(NASA-CR-148205) A UNIQUE FORMULATION OF ELASTIC AIRPLANE LONGITUDINAL EQUATIONS OF MOTION (Purdue Univ.) 25 p HC $3.50 CSCL 01C

N76-26187

Unclas

G3/05 42311

A UNIQUE FORMULATION OF ELASTIC AIRPLANE LONGITUDINAL EQUATIONS OF MOTION

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ABSTRACT

Control-configured vehicle technology has increased the demand for detailed analysis of dynamic stability and control, handling and ride qualities, and control system dynamics at early stages of preliminary design. An approximate, but reasonably accurate, set of equations of motion are needed for these early analyses. Such a formulation is developed for the longitudinal dynamics of elastic airplanes. It makes use of only rigid-body aerodynamic stability derivatives in formulating the forces and moments due to elastic motion. Verification of accuracy using data for the B-1 airplane shows very good agreement. Frequencies and damping ratios of the coupled modes corresponding to complex roots of the characteristic equations agree closely with four symmetric elastic modes included.

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Research supported by NASA Dryden Flight Research Center under grant NSG 4003; Donald T. Berry, Technical Officer.

NOMENCLATURE

c = mean aerodynamic chord (ft)

C_D = drag coefficient

C_{L_α} = lift curve slope stability derivative

$C_{L_{\delta_e}}$ = lift coefficient due to elevator deflection stability derivative.

C_{m_α} = pitching moment coefficient due to angle of attack stability derivative

$C_{m_{\dot{\alpha}}}$ = pitching moment coefficient due to wing downwash on tail stability derivative

C_{m_q} = pitching moment coefficient due to pitch rate stability derivative

$C_{m_{\delta_e}}$ = pitching moment coefficient due to elevator deflection stability derivative

F_{i_w} = ith elastic mode aerodynamic force coefficient in y-direction due to plunge velocity of C.G. (1/sec)

$F_{i_{\xi_j}}$ = ith elastic mode aerodynamic force coefficient in z-direction due to jth mode generalized displacement (1/sec^2)

$F_{i_{\dot{\xi}_j}}$ = ith elastic mode aerodynamic force coefficient in z-direction due to jth mode generalized velocity (1/sec)

$F_{i_{\delta_e}}$ = ith elastic mode aerodynamic force coefficient in z-direction due to elevator deflection (ft/sec^2)

$F_{i_{w_g}}$ = ith elastic mode aerodynamic force coefficient in z-direction due to vertical gust velocity (1/sec)

I_y = mass moment of inertia about y-axis (slug-ft^2)

M = total airplane mass (slugs)

M_w = aerodynamic pitching moment stability derivative due to plunge velocity of C.G. (rad/ft-sec)

$M_{\dot{w}}$ = aerodynamic pitching moment stability derivative due to downwash from wing to tail (rad/ft)

M_q = aerodynamic pitching moment stability derivative due to pitch rate (1/sec)

M_{δ_e} = aerodynamic pitching moment stability derivative due to elevator deflection (1/sec^2)

M_{ξ_i} = aerodynamic pitching moment coefficient due to ith elastic mode generalized displacement (rad/ft-sec^2)

$M_{\dot{\xi}_i}$ = aerodynamic pitching moment coefficient due to ith elastic mode generalized velocity (rad/ft-sec)

$M_{\ddot{\xi}_i}$ = aerodynamic pitching moment coefficient due to ith elastic mode generalized acceleration (rad/ft)

M_i = $\int_y \int_x m(x,y)\phi_i^2(x,y)dxdy$ ith elastic mode generalized mass (slugs)

$q_g(t)$ = pitch gust velocity (rad/sec)

S or S_W = wing planform reference area (ft^2)

S_{HT} = horizontal tail planform reference area (ft^2)

U_0 = trim flight velocity (ft/sec)

$w(x,y,t)$ = local plunge velocity in z-direction (ft/sec)

$w_g(t)$ = vertical gust velocity at C.G. in negative z-direction (ft/sec)

Z_w = aerodynamic force stability derivative in z-direction due to plunge velocity of C.G. (1/sec)

Z_{δ_e} = aerodynamic force stability derivative in z-direction due to elevator deflection (ft/rad-sec^2)

Z_{ξ_i} = aerodynamic force coefficient in z-direction due to ith elastic mode generalized displacement ($1/sec^2$)

$Z_{\dot{\xi}_i}$ = aerodynamic force coefficient in z-direction due to ith elastic mode generalized velocity (1/sec)

$\alpha(x,y,t)$ = local angle of attack (rad)

δ_e = elevator deflection (rad)

ζ_i = ith elastic mode structural damping ratio

$\xi_i(t)$ = ith elastic mode generalized displacement in z-direction (ft)

$\theta(x,y,t)$ = local pitch angle (rad)

ρ_0 = free stream air density ($slugs/ft^3$)

ω_i = free-free undamped natural frequency of ith elastic mode (rad/sec)

$\phi_i(x,y)$ = ith elastic mode normalized mode shape

$\phi_i'(x,y) = \frac{\partial \phi_i(x,y)}{\partial x}$ slope of $\phi_i(x,y)$ with respect to x (1/ft)

$\Xi_{im}(t)$ = ith elastic mode motion-dependent generalized force in z-direction (ft/sec^2)

$\Xi_{ig}(t)$ = ith elastic mode gust-induced generalized force in z-direction (ft/sec^2)

INTRODUCTION

Recent work with control-configured vehicles (CCV) and active control technology (ACT) has improved the performance, stability, and handling qualities of large flexible airplanes and has opened up a new realm of design frontiers[1].

