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THE LONGITUDINAL EQUATIONS OF MOTION OF A TILT PROP/ROTOR AIRCRAFT INCLUDING THE EFFECTS OF WING AND PROP/ROTOR BLADE FLEXIBILITY

by

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SUMMARY

The equations of motion for the longitudinal dynamics of a tilting prop/rotor aircraft are developed. The analysis represents an extension of the equations of motion developed in NASA TM X-62,369 to include the effects of the longitudinal degrees-of-freedom of the body (pitch, heave and horizontal velocity). The development and notation follow that of NASA TM X-62,369 such that, the effects of body freedom can be added to the equations of motion for the flexible wing-propeller combination.

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INTRODUCTION

This report develops the equation of motion for the longitudinal dynamics of a tilting prop rotor aircraft. The effects of wing and prop rotor flexibility are included in the analysis as well as three longitudinal body degrees-of-freedom. The notation and development generally follows that of NASA TM X-62,369 and is formulated in such a way that it modifies that study to include the free longitudinal motion of the complete airframe, such that, the influence of the wing and prop rotor flexibility on the vehicle dynamics, as related to stability and control, can be examined.

The resulting equations of motion developed in NASA TM X-62,369 are expressed in matrix notation as follows:

$$A_2 \ddot{x}_R + A_1 \dot{x}_R + A_0 x_R + \tilde{A}_2 \ddot{\alpha} + \tilde{A}_1 \dot{\alpha} + \tilde{A}_0 \alpha = Bv_R + B_G g \quad (1)$$

$$F = C_2 \ddot{x}_R + C_1 \dot{x}_R + C_0 x_R + \tilde{C}_2 \ddot{\alpha} + \tilde{C}_1 \dot{\alpha} + \tilde{C}_0 \alpha + D_G g \quad (2)$$

$$\alpha = Cx_W \quad (3)$$

$$a_2 \ddot{x}_W + a_1 \dot{x}_W + a_0 x_W = bv_W + b_G g + \tilde{\alpha} F \quad (4)$$

where x_R is the state vector of the rotor degrees-of-freeom; x_W is the state vector of the wing degrees-of-freedom; v_R is the input control vector; and g, is the gust input vector. F is a column matrix representing the forces and moments produced by the prop rotor and α is a column matrix giving the linear and angular displacement of the prop rotor hub due to wing tip motion.

The development here is concerned with adding the effects of vehicle body motion to these equations of motion. The following modifications are developed. Equations (3) must be modified to include the influence of body motion on rotor hub motion. Equations (1) and (2) do not require modification owing to the way in which the equations have been formulated. Equation (4), the wing equations of motion must be modified to account for the influence of fuselage motion and in addition the fuselage motion equations must be developed. This report therefore, is concerned with two items: development of wing/body equations of motion and incorporation of the body motion into the hub displacement expressions.. The analysis of NASA TM X-62,369 is not discussed in detail in this report.

ANALYTICAL DEVELOPMENT

Geometry of Wing/Body Motion

A small disturbance approach is used in developing the equations of motion. The center of gravity of the fuselage is taken as the origin of the axis system. X and Z, in general, refer to geometric distances measured along and perpendicular to the direction of the initial flight velocity. The following specific distances are involved in the formulation:

X_R, Z_R	distance from center of gravity of fuselage to spar location at wing root.
X_{WL}, Z_{WL}	distance from center of gravity of fuselage to local effective elastic axis of wing section in deflected position.
X_{WT}, Z_{WT}	distance from center of gravity of fuselage to wing tip effective elastic axis location in deflected position.
X_P, Z_P	distance from center of gravity of fuselage to pylon center of gravity including wing deflection.

These distances are shown in Figure 1. α_0, θ_0 and γ_0 are respectively the trimmed flight angle of attack, pitch angle, and flight path angle and V_0, is the trimmed flight velocity.

The fuselage center of gravity motion is specified by perturbation velocities $\dot{x}_f$ and $\dot{z}_f$ initially along and perpendicular to the trimmed flight velocity and the rotation of the fuselage is denoted by θ_f.

X_R and Z_R are geometric distances, characteristic of the aircraft, however, they do depend on the trim angle of attack of the aircraft as shown in Figure 1.

All of the other quantities depend upon the wing bending and torsional deflections. The wing orientation with respect to the fuselage is specified by three angles, δw_1, the dihedral, δw_2, the wing incidence plus body initial angle of attack ($\delta w_2 = i_W + \alpha_0$) where i_W is the wing incidence with respect to the body reference, and α_0 is the trimmed flight angle of attack, and δw_3 the wing sweep angle. In order to obtain results which are a modification of NASA TM X-62,369 the wing deflection is expressed in terms of an axis system aligned with the wing.

Assuming that the angles δw_1, δw_2, δw_3 are small, as is characteristic of this aircraft, the relationship among distances in the fuselage coordinate system and the wing coordinate system, where an effective sweep angle δw_3^*, and effective dihedral angle δw_1^* are introduced

$$\begin{Bmatrix} -x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} 1 & +\delta w_3^* & +\delta w_2 \\ -\delta w_3^* & 1 & -\delta w_1 \\ -\delta w_2 & +\delta w_1^* & 1 \end{bmatrix} \begin{Bmatrix} x_W \\ y_W \\ z_W \end{Bmatrix} \tag{5}$$

The minus sign in front of x is required since x_W is positive aft in Reference 1.

$$\begin{aligned} x &= - x_W - \delta w_3^* y_W - \delta w_2 z_W \\ y &= - x_W \delta w_3^* + y_W - \delta w_1^* z_W \\ z &= - x_W \delta w_2 + y_W \delta w_1^* + z_W \end{aligned} \tag{6}$$

δw_1^* and δw_3^* are introduced at this point to reflect the fact that although the wing is swept, the center spar section is not swept as described in Reference 2. δw_1^* and δw_3^* account for an effective change in elastic axis position owing to the torsional deflection at the root. As developed in Reference 2, it is possible to represent this influence on the effective dihedral angle and effective sweep angle of the elastic axis as

$$\begin{aligned} \delta w_3^* &= \delta w_3 (1 - \xi w_B) \\ \delta w_1^* &= \delta w_1 (1 - \xi w_B) \end{aligned} \tag{7}$$

where ξw_B is the torsional deflection at the point where the spar is bent, i.e., at the wing root. Thus we have

$$\begin{aligned} x &= - x_W - y_W \delta w_3 (1 - \xi w_B) - \delta w_2 z_W \\ y &= x_W \delta w_3 (1 - \xi w_B) + y_W - z_W \delta w_1 (1 - \xi w_B) \\ z &= - x_W \delta w_2 + y_W \delta w_1 (1 - \xi w_B) + z_W \end{aligned} \tag{8}$$

Therefore, the position of the local wing elastic axis with respect to the fuselage center of gravity is expressed in terms of wing coordinates measured in the wing axis system.

$$\begin{aligned} X_{WL} &= X_R + x \\ &= X_R - x_W - y_W \delta w_3 (1 - \xi w_B) - \delta w_2 z_W \\ Z_{WL} &= Z_R + z \\ &= Z_R + z_W - x_W \delta w_2 + y_W \delta w_1 (1 - \xi w_B) \end{aligned} \tag{9}$$

The wing tip deflection is expressed in terms of the same quantities with the deflection measured at the tip

$$X_{WT} = X_R - x_{WT} - y_{WT}\,\delta w_3\,(1 - \xi w_B) - \delta w_2\, z_{WT}$$
$$Z_{WT} = Z_R + z_{WT} - x_{WT}\,\delta w_2 + y_{WT}\,\delta w_1\,(1 - \xi w_B) \qquad (10)$$

The pylon center of gravity is located at a fixed distance from the wing tip effective elastic axis location, given by the following distances

$$\Delta X_P = Z_{Pea}$$
$$\Delta Z_P = 0 \qquad (11)$$

where Z_{Pea} is the distance the pylon center of gravity is forward of the wing tip elastic axis measured along the wing reference system. Consequently

$$X_P = X_{WT} + \Delta X_P$$
$$Z_P = Z_{WT} \qquad (12)$$

This completes the expression for the linear displacements of various locations of significance. The hub position will be developed later.

Now the rotation of each point must be considered. The rotation of the fuselage center of gravity is θ_f and consequently the rotation of the wing root is also θ_f. Now this rotation must be expressed in terms of the wing coordinate system. This can be done by using the inverse of the transformation expressed by (5)

$$\begin{Bmatrix} -\Delta\phi_{WL} \\ \Delta\theta_{WL} \\ \Delta\psi_{WL} \end{Bmatrix} = \begin{bmatrix} 1 & -\delta w_3^* & -\delta w_2 \\ \delta w_3^* & 1 & \delta w_1^* \\ \delta w_2 & -\delta w_1^* & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ \theta_f \\ 0 \end{Bmatrix} \qquad (13)$$

Therefore

$$\Delta\phi_{WL} = \delta w_3^*\,\theta_f$$
$$\Delta\theta_{WL} = \theta_f \qquad (14)$$
$$\Delta\psi_{WL} = -\delta w_1^*\,\theta_f$$

where ϕ_{WL} represents a rolling of the wing section, and ψ_{WL} represents a yawing of the wing section. The effect of ϕ_{WL} can be neglected by assuming that the wing is thin. The complete angular motion of the wing section includes the torsional deflection, and is equal to

$$\theta_{WL} = (\theta_f + \theta_W)$$

At the tip of the wing the rotations are expressed back in the fuselage reference frame. The wing tip rotation consists of torsional deflection as well as rotation due to the bending slope. Again using transformation (5)

$$\begin{Bmatrix} -\Delta\Phi_T \\ \Delta\theta_T \\ \Delta\psi_T \end{Bmatrix} = \begin{bmatrix} 1 & \delta w_3^* & \delta w_2 \\ -\delta w_3^* & 1 & -\delta w_1^* \\ -\delta w_2 & \delta w_1^* & 1 \end{bmatrix} \begin{Bmatrix} z'_T \\ \theta_{WT} \\ -x'_T \end{Bmatrix} \qquad (15)$$

where x'_T and z'_T represents the rotation of the tip due to the bending slope

$$\begin{aligned} \Delta\Phi_T &= -z'_T - \delta w_3^* \theta_{WT} + \delta w_2 \; x'_T \\ \Delta\theta_T &= -\delta w_3^* z'_T + \theta_{WT} + \delta w_1^* \; x'_T \\ \Delta\psi_T &= -\delta w_2 z'_T + \delta w_1^* \theta_{WT} - x'_T \end{aligned} \qquad (16)$$

These quantities are the roll angle, pitch angle and yaw angle of the tip expressed in the fuselage axes system. To this must be added the rotation of the body θ_f so that

$$\theta_T = \theta_f - \delta w_3^* \; z'_T + \theta_{WT} + \delta w_1^* \; x'_T \qquad (17)$$

This is also the rotation of the pylon center of gravity. The above expressions describe the linear and angular displacements of the various points of interest.

Kinetic Energy

We now proceed to develop the equations of motion for the body-wing-pylon combination using the Lagrangian approach to evaluate the inertial terms.

a.) Fuselage

The kinetic energy of the fuselage can be expressed as

$$KE_f = \frac{1}{2} m_f (\dot{x}_f^2 + \dot{z}_f^2) + \frac{1}{2} I_f \dot{\theta}_f^2 \qquad (18)$$

where a reference system travelling at the uniform velocity V_o is used.

b.) Wing

A section of the wing has the following kinetic energy

$$KE_W = \frac{1}{2} \int_o^{y_{TW}} m dy_W [(\dot{x}_f + \dot{x} - Z_{WL} \dot{\theta}_f)^2 + (\dot{z}_f + \dot{z} + X_{WL} \dot{\theta}_f)^2] + \frac{1}{2} \int_o^{y_T} I_W (\dot{\theta}_f + \dot{\theta}_W)^2 \, dy + \frac{1}{2} \int_o^{y_{TW}} I_W (- \delta w_1^* \dot{\theta}_f)^2 \, dy \qquad (19)$$

where it has been assumed that the wing is thin such that the roll moment of inertia of the wing section is negligible.

c.) Pylon

The pylon kinetic energy is

$$KE_P = \frac{1}{2} m_P [(\dot{x}_f + \dot{x}_T - Z_P \dot{\theta}_f)^2 + (\dot{z}_f + \dot{z}_T + X_P \dot{\theta}_f + \Delta X_P \Delta\dot{\theta}_T)^2 + (\Delta X_P \Delta\dot{\psi}_T)^2] + \frac{1}{2} I_{P'_y} (\dot{\theta}_f + \Delta\dot{\theta}_T)^2 + \frac{1}{2} I_{P'_x} (\Delta\dot{\psi}_T)^2 \qquad (20)$$

where $I_{P'_y}$ and $I_{P'_x}$ are the pylon pitch and roll moments of inertia without the rotor measured with respect to the pylon center of gravity axis location. The pylon inertia in roll is neglected.

Owing to the manner in which the equations were formulated (Equations (1) through (4)) the rotor effects are included by determining the effects of body motion on rotor hub motion. Thus, to determine the equations of motion for the wing-body combination the kinetic energy of the system is

$$KE = KE_f + KE_W + KE_P$$

Now the modified equations of motion can be developed with the geometrical considerations described above. Prior to developing these equations the modified hub motion equations are developed.

