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NUMERICAL AIRFOIL OPTIMIZATION USING A REDUCED NUMBER OF DESIGN COORDINATES

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16. Abstract

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NOTATION

a_i, b	polynomial coefficient or participation coefficient
A	airfoil cross-sectional area
$\bar{A}$, $\bar{A}_1$, $\bar{A}_2$	vector of participation coefficients
c	chord
C_{D_w}	section wave-drag coefficient
C_L	section lift coefficient
C_M	section pitching-moment coefficient
C_p	pressure coefficient
J	number of chordwise stations at which the coordinates of the airfoil are defined
ℓ	boundary condition number
L	total number of boundary conditions
LS	lower surface
M	Mach number
N	number of basis vectors and participation coefficients
t_{TE}	trailing edge thickness
t/c	thickness to chord ratio
US	upper surface
x	chordwise distance
x/c	fractional chordwise distance
y	airfoil ordinate
$\bar{y}$	basis vector defining airfoil ordinates
$\bar{y}^0$	vector of boundary values
[Y], Y	matrix of basis vectors, $\bar{y}$
Y_{11},Y_{12},Y_{21},Y_{22}	submatrices of Y
α	angle of attack

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SUMMARY

A method is presented for numerical airfoil optimization whereby a reduced number of design coordinates are used to define the airfoil shape. The approach is to define the airfoil as a linear combination of shapes. These basic shapes may be analytically or numerically defined, allowing the designer to use his insight to propose candidate designs. The design problem becomes one of determining the participation of each such function in defining the optimum airfoil. Examples are presented for two-dimensional airfoil design and are compared with previous results based on a polynomial representation of the airfoil shape. Four existing NACA airfoils are used as basic shapes. Solutions equivalent to previous results are achieved with a factor of more than 3 improvement in efficiency, while superior designs are demonstrated with an efficiency greater than 2 over previous methods. With this shape definition, the optimization process is shown to exploit the simplifying assumptions in the inviscid aerodynamic analysis used here, thus demonstrating the need to use more advanced aerodynamics for airfoil optimization.

INTRODUCTION

The design of airfoils using numerical optimization techniques has been the subject of considerable recent interest. The basic design problem addressed is the determination of the optimum airfoil shape which will minimize or maximize a prescribed design objective subject to constraints which the design must satisfy. For example, the design objective may be to minimize the wave drag on a high-speed airfoil. The design constraints may be the requirement that at a prescribed angle of attack, $\bar{\alpha}$, the lift coefficient must be greater than $\bar{C}_L$ and the pitching moment coefficient must not exceed $\bar{C}_M$ in magnitude. Additionally, physical constraints such as a minimum allowable thickness-to-chord ratio of $\overline{t/c}$ may be imposed.

The design approach has been to couple a two-dimensional inviscid aerodynamics program (ref. 1) with an existing optimization program (ref. 2). This design procedure is described in references 3 and 4 and applications to airfoil design are described in references 5 through 7.

Various techniques may be used to define the airfoil shape. The aerodynamic analysis program usually requires a set of upper and lower surface y-coordinates at a specified number of chordwise locations along the airfoil, say at 50 points. Ideally, each of these y-coordinates would be treated as an independent design variable to ensure the widest variety of airfoil shapes. Then the design basis would be the same as the set of analysis coordinates. However, this is too large a number of design variables for the optimization program to handle efficiently. Therefore, it is desirable to use a reduced set of design variables which will adequately define the airfoil shape. A more common approach has been to describe the upper and lower surfaces of the airfoil with polynomials of the form

$$y = a_1\sqrt{\frac{x}{c}} + a_2\frac{x}{c} + a_3\left(\frac{x}{c}\right)^2 + \ldots + a_N\left(\frac{x}{c}\right)^{N-1} \tag{1}$$

One polynomial may be used to describe an entire surface, or several polynomials may be used to describe the surface in a piecewise fashion. The design variables considered by the optimization program are the coefficients of the polynomials. If piecewise polynomials are used, the coordinates and slopes along the surface at the matchpoint between polynomials may also be design variables and some of the polynomial coefficients are eliminated in favor of these physical design variables.

Two problems arise when describing the airfoil surface with polynomials. First, the number of different geometric shapes that can be represented is limited and may not include the shape that corresponds to the true optimum airfoil. The second problem is one of numerical conditioning arising from the fact that a surface described by a polynomial may be quite "wavy." That is, the optimization may produce an airfoil for which the curvature of the surface changes sign at several locations. This results from the fact that the aerodynamic analysis is not sensitive to minor waviness in the airfoil and therefore, the optimization program does not see any design penalty for introducing waviness.

A procedure is presented here for airfoil shape definition which allows for the consideration of a wide range of airfoil geometries. The restriction to shapes which can be described by polynomials is eliminated. While it cannot be shown that surfaces described using the techniques presented here will be smooth, experience has shown that the problem of surface "waviness" is greatly reduced. Also, it cannot be guaranteed that the shapes described by this method will define the true optimum airfoil. However, the generality of the method and the fact that the designer can use his judgment and experience in choosing the airfoil shape parameters produces high quality results.

AIRFOIL SHAPE DEFINITION

First, consider an airfoil surface defined by the polynomial given in equation (1), where one polynomial may be used to describe the upper surface of the airfoil and a second polynomial to describe the lower surface. The coefficients of the polynomial, a_1, a_2 ..., a_N, are treated as design variables and the objective of the design process is to determine the combination of these variables which will provide an optimum airfoil. Most aerodynamic analysis codes require a set of y-coordinates along the upper and lower surface of the airfoil, at J chordwise stations. Therefore, during the optimization process, for each proposed set of design variables the polynomials are evaluated at these J points for each surface. The results are stored in vector $\bar{y}$, for use in aerodynamic analysis.