With increased size of present day airplanes, and with the increased utilization of lighter materials, the elastic behavior of these vehicles is becoming an appreciable influence in their handling and ride qualities. Due to the potential adverse effects of elastic mode interaction with the rigid-body dynamics, there is a need for a simplified method of modeling the dynamic aeroelastic equations of motion for use in preliminary control system design stages of new airplanes.

Usually, only calculated values of the rigid-body aerodynamic stability derivatives are available for the preliminary design from sources such as DATCOM[2], and little, if any, information on the stability derivatives due to elastic modes is available then. However, calculated values of the symmetric and antisymmetric orthogonal elastic vibration mode shapes and natural frequencies are usually available at the preliminary design stage for use in equations of motion formulation.

We have developed a unique formulation of the equations of motion for elastic airplanes that makes use of rigid-body aerodynamic stability derivatives and the elastic mode shapes and frequencies to describe the aerodynamic forces and moments due to the elastic motion of the aircraft. There is no need for unsteady aerodynamic theories or experimental data on elastic mode aerodynamics as with conventional formulations.

This paper describes the longitudinal dynamic formulation and verification of its accuracy using the B-1 aircraft dynamics at a high subsonic flight condition.

EQUATIONS OF MOTION FORMULATION

Since the elastic modes do not produce significant drag force perturbations compared to the rigid-body motion, the longitudinal equation of motion is omitted and only the plunge and pitch rigid-body equations (short-period approximation) are included in what follows. The formulation of the small perturbation aerodynamic forces and moments is based on the local effective angle of attack, $\alpha(x,y,t)$, or effective plunge velocity, $w(x,y,t)$, where $w(x,y,t) = U_0\alpha(x,y,t)$, and the local effective pitch rate, $\dot{\theta}(x,y,t)$.

The elastic vibration characteristics are based on the usual approach of idealizing the structure to a flat plate in the xy-plane, and the symmetric orthogonal free-free elastic vibration mode shapes $\phi_i(x,y)$ are functions of x and y coordinates in the x,y,z body axes system located at the center of gravity[3]. The sign convention for the mode shapes, mode slopes, and generalized displacements is given in Figures 1 and 2.

The two time domain short-period and n elastic mode small perturbation equations of motion about a trim condition are given by (1).

$$
\begin{aligned}
\dot{w}(t) - U_0\dot{\theta}(t) &= \int_y\int_x \frac{\partial^2 Z_w}{\partial x \partial y} w(x,y,t)dxdy + Z_{\delta_e}\delta_e(t) + Z_w w_g(t) \\
\ddot{\theta}(t) &= \int_y\int_x \frac{\partial^2 M_w}{\partial x \partial y} w(x,y,t)dxdy + \int_y\int_x \frac{\partial^2 M_{\dot{w}}}{\partial x \partial y} \dot{w}(x,y,t)dxdy \\
&\quad + \int_y\int_x \frac{\partial^2 M_q}{\partial x \partial y} \dot{\theta}(x,y,t)dxdy + M_{\delta_e}\delta_e(t) + M_w w_g(t) + M_q q_g(t) \\
\ddot{\xi}_i(t) + 2\zeta_i\omega_i\dot{\xi}_i(t) + \omega_i^2\xi_i(t) &= \Xi_{im}(t) + \Xi_{ig}(t) \qquad (i=1,2,\ldots,n)
\end{aligned}
\tag{1}
$$

Z_w, Z_{δ_e}, M_q, M_w, $M_{\dot{w}}$, and M_{δ_e} are the rigid-body dimensional aerodynamic stability derivatives defined in (2). Z_q and $Z_{\dot{w}}$ are assumed to be negligible and are not included in (1).

$$Z_w = -\rho U_0 S(C_{L_\alpha} + C_D)/2M \qquad Z_{\delta_e} = -\rho U_0^2 S C_{L_{\delta_e}}/2M$$

$$M_q = \rho U_0 S c^2 C_{m_q}/4I_y \qquad M_w = \rho U_0 S c C_{m_\alpha}/2I_y$$

$$M_{\dot{w}} = \rho S c^2 C_{m_{\dot{\alpha}}}/4I_y \qquad M_{\delta_e} = \rho U_0^2 S c C_{m_{\delta_e}}/2I_y \tag{2}$$

The integral terms and the generalized force terms in (1) are functions of $w(x,y,t)$, and $\dot{\theta}(x,y,t)$, which can be closely approximated by (3) and (4).

$$w(x,y,t) = w(t) + \sum_{i=1}^{n} \phi_i(x,y)\dot{\xi}_i(t) - \sum_{i=1}^{n} U_0\phi_i'(x,y)\xi_i(t) \tag{3}$$

$$\dot{\theta}(x,y,t) = \dot{\theta}(t) - \sum_{i=1}^{n} \phi_i'(x,y)\dot{\xi}_i(t) \tag{4}$$

The integral terms can be written as in (5) and (6).

$$\int_y \int_x \frac{\partial^2 Z_w}{\partial x \partial y} w(x,y,t)dxdy = Z_w w(t) + \sum_{i=1}^{n} \left[Z_{\xi_i} \xi_i(t) + Z_{\dot{\xi}_i} \dot{\xi}_i(t) \right] \tag{5}$$

$$\int_y \int_x \frac{\partial^2 M_w}{\partial x \partial y} w(x,y,t)dxdy + \int_y \int_x \frac{\partial^2 M_{\dot{w}}}{\partial x \partial y} \dot{w}(x,y,t)dxdy$$
$$+ \int_y \int_x \frac{\partial^2 M_q}{\partial x \partial y} \dot{\theta}(x,y,t)dxdy = M_w w(t) + M_{\dot{w}}\dot{w}(t) + M_q\dot{\theta}(t)$$
$$+ \sum_{i=1}^{n} \left[M_{\xi_i} \xi_i(t) + M_{\dot{\xi}_i} \dot{\xi}_i(t) + M_{\ddot{\xi}_i} \ddot{\xi}_i(t) \right] \tag{6}$$

The expressions for Z_{ξ_i}, $Z_{\dot{\xi}_i}$, M_{ξ_i}, $M_{\dot{\xi}_i}$, and $M_{\ddot{\xi}_i}$ are tabulated in Appendix A.

