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QUANTIZATION AND SYMMETRY IN PERIODIC COVERAGE PATTERNS WITH APPLICATIONS TO EARTH OBSERVATION

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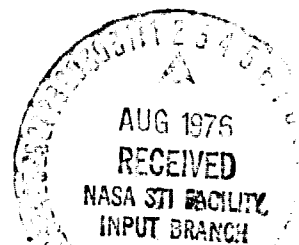
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DECEMBER 1975



GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

Presented at AAS/AIAA Astrodynamics Specialist Conference
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QUANTIZATION AND SYMMETRY
IN PERIODIC COVERAGE PATTERNS
WITH APPLICATIONS TO EARTH OBSERVATION

Joseph C. King
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Abstract

Analytical approaches based on an idealized physical model and concepts from number theory show that in periodic coverage patterns, uniquely* defined by their revolution numbers R (orbital) and N (rotational), the sub-nodal points are earth-fixed, and they divide the equator into R equal segments of length s. The ascending sub-satellite trace crosses each point once (only) each period. The descending subnodal points coincide with the ascending points if the integers N and R have like parity, and bisect the intervals between them if opposite. The interval between consecutive unidirectional crossings is Ns. Symmetries extend the equatorial results to all parallels of latitude. Complete periodic patterns of traces exhibit an overall symmetry, with trace intersections confined to discrete coordinate values which are quantized in longitude (basic s-unit) and symmetric in latitude.

*For a given inclination.

Introduction

The increasing use and importance of earth-oriented satellite applications, together with the expanded demands being placed on satellite functions and observing modes by the variety of uses to be served, emphasize the need for effective approaches to mission analysis and design. A major element of this technical requirement falls in the area of orbit coverage analysis—the determination of geographic areas viewed by a satellite as a function of time, for various choices of orbit parameters.

Although the overall geographic coverage of satellite sensors depends on their viewing angles, the more basic element in determining such coverage is the ground track or nadir trace of the satellite. It is this latter area, the "orbit-coverage problem", which is the subject of this paper. By employing a simplified physical model and introducing analytical approaches derived from basic number theory, some basic and general properties are developed which clarify the general problem of orbit-coverage analysis, and which have immediate application to current mission design problems.

Basic Nadir Trace Definition

In its most basic form, the orbit-coverage problem reduces to one of analyzing the behavior of the nadir trace of a circular-orbiting satellite on a uniformly rotating spherical earth. Such a trace, in one orbital revolution, takes the general form of the curve labeled "A" in Figure 1. Without earth rotation, the trace would be more elongated, like curve X', traversing the full 2π angle in longitude. The longitude difference is designated $\Delta\lambda^1$, which is determined

$$\Delta\lambda = \omega'_e T = 2\pi \frac{T}{D_n} \quad (1)$$

¹Designated "S" in Refs. 1 and 2.

where $\omega'_e = \omega_e - \dot{\Omega}$, the earth rotation rate relative to the orbit node

T = period, orbital

$$D_n = \frac{2\pi}{\omega'_e}, \text{ the "nodal day"}^{2} \quad (2)$$

ω_e = earth rotation rate (inertial)

$\dot{\Omega}$ = rotation rate of orbital line of nodes

Although the two curves resemble sine waves in the distorted planar representations of Figure 1, they actually have the forms, obtained via Napier's rules in spherical trigonometry,

$$\lambda' = \sin^{-1} \left(\frac{\tan \phi}{\tan i} \right) \quad (3)$$

and

$$\lambda = \sin^{-1} \left(\frac{\tan \phi}{\tan i} \right) - \omega'_e t \quad (4)$$

where i = orbital inclination (and trace "amplitude")

As noted on Figure 1, $\Delta\lambda$ is also referred to as the "step," because the subnodal points (1, 2, 3 . . . in Figure 1) appear to step uniformly around the equator—always westward in equal steps.

