 5,600 minutes from the TDDEL graphs for a peak hour demand equal to 112% of saturation capacity) amounted to 87% of the actual delays (i.e. of the 2,966 minutes) as computed by the DELAYS program. Thus, it is recommended that delay estimates from the TDDEL graphs be considered as only rough first-order approximations for cases involving <u>both</u> a 7%-peak or 8%-peak demand profile <u>and</u> a peak hour demand level that exceeds or is very near to the saturation capacity. In such cases the reader should probably take advantage of existing tools (including the DELAYS program on which this handbook is based) to perform a detailed analysis of the particular airport under consideration.

References

1. HENGSBACH, GERD and AMEDEO R. ODONI, Time Dependent Estimates of Delays and Delay Costs at Major Airports, Report R75-4, Flight Transportation Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts (January, 1975).

Chapter III - A User's Guide

This chapter illustrates a simple and practical tool for estimating airside delays at an airport on a daily or annual basis. The delays are those suffered by aircraft waiting for the use of runways. The delays are solely those due to normal runway congestion and do not reflect problems that may be due, for instance, to exceptional weather conditions or to other unusual causes. No distinction is made between delays suffered by landing aircraft which have to queue in the air and those suffered by departing aircraft which wait on the ground.

The basic quantity with which we deal here is that of average total daily delays (TDDEL), i.e. the total delays suffered in the course of a day by all aircraft which attempt to use an airport's runways. Ten sets of curves are provided from which TDDEL can be read for widely varying conditions.

This user's guide consists of two sections: a general discussion of how the TDDEL graphs should be used, including what information is required from the user; and a set of four examples that illustrate the use of the graphs. The reader is strongly advised to review these examples.

A. General Description

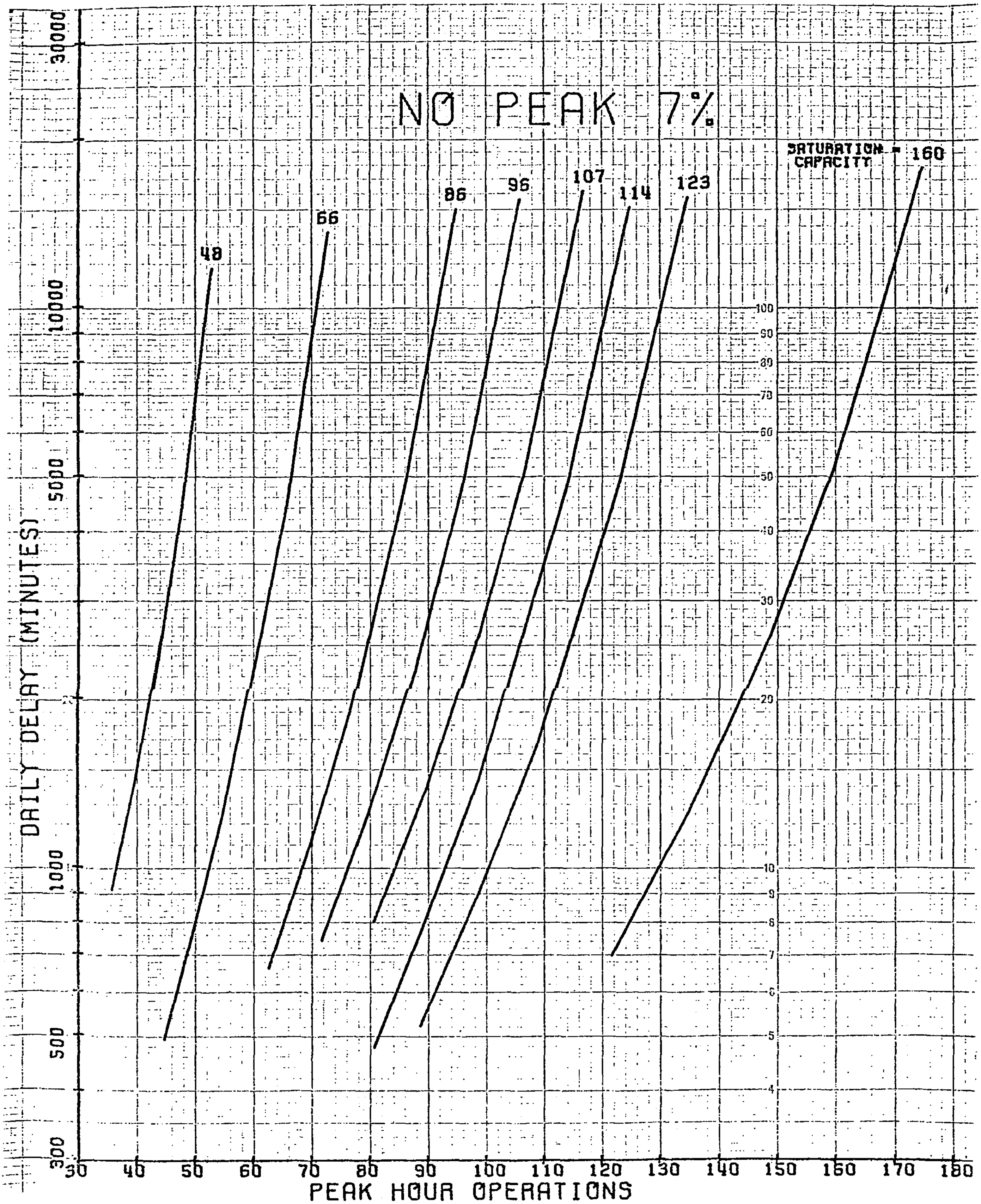
In general terms, the extent of airside delays at an airport, in the course of a day, depends on the relative size of two quantities: the demand for use of the airport and the capacity of the airport. The TDDEL set of curves allows quick estimation of total daily delays at an airport for most common types of demand-to-capacity relationships presently occuring at major airports.

The ten TDDEL graphs (each graph consists of eight curves for eight different levels of airport capacity) which were prepared for this purpose are presented in the next ten pages. Briefly, to estimate the average total daily delays at an airport, the user must, first, choose the one graph (among the ten) which best corresponds to the daily demand pattern at the airport of interest. Then, the appropriate curve must be chosen (or drawn by interpolation) on the basis of the capacity of the airport. Finally, the total daily delay (TDDEL) that corresponds to the peak hour demand (horizontal axis) at the airport can be read from the vertical axis of the graph.

In more detail, use of the TDDEL curves requires that the following four items of information be provided:

(i) The "saturation" hourly capacity of the airport: This capacity is also known as "maximum throughput" or "absolute" capacity. It is defined as the maximum number of aircraft operations that can take place in an hour with the runway configuration in use. As is well known, runway capacity depends on a number of conditions including the prevailing weather conditions, the aircraft mix, the operations mix, the exit taxiway locations, etc. If unknown, the saturation capacities for most runway configurations and for most sets of conditions can be found in the Airfield Capacity and Delay Handbook[1] which has been prepared recently for the FAA. An example in the next section illustrates the use of the TDDEL curves with different levels of capacity (in VFR and IFR conditions) to compute "weighted average" delay estimates.

The table below also provides for easy conversion of "practical hourly capacities" (PHCAP) to saturation capacities. Practical hourly capacity, i.e. the number of hourly operations that imply a 4 minute average delay level, is a concept which may be more familiar to airport planners than saturation capacity due to its use in the currently existing Handbook of Airport Capacities issued by the FAA during the 1960's [2].


NO PEAK 7%
SATURATION CAPACITY = 160
48
66
86
96
107
114
123
DAILY DELAY (MINUTES)
PEAK HOUR OPERATIONS
30000
10000
5000
1000
500
300
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
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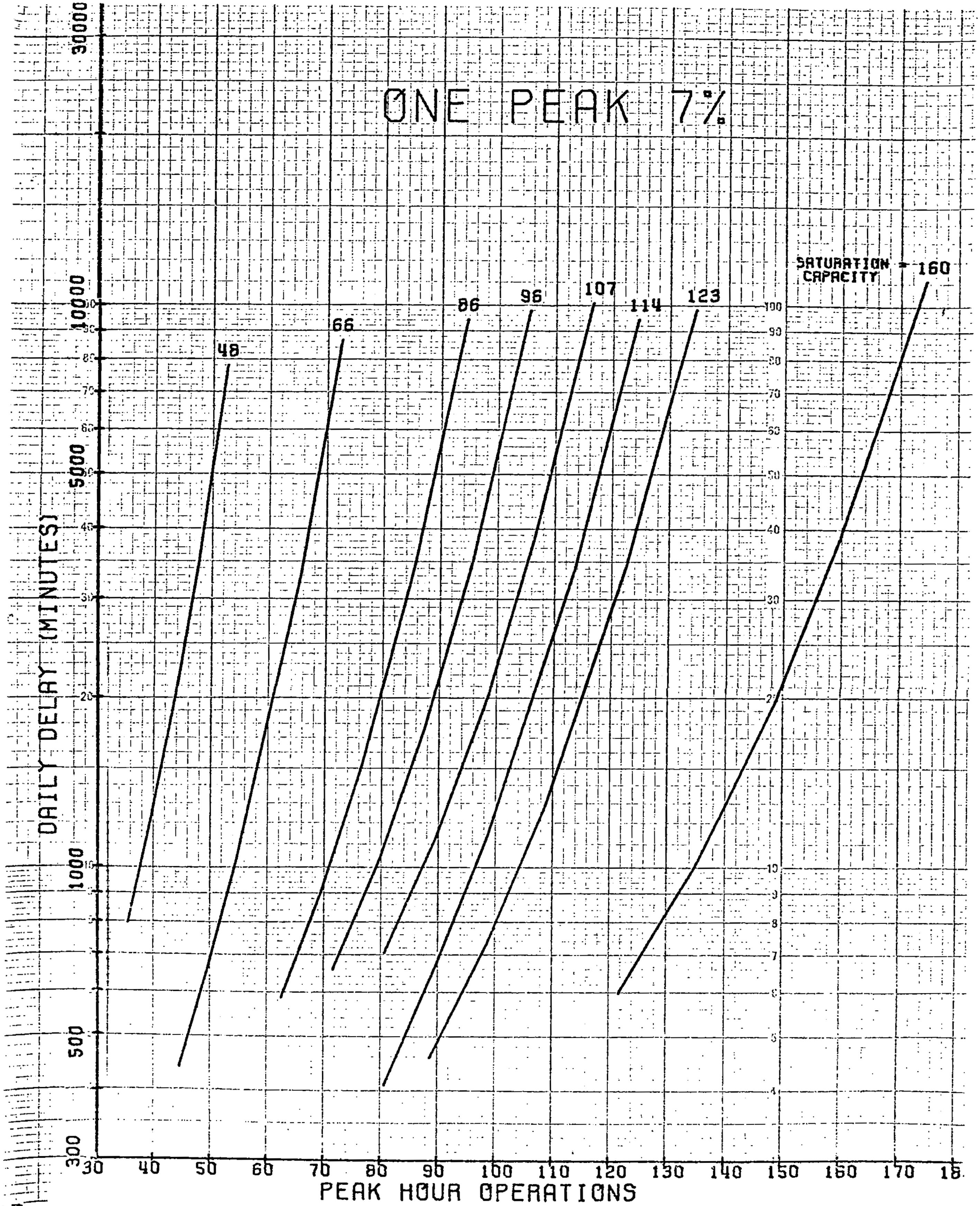
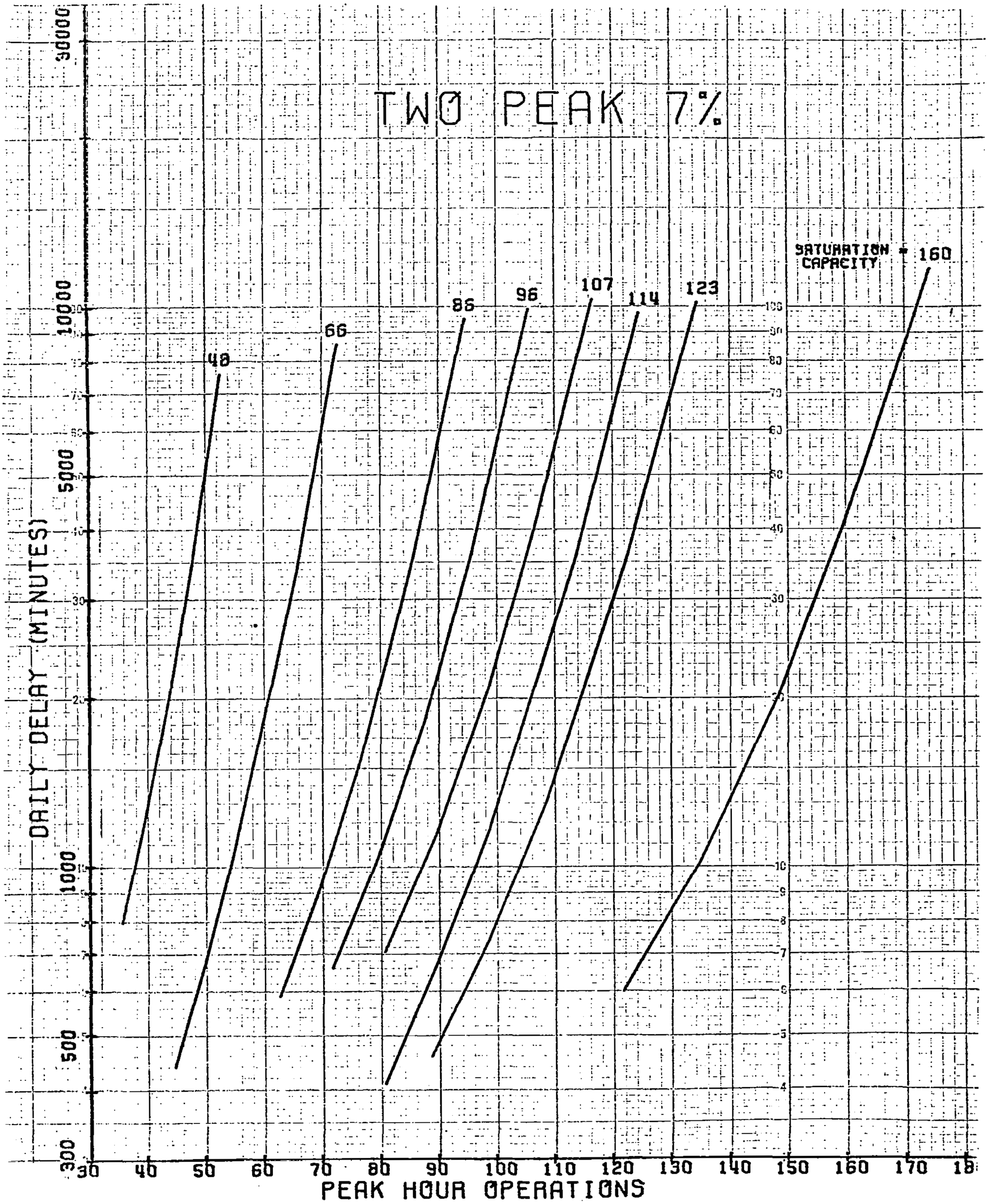
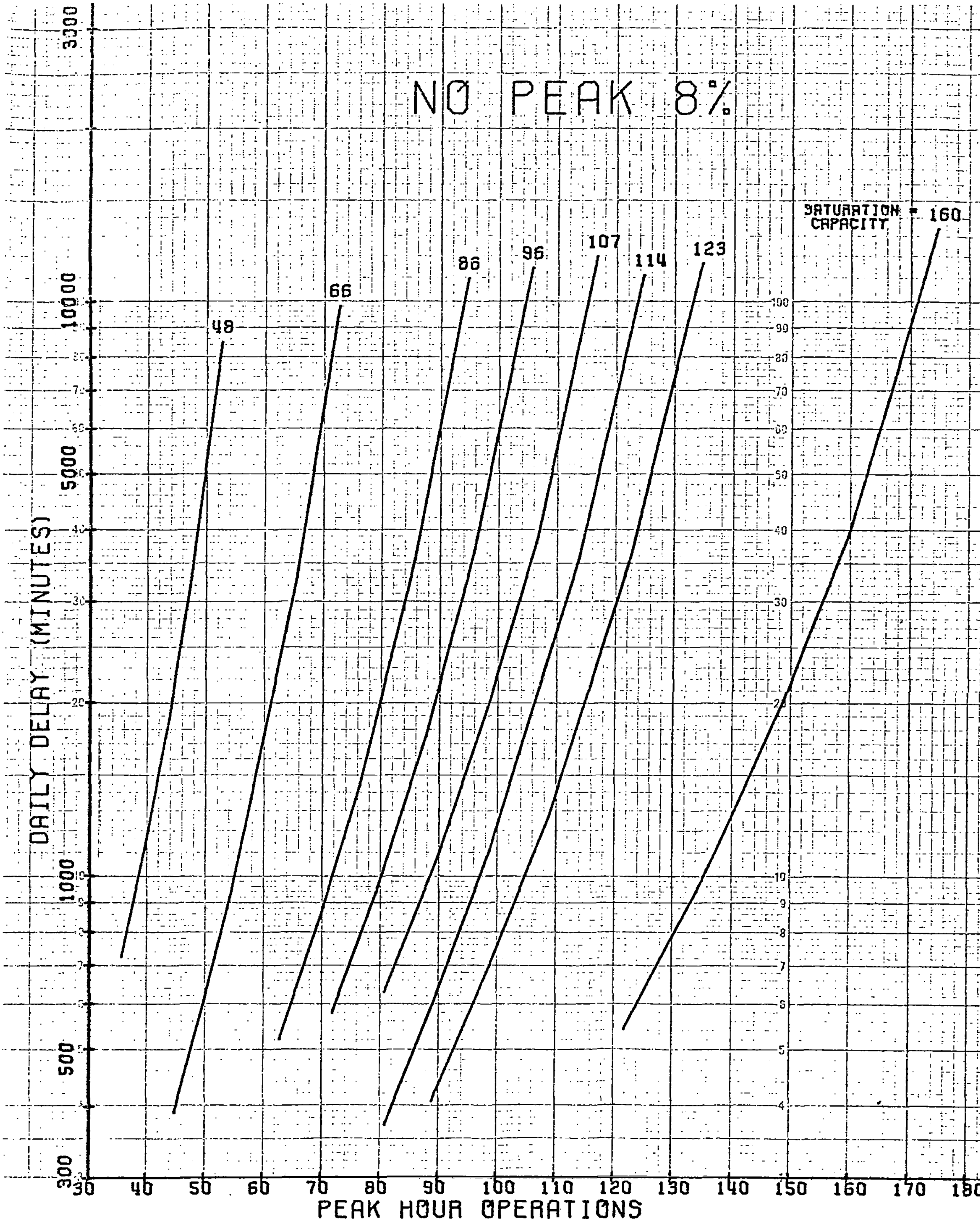
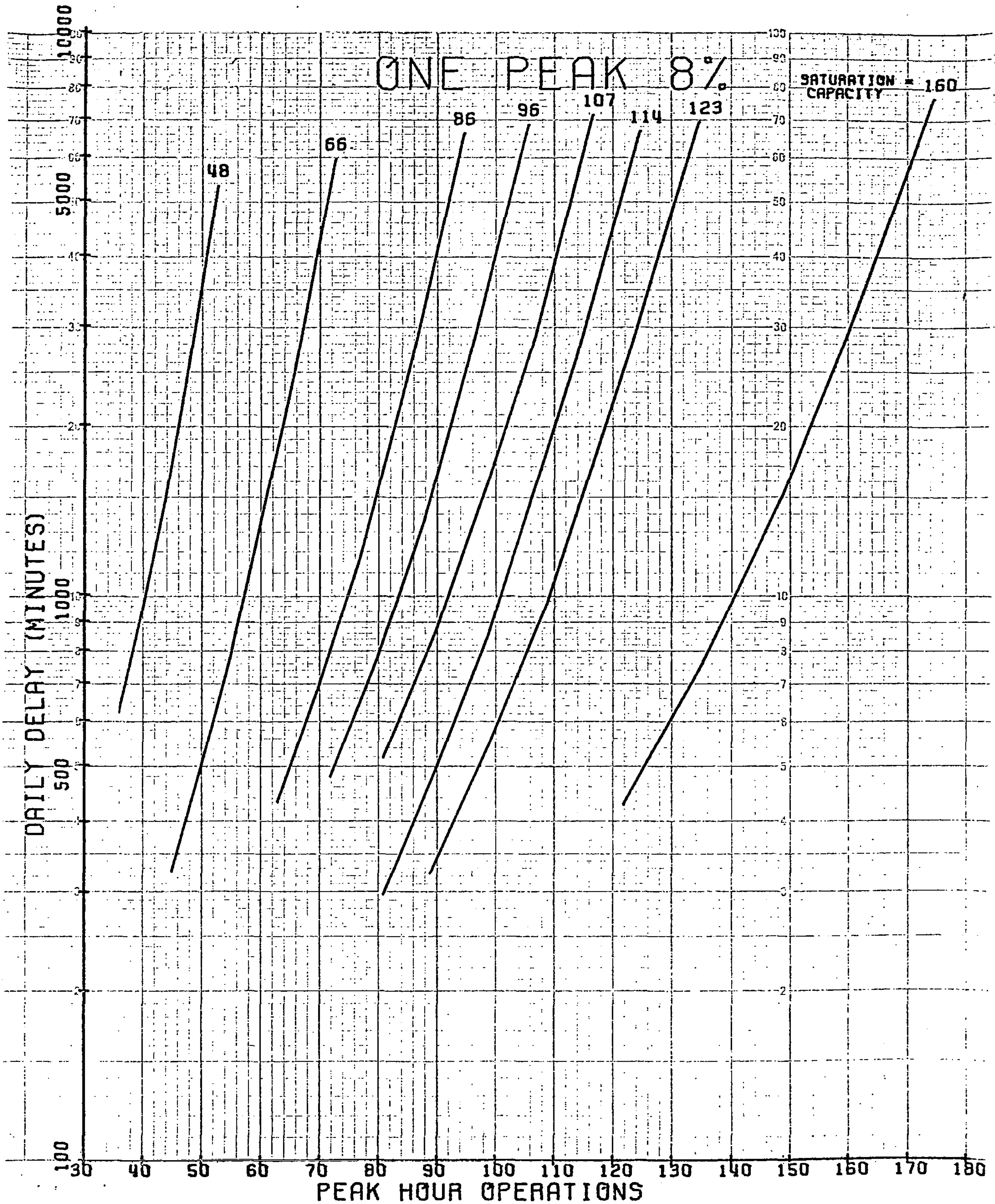
140
150
160
170
180