The expression for the motion-dependent generalized force term in the n elastic mode equations of motion of (1) is given by (7).

$$\Xi_{im}(t) = \frac{1}{M_i} \int_y \int_x \frac{\partial^2 Z(t)}{\partial x \partial y} \phi_i(x,y)dxdy \qquad (7)$$

where

$$Z(t) = M[Z_w w(t) + \sum_{j=1}^{n} [Z_{\xi_j} \xi_j(t) + Z_{\dot{\xi}_j} \dot{\xi}_j(t)] + Z_{\delta_e} \delta_e(t)] \qquad (8)$$

and M_i is the ith mode generalized mass.

Putting (8) into (7),

$$\Xi_{im}(t) = F_{i_w} w(t) + \sum_{j=1}^{n} [F_{i_{\xi_j}} \xi_j(t) + F_{i_{\dot{\xi}_j}} \dot{\xi}_j(t)] + F_{i_{\delta_e}} \delta_e(t) \qquad (9)$$

F_{i_w}, $F_{i_{\xi_j}}$, $F_{i_{\dot{\xi}_j}}$, $F_{i_{\delta_e}}$, and $F_{i_{w_g}}$ are tabulated in Appendix A.

The generalized force term due to C.G. referenced vertical gust velocity $w_g(t)$ is given by (10).

$$\Xi_{ig}(t) = \frac{M}{M_i} \left[\int_y \int_x \frac{\partial^2 Z_w}{\partial x \partial y} \phi_i(x,y)dxdy \right] w_g(t) = F_{i_{w_g}} w_g(t) \qquad (10)$$

Substituting (5), (6), (9), and (10) into (1), Laplace transforming, and putting in matrix form yields (11), where four elastic modes (n=4) have been explicitly included.

$$
\begin{bmatrix}
s-Z_w & -U_0 s & -Z_{\dot{\xi}_1} s - Z_{\xi_1} & -Z_{\dot{\xi}_2} s - Z_{\xi_2} & -Z_{\dot{\xi}_3} s - Z_{\xi_3} & -Z_{\dot{\xi}_4} s - Z_{\xi_4} \\
-M_{\dot{w}} s - M_w & s^2 - M_q s & -M_{\ddot{\xi}_1} s^2 - M_{\dot{\xi}_1} s - M_{\xi_1} & -M_{\ddot{\xi}_2} s^2 - M_{\dot{\xi}_2} s - M_{\xi_2} & -M_{\ddot{\xi}_3} s^2 - M_{\dot{\xi}_3} s - M_{\xi_3} & -M_{\ddot{\xi}_4} s^2 - M_{\dot{\xi}_4} s - M_{\xi_4} \\
-F_{1_w} & 0 & s^2 + (2\zeta_1\omega_1 - F_{1_{\dot{\xi}_1}})s + (\omega_1^2 - F_{1_{\xi_1}}) & -F_{1_{\dot{\xi}_2}} s - F_{1_{\xi_2}} & -F_{1_{\dot{\xi}_3}} s - F_{1_{\xi_3}} & -F_{1_{\dot{\xi}_4}} s - F_{1_{\xi_4}} \\
-F_{2_w} & 0 & -F_{2_{\dot{\xi}_1}} s - F_{2_{\xi_1}} & s^2 + (2\zeta_2\omega_2 - F_{2_{\dot{\xi}_2}})s + (\omega_2^2 - F_{2_{\xi_2}}) & -F_{2_{\dot{\xi}_3}} s - F_{2_{\xi_3}} & -F_{2_{\dot{\xi}_4}} s - F_{2_{\xi_4}} \\
-F_{3_w} & 0 & -F_{3_{\dot{\xi}_1}} s - F_{3_{\xi_1}} & -F_{3_{\dot{\xi}_2}} s - F_{3_{\xi_2}} & s^2 + (2\zeta_3\omega_3 - F_{3_{\dot{\xi}_3}})s + (\omega_3^2 - F_{3_{\xi_3}}) & -F_{3_{\dot{\xi}_4}} s - F_{3_{\xi_4}} \\
-F_{4_w} & 0 & -F_{4_{\dot{\xi}_1}} s - F_{4_{\xi_1}} & -F_{4_{\dot{\xi}_2}} s - F_{4_{\xi_2}} & -F_{4_{\dot{\xi}_3}} s - F_{4_{\xi_3}} & s^2 + (2\zeta_4\omega_4 - F_{4_{\dot{\xi}_4}})s + (\omega_2^2 - F_{4_{\xi_4}})
\end{bmatrix}
\begin{bmatrix} w(s) \\ \theta(s) \\ \xi_1(s) \\ \xi_2(s) \\ \xi_3(s) \\ \xi_4(s) \end{bmatrix}
=
\begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \\ F_{1_{\delta_e}} \\ F_{2_{\delta_e}} \\ F_{3_{\delta_e}} \\ F_{4_{\delta_e}} \end{bmatrix} \delta_e(s) +
\begin{bmatrix} Z_w & 0 \\ M_w & M_q \\ F_{1_{w_g}} & 0 \\ F_{2_{w_g}} & 0 \\ F_{3_{w_g}} & 0 \\ F_{4_{w_g}} & 0 \end{bmatrix}
\begin{bmatrix} w_g(s) \\ q_g(s) \end{bmatrix}
\qquad (11)
$$