Coverage Periodicity

It is helpful now to consider the conditions for coverage periodicity, which is of major interest both for analytical and practical (re-viewing) reasons. The basic condition for periodic coverage is that the nadir trace return to a previously traversed path after some definite period of time, after which the previous pattern is re-traced, and so on indefinitely. In terms of Figure 1, if the series of subnodal points 1, 2, 3, . . . were such that the nth member coincides with the origin, the result would be a "one-day repeater."

More generally, an N-day repeater can be represented analogously by repeating the equator N times in a continuous line, and requiring that the Rth subnodal point fall on the Nth repetition of the origin (end of the Nth day). This representation is illustrated in Figure 2. It is equivalent to viewing the equator as a continuous arc passing under the ascending node, with the arc measure accumulating indefinitely instead of resetting to zero after each earth revolution. In practice, the required coincidence (of points N and R) is obtained by adjusting the orbital period so as to produce exactly R revolutions in exactly N nodal days, i. e.,

$$D_n N = RT \quad (5)$$

²Designated "Q-day" in Ref. 1.

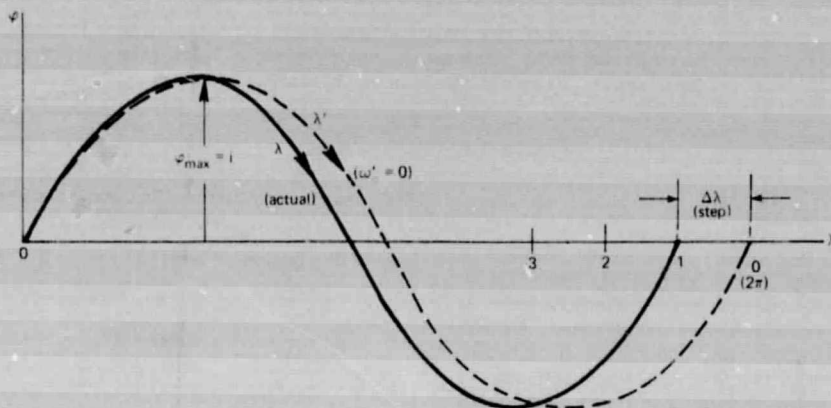


Figure 1. Basic Trace Geometry.

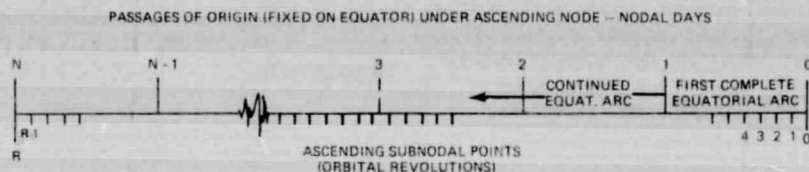


Figure 2. Divisions of Extended Equatorial Arc.

Since the revolution numbers N and R are restricted to integral values, Equation (5) tends to limit the orbit period T to a series of discrete values (Ref. 3). This is true when considering periodic coverage orbits with periods of reasonable durations, say up to a few months. In theory, however, the numbers N and R are unrestricted in positive magnitude, so their ratio N/R , and therefore T , can be considered continuously variable. The coverage patterns also can be considered discrete or continuous, depending on the interpretation given to the ratio N/R .

Integration of Periodicities

It is instructive to note in Figure 2 that the N -fold equatorial arc is evenly divided in two different ways: into N equal parts and also R equal parts. This observation suggests the question: What is the simplest single division mode which includes both the N - and R -fold divisions?

This question is analogous to a more fundamental question in basic number theory: What is the smallest number divisible by two integers (R and N)? The answer, the least common multiple of R and N , is unique, and it is just RN if R and N are relatively prime, i.e., if they have no common factor.¹ They are in fact relatively prime in this case because of the tacit assumption that the N , R coincidence is the first coincidence along the extended arc in Figure 2. Any earlier coincidence, say

after N/m days, would define an N/m -day repeater (with R/m orbital revolutions), making the hypothesized N -day periodicity redundant (just the m -th N/m -day cycle). Similarly, any $>N$ -day periodicity observed would represent redundant multiples of the actual N -day period. Thus the RN -fold division, like the l.c.m. entity itself, is unique. In practical application, the uniqueness requirement is met simply by assuring that only relatively prime N, R pairs are considered (cf., References 1 and 2).