An equivalent surface definition may be formulated by recognizing that the polynomial defined by equation (1) is simply the algebraic sum of curves defined by the terms of the polynomial. Curves defined by these terms are shown in figure 1. Each curve evaluated at J chordwise locations defines a distinct $\bar{y}$. For example, the curve, $\sqrt{x/c}$ will be defined as $\bar{y}_1$. Curve x/c defines $\bar{y}_2$ and curve $(x/c)^{N-1}$ defines $\bar{y}_N$. Therefore, for any set of design variables, a_1, a_2 ..., a_N the vector of y-coordinates defining the airfoil shape is defined by

$$\bar{y} = a_1\bar{y}_1 + a_2\bar{y}_2 + \ldots + a_N\bar{y}_N \tag{2}$$

The individual vectors on the right hand side of equation (2) are referred to as basis vectors.

Note that the fundamental distinction between equations (1) and (2) is that in equation (1) the y-coordinates are defined analytically, whereas in equation (2) the y-coordinates are defined numerically. Therefore, the basis vectors are not restricted to vectors defined by analytical functions, but can be chosen to define any set of shapes which may be expected to define a realistic airfoil. One logical choice of shapes is a set of existing airfoils and this approach is used in the examples presented in this report.

The vectors $\bar{y}$, $\bar{y}_1$, $\bar{y}_2$ ... $\bar{y}_N$ may contain coordinates of both the upper and lower surface of the airfoil so that, in general

$$\bar{y} = \left\{\begin{array}{c} y_{US} \\ --- \\ y_{LS} \end{array}\right\} \tag{3}$$

where $\bar{y}_{US}$ defines the coordinates of the upper surface and $\bar{y}_{LS}$ defines the coordinates of the lower surface.

Equation (2) may be written in matrix form as:

$$\bar{y} = [Y]\bar{A} \tag{4}$$

where

$$[Y]_{2\,xN} = [\bar{y}_1\bar{y}_2 \ldots \bar{y}_N] \;, \quad \bar{A} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix} \tag{5}$$

The design process now entails determining the values of the participation coefficients, a_i, $i = 1, 2, \ldots, N$, which define the optimum airfoil.

Note that, while the airfoil shape is defined as a linear combination of component shapes, the aerodynamics may be quite nonlinear in these variables. Therefore, the design process is a general nonlinear optimization problem.

PARTIAL AIRFOIL MODIFICATION

It is sometimes necessary to modify only a portion of an existing airfoil. For example, it may be desirable to retrofit an existing aircraft with a modified leading edge to improve low-speed characteristics. This is easily accomplished using basis vectors as described by equation (4). The first basis vector y_1 is taken as the vector of y-coordinates defining the existing airfoil. The participation coefficient a_1 is set to unity and is not changed during the design process. That is, a_1 is not a design variable. The remaining basis vectors y_2 through y_N will have nonzero entries over the portion of the airfoil which is to be modified and zero entries for the portion of the airfoil which is not modified. The participation coefficients, a_2 through a_N are then the design variables, which, when multiplied by their corresponding y-vectors, define the modified airfoil shape. As a further example, if it is desired to modify the upper surface of an existing airfoil while leaving the lower surface unchanged, vectors y_2 through y_N will contain nonzero entries corresponding to upper surface coordinates and zero entries corresponding to lower surface coordinates. The initial basis vector, y_1, will contain entries corresponding to the lower surface coordinates of the actual airfoil. Entries in y_1 corresponding to the upper surface may or may not be zero. If these entries correspond to the actual airfoil, the modifications defined by the remaining basis vectors will simply be added to (subtracted if a_i is negative) the existing airfoil. If the upper surface coordinates of vector y_1 are specified as 0, the remaining vectors will actually describe the entire upper surface of the airfoil. It is usually desirable that vector y_1 define the actual airfoil to be modified and that the participation coefficients a_2 through a_N be set initially at zero. Then, the first airfoil analyzed will be the existing airfoil and, as the participation coefficients a_2 through a_N are changed during the optimization process, the design improvements can be readily observed.

The [Y] matrix in equation (4) is presented schematically for upper surface modification as:

$$[Y] = \left[\begin{array}{cccccc} x & x & x & \cdots & & x \\ x & x & x & \cdots & & x \\ x & x & x & \cdots & & x \\ \vdots & \vdots & \vdots & & & \vdots \\ x & x & x & \cdots & & x \\ \hline x & 0 & 0 & \cdots & & 0 \\ x & 0 & 0 & \cdots & & 0 \\ x & 0 & 0 & \cdots & & 0 \\ \vdots & \vdots & \vdots & & & \vdots \\ x & 0 & 0 & \cdots & & 0 \end{array}\right] \begin{array}{l} \text{upper} \\ \text{surface} \\ \text{coordinates} \\ \\ \\ \text{lower} \\ \text{surface} \\ \text{coordinates} \\ \\ \\ \end{array} \qquad (6)$$

In this way, by the appropriate choice of zero and nonzero entries in the basis vectors, one or more segments of an existing airfoil can be readily modified. However, applying this approach directly does not guarantee that the coordinates, slopes, or curvatures of the modified surface will match the existing surface at the point where the two join. These boundary conditions could be imposed as constraints in the optimization problem; however, because these conditions represent simple geometric constraints on the airfoil geometry, they can be imposed directly. The procedure for doing this is described in the following section.

BOUNDARY CONDITIONS

Probably the most common boundary condition is the requirement that the airfoil thickness at the trailing edge be zero or some specified finite thickness. Other common boundary conditions include continuity of coordinates, slopes and curvatures at the match point between the modified and existing airfoil section, and the requirement that the airfoil thickness has a specified value at some chordwise location.