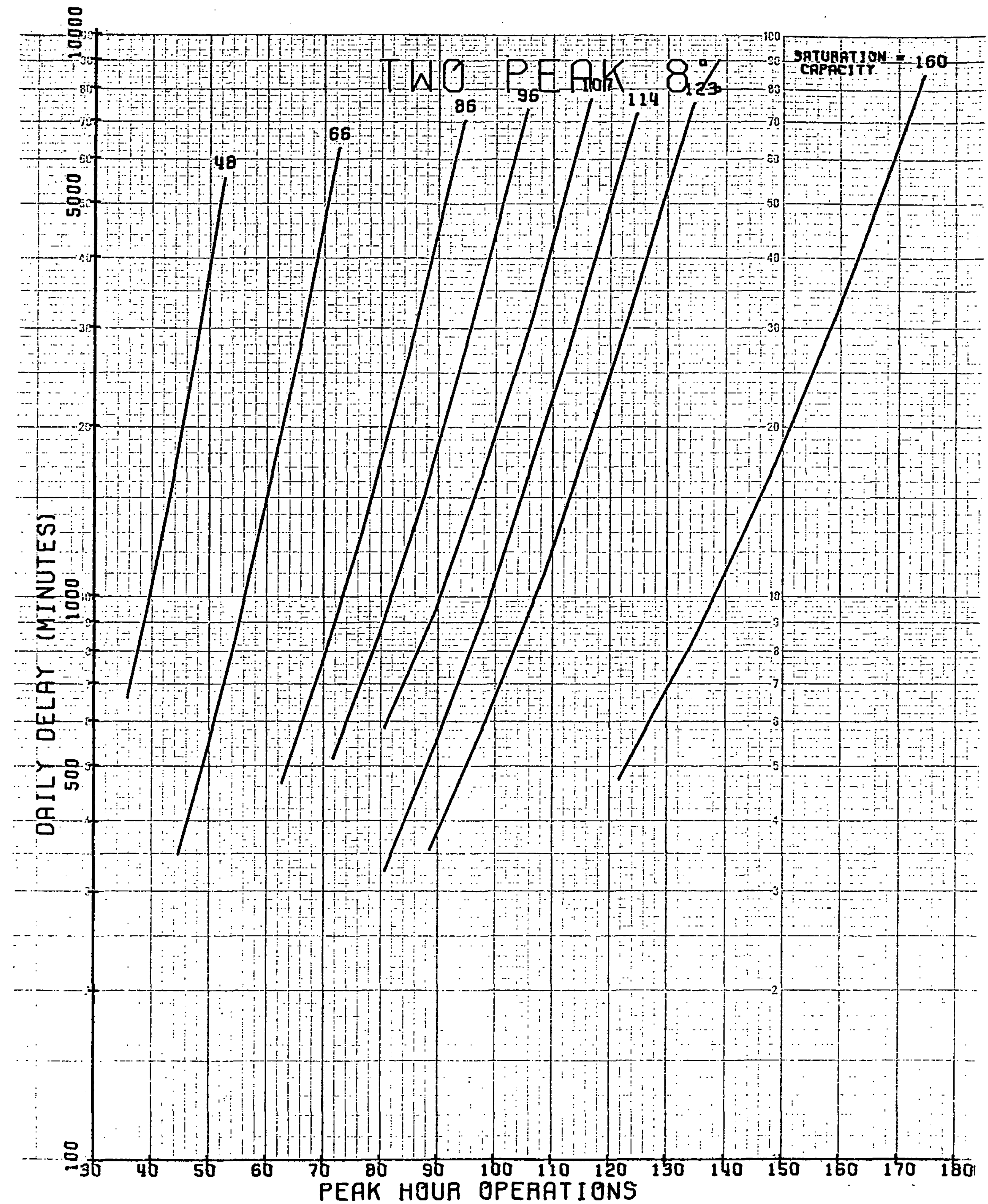

ONE PEAK 7%
SATURATION CAPACITY = 160
48
66
86
96
107
114
123
DAILY DELAY (MINUTES)
PEAK HOUR OPERATIONS
300
500
1000
5000
10000
30000
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170

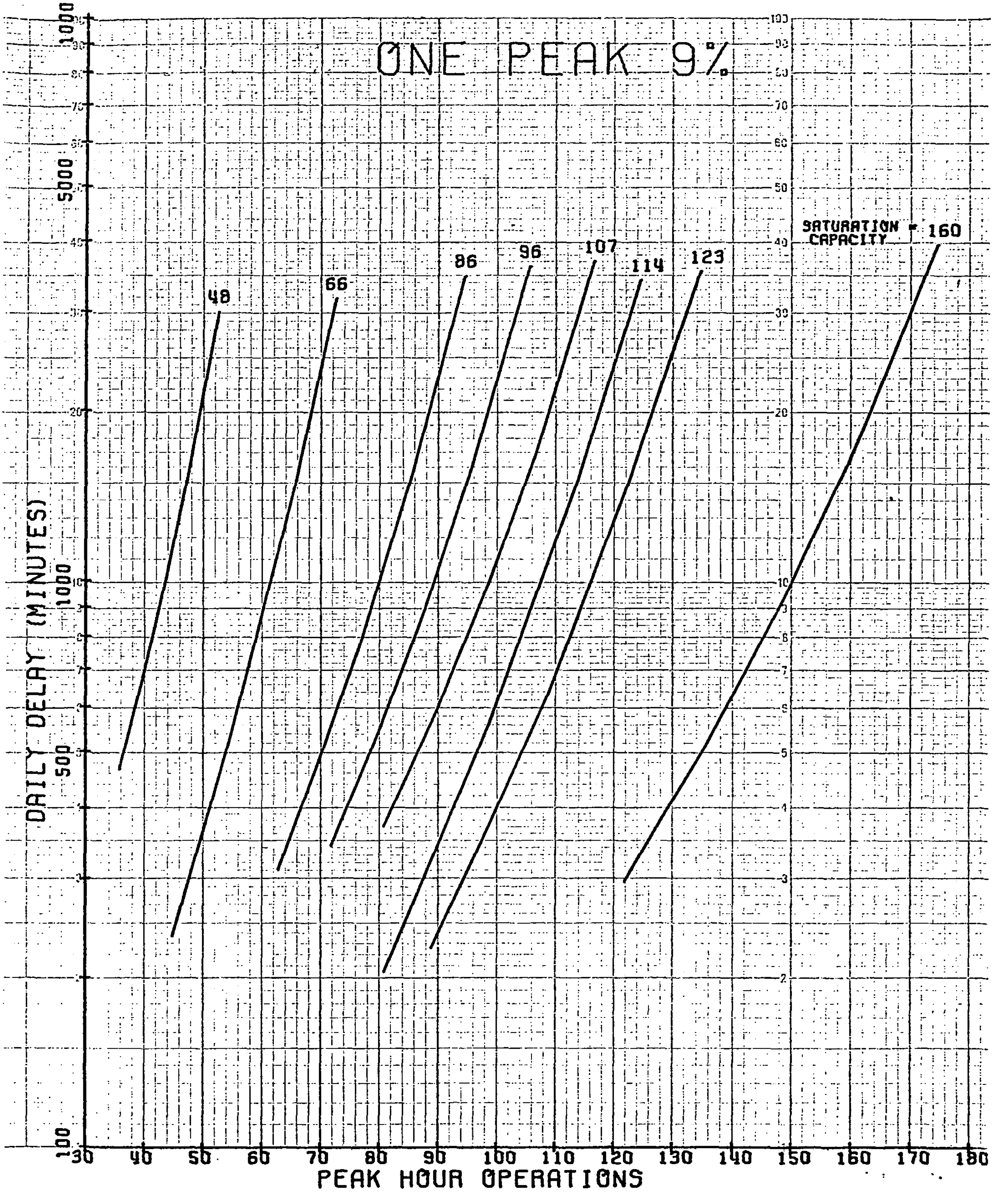

TWO PEAK 7%
SATURATION CAPACITY = 160
48
66
86
96
107
114
123
160
DAILY DELAY (MINUTES)
PEAK HOUR OPERATIONS
300
500
1000
5000
10000
30000
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170