Consequences in Subnodal Point Location

The basic consequence of the above line of reasoning is that the resulting NR -fold division of the N -fold extended equatorial arc implies an R -fold division of the equator, because $NR/N = R$ and all the "equators" represented are alike, including the physical equator. This important result is stated more generally as follows:

- I. For an idealized periodic coverage circular orbit characterized by R orbital revolutions per coverage period of N nodal days (R and N are integers), the R ascending (or descending) subnodal points are earth-fixed, and they divide the equator into R equal segments of arc length (in earth radii) or subtended angle $s = 2\pi/R$. Also:
- II. Each of the fixed subnodal points is intersected once and only once (in one direction) by the nadir trace during each N -day period.
- III. The interval between consecutive subnodal point crossings in one direction (the step) is Ns .

These results are illustrated in Figure 3, which shows the right-hand end of Figure 2 enlarged with the new information added.

¹Note that insertion or removal of a common factor in N and R has the (nil) effect of multiplying Equation (5) by a constant. Only the ratio N/R is effective in specifying T , the single pattern-determining variable (for a given i).

N = NODAL DAYS/PERIOD
 R = ORBITAL REVS./PERIOD
 (N, R INTEGERS)

$s = 2\pi/R$
 $Q = R/N$ (REF. 1,2)
 (Q) = INTEGRAL PART OF Q

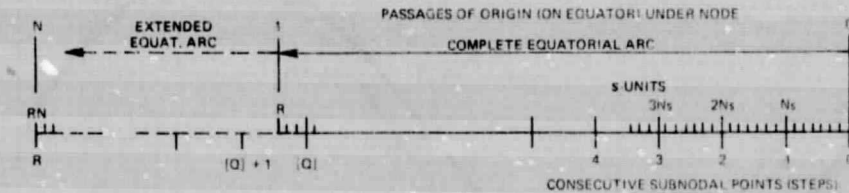


Figure 3. Equal Division of Equator into Basic s-Units by R Fixed Subnodal Points

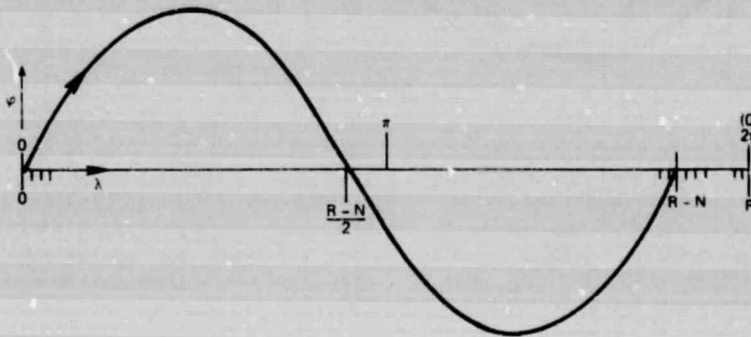


Figure 4. Relative Location of Opposite Node.

Result II relates to the previous discussion regarding the relative primeness of N and R. Any second trace over a given subnodal point during one coverage period would define a new, shorter period, violating the initial hypothesis. And since R revolutions each period are required, all of the R points must be crossed. Result III can be inferred on inspection of Figure 2: The NR divisions (s-units) must be traversed in R equal steps, so each step ($\Delta\lambda$) is $NR/R = N$ s-units. This result is compatible with the statement in Reference 2 to the effect that there are N-1 subsequent crossings of the Ns interval during the remainder of the cycle.