Hub Motion

The hub position with respect to the wing tip effective elastic axis is expressed by ΔX_H, ΔZ_H again measured in the fuselage reference coordinate system. This is the hub location with respect to the fuselage center of gravity. The total hub displacement with respect to the center of gravity of the fuselage is

$$\begin{aligned} X_H &= X_{WT} + \Delta X_H \\ Z_H &= Z_{WT} + \Delta Z_H \end{aligned} \qquad (21)$$

where

$$\begin{aligned} \Delta X_H &= [h\ C - (h - h_{ea})] - [- Z_{ea} + h\ S]\ \delta w_2 \\ \Delta Z_H &= [h\ C - (h - h_{ea})]\ \delta w_2 + [- Z_{ea} + h\ S] \end{aligned} \qquad (22)$$

where

$$C = \cos\ (\delta_P - \delta w_2)$$

$$S = \sin\ (\delta_P - \delta w_2)$$

and $(h - h_{ea})$ and Z_{ea} represent the displacement of the elastic axis from the wing tip location forward and up due to sweep and dihedral. Since $(h - h_{ea})$ and Z_{ea} are proportional to δw_1 and δw_3 these distances are approximately given by

$$\Delta X_H \cong [h\ C - (h - h_{ea})] - h\ S\ \delta w_2$$
$$\Delta Z_H \cong [h\ C]\ \delta w_2 + [- Z_{ea} + h\ S] \qquad (23)$$

The rotation of the hub due to the bending and torsional deflection of the wing tip is given by Equation (16) plus the rotation of the body θ_f. The hub displacements arising from these rotations in the direction of the body reference system are

$$\overline{\Delta x}_{H/R} = - Z_H\ \theta_f - \Delta Z_H\ \Delta\theta_T$$
$$\overline{\Delta z}_{H/R} = X_H\ \theta_f + \Delta X_H\ \Delta\theta_T \qquad (24)$$
$$\overline{\Delta y}_{H/R} = + \Delta Z_H\ \Delta\Phi_T - \Delta X_H\ \Delta\psi_T$$

And in addition we have the center of gravity displacements, Δx_f, Δz_f with respect to the travelling reference system, and the wing tip displacements measured in the body coordinate system direction $\Delta x_{WT/B}$, $\Delta z_{WT/B}$.

The total hub displacements are

$$\overline{\Delta x}_H = \Delta x_f + \Delta x_{WT/B} - Z_H\ \theta_f - \Delta Z_H\ \Delta\theta_T$$
$$\overline{\Delta y}_H = \Delta Z_H\ \Delta\Phi_T - \Delta X_H\ \Delta\psi_T \qquad (25)$$
$$\overline{\Delta z}_H = \Delta z_f + \Delta z_{WT/B} + X_H\ \theta_f + \Delta X_H\ \Delta\theta_T$$

These must now be resolved into the hub direction, i.e., along and perpendicular to the shaft.

In order to remain consistant with the previous development Δx_H and Δz_H are first resolved to the wing direction

$$\overline{\Delta x}_{HW} = \overline{\Delta x}_H + \delta w_2 \, \overline{\Delta z}_H$$
$$\overline{\Delta z}_{HW} = \overline{\Delta z}_H - \delta w_2 \, \overline{\Delta x}_H \tag{26}$$

and then resolved through the angle the prop rotor shaft makes with the wing $(\delta_P - \delta w_2)$

$$\overline{x}_{HS} = \overline{\Delta x}_{HW} \, C + \overline{\Delta z}_{HW} \, S$$
$$\overline{y}'_{HS} = + \Delta Z_H \, \Delta\Phi_T - \Delta X_H \, \Delta\psi_T \tag{27}$$
$$\overline{z}_{HS} = \overline{\Delta z}_{HW} \, C - \overline{\Delta x}_{HW} \, S$$

The rotations also must be expressed in terms of the shaft axes. Again first resolving to the wing direction

$$\Delta\Phi_{TW} = \Delta\Phi_T + \Delta\psi_T \, \delta w_2$$
$$\Delta\psi_{TW} = \Delta\psi_T - \Delta\Phi_T \, \delta w_2 \tag{28}$$

and then in the notation of Reference 1

$$\alpha_x = \Delta\psi_{TW} \, C - \Delta\Phi_{TW} \, S$$
$$\alpha_y = \theta_f + \Delta\theta_T \tag{29}$$
$$\alpha_z = \Delta\Phi_{TW} \, C + \Delta\psi_{TW} \, S$$

Equations (27) and (29) express the motion of the hub in terms of wing motion and body motion.

The rotation angles of the wing can now be expressed as, neglecting products of the wing geometric angles δw_1, δw_2, δw_3, using Equation (16) in Equation (29).

$$\alpha_x = z'_T S - x'_T C + (\delta w_1^* C + \delta w_3^* S)\,\theta_{WT}$$

$$\alpha_y = \theta_f - \delta w_3^* \, z'_T + \delta w_1^* \, x'_T + \theta_{WT} \qquad (30)$$

$$\alpha_z = - z'_T C - x'_T \, S + (\delta w_1^* S - \delta w_3^* C)\,\theta_{WT}$$

The displacement of the hub can be expressed as, using the relationships from Reference 2, pp. 113, for the displacement of the effective elastic axis from the tip elastic axis.

$$Z_{ea} = - y_{TW} \, \delta w_1$$

$$h - h_{ea} = y_W \, \delta w_3$$

and noting a change in axes such that

$$x_H = \bar{z}_{HS}$$

$$z_H = \bar{x}_{HS}, \qquad y_H = \bar{y}_{HS}$$

$$x_H = (\Delta z_f - \delta w_2 \, \Delta x_f)\, C + (- \Delta x_f - \Delta z_f \, \delta w_2)\, S$$

$$+ z_{WT} C + x_{WT} S + [(X_H + \delta w_2 Z_H)\, C + (Z_H - \delta w_2 X_H)\, S]\,\theta_f$$

$$+ [\, h + (h - h_{ea})\, C - Z_{ea} \, S]\,\theta_{WT} + h\,[- \delta w'_3 \, z'_T + \delta w'_1 \, x'_T]$$

$$y_H = h \, S \, z'_T - y_{TW} \, \delta w_1 \, z'_T + h \, C \, x'_T - y_{TW} \, \delta w_3 \, x'_T$$

$$+ \theta_{WT} \, (- h \, C \, \delta w'_1 - h \, S \, \delta w'_3) \qquad (31)$$

$$z_H = (\Delta x_f + \delta w_2 \, \Delta z_f)\, C + (\Delta z_f - \delta w_2 \, \Delta x_f)\, S$$

$$+ [(- Z_H + X_H \, \delta w_2)\, C + (X_H + Z_H \, \delta w_2)\, S]\,\theta_f$$

$$- x_{WT} C + z_{WT} S + [\, Z_{ea} \, C + (h_{ea} - h)\, S]\,\theta_{WT}$$

Again these agree with the formulation of Reference 1, with additional terms for the effect of fuselage displacement and rotation.

These results can be expressed in matrix notation as follows,

$$\alpha = c\, x_W + d\, x_f$$

where

$$\alpha = \begin{Bmatrix} x_h \\ y_h \\ z_h \\ \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix} \qquad x_W = \begin{Bmatrix} q_{W1} \\ q_{W2} \\ p_W \end{Bmatrix} \qquad x_f = \begin{Bmatrix} x_f \\ z_f \\ \theta_f \end{Bmatrix}$$

where

$$z_{WT} = y_{TW} \;\; q_{W1}$$

$$x_{WT} = y_{TW} \;\; q_{W2}$$

$$\theta_{WT} = p_W$$

and

$$z_{WT}' = \eta_W'(y_{TW})\; q_{W1}$$

$$x_{WT}' = \eta_W'(y_{TW})\; q_{W2}$$

The elements of the matrix C are given on page 127 of Reference 1. Assuming that δw_2 is a small angle as has been done in the development, some of the terms above can be simplified. The matrix d which expresses the contribution of the fuselage motion to the hub motion is

$$\Delta_f \begin{Bmatrix} x_H \\ y_H \\ z_H \\ \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix} = \begin{bmatrix} -\sin\delta_P & \cos\delta_P & X_H\cos\delta_P + Z_H\sin\delta_P \\ \cos\delta_P & \sin\delta_P & X_H\sin\delta_P - Z_H\cos\delta_P \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta x_f \\ \Delta z_f \\ \theta_f \end{Bmatrix}$$

Inertial Forces and Moments

The kinetic energy of the fuselage, pylon and wing can be expressed as

$$KE = \tfrac{1}{2} m_f (\dot{x}_f^2 + \dot{z}_f^2) + \tfrac{1}{2} I_f \dot{\theta}_f^2$$

$$+ \tfrac{1}{2} \int_{-y_T}^{y_T} m dy \, [(\dot{x}_f + \dot{x} - Z_{WL} \dot{\theta}_f)^2 + (\dot{z}_f + \dot{z} + X_{WL} \dot{\theta}_f)^2]$$

$$+ \tfrac{1}{2} \int_{-y_T}^{y_T} I_W (\dot{\theta}_f + \dot{\theta}_W)^2 dy$$

$$+ 2 [\tfrac{1}{2} m_P [(\dot{x}_f + \dot{x}_T - Z_P \dot{\theta}_f)^2$$

$$+ (\dot{z}_f + \dot{z}_f + X_P \dot{\theta}_f + \Delta X_P \Delta\dot{\theta}_T)^2]$$

$$\tfrac{1}{2} m_P \Delta X_P^2 (\Delta\dot{\psi}_{T/R})^2 + \tfrac{1}{2} m_P \Delta X_P^2 (\Delta\dot{\psi}_{T/L})^2$$

$$2 [\tfrac{1}{2} I_{Py} (\dot{\theta}_f + \Delta\dot{\theta}_T)^2] + \tfrac{1}{2} I_{PX} (\Delta\dot{\psi}_{T/R})^2$$

$$+ \tfrac{1}{2} I_{PX} (\Delta\dot{\psi}_{T/L})^2$$

The influence of the left and right wings have been included. It is assumed that the only wing motion is symmetrical such that

$$\eta_L = \eta_R$$

$$\eta_L' = - \eta_R'$$

$$p_L = p_R$$

and

$$\delta w_{1L} = - \delta w_{1R}$$

$$\delta w_{2L} = \delta w_{2R}$$

$$\delta w_{3L} = - \delta w_{3R}$$

It can be seen from Equation (16) that

$$\Delta\dot{\theta}_{T/L} = \Delta\dot{\theta}_{T/R}$$

and that

$$\Delta\dot{\psi}_{T/L} = -\Delta\dot{\psi}_{T/R}$$

Therefore as can be seen from the kinetic energy expression, the effect of both wings is simply to double the pylon and wing terms.

The generalized coordinates to be used are the motion of the fuselage measured at its center of gravity x_f, z_f, θ_f and the wing deflection modes in two directions q_{W1} and q_{W2} and the wing torsion p_W. These quantities are related to wing deflection by

$$z_W = \eta_W \; q_{W1}$$

$$x_W = \eta_W \; q_{W2}$$

$$\theta_W = \xi_W \; p_W$$

where the mode shapes η_W and ξ_W have been normalized, such that, at the tip $\eta_{WT} = y_{TW}$ and $\xi_{WT} = 1$. The relationship between the wing deflection coordinates x and z are given by Equation (8) and the relationships between pylon angular motion $\Delta\theta_T$, $\Delta\psi_T$ and $\Delta\phi_T$ and the wing deflection and torsion are given by Equation (16). Precisely speaking, the distances X_{WL}, Z_{WL} and X_P and Z_P are functions of the coordinates, that is the wing deflection, however, they can be considered constant, equal to their trimmed flight value in the following development since the variation in these terms will not contribute any linear terms in the final development. For reference the time derivatives of Equation (8) and (16) are repeated here.

$$\dot{x} = -\eta_W \; \dot{q}_{W2} \; - \delta w_2 \, \eta_W \; \dot{q}_{W1}$$

$$\dot{z} = -\eta_W \; \dot{q}_{W2} \, \delta w_2 + \eta_W \; \dot{q}_{W1} \qquad (8)$$

y is assumed to be constant and does not contribute to the kinetic energy

$\dot{x}_T$ and $\dot{z}_T$ are found by replacing η_W by y_{TW} in the above.

The rotation rates are

$$\Delta\dot{\phi}_T = -\eta'_{WT}\ \dot{q}_{W1} - \delta w_3^*\ \xi_{WT}\ \dot{p}_W + \delta w_2\ \eta'_{WT}\ \dot{q}_{W2}$$

$$\Delta\dot{\theta}_T = \delta w_3^*\ \eta'_{WT}\ \dot{q}_{W1} + \xi_{WT}\ \dot{p}_W + \delta w_1^*\ \eta'_{WT}\ \dot{q}_{W2}$$

$$\Delta\dot{\psi}_T = \delta w_2\ \eta'_{WT}\ \dot{q}_{W1} + \delta w_1^*\ \xi_{WT}\ \dot{p}_W - \eta'_{WT}\ \dot{q}_{W2}$$

and

$$\dot{\theta}_W = \xi_W\ \dot{p}_W$$

The terms of the equations of motion are now evaluated by calculating

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{x}_f}\right), \text{ etc.}$$

The resulting terms in the equations of motion are listed on the following pages in matrix form. The following definitions are employed

$(\hat{\ })$ = quantity normalized by total aircraft mass, M

$(\ )^*$ quantity normalized by $\frac{N}{2}\ I_b$

Therefore,

$$\hat{m}_{fwP} = \frac{m_f + 2m_P + 2m_P}{M}$$

$$\hat{m}_{PZ} = \frac{m_P\ Z_P}{M}$$

$$\hat{m}_{WZ} = \frac{\int_0^{y_{TW}} m\ Z_{WL}\ dy_W}{M}$$

$$\hat{m}_{PX} = \frac{m_P\ X_P}{M}$$

$$\hat{m}_{WX} = \frac{\int^{y_{TW}} m\ X_{WL}\ dy_W}{M}$$

$$\hat{m}_{wx} = \frac{\int^{y_{TW}} m\, X_{WL}\, dy_W}{M}$$

$$m_P^* = \frac{m_P\, y_{TW}^2}{\frac{N}{2} I_b}$$

$$m_{qW}^* = \frac{\left(\int m\eta\, dy\right) y_{TW}}{\frac{N}{2} I_b}$$

$$S_W^* = m_P^* \frac{Z_{P_{ea}}}{y_{TW}}$$

$$\hat{I}_f = \frac{I_f}{M\, y_{TW}^2}$$

$$\hat{I}_{Pf} = \frac{I_{Py} + m_P\,(X_P^2 + Z_P^2)}{M\, y_{TW}^2}$$

$$\hat{I}_{Wf} = \frac{\int I_W\, dy_W + \int m\,(X_{WL}^2 + Z_{WL}^2)\, dy_W}{M\, y_{TW}^2}$$

$$I_{Wqz}^* = \frac{\int m\eta\, Z_{WL}\, dy_W}{\frac{N}{2} I_b}$$

$$I_{Wqx}^* = \frac{\int m\eta\, X_{WL}\, dy_W}{\frac{N}{2} I_b}$$

$$I_{W\zeta}^* = \frac{\int I_W\, \xi_W\, dy_W}{\frac{N}{2} I_b}$$

$$I_{Py}^* = \frac{I'_{Py} + m_P\, \Delta X_P^2}{\frac{N}{2} I_b}$$

$$I^*_{qW} = \frac{\int m\eta_W^2 \, dy_W}{\frac{N}{2} I_b}$$

$$I^*_{PX} = \frac{I'_{PX} + m_P \, \Delta X_P^2}{\frac{N}{2} I_b}$$

$$I^*_{PW} = \frac{\int m \, \xi_W^2 \, dy_W}{\frac{N}{2} I_b}$$

$$M^* = \frac{M \, y_{TW}^2}{\frac{N}{2} I_b}$$

The fuselage X and Z force equations have been divided by the quantity $M\Omega^2 R$ where M is the aircraft mass. The pitching moment equation has been divided by $M \, y_{TW}^2$ and the wing equations have been divided by $\frac{N}{2} I_b$.