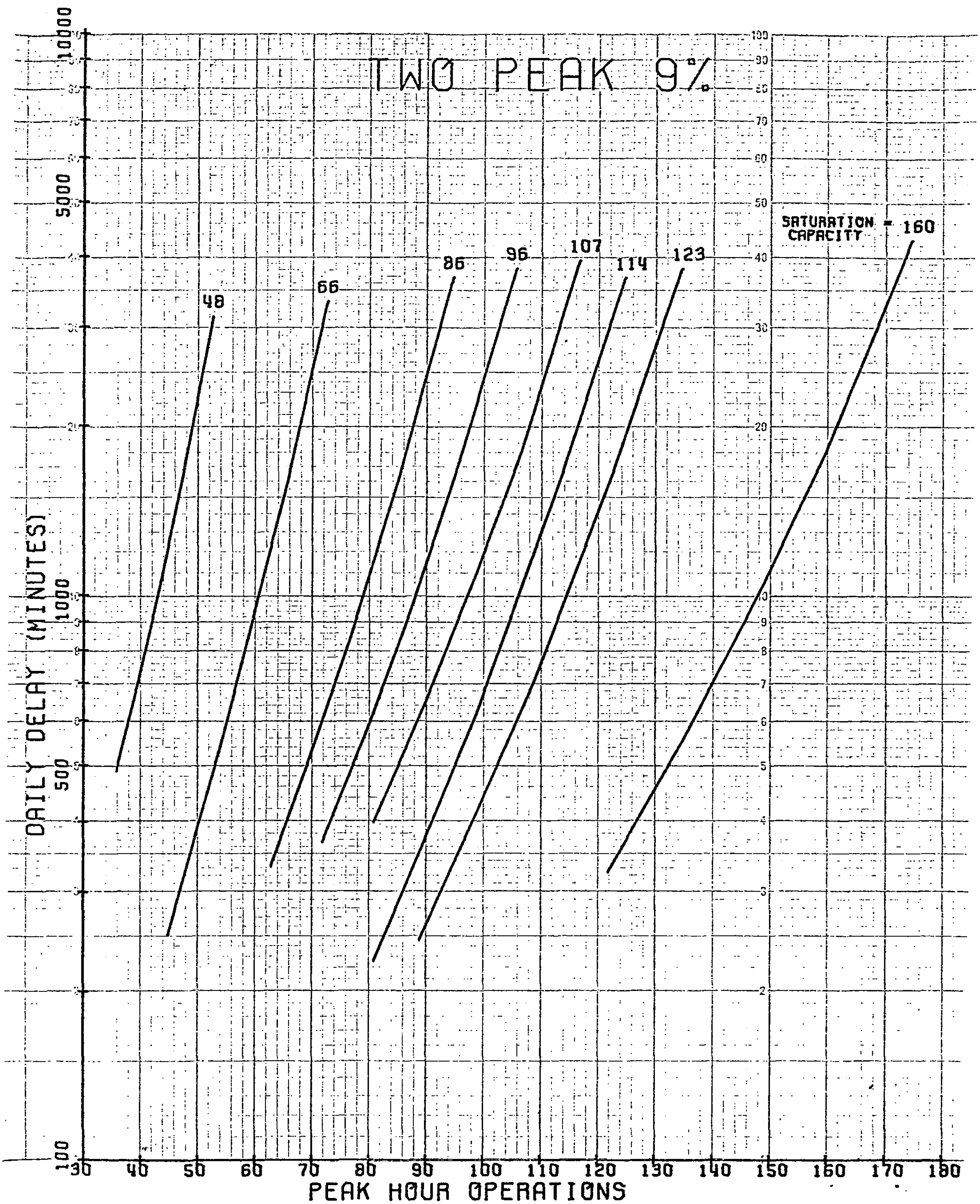


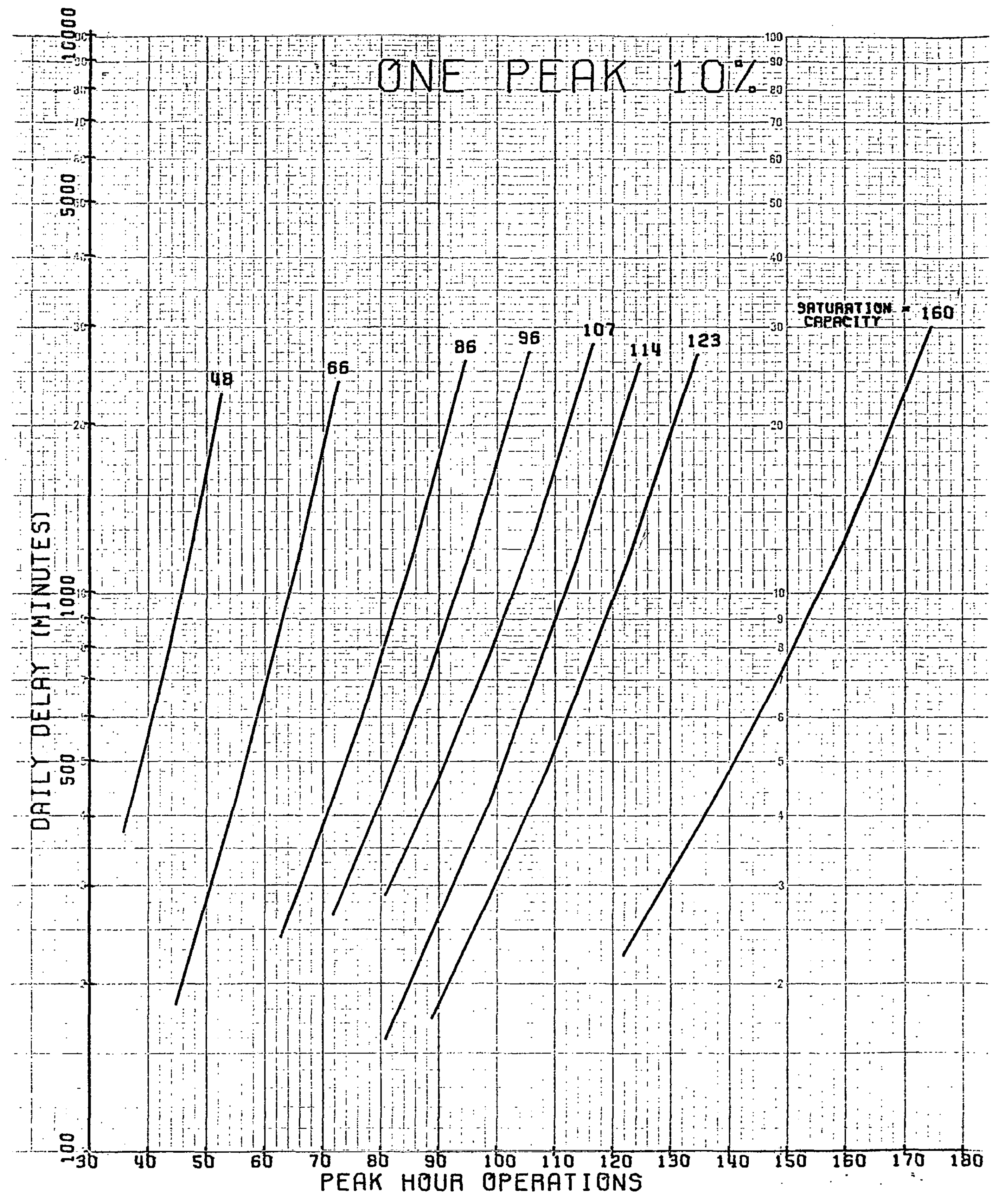
NO PEAK 8%
SATURATION CAPACITY = 160
48
66
86
96
107
114
123
DAILY DELAY (MINUTES)
300
500
1000
5000
10000
3000
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
PEAK HOUR OPERATIONS

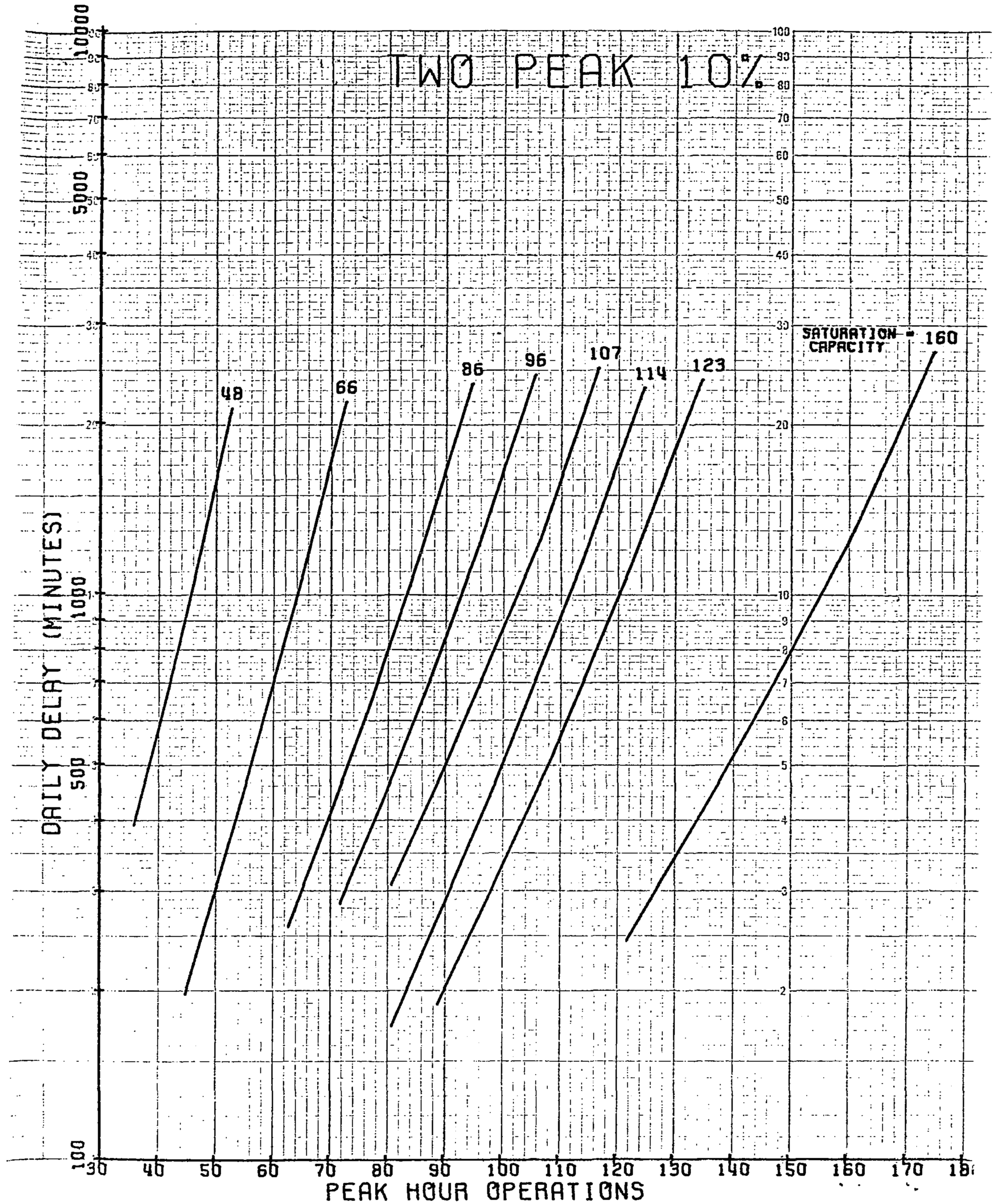

ONE PEAK 8%
SATURATION = CAPACITY
48
66
86
96
107
114
123
160
DAILY DELAY (MINUTES)
100
1000
5000
10000
PEAK HOUR OPERATIONS
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180


TWO PEAK 8%
SATURATION CAPACITY = 160
48
66
86
96
107
114
123
DAILY DELAY (MINUTES)
100
500
1000
5000
10000
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
PEAK HOUR OPERATIONS


ØNE PEAK 9%
SATURATION CAPACITY = 160
48
66
86
96
107
114
123
DAILY DELAY (MINUTES)
100
500
1000
5000
10000
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
PEAK HOUR OPERATIONS


TWO PEAK 9%
SATURATION CAPACITY = 160
48
66
86
96
107
114
123
DAILY DELAY (MINUTES)
10000
5000
1000
500
100
PEAK HOUR OPERATIONS
30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180


ONE PEAK 10%
DAILY DELAY (MINUTES)
100
500
1000
5000
10000
PEAK HOUR OPERATIONS
30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180
48
66
86
96
107
114
123
SATURATION CAPACITY = 160


TWO PEAK 10%
DAILY DELAY (MINUTES)
PEAK HOUR OPERATIONS
SATURATION CAPACITY = 160
48
66
86
96
107
114
123
100
500
1000
5000
10000
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170

Conversion Table

Approximate practical hourly capacity	Equivalent saturation capacity
39	48
53	66
70	86
80	96
90	107
97	114
105	123
138	160

(ii) The hour-by-hour demand during the day of interest: The user should obtain a 24-hour profile of the demand (arrivals plus departures) at the airport under consideration for the day of interest (usually, such a day would be described as "average day during peak season" or "average day during low season" or "peak day, peak season," etc.). Once such a 24-hour profile is available, the following items will be used to determine which TDDEL graph is appropriate to the case:

— the number of operations (arrivals plus departures) demanded during the peak hour(s) of the day.

— the percentage of the total demand represented by the number of operations during the peak hour.

— the number of peak periods during the day, where by a "peak period" is meant a time interval of at least three or four consecutive hours during which demand is appreciably higher than demand during the time periods immediately preceding or following it.

In combination, the last two items above will determine which of the ten TDDEL graphs is the appropriate one for the case being considered. For instance, if the demand pattern at the airport exhibits two main peak periods and the demand during the peak hour of the day is equal to about 9% of the total daily demand, then the graph labeled as "TWO PEAK 9%" should be used.

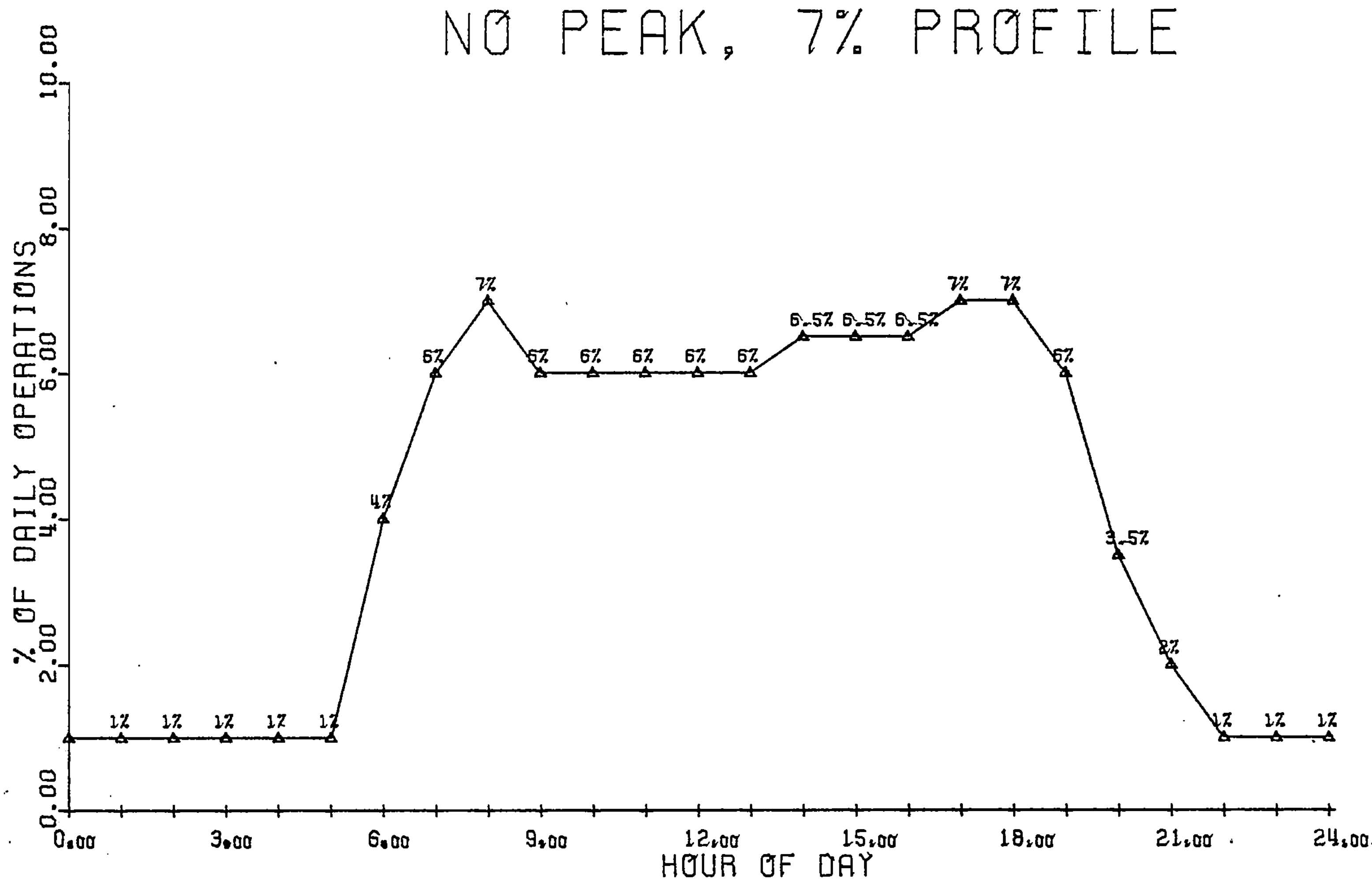
To assist the user in selecting the most appropriate TDDEL graph, ten typical demand profiles (corresponding on a one-to-one basis to each one of the ten TDDEL graphs) are presented in the following pages. The ten profiles are in turn, for the cases of:

1) no particularly outstanding peak period ("no peak") and the peak hour demand is equal to 7% of total daily demand ("7% - peak hour"). This is the "no peak, 7%" (NP7) profile.
2) One peak, 7% peak-hour (OP7)
3) Two peak, 7% peak-hour (TP7)
4) No peak, 8% peak-hour (NP8)
5) One peak, 8% peak-hour (OP8)
6) Two peak, 8% peak-hour (TP8)
7) One peak, 9% peak-hour (OP9)
8) Two peak, 9% peak-hour (TP9)
9) One peak, 10% peak-hour (OP10)
10) Two peak, 10% peak-hour (TP10)

If, for instance, the demand profile of interest most closely resembles the NP8 demand profile, the "NO PEAK, 8%" TDDEL graph should be referred to.

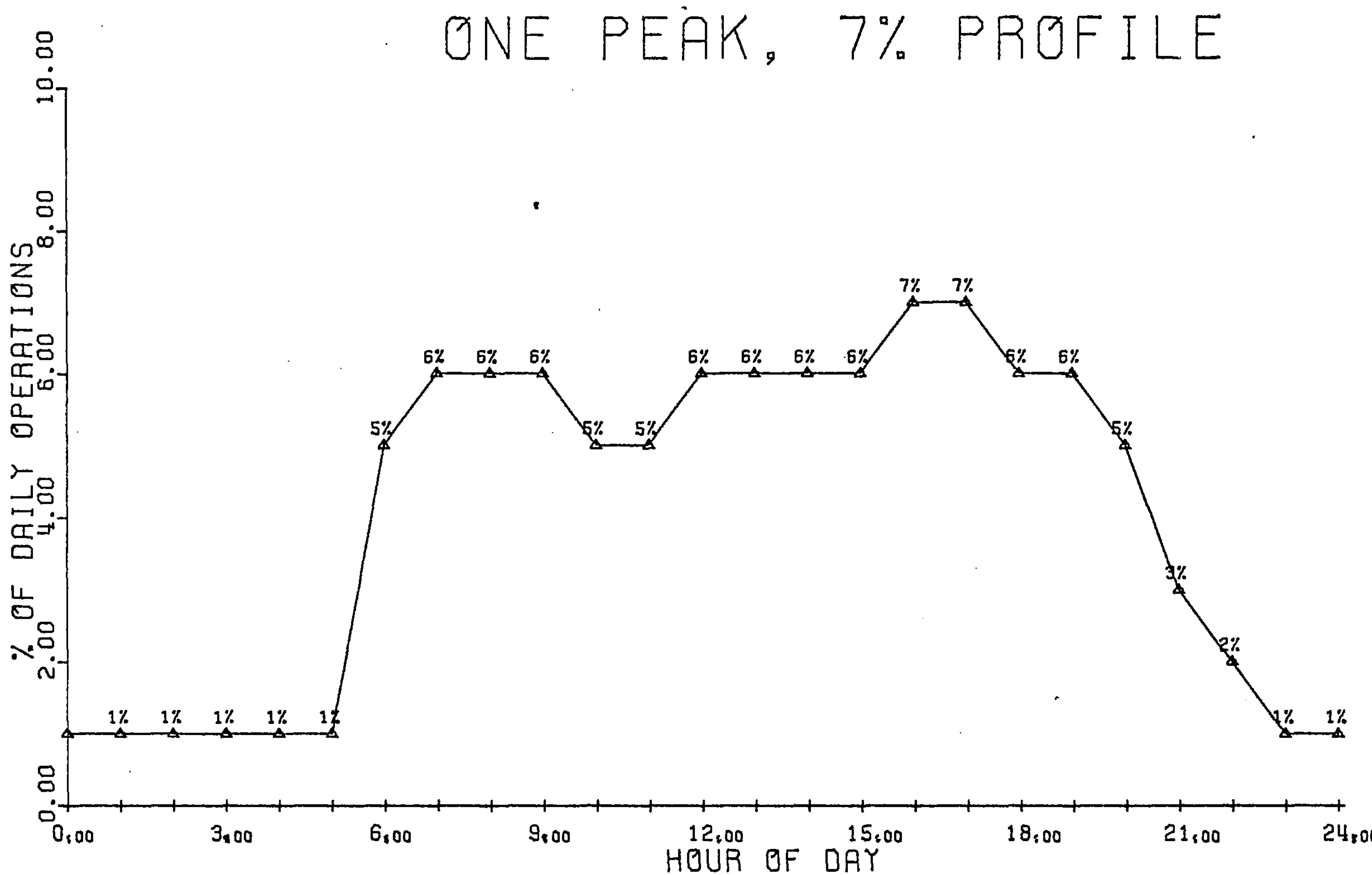
The procedure for estimating total daily delays can now be summarized as follows:

Step 1: From the shape of the demand profile and from the percentage of total daily demand that materializes during the peak demand hour select the appropriate TDDEL graph to use. (A sketch of the demand profile at hand can be helpful in this step).