Location of Opposite Subnodal Points

The discussion so far has pertained to subnodal points defined by equator crossings of the nadir trace in one direction. The results apply equally to either ascending or descending subnodal points, but as yet have not related opposite or "mixed" subnodal point locations. This latter subject can be developed by applying the above new information to Figure 1, as reflected now in Figure 4.

The longitude units shown in Figure 4 are the basic s-units defined above, R of which are contained in the full equator. Since the step ($\Delta\lambda$) contains N of these units, the ascending node spacing (the "wavelength") is R-N. Then the intervening descending node is (R-N)/2 units from either adjacent ascending node, by symmetry.

Since R and N are integers, R-N will be an integer, either even or odd. If R and N have the same parity (both even or both odd), R-N will be even, making (R-N)/2 integral. This condition will place the descending node an integral number of s-units from the ascending node, a location which must coincide with another of the R

ascending subnodal points. If R and N have opposite parity, (R-N)/2 will be an integer plus 1/2, causing the descending node to fall midway between an adjacent pair of ascending node locations.

Thus the descending subnodal points will coincide with the ascending points if N and R have the same parity, and will bisect the interval between the ascending points if N and R have opposite parity. The basic relationships involved are summarized in Table I. The (R + N) variation pertains to retrograde orbits, as explained in the next section.

Table I
Opposite Node Location Key

| R, N Parity | $R + N$ | $\frac{R + N}{2}$ | Ascending- Descending Node Relationship |
|-------------|---------|-------------------|--|
| Same | Even | Integer | Coincident |
| Opposite | Odd | Integer + 1/2 | Intermediate |

Retrograde Orbits

The nadir traces of retrograde orbits ($i > \pi/2$) are similar to those of direct orbits and can be analyzed similarly. The essential difference, a direction reversal in the longitudinal component of motion (to westward, taken negative), transforms the λ' curve Figure 1 to its image in the ϕ -axis, as shown in Figure 5. It now terminates at -R, and the (-N) step, produced by earth-node rotation, now increases the total longitudinal displacement during one orbital revolution to a magnitude of (R + N) instead of (R - N).

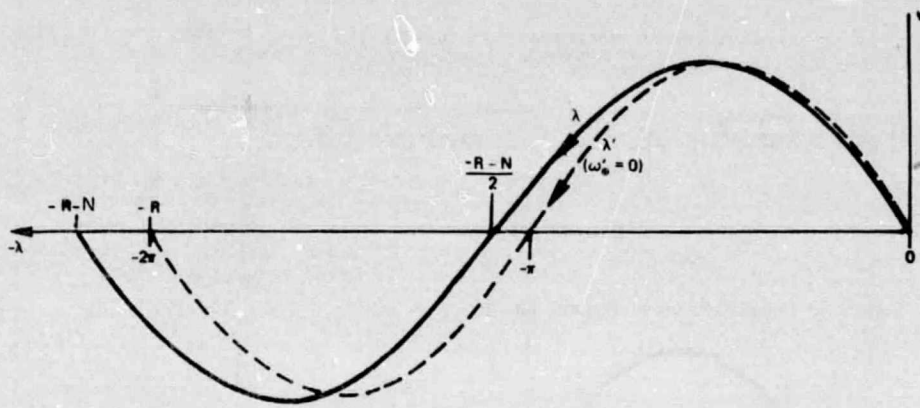


Figure 5. Retrograde Trace.

Note however that the step is still $-N$ s-units (westward), and, while the traces are elongated and reversed, the general pattern-forming properties are unchanged. The relationships of Table I still hold, because they are not affected by the substitution of $-R$ for R . For generality, the longitudinal displacement can now be written " $\pm R-N$," or, when referring to its magnitude or "wave-length", the general expression becomes $|\pm R-N| = R \mp N$.