For example, assume that the airfoil thickness, t_{TE}, at the trailing edge is required to have a specified value t_{TE}^o and that one or more of the basis vectors has a defined finite trailing edge thickness. Let t_{TE_i} be the trailing edge thickness defined by the ith basis vector. One of the basis vectors can now be eliminated in favor of this boundary condition. Assume that basis vector $\bar{y}_N$ defines an airfoil of finite trailing edge thickness and that this basis vector will be eliminated in favor of the boundary condition. Because the design airfoil must have a trailing edge thickness T_{TE}^o, it follows that

$$t_{TE}^o = \sum_{i=1}^{N-1} a_i t_{TE_i} + a_N t_{TE_N} \tag{7}$$

The design variables, a_i, i = 1, 2 . . . N-1, are specified at each stage of the optimization process. Therefore a_N is the only unknown in equation (7). This equation may be rewritten solving for a_N

$$a_N = \frac{1}{t_{TE_N}} \left(t_{TE}^o - \sum_{i=1}^{N-1} a_i t_{TE_i} \right) \tag{8}$$

Note that for a_N to be defined, t_{TE_N} must have a nonzero value; that is, the airfoil defined by the nth basis vector must have a finite trailing edge thickness. Note also that this method of applying boundary conditions applies to equality conditions only. If it is desired to let the trailing edge thickness change during the optimization process so that the only requirement is that the thickness be greater than zero and less than some finite value, this would be handled as an inequality constraint by the optimization program and would not be imposed directly as a boundary condition.

Other boundary conditions, such as thickness at some chordwise location, surface slopes, or curvatures, can be imposed in a similar manner. For the general case, assume that there are L geometric equality constraints to be imposed on the airfoil. Then, L basis vectors will be eliminated in favor of these boundary conditions ($L < N$). Because the coefficients of these basis vectors are determined such that the boundary conditions are satisfied exactly, these coefficients are not design variables in the optimization process.

For the ℓth geometric boundary condition, the associated parameter will be stored in $2J + \ell$ location of each of the basis vectors for $i = 1, N$. The required boundary value will be stored in the $2J + \ell$ location of vector $\bar{y}$ and will be denoted by the superscript 0. Assume that the last L basis vectors will be used to satisfy the boundary conditions. In matrix form, this may be written as

$$\begin{array}{c} \overline{2J} \\ \underline{L} \end{array} \begin{pmatrix} y \\ \hline y^0 \end{pmatrix} = \left[\begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array}\right] \begin{pmatrix} A_1 \\ \hline A_2 \end{pmatrix} \begin{array}{c} \overline{N-L} \\ \underline{L} \end{array} \tag{9}$$

$$2J + LxN$$

Y_{11} and Y_{12} contain the original basis vectors. Y_{21} and Y_{22} contain the geometric values corresponding to these basis vectors for the boundary conditions which must be imposed. $\bar{y}^0$ contains the boundary values. $\bar{A}_1$ and $\bar{A}_2$ contain the participation coefficients for the basis vectors. Because $\bar{A}$, contains the design variables defined by the optimization program, the value of $\bar{A}_1$ is known. It is now necessary to determine $\bar{A}_2$ so that the boundary conditions are precisely satisfied. This is easily accomplished by solving the last L equations defined by the matrix equation (9). These equations are written as

$$\bar{y}^0 = Y_{21}\bar{A}_1 + Y_{22}\bar{A}_2 \tag{10}$$

Solving for $\bar{A}_2$ yields

$$\bar{A}_2 = Y_{22}^{-1}\left(\bar{y}^0 - Y_{21}\bar{A}_1\right) \tag{11}$$

Finally, substituting the values for $\bar{A}_2$ defined by equation (11) into the first 25 equations defined by matrix equation (9) yields

$$\bar{y} = [Y_{11} \mid Y_{12}] \begin{pmatrix} A_1 \\ \hline A_2 \end{pmatrix} = [Y]\bar{A} \tag{12}$$

Equation (12) defines an airfoil which satisfies the boundary conditions precisely. Note that the only requirement for satisfaction of the boundary conditions is that the submatrix Y_{22} be nonsingular. Usually, only a few boundary conditions are imposed on the airfoil and the basis vectors to be eliminated in terms of these boundary conditions can be chosen by inspection so that Y_{22} will satisfy this requirement.

DESIGN EXAMPLES

Examples are presented here to identify the generality and efficiency of the reduced basis concept as applied to numerical airfoil optimization. Four existing airfoils are used as the design basis. These are the NACA 2412, NACA 64_1-412, NACA 65_2-415, and the NACA 64_2A215 airfoils. The coordinates are defined at 50 points along the upper and lower surfaces. The coordinates are approximate, obtained from curve fits of the existing airfoil data (refs. 8 and 9) and no attempt was made to precisely match the data given in the references. Two additional basis vectors were used to impose the boundary conditions at the trailing edge of the airfoil. These are $y_{US} = x/c$, $y_{LS} = 0$ and $y_{US} = 0$, $y_{LS} = -(x/c)$. The shapes defined by these six basis vectors are shown in figure 2. In each of the following examples, except case 4, the NACA 2412 airfoil was chosen as the initial design with an associated participation coefficient of unity. In case 4, the optimum airfoil from case 3 was used as the initial design. The participation coefficients for the basis vectors 2, 3, and 4 were initially 0 and vectors 5 and 6 were used to impose the requirement that the trailing edge thickness be zero for cases 1 and 2 and that the trailing edge thickness be 0.25 percent of the chord length for cases 3 and 4. The initial vector of four independent design variables is

$$A_1^T = (1.0 \quad 0.0 \quad 0.0 \quad 0.0) \tag{13}$$

The boundary condition vector $\bar{y}^0$ in equation (9) which insures a zero trailing edge thickness for cases 1 and 2 is given by

$$\bar{y}^0 = \begin{pmatrix} 0.0 \\ 0.0 \end{pmatrix} \tag{14}$$

The boundary condition vector $\bar{y}^0$ used to define a trailing edge thickness of 0.25 percent in cases 3 and 4 is given by

$$\bar{y}^0 = \begin{pmatrix} 0.00125 \\ -0.00125 \end{pmatrix} \begin{array}{l} \underline{\text{upper surface ordinate}} \\ \underline{\text{lower surface ordinate}} \end{array} \tag{15}$$