NO PEAK, 7% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0.00
3.00
6.00
9.00
12.00
15.00
18.00
21.00
24.00
1%
1%
1%
1%
1%
4%
6%
7%
6%
6%
6%
6%
6%
6.5%
6.5%
6.5%
7%
7%
6%
3.5%
2%
1%
1%
1%

FIGURE 1.



ONE PEAK, 7% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0.00
3.00
6.00
9.00
12.00
15.00
18.00
21.00
24.00
1%
1%
1%
1%
1%
5%
6%
6%
6%
5%
5%
6%
6%
6%
6%
7%
7%
6%
6%
5%
3%
2%
1%
1%

FIGURE 2.

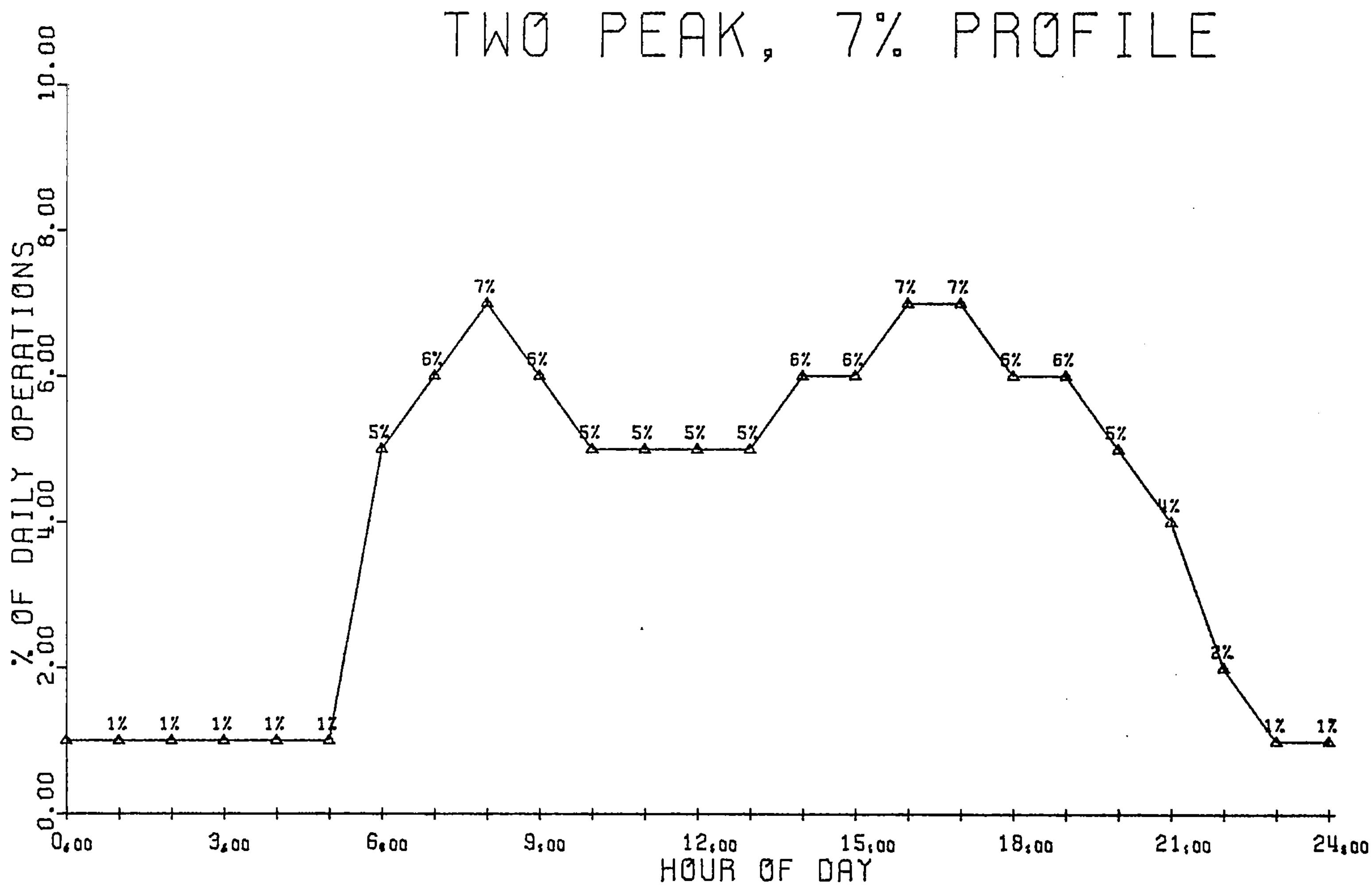

TWO PEAK, 7% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
24:00
1% 1% 1% 1% 1%
5% 6% 7% 6%
5% 5% 5% 5%
6% 6% 7% 7% 6% 6%
5% 4% 2% 1% 1%

FIGURE 3.

NØ PEAK, 8% PRØFILE

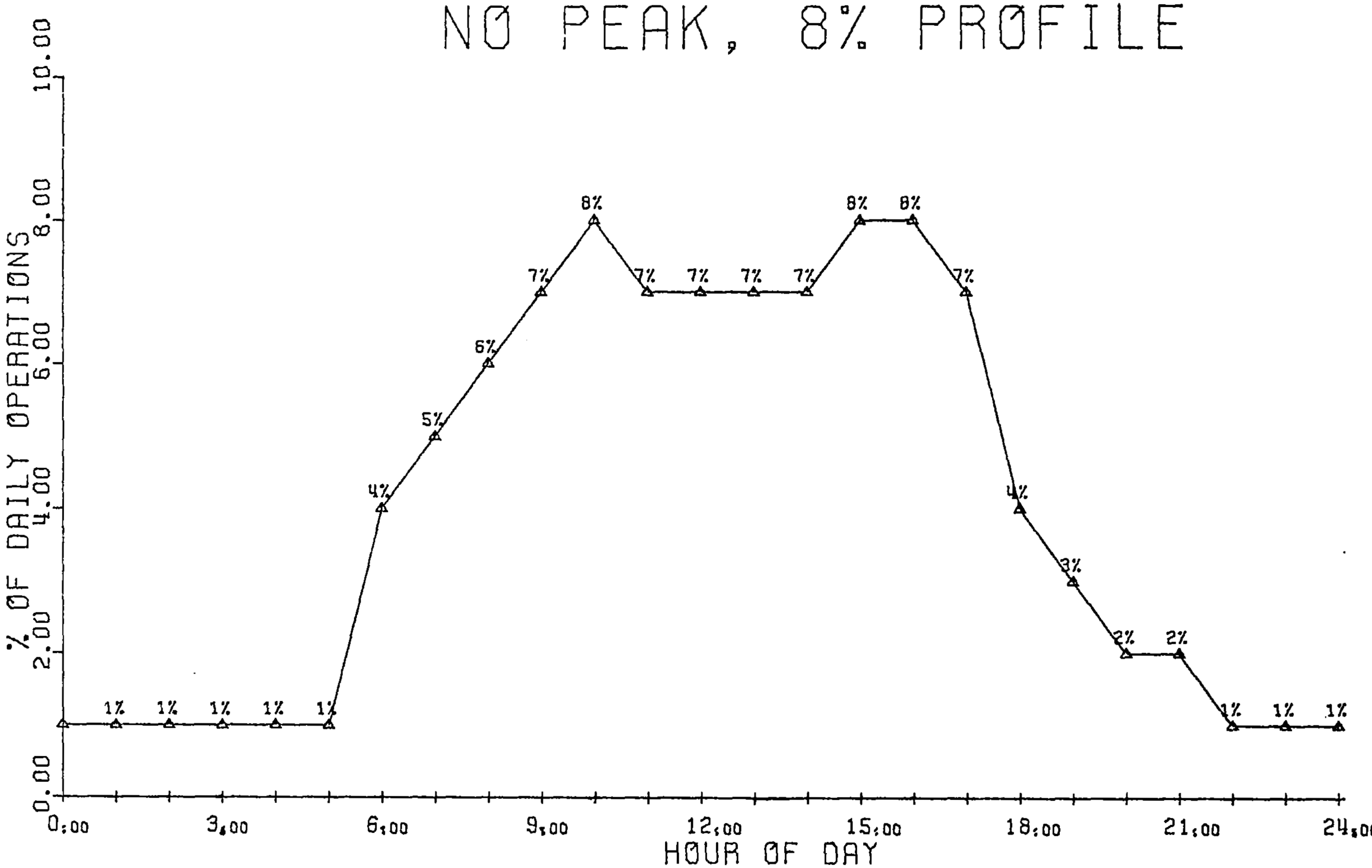
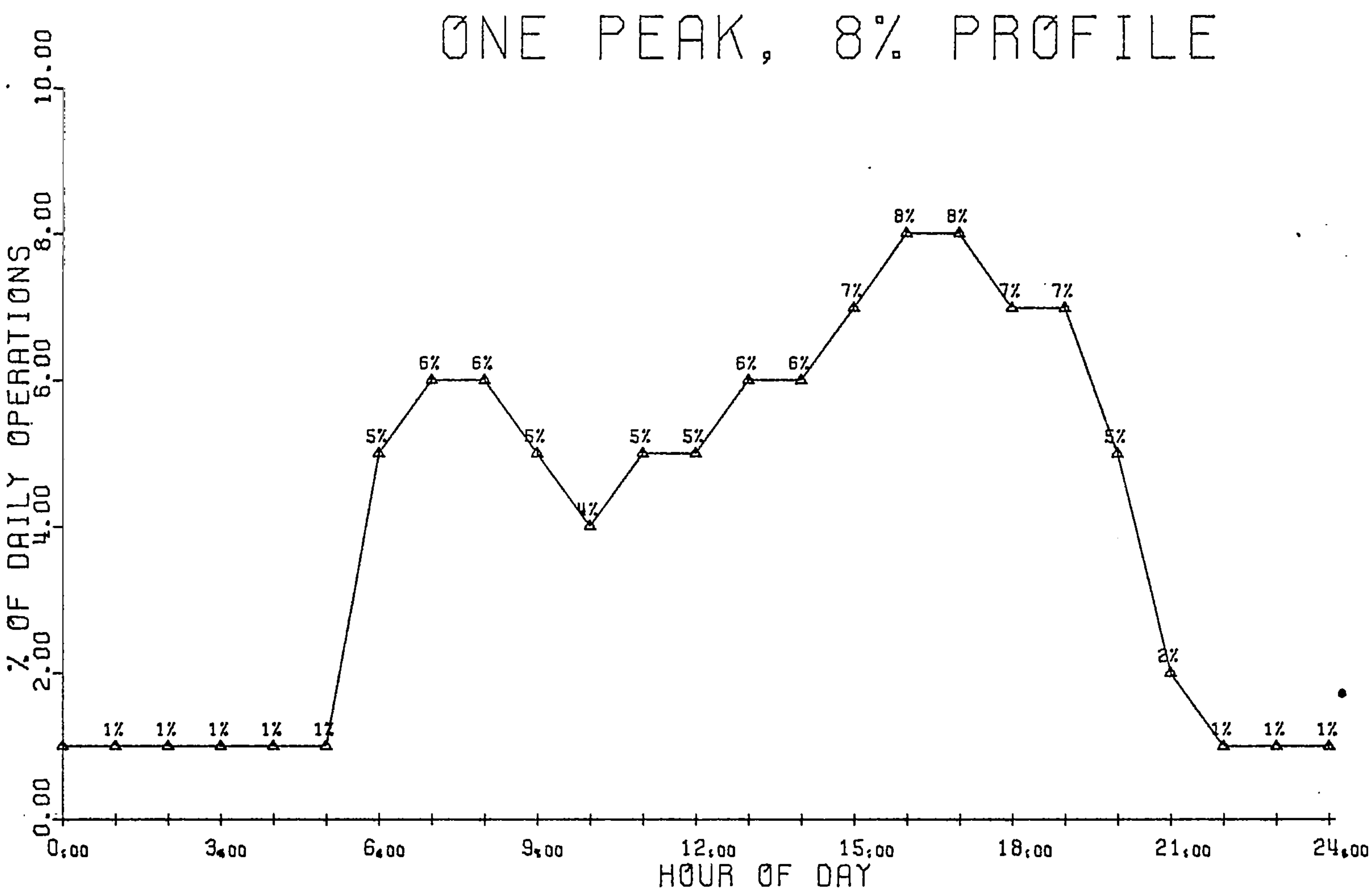


FIGURE 4.



ONE PEAK, 8% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
24:00
1%
1%
1%
1%
1%
5%
6%
6%
5%
4%
5%
5%
6%
6%
7%
8%
8%
7%
7%
5%
2%
1%
1%
1%

FIGURE 5.