The resulting equations of motion for the wing, body are of the form

$$D_2 \ddot{x}_v + D_1 \dot{x}_v + D_o x_v = E v_W + E_g g$$

where

$$x_v = \begin{Bmatrix} x_B \\ x_W \end{Bmatrix} = \begin{Bmatrix} x_f \\ z_f \\ \theta_f \\ q_{W1} \\ q_{W2} \\ p_W \end{Bmatrix}.$$

where D_2 has been developed above. Note that in terms of the Nomenclature of Reference 1,

$$D_2 = \left[\begin{array}{c|c} d_{11} & d_{12} \\ \hline d_{21} & a_2 \end{array}\right]$$

where a_2 was developed in Reference 1. D_2 is given on the accompanying page. The following section develops the aerodynamic force and moment contributions to the matrices D_1 and D_0.

In addition, the influence of the rotor forces and moments must be added to the body equations. The final equations including effects of wing, fuselage and rotor are

$$D_2\, \ddot{x}_v + D_1\, \dot{x}_v + D_0\, x_v = E v_W + E_g g + H_1\, F + H_2\, x_W$$

where

$$F = \left\{ \begin{array}{c} \frac{C_T}{a\sigma} \\ \frac{2C_H}{a\sigma} \\ -\frac{2C_y}{a\sigma} \\ \frac{C_Q}{a\sigma} \\ \frac{2C_{my}}{a\sigma} \\ -\frac{2C_{mx}}{a\sigma} \end{array} \right\} \quad \text{and } H_1 - \left\{ \frac{G_1}{\tilde{\alpha}} \right\} \qquad H_2 = \left\{ \frac{G_2'}{0} \right\}$$

$\tilde{\alpha}$ is developed in Reference 1 and G_1 and G_2' are developed in a following section.

$D_2 =$

$\hat{m}_{fwp}$		$-2\hat{m}_{wz} - 2\hat{m}_{pz}$	$-2\delta_{w2}\frac{m_p^*}{M^*} - 2\delta_{w2}\frac{m_{qw}^*}{M^*}$	$-2\frac{m_{qw}^*}{M^*} - 2\frac{m_p^*}{M^*}$	
	$\hat{m}_{fwp}$	$2\hat{m}_{wx} + 2\hat{m}_{px}$	$2\frac{m_{qw}^*}{M^*} + 2\frac{m_p^*}{M^*} - 2\eta'\frac{\delta_{w3}S_w^*}{M^*}$	$-2\delta_{w2}\frac{m_p^*}{M^*} - 2\delta_{w2}\frac{m_{qw}^*}{M^*} + 2\delta_{w1}\eta'\frac{S_w^*}{M^*}$	$2\frac{S_w^*}{M^*}$
$-2\hat{m}_{pz} - 2\hat{m}_{wz}$	$2\hat{m}_{px} + 2\hat{m}_{wx}$	$\hat{I}_f + 2\hat{I}_{ff} + 2\hat{I}_{wf}$	$2\frac{I_{wqx}^*}{M^*} + 2\delta_{w2}\frac{I_{wqz}^*}{M^*} + 2\frac{m_p^*}{M^*}\left(\frac{X_p + \delta_{w2}Z_p}{Y_{Tw}}\right) - 2\delta_{w3}\eta'\frac{S_w^*}{M^*}\frac{X_p}{Y_{Tw}}$	$2\frac{I_{wqz}^*}{M^*} - 2\delta_{w2}\frac{I_{qwx}^*}{M^*} + 2\frac{m_p^*}{M^*}\left(\frac{Z_p - \delta_{w2}X_p}{Y_{Tw}}\right) + 2\delta_{w1}\eta'\frac{S_w^*}{M^*}\frac{X_p}{Y_{Tw}}$	$2\frac{I_{wy}^*}{M^*} + 2\frac{I_{py}^*}{M^*} + 2\frac{S_w^*}{M^*}\frac{X_{wT}}{Y_{Tw}}$
$-\delta_{w2}m_p^* - \delta_{w2}m_{qw}^*$	$m_p^* + m_{qw}^* - \delta_{w3}S_w^*\eta'$	$I_{wqx}^* + \delta_{w2}I_{wqz}^* + m_p^*\left(\frac{X_p + \delta_{w2}Z_p}{Y_{Tw}}\right) - S_w^*\eta'\delta_{w3}\frac{X_p}{Y_{Tw}}$	$I_{qw}^* + m_p^* - 2S_w^*\eta'\delta_{w3}$	$\delta_{w1}\eta' S_w^* + \eta'^2\frac{S_w^{*2}}{m_p^*}\delta_{w2}$	$S_w^* - \delta_{w3}\eta'\frac{S_w^{*2}}{m_p^*}$
$-m_p^* - m_{qw}^*$	$-\delta_{w2}m_p^* - \delta_{w2}m_{qw}^* + \delta_{w1}\eta' S_w^*$	$I_{wqz}^* - \delta_{w2}I_{wqx}^* + m_p^*\left(\frac{Z_p - \delta_{w2}X_p}{Y_{Tw}}\right) + S_w^*\frac{X_p}{Y_{Tw}}\eta'\delta_{w1}$	$\eta'\delta_{w1}S_w^* + \eta'^2\delta_{w2}\frac{S_w^{*2}}{m_p}$	$I_{qw}^* + m_p^* + \eta'^2 I_{px}^*$	$-\delta_{w2}S_w^*$
	S_w^*	$I_{wy}^* + I_{py}^* + S_w^*\frac{X_{wT}}{Y_{Tw}}$	$S_w^* - \delta_{w3}\eta'\frac{S_w^{*2}}{m_p}$	$-\delta_{w2}S_w^*$	$I_{py}^* + I_{pw}^*$

Wing Aerodynamic Forces and Moments

This section develops the wing forces and moments to be added to the wing and fuselage equations of motion. The formulation is complicated by the fact that the wing has a sweep angle δw_3. The geometry of the wing is shown in Figure 4. The lift, drag and pitching moment on the wing section can be expressed as

$$\bar{\ell} = \frac{1}{2} \rho V_N^2 \; \bar{c} \, a \, \bar{\alpha}$$

$$\bar{d} = \frac{1}{2} \rho V_N^2 \; \bar{c} \, (C_{D0} + C_{D\alpha} \bar{\alpha})$$

$$\bar{m}_{ea} = \frac{1}{2} \rho V_N^2 \; \bar{c}^2 \, (C_{mac} - \frac{x_{AW}}{c_W} C_L)$$

The total lift, drag and pitching moment acting on the wing is

$$L = \int \bar{\ell} \, d\bar{y}$$

$$D = \int \bar{d} \, d\bar{y}$$

$$M = \int \bar{m}_{ea} \, d\bar{y}$$

The normal and chordwise forces, $\bar{n}$ and $\bar{c}$ are

$$\bar{n} \cong \bar{\ell}$$

$$\bar{c} = \bar{d} - \bar{\ell} \bar{\alpha}$$

The exciting forces and moments for the wing bending and torsion equations are

$$N = \int \bar{\ell} \, \eta \, d\bar{y}$$

$$C = \int (\bar{d} - \bar{\ell} \bar{\alpha}) \, \eta \, d\bar{y}$$

$$M = \int \bar{m}_{ea} \, \xi \, d\bar{y}$$

Now, noting from Figure 4 that

$$\bar{c} = c \cos \Lambda$$

$$d\bar{y} = \frac{dy}{\cos \Lambda}$$

We can write

$$L = \frac{1}{2} \rho\, c_W a_W \int \bar{V}_N^2\, \bar{\alpha}\, dy$$

$$D = \frac{1}{2} \rho\, c_W \int V_N^2 \left(C_{D0} + C_{D\alpha}\, \bar{\alpha}\right) dy$$

$$M = \frac{1}{2} \rho\, c_W^2 \cos \Lambda \int V_N^2 \left(C_{mac} - \frac{x_{aW}}{c_W}\, C_L\right) dy$$

and

$$N = \frac{1}{2} \rho\, c_W a_W \int V_N^2\, \bar{\alpha}\, \eta\, dy$$

$$C = \frac{1}{2} \rho\, c_W \int V_N^2 \left(C_{D0} - (C_L - C_{D\alpha})\, \bar{\alpha}\right) \eta\, dy$$

$$M = \frac{1}{2} \rho\, c^2 \cos \Lambda \int V_N^2 \left(C_{mac} - \frac{x_{aW}}{c_W}\, C_L\right) \xi\, dy$$

These last three expressions are nondimensionalized by dividing by $\gamma \frac{N}{2} I_b$, which yields

$$M_{qW1} = \phi_{12}\, a_W \int V_N^2\, \bar{\alpha}\, \frac{\eta\, dy}{y_{TW}^2}$$

$$M_{qW2} = \phi_{12} \int V_N^2 \left(C_{D0} - (C_L - C_{D\alpha})\, \bar{\alpha}\right) \frac{\eta\, dy}{y_{TW}^2}$$

$$M_P = \phi_{21} \cos \Lambda \int V_N^2 \left(C_{mac} - \frac{x_{aW}}{c_W}\, C_L\right) \xi\, \frac{dy}{y_{TW}}$$

where

$$\phi_{nm} = \frac{c_W^n\, y_{TW}^m}{\pi\, \sigma\, a}$$

Note that c_W and y_{TW} have been nondimensionalized by the rotor radius R, similarly L, D, and M are normalized by $M\Omega^2 R$ and $M\, y_{TF}^2\, \Omega^2$ to yield L^*, D^*, and M^*.

$$L^* = \frac{\gamma}{M^*}\phi_{13}\ a_W \int V_N^2\, \bar{\alpha}\, \frac{dy}{y_{TW}}$$

$$D^* = \frac{\gamma}{M^*}\phi_{13} \int V_N^2\, (C_{D0} + C_{D\alpha}\bar{\alpha})\, \frac{dy}{y_{TW}}$$

$$M^* = \frac{\gamma}{M^*}\phi_{21}\ \cos\Lambda \int V_N^2\, \left(C_{mac} - \frac{x_{aW}}{c_W} C_L\right) \frac{dy}{y_{TW}}$$

From the geometry of the figure, assuming that the inflow angle, ϕ_W, is small, the velocity normal to the wing leading edge is

$$V_N = [(V_0 + \dot{x}_f)\cos\Lambda - (\dot{x}_W + \dot{z}_W\,\delta_{W2}) - Z_{WL}\,\dot{\theta}_f]$$

and the angle of attack of the section is

$$\bar{\alpha} = \theta_W + \frac{V_T\ \sin\phi_W - [\dot{z}_f + \dot{z}_W - \dot{x}_W\,\delta_{W2}] - X_{WL}\,\dot{\theta}_f}{V_N}$$

where

$$\Lambda = \delta_{W3} + x'$$

$$\phi_W = (\delta_{W2} + \theta_f)\sin\Lambda - z'$$

$$\theta_W = (\delta_{W2} + \theta_f)\cos\Lambda + \xi_W\, p$$

$$V_T = (V_0 + \dot{x}_f)\sin\Lambda$$

Since many of these quantities are perturbation quantities we may express the angle of attack as, retaining only linear terms

$$\bar{\alpha}' = \frac{\delta_{W2} + \theta_f}{\cos \Lambda} + \xi_W p - z'\delta_{W3} - \{\frac{\dot{z}_f + \dot{z}_W - \dot{x}_W \delta_{W2} - X_{WL}\dot{\theta}_f}{V_0}\},$$

There are in addition, effects of angular rates, $\dot{\alpha}$ and $\dot{\theta}_f$ accounted for with the following increments to C_ℓ and C_m

$$\Delta C_\ell = \frac{a_W\, c_W}{V_0}\ [\frac{3}{4} + \frac{x_{aW}}{c_W}]\ (\xi_W \dot{p} + \frac{\dot{\theta}_f}{\cos \Lambda})$$

and

$$\Delta C_m = \frac{a_W\, c_W}{V_N}\ [-\frac{1}{8} - \frac{3}{4}\frac{x_{aW}}{c_W} - (\frac{x_{aW}}{c_W})^2]\ [\xi_W \dot{p} + \frac{\dot{\theta}_f}{\cos \Lambda}]$$