Complete Patterns

Having determined the essential properties of single-revolution traces and their extended patterns of subnodal points, it remains now to examine the general characteristics of complete, full-period patterns of traces. Since

these patterns fall basically into two categories (Table I), it is desirable to examine both types and compare. In the discussion which follows, the two categories are designated "Coincident" and "Intermediate," according to the relative position of their opposite sets of subnodal points.

Intermediate-Type Patterns

A representative intermediate-type pattern is illustrated in Figure 6. This particular pattern is associated with the characteristic revolution numbers $N = 3$ and $R = 26$, of opposite parity as required for an intermediate-type pattern. Note the alternating (intermediate) placement of opposite subnodal points.

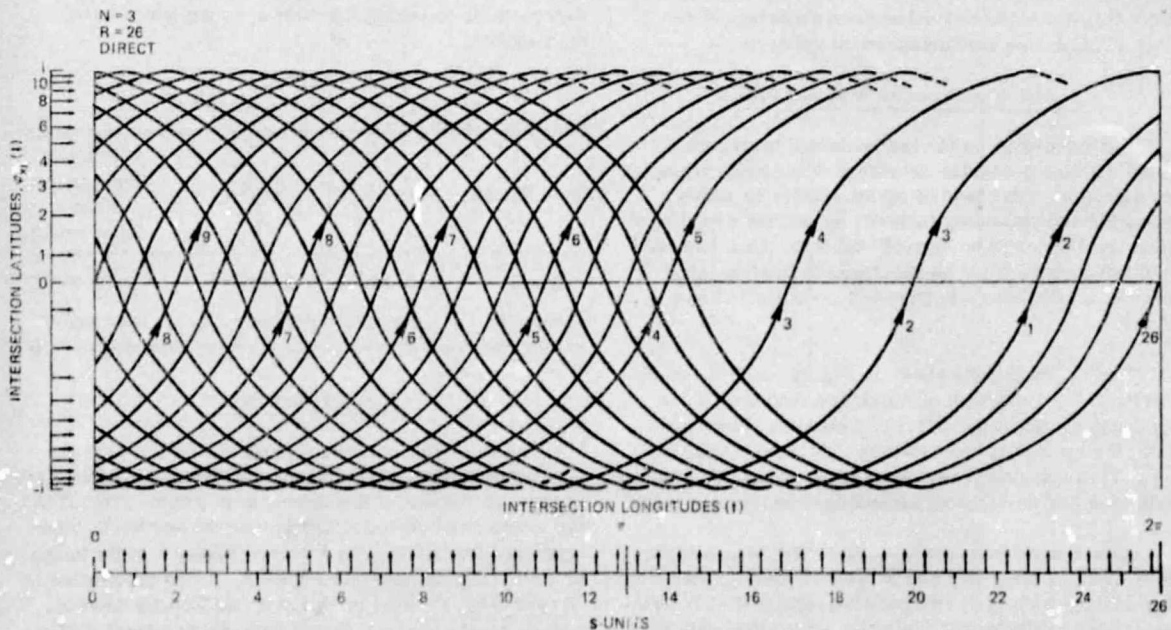


Figure 6. Complete Intermediate-Type Pattern.

The most notable feature of the pattern is its overall symmetry, particularly in the pattern of intersections. The intersections occur at discrete values of the coordinates, both in longitude and latitude, forming a rectangular grid of intersection coordinate lines. The intersection meridians are spaced $1/2$ unit ($s/2$) apart, and are placed at integer $\pm 1/4$ locations between the integer meridians (containing the ascending nodes).

The quantized longitudinal placement of the intersections results from the basic symmetry of the ascending and descending traces. Each intersection is at the peak of a symmetric arch, the bases of which are defined by subnodal points.

The spacing of intersection latitudes decreases* with distance from the equator, about which the pattern of intersection latitudes is symmetric. Note that the intersections, and in fact all corresponding points (same "phase") along any adjacent pair of traces, are spaced longitudinally at the basic quantized interval s (one unit on the diagram). It follows then that all parallels within the pattern are equally divided, like the equator, into R

s -units (angular) by the ascending traces, and again separately by the descending traces (separately except when the two sets intersect on a particular parallel).