STABILITY DERIVATIVES

Since elastic mode shape and slope data is usually given as a function of lumped mass stations, which themselves are given in xy-coordinates, the double integrals in the terms of Appendix A can be conveniently represented as summations over incremental areas ($\Delta x \Delta y$) associated with each lumped mass point. Thus, it is necessary to develop methods for evaluating the following partial derivative terms at each point:

$$\frac{\partial^2 Z_{\delta_e}}{\partial x \partial y}, \quad \frac{\partial^2 Z_w}{\partial x \partial y}, \quad \frac{\partial^2 M_w}{\partial x \partial y}, \quad \frac{\partial^2 M_{\dot{w}}}{\partial x \partial y}, \quad \frac{\partial^2 M_q}{\partial x \partial y}$$

Using (2), these become (12) through (16).

$$\frac{\partial^2 Z_{\delta_e}}{\partial x \partial y} = \frac{-\rho U_0^2 S}{2M} \frac{\partial^2 C_{L_{\delta_e}}}{\partial x \partial y} \qquad (12)$$

$$\frac{\partial^2 Z_w}{\partial x \partial y} = \frac{-\rho U_0 S}{2M} \left[\frac{\partial^2 C_{L_\alpha}}{\partial x \partial y} + \frac{\partial^2 C_D}{\partial x \partial y}\right] \qquad (13)$$

$$\frac{\partial^2 M_w}{\partial x \partial y} = \frac{\rho U_0 S c}{2 I_y} \frac{\partial^2 C_{m_\alpha}}{\partial x \partial y} \qquad (14)$$

$$\frac{\partial^2 M_{\dot{w}}}{\partial x \partial y} = \frac{\rho S c^2}{4 I_y} \frac{\partial^2 C_{m_{\dot{\alpha}}}}{\partial x \partial y} \qquad (15)$$

$$\frac{\partial^2 M_q}{\partial x \partial y} = \frac{\rho U_0 S c^2}{4 I_y} \frac{\partial^2 C_{m_q}}{\partial x \partial y} \qquad (16)$$

C_{L_α}, C_D, C_{m_α}, $C_{m_{\dot{\alpha}}}$, C_{m_q}, and $C_{L_{\delta_e}}$ are the total-airplane rigid-body nondimensional stability derivatives, which are known constants for a trim flight condition. We need to determine the xy area distribution of these; i.e., the second partials in (12) through (16).

For conventional-tailed airplanes, the lift curve slope can be reasonably approximated by

$$C_{L_\alpha} = C_{L_{\alpha_W}} + C_{L_{\alpha_{HT}}} \tag{17}$$

where $C_{L_{\alpha_W}}$ and $C_{L_{\alpha_{HT}}}$ are the wing and horizontal tail contributions. Fuselage lift is neglected as small. Methods for computing these can be found in reference 2. The tail contribution is about ten percent of the total. Thus,

$$\begin{aligned} C_{L_{\alpha_W}} &= 0.9\, C_{L_\alpha} \\ C_{L_{\alpha_{HT}}} &= 0.1\, C_{L_\alpha} \end{aligned} \tag{18}$$

A crude approximation, but one found to be adequate for this formulation, is to assume the derivatives to be uniformly distributed over the xy-plane representation of each component (i.e., wing, tail, fuselage). More accurate elliptical lift distributions were tried, but resulted in very little difference to transfer function dynamics obtained from (11) over that for the uniform distributions. Thus,

$$\begin{aligned} \frac{\partial^2 C_{L_\alpha}}{\partial x \partial y} &= 0, \text{ for fuselage stations } x \ (y=0) \\ &= \frac{C_{L_{\alpha_W}}}{S_W} = \frac{0.9}{S_W} C_{L_\alpha}, \text{ for wing stations } x,y \\ &= \frac{C_{L_{\alpha_{HT}}}}{S_{HT}} = \frac{0.1}{S_{HT}} C_{L_\alpha}, \text{ for tail stations } x,y \end{aligned} \tag{19}$$

where S_W and S_{HT} are wing and horizontal tail x,y planform areas. $C_D \ll C_{L_\alpha}$ and, therefore, C_D will be neglected in evaluation of (13).

Since $C_{L_{\delta_e}}$ is due only to elevator deflection,

$$\frac{\partial^2 C_{L_{\delta_e}}}{\partial x \partial y} = \frac{C_{L_{\delta_e}}}{S_{HT}}, \text{ for tail stations } x,y$$

$$= 0 \quad , \text{ for wing and fuselage stations } x,y \qquad (20)$$

Neglecting small effects due to the fuselage,

$$C_{m_\alpha} = C_{m_{\alpha_W}} + C_{m_{\alpha_{HT}}} \qquad (21)$$

$$\frac{\partial^2 C_{m_\alpha}}{\partial x \partial y} = \frac{C_{m_{\alpha_W}}}{S_W}, \text{ for wing stations } x,y$$

$$= \frac{C_{m_{\alpha_{HT}}}}{S_{HT}}, \text{ for tail stations } x,y \qquad (22)$$

$$= 0 \quad , \text{ for fuselage stations } x \ (y=0)$$

For the pitch damping derivatives,

$$C_{m_{\dot\alpha}} = C_{m_{\dot\alpha_W}} + C_{m_{\dot\alpha_{HT}}} \qquad (23)$$

$$C_{m_q} = C_{m_{q_{WF}}} + C_{m_{q_{HT}}} \qquad (24)$$

where WF indicates wing and fuselage combination.