We can now determine the terms in the six equations of motion by taking derivatives of the above expressions. Denoting the derivatives by

$$C_{q1\,\dot{x}_f} = \frac{\partial M_{q1}}{\partial \dot{x}_f}$$

The following expressions result:

$$C_{q1\,\dot{x}_f} = \phi_{12}\ 2C_{Lo}\, V_0\ e_1 = C_{q_1 u_g}$$

$$C_{q1\,\dot{z}_f} = -\phi_{12}\ V_0\ a_W\, e_1 = -C_{q_1 w_g}$$

$$C_{q1\,\dot{q}1} = \phi_{13}\ (-V_0(a_W + 2\frac{C_{Lo}^2}{a_W}))\ e_2$$

$$C_{q1\,\dot{q}2} = -\phi_{13}\ V_0\ C_{Lo}\ e_2$$

$$C_{q1\,q1} = -\phi_{12}\ V_0^2\ a_W\, \delta_{W3}\ e_3$$

$$C_{q1\,q2} = -\phi_{12}\ C_{Lo}\, V_0^2\ \delta_{W3}\ e_2$$

$$C_{q1\,\dot{p}} = \phi_{22}\ a_W\, [\frac{3}{4} + \frac{x_{aW}}{c_W}]\ V_0\ e_4$$

$$C_{q1\,p} = \phi_{12}\ V_0^2\ a_W\, e_4$$

$$C_{q_1 \theta_f} = \phi_{12} V_0^2 a_W e_1$$

$$C_{q_1 \dot{\theta}_f} = \phi_{22} V_0 \left(- 2C_{L0} \frac{Z_{WL}}{c_W} + a_W \left[\frac{X_{WL}}{c_W} + \frac{3}{4} + \frac{xa_W}{c}\right]\right)$$

The chordwise equation terms are:

$$C_{q_2 \dot{x}_f} = \phi_{12}\ 2 V_0 \ (C_{D0} - C_{L0} \delta_{W2}) e_1 = C_{q_2 u_g}$$

$$C_{q_2 \dot{z}_f} = \phi_{12} V_0 (C_{D\alpha} - 2C_{L0}) e_1 = - C_{q_2 w_g}$$

$$C_{q_2 \dot{q}_1} = - \phi_{13} V_0 (C_{D\alpha} - 2C_{L0}) e_2$$

$$C_{q_2 \dot{q}_2} = \phi_{13} V_0 (- 2 C_{D0} + C_{D\alpha} \delta_{W2}) e_2$$

$$C_{q_2 q_1} = - \phi_{12} V_0^2 (C_{D\alpha} - C_{L0}) \delta_{W3}\ e_3$$

$$C_{q_2 q_2} = - \phi_{12} V_0^2 \delta_{W3} [2C_{D\alpha} \delta_{W3}]$$

$$C_{q_2 \dot{p}} = \phi_{22} V \left[\left(\frac{1}{2} + \frac{xa_W}{c_W}\right) (C_{D\alpha} - 2C_{L0}) - \frac{1}{4} C_{L0}\right] e_4$$

$$C_{q_2 p} = \phi_{12} V_0^2 (C_{D\alpha} - 2C_{L0}) e_4$$

$$C_{q_2 \theta_f} = \phi_{12} V_0^2 (C_{D\alpha} - 2C_{L0}) e_1$$

$$C_{q_2 \dot{\theta}_f} = \phi_{22} V_0 \left[-2 \frac{Z_{WL}}{c_W} (C_{D0} - (C_{L0} - C_{D\alpha}) \delta_{W2}) + \frac{X_{WL}}{c_W} C_{L0} + (C_L - C_{D\alpha}) \left(\frac{X_{WL}}{c_W} - \frac{x_{aW}}{c_W} - \frac{1}{2}\right)\right] e_1$$

The torsion equation terms are:

$$C_{p\dot{x}_f} = \phi_{21}\ 2 V_0 \left(C_{mac} - \frac{x_{aW}}{c_W} C_{L0}\right) f_1 = C_{pu_g}$$

$$C_{p\dot{z}_f} = \phi_{21} V_0 \frac{xa_W}{c_W} a_W\ f_1 = - C_{pw_g}$$

$$C_{p\dot{q}_1} = \phi_{22}\ [V_0 \frac{x_{aw}}{c_w} a - 2V_0 \frac{C_{L0}}{a_w} (C_{mac} - \frac{x_{aw}}{c_w} C_{L0})]\ e_4$$

$$C_{p\dot{q}_2} = \phi_{22}\ [- 2V_0\ C_{mac} + V_0 \frac{x_{aw}}{c_w} C_{L0}]\ e_4$$

$$C_{p\dot{q}_2} = \phi_{12}\ V_0^2\ C_{mac}\ f_2 + \phi_{21}\ V_0^2\ a_w\ \delta_{w3} \frac{x_{aw}}{c_w} f_5$$

$$C_{pq_2} = - \phi_{12}\ V^2\ C_{L0}\ f_2 - \phi_{21}\ V_0^2\ \delta_{w3}\ [- 2C_{mac} + \frac{x_{aw}}{c_w} C_{L0}]\ f_5$$

$$C_{p\dot{p}} = - \phi_{31}\ (\frac{1}{8} + \frac{3}{4} \frac{x_{aw}}{c_w} + (\frac{x_{aw}}{2})^2\ )\ a_w\ f_3$$

$$C_{pp} = - \phi_{21}\ V_0^2 \frac{x_{aw}}{c_w}\ a_w\ f_3$$

$$C_{p\theta_f} = - \phi_{21}\ V_0^2 \frac{x_{aw}}{c_w} a \quad f_1$$

$$C_{p\dot{\theta}_f} = - \phi_{31}\ (\frac{1}{8} + \frac{3}{4} \frac{x_{aw}}{c_w} + (\frac{x_{aw}}{c_w})^2)\ \ a_w\ f_1$$

These are the terms in the wing equations of motion.

For the body equations of motion, the lift terms will be similar to the normal force terms applied to the wing with the mode shape not present in the integral. Following a similar notation

$$C_{z\dot{x}_f} = \phi_{12}\ 2C_{L0}\ V_0 = C_{zu_g}$$

$$C_{z\dot{z}_f} = - \phi_{12}\ V_0\ a_w = - C_{zw_g}$$

$$C_{zq_1} = \phi_{13}\ (- V_0\ (a_w + 2 \frac{C_{L0}^2}{a_w}))\ e_1$$

$$C_{z\dot{q}_2} = - \phi_{13} V_0 C_{L0} e_1$$

$$C_{zq_1} = - \phi_{12} V_0^2 a_W \delta_{W3}$$

$$C_{zq_2} = - \phi_{12} C_{L0} V_0^2 \delta_{W3} e_1$$

$$C_{z\dot{p}} = \phi_{22} a_W [\frac{3}{4} + \frac{x_{aW}}{c_W}] V_0 f_1$$

$$C_{z\theta_f} = \phi_{12} V_0^2 a_W$$

$$C_{z\dot{\theta}_f} = \phi_{22} V_0 [- 2C_{L0} \frac{Z_{WL}}{c_W} + a_W (\frac{X_{WL}}{c_W} + \frac{3}{4} + \frac{x_{aW}}{c_W})]$$

The terms in the x force equation do not involve quite the same terms as the chordwise wing terms since the resolution is along the fuselage axes which are initially aligned with the wind.

$$C_{x\dot{x}_f} = \phi_{12} 2 V_0 C_{D0} = C_{xu_g}$$

$$C_{x\dot{z}_f} = \phi_{12} V_0 (C_{D\alpha} - C_{L0}) = - C_{xw_g}$$

$$Cx\dot{q}_1 = - \phi_{13} V_0 (C_{D\alpha} - C_{L0} + 2C_{D0} \delta_{W2}) e_1$$

$$C_{x\dot{q}_2} = \phi_{13} V_0 [- 2C_{D0} + \delta_{W2} (C_{D\alpha} - C_{L0})] e_1$$

$$C_{xq_1} = \phi_{12} V_0 (C_{L0} - C_{D\alpha}) \delta_{W3}$$

$$C_{xq_2} = - \phi_{12} V_0^2 \delta_{W3} (2C_{D0} - (C_{D\alpha} - C_{L0}) \delta_{W2})$$

$$C_{x\dot{p}} = \phi_{22}\ V_0\ [(C_{D\alpha} - C_{L0})\ (x_{aW} + \tfrac{1}{2} c_W)]\ f_1$$

$$C_{xp} = \phi_{12}\ V_0^2\ (C_{D\alpha} - C_{L0})\ f_1$$

$$C_{x\theta_f} = \phi_{12}\ V_0^2\ (C_{D\alpha} - C_{L0})$$

$$C_{x\dot{\theta}_f} = \phi_{22}\ V_0\ [-\ 2\ \frac{Z_{WL}}{c_W} C_{D0} + (c_{L0} - C_{D\alpha})(\frac{X_{WL}}{c_W} - \frac{x_{aW}}{c_W} - \tfrac{1}{2})]$$

The pitching moment about the fuselage center of gravity is

$$M_{CG} = M_{ea} + L\ X_{WL} + D\ Z_{WL}$$

The nondimensionalized coefficients are

$$C_{m()} = C_{m_{ea()}} + C_{z()} \frac{X_{WL}}{c_W} + C_{x()} \frac{Z_{WL}}{c_W}$$

that is

$$C_{m\dot{x}_f} = \phi_{21}\ 2\ V_0\ (C_{mac} - \frac{x_{aW}}{c_W} C_{L0}) + C_{z\dot{x}_f} \frac{X_{WL}}{c_W} + C_{x\dot{x}_f} \frac{Z_{WL}}{c_W} = C_{mu_g}$$

$$C_{m\dot{z}_f} = \phi_{22}\ V_0\ \frac{x_{aW}}{c_W} a_W + C_{z\dot{z}_f} \frac{X_{WL}}{c_W} + C_{x\dot{x}_f} \frac{Z_{WL}}{c_W} = -\ C_{mw_g}$$

$$C_{m\dot{q}_1} = \phi_{22}\ [V_0\ \frac{x_{aW}}{c} a_W - 2V_0\ \frac{C_{L0}}{a_W} (C_{mac} - \frac{x_{aW}}{c_W} C_{L0})\ e_1$$

$$+\ C_{z\dot{q}_1} \frac{X_{WL}}{c_W} + C_{x\dot{q}_1} \frac{Z_{WL}}{c_W}$$

$$C_{mq_2} = \phi_{22}\ [-\ 2V_0\ C_{mac} + V_0\ \frac{x_{aw}}{c_w} C_{L0}]\ e_1 + C_{z\dot{q}_2} \frac{X_{WL}}{c_w} + C_{x\dot{q}_1}\ \frac{Z_{WL}}{c_W}$$

$$C_{mq_1} = \phi_{12}\ V_0\ C_{mac}\ f_6 + \phi_{21}\ V_0^2\ a_w\ \delta_{w3}\ \frac{x_{aw}}{c_w} + C_{zq_1} \frac{X_{WL}}{c_w} + C_{xq_1}\ \frac{Z_{WL}}{c_w}$$

$$C_{mq_2} = -\ \phi_{12}\ V_0^2\ C_{L0}\ f_6\ -\ \phi_{21}\ V_0^2\ \delta_{w3}\ [-\ 2C_{mac} + \frac{x_{aw}}{c_w} C_{L0}]$$

$$+ C_{zq_2}\ \frac{X_{WL}}{c_w} + C_{xq_2}\ \frac{Z_{WL}}{c_w}$$

$$C_{m\dot{p}} = -\ \phi_{31}\ (\frac{1}{8} + \frac{3}{4}\frac{x_{aw}}{c_w} + (\frac{x_{aw}}{c_w})^2)\ a_w\ f_1 + C_{z\dot{p}} \frac{X_{WL}}{c_w} + C_{x\dot{p}} \frac{Z_{WL}}{c_L}$$

$$C_{mp} = -\ \phi_{21}\ V_0^2\ \frac{x_{aw}}{c_w}\ a_w\ f_1 + C_{zp} \frac{X_{WL}}{c_w} + C_{xp} \frac{Z_{WL}}{c_w}$$

$$C_{m\theta_f} = -\ \phi_{21}\ V_0^2\ \frac{x_{aw}}{c_w}\ a_w + C_{z\theta_f} \frac{X_{WL}}{c_w} + C_{x\theta_f} \frac{Z_{WL}}{c_w}$$

$$C_{m\dot{\theta}_f} == -\ \phi_{31}\ (\frac{1}{8} + \frac{3}{4}\frac{x_{aw}}{c_w} + (\frac{x_{aw}}{c_w})^2)\ a_w + C_{z\dot{\theta}_f} \frac{X_{WL}}{c_w} + C_{x\dot{\theta}_f} \frac{Z_{WL}}{c_w}$$

The various integrals involved in these equations are:

$$e_1 = \int_0^{y_{Tw}} \eta_w \frac{dy_w}{y_{Tw}^2} \cong \frac{1}{3}$$

$$e_3 = \int_0^{y_{Tw}} \eta_w^2 \frac{dy_w}{y_{Tw}^2} \cong \frac{1}{5}$$

$$e_3 = \int_0^{y_{TW}} \eta_W \, \eta_W' \, \frac{dy_W}{y_{TW}^2} \cong \frac{1}{2}$$

$$e_4 = \int_0^{y_{TW}} \eta_W \, \xi_W \, \frac{dy_W}{y_{TW}} \cong \frac{1}{4}$$

$$e_5 = \int_{y_{F1}}^{y_{F0}} \eta_W \, \frac{dy_W}{y_{TW}} \cong \frac{1}{3} \left[\left(\frac{y_{F0}}{y_{TW}}\right)^3 - \left(\frac{y_{F1}}{y_{TW}}\right)^2 \right]$$

$$f_1 = \int_0^{y_{TW}} \xi_W \, \frac{dy_W}{y_{TW}} \cong \frac{1}{2}$$

$$f_2 = \int_0^{y_{TW}} \xi_W \, \eta_W'' \, \frac{1}{2} \, (y_{TW} - y_W)^2 \, \frac{dy_W}{y_{TW}} \cong \frac{1}{12}$$

$$f_3 = \int_0^{y_{TW}} \xi_W^2 \, \frac{dy_W}{y_{TW}} \cong \frac{1}{3}$$

$$f_4 = \int_{y_{F1}}^{y_{F0}} \xi_W \, \frac{dy_W}{y_{TW}} \cong \frac{1}{2} \left[\left(\frac{y_{F0}}{y_{TW}}\right)^2 - \left(\frac{y_{F1}}{y_{TW}}\right)^2 \right]$$

$$f_5 = \int_0^{y_{TW}} \xi \, \eta' \, \frac{dy}{y_{TW}} = \frac{2}{3}$$

$$f_6 = \int_0^{y_{TW}} \eta'' \, \left(\frac{1}{2}\right) \, (y_{TW} - y_W)^2 \, \frac{dy_W}{y_{TW}^2} = \frac{1}{3} \, .$$

and

$$\phi_{nm} = \frac{c_W^n \, y_{TW}^m}{\pi \, \sigma \, a}$$

The control terms for the wing body equations of motion at this point involve only the effect of flap delfection.