Thus a regular, symmetric pattern exists, quantized in longitude and symmetric in latitude. The principal quantitative parameters of the pattern of intersections are given in Table II.

Coincident-Type Patterns

The corresponding properties of the coincident-type pattern (Figure 7) are generally predictable at this stage. The opposite nodes coincide, as determined by the like parity of the revolution numbers $N = 3$ and $R = 23$ (both odd). The intersection longitudes are spaced at $s/2$ intervals as before, but in this pattern type they are located on the integer and integer $+ 1/2$ meridians. The intersection latitudes occur in a pattern similar to the previous case, except here there is an additional line of intersections on the equator. Otherwise, the coincident and intermediate-type patterns have similar characteristics.

Table II provides a comparison of their quantitative parameters. The secondary classification of the coincident patterns, by $(R + N)/2$ even or odd, determines

*Relationships for calculating the series of intersection latitudes are given in Appendix I.

Table II
Intersection Pattern Characteristics

| | Intermediate Patterns *($R \mp N$ Odd) | Coincident Patterns *($R \mp N$ Even) | |
|--|--|---|--------------------------|
| | | $\frac{R \mp N}{2}$ Odd | $\frac{R \mp N}{2}$ Even |
| <u>Longitudinal</u> | | | |
| Location of Intersection Meridians | Integer $\pm \frac{1}{4}$ | Integer and Integer $+\frac{1}{2}$ | |
| Number of Intersection Meridians | total ($\lambda = 0 - 2\pi$) per wavelength ($t = 0 - T$) | 2R 2($R \mp N$) | |
| Number of Intersections per Meridian | $\frac{R \mp N - 1}{2}$ | Integer: $\frac{R \mp N}{2}$ | $\frac{R \mp N}{2} - 1$ |
| Number of Trace Crossings per Meridian | | Integer $+\frac{1}{2}$: $\frac{R \mp N}{2} - 1$ | $\frac{R \mp N}{2}$ |
| | | R \mp N | |
| <u>Latitudinal</u> | | | |
| Location of Intersection Parallels | | Symmetric about equator, decreasing interval (Compute, Appendix I) | |
| Number of Intersection Parallels | | R \mp N - 1 | |
| Number of Intersections per Parallel | | R | |
| Number of Trace Crossings per Parallel (non-intersection parallels) | | 2R | |

*Use minus for direct ($i < \pi/2$) and plus for retrograde ($i > \pi/2$) orbits. $R \mp N (= |\pm R - N|)$ is used here instead of $\pm R - N$ to give positive values to basically positive quantities.