$$\frac{\partial^2 C_{m_{\dot\alpha}}}{\partial x \partial y} = \frac{C_{m_{\dot\alpha_W}}}{S_W} \text{, for wing stations } x,y$$

$$= \frac{C_{m_{\dot\alpha_{HT}}}}{S_{HT}} \text{, for tail stations } x,y \qquad (25)$$

$$= 0 \text{, for fuselage stations } x \ (y=0)$$

$$\frac{\partial^2 C_{m_q}}{\partial x \partial y} = \frac{C_{m_{q_{WF}}}}{S_W + S_F} \text{, for wing and fuselage stations } x,y$$

$$= \frac{C_{m_{q_{HT}}}}{S_{HT}} \text{, for tail stations } x,y \qquad (26)$$

Methods for estimating $C_{m_{\dot\alpha_W}}$, $C_{m_{\dot\alpha_{HT}}}$, $C_{m_{q_{WF}}}$, and $C_{m_{q_{HT}}}$ are given in references 3 and 4.

Knowing the airplane elastic mode shapes, slopes, and five rigid-body total-airplane stability derivatives, all the terms in Appendix A and thus the coefficients in (11) can be computed.

VERIFICATION WITH B-1 AIRPLANE DYNAMICS

To verify the accuracy of the unique formulation of elastic airplane small perturbation dynamic equations of motion developed above, the terms in equation (11) are calculated by this method for the B-1 at a sea level, Mach 0.85 flight condition and compared with the corresponding terms in equations provided by Rockwell in references 5 and 6, which were generated by other methods.

The xy-planform and incremental area divisions for each mass point for the B-1 is depicted in Figure 3. Since the elastic modes used in longitudinal dynamic equations are symmetric about the x-axis, only half of the planform is shown in the figure.

As an example of how the double integrals are evaluated, consider the term for the generalized force of the second elastic mode due to the third mode. From Appendix A, it is

$$F_{2_{\xi_3}} = \frac{-U_0 M}{M_2} \int_y \int_x \frac{\partial^2 Z_w}{\partial x \partial y} \phi_2(x,y)\phi_3'(x,y)dxdy \tag{27}$$

One half of the value is obtained by summing over the 97 stations in Figure 3; the other half coming from the symmetric right side planform not shown.

$$\frac{1}{2} F_{2_{\xi_3}} = \frac{-U_0 M}{M_2} \sum_{i=1}^{97} \left(\frac{\partial^2 Z_w}{\partial x \partial y}\right)_i \phi_2(i)\phi_3'(i)(\Delta x \Delta y)_i \tag{28}$$

The $(\Delta x \Delta y)_i$ term is the area associated with each lumped mass point in Figure 3. $\phi_2(i)$ and $\phi_3'(i)$ are the values of the second mode shape and third mode slope in the chordwise x-direction at the ith mass point. $\left(\frac{\partial^2 Z_w}{\partial x \partial y}\right)_i$ has three constant values; one for fuselage stations, one for wing stations, and one for horizontal tail stations. $C_{L_\alpha} = 3.94$ for this B-1 flight condition. Wing and tail areas are $S = S_W = 1946$ ft^2, $S_{HT} = 502$ ft^2. From (19),

$$\frac{\partial^2 C_{L_\alpha}}{\partial x \partial y} = \begin{cases} 0, \text{ fuselage stations} \\ 18.42 \times 10^{-4}/\text{ft}^2, \text{ wing stations} \\ 7.14 \times 10^{-4}/\text{ft}^2, \text{ tail stations} \end{cases} \tag{29}$$

From (13), with C_D neglected as small,

$$\left(\frac{\partial^2 Z_w}{\partial x \partial y}\right)_i = \begin{cases} 0, \text{ fuselage stations} \\ -5.708 \times 10^{-4}/\text{ft}^2\text{-sec, wing stations} \\ -2.213 \times 10^{-4}/\text{ft}^2\text{-sec, tail stations} \end{cases} \quad (30)$$

The calculation of (28) gives $F_{2_{\xi_3}} = 5.8459$, which compares with 6.6257 obtained from Rockwell's formulation of the equations.

All other coefficients in (11) were similarly evaluated and are tabulated, along with the values from Rockwell's B-1 equations of motion, in Appendix B. Approximately 80 percent of the terms show good agreement with the B-1 data. In view of the approximate nature of the formulation, this is reasonable confirmation of the validity of the method.

A further check was made by comparing the roots of the characteristic equations for the B-1 data and this formulation by expanding the determinant of the 6x6 matrix of polynomials and coefficients in (11). The coupled frequencies in rad/s and damping ratios were calculated for each pair of complex roots. The comparisons are shown in Table 1. It is

TABLE 1 FREQUENCIES AND DAMPING RATIOS

B-1 Data		Unique Formulation	
Frequency	Damping	Frequency	Damping
2.868	0.489	3.103	0.492
13.298	0.053	13.709	0.034
21.375	0.031	21.221	0.025
22.020	0.020	22.020	0.020
22.480	0.206	25.366	0.233

evident from the data in this table that the new formulation of the equations of motion is surprisingly accurate considering the level of approximations made. The four symmetric elastic modes of the B-1 had free-free undamped natural frequencies of 13.591, 14.123, 21.198, and 22.055 rad/s. All had 0.02 structural damping ratios. The first line of numbers in Table 1 corresponds to the short-period frequency and damping ratio.