The wing equation terms were given in Reference 1 as

$$C_{q_1\delta} = \phi_{12} V_0^2 C_{L\alpha} C^*_{L\delta} e_5$$

$$C_{q_2\delta} = \phi_{12} V_0^2 (C_{d\delta} + C_{D\alpha} - C_{L0}) C^*_{L\delta}) e_5$$

$$C_{p\delta} = -\phi_{21} V_0^2 (\frac{x_{aW}}{c_W} C^*_{\ell\delta} - C^*_{m\delta}) C_{L\alpha} f_4$$

The terms in the body equations of motion are

$$C_{z\delta} = \phi_{12} V_0^2 C_{L\alpha} C^*_{L\delta} (\frac{y_{F0}}{y_{TW}} - \frac{y_{F1}}{y_{TW}})$$

$$C_{x\delta} = \phi_{12} V_0^2 C_{D\delta} (\frac{y_{F0}}{y_{TW}} - \frac{y_{F1}}{y_{TW}})$$

$$C_{m\delta} = -\phi_{21} V_0^2 (\frac{x_{aW}}{c_W} C^*_{L\delta} - C^*_{m\delta}) C_{L\alpha} (\frac{y_{F0}}{y_{TW}} - \frac{y_{F1}}{y_{TW}})$$

$$+ C_{z\delta} \frac{X_{WL}}{c_W} + C_{x\delta} \frac{Z_{WL}}{c_W}$$

Thus the control matrix is

$$E_g = \left\{ \begin{array}{c} \gamma C_{x\delta} \\ \gamma C_{z\delta} \\ \gamma C_{m\delta} \\ \gamma C_{q_1\delta} \\ \gamma C_{q_2\delta} \\ \gamma C_{p\delta} \end{array} \right\}$$

The input matrices E and E_g are modified as follows. Since only longitudinal motion is being considered, lateral gust terms are not included.

$$E_g = \begin{bmatrix} h_g \\ \hline b_g \end{bmatrix}$$

This matrix is given on the accompanying pages.

Because of the normalization procedure, in the wing equation the derivatives appear multiplied by γ and in the body equations by $\frac{\gamma}{M^*}$. Since there are two wings all of the wing terms in the body equations appear multiplied by 2. This completes the development of the aerodynamic contributions of the matrices D_1 and D_0 from the wing. The wing stiffness terms carry over directly from Reference 1 along with the structural damping and the influence of rotor thrust.

To be added to these matrices are the influence of the horizontal tail, fuselage, gravity forces.

$D_1 =$

$-2\frac{\gamma}{M^*} C_{x\dot{x}_f}$	$-2\frac{\gamma}{M^*} C_{x\dot{z}_f}$	$-2\frac{\gamma}{M^*} C_{x\dot{\theta}_f}$	$-2\frac{\gamma}{M^*} C_{x\dot{q}_1}$	$-2\frac{\gamma}{M^*} C_{x\dot{q}_2}$	$-2\frac{\gamma}{M^*} C_{x\dot{p}}$
$-2\frac{\gamma}{M^*} C_{z\dot{x}_f}$	$-2\frac{\gamma}{M^*} C_{z\dot{z}_f}$	$-2\frac{\gamma}{M^*} C_{z\dot{\theta}_f}$	$-2\frac{\gamma}{M^*} C_{z\dot{q}_1}$	$-2\frac{\gamma}{M^*} C_{z\dot{q}_2}$	$-2\frac{\gamma}{M^*} C_{z\dot{p}}$
$-2\frac{\gamma}{M^*} C_{M\dot{x}_f}$	$-2\frac{\gamma}{M^*} C_{M\dot{z}_f}$	$-2\frac{\gamma}{M^*} C_{M\dot{\theta}_f}$	$-2\frac{\gamma}{M^*} C_{M\dot{q}_1}$	$-2\frac{\gamma}{M^*} C_{M\dot{q}_2}$	$-2\frac{\gamma}{M^*} C_{M\dot{p}}$
$-\gamma C_{q_1\dot{x}_f}$	$-\gamma C_{q_1\dot{z}_f}$	$-\gamma C_{q_1\dot{\theta}_f}$	$C_{q_1}^*$ $-\gamma C_{q_1\dot{q}_1}$	$-\gamma C_{q_1\dot{q}_2}$	$-\gamma C_{q_1\dot{p}}$
$-\gamma C_{q_2\dot{x}_f}$	$-\gamma C_{q_2\dot{z}_f}$	$-\gamma C_{q_2\dot{\theta}_f}$	$-\gamma C_{q_2\dot{q}_1}$	$C_{q_2}^*$ $-\gamma C_{q_2\dot{q}_2}$	$-\gamma C_{q_2\dot{p}}$
$-\gamma C_{p\dot{x}_f}$	$-\gamma C_{p\dot{z}_f}$	$-\gamma C_{p\dot{\theta}_f}$	$-\gamma C_{p\dot{q}_1}$	$-\gamma C_{p\dot{q}_2}$	C_p^* $-\gamma C_{p\dot{p}}$

$D_0 =$

		$-2\frac{\gamma}{M^*} C_{x\theta_f}$	$-2\frac{\gamma}{M^*} C_{xq_1}$	$-2\frac{\gamma}{M^*} C_{xq_2}$	$-2\frac{\gamma}{M^*} C_{xp}$
		$-2\frac{\gamma}{M^*} C_{z\theta_f}$	$-2\frac{\gamma}{M^*} C_{zq_1}$	$-2\frac{\gamma}{M^*} C_{zq_2}$	$-2\frac{\gamma}{M^*} C_{zp}$
		$-2\frac{\gamma}{M^*} C_{m\theta_f}$	$-2\frac{\gamma}{M^*} C_{mq_1}$	$-2\frac{\gamma}{M^*} C_{mq_2}$	$-2\frac{\gamma}{M^*} C_{mp}$
		$-\gamma C_{q_1\theta_f}$	$k^*_{q_1}$ $-\gamma C_{q_1q_1}$	$-\gamma C_{q_1q_2}$	$-\gamma C_{q_1p}$
		$-\gamma C_{q_2\theta_f}$	$-\gamma C_{q_2q_1}$	$k^*_{q_2}$ $-\gamma C_{q_2q_2}$	$-\gamma C_{q_2p}$
		$-\gamma C_{p\theta_f}$	$-\gamma C_{pq_1}$ $C^*_{pq}\, y\, \gamma\, \frac{2G}{a\sigma}\, C$	$-\gamma C_{pq_2}$ $C^*_{pq}\, y\, \gamma\, \frac{2G}{a\sigma}\, S$	k^*_p $-\gamma C_{pp}$

$E_g =$

$2\frac{\gamma}{M^*}C_{xu_g}$		$2\frac{\gamma}{M^*}C_{xw_g}$
$2\frac{\gamma}{M^*}C_{zu_g}$		$2\frac{\gamma}{M^*}C_{zw_g}$
$2\frac{\gamma}{M^*}C_{mu_g}$		$2\frac{\gamma}{M^*}C_{mw_g}$
$\gamma C_{q_1 u}$	$\gamma C_{q_1 v}$	$\gamma C_{q_1 w}$
$\gamma C_{q_2 u}$	$\gamma C_{q_2 v}$	$\gamma C_{q_2 w}$
γC_{pu}	γC_{pv}	γC_{pw}

Rotor Force and Moment Contributions

The contributions of the rotor forces and moments to the body equations of motion are calculated. The rotor forces and moments are formulated in a shaft axis system and so the deflections of the wing tip must be included in the effect of the rotor forces on the body. Linear deflections of the tip are assumed to have a negligible effect, however, the influence of the rotation of the tip of the wing is included.

The rotation of the rotor shaft in terms of the torsional deflection of the wing tip and the bending slopes of the wing can be expressed as follows:

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix} = \begin{bmatrix} \eta' S & -\eta' C & \delta_1 C + \delta_3 S \\ -\eta' \delta_3 & \eta' \delta_1 & 1 \\ -\eta' C & -\eta' S & \delta_1 S - \delta_3 C \end{bmatrix} \begin{Bmatrix} q_{W1} \\ q_{W2} \\ p_W \end{Bmatrix}$$

or

$$\alpha_R = R x_W$$

where

$$C = \cos\,(\delta_{WP} - \delta_{W2})$$

$$S = \sin\,(\delta_{WP} - \delta_{W2})$$

This relationship is that given by Equation (30) without the influence of body attitude, since the forces and moments are being calculated in a body axis system.

Forces and moments in the undeflected system (the body axis system) are related to forces and moments in the shaft system which is rotated by α_x, α_y, and α_z by the following relationship

$$\begin{Bmatrix} X' \\ Y' \\ Z' \end{Bmatrix} = \begin{bmatrix} 1 & -\alpha_z & \alpha_y \\ \alpha_z & 1 & -\alpha_x \\ -\alpha_y & \alpha_x & 1 \end{bmatrix} \begin{Bmatrix} x_s \\ y_s \\ z_s \end{Bmatrix}$$

where $\begin{Bmatrix} x_s \\ y_s \\ z_s \end{Bmatrix}$ may be interpreted as $\begin{Bmatrix} H \\ Y \\ T \end{Bmatrix}$

or as

$$\begin{Bmatrix} M_x \\ M_y \\ -Q \end{Bmatrix}$$

The forces and moments relevant to the longitudinal dynamics are expressed in terms of the rotor forces in the deflected position as

$$X' = H - \alpha_z Y + \alpha_y T$$

$$Z' = -\alpha_y H + \alpha_x Y + T$$

$$M' = \alpha_z M_x + M_y + \alpha_x Q$$

where M_y is measured about the rotor hub. The forces are now resolved in the body axis directions, parallel and perpendicular to the initial velocity direction, and the moment is expressed about the fuselage center of gravity

$$X = Z' \cos \delta_p - X' \sin \delta_p$$

$$Z = Z' \sin \delta_p + X' \cos \delta_p$$

$$M' = M_y + X_H Z' - Z_H X'$$

Now perturbation equations are formulated for the variations in X, Z and M. For example

$$\delta X = \delta Z' \cos \delta_p - \delta X' \sin \delta_p$$

where

$$\delta Z' = -\delta\alpha_y H_0 - \alpha_{y_0} \delta H + \delta\alpha_x Y_0 + \alpha_{x_0} \delta Y + \delta T$$

and

$$\delta X' = \delta H - \delta\alpha_z Y_0 - \alpha_{z_0} \delta Y + \delta\alpha_y T_0 + \alpha_{y_0} \delta T$$

where $()_0$ indicates an equilibrium flight value. Proceeding with this development the force and moment perturbations applied to the vehicle by the rotor can be expressed as:

$$F_B = G_1 F + G_2 \alpha_R$$

where

$$F_B = \gamma \begin{Bmatrix} \frac{2C_x}{a\sigma} \\ \frac{2C_z}{a\sigma} \\ \frac{2C_M}{a\sigma} \end{Bmatrix} \qquad F = \gamma \begin{Bmatrix} \frac{C_T}{\sigma a} \\ \frac{2C_H}{a\sigma} \\ \frac{-2C_y}{a\sigma} \\ \frac{C_Q}{a\sigma} \\ \frac{2C_{my}}{a\sigma} \\ \frac{-2C_{Mx}}{a\sigma} \end{Bmatrix}$$

and $\alpha_R = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix}$

α_R is related to the wing deflection as given above

$$\alpha_R = R\ x_W$$

therefore

$$F_B = G_1\ F + G_2{}'x_W$$

when

$$G_2' = G_2\ R \quad \text{and } x_W = \begin{Bmatrix} q_{W1} \\ q_{W2} \\ p_W \end{Bmatrix}$$

The matrices G_1 and $G_2{}'$ are given on the accompanying pages. The following notation is used

$$C' = \cos\ \delta_P$$

$$S' = \sin\ \delta_P$$

and the subscript zero is dropped with the understanding that all matrix elements are evaluated at the trim flight condition. Both rotors are accounted for in the matrix elements.