Case 1 - Lift Maximization, M = 0.1, $\alpha = 6°$ - Figure 3 shows results from the optimization of an airfoil for maximum lift. Constraints were imposed on the area enclosed by the airfoil, the upper surface pressure coefficient near the leading edge, and the magnitude of the pitching moment coefficient. The initial airfoil is shown as a solid line in figure 3 and the final airfoil is shown as a dashed line. This figure also represents initial and final pressure distributions and airfoil characteristics. The results of this design example are significant in two respects. First, it is highly improbable that a lift coefficient of 1.478, as predicted by the inviscid aerodynamics program, could be achieved for the highly unconventional airfoil shape obtained by the optimization program. The optimization process was arbitrarily terminated after 10 design iterations. Had it been allowed to continue, even further mathematical improvements in the design lift coefficient could have been achieved. This suggests that the optimization process is taking undue advantage of simplifying assumptions in the inviscid aerodynamic analysis. This is verified by the fact that when viscosity terms are included in the aerodynamic analysis by methods described in reference 10, upper surface separation is predicted at the 65 percent chordwise location. The second noteworthy point of this result is that the use of basis vectors appears to provide a higher degree of generality in airfoil shapes than was previously obtainable using polynomial representations. The airfoil obtained for the same design conditions in reference 4 using a polynomial representation with 14 independent design variables did not exploit the simplifying assumptions of the aerodynamic code. A lift coefficient of 1.478 was obtained in the present study as compared to 1.085 reported in reference 4. Using the basis vector approach of the current study, this shape was obtained using only four independent design variables. This design required 72 aerodynamic analyses as compared to 103 analyses reported in reference 4. The final vector of independent participation coefficients obtained from the optimization process is

$$A_1^T = (6.49 \quad -4.65 \quad 6.59 \quad -6.47) \tag{16}$$

Case 2 - Lift Maximization, $M = 0.1$, $\alpha = 6°$ - The airfoil designed as case 1 was reoptimized with the addition of two geometric constraints. The maximum thickness-to-chord ratio was not allowed to exceed 15 percent and the maximum camber was not allowed to exceed 4 percent. The results of the optimization are shown in figure 4 together with the airfoil design of reference 4. The airfoil shapes and pressure distributions are somewhat different, even though the airfoil section coefficients are nearly the same. This suggests that the optimum airfoil shape is not unique and that a variety of airfoils may exist which provide the same section coefficients. This optimization required 44 aerodynamic analyses representing an improvement in efficiency of more than a factor of 2 over reference 4. The final vector of independent participation coefficients for this case is

$$A_1^T = (1.89 \quad -0.54 \quad 0.89 \quad -0.96) \tag{17}$$

Case 3 - Lift Maximization, $M = 0.75$, $\alpha = 0°$ - Figure 5 presents the results of lift maximization for a high-speed airfoil with a wave drag constraint. The airfoil was required to have a finite trailing edge thickness of 0.25 percent of the chord length. The initial airfoil violates the drag constraint by nearly a factor of 3. The optimum airfoil satisfies this drag constraint with a lift coefficient of $C_L = 0.4188$; a more than 7 percent improvement over the value of $C_L = 0.3881$ reported in reference 4. The basis vectors used here are for airfoils intended for low speed applications. Further design improvements may be possible by utilizing existing high-speed airfoils as basis vectors. The optimum airfoil obtained here required 70 aerodynamic analyses as compared to 143 analyses reported in reference 4, again representing a factor of 2 improvement in efficiency. A lift coefficient of $C_L = 0.3896$ was obtained after 42 aerodynamic analyses, representing a factor of more than three gain in efficiency to achieve a design equivalent to that of reference 4. The final vector of independent participation coefficients is

$$A_1^T = (0.47 \quad 0.53 \quad -1.00 \quad 0.99) \tag{18}$$

Case 4 - Wave Drag Minimization, $M = 0.75$, $\alpha = 0$ — The high-speed airfoil presented as case 3 was reoptimized, where now the wave drag was minimized subject to the requirement that the lift coefficient be 0.30 or greater. The initial design was taken as the optimum obtained in case 3 and is defined by equation (18). The results of this optimization are shown in figure 6. This design required 44 aerodynamic analyses and yielded a final vector of independent participation coefficients of

$$A_1^T = (0.31 \quad 0.37 \quad -1.03 \quad 1.26) \tag{19}$$

CONCLUDING REMARKS

A procedure has been presented for airfoil shape definition which allows for the consideration of a wide range of airfoil geometries. The set of basis vectors used to define the airfoil may be any analytically or numerically defined coordinate representations. It is noteworthy that this concept is equivalent to the reduced basis concept for structural optimization presented by Pickett *et al.* in reference 11 and by Schmit and Miura in reference 12. Indeed, the idea of applying reduced basis concepts to airfoil optimization originated from private communications with the authors of reference 12.* Equality boundary conditions were not required for the structural optimization problem of reference 12. The boundary condition requirements in airfoil optimization could be handled as natural boundary conditions by choosing each basis vector such that the boundary conditions are satisfied. However, the approach presented here is less restrictive, allowing the engineer to change the boundary conditions without respecifying the basis vectors.

The procedure presented here is in no way restricted to two-dimensional airfoil design. The reduced basis concept is directly applicable to three-dimensional aerodynamic design of wings or of propulsion system inlets.

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*ARC/UCLA Consortium on Structural Optimization.