N = 3
R = 23
DIRECT

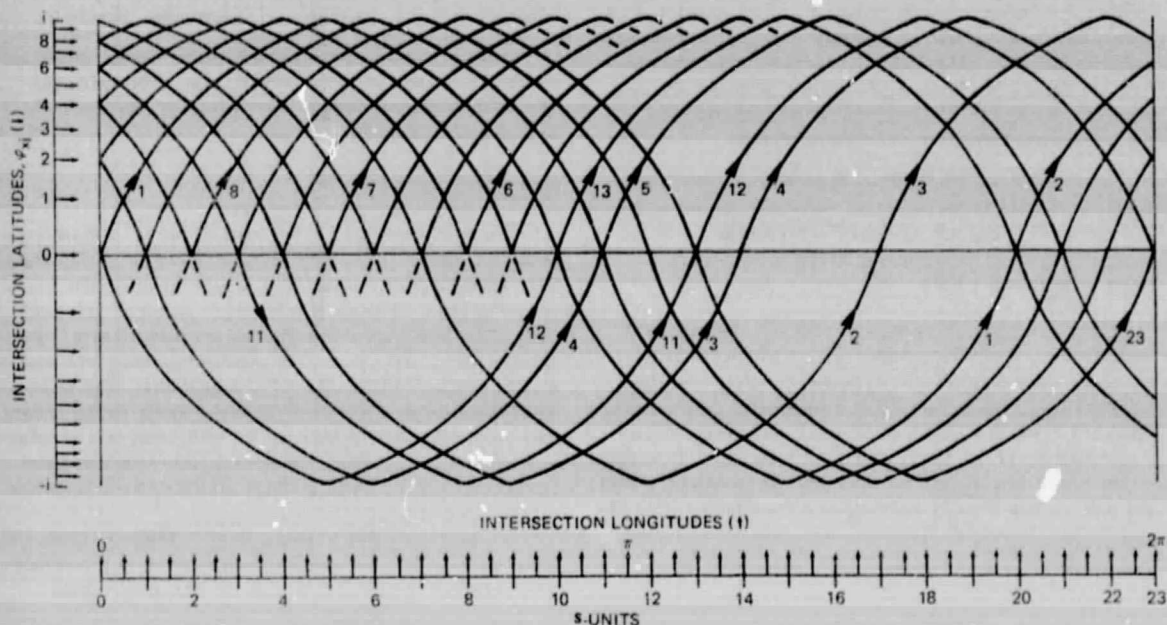


Figure 7. Complete Coincident-Type Pattern.

whether the extreme latitude points fall on integer or integer + 1/2 meridians, respectively. The Figure 7 pattern is in the even (integer meridians) category. Note that the alternating numbers of intersections per meridian in the coincident patterns have an average value equal to the single number for the intermediate pattern.

Summary

By way of summary, the principal facts developed herein are restated below in abbreviated form. For a periodic coverage pattern defined* by its revolution numbers N and R:

- The R ascending (or descending) subnodal points are earth-fixed, and they divide the equator into R equal segments $s = 2\pi/R$
- Each of the fixed subnodal points is intersected once and only once (in one direction) by the nadir trace during each N-day period
- The interval between consecutive subnodal point crossings is $N \cdot s$
- The two opposite sets of subnodal points (ascending and descending) coincide if N and R have the same parity and alternate (spaced $s/2$) if N and R have opposite parity
- All parallels of latitude within the pattern are divided equally into the angular s-units by each set of trace crossings (ascending and descending)
- All trace intersections occur on a discrete set of meridians which is evenly spaced and quantized by the basic s-unit.

- All trace intersections occur on a discrete set of parallels which is symmetric about the equator and spaced at intervals which decrease with latitude

In overview, the above analysis has defined a system of basic and general properties by which idealized periodic coverage patterns can be characterized, classified, and delineated. The principal common features of these patterns are their longitudinal quantization, determined by the revolution number R, and their overall symmetry.

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*Uniquely, for any given inclination i .

Determining Discrete Intersection Latitude Levels

As noted earlier under Intermediate-Type Patterns, each trace intersection is at the peak of a symmetric arch, illustrated by the solid lines in Figure 8. The base vertices are defined by subnodal points, which occur at integer locations (s-units) on coincident patterns and at integer and integer + 1/2 locations on intermediate patterns.

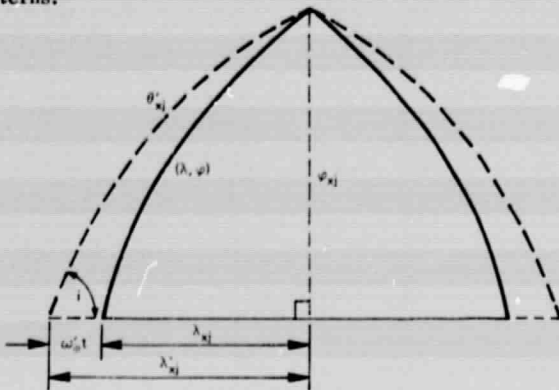


Figure 8. Intersection Location Geometry.