$G_1 =$

$4C' - 4\alpha_y S'$	$-2\alpha_y C' - 2S'$	$2\alpha_x C' - 2\alpha_z S'$	0	0	0
$4S' + 4\alpha_y C'$	$-2\alpha_y S' + C'$	$2\alpha_x S' + 2\alpha_z C'$	0	0	0
$4 X_H(S' + \alpha_y C') - 4 Z_H(C' - \alpha_y S')$	$2 X_H(-\alpha_y S' + C') - 2 Z_H(-\alpha_y C' - S')$	$2 X_H(\alpha_x S' + \alpha_z C') + 2 Z_H(-\alpha_x C' + \alpha_z S')$	$-2\alpha_x$	2	$-2\alpha_z$

$G_2^1 =$

$2\left(\frac{2C_Y}{a\sigma}\right)\eta'(c's - cs') + 2\eta'\delta_3\left(\frac{2C_H}{a\sigma}c' + \frac{2C_T}{a\sigma}s'\right)$	$-2\left(\frac{2C_Y}{a\sigma}\right)(cc'+ss') - 2\eta'\delta_1\left(\frac{2C_H}{a\sigma}c' + \frac{2C_T}{a\sigma}s'\right)$	$2\left(\frac{2C_Y}{a\sigma}\right)\left(\delta_1(cc'+ss') + \delta_3(c's-s'c)\right) - 2\left[\frac{2C_H}{a\sigma}c' + \frac{2C_T}{a\sigma}s'\right]$
$2\left(\frac{2C_Y}{a\sigma}\right)\eta'(ss'+cc') + 2\eta'\delta_3\left(-\frac{2C_H}{a\sigma}s' + \frac{2C_T}{a\sigma}c'\right)$	$2\left(\frac{2C_Y}{a\sigma}\right)\eta'(sc'-s'c) + 2\eta'\delta_1\left(-\frac{2C_H}{a\sigma}s' + \frac{2C_T}{a\sigma}c'\right)$	$2\left(\frac{2C_Y}{a\sigma}\right)\left(\delta_1(s'c-c's) + \delta_3(s's+c'c)\right) + 2\left(-\frac{2C_H}{a\sigma}s' + \frac{2C_T}{a\sigma}c'\right)$
$2\left(\frac{2C_Q}{a\sigma}\eta's - \frac{2C_{mx}}{a\sigma}\eta'c\right)$	$2\left(-\frac{2C_Q}{a\sigma}\eta'c - \frac{2C_{mx}}{a\sigma}\eta's\right)$	$2\left(\frac{2C_Q}{a\sigma}\right)(\delta_1 c + \delta_3 s) + 2\left(\frac{2C_{mx}}{a\sigma}\right)(\delta_1 s - \delta_3 c)$

Gravity Terms

Also the effect of the weight of the aircraft must be added in the fuselage equations. The forces and moments to be added are

$$Z = - W \cos (\gamma_0 + \theta_f)$$

$$X = W \sin (\gamma_0 + \theta_f)$$

$$M = W [Z_{CG} \sin (\gamma_0 + \theta_f) - X_{CG} \cos (\gamma_0 + \theta_f)$$

where γ_0 is the initial flight path angle and X_{CG}, Z_{CG} is the location of the total aircraft center of gravity from the fuselage center of gravity. Assuming that θ_f is a small angle these forces and moments can be written in perturbation form as

$$Z = - W \cos \gamma_0 + W \sin \gamma_0 \, \theta_f$$

$$X = W \sin \gamma_0 + W \cos \gamma_0 \, \theta_f$$

$$M = W (Z_{CG} \sin \gamma_0 - X_{CG} \cos \gamma_0)$$
$$+ W (Z_{CG} \cos \gamma_0 + X_{CG} \sin \gamma_0) \, \theta_f$$

Fuselage Aerodynamics

The fuselage is assumed to experience only a drag force and a pitching moment which can be expressed in normalized form as

$$D^* = \frac{\gamma}{M^*} S_f \phi_{o2} V_0^2 C_{D f}$$

$$M_f^* = \frac{\gamma}{M^*} S_f \phi_{10} V_0^2 C_{mf}$$

The perturbation terms for the equations of motion would be

$$\delta D^* = 2 \frac{\gamma}{M^*} S_f \phi_{o2} V_0 C_{D f} \delta V + \frac{\gamma}{M^*} S_f \phi_{o2} V_0^2 C_{D f_\alpha} \delta\alpha$$

$$\delta M^* = 2 \frac{\gamma}{M^*} S_f \phi_{10} V_0 C_{mf} \delta V + \frac{\gamma}{M^*} S_f \phi_{10} V_0^2 C_{mf_\alpha} \delta\alpha$$

and

$$\delta V = \dot{x}_f$$

$$\delta\alpha = \theta_f - \frac{\dot{z}_f}{V_0}$$

Horizontal Tail Aerodynamics

The horizontal tail is taken to be located a distance ℓ_T behind the fuselage center of gravity and a distance Z_{HT} above the fuselage center of gravity. Only the lift of the horizontal tail is considered as contributing to the motion equations. The drag of the tail surfaces is included with the drag of the fuselage. The contributions to the vertical force and the pitching moment are

$$Z_{HT} = L_{HT}$$

$$M_{HT} = -\ell_T L_{HT}$$

The lift of the horizontal tail suitably normalized may be expressed as

$$L^*_{HT} = \frac{\gamma}{M^*} S_T \phi_{02} V_0^2 a_T \left(i_T + \alpha_0 - \epsilon + \theta_f - \frac{\dot{z}_f}{V_0} + \frac{\ell_T \theta_f}{V_0} - \frac{\ell_T}{V_0}\frac{d\epsilon}{d\alpha}\left(\dot{\theta}_f - \frac{\ddot{z}_f}{V_0}\right) + \tau\,\delta_e \right)$$

The next to the last term accounts for the downwash lag at the tail, and τ is the elevator effectiveness. As a consequence we have the following terms in perturbation form,

$$(C_{z\theta_f})_{HT} = \frac{\gamma}{M^*} S_T \phi_{02} V_0^2 a_T \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$$(C_{z\dot{x}_f})_{HT} = 2\,\frac{\gamma}{M^*}\,S_T\,\phi_{o2}\,V_0\,C_{L\,T_0} = (C_{zu_g})_{HT}$$

$$(C_{z\dot{z}_f})_{HT} = -\,\frac{\gamma}{M^*}\,S_T\,\phi_{o2}\,V_0\,a_T\,(1 - \frac{d\epsilon}{d\alpha})$$

$$(C_{z\dot{\theta}_f})_{HT} = \frac{\gamma}{M^*}\,\ell_T S_T\,\phi_{o2}\,V_0\,a_T\,(1 - \frac{d\epsilon}{d\alpha})$$

$$(C_{z\ddot{z}_f})_{HT} = \frac{\gamma}{M^*}\,\phi_{o2}\,V_0\,S_T\,a_T\,\frac{d\epsilon}{d\alpha}$$

$$(C_{z\delta_e})_{HT} = \frac{\gamma}{M^*}\,S_T\,\phi_{o2}\,V_0^2\,a_T\,\tau$$

The pitching moment derivatives follow directly from these expressions. The pitching moment derivatives are related to the lift derivatives by

$$(C_{m()})_{HT} = -\,\frac{\ell_T}{y_{T\,W}^2}\,(C_{z()})_{HT}$$

The trim tail lift coefficient is

$$C_{L\,T_0} = a_T\,(i_T + \alpha_0 - \epsilon + \tau\,\delta_{e_0})$$

The downwash model used here is satisfactory for forward flight but is probably too simple for the low speed condition where the downwash conditions at the tail are quite complex.

The matrix terms comprising ΔD_1, ΔD_0 and ΔE_g are given on the following pages. $(C_{z\ddot{z}_f})_{HT}$ contributes to the matrix D_2.

EQUATIONS OF MOTION

The final form of the wing body equations of motion are

$$D_2 \ddot{x}_v + D_1' \dot{x}_v + D_0' x_v = E v_w + E_g' g + E_\delta \delta_e + H_1 F + H_2 x_w$$

The matrices D_1', D_0' and E_g' include the effects of the fuselage, horizontal tail, and the qravity forces and moments. E_δ is the influence of the elevator. Thus

$$D_1' = D_1 + \Delta D_1$$

$$D_2' = D_2 + \Delta D_2$$

$$E_g' = E_g + \Delta E_g$$

and

$$x_v = \begin{Bmatrix} x_f \\ z_f \\ \theta_f \\ q_{w1} \\ q_{w2} \\ p_w \end{Bmatrix} \qquad H_1 = \begin{bmatrix} G_1 \\ \hline \tilde{\alpha} \end{bmatrix} \qquad H_2 = \begin{bmatrix} G_2' \\ \hline 0 \end{bmatrix}$$

The matrices ΔD_1, ΔD_2 and ΔE_g are given on the accompanying pages. The equations relating hub motion α to wing motion x_w and x_f are

$$\alpha = c\, x_w + d\, x_f$$

These two equations replace the lower two sets of equations given on page 141 of Reference 1 for the dynamic motion of the vehicle with the fuselage free to move in the longitudinal plane.

The complete set of equations of motion are:

$$A_2 \ddot{x}_R + A_1 \dot{x}_R + A_0 x_R + \tilde{A}_2 \ddot{\alpha} + \tilde{A}_1 \dot{\alpha} + \tilde{A}_0 \alpha = Bv_R + B_G g$$

$$F = C_2 \ddot{x}_R + C_1 \dot{x}_R + C_0 x_R + \tilde{C}_2 \ddot{\alpha} + \tilde{C}_1 \dot{\alpha} + \tilde{C}_0 \alpha + D_G g$$

$$\alpha = c x_W + d x_f$$

$$D_2 \ddot{x}_v + D_1' \dot{x}_v + D_0' x_v = Ev_W + E_g g' + E_e \delta_e + H_1 F + H_2 x_W$$

$\Delta D_1 \doteq$

$2\frac{\gamma}{M^*} S_f \rho_{02} V_0 C_{D_f}$	$-\frac{\gamma}{M^*} S_f \rho_{02} V_0 C_{D_{f\alpha}}$	$\frac{\gamma}{M^*} S_f \rho_{02} V_0^2 C_{D_{f\alpha}}$			
$2\frac{\gamma}{M^*} S_T \rho_{02} V_0 C_{L_T}$	$-\frac{\gamma}{M^*} S_T \rho_{02} V_0 a_T \left(1-\frac{d\epsilon}{d\alpha}\right)$	$\frac{\gamma}{M^*} l_T S_T \rho_{02} V_0 a_T \left(1-\frac{d\epsilon}{d\alpha}\right)$			
$-2\frac{\gamma}{M^*} \frac{l_T}{y_{TW}^2} S_T \rho_{02} V_0 C_{L_T}$ $+2\frac{\gamma}{M^*} S_f \rho_{10} V_0 C_{m_f}$	$-\frac{\gamma}{M^*} \frac{l_T}{y_{TW}^2} S_T \rho_{02} V_0 a_T \left(1-\frac{d\epsilon}{d\alpha}\right)$ $-\frac{\gamma}{M^*} S_f \rho_{10} V_0 C_{m_{f\alpha}}$	$\frac{\gamma}{M^*} \frac{l_T^2}{y_{TW}^2} S_T \rho_{02} V_0 a_T \left(1-\frac{d\epsilon}{d\alpha}\right)$ $\frac{\gamma}{M^*} S_f \rho_{10} V_0^2 C_{m_{f\alpha}}$			

$\Delta D_0 =$

0	0	$\frac{g}{\Omega^2 R}\cos\gamma_0 + \frac{\gamma}{M^*} S_f \rho_{02} V_0^2 C_{D_{f\alpha}}$	0	0	0
0	0	$\frac{g}{\Omega^2 R}\sin\gamma_0 + \frac{\gamma}{M^*} S_T \rho_{02} V_0^2 a_T\left(1-\frac{d\epsilon}{d\alpha}\right)$	0	0	0
0	0	$\frac{g}{\Omega^2 R}\left(\frac{z_{CG}}{y_{TW}^2}\cos\gamma_0 + \frac{x_{CG}}{y_{TW}^2}\sin\gamma_0\right) + \frac{\gamma}{M^*} S_f \rho_{10} V_0^2 C_{m_{f\alpha}} - \frac{\gamma}{M^*}\frac{l_T}{y_{TW}^2} S_T \rho_{02} V_0^2 a_T\left(1-\frac{d\epsilon}{d\alpha}\right)$	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$\Delta E_g =$

$2\frac{\gamma}{M^*} S_f \varphi_{02} V_0 C_{D_f}$		$\frac{\gamma}{M^*} S_f \varphi_{02} V_0 C_{D_{f\alpha}}$
$2\frac{\gamma}{M^*} S_T \varphi_{02} V_0 C_{L_T}$		$\frac{\gamma}{M^*} S_T \varphi_{02} V_0 a_T \left(1-\frac{d\epsilon}{d\alpha}\right)$
$-2\frac{\gamma}{M^*} \frac{l_T}{y_{TW}} S_T \varphi_{02} V_0 C_{L_T}$ $+2\frac{\gamma}{M^*} S_f \varphi_{10} V_0 C_{m_f}$		$\frac{\gamma}{M^*} \frac{l_T}{y_{TW}^2} S_T \varphi_{02} V_0 a_T \left(1-\frac{d\epsilon}{d\alpha}\right)$ $+\frac{\gamma}{M^*} S_f \varphi_{10} V_0 C_{m_{f\alpha}}$

$E_\delta =$

$\frac{\gamma}{M^*} S_T \varphi_{02} V_0^2 a_T \tau$
$-\frac{\gamma}{M^*} \frac{l_T}{y_{TW}^2} S_T \varphi_{02} V_0^2 a_T \tau$