CONCLUSIONS

The unique method of formulation of the longitudinal small perturbation equations of motion for elastic airplanes described herein allows the expression of aerodynamic forces and moments, due to elastic vibration, in terms of rigid-body aerodynamic stability derivatives. Thus, it serves as a useful preliminary design tool for airplane stability and control, handling and ride qualities, and control system design studies.

The good accuracy of the method has been established by comparison with more accurate data for the B-1 airplane. The lack of complete information on the planform geometry of the B-1 and our having to analytically calculate mode slopes by curve fits from the mode shape data, probably accounts for much of what differences appear in the term by term comparisons in Appendix B. Therefore, the new method is probably even more accurate than this one example comparison indicates.

We have developed a similar formulation for the lateral-directional dynamics with the antisymmetric elastic modes included and are presently checking its accuracy. This formulation will be published in a sequel paper to the present one.

APPENDIX A. EQUATION COEFFICIENTS

$$Z_{\xi_i} = -U_0 \int_y \int_x \frac{\partial^2 Z_w}{\partial x \partial y} \phi_i'(x,y)dxdy$$

$$Z_{\dot{\xi}_i} = \int_y \int_x \frac{\partial^2 Z_{\dot{w}}}{\partial x \partial y} \phi_i(x,y)dxdy$$

$$M_{\ddot{\xi}_i} = \int_y \int_x \frac{\partial^2 M_{\dot{w}}}{\partial x \partial y} \phi_i(x,y)dxdy$$

$$M_{\dot{\xi}_i} = \int_y \int_x \left\{\frac{\partial^2 M_w}{\partial x \partial y} \phi_i(x,y) - [U_0 \frac{\partial^2 M_{\dot{w}}}{\partial x \partial y} + \frac{\partial^2 M_q}{\partial x \partial y}]\phi_i'(x,y)\right\}dxdy$$

$$M_{\xi_i} = -U_0 \int_y \int_x \frac{\partial^2 M_w}{\partial x \partial y} \phi_i'(x,y)dxdy$$

$$F_{i_w} = \frac{M}{M_i} \int_y \int_x \frac{\partial^2 Z_w}{\partial x \partial y} \phi_i(x,y)dxdy$$

$$F_{i_{\xi_j}} = -\frac{U_0 M}{M_i} \int_y \int_x \frac{\partial^2 Z_w}{\partial x \partial y} \phi_i(x,y)\phi_j'(x,y)dxdy$$

$$F_{i_{\dot{\xi}_j}} = \frac{M}{M_i} \int_y \int_x \frac{\partial^2 Z_w}{\partial x \partial y} \phi_i(x,y)\phi_j(x,y)dxdy$$

$$F_{i_{\delta_e}} = \frac{M}{M_i} \int_y \int_x \frac{\partial^2 Z_{\delta_e}}{\partial x \partial y} \phi_i(x,y)dxdy$$

$$F_{i_{w_g}} = \frac{M}{M_i} \int_y \int_x \frac{\partial^2 Z_w}{\partial x \partial y} \phi_i(x,y)dxdy = F_{i_w}$$

APPENDIX B. COEFFICIENT VALUES

Term	From B-1 Equations	From Unique Formulation
F_{1_w}	-0.77480	-0.69335
F_{2_w}	1.3590	1.4180
F_{3_w}	0.80586	0.81559
F_{4_w}	1.7902×10^{-3}	1.8274×10^{-3}
$Z_{\dot{\xi}_1}$	-0.20177	-0.01798
$Z_{\dot{\xi}_2}$	2.4702	1.9202
$Z_{\dot{\xi}_3}$	0.14486	0.15373
$Z_{\dot{\xi}_4}$	-4.7412×10^{-3}	0.11254
Z_{ξ_1}	-8.4911	-6.7715
Z_{ξ_2}	90.322	103.32
Z_{ξ_3}	4.3792	-1.3083
Z_{ξ_4}	-4.1323	-5.3236
$M_{\ddot{\xi}_1}$	0	-0.72033×10^{-4}
$M_{\ddot{\xi}_2}$	0	-3.6129×10^{-4}
$M_{\ddot{\xi}_3}$	0	1.4315×10^{-4}
$M_{\ddot{\xi}_4}$	0	0.06947×10^{-4}
$M_{\dot{\xi}_1}$	-7.5404×10^{-3}	-10.823×10^{-3}
$M_{\dot{\xi}_2}$	50.866×10^{-3}	2.9502×10^{-3}
$M_{\dot{\xi}_3}$	7.0361×10^{-3}	16.427×10^{-3}
$M_{\dot{\xi}_4}$	-2.0060×10^{-3}	-3.0665×10^{-3}

Term	From B-1 Equations	From Unique Formulation
M_{ξ_1}	-0.18849	-0.04194
M_{ξ_2}	-0.07903	-0.01592
M_{ξ_3}	0.20945	0.07459
M_{ξ_4}	-0.09381	-0.01065
$F_{1\dot{\xi}_1}$	-0.86630	-1.1180
$F_{1\dot{\xi}_2}$	-0.28286	17.818
$F_{1\dot{\xi}_3}$	0.91557	-0.07921
$F_{1\dot{\xi}_4}$	-0.19759	-0.69559
$F_{1\xi_1}$	7.2681	-31.795
$F_{1\xi_2}$	-15.897	643.69
$F_{1\xi_3}$	61.886	9.640
$F_{1\xi_4}$	20.894	-18.803
$F_{2\dot{\xi}_1}$	0.23360	0.34121
$F_{2\dot{\xi}_2}$	-8.2949	-10.334
$F_{2\dot{\xi}_3}$	0.11424	0.00560
$F_{2\dot{\xi}_4}$	0.11188	0.11583
$F_{2\xi_1}$	14.002	18.740
$F_{2\xi_2}$	-306.000	-415.280
$F_{2\xi_3}$	6.6257	5.8459
$F_{2\xi_4}$	11.211	14.696
$F_{3\dot{\xi}_1}$	-0.12060	-0.010897
$F_{3\dot{\xi}_2}$	3.7684	0.04020
$F_{3\dot{\xi}_3}$	-0.42578	-0.18203
$F_{3\dot{\xi}_4}$	-0.25330	-0.11246