In the examples presented here, existing airfoils were used as basis vectors. The results, using only 4 independent design variables, were compared with results previously obtained using a polynomial representation for the airfoil with 14 independent design variables. In each case, the number of aerodynamic analyses required to obtain a design equivalent to the previous results was reduced by a factor of more than 3. This is consistent with the observation that the required number of analyses is approximately linearly proportional to the number of independent design variables. Therefore, any procedure which can be used to reduce the number of design variables without introducing numerical ill-conditioning can be expected to improve design efficiency. This efficiency improvement, with its associated reduction in computational time, will allow for the use of more sophisticated (and time consuming) aerodynamic analyses. The fact that the optimization process exploits the simplifying assumptions in the aerodynamic analysis suggests that more sophisticated analyses are required if a truly practical airfoil optimization capability is to be developed.

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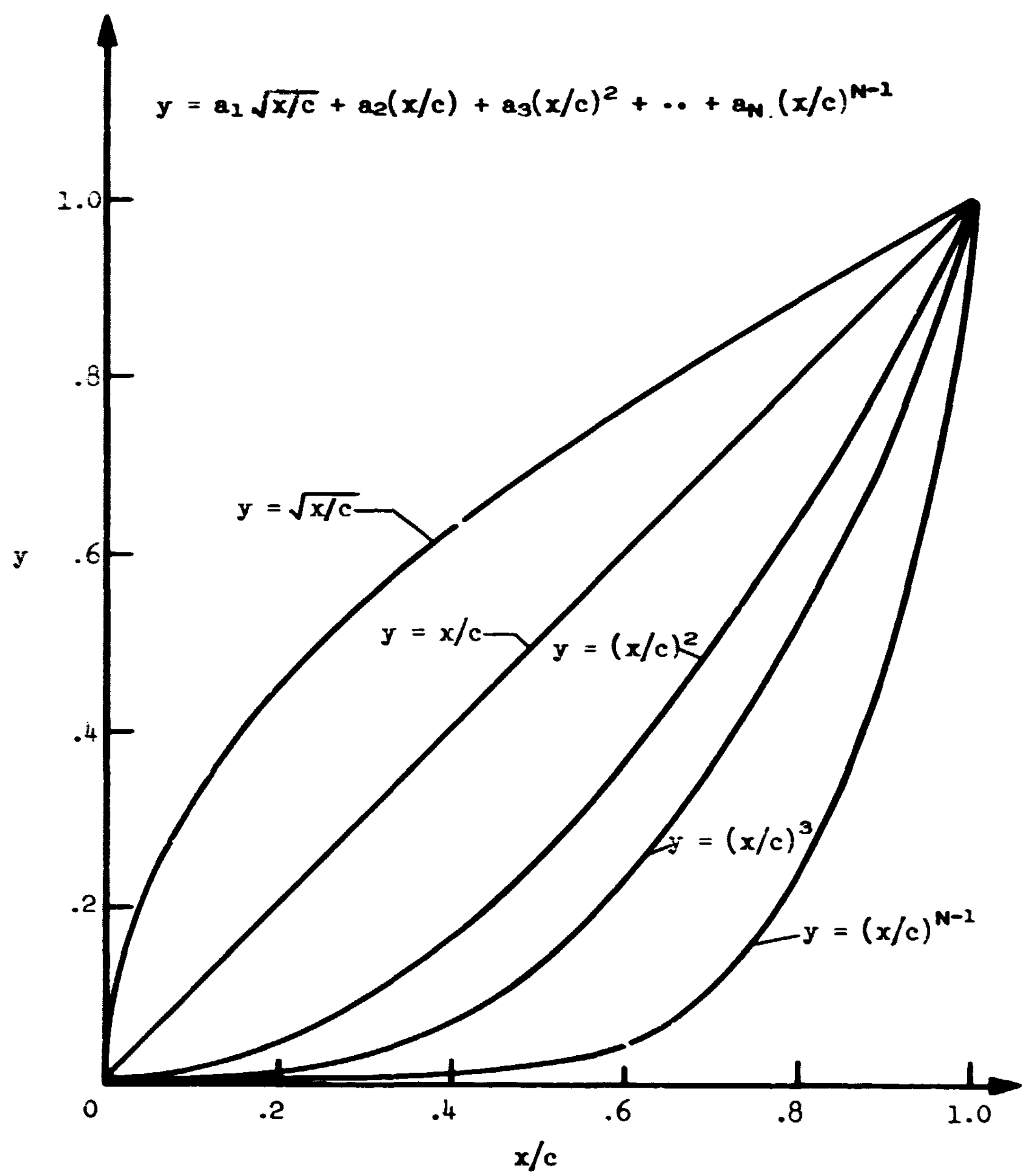
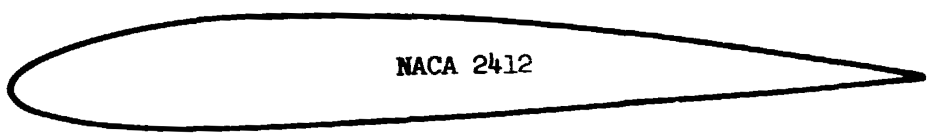
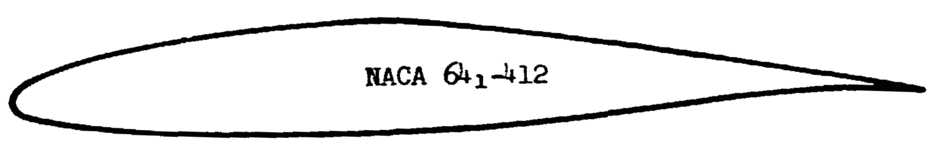


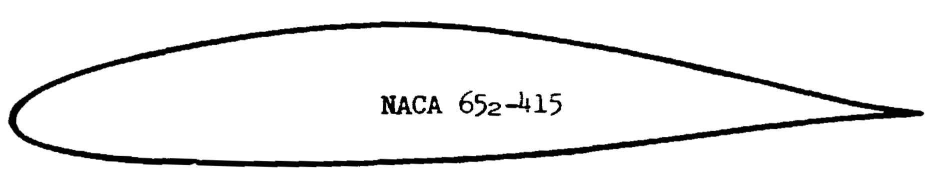
Figure 1.- Components of polynomial shape.