TRIM CALCULATIONS

The equations which determine the equilibrium flight condition of the aircraft in normalized form may be written as follows:

$$\frac{g}{\Omega^2 R} \sin \gamma_0 + \frac{\gamma}{M*} \phi_{02} V_0^2 (S_f C_{Df} + 2 S_W C_{DW})$$
$$+ 2 \frac{\gamma}{M*} \left(\frac{2C_z'}{a\sigma} \cos \delta_p - \frac{2C_x'}{a\sigma} \sin \delta_p \right) = 0$$

$$- \frac{g}{\Omega^2 R} \cos \gamma_0 + \frac{\gamma}{M*} \phi_{02} V_0^2 (S_T C_{LT} + 2 S_W C_{LW})$$
$$+ 2 \frac{\gamma}{M*} \left(\frac{2C_z'}{a\sigma} \sin \delta_p + \frac{2C_x'}{a\sigma} \cos \delta_p \right) = 0$$

$$\frac{g}{\Omega^2 R} \left(\frac{Z_{CG}}{y_{TW}^2} \sin \gamma_0 - \frac{X_{CG}}{y_{TW}^2} \cos \gamma_0 \right) + \frac{\gamma}{M*} \left(\phi_{10} S_f C_{mf} - \frac{\ell_T S_T}{y_{TW}^2} \phi_{10} C_{LT} \right.$$
$$+ 2 S_W \phi_{10} \left(C_{mac} - \frac{x_{aW}}{c_W} C_{LW} + \frac{X_{WL}}{c_W} C_{LW} - \frac{Z_{WL}}{c_W} C_{DW} \right) \Bigg)$$
$$+ \frac{2\gamma}{M*} \left(\frac{2C_m'}{a\sigma} + x_H \frac{2C_z'}{a\sigma} - z_H \frac{2C_x'}{a\sigma} \right) = 0$$

where

$$\frac{2C_x'}{a\sigma} = \frac{2C_H}{a\sigma} - \alpha_z \frac{2C_y}{a\sigma} + \alpha_y \frac{2C_T}{a\sigma}$$

$$\frac{2C_z'}{a\sigma} = -\alpha_y \frac{2C_H}{a\sigma} + \alpha_x \frac{2C_y}{a\sigma} + \frac{2C_T}{a\sigma}$$

$$\frac{2C_m'}{a\sigma} = \alpha_z \frac{2C_{mx}}{a\sigma} + \frac{2C_{my}}{a\sigma} + \alpha_x \frac{2C_Q}{a\sigma}$$

$$C_{LW} = a_W \left((i_W + \alpha_0) \cos\Lambda + (\xi_W p - Z' \delta_{W3}) \cos^2 \Lambda \right)$$

$$C_{LT} = a_T (i_T + \alpha_0 - \varepsilon + \tau \delta_e)$$

$$C_{mf} = C_{mf_0} + C_{mf_\alpha} \alpha_0$$

$$C_{DW} = C_{DW_0} + \frac{C_{LW}^2}{\pi A\!\!R\, e}$$

$$\delta_P = i_P + i_W + \alpha_0$$

If the equilibrium deflections of the wing are included in the trim calculation then the solution for trim becomes quite complex. It seems unlikely that the equilibrum deflections will have a significant influence on the trim of the aircraft and therefore they will be neglected in the solution for trim. With this assumption,

$$\frac{2C_X'}{a\sigma} \cong \frac{2C_H}{a\sigma}$$

$$\frac{2C_Z'}{a\sigma} = \frac{2C_T}{a\sigma}$$

$$\frac{2C_m'}{a\sigma} = \frac{2C_{my}}{a\sigma}$$

$$C_{LW} \cong a_W (i_W + \alpha_0)$$

The simplest trim problem is the case in which the rotor shaft is aligned with the initial velocity, that is, when $\delta_P = 0$, or in other words when $i_P = -(i_W + \alpha_0)$. In this case, it is further assumed that cyclic pitch is not used for control such that also in trim θ_{1c} and $\theta_{1s} = 0$ then the rotor trim condition is a perfectly axisymmetric case and

$$\frac{2C_H}{a\,\sigma} = 0 \qquad \text{and} \qquad \frac{2C_m}{a\,\sigma} = 0 \ .$$

Given the geometry of the aircraft, and selecting the initial flight velocity V_0 (actually the advance ratio) and the flight path angle γ_0, the three equilibrium equations may be solved to determine the trim values of the airplane angle of attack α_0, the elevator angle δ_e, and the rotor thrust, $\frac{2C_T}{a\,\sigma}$.

For the more general case in which δ_P is not equal to zero, but is still a small angle, that is, when airplane flight is being considered, linearized expressions are employed to calculate the rotor inplane force and the rotor pitching moment.

For this more complex case in which the blades are assumed to be torsionally rigid and no cyclic is applied for control, the following equations are involved. The gimbal motion is determined from

$$(I_0^* (\nu_G^2 - 1) + K_P \gamma M_P i)\, \beta_{GC} = \gamma M_\beta \,\beta_{GC} + \gamma M_{\dot\beta} \,\beta_{GS}$$

$$(I_0^* (\nu_G^2 - 1) + K_P \gamma M_P i)\, \beta_{GC} = \gamma M_{\dot\beta} \,\beta_{GC} + \gamma M_\beta \,\beta_{GS} + \gamma \lambda M_\mu \,\delta_P$$

$$\frac{2C_m}{a\,\sigma} = - I_0^* (\nu_G^2 - 1)\, \beta_{GC}$$

$$\frac{2C_H}{a\,\sigma} = \gamma \lambda (H_\mu + R_\mu)\, \delta_P + (\gamma R_\beta - \gamma H_{\dot\beta} - \gamma K_P R_P i)\, \beta_{GC}$$
$$+ (\gamma H_\beta + \gamma R_{\dot\beta} - \gamma K_P H_P i)\, \beta_{GS}$$

The aerodynamic derivatives in the above expressions do depend upon the trim thrust to some degree. However, it would be expected that the effects of rotor inplane force have only a small effect on the force equilibrium.

The solution for trim can therefore proceed as in the purely axial flow case with the selcted small incidence of the shaft. Once the angle of attack of the aircraft is computed such that an initial value of δ_p is obtained, the flapping coefficients can be calculated and consequently the equilibrium values of the inplane force and the hub moment can be calculated and the trim calculation repeated to account for these effects.

Once this procedure is completed, expressions from Reference 1 can be used to calculate the trim values of the remaining rotor forces and moments which are used in the equations of motion.

If rotor cyclic is introduced into the trim calculation, then the relationship between cyclic and elevator angle must be selected.

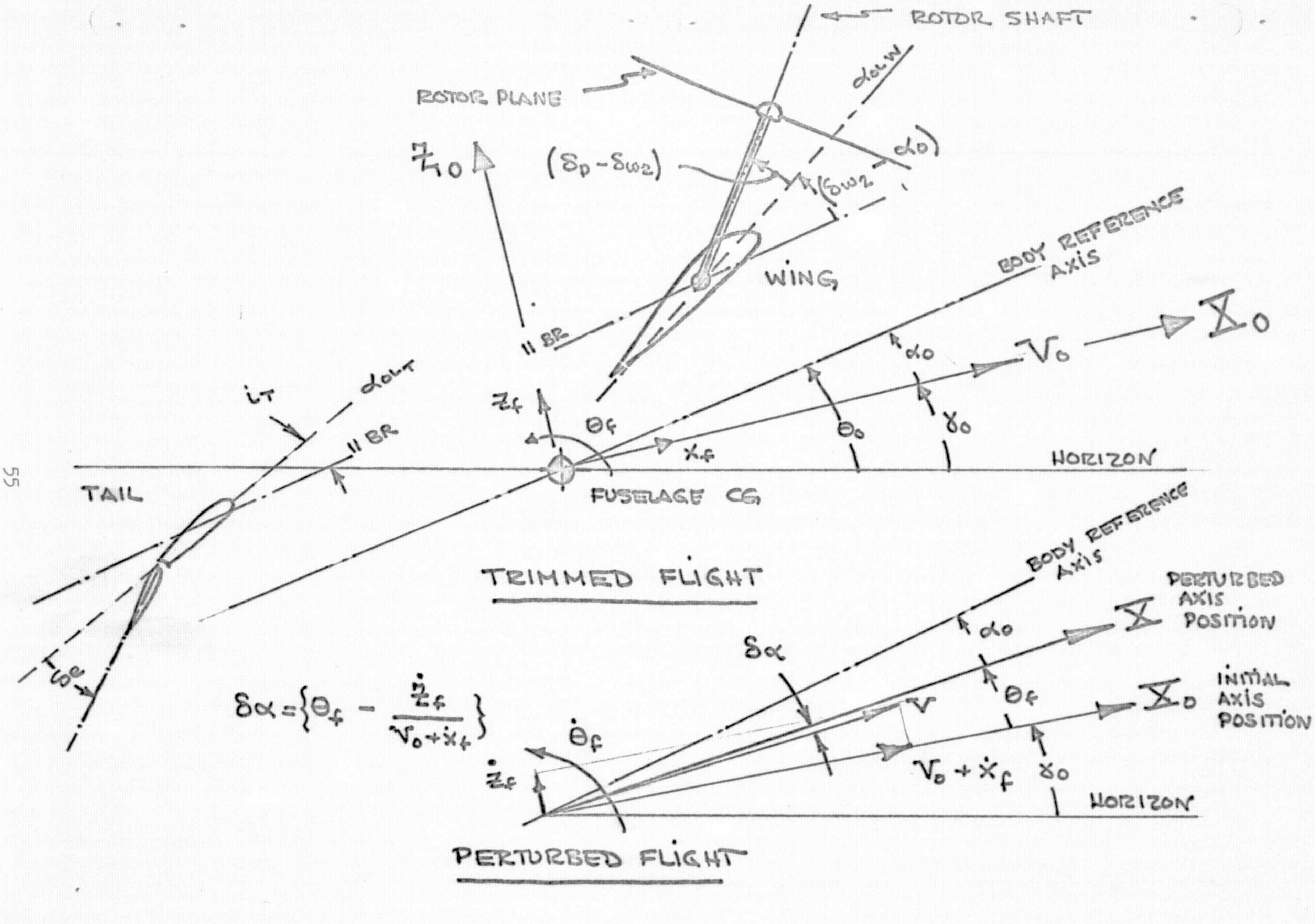


FIGURE 1: AXIS SYSTEM FOR VEHICLE DYNAMICS

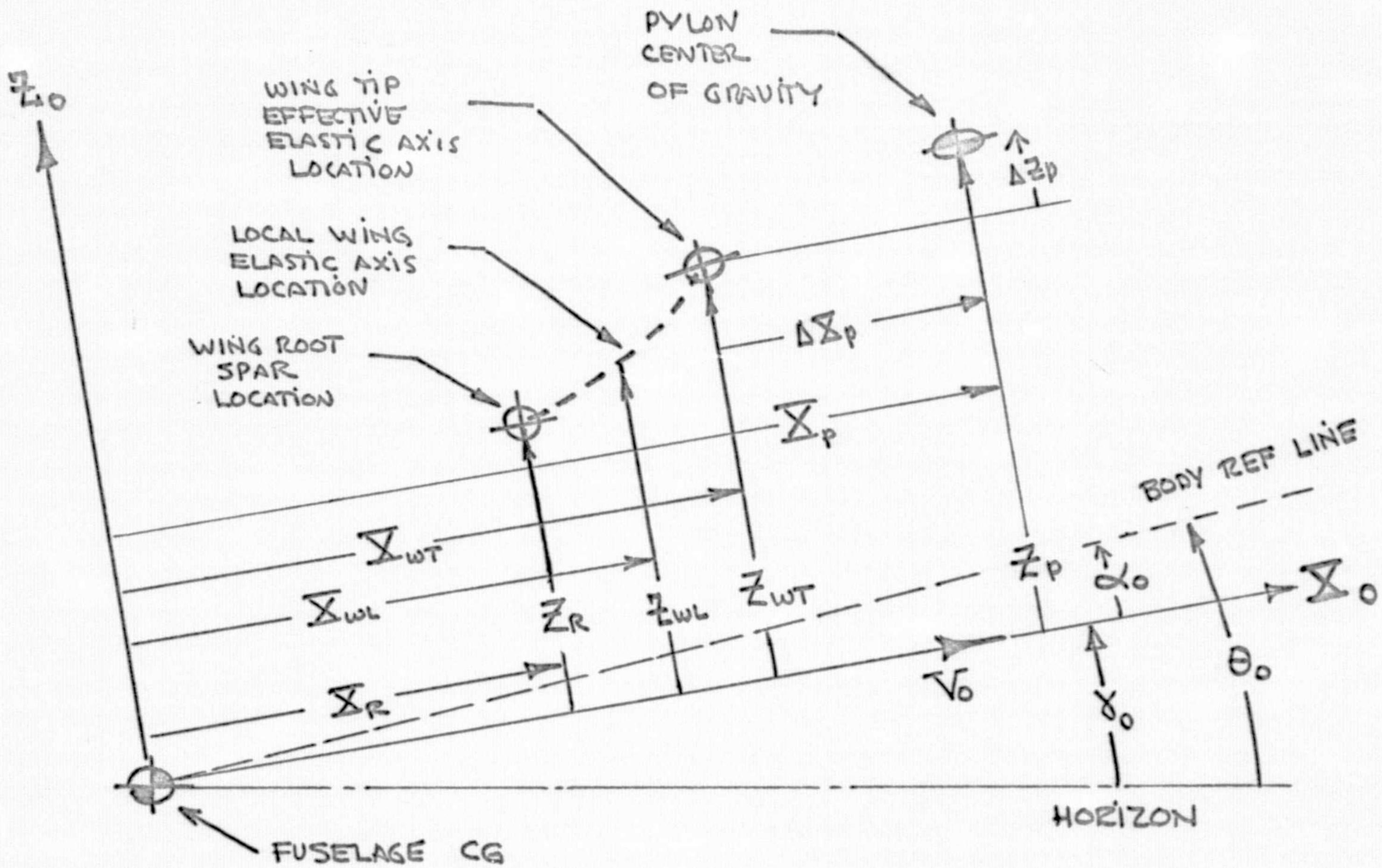


FIGURE 2: DEFINITION OF VARIOUS GEOMETRIC DISTANCES. (NOTE THAT THE VARIOUS DISTANCES INVOLVE THE TRIMMED FLIGHT ANGLE OF ATTACK.)