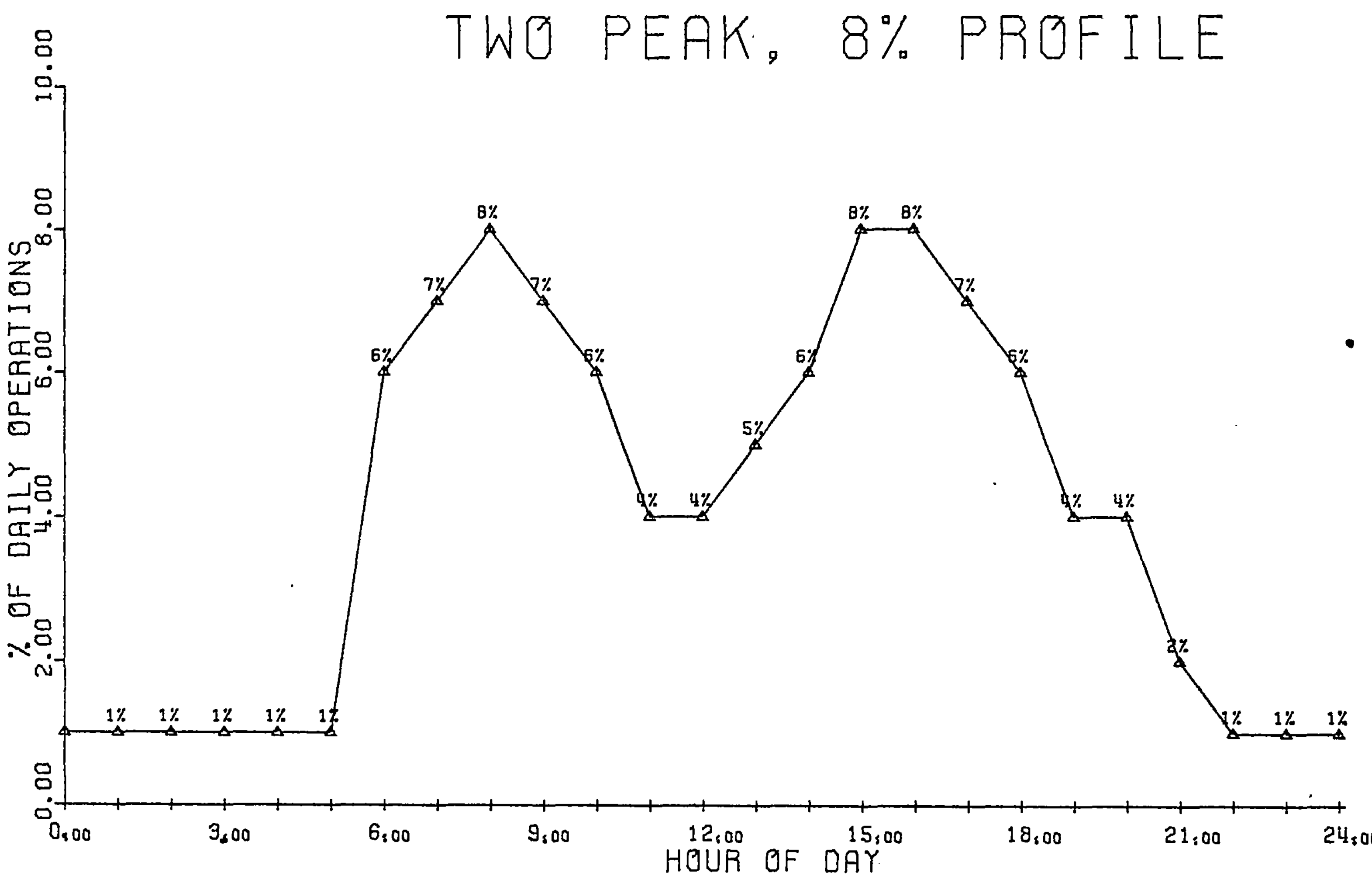
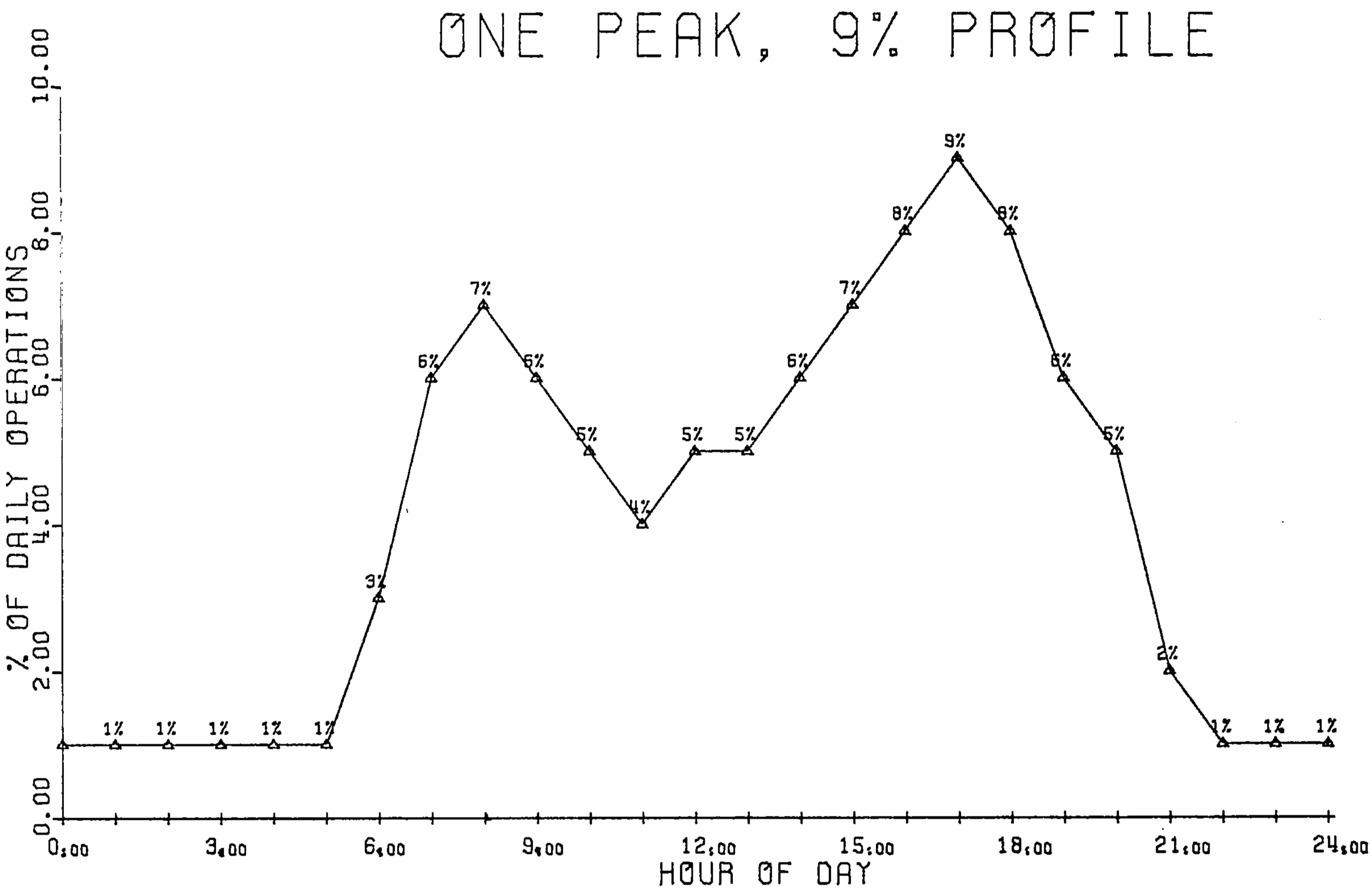
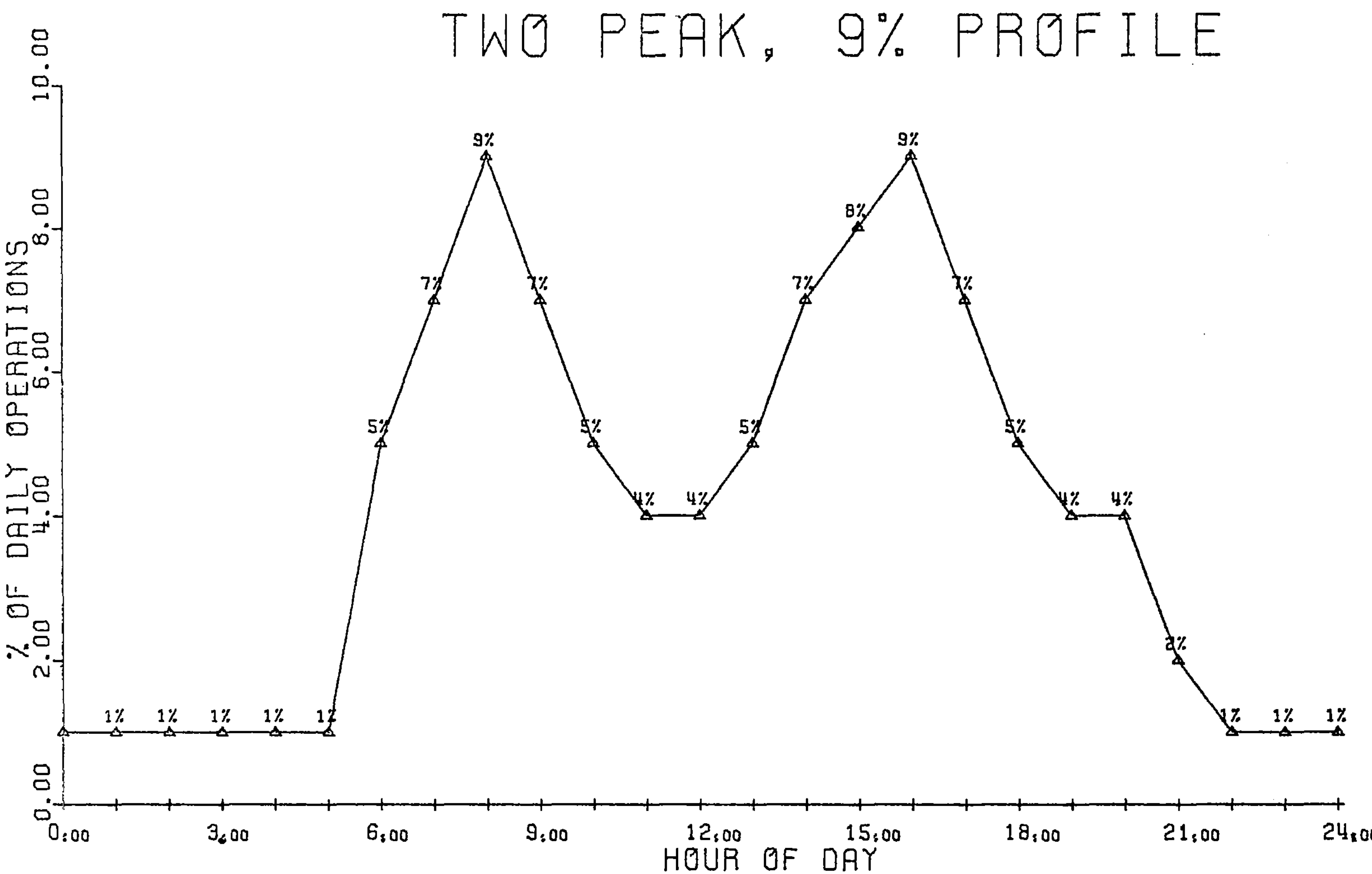

TWO PEAK, 8% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
24:00
1%
1%
1%
1%
1%
6%
7%
8%
7%
6%
4%
4%
5%
6%
8%
8%
7%
6%
4%
4%
2%
1%
1%
1%

FIGURE 6.



ONE PEAK, 9% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
24:00
1%
1%
1%
1%
1%
3%
6%
7%
6%
5%
4%
5%
5%
6%
7%
8%
9%
8%
6%
5%
2%
1%
1%
1%

FIGURE 7.



TWO PEAK, 9% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
1%
1%
1%
1%
1%
1%
5%
7%
9%
7%
5%
4%
4%
5%
7%
8%
9%
7%
5%
4%
4%
2%
1%
1%
1%
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
24:00
HOUR OF DAY

FIGURE 8.

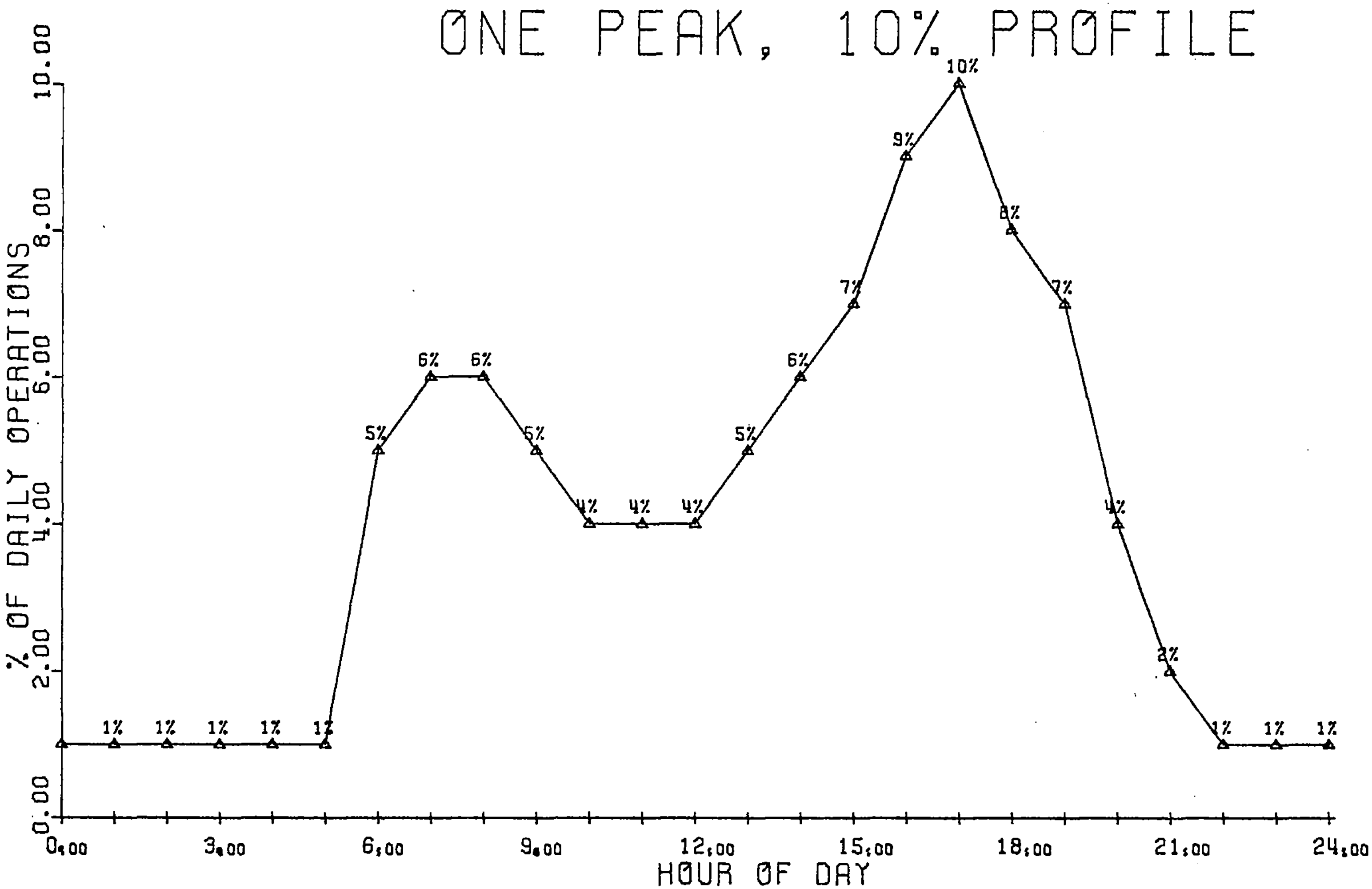

ØNE PEAK, 10% PRØFILE
% ØF DAILY ØPERATIØNS
0.00
2.00
4.00
6.00
8.00
10.00
HØUR ØF DAY
0.00
3.00
6.00
9.00
12.00
15.00
18.00
21.00
24.00
1%
1%
1%
1%
1%
5%
6%
6%
5%
4%
4%
4%
5%
6%
7%
9%
10%
8%
7%
4%
2%
1%
1%
1%

FIGURE 9.

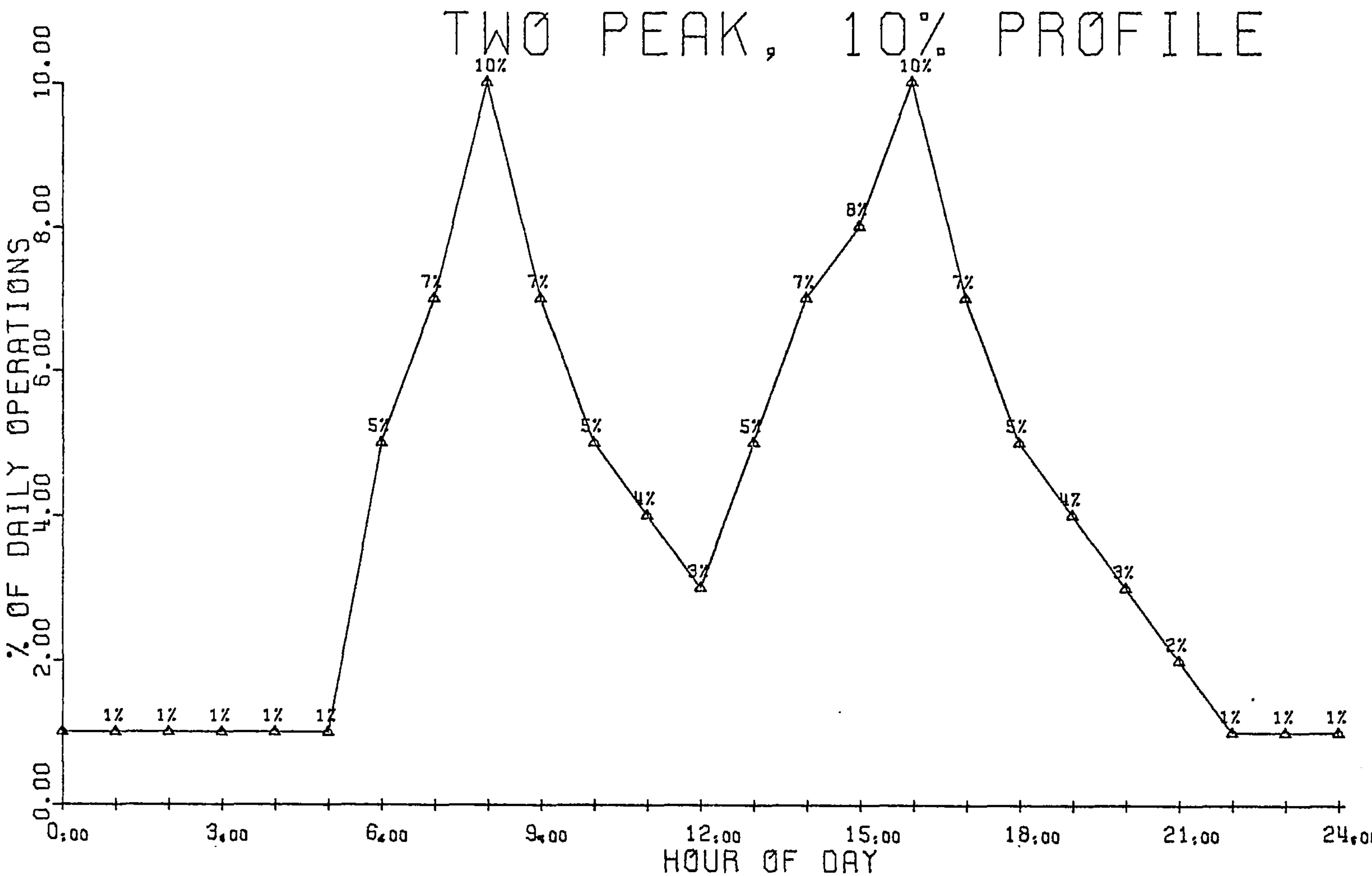

TWO PEAK, 10% PROFILE
% OF DAILY OPERATIONS
0.00
2.00
4.00
6.00
8.00
10.00
HOUR OF DAY
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
24:00
1%
1%
1%
1%
1%
5%
7%
10%
7%
5%
4%
3%
5%
7%
8%
10%
7%
5%
4%
3%
2%
1%
1%
1%

FIGURE 10.

Step 2: Use the saturation capacity of the airport (or convert the practical hourly capacity to saturation capacity by using the conversion table provided earlier) to identify the TDDEL curve to be used on the TDDEL graph already selected in Step 1. (FAA handbooks of airport capacities [1,2] list the saturation or the practical hourly capacities for most common airport configurations).

Step 3: Find the total daily delay at the airport by using the peak hour demand (horizontal axis) and the TDDEL curve selected in Step 2.