Basis shape 1.



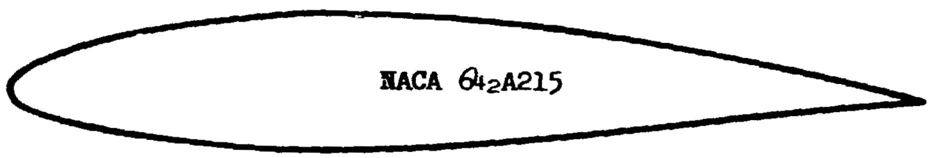
Basis shape 2.



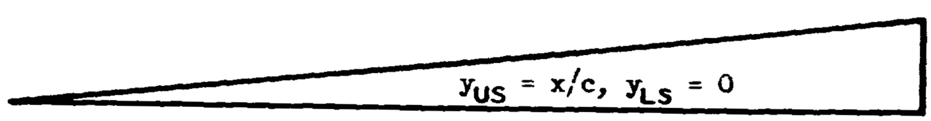
Basis shape 3.

(a) Shapes 1 through 3.

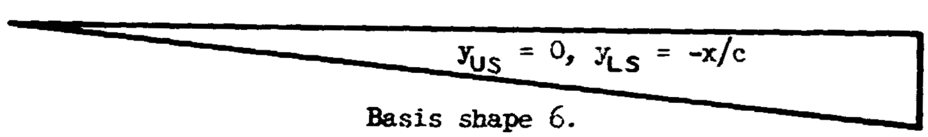
Figure 2.- Basis vectors.



Basis shape 4.



Basis shape 5.



Basis shape 6.

(b) Shapes 4 through 6.

Figure 2.- Concluded.

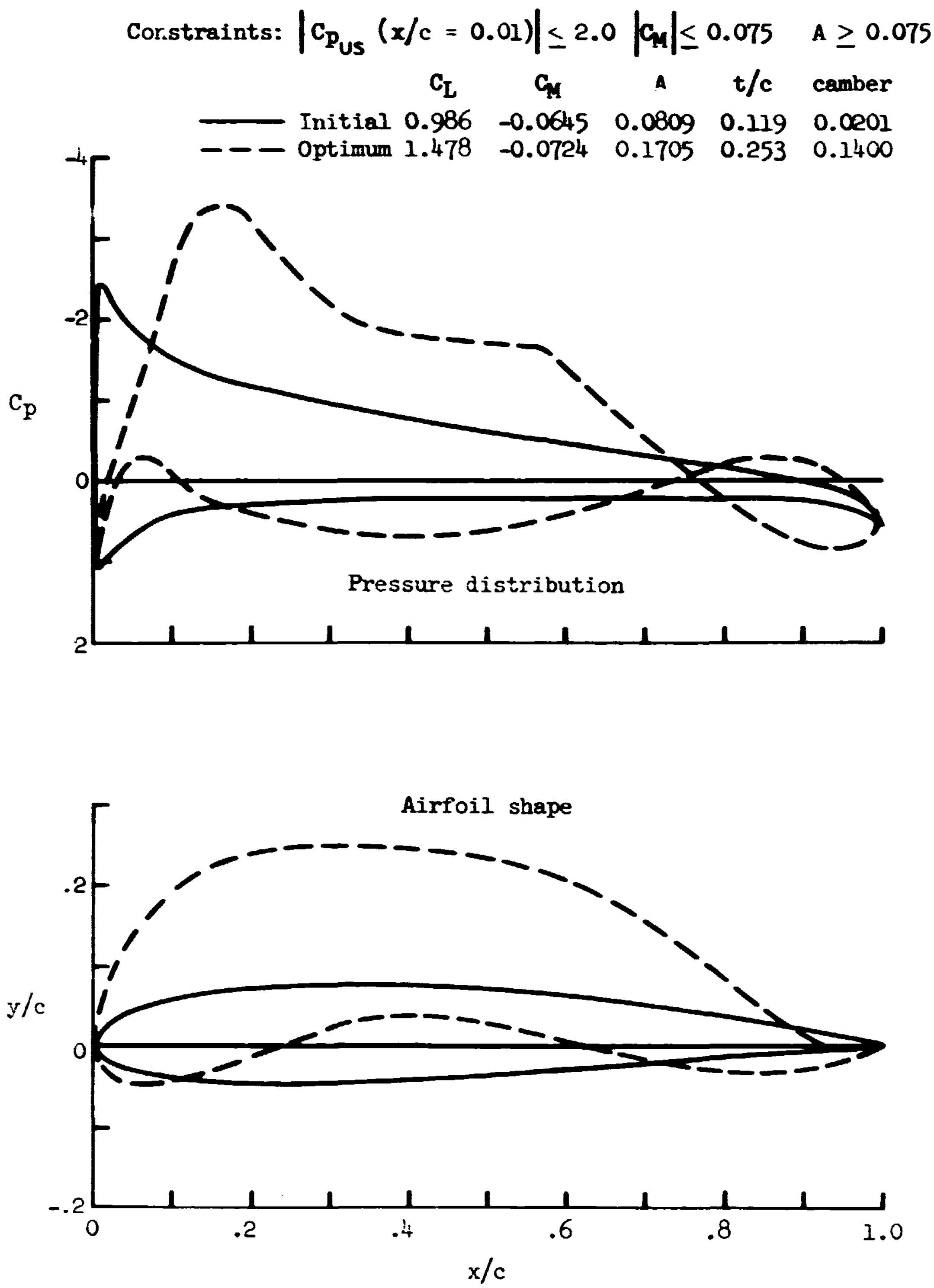


Figure 3.- Case 1 lift maximization; M = 0.1, $\alpha = 6°$.