Term	From B-1 Equations	From Unique Formulation
$F_{3\xi_1}$	7.0455	2.6015
$F_{3\xi_2}$	33.993	-15.509
$F_{3\xi_3}$	-7.9516	-1.5492
$F_{3\xi_4}$	3.4837	2.0596
$F_{4\dot{\xi}_1}$	-3.2701×10^{-4}	-2.9290×10^{-4}
$F_{4\dot{\xi}_2}$	24.031×10^{-4}	25.469×10^{-4}
$F_{4\dot{\xi}_3}$	-2.7776×10^{-4}	-3.4422×10^{-4}
$F_{4\dot{\xi}_4}$	-4.0417×10^{-4}	-4.2249×10^{-4}
$F_{4\xi_1}$	1.6305×10^{-2}	-0.04974×10^{-2}
$F_{4\xi_2}$	-9.7878×10^{-2}	3.6896×10^{-2}
$F_{4\xi_3}$	0.70767×10^{-2}	0.24563×10^{-2}
$F_{4\xi_4}$	1.3340×10^{-2}	0.18614×10^{-2}
$F_{1\delta_e}$	-22.296×10^{2}	-15.978×10^{2}
$F_{2\delta_e}$	-2.1741×10^{2}	-1.5347×10^{2}
$F_{3\delta_e}$	6.1537×10^{2}	4.3685×10^{2}
$F_{4\delta_e}$	0.11048	0.06489
F_{1w_g}	-0.77350	-0.69335
F_{2w_g}	1.3567	1.4180
F_{3w_g}	0.80450	0.81559
F_{4w_g}	1.7872×10^{-3}	1.8274×10^{-3}

REFERENCES

1. Proceedings of the NASA Symposium on Advanced Control Technology and Its Potential for Future Transport Aircraft, July 9-11, 1974, Los Angeles, Calif.

2. Ellison, D. E., Finck, R. D., and Hoak, D. E., USAF Stability and Control Datcom, October 1960 (revised August 1968), Wright-Patterson AFB, Ohio.

3. Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., Aeroelasticity, Addison-Wesley, Reading, Mass., 1955, pp. 106-114.

4. Roskam, J., "Methods for Estimating Stability and Control Derivatives of Conventional Subsonic Airplanes," 1971, Roskam Aviation and Engineering Corp., Lawrence, Kansas.

5. Wykes, J. H., "B-1 Flexible Vehicle Equations of Motion for Ride Quality, Terrain Following, and Handling Quality Studies," TFD-71-430-1, January 1973, Rockwell International B-1 Division, Los Angeles, Calif.

6. Freeman, R. C., and Rozsa, T. I., "Basic Modal Data Package for -55B Mid-Penetration Weight 65 Degree Wing Sweep," TFD-73-362, March 1973, Rockwell International B-1 Division, Los Angeles, Calif.