Finally, the following notes provide additional important information:

a) Interpolation between TDDEL curves (i.e. for airport capacities different than those listed) and between TDDEL graphs (i.e. for demand profiles "in-between" the ten demand profiles used) is valid. This is illustrated through the examples in the next section.

b) Extrapolation within reasonable limits is also acceptable. However the reader should be cautioned that delay estimates obtained through extrapolation for cases when the peak hour demand far exceeds the saturation capacity of an airport are subject to large errors (see also note c below). For extrapolation purposes, it should be noted that the slope of any given TDDEL curve is everywhere increasing, and that the slope of the extrapolated segment should be likewise shallower or steeper (depending on whether one is concerned with the lower or upper portion of the curve) than the adjacent segment.

c) Total daily delay estimates are particularly sensitive to the details of the demand profile during peak demand periods. The sensitivity is especially acute whenever the demand during peak demand periods reaches or exceeds the saturation capacity of the airport. Consequently, whenever (i) the demand profile for a given airport during peak demand periods is appreciably different from all those in the ten typical profiles and (ii) the demand level is close to or above the saturation capacity level, then the TDDEL estimates obtained through this handbook should be viewed only as rough approximations. In all other cases, the estimates obtained through the TDDEL graphs will be quite accurate.

d) Calculation of annual delays at the airport, average delays per aircraft, delays under IFR or VFR conditions, etc. can all be performed with the aid of the TDDEL graphs. Example 4 in the next section illustrates this.

e) As a last remark, the user is encouraged to scan again the ten typical profiles presented earlier in order to clarify the concept of a "peak," especially in the "no peak" and "one peak" cases. It should be noted that "no peak" does not imply a perfectly flat demand profile. Similarly, "one peak" simply means that there is one main demand periods during the course of a day at an airport. This, however, does not preclude the existence of secondary peaks in the demand pattern.

B. Illustrative Examples

Example 1:

Assume that after hourly totals of takeoffs and landings at an airport have been combined, the plot of operations versus hour of the day is as shown in Figure 1. If the saturation capacity is 66 operations/hour, what is the average total daily delay under these traffic conditions.

Solution: From the figure the total daily traffic is calculated to be 560 operations, implying that peak hour traffic is $\frac{56}{560}$ or 10% of the daily total. Comparison with the 10 standard profiles shows that the airport can be said to have a one peak profile. Using this and the 10% figure for peak hour traffic, the "one peak 10%" graph should be consulted in the procedure that follows: First, locate the point - 56 operations - on the abscissa of the graph and trace up vertically to the intersection with the curve corresponding to a saturation capacity of 66 operations/hour. From the intersection look across horizontally to the ordinate of the graph to obtain 460 minutes of delay per day.

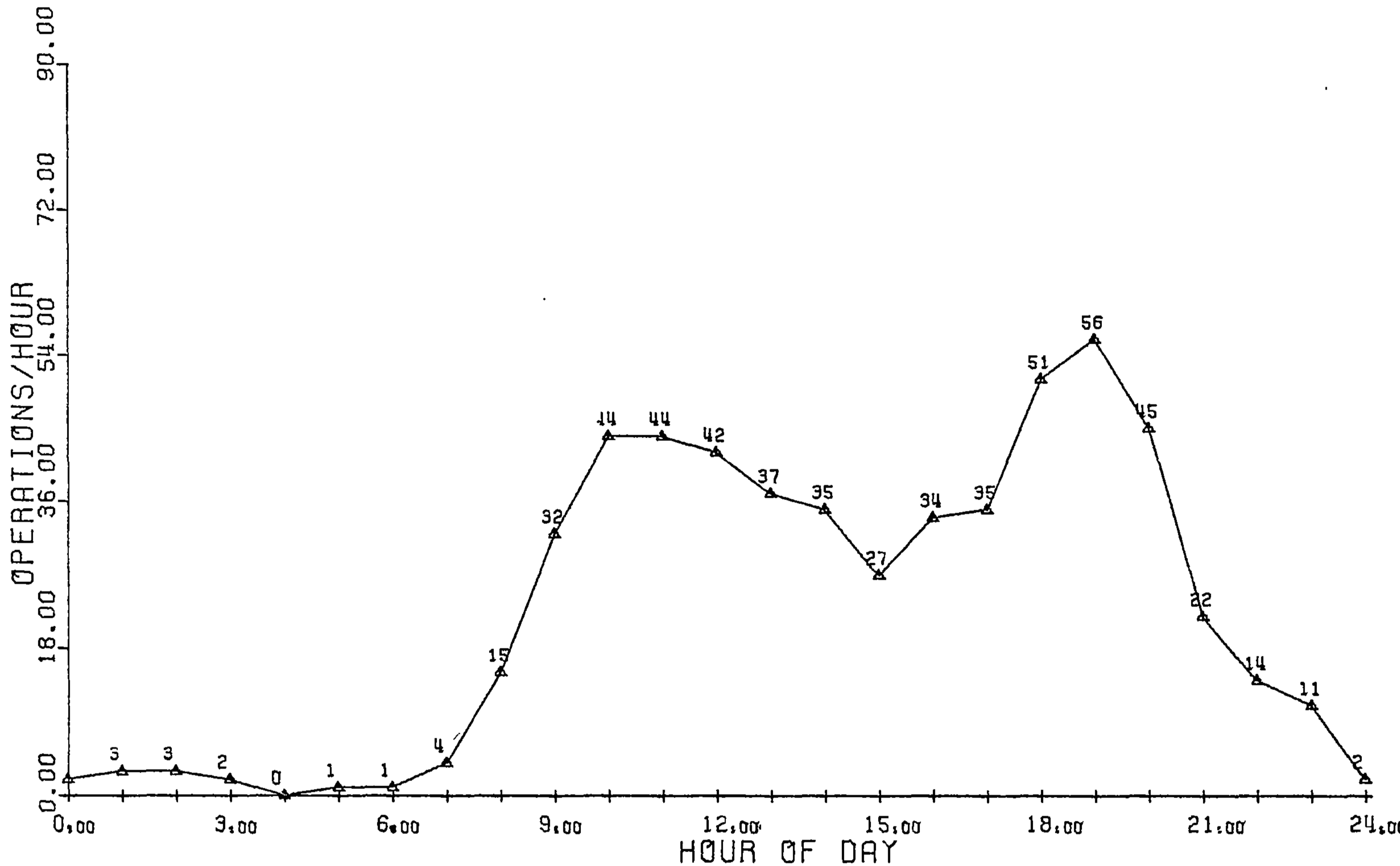
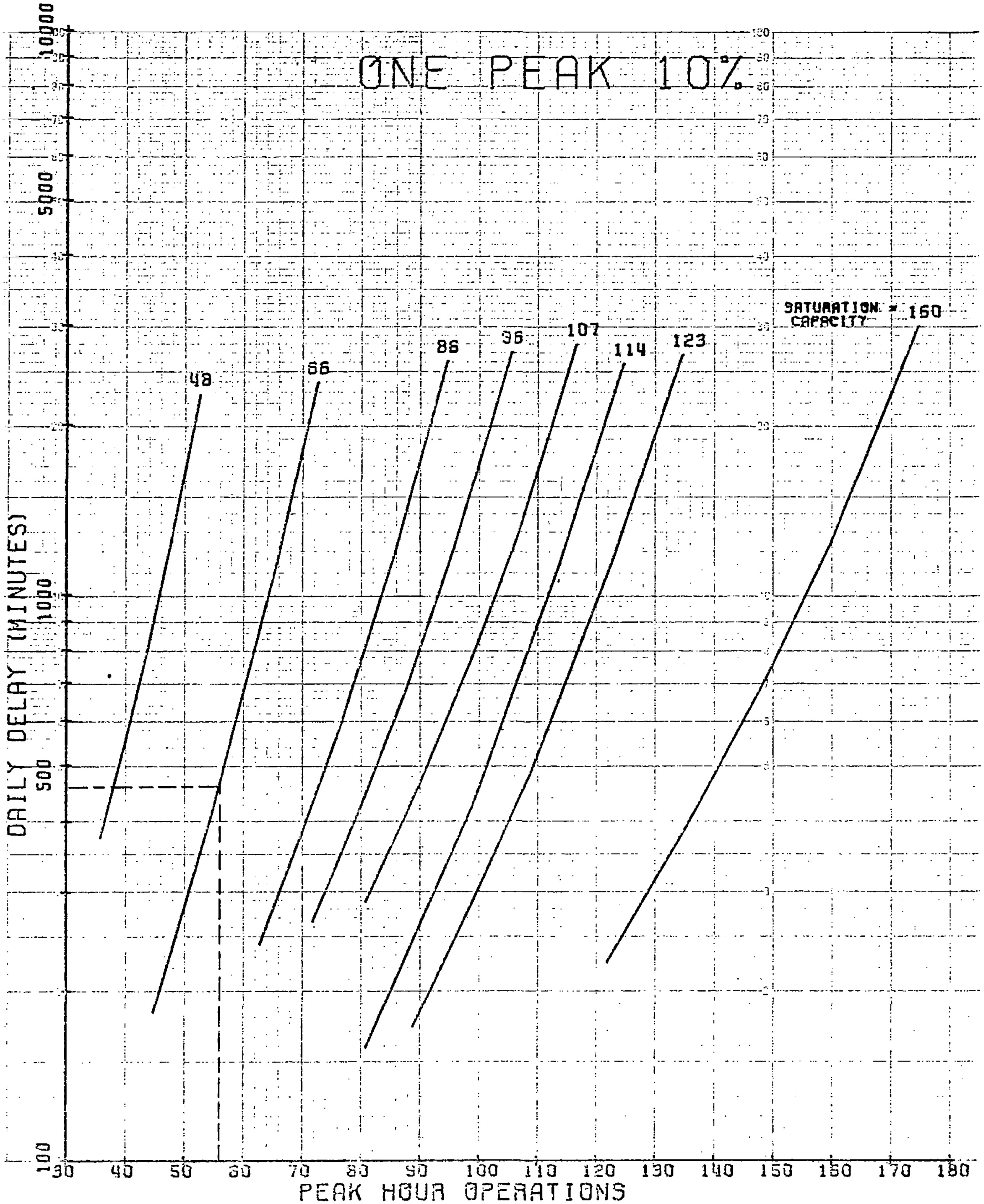


FIGURE FOR EXAMPLE 1.


ONE PEAK 10%
SATURATION CAPACITY = 160
48
66
86
96
107
114
123
DAILY DELAY (MINUTES)
10000
5000
1000
500
100
PEAK HOUR OPERATIONS
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180

Example 2:

Assume, as in Example 1, that after hourly operations have been totaled the plot of operations versus hour of the day is as shown in Figure 2. Let the saturation capacity be 86 operations/hour. What is the expected daily delay under these traffic conditions.

Solution: The procedure is almost identical to that of Example 1. Comparison with the 10 standard profiles indicates that the demand is best approximated by a two peak profile. However, the calculation of the percentage of daily traffic handled at the peak - hour yields $\frac{86}{1175}$ or 7.3 %, a percentage for which a two peak graph does not exist. Therefore, interpolation using the two 2-peak graphs corresponding to the two closest available percentages is necessary. These turn out to be the "two peak 7%" and the "two peak 8%" graphs. Following the procedure of Example 1 on both graphs, the expected delay for peak hour operations of 86 for the 7% case is 3650 minutes and 3000 minutes for the 8% case. Interpolating, the expectation is for 3455 minutes of delay per day.

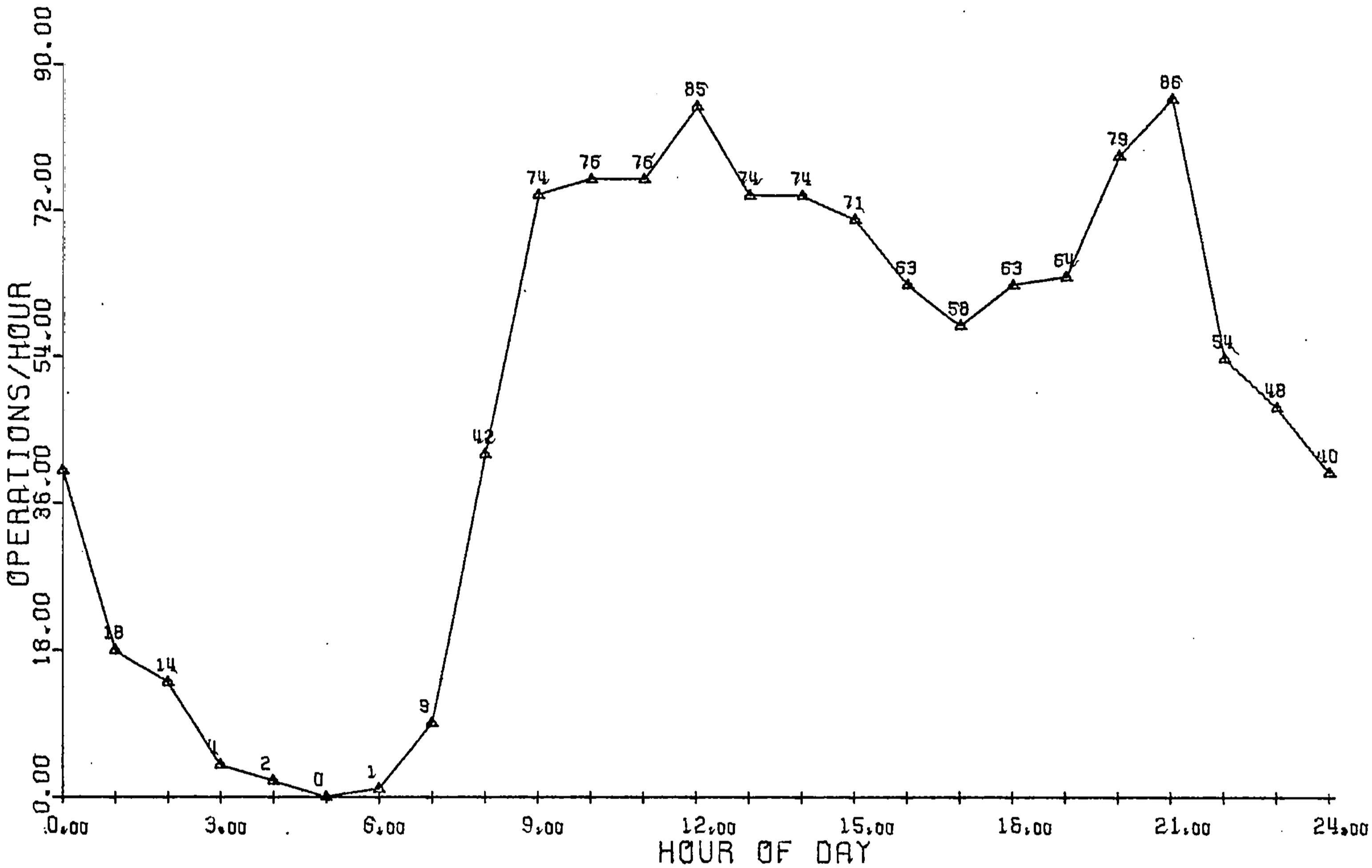
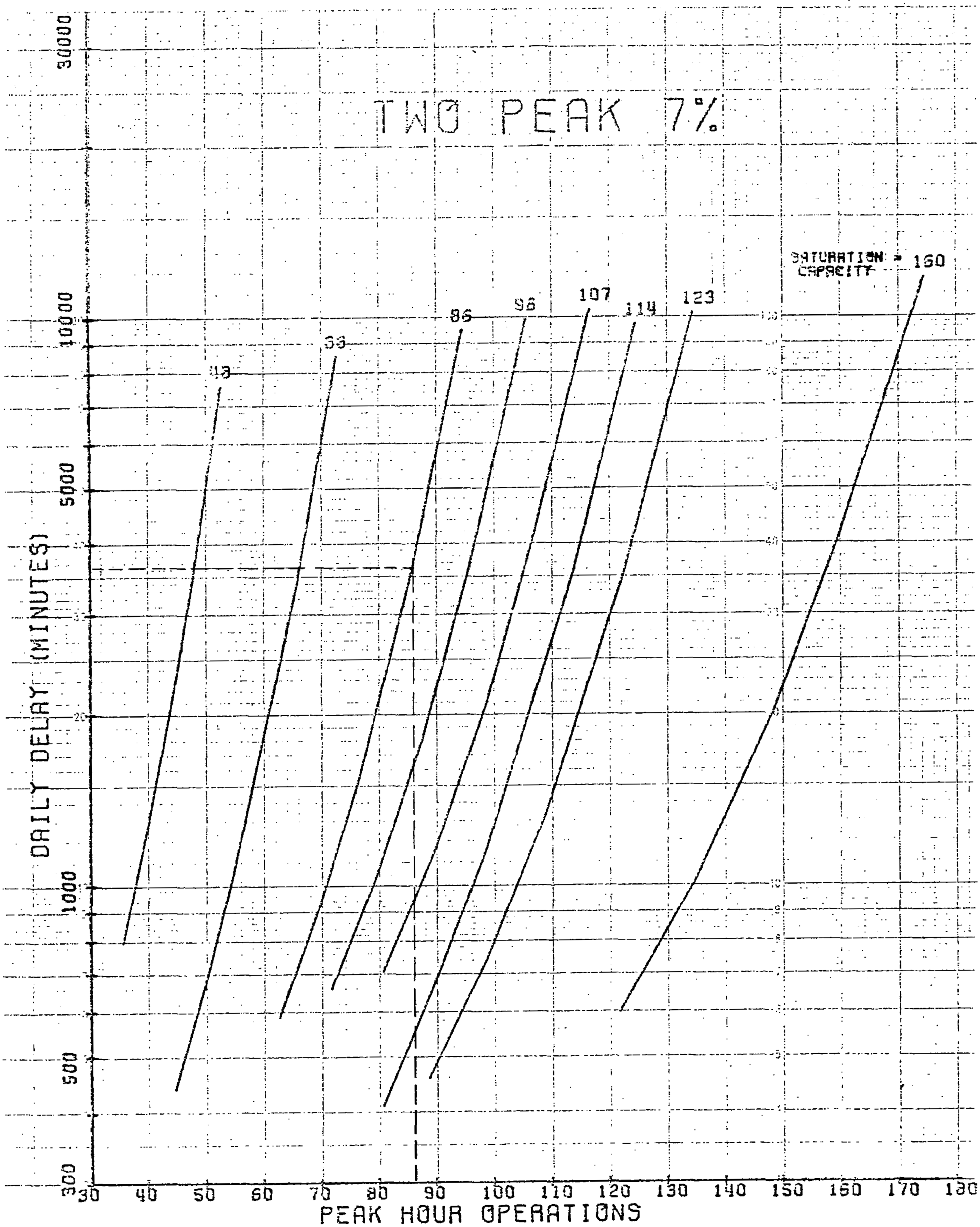
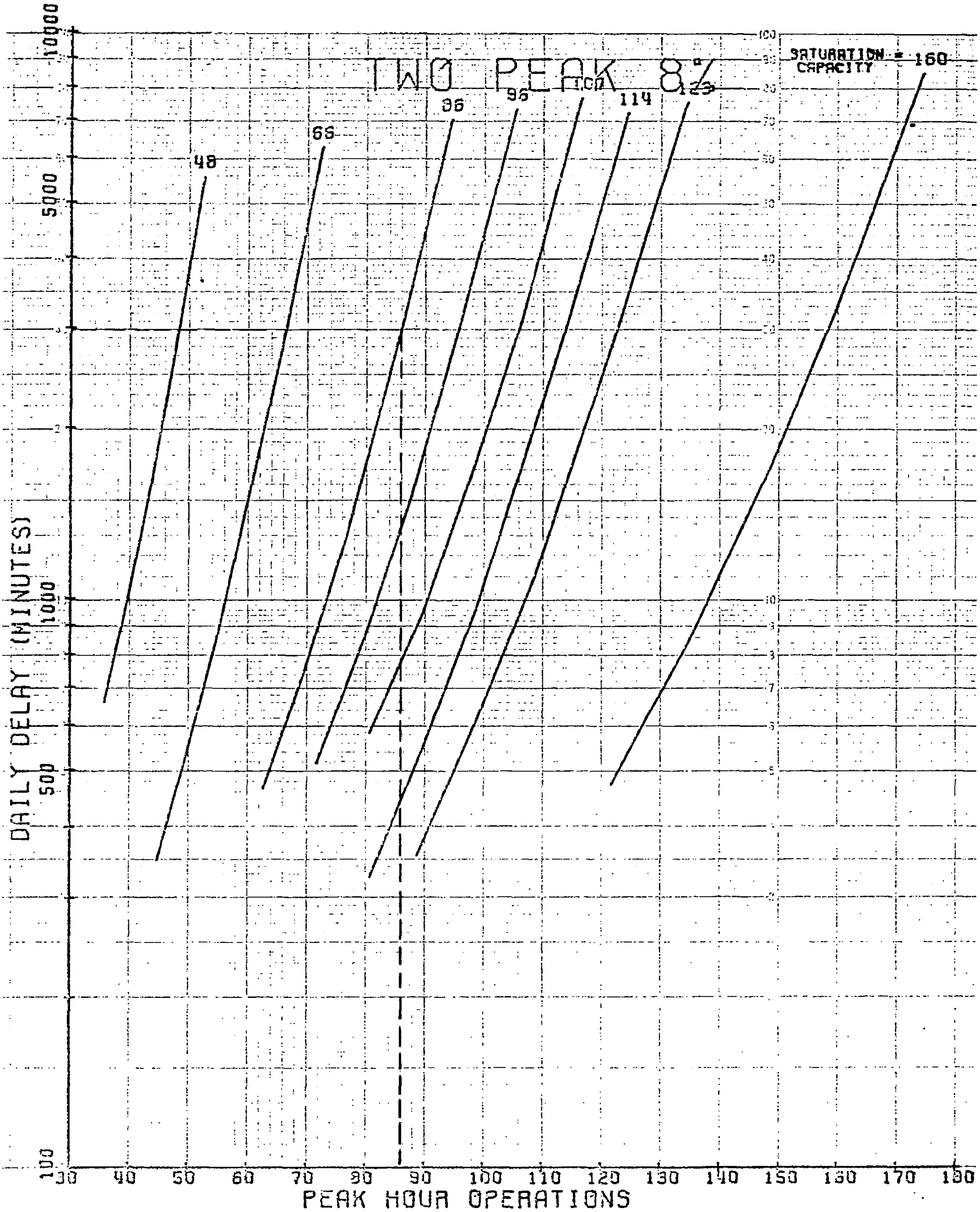