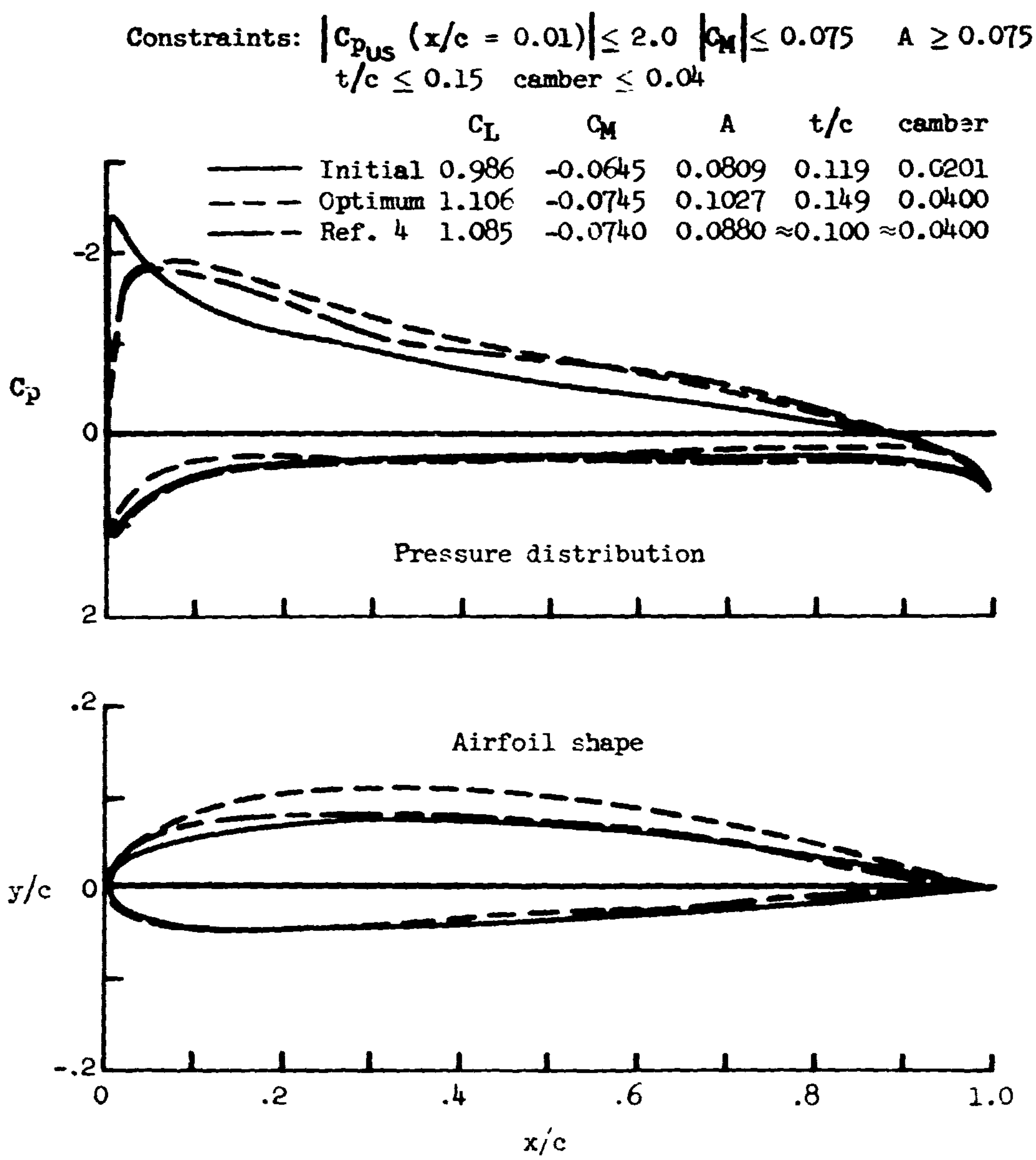


Figure 4.- Case 2 lift maximization: $M = 0.1$, $\alpha = 6°$.