Referring to Figures 6 and 7 and Table II, it is apparent that the series of longitude increments, λ_{xj} , for the two pattern types are as follows (in s-units):

$$\text{Intermediate pattern: } \lambda_{xj} = \frac{2j - 1}{4} \left(j = 1, 2, 3, \dots, \frac{R+N-1}{2} \right)$$

$$= \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \dots, \frac{R+N-2}{4}$$

$$\text{Coincident pattern: } \lambda_{xj} = \frac{j}{2} \left(j = 1, 2, 3, \dots, \frac{R+N-2}{3} \right)$$

$$= \frac{1}{2}, 1, \frac{3}{2}, \dots, \frac{R+N-2}{4}$$

The above series cover only the northern half ((R+N-1)/2 values of j instead of R+N-1 per Table II) of the full latitude range, since the patterns are symmetrical about the equator, and the same numerical values of latitude will apply north and south. In addition, the Table II number is reduced by one in the coincident case because one of the intersection parallels is the equator.

In applying the above values to finding corresponding values of ϕ_{xj} in Figure 8, note that the solid-line arch is not subject to spherical trigonometric solution because the traces (λ, ϕ) are not circles, having been modified by earth-node rotation. The dashed curves, however, corresponding to the non-rotating model, are great circles.* Then the dashed right spherical triangles can be solved for the desired latitudes ϕ_{xj} , based on the known series

*As in Reference 4 analysis. This distinction illustrates an essential difference herein from Reference 4 and related earlier studies, in which longitudinal coverage is obtained using multiple satellites distributed in right ascension. That configuration permits a generalized treatment of longitudinal effects without reference to specific ground traces and earth-fixed coordinates.

of longitude segments λ_{xj} . By Equations (2), (3), (4), and (5):

$$\lambda_{xj} = \lambda'_{xj} - \omega'_{\phi} t$$

$$= \lambda'_{xj} - \frac{2\pi}{D_n} t$$

$$= \lambda'_{xj} - \frac{N}{R} \theta'_{xj} \quad (6)$$

Equation (6) results from the circular orbit relationship $2\pi t/T = \theta'$. The "nonrotating" longitude increment λ'_{xj} in Equation (6) can be expressed in terms of θ'_{xj} by applying Napier's rules to Figure 8, modifying Equation (6) to:

$$\lambda_{xj} = \tan^{-1}(\cos i \tan \theta'_{xj}) - \frac{N}{R} \theta'_{xj} \quad (7)$$

Substituting the known values of λ_{xj} (above) into Equation (7), it can be solved (numerically) for θ'_{xj} , which in turn yields the desired ϕ_{xj} values via an additional application of Napier's rules to the dashed right triangle in Figure 8:

$$\phi_{xj} = \sin^{-1}(\sin i \sin \theta'_{xj}) \quad (8)$$

Appendix II

Nomenclature

| Symbol | Definition |
|------------------|---|
| D_n | nodal day ($D_n = 2\pi/\omega'_{\phi}$) |
| i | orbital inclination (and trace "amplitude") |
| N | number of nodal days (D_n) per coverage period |
| Q | orbital revolutions per day (Refs. 1 and 2) |
| [Q] | integral part of Q |
| R | number of orbital revolutions per coverage period |
| s | basic longitudinal unit ($s = 2\pi/R$) |
| t | time variable |
| T | orbital period |
| θ' | orbital displacement angle (measured from ascending node) |
| λ | longitude coordinate |
| λ' | longitude coordinate in nonrotating model |
| $\Delta\lambda$ | longitude displacement between consecutive subnodal points in same direction (the "step") |
| ϕ | latitude coordinate |
| ω_{ϕ} | earth rotation rate (inertial) |
| ω'_{ϕ} | earth rotation rate relative to satellite orbit line of nodes |
| $\dot{\Omega}$ | rotation rate of satellite orbit line of nodes |

Subscripts

| | |
|-----|--------------|
| j | free index |
| x | intersection |