FIGURE 1:

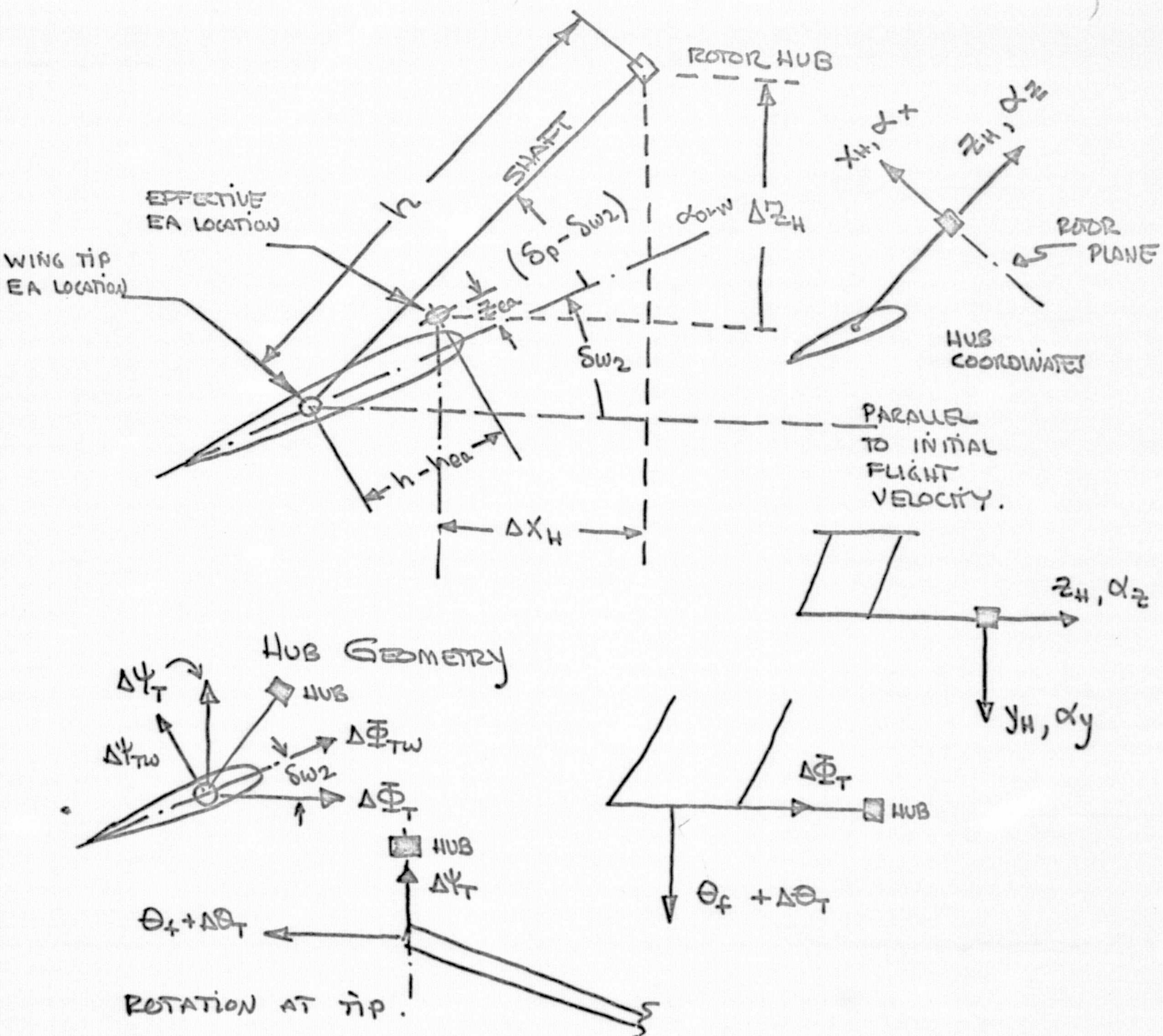
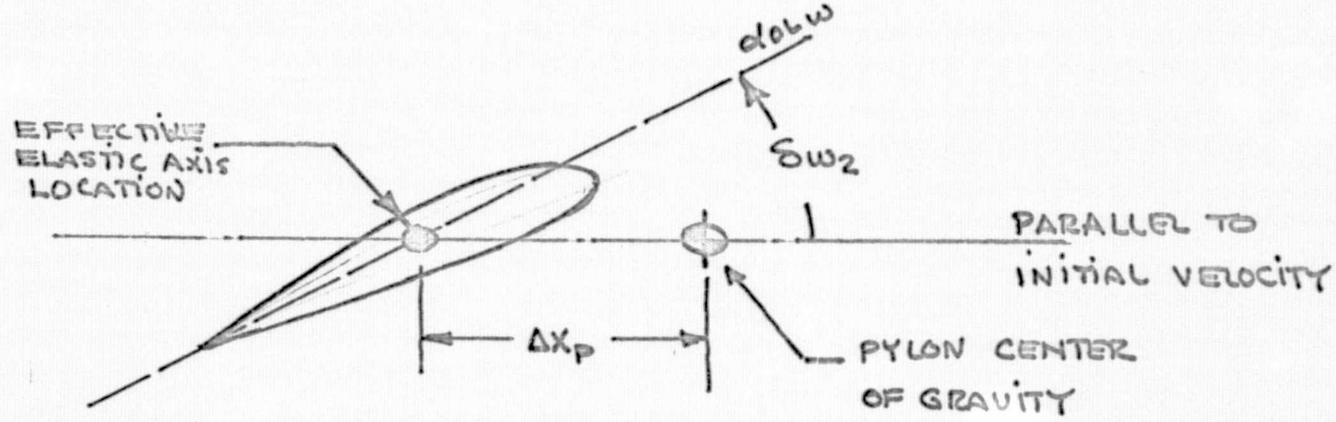


FIGURE 3: HUB GEOMETRY AND ELASTIC AXIS LOCATION.



PYLON CENTER OF GRAVITY LOCATION

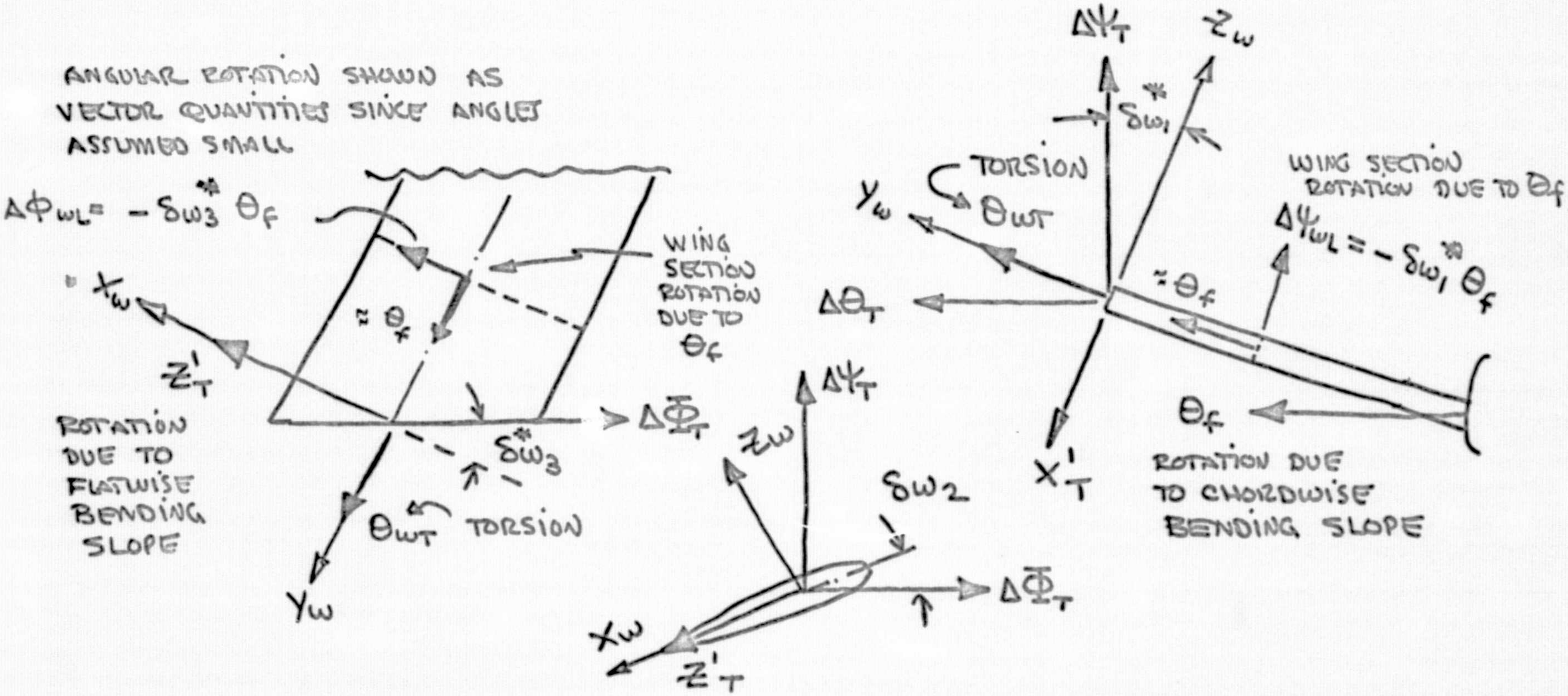


FIGURE 4: ROTATION AT WING TIP AND AT SECTION

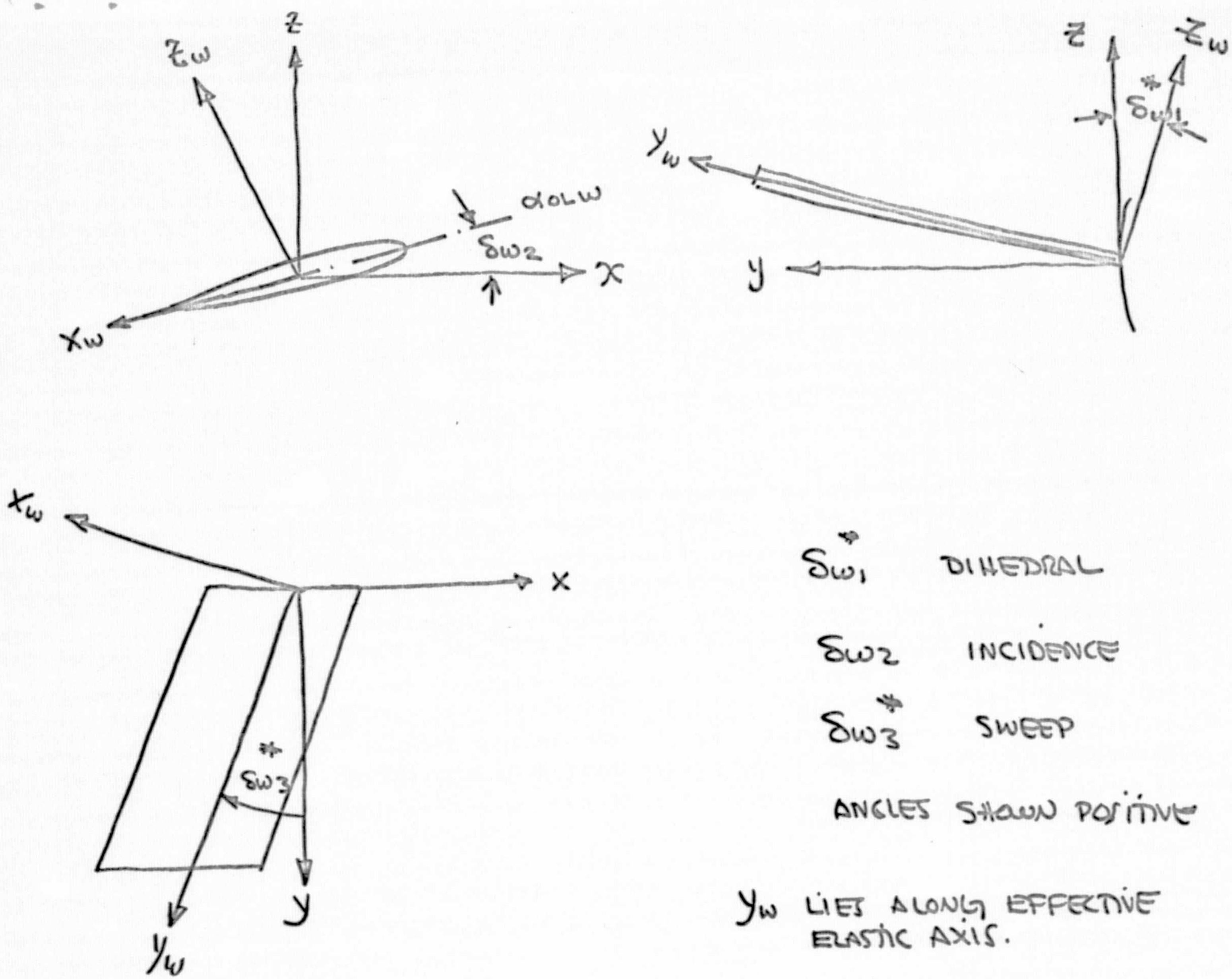


FIGURE 5 WING COORDINATE SYSTEM.