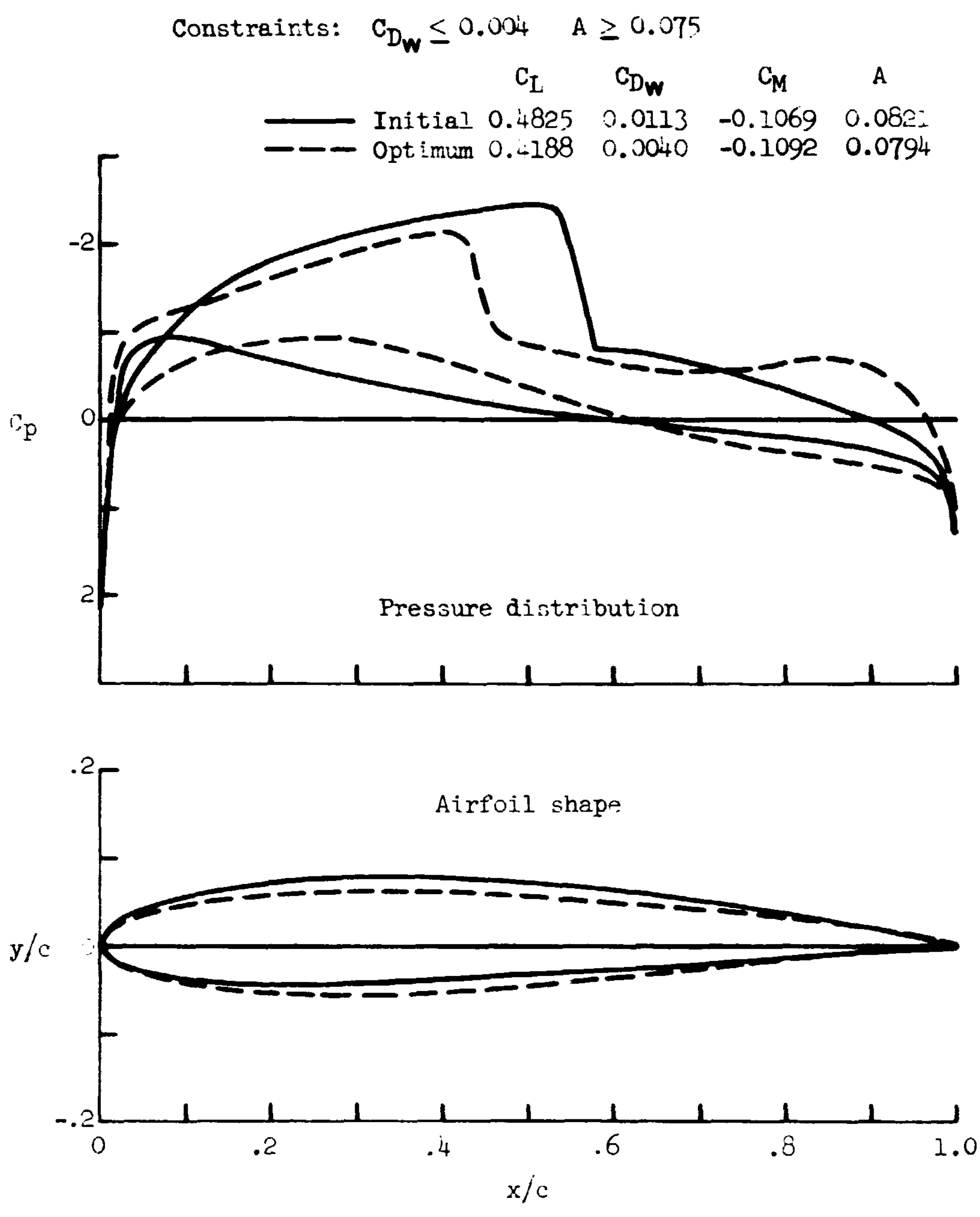


Figure 5.- Case 3 lift maximization; M = 0.75, α = 0°.

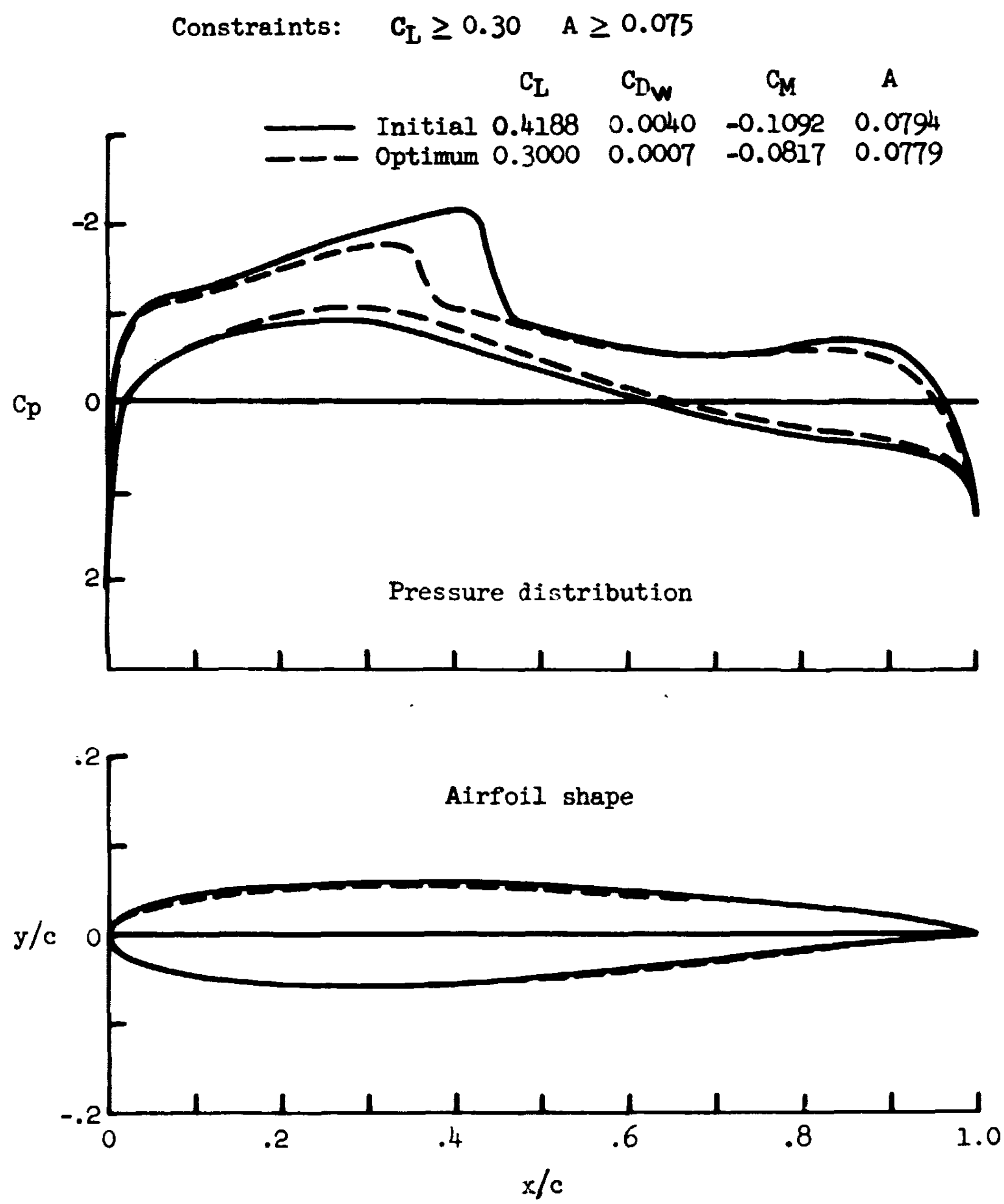


Figure 6.- Case 4 wave drag minimization; M = 0.75, α = 0°.