Figure Captions

Figure 1. Fuselage Vertical Bending Sign Convention

Figure 2. Wing Deflection Sign Convention

Figure 3. B-1 Mass and Area Distribution

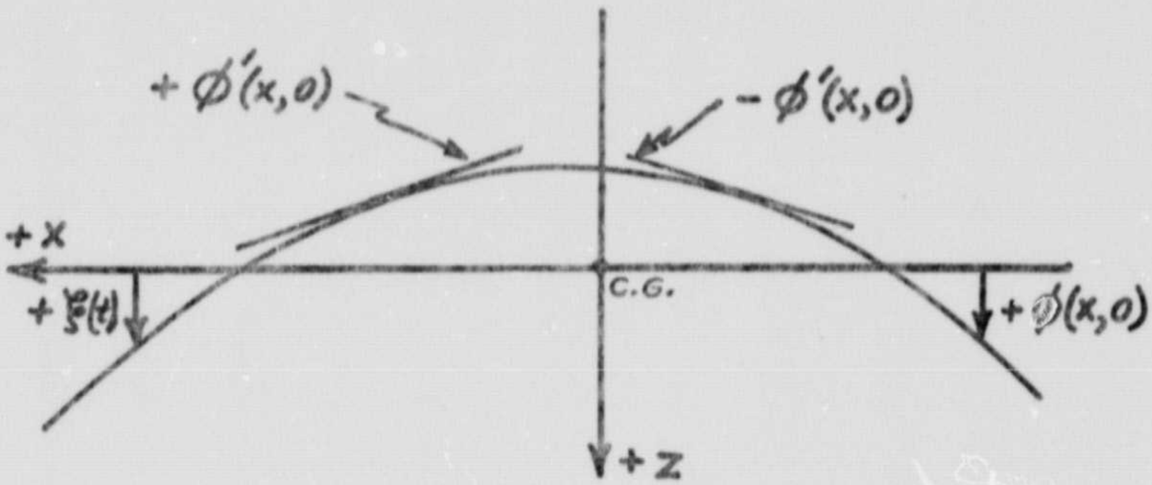

Fig. 1
R. L. Swaim

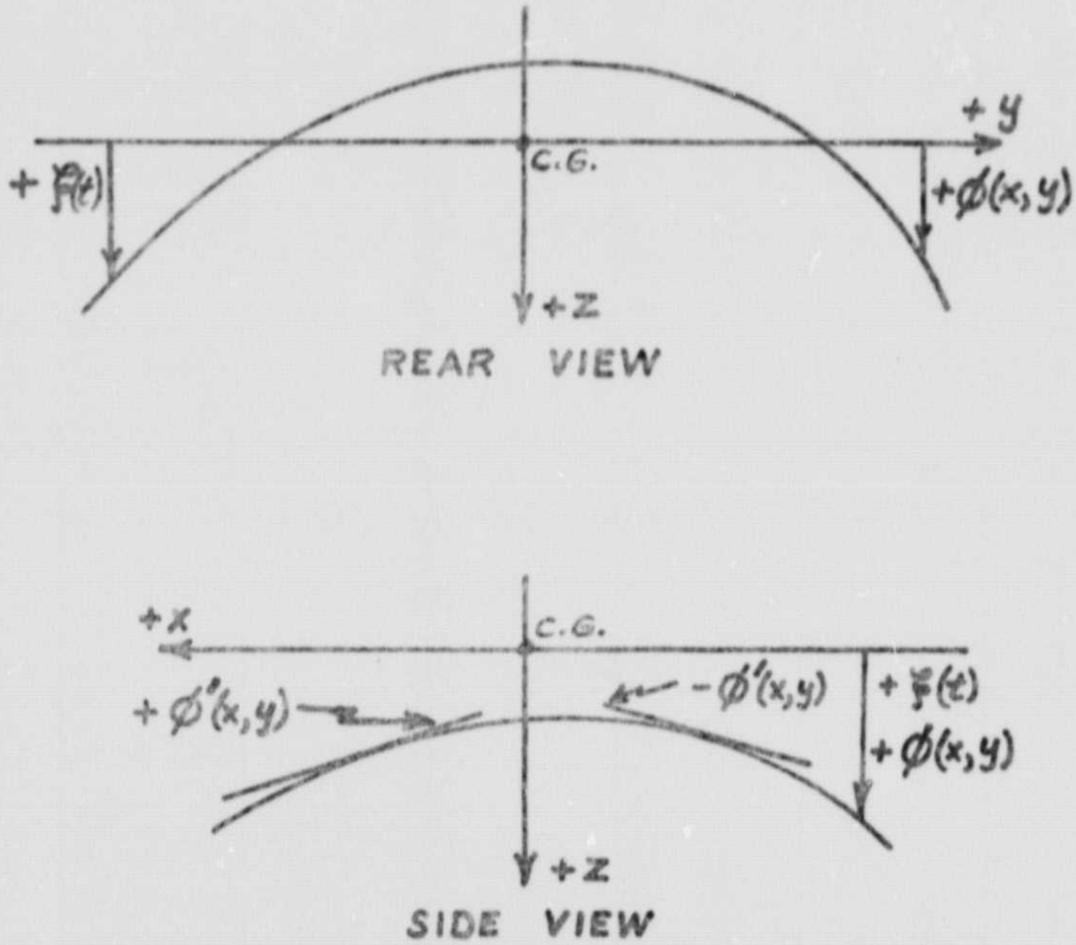

Fig. 2
R. L. Swain

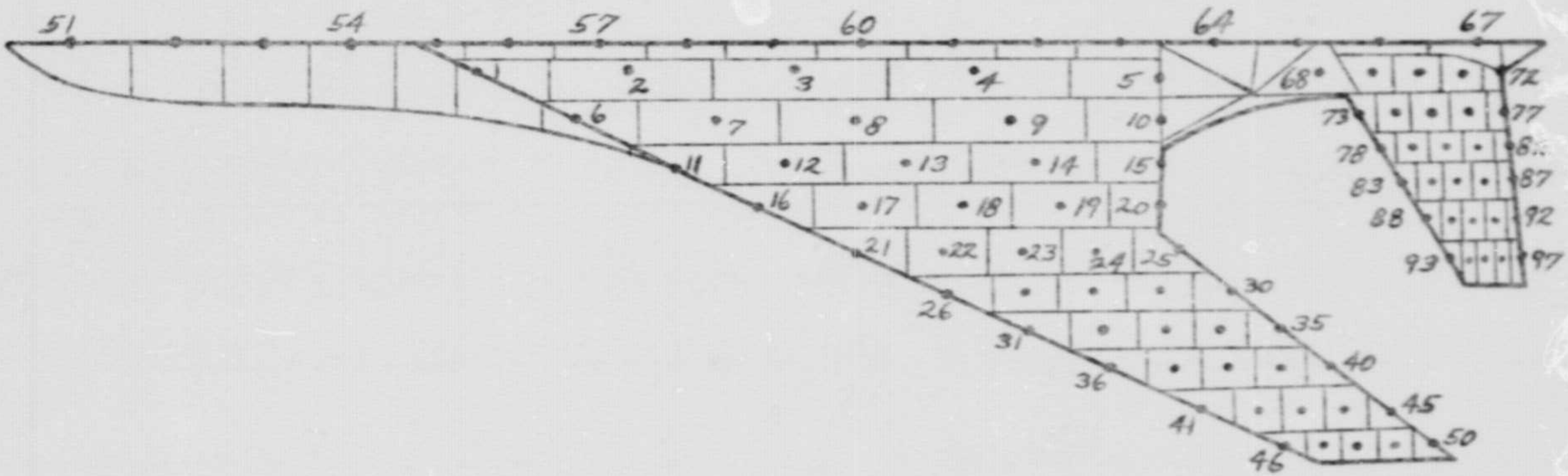
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Fig. 3