FIGURE FOR EXAMPLE 2.


TWO PEAK 7%
SATURATION CAPACITY = 180
48
68
86
96
107
114
123
30000
10000
5000
1000
500
300
DAILY DELAY (MINUTES)
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
PEAK HOUR OPERATIONS



TWO PEAK 8%
SATURATION CAPACITY = 160
48
66
86
96
100
114
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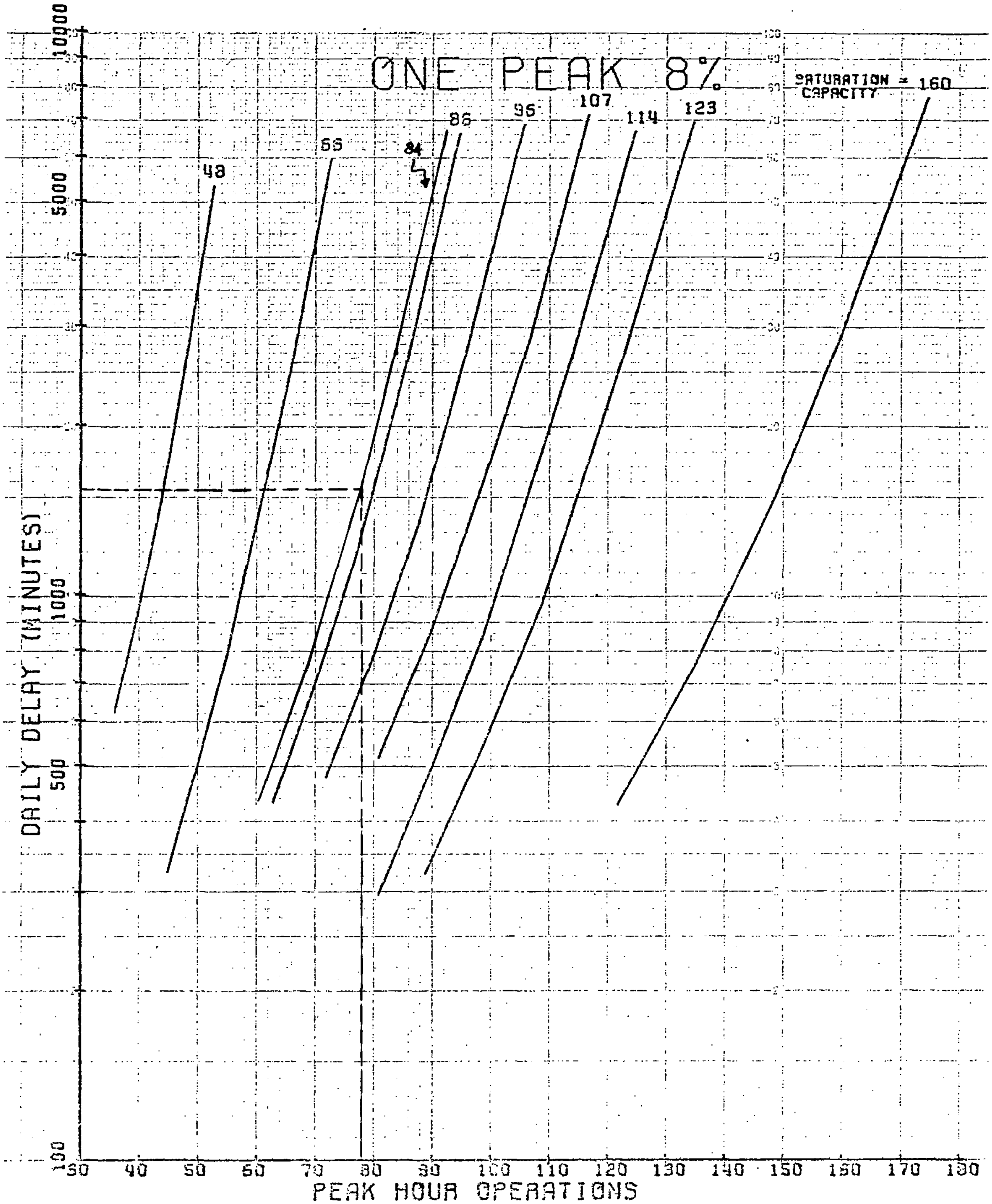
PEAK HOUR OPERATIONS

Example 3:

Assume the airport is best described as having one daily traffic peak with peak hour demand of 78 comprising 8% of total daily traffic. The airport has a saturation capacity of 84 operations/hour. Find the expected daily delay.

Solution: Select from the graphs the one titled "one peak 8%." Since a curve corresponding to saturation capacity of 84 operations is not available, construct this curve by interpolation on the graph. Locate the point - 78 operations- on the abscissa of the graph and trace up vertically to the corresponding point on the 84 operations/ hour curve just constructed. Look across horizontally to the ordinate of the graph to obtain 1550 minutes of delay per day.


ONE PEAK 8%
SATURATION ≈ CAPACITY
48
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DAILY DELAY (MINUTES)
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PEAK HOUR OPERATIONS

Example 4:

For the purpose of computing annual delay assume that an airport undergoes two readily identifiable half year cycles which we shall term "peak season" and "low season." Low season peak hour (p.h.) demand will in all cases be assumed to be given as .85 of the peak season p.h. demand. Within these cycles assume that the weekly operations pattern (daily fluctuations in p.h. demand) is identical for all weeks throughout the year and given by the following ratios of p.h. demand on the day of the week to the greatest p.h. demand (assume 78 operations) which is set ot occur on Friday:

Monday through Thursday	0.95
Friday	1.00
Saturday	0.80
Sunday	0.90

Further assume that in both seasons airport capacity is a constant dependent only on the prevailing weather conditions with 85% of the time VFR weather with airport capacity 84 operations/hour. Assume for simplicity that the demand profile is invariant under the various operating conditions and best approximated by the 8%, One Peak typical profile. Determine the total annual delay at the airport.

Solution: From the above information, since the ratios of the p.h. demand to Friday p.h. demand are specified, the p.h. demand for each day of the week for the peak season weeks can be calculated. Corresponding figures can be obtained for low season weeks by multiplying the values obtained for peak season weeks by .85 as hypothesized. Since the demand profile is unchanged throughout the year, we need only work with the "one Peak 8%" graph. On it, we construct by interpolation the two lines of interest, capacity (VFR) = 84 and capacity (IFR) = 64. On the VFR line we obtain the delay for each day of the week for both peak and low seasons using the set of p.h. demand figures calculated earlier. Likewise, with the same set of p.h. demands, for the IFR line (assuming that IFR weather does not change the volume of traffic). By adding up the results for each of the four groups of seven days, multiplying each of the two (peak, low) VFR results by .85 and adding to the two (peak, low) IFR results multiplied by .15, we compute the expected delays for a peak and low week of the year respectively. After multiplying each result by 26 weeks, the annual delay is finally derived.

A summary of the data and calculations follows.

Summary of Example 4

Scenario:

Peak day (Friday), peak season demand: 970 operations

Peak hour traffic: 78 operations

% peak hour: 8%

Demand distribution: Friday = 1.0, Mon. - Thurs. = 0.95, Sun. = .90

Sat. = .80

Low season demand = (0.85)•(Peak season demand)

Peak season = 6 months

Low season = 6 months

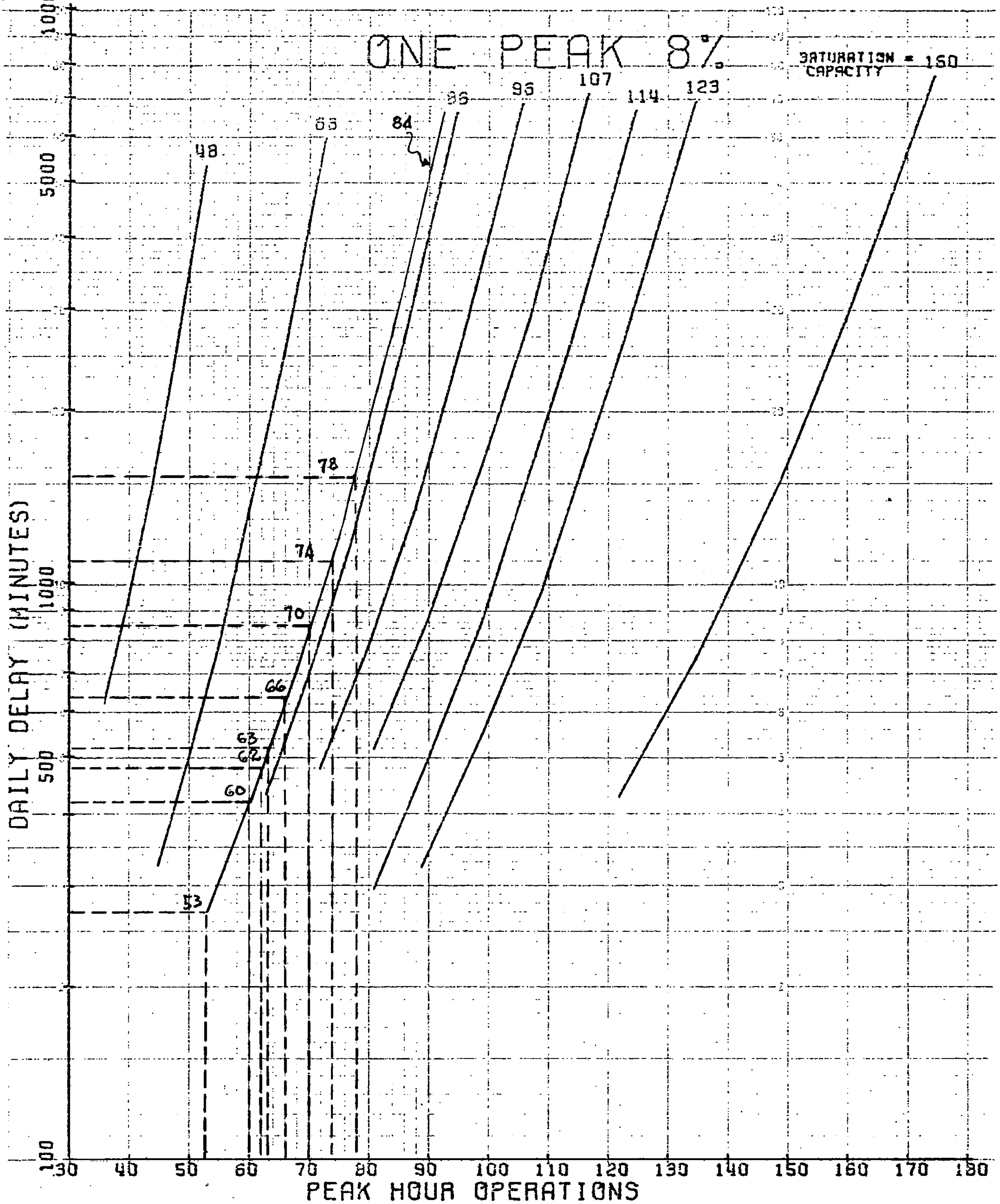
Saturation capacity: VFR weather = 84 operations/hour

IFR weather = 64 operations/hour

Weather distribution: 85% VFR, 15% IFR

Two runways in use at all times

NOTE: Use 8%, one peak graph


ONE PEAK 8%
SATURATION CAPACITY = 160
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DAILY DELAY (MINUTES)
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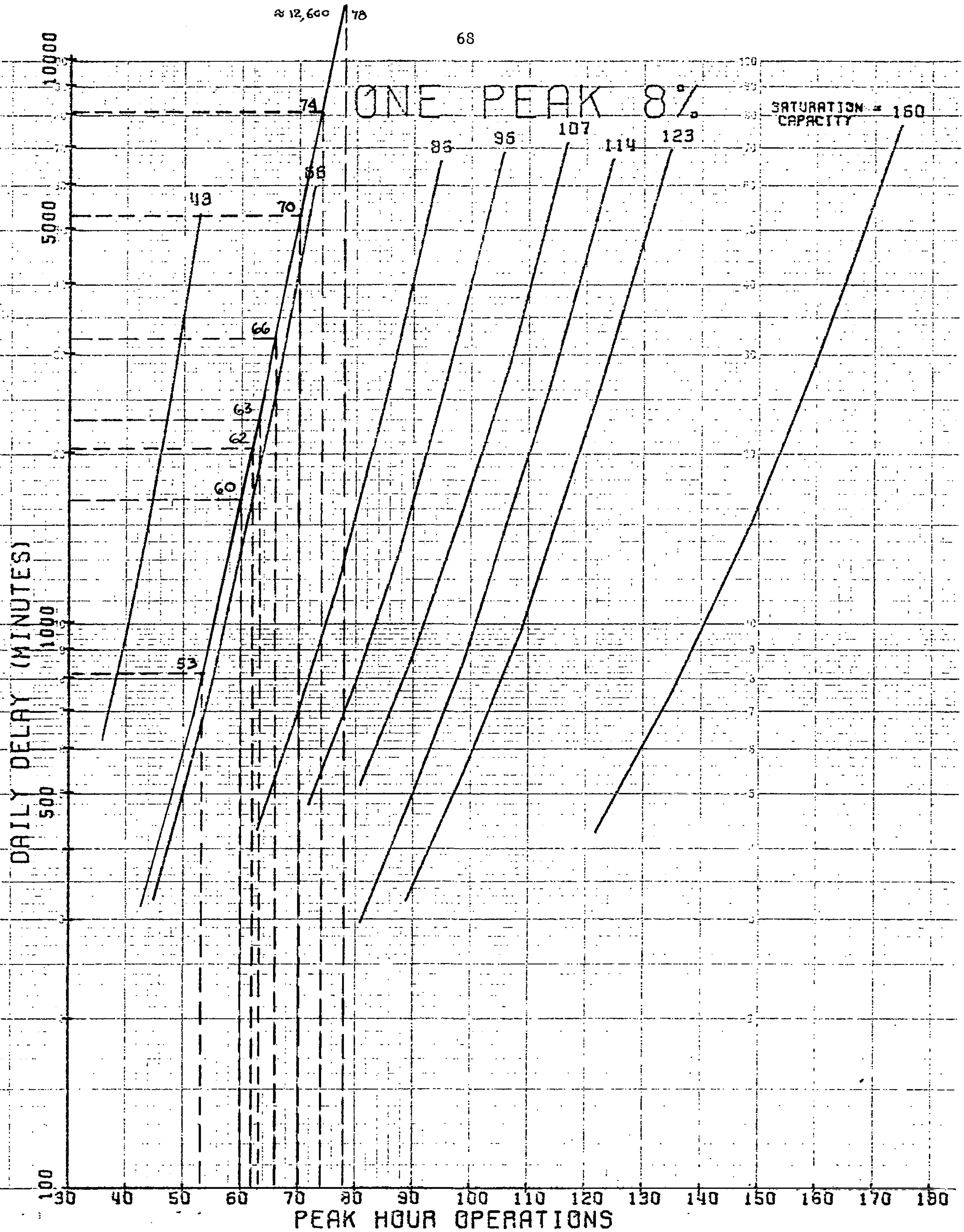
PEAK HOUR OPERATIONS


ONE PEAK 8%
SATURATION CAPACITY = 160
≈ 12,600
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DAILY DELAY (MINUTES)
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PEAK HOUR OPERATIONS

Computations

	Peak Season			Low Season		
	Peak Hr. Demand	VFR Day Delay	IFR Day Delay	Peak Hr. Demand	VFR Day Delay	IFR Day Delay
Friday	78 opers	1,550 mins/day	12,000 mins/day	66 opers	640 mins/day	3,200 mins/day
Mon.-Thur.	74 "	1,100 "	8,100 "	63 "	520 "	2,300 "
Sunday	70 "	850 "	5,300 "	60 "	420 "	1,650 "
Saturday	62 "	480 "	2,050 "	53 "	270 "	820 "

Peak season total delay per week: 14,040 mins.

Low season total delay per week: 5,300 mins.

Total average annual delay = 498,400 mins.

REFERENCES

1. Airfield Capacity and Delay Handbook, prepared for Department of Transportation, Federal Aviation Administration under contract with DOT FA72WA-2897, FAA HBK 74-124, draft final report May 1975.

2. Airport Capacity Handbook (2nd Edition), prepared for the Federal Aviation Administration, Airborne Instrument Laboratory, Farmingdale, New York (1967).

Appendix I

(This Appendix is excerpted from the report Time Dependent Estimates of Delays and Delay Costs at Major Airports by Gerd Hengsbach and Amedeo R. Odoni, Flight Transportation Laboratory, M.I.T. Report R 75-4, Cambridge, Mass., January 1975.)

THE MODELS

The theoretical model presented here is based on the earlier work of KOOPMAN [2] and is a quite straight-forward extension of that work to the case of multiple servers (i.e., multiple runway airports). For this reason we shall only describe the bare essentials of the theoretical foundations here and, instead, concentrate on providing an intuitive explanation of the basic rationale, of the assumptions used, and of the limitations of the models. For a rigorous treatment of the theoretical questions, the reader is referred to [2].

The model considers an airport as a set of independent, parallel servers (the runways). A schematic representation of this system is shown in figure 1.

It is assumed that the total demand at the airport - that is, the sum of the demands for landings and for take-offs - is a Poisson process with a time-dependent average demand rate, given by $\lambda(t)$. The Poisson assumption for airport demand is consistent with actual observations at several major airports and has been used extensively in the literature [1], [3], [6].

By contrast, the form of the probability law describing the duration of a service at the runways is still a matter for speculation [1], [3], [4]. The duration of the period during which a runway is busy with an aircraft depends on such diverse factors as type of operation being conducted, weather, aircraft mix, runway configuration in use, runway surface conditions, location of runway exits, air traffic control equipment, requirements for minimum separations

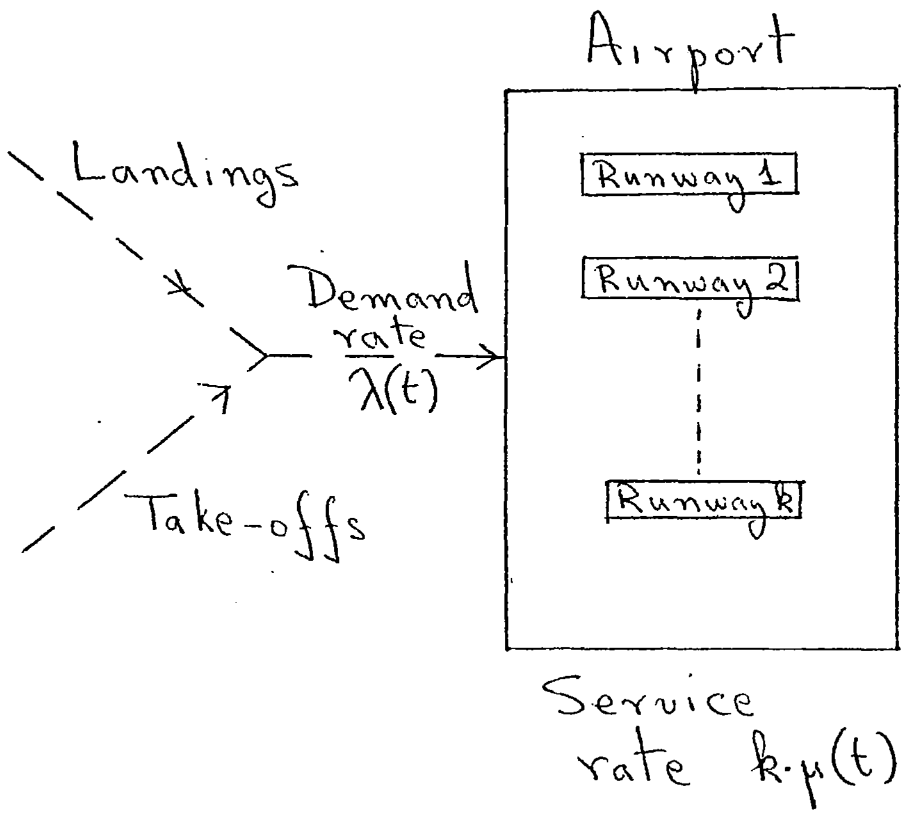

Figure 1: Schematic representation of the model.

between aircraft, pilot and air traffic controller performance, etc. Following the example of [2], we shall sidestep this issue by making this intuitively reasonable observation: the duration of the service times must be "less random" than the perfect randomness described by the negative exponential probability density function and "less regular" than the perfect regularity described by deterministic service times.

This last point is a crucial one as it drives our whole approach to the problem: we shall seek to obtain upper and lower bounds on congestion-related statistics by noting that a worst case is provided by the negative exponential service assumption and a best case by the deterministic service assumption. The rationale, of course, is that, if - for the set of parameter values prevalent in the systems under consideration, i.e. the major commercial airports - the upper and lower bounds turn out to be reasonably close to each other, then either bound (or any reasonably weighted combination of the two) can be used as a good approximation of the actual statistics desired. As will be seen in what follows, the bounds do indeed turn out to be close for all practical purposes, and under widely varying sets of conditions.

Here then is the strategy to be followed: Given an airport with k independent runways each of which has a time-dependent average service rate $\mu(t)$, we shall solve iteratively and for the desired period of time two systems of equations, one describing an M/M/k queuing system and the other an M/D/k queuing system. The actual values of interest will then be bounded from above and below by he values obtained from these two queuing models. This whole ap-roach is dictated by the fact that the integro-differential equa-ions that describe an M/G/k queuing system - a more realistic odel for the case of interest - are unwieldy even for the purpose f obtaining numerical solutions.

ssumptions in the Model

To complete the description of our queuing models, we now list ome assumptions that were made, mostly for reasons of computational easibility. The most important of these, from a practical viewpoint, s the assumption of the existence of a single queue of aircraft waiting use of the runways on a strictly first-come, first-erved basis. Thus, we make no distinction between landing and eparting aircraft but are instead interested only in overall measures f congestion. While, in practice, the average service times (and the robability distributions) for landings and take-offs are different we use here what is in effect a single weighted verage service time for both kinds of operations.

Another assumption is that all active runways (or, all the arallel servers in figure 1) operate independently and are identical. n practice, runways often can not be operated independently, since perations at one may affect those on another, due to airport geometry. gain, from the practical viewpoint, this assumption is not too re-trictive since dependencies among the servers, if they exist, can e accounted for by adjusting the service rates accordingly. As an xample, consider an airport with a single runway which can handle, say, 50 aircraft movements per hour, i.e. the average service time is 72 seconds. Suppose now that operations are begun at a second runway which intersects the first one. Then, the overall airport capacity might increase to, say, 80 operations per hour, and not to 100 as it would if the two runways were independent. To account for this in our model, we would then assume the existence of a single independent server, with an average service time of 45 seconds for an overall airport capacity of 80 movements per hour.

Obviously, the number of state-transition equations, describing the queuing models and being iteratively solved by the computer, must be finite. Since the number of such equations is equal to the number of states in the queuing model, a futher condition must be that the capacity of the airport queue is finite. Thus, it is assumed that the queuing system of figure 1, can accomodate up to a maximum of m aircraft (including the ones in service at the k servers In practice, this is entirely inconsequential since m can be selected large enough to make it highly unlikely that the number of aircraft in the terminal area at any given instant will be equal to m. This is further discussed later in this paper.

Finally, it is assumed that successive service times are statistically independent. This is substantially true in reality, as little attempt is made, under today's air traffic control regime, to sequence operations in anything but a first-come, first-served way. Successive service times are, therefore, randomly mixed according to the mix of aircraft with little or no inter-dependence among them.

ıe M/M/k System Equations

We now list the equations that describe the two queuing systems ıder consideration here. First, for the M/M/k model, we have ›isson arrivals at a time-dependent average rate of $\lambda(t)$. These :rivals are served by k parallel servers, each operating at an /erage service rate, $\mu(t)$. It is assumed, that individual service imes are distributed as negative exponential random variables with :ponent equal to the value of $\mu(t)$ at the instant t when service is ıitiated. The queue capacity is equal to m.

Let us define by $P_i(t)$, $i = 0,1,2,\ldots,m$, the probability that time t there are i aircraft in the terminal area. Then, for any we can write the well-known set of Chapman-Kolmogorov equations ›r the derivatives $P_i'(t)$ of the state probabilities. Suppressing, ›r reasons of conciseness, the time-dependence of the arrival and :rvice rates, i.e. writing $\lambda = \lambda(t)$ and $\mu = \mu(t)$, we have:

$$(t) = -\lambda P_0(t) + \mu P_1(t) \qquad (1.1)$$

$$(t) = \lambda P_{i-1}(t) - (\lambda + i\mu)P_i(t) + (i+1)\mu P_{i+1}(t) \quad \text{for } 1 \le i \le k-1 \qquad (1.2)$$

$$(t) = \lambda P_{i-1}(t) - (\lambda + k\mu)P_i(t) + k\mu P_{i+1}(t) \quad \text{for } k \le i \le m-1 \qquad (1.3)$$

$$(t) = \lambda P_{m-1}(t) - k\mu P_m(t) \qquad (1.4)$$

The above m + 1 equations can be solved iteratively for any :sired period of time T, using the approximation $P_i(t+\Delta t)=P_i(t)+P_i'(t)\cdot\Delta t$, ıere Δt is a time interval chosen sufficiently small to be consistent with the Poisson assumptions regarding the arrival and service processes. A boundary set of values $P_i(0)$, $i = 0,1,2,\ldots,m$, and the functions $\lambda(t)$ and $\mu(t)$ for $0 \leq t \leq T$ must be provided.

The M/D/k System Equations

Turning to the corresponding system of equations for the model in which service is assumed to be deterministic, we define the increment of time as equal to the duration of a single service time. We assume further that all k parallel servers begin and end service simultaneously. It is then possible to write equations relating the sets of state probabilities $P_i(t)$ and $P_i(t+1)$ - remember that t is now being increased at discrete intervals equal to the average service time. (Since time intervals are normalized to $1/\mu$, the demand rate must also be normalized to $\rho = \lambda/\mu$, the demand per unit of service.) These equations are based on the fact that the probability that exactly n aircraft will attempt to join the system between t and t+1 is equal to $\rho^n \cdot \exp(-\rho)/n!$ due to the Poisson law for the demand pattern.

We then have:

$$P_0(t+1) = \exp(-\rho)\, q_k(t) \tag{2.1}$$

$$P_i(t+1) = \exp(-\rho)\left[q_k(t)\frac{\rho^i}{i!} + P_{k+1}(t)\frac{\rho^{i-1}}{(i-1)!} + P_{k+i}(t)\right] \quad \text{for } 1 \leq i \leq m-k \tag{2.2}$$

$$P_i(t+1) = \exp(-\rho)\left[q_k(t)\frac{\rho^i}{i!} + P_{k+1}(t)\frac{\rho^{i-1}}{(i-1)!} + \cdots\cdots + P_m(t)\frac{\rho^{i+k-m}}{(i+k-m)!}\right] \quad \text{for } m-k+1 \leq i \leq m-1 \tag{2.3}$$

$$P_m(t+1) = q_k(t)b_m + P_{k+1}(t) \cdot b_{m-1} + \ldots + P_m(t) \cdot b_k \quad (2.4)$$

where $q_k(t) = \sum_{i=0}^{k} P_i(t)$ and $b_j = \exp(-\rho)\sum_{i=j}^{\infty} \frac{\rho^i}{i!}$.

Strictly speaking, (2) assumes that the new arrivals during a unit of time join the queue at the end of the service unit at which time the capacity limit, m, applies.

Again, beginning with a set of initial conditions $P_i(0)$, $i = 0, 1, 2, \ldots, m$, the above set of equations can be solved iteratively to obtain numerical answers for demand and service rate profiles, $\lambda(t)$ and $\mu(t)$ (we have, for conciseness, suppressed the time variable in the equations).

Related Quantities

KOOPMAN [2] has shown that for "relatively slow varying" $\lambda(t)$ and $\mu(t)$ the sets of equations for the M/M/k and M/D/k systems possess unique periodic solutions with period T whenever the demand and service rates are both periodic with period T. In the case of airports, demand and service rates can indeed be considered to be periodic quantities with period T=24 hours. It remains, therefore, to solve the two sets of equations numerically to obtain estimates of the state probilities, $P_i(t)$, for all $0 \le t \le T$. The state probabilities, in turn, can be used to compute other quantities of interest. Of those, we shall specifically refer to:

i) The probability that all runways are busy and, therefore, that a newly- arriving aircraft will experience positive delay,

$$B(t) = 1 - \sum_{i=0}^{k} P_i(t) \quad (3)$$

ii) The expected number of aircraft in the queue at time t,

$$Q(t) = \sum_{i=k+1}^{m} (i-k)P_i(t) \qquad (4)$$

iii) The average waiting time in the queue for aircraft that arrive at time t (see Note 5)

$$W(t) = \frac{1}{k \cdot \mu(t)} \sum_{i=k}^{m} (i-k+1)P_i(t) \qquad (5)$$

This last quantity is only an approximation in the case when $\mu(t)$ is a function of time. The reason is that the rate of service, $\mu(t)$, may change in the future if the waiting time is long.

In all cases, two estimates of these parameters of interest are obtained, one based on the M/M/k and the other based on the M/D/k model.

REFERENCES

1. R.M. HARRIS, Models for Runway Capacity Analysis, Report MITRE 4102, Rev. 2, the MITRE Corporation, McLeans, Virginia, December, 1972.

2. B.O. KOOPMAN, "Air Terminal Queues under Time-Dependent Conditions," Operations Research 20, 1089-1114 (1972).

3. Analysis of Capacity Concept for Runway and Final Approach Path Airspace, Report No. NBS-10111, AD-698-521, U.S. National Bureau of Standards, November, 1969.

4. A.R. ODONI, An Analytic Investigation of Air Traffic in the Vicinity of Terminal Areas, Technical Report No. 46, Operations Research Center, MIT, Cambridge, Massachusetts, December, 1969.

NASA-Langley, 1976 CR-2644